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Abstract

This paper suggests an indirect method for making inference on the parameter region of the elasticity of substitution between capital and labor. The idea is to investigate if the observed behavior of the data is consistent with theoretical relationships that depend on the particular parameter region of the elasticity of substitution. The advantage of this approach is that it allows for more data oriented modeling. Long-run empirical relationships, consistent with a modeling approach for the economy that assumes a CES production function with an elasticity of substitution above one, are established using Finnish manufacturing data

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1 Introduction

There has recently been a renewed interest in the functional form of the aggregate production function. This development has partly been due to the empirical observation that factor shares have not remained constant (see for example Bentalila and Saint-Paul (2003)) casting doubts on the validity of the popular Cobb-Douglas specification. More importantly, the revival of growth theory during the last decade (see Quah (1996) and Solow (1994)) makes the issue topical since other functional specifications can lead to different and richer theoretical results. For instance, assuming a more general CES (Constant Elasticity of Substitution) technology allows for endogenous growth (see for instance Jones and Manuelli (1990)) for some parameter values and multiple stable equilibrium configurations with respect to the labor share for others (see Azariadis (1996)). Also, in a recent paper Kauppi et al. (2004) establish relationships between capital and labor market variables by assuming a CES production function coupled with imperfections in both labor and product markets.

With CES production functions, the theoretical nature of many models are dependent on the parameter value of the elasticity of substitution¹. Attempts at estimating this elasticity for the CES specification have been the focus of some previous empirical studies. The predominant approach is to estimate economic first order conditions directly and then derive estimates of the substitution elasticity from the estimated parameters. The seminal contribution in this respect is Arrow et al. (1961). Recent contributions are provided by Duffy and Papageorgiou (2000), Chirinko et al. (2004), Klump et al. (2004) and Antras (2004). In the study by Duffy and Papageorgiou (2000) a time panel of 82 countries over 28 years is used to estimate a CES specification. They find that a CES production function with an elasticity of substitution above one fits the data quite well for the most developed countries in their sample (including Finland). They do not, however, address the time series problems of the data². Chirinko et al. (2004) use an extensive time panel containing data on 1860 US firms over 20 years. They are concerned with the potential time series problems of the data. Their solution is splitting the period, taking averages over the sub periods and finally differencing, essentially collapsing the time dimension. By this technique they obtain estimates of the elasticity of substitution at around 0.40. Klump et al. (2004) estimate a supply side system using non-linear estimation techniques for US time series data. They estimate the substitution elasticity at around 0.60. Antras (2004) follows the approach set forth by Berndt (1976) using time series data from the US private sector to estimate the elasticity of substitution. He

¹The Cobb-Douglas functional form is a special case of the CES function corresponding to the case where the elasticity of substitution is equal to one, as is well known. Whether the elasticity of substitution is above or below unity is of special theoretical interest.

²Another problem with using a panel of countries is that it requires strong assumptions on the similarities of the countries in sample.

finds that this parameter is well below unity if one allows for biased technological progress. He also investigates the time series properties of the data and find non-stationarity and cointegration but does not analyze these problems any further. Ripatti and Vilmunen (2001) estimate first order conditions, based on a CES specification, on Finnish data using non-linear cointegration techniques. They find an elasticity of substitution well below one, but are forced to make some strong assumptions on the processes of the unobservables in order to obtain their results. As a consequence they implicitly restrict the number of common trends and hence the number of cointegration vectors. Furthermore, their results are derived under the assumption of perfectly competitive capital and labor markets although they allow for monopolistic competition in the product market.

Overall, the evidence on the elasticity of substitution from previous studies is mixed, with estimated values both above and below one³. Klump et al. (2004) suggest that the reasons for these disparities can be attributed to differences in data construction, different *a priori* assumptions about the nature of technological change and differences in the methods applied in estimating the parameters of the CES functions. In addition, it seems likely that the technological parameters differ substantially across countries (for instance as consequences of different stages of development and country level specialization in more advanced economies). In this case country-specific estimates should be the preferred alternative. At the macro level this is only possible in a time series framework. However, most estimation strategies based on first order conditions are restrictive in the sense that it becomes difficult to model non-stationary data in a persuasive way.

This paper suggests an indirect way of making empirical inference on the parameter region of the elasticity of substitution. The idea is to investigate if the observed behavior of the data is consistent with theoretical relationships that depend on the particular parameter region of the elasticity of substitution. This approach is applied to Finnish manufacturing time series data. The advantage of this approach is that it is flexible enough to allow for a more data oriented modeling approach while retaining a firm connection to economic theory⁴. The drawback, however, is that precise parameter estimates cannot be obtained.

To this end, a theoretical relationship between wages and the capital-labor share and a relationship between the unemployment rate and the interest rate, derived by Kauppi et al. (2004), will be utilized. They show that the nature of these relationships depends crucially on the elasticity of substitution. Furthermore, the relationships vanish in the Cobb-Douglas case, i.e in the case when the

³An excellent review of the findings in previous studies that estimate the elasticity of substitution is provided in Klump et al. (2004).

⁴For example, the time series properties of the data can be handled in a convincing way. Furthermore, economic theory usually describe long-run (equilibrium) behavior but is silent about the short-run. The present approach makes it possible to model both within a cointegration framework.

elasticity of substitution is one. In particular, the relationship between wages and the capital-labor share is negative (positive) when the elasticity of substitution is below (above) one, while the relationship between the unemployment rate and the interest rate is positive (negative) when the elasticity of substitution is above (below) one. The intuition for these relationships is that the capital stock acts as a strategic commitment device with respect to wage formation and thereby affecting unemployment. Thus, if it is possible to establish similar long-run⁵ relationships empirically, it would be consistent with the CES production function (and inconsistent with a Cobb-Douglas technology). Moreover, the nature of the relationships allow us to make predictions on the parameter values of the elasticity of substitution⁶.

The statistical workhorse utilized in this paper is the cointegrated VAR (Vector Auto-Regression) model (see Johansen (1995)). This model is particularly well suited since it does not require assumptions on causality and it allows for non-stationarity in the data and hence for distinguishing between long-run economic relationships and (i.e. cointegration) and short-run adjustments (see Engle and Granger (1987)). Furthermore, the estimates also provide insights into the processes that determine the variables at hand, which is of interest in itself.

The cointegrated VAR model is estimated on quarterly Finnish manufacturing data (using an information set that captures the main features of the model in Kauppi et al. (2004)) over the years 1980:1-2001:4. I find a positive long-run relationship between manufacturing wages and the capital-labor ratio and a negative long-run relationship between the unemployment rate and the interest rate. These results have two implications. First, the Cobb-Douglas production function specification seems to be inappropriate for the case of Finland. As an alternative, the CES specification coupled with imperfections in both product and labor markets is more consistent with the observed behavior of the data. Second, the nature of these relationships is broadly consistent with an elasticity of substitution above one. Furthermore, utilizing the methods proposed by Arrow et al. (1961), I also obtain a crude estimate of the elasticity of substitution equal to 1.39. Estimates of the processes determining the changes in the manufacturing output, hours worked in manufacturing, long-term interest rates, and price inflation are also obtained as byproducts of the analysis.

The paper is organized in the following manner. In section 2 some properties of the CES aggregate production function and the most relevant results from Kauppi et al. (2004) are discussed. Section 3 provides a short summary of the main properties of the cointegrated VAR model. Section 4 is concerned with the statistical analysis of the data. The empirical results are then discussed in section

⁵The relationship in Kauppi et al. (2004) is derived in the presence of product and labor market imperfections and should be viewed as describing “long-run” relationships. Hence, the empirical relationship should have the same property.

⁶Bentolila and Saint-Paul (2003) uses similar indirect arguments to make inference on the parameter value of the elasticity of substitution based on a fitted equation for the labor share.

5 and section 6 finally concludes.

2 The CES production function and relationships dependent on the elasticity of substitution

This section presents two theoretical relationships that depend on the elasticity of substitution, as established in Kauppi et al. (2004). The first relationship is between wages and the capital-labor share while the second is a relationship between unemployment and the interest rate. These relationships can be utilized to make empirical inference on the value of the elasticity of substitution. The relationships are dependent on the assumption of a CES production function and indeed on imperfections in both product and labor markets. We begin by discussing the CES production function and then turn to the results from Kauppi et al. (2004).

2.1 The CES production function

The concept of an aggregate production function is often a useful (theoretical) point of departure. Although this concept is somewhat problematic⁷, most authors assume that the economy's production technology is represented by some (two factor) production function like

$$Y = AF(K, L) \tag{1}$$

where Y is output, K is capital, L is labor and A captures technological progress⁸. The function F is usually assumed to be twice continuously differentiable and to satisfy INADA conditions. Since its introduction by Solow (1956), the undoubtedly most popular choice has been the Cobb-Douglas production function

$$Y = AK^\beta L^{1-\beta} \tag{2}$$

where $0 < \beta < 1$. The popularity of the Cobb-Douglas specification is probably mostly due to its analytical convenience rather than its empirical realism⁹. A

⁷The literature on aggregation suggests that the assumption of an aggregate production function might be hard to justify. For a nice overview see Felipe and Fisher (2003).

⁸In (1) the technological progress is in the Hicks neutral form. However, allowing for biased technological progress may be important in empirical investigations as noted in Antras (2004).

⁹The Cobb-Douglas functional form is usually motivated by the Kaldor's 'stylized facts', for instance that factor shares have been largely constant (in fact, the Cobb-Douglas production function implies *absolutely* constant factor shares). However, this is clearly not the case for many economies as demonstrated by Bentolila and Saint-Paul (2003). Also, allowing for biased technological progress makes Kaldor's 'stylized facts' consistent with any production function specification Antras (2004).

considerably more general alternative is provided by the CES production function

$$Y = \gamma \left[\beta K^{\frac{\sigma-1}{\sigma}} + (1 - \beta) L^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (3)$$

where σ is the elasticity of substitution and γ plays a similar role as A in (1) and β captures the functional distribution of income. (3) embeds the popular Cobb-Douglas form as a special case when $\sigma = 1$. As noted in the introduction this functional form allows for much richer theoretical results, for example in contexts of endogenous growth. Naturally, many other functional forms would do as well. However, as the theoretical literature has predominately focused on the Cobb-Douglas or the CES function the same focus is maintained in this paper.

2.2 Relationships between capital and labor markets

Kauppi et al. (2004) analyze equilibrium wage formation, unemployment and investments under the assumptions of product market imperfections à la Dixit and Stiglitz (1977) and a 'right-to-manage' union bargaining approach to labor market imperfections. They generalize previous contributions by Blanchard and Giavazzi (2003) and Spector (2004) by assuming a CES production function which allows them to study the role of investments. Under these assumptions they establish the following relationship between wages and the capital-labor share

$$\frac{\partial w^N}{\partial k} \begin{cases} < \\ = \\ > \end{cases} 0 \text{ as } \begin{cases} \sigma < 1 \\ \sigma = 1 \\ 1 < \sigma < s \end{cases} \quad (4)$$

where w^N is the Nash bargaining solution of the wage rate, k is the capital-labor share (defined as K/L), σ is the elasticity of substitution from the CES production function and $s > 1$ is the elasticity of substitution between products in a CES-utility function. Thus, s can be viewed as a measure of the degree of product market competition. As can be seen from (4), the relationship between wages and the capital-labor share is negative if the elasticity of substitution is less than one (i.e. gross complementary). In the Cobb-Douglas case, where $\sigma = 1$ there is no relation between wages and the capital-labor share. If on the other hand the elasticity of substitution is above one (gross substitutability) but less than the elasticity of product demand, the relationship is positive. The intuition for these relationships is that the capital stock acts as a strategic commitment device, inducing wage moderation when $\sigma < 1$ while the opposite is true when $\sigma > s > 1$. Kauppi et al. (2004) argue that the case where $1 < s < \sigma$ is unlikely to be empirically relevant and hence disregard this particular case.

Kauppi et al. (2004) also establish another useful relationship. Since unemployment is positively related to the wage rate and capital is negatively related to the rate of interest, unemployment and the rate of interest are either positively or negatively related as well, if the elasticity of substitution is different

from one. In fact, from proposition 6 in Kauppi et al. (2004) this relationship can be summarized as

$$\frac{\partial u}{\partial r} \begin{cases} > \\ = \\ < \end{cases} 0 \text{ as } \begin{cases} \sigma < 1 \\ \sigma = 1 \\ 1 < \sigma < s \end{cases} \quad (5)$$

where u is the unemployment rate and r is the rate of interest. Hence, there is a positive (negative) relationship between the unemployment rate and the interest rate if the elasticity of substitution is below (above) one. The unemployment rate and the interest rate are not related in the Cobb-Douglas case.

This framework provides a realistic setting within which it is possible to make inferences on the functional form of the production function for the Finnish economy. The Finnish economy is indisputably best characterized by labor and product market imperfections (see for example Nickell (1997)). The empirical problem is to establish long-run relationships between wages and the capital-labor share on one hand and between the unemployment rate and the interest rate on the other hand. The absence of such relationships would support the Cobb-Douglas specification. The nature of these relationships enables us to make predictions on the parameter value of the elasticity of substitution according to (4) and (5).

3 The statistical model

The baseline statistical model is the p -dimensional cointegrated VAR-model with k lags

$$\Delta X_t = \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \Pi X_{t-1} + \mu + \Psi D_t + \varepsilon_t \quad (6)$$

where the vector process X_t is assumed to be $I(1)$, μ is a vector of constants, D_t consist of the other deterministic components and $\varepsilon_t \sim N_p(0, \Sigma)$. If there exists cointegration $\Pi = \alpha\beta'$ and $\mu = \alpha\beta_0 + \alpha_\perp\gamma_0$, where α and β are two $(p \times r)$ matrixes such that $r < p$, $\alpha_\perp$ and $\beta_\perp$ are two $p \times (p - r)$ matrixes such that $\alpha'\alpha_\perp = 0 = \beta'\beta_\perp$, then the moving average representation of the model is given by

$$X_t = B + C \left(\sum_{i=1}^t (\varepsilon_i + \Psi D_i) + \mu t \right) + C^*(L)(\varepsilon_t + \Psi D_t) \quad (7)$$

where B depends on the initial values and $C = \beta_\perp(\alpha'_\perp(I - \sum_{i=1}^{k-1} \Gamma_i)\beta_\perp)^{-1}\alpha_\perp$, and the polynomial $C^*(z)$ is convergent for some $z < 1 + \delta$ for some $\delta > 0$. The main properties of the model are investigated in Johansen (1995) and will not be repeated here¹⁰. However, for the sake of clarity some of the most crucial results are reproduced.

¹⁰See Hendry and Juselius (1999, 2000) for a helpful overview.

3.1 Linear restrictions on the β -vectors

An important part of the analysis consists of testing linear restrictions on the β -vectors with the aim of identifying a structure of empirically relevant relations. The restrictions can either be applied to all the cointegrating relations simultaneously or to the individual vectors separately. Given $\Pi = \alpha\beta'$ Johansen and Juselius (1990) show that the same restriction on all the vectors can be tested by formulating the null hypothesis

$$\mathcal{H}_{0.1} : \Pi = \alpha\varphi'H', \quad (8)$$

that is $\beta = H\varphi$, where H is a $(p \times s)$ with $r \leq s \leq p$ matrix defining linear restrictions on β . Restrictions on the individual vectors are formulated as

$$\mathcal{H}_{0.2} : \begin{array}{lcl} \Pi & = & \alpha\beta' \\ \beta & = & \{H_1\varphi_1, \dots, H_r\varphi_r\} \end{array} \quad (9)$$

where H_i is $p \times s_i$, defining linear restrictions on the individual vectors. Johansen (1995) derives the LR-tests for testing the hypothesis in the form (8) and (9).

3.2 Linear restrictions on the α -matrix and weak exogeneity

The α -vectors can also be restricted in a similar fashion as in the last section. Of special interest is the case where one or several rows in α consist of zeros. A variable with a zero row in α is not affected by the long-run relationships and is hence treated as weakly exogenous. In this case one estimates

$$\Delta X_{1,t} = A\Delta X_{2,t} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \tilde{\alpha}\beta'X_{t-1} + \mu + \psi D_t + \varepsilon_t \quad (10)$$

where $X_{1,t}$ consist of the endogenous variables while $X_{2,t}$ consist of the weakly exogenous variables and $X_t = \{X_{1,t}, X_{2,t}\}$. The dimension is now $p - h$, where h is the number of exogenous variables, Hendry and Juselius (2000).

4 Empirical analysis

The data used in the analysis is Finnish quarterly manufacturing data spanning over the years 1980:1-2001:4 (88 observations). In accordance with Kauppi et al. (2004), the process determining Finnish manufacturing production, wages and unemployment is assumed to be described by the information set $I_t = \{w_t, u_t, c_t, y_t, l_t, r_t, \Delta p_t\}$ where w_t is real manufacturing wages ($\log(W_t/P_t^p)$), where W_t is a index of manufacturing wages and P_t^p is the manufacturing producer price index), u_t is the manufacturing unemployment rate, c_t is (the log

of) the real capital stock, y_t is (the log of) real manufacturing output, l_t is (the log of) hours worked in manufacturing, r_t is the nominal yield on government 10 year bonds and $\Delta p_t = \Delta \log(P_t)$ is the inflation rate. For future reference we also define the capital labor share, k_t , and the output labor share, q_t , as the restrictions $k_t = c_t - l_t$ and $q_t = y_t - l_t$. The analysis of the long-run relations was conducted using the computer package *CATS* in *RATS* while the short-run structure was estimated using the computer package *PC-Fiml*.

4.1 Dynamic long-run relations

The long-run properties (i.e. cointegration relations) of the data were investigated by estimating model (6) with $X_t = [w_t, u_t, c_t, y_t, l_t, r_t, \Delta p_t]'$, $k = 2$ and a linear trend restricted to the cointegration space. The reduced rank hypothesis (trace test, see Johansen (1995)) is reported in table 1 and indicates that the choice of rank should be 4 . However, note also that $r = 3$ is almost accepted at

Table 1: The reduced rank hypothesis. The λ_i are the eigenvalues corresponding to the estimates of the cointegration vectors.

$H_o : r$	Trace	Trace(95)	λ_i
0	217.1	150.3	0.52
1	154.8	117.5	0.42
2	108.0	88.6	0.40
3	64.9	63.7	0.28
4	37.3	42.8	0.18
5	20.7	25.7	0.16

the 5% significance level. As a sensitivity precaution, the model was analyzed with this choice of rank as well, but it did not alter the main results. Given $r = 4$, the variables were tested for stationarity, long-run exclusion and weak exogeneity. Stationarity and long-run exclusion were rejected in all variables while the capital stock, c_t , was found to be weakly exogenous (with LR value 2.91 against $\chi^2_{0.95}(4) = 9.49$). This result is natural, since our empirical perspective is likely to be the medium run whereas capital is endogenous only in the long-run in Kauppi et al. (2004). Also, the literature on investment modeling suggests that the investment process is rather complicated (for a overview see Mairesse et al. (1999)). Thus, it seems unlikely that the information set used in this paper would capture the essential features of this process.

Moving to a partial system, model (10) was estimated with $X_{1,t} = [w_t, u_t, y_t, l_t, r_t, \Delta p_t]'$, $X_{2,t} = [c_t]$, $k = 2$. Although the trace test is not appropriate for a partial system, collateral evidence such as the roots of the

companion matrix and the t-values of the α matrix supported the previous choice of $r = 4$, which consequently was imposed.

Given this choice of rank, some structural hypotheses of the form (9) were tested on the cointegration space. The null hypothesis is that a particular relation is in the cointegration space, i.e. that it is stationary. The results are reported in table 2, where c_{ij} are positive constants corresponding to the j :th unrestricted

Table 2: Structural hypothesis on the estimated β -vectors.

Hypothesis	w	u	y	l	r	Δp	c	t	LR, $\chi^2(df)$	p-value
$\mathcal{H}_1$	1	0	0	c_{11}	c_{12}	0	$-c_{11}$	$-c_{13}$	0.97(1)	0.33
$\mathcal{H}_2$	0	1	c_{21}	0	c_{22}	c_{23}	0	0	1.32(1)	0.25
$\mathcal{H}_3$	0	0	0	0	1	0	$-c_{32}$	c_{33}	0.61(2)	0.74
$\mathcal{H}_4$	0	0	$-c_{41}$	1	0	0	0	c_{42}	4.38(2)	0.11
$\mathcal{H}_5$	0	0	-1	1	0	0	0	c_{52}	12.69(3)	0.005
$\mathcal{H}_6$	$-c_{61}$	0	1	-1	0	0	0	0	4.62(3)	0.20

estimate of the parameter in the i :th relation (the signs of the estimated coefficients are reported for clarity). Each vector is normalized on variables that are error correcting¹¹ (see table 5). The relation tested in the first hypothesis, $\mathcal{H}_1$, describes an error correcting mechanism for manufacturing wages (see equation (12) and table 5). The stationarity of this relation cannot be rejected (p-value 0.33, i.e. the relation belongs to the cointegration space). This relation turns out to be important for making inferences about the functional form of the production function and the value of the elasticity of substitution. Note that the coefficients on c_t and l_t are restricted to take the same value with opposite signs. Thus, we can write this relation as, $w_t - c_{11}(c_t - l_t) + c_{12}r_t$ or $w_t - c_{11}k_t + c_{12}r_t$. Hence, k_t enters this relation with a negative sign, which implies a positive relationship between w_t and k_t . To see this, assume that we have initial equilibrium such that the relation in $\mathcal{H}_1$ holds exactly. If we now change k_t by Δk_t the equilibrium error will be negative, and since the relation is error correcting in wages the change in the wage rate, Δw_t , will be positive (see equation (12) below). The relation also imply a negative relationship between wages and the interest rate, by similar reasoning.

The relation in the second hypothesis, $\mathcal{H}_2$, is error correcting in unemployment. The interesting feature of this relation is the positive coefficient on the interest rate, which implies a negative relationship between unemployment and

¹¹A variable is error correcting if the linear combination implied by a cointegrating vector contains the variable with a positive (negative) sign and enters the equation of the variable with a negative (positive) sign. For example, if $y - \beta x$ is a linear combination and $\Delta y_t = -\alpha(y - \beta x)_{t-1}$ then y is error correcting in $y - \beta x$.

the interest rate. Similarly, the relation in $\mathcal{H}_2$ also implies that the effects of inflation and real output on unemployment are negative, which seems natural. The stationarity of the relation in $\mathcal{H}_2$ cannot be rejected (p-value 0.25).

The third relation, tested in $\mathcal{H}_3$, describes the interaction between the interest rate and the capital stock (adjusted by a trend). A higher demand for capital would *ceteris paribus* increase interest rates. The stationarity of the relation cannot be rejected.

The relation tested in $\mathcal{H}_4$ describes the interaction between manufacturing output and the hour worked (the trend captures the fact that the labor share of output has decreased during the last two decades). The stationarity of the relation cannot be rejected.

In $\mathcal{H}_5$ the relation in $\mathcal{H}_4$ is restricted so that output and labor input are one to one, i.e. $\mathcal{H}_5$ test the stationarity of the output labor share, q_t , around a trend. However, this relation is not stationary.

The relation in the last hypothesis, $\mathcal{H}_6$, is of interest since c_{61} corresponds to a direct estimate of the elasticity of substitution, σ , under the additional assumption of perfect competition¹² (which is hardly the case for Finland). The stationarity of this relation cannot be rejected (but as it turns out, it cannot be combined with the other relations to span an identified cointegration space). The estimate of c_{61} in $\mathcal{H}_6$ is 1.39, i.e. well above one, although this estimate should be viewed with caution since it is only valid under perfect competition.

Finally, the hypothesis that the relations tested in the first four hypotheses jointly span the cointegration space cannot be rejected (p-value 0.09). The restricted estimates of the cointegration vectors are reported in table 3 and figure 1 shows the centered and normalized linear combinations of the variables, $\hat{\beta}'_1 X_t$, $\hat{\beta}'_2 X_t$, $\hat{\beta}'_3 X_t$ and $\hat{\beta}'_4 X_t$ labeled as *ecm1*, *ecm2*, *ecm3* and *ecm4* respectively. Note that the vectors in table 3 span an identified system. The identified cointegration space was also tested for parameter constancy by a recursive test described in Hansen and Johansen (1993). The results are reported in figure 2 where the recursions have been performed both forwards and backwards to cover the complete period. The tests indicate that the parameters of the identified cointegration space have been constant for the time period of the analysis¹³.

¹²Arrow et al. (1961) show that b in the regression $q_t = a + bw_t + \epsilon_t$ provides an estimate of the elasticity of substitution when labor and product markets are competitive. Taking the time series properties of the data into account, this would correspond to testing the stationarity of the relation in $\mathcal{H}_4$.

¹³The same is not true for the short-run parameters which have fluctuated in particular during the late 80's and the beginning of the 90's when Finland experienced a strong boom followed by a deep recession. Thus, more care should be taken when interpreting the short-run effects.

Figure 1: Centered and normalized cointegration vectors.

Figure 2: Test of beta constancy over the period 1985:3-2001:4. The line at 1 indicates rejection at the 5% significance level.

Table 3: Identified β -vectors (t-values in brackets).

	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$
w	1	0	0	0
u	0	1	0	0
y	0	0.05 (12.69)	0	-0.81 (-18.32)
l	0.13 (2.21)	0	0	1
r	6.72 (4.83)	0.84 (7.83)	1	0
Δp	0	0.74 (12.28)	0	0
c	-0.13 (-2.21)	0	-0.05 (-9.80)	0
t	-0.01 (-10.87)	0	0.001 (15.95)	0.01 (23.21)

4.2 The short-run structure of the model

The short run structure of the model was estimated by taking the long-run relations as given from the previous section. The short-run structure was modeled by

$$\Delta X_{1,t} = \Gamma_1 \Delta X_{2,t} + \Gamma_2 \Delta X_{t-1} + \alpha ECM_{t-1} + \mu + \Psi D_t + \epsilon_t \quad (11)$$

where $X_{1,t}$ contains the endogenous variables and $X_{2,t}$ the exogenous variable, Γ_1 is a (6×1) matrix, Γ_2 is a (6×7) matrix and α is a (6×4) matrix, ECM_t is a column vector with $ecm_{i,t}$ as elements and D_t includes seasons and an unrestricted constant. Initial testing of zero restrictions on the parameters of the complete system by F-tests indicated that Δw_{t-1} , Δu_{t-1} , Δy_t and Δl_t could be excluded (p -values 0.14, 0.32, 0.31 and 0.48 respectively). The system was then re-estimated with these changes. The results from misspecification tests on the estimated residuals from the equations are reported in table 4 (the numbers are p -values). By and large, the model seems to fit the data quite well, although normality is rejected in the unemployment equation. This is due to a large outlier in the first quarter of 1987. However, modeling this outlier does not alter the results significantly. There might also be a small problem with ARCH in the interest rate equation (the null of no ARCH is rejected at the 5% significance level but not at the 1% level).

The result from imposing zero restrictions on the individual equations are

Table 4: Misspecification tests on the estimated residuals from each equation. The tests are described in Doornik and Hendry (2001). The normality test is derived under the null of normality. The null in the ARCH test is no conditional heteroscedasticity and the null in the AR 1-5 test is no autocorrelation in the first 5 lags. Bold values indicate rejection at 5% significance.

Equ.	“normality χ^2 ”	ARCH	AR 1-5	correlation(obs/est)
Δw	0.20	0.79	0.10	0.76
Δu	0.001	0.86	0.87	0.81
Δy	0.23	0.24	0.10	0.96
Δl	0.90	0.95	0.09	0.97
Δr	0.49	0.02	0.38	0.70
$\Delta^2 p$	0.08	0.83	0.76	0.84

reported in table 5. As noted in section 2, our primary interest is in the equations determining the change in the real wages and the change in the unemployment rate (equations 1 and 2 in table 5). Writing out these equations explicitly we have

$$\Delta w_t = -0.27(w + 6.72r - 0.13k)_{t-1} + 0.22(l - 0.81y + 0.01t)_{t-1} \quad (12)$$

and

$$\begin{aligned} \Delta u_t = & -0.2\Delta c_t - 0.31\Delta r_{t-1} + 0.09\Delta^2 p_{t-1} + 0.48(r - 0.05c + 0.01t)_{t-1} \\ & -0.29(u + 0.05y + 0.84r + 0.74\Delta p)_{t-1} \end{aligned} \quad (13)$$

where *ecm1* and *ecm4* in (12) and *ecm2* and *ecm3* in (13) are written out in full.

5 Interpretation of the results

5.1 The elasticity of substitution

Equation (12) describes the change in real manufacturing wages due to imbalances in the long-run relationships *ecm1* and *ecm4*. Note that only *ecm1* is error correcting in the wage equation (since w_t does not appear in *ecm4*), so that the other variables, k_t and r_t , are determining the change in wages in that relation. Since the capital-labor share enter in *ecm1*, the long-run relationship between the real wages and the capital-labor share *is empirically established*. This relationship is consistent with $\sigma \neq 1$ in (3) and therefore inconsistent with the standard Cobb-Douglas production function. Furthermore, the coefficients in equation (12) imply that this relationship is positive. To see this consider a small positive increase

Table 5: The short-run structure of the model. Coefficients in boldface indicate significance at the 5% level of the deterministic components (lowest part of the table).

	Equ1 Δw_t	Equ2 Δu_t	Equ3 Δy_t	Equ4 Δl_t	Equ5 Δr_t	Equ6 $\Delta^2 p_t$
Δc_t	—	-0.20 (-4.26)	4.71 (4.18)	2.25 (3.12)	—	-0.39 (-3.46)
Δc_{t-1}	—	— —	-6.74 (-6.27)	—	—	—
Δr_{t-1}	—	-0.31 (-2.29)	—	—	-0.58 (-6.58)	0.96 (2.99)
$\Delta^2 p_{t-1}$	—	0.09 (2.37)	—	1.35 (2.33)	—	—
$ecm1_{t-1}$	-0.27 (-4.34)	—	-0.33 (-4.40)	—	—	—
$ecm2_{t-1}$	—	-0.29 (-5.55)	—	-2.04 (-3.01)	—	-0.86 (-7.08)
$ecm3_{t-1}$	—	0.48 (5.25)	—	—	-0.26 (-5.20)	0.54 (-2.48)
$ecm4_{t-1}$	0.22 (3.41)	—	0.29 (3.00)	-0.78 (-8.33)	—	-0.06 (-4.87)
<i>Constant</i>	0.01	0.00	0.02	-0.01	-0.00	0.00
<i>Season_t</i>	0.03	-0.00	-0.12	-0.11	-0.00	0.01
<i>Season_{t-1}</i>	0.01	0.00	-0.08	-0.06	0.00	0.01
<i>Season_{t-2}</i>	-0.01	0.00	-0.21	-0.21	0.00	0.00

in the capital-labor share ($\Delta k_t > 0$) assuming initial equilibrium. This would make *ecm1* negative by the negative coefficient of k_t in the first cointegration relationship and hence imply a *positive* change in the real wages (by the negative coefficient to *ecm1* in equation (12)). Thus, in the long-run the effect would be

$$\frac{\Delta w}{\Delta k} > 0$$

which in terms of the model by Kauppi et al. (2004) (see equation (4)) corresponds to the case where $1 < \sigma < s$. In fact, since the long run structure of the model is identified we can use the other variables as instruments to increase k_t by one unit such that w_t changes by 0.13 (see Johansen (2002)). The long-run effects from the interest rate in *ecm1* is also consistent with $1 < \sigma < s$. To see this, note that capital is a decreasing function in interest rates and hence that the sign on the interest rate should be the opposite to that of k_t . Finally, the change in the wage rate is also driven by disequilibrium in *ecm4*. However, the effects from *ecm4* are not properly described as long-run effects since *ecm4* is not error correcting in wages. Hence, it becomes difficult to judge the long-run impact from *ecm4* on wages. Tentatively, it would appear that a rise in the hours worked would increase wages, which seems natural. Similarly, a rise in the output level would decrease wages, which is counter intuitive.

Further evidence on the elasticity of substitution is provided by equation (13). Equation (13) describes the change in manufacturing unemployment due to short-run effects (the differenced terms) and imbalances in the long-run relationships *ecm2* and *ecm3*. Only *ecm2* is error correcting in the unemployment equation, so we have the same difficulties of interpreting *ecm3* as above. Since the interest, r_t , appears in *ecm2* a long-run relationship between the unemployment rate and the interest rates *is empirically established*, again consistent with $\sigma \neq 1$. Furthermore, the coefficients imply that this relationship is negative and by similar reasoning as above. Thus, in the long-run we would have something similar to

$$\frac{\Delta u}{\Delta r} < 0$$

which again correspond to the case where $1 < \sigma < s$ (see equation (5)). Additionally, *ecm2* relates unemployment to the inflation rate and the real output level in the long-run. Both of these affect unemployment negatively, which accords with intuition. Once again it is difficult to interpret the effects from *ecm3* on unemployment since *ecm3* is not error correcting. However, it should be noted that the interest rate also appears in *ecm3* and affects unemployment positively from that relation, somewhat weakening the previous conclusions.

Finally, the coefficient estimate c_{61} in $\mathcal{H}_6$ in table 2 provides a direct estimate of $\sigma = 1.39$. Although this crude estimate should be viewed with caution it is reassuring that it is well above one, consistent with the findings above. Taken together, the empirical results provide consistent evidence that the elasticity of substitution in Finland is above one.

5.2 A brief discussion of the remaining equations of the model

The remaining equations in table 5 are briefly discussed in this section. The focus will be on the empirical consistency of the long-run effects in each of the remaining equations, i.e. on the error correcting equilibrium relations. Equation 3 explains the changes in manufacturing output where only *ecm4* is error correcting. The interpretation from this equilibrium relationship is straightforward. An increase in the hours worked in manufacturing leads to a long-run increase in output (however, output increases slightly less than the initial increase in hours worked). Additional effects on the change in output stem from *ecm1* and the short-run effects.

Equation 4 explains the changes in hours worked in manufacturing. As with output, only *ecm4* is error correcting in that equation and the effects are similar, i.e. increased output leads to more hours worked. Thus, there exists a two way relationship between the hours worked and the output. In addition, the hours worked are affected by disequilibrium in *ecm2* and other short-run effects.

Equation 5 explains the change in the long-term interest rate where only *ecm3* is error correcting. The effects from increased real capital is higher interest rates, which is somewhat counterintuitive. However, note that since capital is exogenous in the time frame of this analysis, this might a consequence of excess capital demand on the interest rates in the medium term. Intuitively, when there is a high demand for capital, interest rates start to increase.

Finally, equation 6 describes the change in inflation where only *ecm2* is error correcting. An increase in unemployment decreases inflation, which seems natural. However, increases in output or the interest rates appears to decrease inflation, which is counter intuitive. In addition the change in inflation is also affected by disequilibrium in *ecm3* and *ecm4* and by other short-run effects.

6 Conclusions

In this paper, an indirect method for making econometric inference on the parameter region of the elasticity of substitution, utilizing functional relationships that depend on this elasticity, was proposed. Compared with alternative methods of estimating the elasticity, the present approach provides enough flexibility to allow for more data oriented modeling while still retaining a firm connection to economic theory. A positive long-run relationship between real manufacturing wages and the capital-labor share and a negative long-run relationship between the unemployment rate and the interest rate were established by estimating a cointegrated VAR-model on Finnish manufacturing data. These relationships have implications for the parameter region of the elasticity of substitution as demonstrated in Kauppi et al. (2004). In particular, the results are consistent

with a modeling approach for the Finnish economy that uses a CES production technology with an elasticity of substitution above one coupled with imperfections in both product and labor markets. A rough estimate of the elasticity of substitution equal to 1.39 was also obtained.

References

- Antras, P., 2004. Is the U.S. Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution. *Contributions to Macroeconomics* 4 (1).
- Arrow, K., Chenery, H., Minhas, B., Solow, R., 1961. Capital-Labor Substitution and Economic Efficiency. *Review of Economics and Statistics* 43, 225–250.
- Azariadis, C., 1996. The Economics of Poverty Traps Part One: Complete markets. *Journal of Economic Growth* (1), 449–486.
- Bentolila, S., Saint-Paul, G., 2003. Explaining Movements in the Labor Share. *Contributions to Macroeconomics* 3 (1).
- Berndt, E., 1976. Reconciling Alternative Estimates of the Elasticity of Substitution. *Review of Economics and Statistics* 58 (1), 81–114.
- Blanchard, O., Giavazzi, F., 2003. Macroeconomic Effects of Regulation and Deregulation in Goods and Labor Markets. *Quarterly Journal of Economics*, 879–907.
- Chirinko, R., Fazzari, S., Meyer, A., 2004. That Elusive Elasticity: A Long-Panel Approach to Estimating the Capital-Labor Substitution Elasticity. CE-Sifo working paper 1240.
- Dixit, A., Stiglitz, J., 1977. Monopolistic Competition and Optimum Product Diversity. *American Economic Review* 67 (3), 297–308.
- Doornik, J., Hendry, D., 2001. *Empirical Econometric Modelling Using PcGive*. London: Timberlake Consultants Press.
- Duffy, J., Papageorgiou, C., Mar. 2000. A Cross-Country Empirical Investigation of the Aggregate Production Function Specification. *Journal of Economic Growth* 5, 87–120.
- Engle, R., Granger, C., Mar. 1987. Co-integration and Error Correction: Representation, Estimation and Testing. *Econometrica* 55 (2), 251–76.
- Felipe, J., Fisher, F., May 2003. Aggregation in Production Functions: What Applied Economists should Know. *Metroeconomica* 54 (2), 208–263.

- Hansen, H., Johansen, S., 1993. Recursive Estimation in Cointegrated VAR-models. Preprint 1993, No. 1, Institute of Mathematical Statistics, University of Copenhagen .
- Hendry, D., Juselius, K., 1999. Explaining Cointegration Analysis: Part I. *Energy Journal* 21 (1), 1–42.
- Hendry, D., Juselius, K., 2000. Explaining Cointegration Analysis: Part II. *Energy Journal* forthcoming.
- Johansen, S., 1995. Likelihood-based Inference in Cointegrated Vector Autoregressive Models. Oxford University Press.
- Johansen, S., 2002. The Interpretation of Cointegrating Coefficients in the Cointegrated Vector Autoregressive Model. Preprint .
- Johansen, S., Juselius, K., 1990. Maximum Likelihood Estimation and Inference on Cointegration – With Applications to the Demand for Money. *Oxford Bulletin of Economics and Statistics* 52 (2), 169–211.
- Jones, L., Manuelli, R., 1990. A Convex Model of Equilibrium Growth: Theory and Policy Implications. *Journal of Political Economy* 98.
- Kauppi, H., Koskela, E., Stenbacka, R., Oct. 2004. Equilibrium Unemployment and Capital Intensity Under Product and Labour Market Imperfections. Helsinki Center of Economic Research, Discussion Paper 24.
- Klump, R., McAdam, P., Willman, A., Jun. 2004. Factor Substitution and Factor Augmenting Technical Progress in the US: A Normalized Supply-Side System Approach. ECB Working Paper Series 367.
- Mairesse, J., Hall, B., Mulkay, B., 1999. Firm-Level investment in France and the United States: An Exploration of What We Have Learned in Twenty Years. *Annales d'Economie et de Statistique* (55-56), 27–67.
- Nickell, S., 1997. Unemployment and Labor Market Rigidities: Europe versus North America. *Journal of Economic Perspectives* 11 (3), 55–74.
- Quah, D., Jun. 1996. Empirics for Economic Growth and Convergence. *European Economic Review* 40 (6), 1353–76.
- Ripatti, A., Vilmunen, J., 2001. Declining Labour Share - Evidence of a Change in the Underlying Production Technology? Bank of Finland Discussion Papers (10).
- Solow, R., Feb. 1956. A Contribution to the Theory of Economic Growth. *Quarterly Journal of Economics* 70, 65–94.

Solow, R., 1994. Perspectives on Growth Theory. *Journal of Economic Perspectives* 8 (1), 45–54.

Spector, D., Feb. 2004. Competition and the Capital-Labor Conflict. *European Economic Review* 48, 25–38.