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Strategic R&D and Network Compatibility*

Abstract

This paper analyses the effects of network externalities in strategic R&D competition. We present a model of two firms competing with R&D investments and prices in a differentiated consumer market. Buyers form firm-specific networks which can be compatible. A high degree of compatibility and large spillovers moderate price competition due to weak strategic value of firm-specific networks and R&D investments respectively. Asymmetry in product qualities brings out network effects that cancel out in conventional symmetric settings. The lower quality firm increases R&D and decreases its price as spillovers or network compatibility is increased. This happens when R&D and firm-specific network size have high strategic value.

JEL Classification: L13, L15, O32.

Keywords: Research and development (R&D), spillovers, network compatibility.

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1 Introduction

Network structures are pervasive in modern economies: people spend increasing amounts of time and money on internet services, organisations link themselves to other organisations with various cooperational relationships. At the same time, competition in many network industries is undertaken on various levels that mix strategic investments and price competition. This complexity generates interesting competitive firm behaviour. In this paper, we examine the effects of demand side network externalities on strategic cost reducing R&D investments. We augment a standard horizontal differentiation model by introducing network externalities into the demand side and involuntary spillovers into R&D production. Technological change has been an element in network economics since Farrell & Saloner (1985), David (1985) and Arthur (1989), but the feature that technological investments are imperfectly appropriable has been overlooked. The merger of strategic R&D and networks brings up new kinds of firm behaviour that refines the results of earlier literature.

Imperfect appropriability of R&D and its consequences on industrial competition, as well as the performance of different forms of R&D cooperation, in terms of welfare, have been studied by, among others: Spence (1984), d'Aspremont & Jacquemin (1988, 1990), Henriques (1990), Kamien et al. (1992), Suzumura (1992), Suzumura & Yanagawa (1993), and Amir (2000). De Bondt (1997) is a survey of R&D appropriability literature. In strategic investment games, the total effect of R&D spillovers consists of a market expansion effect that encourages R&D investments as well as a competitive effect that discourages R&D. De Bondt (1997) generalises that the competitive effect dominates the market expansion effect, and therefore, spillovers tend to have a discouraging effect on an individual firm's willingness to invest in strategic R&D. Yet, De Bondt (1997) claims that the negative relationship may reverse with a small number of rivals and sufficient product differentiation. Asymmetry between firms can result in increasing R&D in spillovers. For example, if one firm is better in appropriating R&D than other firms, it may increase its efforts when

spillovers increase. On the other hand, effective R&D that measures total cost reduction for a firm, i.e. firm-specific R&D plus the spillover benefits from other firms, tends to be increasing up to a certain spillovers threshold. When spillovers exceed the threshold, also effective R&D decreases in spillovers. Product differentiation raises the threshold.

Even though the negative association between firm-specific R&D and spillovers is the general result in the strategic R&D literature, other results have a tendency to be dependent on the chosen model set-up. Hence, many results appear not to be too robust. For example, Amir (2000) reports that the effective R&D level increases in the d'Aspremont & Jacquemin (1988, 1990) set-up when spillovers are relatively small and decreases when spillovers are large. But, the set-up used by Kamien et al. (1992) produces decreasing effective R&D for all spillover levels.

Network incompatibility may arise from many sources such as technical product features or personal tastes. Following Katz & Shapiro (1985), the compatibility literature has focused on the technical interpretation and analysed private and social incentives for compatibility. Incompatibilities create implicit switching costs. Beggs & Klemperer (1992) show that consumer prices are higher with switching costs than without. Besen & Farrell (1994) interpret this result as a tendency of incompatibility to tone down price competition. Incompatibility represents a degree of consumer lock-in, which allows the firm to charge above the price of perfectly compatible goods. Even though Besen & Farrell (1994) leave some room for doubt, they report the literature generally agreeing on that incompatibilities reduce (price) competition. Bental & Spiegel (1995) analyse a network competition model with income-wise differentiated consumers. Richer consumers are willing to pay more for a network of a given size. They show that consumers prefer compatible networks as market coverage is the highest and price the lowest under compatibility. The result, however, comes from free entry of firms. In contrast, Shy (2001) shows how "*compatibility is anticompetitive*". The key to his conclusion is that, with incompatible goods, market share competition is the dominating feature, which drives prices down. Shy (2001) explains that, with incompatibility, consumers care about the price difference between goods and about the sizes of

firm-specific networks. Under perfect compatibility, price competition is relaxed, since the sizes of firm-specific networks become irrelevant as all consumers attain the benefits of the whole network. Firm-specific market shares carry strategic value only under incompatibility.

It is interesting to expose the results of R&D models to network externalities. The combination is more than the sum of parts. In this paper, we employ demand side network externalities that resemble the telecommunications interconnection traffic in Laffont et al.'s (1997, 1998) and Armstrong's (1998) models. We extend the game with a stage in which firms choose R&D investments. Our focus is on cost-reducing (process) R&D and its implications on price competition¹. The main item of interest in our model is consumer interaction; each consumer gets utility from interaction with others. Two firms each serve a network of consumers. Networks are vertically differentiated and they may be linked with different degrees of compatibility. Firms choose R&D investment levels in the first period. In the second period, they set prices.

Network externalities are prone to produce multiple equilibria. Multiplicity causes that equilibrium analysis does not yield determinate predictions. We overcome the multiplicity problem by differentiating consumers à la Hotelling (1929). With sufficient price insensitivity, we get a unique interior equilibrium. We also consider a conceptually more interesting vertical differentiation of networks. Vertical differentiation is exogenous, which makes possible the analysis of asymmetric set-ups.

We show how asymmetric equilibria differ from a symmetric one, and when the asymmetric equilibria involve firm behaviour which disagrees with the conventional results of strategic R&D and network compatibility models. Our model adds to the complexity in results of conventional models, rather than generalises them. The main findings are:

1. We derive a case where a firm increases firm-specific R&D investments under a marginal increase in spillovers: the firm with lower quality product tends to increase R&D under an

¹ For a network model à la Armstrong (1998) - Laffont et al. (1998) with product R&D that improves the quality of the good, see Valletti & Cambini (2003).

increase in spillovers if R&D and firm-specific network size have high strategic value. This happens when networks are (almost) perfectly incompatible and spillovers small.

2. We characterise conditions where Shy's (2001) result "*compatibility is anticompetitive*" fails in its strongest form: the firm with lower quality product tends to decrease its price with an increase in compatibility, again, if R&D and firm-specific network size have high strategic value. Price decreases because the firm chooses to increase R&D investments in order to cut costs.

Under competition, private and social incentives for network compatibility are aligned, but they differ for R&D appropriability in general. In the situations where we have the unorthodox behaviour detailed in 1 and 2 above, consumer surplus and the higher quality firm's profits move together and dominate the opposite change in the lower quality firm's profits with marginal changes in compatibility and R&D appropriability.

Findings from a symmetric model agree with the existing literature. Because asymmetry can produce unorthodox results, literature which focuses on symmetric equilibria fail to capture all prevailing effects.

In section 2, the model is constructed and equilibrium is derived. Then the equilibrium is subjected to comparative statics analysis. We illustrate the unorthodox results with a numerical example. After that we bring forth some issues on total surplus. We conclude with a discussion. All proofs of results are straightforward and are relegated to the appendix.

2 Model

We are interested in industries that present demand side network externalities and some degree of product compatibility. We will later formally propose a compatibility measure that incorporates personal tastes as well as pure technical compatibility.

As an example of our model, consider instant messaging (IM) software that allows people to

communicate over the Internet. A user of IM software benefits when there are more people on the IM network. Yahoo!, MSN or AOL IM do not interoperate, but users of Gaim, Adium or Trillian IM can communicate with users of (practically) any other IM software². In addition to basic instant messaging, some IM software includes more advanced communication, file swapping, and entertainment features. The choice of IM software depends heavily on the versatility of the software as well as on the brand factor (e.g. some consumers have preferences to use only open-source rather than commercial software). But if software is incompatible, then also the size of the user network matters. The second example is software, such as spreadsheets, that allows consumers to share work with other people. Users of a particular brand of spreadsheet may use files created by different brands. Brands may not be perfectly compatible so that some software functionalities do not work thoroughly. Some users may dislike the way information is handled in one brand of software, and therefore may swap files less with users of that brand. Another example is game consoles (e.g. PlayStation 2, Xbox, or Game Cube) that allow playing against other people over an Internet connection. Consoles tend to support little interoperability. A particular game seldom works on more than one brand, even if there exists many versions of the game (which cover the whole console spectrum). The final example is mobile telecommunications. Here, the demand side externalities arising from person to person communication are obvious, but brand preference in interaction is (almost) negligible³.

There are two firms in the market, say A and B . The market is closed and the number of consumers fixed. Consumers are evenly distributed over a unit line according to their subjective taste preferences. Each firm is exogenously located at one extreme of the line and the locations are inherited from outside this model, so that firm A is located at 0 and firm B at 1.

² Some multi-medium software, such as Trillian, actually allows the users to log-on simultaneously to "host" IM networks of Yahoo!, MSN or AOL (among others), rather than being standalone IM networks. However, the multi-medium IM software incorporate features that are not supported by the host networks making them more than pure adapters. Usually the basic versions of IM software are free of charge, but more advanced features have to be bought.

³ We assume that firms charge simple flat rate fees without interconnection payments. This reduces applicability of our model to (voice) telecommunications. However, SMS services and similar new services are a better fit compared to pure voice services.

Consumers have unit demand. The product yields intrinsic utility and utility contingent on all other consumers who have bought the product. The idea is that utility is driven by peer-to-peer types of services that enable interaction between consumers, and interaction utility can be split into two parts. First, the consumer gets utility from consumers who have bought from the same firm as he has. This network of consumers is referred to as a "home network" and the utility is labelled as an "intra-network utility". Secondly, the consumer may also get utility from the consumers who have bought from the other firm. This network is referred to as a "rival network", and the utility as an "inter-network utility". Interaction utility depends on the sizes of home and rival networks and on the compatibility between networks. Consumers who have not bought any product give zero utility to those that have. If the consumer located at $s \in [0, 1]$ on the Hotelling beach, buys the product, he gets intrinsic utility $V(s) = v - l(s)z$, where $v > 0$ is a fixed base utility, and $l(s)$ is the distance to the supplier. Transportation cost parameter z measures how well the product matches the consumer's subjective preferences. Consumers are assumed not to be constrained by their budgets, and v is large enough so that all consumers opt for purchasing the product independent of other consumers' decisions. As a result, the consumer's problem is reduced to choosing from which firm he buys.

Firm i charges a flat rate p_i . Net utility for a consumer located at s who buys from firm $i = A, B$, $i \neq j$ is then

$$U_i(s) = v - l(s)z + n_i v_i + n_j \bar{v}_i - p_i. \quad (1)$$

The principal item of interest in equation (1) is the interaction utility given by $n_i v_i + n_j \bar{v}_i$. For the consumer that has decided to buy from firm i , intra-network utility is $n_i v_i$, where n_i is the number of consumers on the home network. Parameter v_i measures the objective value associated with each network member. It gives the usage value of services used in interaction. This objective valuation is shared by all consumers and it is independent of subjective taste. A consumer located in the middle of the Hotelling beach can be indifferent between the goods in terms of subjective attractiveness but strictly prefer one good to the other in terms of objective quality.

Term $n_j \bar{v}_i$ gives the respective inter-network utility from a rival network of size n_j with objective value $\bar{v}_i$. Our specification of network externalities implicitly assumes that the underlying social network characterising consumers' relationships is a completely connected graph. Each consumer is connected to everyone else.

The consumer gains at most equal utility from a single rival network member compared to a home network member. The rationale behind this assumption is that it is likely that similar types of consumers choose to buy from the same firm, and that consumers interact more with people similar to them. In the absence of usage fees, the rival network is less regularly accessed⁴. Furthermore, there might be technical incompatibilities between rival goods which hamper inter-network interaction. From the perspective of a consumer, the assumption that an individual consumer on the home network is at least valuable as one on the rival network translates into a condition $v_i \geq \bar{v}_i$, which can be parameterised as $v_i \geq \bar{v}_i (1 - t)$, where $t \in [0, 1]$ measures network compatibility. The proposed concept of network compatibility should be understood arising from consumers' tastes and from technical compatibility features. The following example clarifies this. A consumer, who has bought from firm A , located at s interacts more with people located close to s (call these people s 's friends). He also has a bias towards interacting more with friends who are on his home network. At the margin, where half of the consumer's friends are on his home network and half on the rival network, the bias in favour of home network is captured in t . If there is no bias between networks, we have $t = 0$. If, however, network brand determines perfectly with whom he interacts with, $t = 1$.

Our assumption of covered and closed markets, removes network expansion effects of R&D. An improvement in network compatibility, however, captures some forms of market expansion. Define the efficient network size as $n_i v_i + n_j v_i (1 - t)$. As t is lowered, the effective size grows.

If networks are perfectly incompatible ($t = 1$), the rival network does not yield utility. In

⁴ Note that we do not need to make any formal assumptions on the balance of access between networks since consumers make a single flat rate payment to the firm. Compare this with telecommunications industry models by Armstrong (1998) or Laffont et al. (1997, 1998) who assume a neutral calling pattern.

the typology of Besen & Farrell (1994), perfect incompatibility makes the firms compete for the market. In this case, firm-specific market shares are important in consumers' decision making. The polar case, perfect compatibility ($t = 0$), produces competition within the market. In this case, consumers get the full benefits of the total network, and firm-specific network sizes are irrelevant in consumers' decision making.

Consumers' expectations on network sizes are fulfilled in the equilibrium. The indifference condition $U_A(s) = U_B(s)$ determines market shares uniquely. The market share for firm A is

$$s = \frac{z - p_A + p_B + (1 - t)v_A - v_B}{2z - t(v_A + v_B)}. \quad (2)$$

The firms' problem is to maximise profits by choosing R&D investments and setting unit prices. Investments in R&D reduce unit production costs capturing the idea of process R&D. Let x_i be the autonomous, or firm-specific, output of firm i 's R&D investment. We eliminate the case in which R&D spillovers would flow only from the R&D leader to the laggard by interpreting x_i so that it represents both R&D output and total input including all trials and errors. A fraction of R&D output, ξx_i , is spilled over to the rival without any cost or compensation. The spillover parameter, $\xi \in [0, 1]$, is symmetric between firms. The effective cost reduction for firm i is then $X_i = x_i + \xi x_j$. Productivity of R&D is independent of spillovers, and the firm's own and the rival's R&D are substitutes. The potentially undesirable possibility that one firm can benefit passively from the rival's R&D is not dealt with. A firm can enjoy cost reduction even if it does not invest in R&D at all. The unit cost per sale for firm i is $C_i = c - X_i$, where $c > 0$ is assumed to be symmetric between firms⁵.

⁵ Different approaches to modeling R&D and spillovers are abundant: for example, Levin & Reiss (1988) and Kesteloot & De Bondt (1993) assume imperfect substitutability of autonomous R&D and an industry-wide pool of R&D. Of other variants Cohen & Levinthal (1989) (variable learning capacities and intra-industry R&D spillovers), De Bondt & Henriques (1995) (asymmetric spillovers and R&D absorption capabilities) and Katsoulacos & Ulph (1998) (endogenous spillovers) are among the most interesting ones. See also Amir (2000) who looks at the differences between d'Aspremont & Jacquemin's (1988, 1990) and Kamien et al.'s (1992) formulations of R&D productivity. However, we find symmetry and the absence of industry R&D pools in our set-up appropriate considering the small number of firms. We also consciously ignore other more sophisticated treatments of R&D and spillovers.

The objective functions for firm A and B are

$$\begin{aligned}\pi_A &= s [p_A - (c - x_A - \xi x_B)] - \frac{1}{2}x_A^2 \\ \pi_B &= (1 - s) [p_B - (c - x_B - \xi x_A)] - \frac{1}{2}x_B^2,\end{aligned}\tag{3}$$

where R&D costs are given by $-\frac{1}{2}x_i^2$, $i = A, B$.

3 Equilibrium

The firms maximise profits (3) in two stages. In the first stage, they choose simultaneous R&D investments (x_A, x_B) , and in the second stage they set prices (p_A, p_B) simultaneously. We solve the problem for a sub-game perfect Nash equilibrium (NE).

Second stage best responses are

$$p_i(p_j) = \frac{1}{2} [p_j + c - (x_i + \xi x_j) + (1 - t) v_i - v_j + z].\tag{4}$$

Reactions (4) are upward sloping, characteristic of Bertrand price competition. They cross correctly to produce a stable equilibrium⁶. Note that the reaction to a marginal increase in the value of rival product is tougher than to an increase in the value of the firm's own product.

NE prices are

$$p_i^{NE} = c + z - \frac{1}{3} [(2 + \xi) x_i + (1 + 2\xi) x_j - (1 - 2t) v_i + (1 + t) v_j].\tag{5}$$

The firm's own R&D causes a larger drop in price than the rival's R&D does, $\frac{\partial p_i^{NE}}{\partial x_i} \leq \frac{\partial p_i^{NE}}{\partial x_j}$, with equality at $\xi = 1$. Hence, all things constant, an increase in R&D by one firm increases its market share.

First stage best responses are

$$x_i(x_j) = \frac{(1 - 2t) v_i - (1 + t) v_j + 3z - (1 - \xi) x_j}{A - (1 - \xi)},\tag{6}$$

where $A = \frac{9}{2(1-\xi)} [2z - t(v_A + v_B)]$. First stage reaction functions have a cut-off point where R&D investments change from being strategic complements to strategic substitutes. Strategic

⁶ Stability is ensured since $\left| \frac{\partial p_i}{\partial p_j} \right| = \frac{1}{2} < 1$. Second stage second order conditions require that $z > \frac{1}{2}t(v_A + v_B)$.

complementarity, however, is ruled out by the second order condition for a maximum⁷. Since the reaction functions are linear, they cross at most once. Thus if there is an interior equilibrium, it is unique and corresponds to the fulfilled expectations equilibrium of consumers' problem.

NE investments are given by equation (7).

$$x_i^{NE} = \frac{(1 - 2t) v_i - (1 + t) v_j + 3z - \frac{2}{3}(1 - \xi)^2}{A - 2(1 - \xi)}. \quad (7)$$

NE investments can be presented as $x_A^{NE} = \frac{2}{3}(1 - \xi) s^{NE}$ and $x_B^{NE} = \frac{2}{3}(1 - \xi)(1 - s^{NE})$. In the case of symmetric market shares, investments are equal. If one firm has smaller market share, it also invests less in R&D. Industry-wide R&D effort depends only on spillovers, $x_A^{NE} + x_B^{NE} = \frac{2}{3}(1 - \xi)$, underlining the absence of the market expansion effect. The industry-wide effective cost reduction is $(1 + \xi)(x_A^{NE} + x_B^{NE}) = \frac{2}{3}(1 - \xi^2)$. Obviously, both industry-wide investment and effective output are decreasing on the whole range $\xi \in [0, 1]$, and they drop to zero with perfect spillovers. There are no incentives to do R&D when it does not yield competitive advantage.

A general condition for equilibrium stability is $\left| \frac{\partial x_i(x_j)}{\partial x_j} \right| < 1$ (Tirole 1988). This condition requires in the current model that $A - 2(1 - \xi) > 0$. Effectively, this condition sets a lower limit for the transportation costs. The condition guarantees also that second order conditions for maximum hold. Hence, the following condition (A1) is required to hold in equilibrium⁸.

Assumption (A1) Minimum price insensitivity: $z > \frac{1}{2} \left[t(v_A + v_B) + \frac{2}{9}(1 - \xi)^2 \right]$.

Using NE prices and investments, firm i 's NE market share can be expressed as

$$s_i^{NE} = \frac{1}{2} - \frac{(v_j - v_i) \left(1 - \frac{1}{2}t\right)}{\frac{2}{3}(1 - \xi) [A - 2(1 - \xi)]}. \quad (8)$$

⁷ Strategic complementarity and substitutability are defined as in Bulow et al. (1985). First stage second order conditions require that $z > \frac{1}{2}t(v_A + v_B) + \frac{1}{9}(1 - \xi)^2 \Leftrightarrow A > (1 - \xi)$, which is more stringent condition than the second stage second order conditions. Now, the second order conditions for the first stage rule out the case in which $\frac{\partial x_A}{\partial x_B}$ would be positive. Consequently, investments are always strategic substitutes. Unique strategic substitutability is a simplification of the general tendency of mixed strategic substitutability and complementarity in strategic R&D investments. De Bondt (1997) states that, in general with quadratic payoffs, first stage investments are strategic substitutes when spillovers are below a certain critical level, and strategic complements with spillovers that exceed the critical level. Here, there is no critical threshold in that sense.

⁸ Fudenberg & Tirole's (1991) sufficient condition for asymptotic equilibrium stability is $\left| \frac{\partial x_i}{\partial x_j} \right| \left| \frac{\partial x_j}{\partial x_i} \right| < 1$. For the current model, this stability condition is satisfied when $A^2 - 2(1 - \xi) > 0$. The results of the model do not change if we require $A^2 - 2(1 - \xi) > 0$ instead of $A - 2(1 - \xi) > 0$.

Note that the denominator in equation (8) is positive by condition (A1).

4 Comparative statics

4.1 Efficiency benchmark

A competitive duopoly produces industry-wide R&D levels which are lower than the social optimum. We show this with a comparison between the competitive industry and a Ramsey benchmark. In the Ramsey case, we maximise consumer surplus conditional on the industry breaking even. When qualities are asymmetric, this may involve transfers between firms. Consumer surplus is

$$CS = v + \int_0^s [sv_A + (1-s)(1-t)v_A - p_A - zl] dl + \int_s^1 [s(1-t)v_B + (1-s)v_B - p_B - z(1-l)] dl \quad (9)$$

The Lagrangian of the Ramsey maximisation problem is

$$\mathcal{L} = CS - \lambda(\pi_A + \pi_B), \quad (10)$$

where λ is the Lagrange multiplier. Second stage optimisation of (10) gives the socially optimal prices (p_A^R, p_B^R) . Prices are the same if qualities are identical, otherwise they differ. First stage optimisation of (10) gives socially optimal R&D levels (x_A^R, x_B^R) . The Ramsey industry-wide R&D equals to

$$x_A^R + x_B^R = 1 + \xi. \quad (11)$$

In the competitive duopoly, the industry-wide R&D level is

$$x_A^{NE} + x_B^{NE} = \frac{2}{3}(1 - \xi). \quad (12)$$

It is evident that the competitive industry always produces socially too little R&D.

4.2 Symmetric qualities

We start the analysis of the competitive duopoly equilibrium with a case of symmetric qualities ($v_A = v_B = \tilde{v}$). Symmetric firms split the market 50/50. Market share effects due to changes in

ξ and t are neutralised with symmetric qualities, $\frac{ds_{SYM}^{NE}}{d\xi} = \frac{ds_{SYM}^{NE}}{dt} = 0$. With symmetry, the first stage NE degenerates into a simple relationship between R&D and spillovers,

$$x_{SYM}^{NE} = \frac{1}{3} (1 - \xi). \quad (13)$$

The two comparative statics that are of interest, namely $\frac{dx_{SYM}^{NE}}{d\xi}$ and $\frac{dx_{SYM}^{NE}}{dt}$, are trivial. Both firm-specific and effective R&D are decreasing in $\xi \in [0, 1]$. On the other hand, $\frac{dx_{SYM}^{NE}}{dt}$ is zero. In this case, neither firm can take advantage of the other firm's network due to symmetric behaviour that cancels out.

NE prices are

$$p_{SYM}^{NE} = c + z - t\tilde{v} - \frac{1}{3} (1 - \xi^2). \quad (14)$$

The last term in equation (14) equals effective R&D. Not surprisingly, NE prices increase as spillovers increase. This is due to decreasing investments in the first stage. More interestingly, higher levels of network compatibility are associated with higher prices. With some degree of incompatibility ($t > 0$) firms are involved in competition for market share. This competition pushes prices down. Price competition is at the most intense level when consumer networks are completely incompatible ($t = 1$). Whereas the highest profits are attained with perfectly compatible goods ($t = 0$). Incompatibility represents a degree of lock-in of consumers, which softens price competition, but only imperfectly. Despite consumer utility would look likely to increase as parameter t decreases; the benefit is offset by the increase in prices. In this sense, the parameter t acts as a device of tacit collusion, similar to the interconnection charge in telecommunications models (à la Laffont et al. 1998). Finally, profits increase in spillovers because of smaller outlays in R&D.

4.3 Asymmetric qualities

Like the NE investment functions point out, market share dynamics have a central role in the model. Therefore, it is useful to derive comparative statics for the NE market shares. Lemmas 1 and 2 summarise these statics.

Lemma 1 (i) *The market share of the firm with a lower quality good is increasing in spillovers*

$$\frac{ds^{NE}}{d\xi} > 0 \Leftrightarrow v_A < v_B.$$

(ii) *The market share of the smaller firm is increasing in spillovers.*

Lemma 2 (i) *With sufficiently high price sensitivity, $z < v_A + v_B - \frac{2}{9}(1 - \xi)^2$, the market share of the firm with a lower quality good is increasing in network compatibility*

$$\frac{ds^{NE}}{dt} < 0 \Leftrightarrow v_A < v_B.$$

(ii) *With sufficiently low price sensitivity, $z > v_A + v_B - \frac{2}{9}(1 - \xi)^2$, the market share of the firm with a lower quality good is decreasing in network compatibility*

$$\frac{ds^{NE}}{dt} > 0 \Leftrightarrow v_A < v_B.$$

Because network compatibility is foremost associated with consumer's utility, and firms' profits depend on it only indirectly, it is worthwhile to study what kind of impact a change in the parameter t has on intra- and inter-network utilities, which equal to $sv_A + (1 - s)(1 - t)v_A$ for firm A 's customers.

First, if the firm's market share is decreasing in network compatibility, then intra-network utility is also decreasing in compatibility. Smaller home network yields less intra-network utility.

Second, a decrease in compatibility has both a direct effect and an indirect effect on inter-network utility. These two effects are $\frac{d[(1-s)(1-t)v_A]}{dt} = \underbrace{-(1-s)v_A}_{\text{direct effect}} - \underbrace{(1-t)v_A \frac{ds}{dt}}_{\text{indirect effect}}$ for firm A . The direct effect of is always negative. If a decrease in compatibility increases home network size ($\frac{ds}{dt} > 0$), then the indirect effect is negative as well. Smaller rival network yields less inter-network utility.

Consider a case with high price sensitivity (as defined in Lemma 2) and a reduction in network compatibility ($dt > 0$). The market share of the lower quality good firm decreases. Its customers get a negative utility effect through intra-network utility. They also get a negative direct effect through the inter-network utility. Negative effects are partly compensated by a positive indirect inter-network effect due to the increase in the size of the rival network. If networks are relatively incompatible, the positive effect is not very strong. It becomes stronger with higher compatibility

levels. At the extreme, with perfectly compatible networks, the intra-network effect is cancelled by the positive indirect effect of inter-network utility, i.e. the terms with $\frac{ds}{dt}$ cancel out. Perfect compatibility eliminates the strategic role of firm-specific network size.

4.3.1 Comparative statics with respect to spillovers

Conventionally, in strategic investment games, the dominant competitive effect of spillovers guarantees that firm-specific R&D unambiguously decreases as spillovers increase. Since, the market expansion effect is absent in the current model, the competitive effect should guarantee a negative relationship between R&D investments and spillovers. Even if this relation still holds in most cases with asymmetric qualities as well as in the symmetric case, it is not true universally. With asymmetry in the product qualities and high strategic value of R&D and firm-specific networks, the disadvantaged firm increases its investments under a marginal increase in spillovers.

Direct differentiation of x_A^{NE} with respect to ξ gives the following formula

$$\frac{dx_A^{NE}}{d\xi} = -\frac{2}{3}s^{NE} \left[1 + \frac{1-\xi}{\xi} \varepsilon_\xi^s \right], \quad (15)$$

where $\varepsilon_\xi^s = -\left(\frac{\xi}{s^{NE}}\right) \left(\frac{ds^{NE}}{d\xi}\right)$ is the elasticity of market share with respect to spillovers⁹. Increase in spillovers always induces a direct effect to reduce investments. There is also an indirect effect through market share. The firm with higher market share always cuts back investments since its market share decreases as spillovers increase. Smaller firm's market share is growing in spillovers. When its market share is sufficiently elastic, the positive effect can dominate, and the firm increases its R&D investments with an increase in spillovers.

Proposition 3 (i) *The firm increases its autonomous NE R&D investments with a marginal increase in the level of R&D spillovers, if the elasticity of its market share with respect to spillovers is sufficiently high*

$$\frac{dx_A^{NE}}{d\xi} > 0 \Leftrightarrow \varepsilon_\xi^s < \frac{\xi}{\xi-1}.$$

(ii) *The firm with a higher market share always decreases R&D investments under a marginal increase in spillovers.*

(iii) *The positive relation $\frac{dx_A^{NE}}{d\xi} > 0$ is more likely with high quality difference, and with low absolute levels of spillovers. The positive relation is also more likely with low levels of network compatibility, conditional on sufficiently high price sensitivity.*

⁹ Since consumers have unit demand, the elasticity of market share corresponds to the elasticity of demand.

The idea in Proposition 3 is that the smaller firm can take advantage of the possibility to grow its home network (when t is large). The larger firm always invests more in R&D than its rival, but once spillovers are increased, it wants to limit the leakage. It reduces R&D, which increases its costs and subsequently drives its price up. As the larger firm becomes relatively less attractive, the smaller firm can afford to attack. It invests more. In the new situation, network externalities generated by a larger home network outweigh the quality disadvantage, though the smaller firm remains smaller. Higher network externalities compensate for lower quality.

The comparative static for the NE price of firm A is given by equation (16).

$$\frac{dp_A^{NE}}{d\xi} = \frac{-[(1 - 2s^{NE}) - 2\xi(2 - s^{NE})][2z - t(v_A + v_B)] - \frac{3}{4}\xi(1 - \xi)^2}{(1 - \xi)[A - 2(1 - \xi)]}. \quad (16)$$

Proposition 4 *There exists (at least one) threshold level ξ^* , above which both firms increase their NE prices under a marginal increase in spillovers. Below the threshold, the smaller firm (in terms of market share) reduces and the larger firm increases its NE price.*

Since firm size is directly related to the quality difference of the goods, the firm with a lower quality good decreases its price under a marginal increase in spillovers when $\xi < \xi^*$. Otherwise both firms increase price due to smaller outlays in R&D.

4.3.2 Comparative statics with respect to network compatibility

In the symmetric case, network compatibility is neutralised in the investment decisions. The situation becomes more interesting with asymmetric qualities, which reintroduce the competitive nature of network size and network compatibility into the game. By differentiating firm A 's NE investments, we get

$$\frac{dx_A^{NE}}{dt} = \frac{2}{3}(1 - \xi) \frac{ds^{NE}}{dt}. \quad (17)$$

Proposition 5 gives the equilibrium behaviour of the lower quality firm. The investment behaviour of the high quality firm is of opposite sign.

Proposition 5 *(i) With sufficiently high price sensitivity, $z < v_A + v_B - \frac{2}{9}(1 - \xi)^2$, the low quality firm decreases R&D under a marginal decrease in network compatibility*

$$\frac{dx_A^{NE}}{dt} < 0 \Leftrightarrow v_A < v_B.$$

(ii) With sufficiently low price sensitivity, $z > v_A + v_B - \frac{2}{9}(1 - \xi)^2$, the low quality firm increases R&D under a marginal decrease in network compatibility

$$\frac{dx_A^{NE}}{dt} > 0 \Leftrightarrow v_A < v_B.$$

Consider an asymmetric case where $\frac{ds^{NE}}{dt} < 0$. An increase in network compatibility results in a gain in intra-network utility for firm A 's customers due to the increase in firm A 's market share. In addition, the direct effect of the inter-network utility is positive as well, but the indirect effect is negative. However, the negative indirect effect never dominates the positive effects. Hence, firm A 's customers face a positive impact on their utility in total. The competitive position of firm A is improved, which motivates it to increase investments.

Although the firms increase prices with an increase in network compatibility in general, the opposite reaction is possible. A firm may decrease its price if its market share is elastic enough. Firm A 's price response to a decrease in network compatibility is

$$\frac{dp_A^{NE}}{dt} = -\frac{1}{3} \left[\frac{2}{3} (1 - \xi)^2 \frac{ds^{NE}}{dt} + 2v_A + v_B \right]. \quad (18)$$

With high price sensitivity defined as in Lemma 2, Proposition 6 summarises price changes.

Proposition 6 *Let $v_A < v_B$, then under a marginal decrease in network compatibility ($dt > 0$):*

$$\begin{aligned} (i) \text{ High quality firm } & \begin{cases} \text{decreases its price when price sensitivity is high} \\ \text{decreases its price when price sensitivity is low and } 0 < \frac{ds^{NE}}{dt} < \frac{2v_B + v_A}{\frac{2}{3}(1-\xi)^2} \\ \text{increases its price when price sensitivity is low and } \frac{ds^{NE}}{dt} > \frac{2v_B + v_A}{\frac{2}{3}(1-\xi)^2} \end{cases} \\ (ii) \text{ Low quality firm } & \begin{cases} \text{decreases its price when price sensitivity is high and } 0 > \frac{ds^{NE}}{dt} > -\frac{2v_A + v_B}{\frac{2}{3}(1-\xi)^2} \\ \text{increases its price when price sensitivity is high and } \frac{ds^{NE}}{dt} < -\frac{2v_A + v_B}{\frac{2}{3}(1-\xi)^2} \\ \text{decreases its price when price sensitivity is low} \end{cases} \end{aligned}$$

Note that the higher are R&D spillovers, the greater must the change in market share be in order to obtain the positive result $\frac{dp_A^{NE}}{dt} > 0$. In general, the case $\frac{dp_A^{NE}}{dt} < 0$ is dominant.

5 Numerical example

We have derived two results that work against the general findings. The first one is that a firm can increase its autonomous R&D investments if spillovers are marginally increased. The second result is that a firm can decrease its price when network compatibility is marginally increased.

Figure 1: First stage reactions, $t = 1$.

The origin of both results is in network externalities. The unorthodox behaviour appears only when R&D and firm-specific networks have high strategic value.

We can clarify the results with a numerical example. Consider a duopoly with the following set of parameters: $v = 2$, $v_A = 0.72$, $v_B = 0.8$, $z = 1$, $c = 0.25$. Firm A has a 10% disadvantage in terms of product quality. Consumers are relatively price sensitive as the transportation cost is set at a low level (as defined in Lemma 2) motivating the firms to engage in harsher price competition as undercutting is more effective. The parameter configuration characterises a market with an incumbent and an entrant. Entrant's product suffers from early development phase problems, so that its objective quality is slightly lower than incumbent's. However, the entrant has established an equally attractive brand (captured in the uniform distribution on Hotelling beach).

5.1 Changes in spillovers

We first demonstrate the case in which $\frac{dx_i^{NE}}{d\xi} > 0$ holds for the smaller firm. In the first situation, consumer networks are incompatible, $t = 1$. This is the archetypical case for competition for the market as categorised by Besen & Farrell (1994). Figures 1 and 2 show the reaction functions in both stages as spillovers change from $\xi_1 = 0.05$ to $\xi_2 = 0.15$.

The initial equilibrium E^1 , corresponding to $\xi_1 = 0.05$, is given by the crossing of the solid reaction curves. The equilibrium after the change is E^2 , at the crossing of the dashed lines. Firm

Figure 2: Second stage reactions, $t = 1$.

A has increased its investments. Firm B reduces its investments because an invested unit of R&D becomes strategically less effective. Since firm B invests more in absolute terms, its own investments dominate its behaviour in the second stage. Cut down of investments leads to a higher price for firm B . The positive spillover effect caused by an increase in firm A 's investments never dominates firm B 's investments. Firm A decreases its price due to an increase in investments. Because networks are incompatible, home network has a high strategic value which compensates for (low) quality. Firm A realises the possibility to increase home network size as firm B becomes less attractive.

In the initial situation, firms' profits are $\pi_A^1 = 0.0306$ and $\pi_B^1 = 0.1251$, and firm A has a market share of $s^{NE^1} = 0.3310$. After the change in appropriability conditions, firms' profits are $\pi_A^2 = 0.0553$ and $\pi_B^2 = 0.1089$, and firm A has increased its market share to $s^{NE^2} = 0.4161$.

In the symmetric case, when networks were fully compatible, price competition was relaxed as firm-specific network sizes became irrelevant. Hence, it would be logical to expect that as network compatibility increases, the unorthodox aggressive behaviour of the underdog illustrated in Figures 1 and 2 would soften. This in fact is true. Only the extreme case $t = 1$ and its proximate values produce the unorthodox behaviour of firm A . The typical relation of decreasing R&D in spillovers emerges with higher network compatibility. As the parameter t is decreased, the elasticity of market share gets closer to zero, and any market share expansion triggered by a change

in spillovers becomes too small to justify aggressive investment behaviour. With higher levels of network compatibility, home network size has less strategic value, and therefore the underdog wins more by concurring with the dominant firm's strategy.

At the extreme when networks are perfectly compatible ($t = 0$), we have the archetypical case for competition within market. Consumers do not distinguish between home and rival networks eliminating any strategic motives for market share competition. Consider the same parameter set and perfect compatibility. Again, the change in spillovers is from $\xi_1 = 0.05$ to $\xi_2 = 0.15$. Firm A cuts its investments from $x_A^{NE^1} = 0.3061$ to $x_A^{NE^2} = 0.2743$. Firm B 's investments decrease from $x_B^{NE^1} = 0.3272$ to $x_B^{NE^2} = 0.2923$. Firm A 's market share increases from $s^{NE^1} = 0.4833$ to $s^{NE^2} = 0.4841$. This happens because it benefits from a larger share of rival's R&D. Profits increase for both firms due to savings in R&D and increased price level (tacit collusion effect). For firm A , profits go up from $\pi_A^{NE^1} = 0.4204$ to $\pi_A^{NE^2} = 0.4311$, and for firm B from $\pi_B^{NE^1} = 0.4804$ to $\pi_B^{NE^2} = 0.4895$. Hence, firm behaviour is regular.

5.2 Changes in compatibility

The case where both firms raise prices if compatibility is increased, is the most common case, which underlines the general result "*compatibility is anticompetitive*" by Shy (2001). Still, we can construct situations where the underdog firm has incentives to decrease its price under a marginal increase in compatibility. This happens only when R&D and firm-specific networks have high strategic value. Price drop results from a boost in R&D output. Consider the same parameter set, as in previous section, with zero spillovers, $\xi = 0$. Table (19) gives the model output for cases $t = 1$, $t = 0.95$, and $t = 0.9$.

	x_A^{NE}	x_B^{NE}	p_A^{NE}	p_B^{NE}	π_A^{NE}	π_B^{NE}	s^{NE}
$t = 1$	0.0833	0.5833	0.2267	0.0867	0.0040	0.1974	0.1250
$t = 0.95$	0.2497	0.4170	0.2086	0.1808	0.0468	0.1306	0.3745
$t = 0.9$	0.2812	0.3855	0.2354	0.2300	0.0729	0.1370	0.4218

(19)

Firm A prices above firm B , even though its product has lower quality. It can do so thanks to brand loyal customers (horizontal differentiation). It does not payoff to battle over consumers located in the middle of the Hotelling beach; it is more profitable to charge a higher price for brand loyal consumers. Competitive pressure from firm B though destroys its profits.

As we increase network compatibility marginally (to $t = 0.95$), we open up competition within the market, which benefits firm A because of the large size of the rival network. Firm A 's offering becomes more attractive, despite lower quality. Firm A can increase market share significantly by lowering its price. At the same time, firm B increases its price in order to temper price competition. If we increase compatibility even further (to $t = 0.9$), then firm A 's aggressive behaviour is moderated and it concurs with firm B by increasing its price. It is no more profitable to fiercely compete against the higher quality firm B . Note that firm B 's profits are not monotonic in t . The large firm prefers either full incompatibility or high level of compatibility, as there is a dip in profits initially when we depart from perfect incompatibility.

If compatibility was further increased, firms would further increase prices (tacit collusion effect) and profits would be driven up. Firm A 's aggressive pricing characterised above would be eliminated also if spillovers were large. R&D would be relatively too expensive with regard to the potential market share gain.

6 Surplus

We conclude our analysis with few comments on surplus. It was earlier shown that competitive industry produces too little R&D compared with the social optimum. The Ramsey surplus for symmetric qualities ($v_A = v_B \equiv \tilde{v}$) is obtained from the optimisation problem (10)

$$CS^R = \tilde{v} \left(1 - \frac{1}{2}t \right) + \frac{1}{2}\xi + \frac{1}{4}\xi^2 + \frac{7}{4}. \quad (20)$$

CS^R equals total surplus generated in the industry. It is increasing in compatibility and in spillovers.

In the competitive symmetric duopoly, NE profits of a firm are

$$\pi_{SYM}^{NE} = \frac{1}{2} \left[z - t\tilde{v} - \frac{1}{9} (1 - \xi)^2 \right]. \quad (21)$$

It is easy to see now that industry profits increase as spillovers increase. This is because of cut backs in R&D outlays and subsequently higher prices. Industry profits increase as network compatibility increases. This is due to relaxed price competition. Hence, private (firms') incentives for compatibility and R&D appropriability are aligned with the Ramsey case.

Consumer surplus in the symmetric competitive duopoly is

$$CS_{SYM}^{NE} = v + \tilde{v} \left(1 + \frac{1}{2}t \right) - c - \frac{5}{4}z + \frac{1}{3} (1 - \xi^2). \quad (22)$$

It is straightforward to see that consumer surplus decreases as spillovers increase or as network compatibility increases. Both features are consequences of firms' decisions to raise prices.

Total surplus in the competitive duopoly is given by $W_{SYM}^{NE} = CS_{SYM}^{NE} + 2\pi_{SYM}^{NE}$, which equals to

$$W_{SYM}^{NE} = v - \frac{1}{4}z - c + \tilde{v} \left(1 - \frac{1}{2}t \right) + \frac{2}{9} (1 - \xi) (1 + 2\xi). \quad (23)$$

When considering the total surplus, the sign of the comparative statics with respect to spillovers is not constant. Total surplus is maximised with $\xi = \frac{1}{4}$. When spillovers are below the cut-off point of $\frac{1}{4}$, increase in spillovers increases total surplus. In this region, firms' profits increase more than the consumer surplus decreases. With spillovers above the cut-off point, a marginal increase in spillovers causes a reduction in total surplus. Firms' incentives for R&D appropriability differ from social incentives under competition.

Total surplus increases in network compatibility. The increase in firms' profits outweighs the decrease in consumer surplus. Hence, total surplus is maximised with $t = 0$. Private and social incentives for compatibility match. However, consumers would be better off with incompatible networks and intense price competition.

The asymmetric qualities case examined with the numerical example provides similar results. Consumer surplus decreases in spillovers and network compatibility everywhere due to relaxed

price competition. Total surplus tends to increase in network compatibility because the positive change in profits dominates. There is also a spillovers threshold that maximises total surplus. The only region where these results break is the area where R&D is (almost) perfectly appropriated ($\xi = 0$) and networks (almost) completely incompatible ($t = 1$). This is the region where the lower quality firm has unconventional behaviour illustrated in the numerical example. When spillovers or network compatibility is increased in that region, lower quality firm aggressively increases R&D and lowers its price. Its profits go up, but the positive effect is dominated by a decrease in higher quality firm's profits and in consumer surplus.

7 Conclusions

How spillovers and compatibility jointly affect firm behaviour in strategic games with network externalities has been overlooked in the literature. Network effects have also been overlooked by focusing on symmetric equilibria. We studied the interplay between demand side network effects and strategic R&D. We constructed a duopoly model with horizontal differentiation and exogenously given product qualities. Horizontal differentiation provided us means to derive a unique equilibrium, which is needed to obtain determinate comparative statics. Exogenously given quality difference helped to study situations where firms start in asymmetric positions. An alternative, but principally analogous, way to establish asymmetric positions is via fixed installed customer bases à la Crémer et al. (2000)¹⁰.

The role of R&D spillovers and network compatibility in price competition has been covered in this paper. Higher spillovers have a tendency to reduce incentives to invest in R&D and push up prices. Network compatibility tends to moderate price competition by reducing market share competition. Both effects have a similar background. Spillovers reduce the competitive advantage a firm can get by investing more than its rival in R&D. Network compatibility reduces the strategic

¹⁰ Crémer et al. (2000) is a network competition model with endogenous quality and fixed captive customer bases. They analyse internet backbone operator competition, where the quality of interconnection is chosen by the operators.

value of firm-specific customer networks. Thus, the "tacit collusion effects" dominant in pure price setting models carry over to two-stage games in general.

We also showed how a firm may take up reverse actions as R&D appropriability or network compatibility conditions are altered compared to what the standard models would predict. Symmetry was found to support regular firm behaviour. This is natural as the strategic positions of the firms are levelled, which inflicts symmetric behaviour that cancels out. Hence, asymmetry is required if a firm's behaviour is to differ from the norm. Once we considered asymmetric firms, we found cases where the underdog firm takes reverse actions compared to conventional predictions. As R&D spillovers or network compatibility were increased, the underdog firm increases its investments and decreases its price. However, this unorthodox behaviour is not universal, even in asymmetric settings. The strategic variables must have sufficient power. Only when spillovers and compatibility level are low, such reverse behaviour exists. In that case, network externalities intensify the effect induced by strategic investments. In contrast, if compatibility (or spillovers) are very high, the strategic value of firm-specific market shares (or R&D investments) are diminished, supporting regular behaviour.

Although the newly found unconventional results are less than universal, their existence is interesting. Asymmetry between firms is more common than symmetry. Consequently, firm behaviour in industries with network externalities can differ from conventional predictions. In order to have it *not surprising*, the reverse behaviour should be seen as a refinement to theory.

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9 Appendix

Proof of Lemma 1. Differentiation of equation (8) gives $\frac{ds^{NE}}{d\xi} = \frac{2}{A-2(1-\xi)} (1 - 2s^{NE})$. The term $\frac{2}{A-2(1-\xi)}$ is positive by condition (A1). Therefore, the derivative $\frac{ds^{NE}}{d\xi}$ is positive only if $s^{NE} < \frac{1}{2}$ holds. Since we have $s^{NE} = \frac{1}{2} - \frac{(v_B - v_A)(1 - \frac{1}{2}\xi)}{\frac{2}{3}(1-\xi)[A-2(1-\xi)]}$, the firm's market share is less than half, only if its quality is lower than rival's quality, $s^{NE} < \frac{1}{2} \Leftrightarrow v_A < v_B$. Hence, we have that, if $v_A < v_B$ holds, then $\frac{ds^{NE}}{d\xi} > 0$ follows. By symmetry, a respective rule holds for firm B . ■

Proof of Lemma 2. Differentiation of equation (8) yields

$$\frac{ds^{NE}}{dt} = \frac{(v_B - v_A) \left[z - (v_A + v_B) - \frac{2}{9} (1 - \xi)^2 \right]}{\frac{1}{3} \left\{ \frac{2}{3} (1 - \xi) [A - 2(1 - \xi)] \right\}^2}. \quad (24)$$

The sign of the numerator depends now on the relative qualities (v_A, v_B) , on the transportation cost z , and on spillovers ξ . For a given pair (v_A, v_B) , the sign of the derivative $\frac{ds^{NE}}{dt}$ is different for high and low price sensitivity. Equation (24) gives the following rule

$$\begin{aligned} (i) \quad z < v_A + v_B + \frac{2}{9}(1-\xi)^2 &\Rightarrow \begin{cases} v_A < v_B \Rightarrow \frac{ds^{NE}}{dt} < 0 \\ v_A > v_B \Rightarrow \frac{ds^{NE}}{dt} > 0 \end{cases} \\ (ii) \quad z > v_A + v_B + \frac{2}{9}(1-\xi)^2 &\Rightarrow \begin{cases} v_A < v_B \Rightarrow \frac{ds^{NE}}{dt} > 0 \\ v_A > v_B \Rightarrow \frac{ds^{NE}}{dt} < 0 \end{cases}. \end{aligned}$$

■

Proof of Proposition 3. (i) Part one of Proposition 3 follows directly from equation (15).

(ii) Write the comparative statics as $\frac{dx_A^{NE}}{d\xi} = -\frac{2}{3}s^{NE} + \frac{2}{3}(1-\xi)\frac{ds^{NE}}{d\xi}$. The RHS comprises the direct effect and the indirect effect of a marginal change in R&D spillovers. The direct effect equals to $-\frac{2}{3}s^{NE}$, which is always non-positive. The indirect effect equals to $-\frac{2}{3}(1-\xi)\frac{ds^{NE}}{d\xi}$. By Lemma 1, we have $\frac{ds^{NE}}{d\xi} > 0 \Leftrightarrow v_A < v_B$.

(iii) Define

$$h(\cdot) = -\frac{2}{3}s^{NE} + \frac{2}{3}(1-\xi)\frac{ds^{NE}}{d\xi}. \quad (25)$$

Differentiating $h(\cdot)$ with respect to quality relation $\alpha = \frac{v_B}{v_A}$ gives $\frac{\partial h}{\partial \alpha} = \frac{2}{3}(1-\xi)\frac{\partial^2 s^{NE}}{\partial \xi \partial \alpha} - \frac{2}{3}\frac{\partial s^{NE}}{\partial \alpha}$. Next, we differentiate s^{NE} with respect to α . This gives $\frac{ds^{NE}}{d\alpha} = \frac{v_A[-(1+t)+3ts^{NE}]}{\frac{2}{3}(1-\xi)[A-2(1-\xi)]} > 0 \Leftrightarrow s^{NE} > \frac{1+t}{3t}$. Hence firm A 's market share increases with an increase in the quality of the rival good only if its market share is higher than $\frac{1+t}{3t}$. Term $\frac{1+t}{3t}$ is decreasing in t , and with perfect incompatibility it becomes $\frac{2}{3}$. When the positive relation $\frac{dx_A^{NE}}{d\xi} > 0$ holds, we know that $s^{NE} < \frac{1}{2}$ from part (ii). Consequently, if $\frac{dx_A^{NE}}{d\xi} > 0$ holds, then $\frac{\partial s^{NE}}{\partial \alpha} < 0$. Next we compute $\frac{\partial^2 s^{NE}}{\partial \xi \partial \alpha} = \frac{9(1-\xi)^{-1}tv_A(1-2s^{NE})-4[A-2(1-\xi)]\frac{ds^{NE}}{d\alpha}}{[A-2(1-\xi)]^2}$. We get the result $s^{NE} < \frac{1}{2} \Rightarrow \frac{\partial^2 s^{NE}}{\partial \xi \partial \alpha} > 0$. Hence, if $s^{NE} < \frac{1}{2}$ holds, then we have $\frac{\partial h}{\partial \alpha} > 0$, which implies that higher quality difference makes the positive relation $\frac{dx_A^{NE}}{d\xi} > 0$ more likely.

Next, we differentiate $h(\cdot)$ with respect to spillovers. We get $\frac{\partial h}{\partial \xi} = \frac{2}{3}(1-\xi)\frac{\partial^2 s^{NE}}{\partial \xi^2} - \frac{4}{3}\frac{\partial s^{NE}}{\partial \xi}$. The second derivative is $\frac{\partial^2 s^{NE}}{\partial \xi^2} = \frac{-2(\frac{A}{1-\xi}+2)(1-2s^{NE})-4[A-2(1-\xi)]\frac{ds^{NE}}{d\xi}}{[A-2(1-\xi)]^2}$. By Lemma 1, we have

$\frac{\partial s^{NE}}{\partial \xi} > 0 \Leftrightarrow s^{NE} < \frac{1}{2}$, which implies that if $s^{NE} < \frac{1}{2} \Rightarrow \frac{\partial^2 s^{NE}}{\partial \xi^2} < 0$. In total, if $s^{NE} < \frac{1}{2}$ holds, then we have $\frac{\partial h}{\partial \xi} < 0$, which implies that lower spillovers level makes the positive relation $\frac{dx_A^{NE}}{d\xi} > 0$ more likely.

Finally, we differentiate $h(\cdot)$ with respect to t . We get $\frac{\partial h}{\partial t} = \frac{9(v_A+v_B)(1-\xi)^{-1}(1-2s^{NE})-4[A-2(1-\xi)]\frac{ds^{NE}}{dt}}{[A-2(1-\xi)]^2}$.

When transportation costs are sufficiently low (as defined in Lemma 2), $z < v_A + v_B + \frac{2}{9}(1-\xi)^2$, $\frac{\partial h}{\partial t} > 0$ holds if $v_A < v_B$. With higher transportation costs, the sign of $\frac{\partial h}{\partial t}$ becomes ambiguous. Hence, conditional on sufficiently low transportation costs, if $s^{NE} < \frac{1}{2}$, then the case $\frac{dx_A^{NE}}{d\xi} > 0$ is more likely with low levels of network compatibility. ■

Proof of Proposition 4. Start with symmetric qualities, thus with equal NE firm sizes. Equation (16) can be expressed as $\frac{dp_A^{NE}}{d\xi} \big|_{v_A=v_B} = \frac{2}{3}\xi \geq 0$. Differentiation of the numerator of equation (16) with respect to s yields $2(1-\xi)[2z - t(v_A + v_B)]$, which is always positive. Hence, the numerator increases in s . This proves that, for firm with larger market share always increases its price under a marginal increase in spillovers.

Term $-\frac{3}{4}\xi(1-\xi)^2$ in equation (16) is always negative, but the sign of the first term depends on parameter values. If $s^{NE} < \frac{1}{2}$, the numerator is negative for low levels of spillovers. With zero spillovers, the numerator equals $-(1-2s^{NE})[2z - t(v_A + v_B)] < 0$. For high levels of spillovers, the numerator is positive. With perfect spillovers, the numerator becomes $3[2z - t(v_A + v_B)] > 0$. Hence, there exists at least one spillover level, ξ^* , below which smaller firm decreases its price under a marginal increase in spillovers, and above which it increases its price as spillovers are marginally increased. ■

Proof of Proposition 5. Proof follows directly from Lemma 2. ■

Proof of Proposition 6. Let $v_A < v_B$. Firm A 's NE price can be expressed as

$$p_A^{NE} = c + z - \frac{1}{3} \left[\frac{2}{3}(1-\xi)(1+2\xi) + \frac{2}{3}(1-\xi)^2 s^{NE} - (1-2t)v_A + (1+t)v_B \right]. \quad (26)$$

The derivative of p_A^{NE} with respect to t is

$$\frac{dp_A^{NE}}{dt} = -\frac{1}{3} \left[\frac{2}{3}(1-\xi)^2 \frac{ds^{NE}}{dt} + 2v_A + v_B \right]. \quad (27)$$

First observation is that whenever $\frac{ds^{NE}}{dt}$ is positive, the sign of $\frac{dp_A^{NE}}{dt}$ is always negative. In such a case, firm A always increases its NE price if network compatibility is marginally increased.

$$\frac{dp_A^{NE}}{dt} < 0 \Leftrightarrow \frac{ds^{NE}}{dt} > 0.$$

Next, consider the case when $\frac{ds^{NE}}{dt}$ is negative. In this case, firm A may increase or decrease its NE price conditional on the magnitude of the change in its market share. By rearranging the square bracketed term in equation (27), the following rule is established

$$\begin{aligned} \frac{dp_A^{NE}}{dt} < 0 &\Leftrightarrow \frac{ds^{NE}}{dt} > -\frac{2v_A + v_B}{\frac{2}{3}(1-\xi)^2} \\ \frac{dp_A^{NE}}{dt} > 0 &\Leftrightarrow \frac{ds^{NE}}{dt} < -\frac{2v_A + v_B}{\frac{2}{3}(1-\xi)^2}. \end{aligned}$$

Firm B 's market share moves in the opposite direction. Its NE price is

$$p_B^{NE} = c + z - \frac{1}{3} \left[\frac{2}{3}(1-\xi)(2+\xi) - \frac{2}{3}(1-\xi)^2 s^{NE} - (1-2t)v_B + (1+t)v_A \right]. \quad (28)$$

The derivative with respect to t is

$$\frac{dp_B^{NE}}{dt} = -\frac{1}{3} \left[-\frac{2}{3}(1-\xi)^2 \frac{ds^{NE}}{dt} + 2v_B + v_A \right], \quad (29)$$

which yields respective comparative statics

$$\begin{aligned} \frac{ds^{NE}}{dt} < 0 &\Rightarrow \frac{dp_B^{NE}}{dt} < 0 \\ 0 < \frac{ds^{NE}}{dt} < \frac{2v_B + v_A}{\frac{2}{3}(1-\xi)^2} &\Rightarrow \frac{dp_B^{NE}}{dt} < 0 \\ \frac{ds^{NE}}{dt} > \frac{2v_B + v_A}{\frac{2}{3}(1-\xi)^2} &\Rightarrow \frac{dp_B^{NE}}{dt} > 0. \end{aligned}$$

■