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Monopoly Pricing of Social Goods*

Abstract

A product has a social network dimension when its use involves interaction between people. We analyse monopoly pricing in a market where consumers are characterised by their social relations. Consumers get utility from interacting with other people with whom they have a social relation. The monopolist sells a device that enables efficient interaction. This paper introduces two novel features to social relations literature. One, we make players' payoffs endogenous by setting a monopoly pricing problem on top of a network coordination game. Two, we abandon the perfect information assumption by limiting players' capacity to observe prevailing information. Asymmetric information eliminates much of the complexity inherent in the perfect information variant: the role of consumer identity is eliminated, but the role of network structure is maintained. We analyse the roles of network topology and size on the monopoly price and surplus generated in the network. In markets where social relations are important, the implicit assumption on total connectedness of conventional network externalities models exaggerates the value of the network. The topological effect works against, and dominates the size effect. Therefore, the monopolist incorporates network topology in its price. Under asymmetric information, the monopoly prefers symmetric networks, but the social optimum is an asymmetric network. If the firm is allowed to price discriminate, its profits increase to the same level that it obtains in symmetric networks. Monopoly rents and consumer surplus decrease as consumer heterogeneity is increased. This does not necessarily happen under perfect information; it depends on the network topology.

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1 Introduction

In the theory of network economics, network effects have been predominantly synthesised in positive externalities: agent's utility increases as an additional member joins the network. What this approach has overlooked are the effects of network topology. These effects have only recently been incorporated in the research. This paper shows how industry performance depends on both the size effects (positive externalities) and the topological effects.

People have a varying number of social relations. Some people maintain a small number of close relations, whereas some people have a large number of acquaintances that are more shallow. In some collectivist cultures, family constitutes the main social reference group, whereas in more individualist cultures the most important social relations can be friends outside the family. Diversity in people's social relations extends to group level behaviour directly. Cooperation in Japanese keiretsu-groups exceeds that of pure supplier-buyer relationships. In high tech industries, firms have formed R&D alliances where varying levels of interest often include government participation. Conventional economic models of networks have abstracted this kind of diversity away. The conventional externalities model building on the seminal work by Farrell & Saloner (1985, 1986), Katz & Shapiro (1985), David (1985) and Arthur (1989) assumes a functional form for network effects: a network member's utility increases directly, the more people join the network¹. This approach involves an implicit assumption that takes the underlying relations network as a completely connected graph. What it means is that any kind of heterogeneity in terms of social relations is absent. Network members are symmetric in terms of connectedness, therefore, in markets where (asymmetric) social relations are important, conventional models fall short and need to be corrected. In this paper, we examine how two static properties of social networks, size

¹ Network externalities appear most commonly as a linear function of the number of network members. Consider the example by Mason & Valletti (2001): Assume that a link between two network members corresponds to utility equal to 1. When a member indexed as n joins the network with $n - 1$ existing members, the total utility generated in the network increases by amount $2(n - 1)$. The total utility equals $n(n - 1)$ in the network of n members. When n is large, we have $n(n - 1) \approx n^2$. This corresponds to the famous Metcalfe's Law, which states that the value of the network equals the square of the number of network members.

and topology, affect monopoly's pricing strategy and total surplus generated in the network.

Recent work on economics of social relations has moved beyond the traditional functional forms of network externalities. Modern models consider richer forms of underlying networks. The emphasis is on the topology of the underlying social network. In these models it is important to know who is connected to whom. In a social network, there can be well-connected members and members with very little relations. Such heterogeneity leads to asymmetric behaviour.

Social relations literature has thus far focused on models where the interest is in outcomes of games that are structured in a form of network. The underlying network is (usually) fixed so that agents inherit their social characteristics from outside the model. On top of the network, agents play a game of perfect information. The interesting question is how network members can benefit from their network position? In this paper, we study the role of social relations in markets for network goods such as personal telecommunications equipment, e-mail clients, and online game consoles. Our model differs from the previous work in two aspects. First, we introduce a monopoly pricing decision which makes players' payoffs endogenous. The question we are interested in is how an external player (the firm) can take advantage of the network structure. Second, we introduce imperfect information.

Communications networks, rural village economies, and job markets are the most obvious examples of markets where social relations have a non-trivial role. Goldstein et al. (2002) and Udry & Conley (2004) study different overlapping social networks including mutual insurance and information sharing in Ghanaian villages. Gaduh (2002) surveys work on social learning networks in village economies. There is a rapidly growing amount of literature on the role of social networks in labour economics. Applications include Calvó-Armengol & Jackson (2004) and Bramoullé & Saint-Paul (2004) who analyse the interdependence between social relations and unemployment. Bentolila et al. (2004) and Labini (2004) compare wage differentials between employees who find their jobs through either formal or informal channels (social relations). Ioannides & Loury (2004) is a survey of the literature on social relations in labour markets. Bramoullé & Kranton (2004)

study public good provision, particularly innovation and experimentation, in different network settings. Kranton & Minehart (2001) discuss buyer-seller networks, and how buyers (sellers) can hedge against too high dependency on a single subcontractor (client) by forming links with other subcontractors (clients). Goyal et al. (2003) present a model of R&D where different levels of R&D collaboration are mapped as networks. Glaeser et al. (1996) and Ballester et al. (2004) study how crime rates are affected by social relations. Chwe (2000) analyses how the diffusion speed of political action depends on the social network. Glaeser & Scheinkman (2003) discuss various models of non-market social interactions.

Our model is also related to a more dynamic class of games, namely local interaction models (see Ellison 1993, Young 1998 ch.6, Lee & Valentinyi 2000 or Morris 2000). Local interaction models analyse how a particular equilibrium play becomes adopted in the long run. Key features of local interaction games are fixed network structures, imperfect rationality of agents, and exogenous payoffs.

There exists two classes of social relations models. One class treats network structures exogenous to the model, and the other studies endogenous network formation. Jackson (2003) is a survey on endogenous (undirected) network formation models. Endogenous network formation models tend to be more abstract and less applicable to problems associated with personal social relations. When link formation is endogenous, it must comprise all relevant aspects. For example, a decision to form a personal link can be based on family ties, friendships, occupational and economic issues, always encompassing a vast number of personal characteristics. In most cases, the economic dimension of a personal link is difficult to isolate. In contrast, if the network of social relations is exogenous, we can focus on a specific economic problem, such as whether to buy or not a mobile phone. The applied fixed network structure can reflect personal relations which give utility that is hard to measure against utility from consumption of mobile services. Moreover, social relations in many cases exist prior to the decision making. In the mobile phone example: when we think about buying the phone, we think about with whom of our acquaintances we can

use it; not how many new contacts we make when using it. Hence, there is a reason for separating social aspects from economic decision making and taking them as exogenous parameters. But, separation of social relations from economic decisions does not mean that they are irrelevant. In models where the network does not characterise personal relations, such as firm-level R&D networks, endogenous link formation fits well. All link formation decisions involve payoff of the same kind. As predicted Goyal et al. (2003) and Kranton & Minehart (2001), who analyse firms as decision makers, consider endogenous network formation. The payoffs from link formation for firms are better comparable and comprehensive.

In this paper, we depart from the implicit assumption of symmetric complete graphs of conventional network externalities models. Players are characterised by their exogenous personal social networks. Each person is interested only in interaction with a subset of the population, called his neighbourhood. The idea is that the social relations are determined outside of the model. A social link between consumers could mean for example that they are friends, relatives, or colleagues that tend to do things together, thus have a need to interact. We analyse a monopoly market for pure coordination goods. The product does not have any standalone value, but all utility is generated in interaction between people. Consumers need to coordinate their purchases as the only way to benefit from the product is through interaction (efficient interaction is possible only if all parties have the device). They must decide whether to switch to the new good or to stay using the legacy system. The vendor must decide on an (introductory) price. What is important is that consumers cannot tailor their actions vis-à-vis each neighbour. They need to take a single action that applies to every neighbour. This way consumers must consider the overall network structure, rather than each particular link separately.

We consider two informational regimes. One, where all information is perfect to all players. In the other case, buyers' valuations of the good are private information. We give general characterisations of both cases. Then we apply the general results to three basic network structures: complete graph, circle, and star. The complete graph and the circle are symmetric networks,

whereas the star is asymmetric.

We show how the topology of the social network affects the firm's pricing strategy and total surplus generated in the network. Under perfect information, the monopolist chooses to cover the whole market even if it is unable to price discriminate in some network structures. In identical networks, except in terms of who is connected to whom, the firm may choose to limit supply. It is shown how some agents have preferential roles through their connections. These critical agents are able to capture higher surplus than other agents. Interestingly, critical positions exist in asymmetric and symmetric networks under perfect information. In symmetric networks critical positions are due to consumer heterogeneity. Agents who have links *with* high types are critical, as opposed to the high types themselves.

When information is reduced to asymmetric, critical agents lose their market power in symmetric networks. On the other hand, in asymmetric networks, the topologically central agents now always capture higher utility. This is not true necessarily with perfect information.

There are three main findings in the paper. One, network topology matters. Two, the implicit complete graph assumption of the conventional network externality model risks seriously overestimating the value of network effects. Three, with private information, asymmetric networks yield lower profits, but higher total surplus, than symmetric networks of a given link value. However, the firm can match the profits generated in symmetric networks by price discriminating according to the network position.

The paper is organised in the following way. In section 2, we formulate the utility function and formalise the social network. In section 3 we study the perfect information case. In section 4 we analyse the asymmetric information case. Section 5 presents some interesting extensions to the basic model. Section 6 concludes the paper.

2 Network structure and actions

2.1 Informal characterisation

We start with an informal characterisation of the model. The firm launches an innovative device that constitutes an efficient medium for interaction. The product supersedes older generations of products serving similar interaction needs. The product itself has no intrinsic value as it is used only when two people are interacting with each other. As a consequence, consumers who consider buying the product need to estimate what proportion of other people in the market buy it. People are heterogeneous with respect to attainable utility. For example, some people prefer to write letters (the conventional way to interact) whereas some people prefer to use mobile phones (the novel product). The firm on the other hand has to decide on the price of the product. A low price may help solving buyers' coordination problem, but it erodes margins.

We think of products such as the fax machine or e-mail client software. These products are relatively drastic innovations in the sense that they are not compatible with older generation products (fax is not compatible with postal service or courier service, and early e-mail client versions were not compatible with fax machines). Another, but less fitting, example is the first generation mobile phones (in fact, the fixed line telephony in late 1800's is a more suitable example to our model). In the context of this model, mobile phone's capacity to call fixed network and all standalone services should be abstracted away. However, the addition of some intrinsic value would not change the results qualitatively².

The social relations of the population are represented by a network of nodes and links. Consumers are located on the nodes. A link between two consumers (nodes) represents a social relation. The origin for this relation is exogenous. It could base on e.g. family ties, friendships, or occupational contacts. The mapping, or graph, of all social relations gives information about who is interested in interacting with whom. If both end nodes buy the new product, we say that

² See section 5.2.

the link between them becomes active. An active link represents efficiently mediated interaction between consumers. Interaction forms the primary source of utility.

There tends to be multiple equilibria, because like any network model with positive externalities, our model is inherently a coordination game. Whenever coordination fails to reach the Pareto-efficient outcome, we say that there is a coordination failure in the market. Our model relates coordination failure to situations where ex post demand falls short of the forecasted demand. If this problem is extrapolated with a dynamic perspective, coordination failure occurs when demand fails to grow above a critical level. The firm's problem is how to bridge this "chasm" between low and high equilibria³. Multiplicity of equilibria has an economic perspective in this paper, and therefore we do not seek to solve the problem⁴. Instead, we justify theoretically the use of the maximal coordination equilibrium. In our (positive) network externalities model, the underlying coordination game is supermodular with positive spillovers. Consequently, the maximal coordination equilibrium Pareto-dominates other equilibria, which, we argue, focalises the equilibrium.

2.2 Formal model

Let the population of individuals $\mathcal{I} = (1, \dots, I)$, $I \in \mathbb{N}$ be located on the graph $\mathcal{G}$ so that there is a unique individual located on each node of the graph. The set of undirected edges between the nodes of $\mathcal{G}$ is $\mathcal{E}$. An edge represents a social relation. Two consumers i and j are neighbours if they are connected by an edge, $\{i, j\} \in \mathcal{E}$. Undirectedness of all edges guarantees symmetry so that, if $(i, j) \in \mathcal{E} \Rightarrow (j, i) \in \mathcal{E}$. The set of neighbours of consumer i is $\mathcal{N}_i = \{j \in \mathcal{I} \setminus i\}$, with $\mathcal{N}_i \neq \emptyset$ so that there are no isolated nodes. The consumer cannot be his own neighbour, $i \notin \mathcal{N}_i$. The neighbourhood $\mathcal{H}_i = \{i, \mathcal{N}_i\}$ of consumer $i \in \mathcal{G}$ is defined as a collection of agent i himself

³ The taxonomy of bridging "the chasm" between early and mass market adoption is due Moore (1999).

⁴ Our related paper, Sääskilähti (2005), focuses on solving the multiplicity problem. In that paper, we analyse how equilibrium uniqueness is attainable in a monopoly model of network goods under perfect and imperfect information. In both information regimes, key to uniqueness is that one group of agents has a strictly dominating strategy to buy at the same time as another group has a strictly dominant strategy not to buy.

and the set of his neighbours $\mathcal{N}_i$. Consumer i has an interest in interacting only with the people in his neighbourhood. The structure of the graph $\mathcal{G}$ is common knowledge.

The problem for the consumer $i \in \mathcal{G}$ is to choose action $a_i \in \{B, N\}$, where B = buy the new device and N = do not buy. A link between neighbours becomes automatically active if both end nodes buy the goods. The activity of link between i and j is represented by $e(a_i, a_j) \equiv e_{ij}$. Define an active link between agents i and j as $e_{ij} = 1$. If only one agent buys or neither buy, the edge remains inactive, $e_{ij} = 0$. An active link represents interaction between consumers mediated by the new good.

Let the value of an inactive link be normalised to zero. This value represents the utility from interaction with the help of older generation systems. Interaction generates positive utility when it is facilitated by the new device. This can be thought as an efficiency gain or additional utility obtained from type of interaction not previously possible. Consumer i gets utility θ_i from each activated link. The value θ_i is an i.i.d. random variable across consumers $i \in \mathcal{G}$. It is drawn from a uniform distribution $F(\theta)$ with the support $[\theta^-, \theta^+]$, with $\theta^- \geq 0$. The distribution $F(\theta)$ is common knowledge. We assume that the valuation θ_i for consumer $i \in \mathcal{G}$ is independent of the network location he occupies. Why is this so? The network inherits its structure from outside the model. Links represent personal relationships with family members, friends and colleagues. These contacts are formed prior to the model and they are independent of the value the consumer puts on the new device. Exogeneity of the network rules out those cases where the new device (say a mobile phone) would create a new link with a person formerly unknown to the consumer⁵.

The question whether a link is active or not, builds another (technical) layer on top of the inherent (social) network. This way, we differentiate between the exogenous social network and the endogenous technical network. The technical network can be completely active when all links are activated, or it can be partially or totally inactive. On the contrary, the social network is always a fully connected graph without isolated nodes. The following definitions characterise the

⁵ See section 5.2 for discussion on more complex utility specifications.

degree of activity on the technical network.

Definition 1 *Technical network is said to be*

- (i) *a complete network, when $a_i = B$ for all $i \in \mathcal{G}$.*
- (ii) *an empty network, when $a_i = N$ for all $i \in \mathcal{G}$.*
- (iii) *a partial network, when $a_i = B$ for at least one $i \in \mathcal{G}$ and $a_j = N$ for at least one $j \in \mathcal{G}$, $i \neq j$ simultaneously.*

A complete network corresponds to a network where all interaction is mediated by the new product. A partial network is a network where some interaction is mediated by the new product. In the empty network no-one uses the new product.

Throughout the paper we are interested in the role of the social network's structure on the activity level on the technical network. We focus our attention on three different social networks.

- Complete graph, where each consumer is connected to everybody else, $\mathcal{N}_i = \{\mathcal{G} \setminus i\}$ for all $i \in \mathcal{G}$. The complete graph is the structure used implicitly by conventional network externalities models.
- Circle, where each consumer is connected to exactly two neighbours. When agents are indexed in ascending order, the consumer labelled i has neighbours $\mathcal{N}_i = \{i - 1, i + 1\}$. The links form a circle, as consumer labelled I is connected to consumers $I - 1$ and 1 .
- Star, where one consumer is a central agent with connections to everybody else, and where peripheral agents are linked only to the centre. Centre's set of neighbours is $\mathcal{N}_C = \{\mathcal{G} \setminus C\}$, where C is the index for the centre. A peripheral consumer's only neighbour is the centre, $\mathcal{N}_i = \{C\}$, $i \in \{\mathcal{G} \setminus C\}$.

The network is symmetric if all consumers have identical number of links. Network symmetry implies that any two neighbourhoods are symmetric, but the reverse is not necessarily true. The complete graph and the circle are symmetric and the star asymmetric. Complete graph, circle and star are of course very primitive social networks. In spite of primitivity, they bring out the

topological effects missing in conventional externalities models, and effects that are incorporated in more general networks⁶.

With I consumers, the complete graph has $I(I-1)$ directed links, whereas the circle has $2I$ and the star $2(I-1)$ directed links. We can do a comparison across different networks either by keeping the number of agents fixed or keeping the number of links fixed. By construction, the complete graph generates the highest maximal value for a given number of consumers, because it has the highest number of links. The second alternative holds the maximal value in the network (ex ante) fixed. There are $I(I-1)$ links in total in the complete graph. Now, let us fix this number. The corresponding compensated number of consumers in the circle is $\frac{I(I-1)}{2}$. Respectively, the compensated star has $1 + \frac{I(I-1)}{2}$ consumers.

3 Perfect information

We start with the case where all players observe perfectly all available information. The consumer i receives utility $u_i(\theta_i, B)$ if he buys the product

$$u_i(\theta_i, B) = \sum_{j \in \mathcal{N}_i} e_{ij} \theta_i - p, \quad (1)$$

where $e_{ij} = \{0, 1\}$ captures link activity, and p is the unit price for the device⁷. If the consumer does not buy, he receives zero utility. At the margin, the agent is indifferent between buying and not when his valuation is

$$\tilde{\theta}_i = \frac{p}{\sum_{j \in \mathcal{N}_i} e_{ij}}.$$

The better connected the agent is, the lower is his marginal value. A low valuation is compensated by high number of (active) neighbours.

⁶ We discuss two general network types, namely the random network and the scale-free network, in section 5.3.

⁷ To be precise, e_{ij} indicates if the link becomes active when i buys the good, given that j buys the good. If $e_{ij} = 1$, the link between i and j is potentially active, and it becomes active when i buys. Expectations on e_{ij} are fulfilled in equilibrium. We can also write the utility with social relations explicitly expressed, $u_i(\theta_i, B) = \sum_{j \in \{\mathcal{G} \setminus i\}} g_{ij} e_{ij} \theta_i - p$, where $g_{ij} = \{0, 1\}$ indicates whether i and j are neighbours ($g_{ij} = 1$) or not ($g_{ij} = 0$).

If we write the utility as $u_i(\theta_i, B) = \alpha + \sum_{j \in \mathcal{N}_i} e_{ij} \theta_i - p$, where $\alpha = 0$ is the intrinsic utility from the good, we see that the utility function is of type where consumers have differentiated valuation of network benefits. Such utility formulation has been used by de Palma & Leruth (1996). Compare this with Katz & Shapiro (1985) specification where consumers are differentiated according to the intrinsic utility α .

The coordination game Γ consists of consumers $\mathcal{I}$ arranged on the graph $\mathcal{G}$, pure actions $a \in \{B, N\}$, and payoffs $u_i(N) = 0$ and $u_i(B)$ given by equation (1) for all $i \in \mathcal{G}$, and it is parameterised by the unit price p . Let $\mathbf{a}_{\mathcal{N}_i} = (a_j \mid j \in \mathcal{N}_i)$ be the vector of actions taken by consumer i 's neighbours. Consumer i 's best response is $a_i^* \in \arg \max_{a_i \in \{B, N\}} u_i(\theta_i, a_i, \mathbf{a}_{\mathcal{N}_i})$. The best response depends on the price level, the realisation of θ , and on neighbours. Nash equilibrium (NE) of Γ is the strategy profile $\mathbf{a}^* = (a_1^*, \dots, a_I^*)$ which maximises consumer's utility, $u_i(\theta_i, a_i^*, \mathbf{a}_{\mathcal{N}_i}^*) \geq u_i(\theta_i, a_i, \mathbf{a}_{\mathcal{N}_i}^*)$ for all $i \in \mathcal{G}$.

Definition 2 *Nash equilibrium with perfect information is the action profile*

$$\begin{aligned} a_i^* &= B \Leftrightarrow \theta_i \geq \tilde{\theta}_i \\ a_i^* &= N \Leftrightarrow \theta_i < \tilde{\theta}_i \end{aligned} ,$$

where $\tilde{\theta}_i = \frac{p}{\sum_{j \in \mathcal{N}_i} e_{ij}^*}$ and $e_{ij}^* = e(a_i^*, a_j^*)$ for all $i \in \mathcal{G}$.

The coordination game has multiple equilibria. In particular, the empty network is always NE. A total coordination failure occurs when all consumers play systematically, or "stubbornly", $a = N$ irrespective of valuations⁸. When all consumers expect that no-one will buy, no-one will buy in equilibrium. Due to an exogenous network structure, equilibria impaired with coordination failure of smaller sets of consumers (than the total population) are also possible.

We argue that equilibrium selection is likely to favour efficient coordination, although it is impossible to provide a full proof of it.

Lemma 3 *The coordination game Γ is supermodular with positive spillovers (action complementarity).*

Proof.

- (i) *Action set $a = \{B, N\}$ is a compact subset of $\mathbb{R}$.*
- (ii) *The payoffs show increasing differences. If proportion $k = |a_j = B|$, $j \in \mathcal{N}_i$ of i 's neighbours play B , the number of active links is $\sum_{j \in \mathcal{N}_i} e_{ij} = k$ when i plays also B . The payoff of $a_i = B$ versus $a_i = N$ is $v_i(\theta_i, k) = u_i(\theta_i, k, B) - u_i(\theta_i, k, N) = k\theta_i - p$. Then i 's payoff gain $v_i(\theta_i, k)$ is strictly increasing in θ_i for all $i \in \mathcal{G}$.*

⁸ Consider a duopoly where competition is in introduction of new products. Products are differentiated by quality. Farrell & Katz (1998) call consumers' expectations "stubborn in favor of firm k " when a consumer expects that all other consumers prefer firm k 's product irrespective of current market prices. All consumers buy always from firm k , except in the cases where rival l 's quality advantage is large enough to overcome the expected network benefits from the total network. Motivation for such stubborn expectations is in exogenous conditions, e.g. when firm k has a strong financial position compared to rival or it has a good reputation. Note that consumers are perfectly rational, and the resulting equilibrium belongs to the class of fulfilled expectations.

(iii) *The payoff function $u_i : \{B, N\} \times \theta \rightarrow \mathbb{R}$ is continuous.*

(iv) *The payoff gain $v_i(\theta, k)$ is strictly increasing in k .*

Steps (i) - (iii) guarantee that the game Γ is supermodular. Positive spillovers result from (iv).

■

Topkis' theorem guarantees that the supermodular game Γ has largest and smallest NE elements (Vives 2001, p. 33). The smallest NE is the empty network. The largest equilibrium, on the other hand, depends on price p and corresponds to efficient coordination. Due to positive spillovers, the largest NE is Pareto-dominating (Vives 2001, p. 34). Note that positive spillovers are limited to the consumer's neighbourhood only. If there were higher order interaction benefits, e.g. if the consumer gets utility from interaction that takes place between his neighbours or neighbours' neighbours, then positive spillovers would exceed one's neighbourhood. Supermodularity with positive spillovers apply to both symmetric and asymmetric social networks as long as the underlying network is completely connected.

Pareto-dominance makes the equilibrium focal. Especially, allowing pre-game communication, efficient coordination should be more likely, although it is not guaranteed. Also the firm could help coordination. There could be other focal points that favour efficient coordination as well. Some neighbours might be known to work in the high tech industry or be otherwise pro new technology. Alternatively, macrofactors such as a technology boom could trigger efficient outcomes. On the other hand, technology antagonism works against efficient coordination.

When considering the firm's problem, we focus on the maximal NE. Denote $b(p)$ as the largest possible number of consumers who buy (the maximal NE). The function $b(p)$ is confined in the interval $b(p) \in [0, I]$. $b(p)$ is decreasing in p with possible large discontinuities (drops).

The firm observes the realisations of θ and sets the price p . The firm cannot price discriminate between consumers. If price discrimination was allowed, the firm would capture all surplus from every consumer. The resulting technical network would always be a complete network. The pricing problem becomes interesting when the firm must choose one price that applies to everyone.

The firm's problem is to maximise profits $V = b(p)(p - c)$. Marginal cost is constant $c > 0$, and there are no fixed costs. The optimal price is given by equation (2).

$$p^* = \arg \max_p \{b(p)(p - c)\}. \quad (2)$$

We have now characterised the model under perfect information in general terms. Next we apply the general framework to three classes of social networks: complete graph, circle and star. We first describe the NE in each separate case. In section 3.4, we compare how NE prices and surpluses differ in different social networks. Detailed analyses of the cases discussed in the comparison are provided in appendix 8.1.

3.1 Complete graph

The specific location of a consumer on the underlying social network is irrelevant in a complete graph because each consumer is connected to everybody else. Utility for consumer i can be written as $u_i(\theta_i, B) = \sum_{j \in \{\mathcal{G} \setminus i\}} e_{ij} \theta_i - p$. NE of the coordination game when the network is a complete graph can be expressed as in definition 4.

Definition 4 *Nash equilibrium is the action profile*

$$\begin{aligned} a_i^* = N &\Leftrightarrow \theta_i < \frac{p}{\sum_{j \in \{\mathcal{G} \setminus i\}} e_{ij}^*} \\ a_i^* = B &\Leftrightarrow \theta_i \geq \frac{p}{\sum_{j \in \{\mathcal{G} \setminus i\}} e_{ij}^*} \end{aligned} ,$$

for all $i \in \mathcal{G}$.

All network forms are sustainable in equilibrium, conditional on price p and the realisations of θ . Empty network is NE for example when all agents face $\mathbf{a}_{\mathcal{N}_i = \{\mathcal{G} \setminus i\}} = (N, \dots, N)$ or, if for all $i : \theta_i < \frac{p}{I-1}$. Complete network is a feasible NE only if for all $i : \theta_i \geq \frac{p}{I-1}$. Partial network is a feasible NE if for at least one agent has $\theta_i < \frac{p}{\sum_{k \in \{\mathcal{G} \setminus i\}} e_{ik}}$, and at least one agent $\theta_j \geq \frac{p}{\sum_{l \in \{\mathcal{G} \setminus j\}} e_{jl}}$, $i \neq j$ simultaneously.

In the price range $p \in [(I-1)\theta^-, (I-1)\theta^+]$ the game can produce multiple equilibria. Consider a complete graph of four agents with valuations $\theta_1 < \theta_2 < \theta_3 < \theta_4$. Let $\theta_3 > p$, and assume that θ_3 and θ_4 buy. If $3\theta_1 > p$, then the Pareto optimal NE is with all four agents buying.

However, if also $2\theta_2 < p$ holds, then we have two possible non-empty NE (and the empty network NE). One where all four agents buy, and the other where only agents θ_3 and θ_4 buy.

The firm maximises profits $V = b(p)(p - c)$ with price $p^* \in [(I - 1)\theta^-, (I - 1)\theta^+]$. Function $b(p)$ gives the largest number of agents who buy for a given price p . The function $b(p)$ is decreasing in p , with a ceiling $b((I - 1)\theta^-) = I$ and a floor $b((I - 1)\theta^+ + \varepsilon) = 0$, where $\varepsilon > 0$ is small. Price $p = (I - 1)\theta^-$ guarantees that all agents buy in the maximal NE, and $p = (I - 1)\theta^+ + \varepsilon$ guarantees that nobody buys. Example 8.1.1 in the appendix analyses how the firm sets price in a four consumer complete graph.

3.2 Circle

In the circular network each consumer has exactly two neighbours. Utility from $a = B$ can be written as $u_i(\theta_i, B) = (e_{i,i-1} + e_{i,i+1})\theta_i - p$. We obtain a three-partition of types. Low types are consumers who never buy, their valuation satisfies $2\theta < p$. Medium types are those who buy only if both of their neighbours buy. Their valuations satisfy $\frac{1}{2}p \leq \theta < p$. High types are those who buy if at least one of their neighbours buys, $\theta \geq p$. To define the NE fully, we need to consider these three classes only.

Definition 5 *Let a low type have a valuation $\theta < \frac{1}{2}p$. Similarly, let a medium and high type have valuations $\frac{1}{2}p \leq \theta < p$ and $\theta \geq p$ respectively. The following action profiles constitute NE:*

- (i) $\mathbf{a}_{\mathcal{N}_i}^* = (N, N) \Rightarrow a_i^* = N$ for all $i \in \mathcal{G}$.
- (ii) $\begin{cases} \mathbf{a}_{\mathcal{N}_i}^* = (B, N) \text{ or } (N, B) \Rightarrow a_i^* = N \text{ for low and medium types.} \\ \mathbf{a}_{\mathcal{N}_i}^* = (B, N) \text{ or } (N, B) \Rightarrow a_i^* = B \text{ for high types.} \end{cases}$
- (iii) $\begin{cases} \mathbf{a}_{\mathcal{N}_i}^* = (B, B) \Rightarrow a_i^* = N \text{ for low types.} \\ \mathbf{a}_{\mathcal{N}_i}^* = (B, B) \Rightarrow a_i^* = N \text{ for medium and high types.} \end{cases}$

We can infer from definition 5 that all activity levels are feasible as NE, conditional on price p and realisations of θ . It is also evident that network structure matters more than in the case of a complete network. The consumer's action depends on the fact which types his neighbours happen to be.

As an example of multiplicity of equilibria, consider a sequence of four agents of a circle, and assume that the price is $p \in (2\theta^-, \theta^+)$. Assume that the agents at the ends of the sequence are

high types and they play B in equilibrium, and the middle agents are of medium type. Then, the middle agents can either both play B or N . Both $(..., B, B, B, B, ...)$ and $(..., B, N, N, B, ...)$ constitute NE, but which one occurs is indeterminate.

The firm maximises profits $V = b(p)(p - c)$ with price $p^* \in [2\theta^-, 2\theta^+]$. Function $b(p)$ gives the number of buyers in the maximal equilibrium. It is decreasing in p , with upper bound, $b(2\theta^-) = I$, and lower bound $b(2\theta^+ + \varepsilon) = 0$ (ε small and positive). See example 8.1.2 in the appendix for an example how the monopolist sets the price in a four consumer circle.

3.3 Star

The star formation is asymmetric with a single central agent who is connected to $I - 1$ peripheral agents. The peripheral consumers are connected only to the centre. Centre's utility from buying is $u_C(\theta_C, B) = \sum_{i \in \mathcal{N}_C} e_{Ci}\theta_C - p$, $\mathcal{N}_C = \{\mathcal{G} \setminus C\}$, where the index C stands for "centre". Peripheral consumer's utility is $u_i(\theta_i, B) = e_{iC}\theta_i - p$, for all $i \neq C$. In any non-empty equilibrium the centre buys the device.

Definition 6 *Nash equilibrium is the action profile*

(i) *For centre $C \in \mathcal{G}$:*

$$\begin{aligned} \mathbf{a}_{\mathcal{N}_C}^* &= (N)^{\mathcal{N}_C} \Rightarrow a_C^* = N. \\ \mathbf{a}_{\mathcal{N}_C}^* &= (a_i)^{i \in \mathcal{N}_C}, \text{ and not all } a_i^* = N \Rightarrow a_C^* = N \text{ if } \theta_C < \frac{p}{\sum_{i \in \mathcal{N}_C} e_{Ci}^*}. \\ \mathbf{a}_{\mathcal{N}_C}^* &= (a_i)^{i \in \mathcal{N}_C}, \text{ and not all } a_i^* = N \Rightarrow a_C^* = B \text{ if } \theta_C \geq \frac{p}{\sum_{i \in \mathcal{N}_C} e_{Ci}^*}. \end{aligned}$$

(ii) *For all peripheral agents $i \in \{\mathcal{G} \setminus C\}$:*

$$\begin{aligned} a_C^* &= (N) \Rightarrow a_i^* = N. \\ a_C^* &= (B) \Rightarrow a_i^* = N \text{ if } \theta_i < p. \\ a_C^* &= (B) \Rightarrow a_i^* = B \text{ if } \theta_i \geq p. \end{aligned}$$

The firm has to set the price low enough to attract the central agent and at least one peripheral consumer to buy. Let $b_C(p)$ be centre's quasi-demand, and $b(p)$ the largest number of peripheral agents who buy for a given price p . Centre's quasi-demand is a step-function

$$b_C(p) = \begin{cases} 0, & \text{if } p > \bar{u}_C \\ 1, & \text{if } p \leq \bar{u}_C \end{cases},$$

where $\bar{u}_C = b(p) \theta_C$ is the utility sum from active links. The lower and upper bounds for $b(p)$ are $b(\min\{\theta^+, (I-1)\theta_C\} + \varepsilon) = 0$, and $b(\theta^-) = I-1$, which take into account centre's and periphery's topological differences. Between the limits, the function $b(p)$ is decreasing in p with possible large drops. In order to evade the empty network, the firm must set $b_C(p) = 1$. Hence, the firm's problem is to maximise profits, $V = [1 + b(p)](p - c)$ subject to $p \leq \bar{u}_C$. See example 8.1.3 in the appendix how the monopolist sets the price in a four consumer star.

3.4 Comparison of networks

In this section, we study the differences between complete graph, circle, and star. It is a matter of substance whether we should take the number of consumers or the value generated in the network as the primitive of the model. In most cases, a fixed number of consumers is the appropriate set-up, since it is the consumer who makes the decision. However, the comparison across different network types when the number of consumers is fixed, comprises the size effect (number of links) and the topological effect (link wiring). If we fix the value of the network, we can isolate the topological effect on monopoly's pricing strategy. Due to the overwhelming number of different cases under perfect information, a comparison of compensated networks is insensible to carry out. For example, a complete graph with four consumers corresponds to a compensated circle with six consumers. A circle of six consumers has 720 permutations (of which half are mirror images). Fortunately, it is easy to distinguish between the topological effect and the size effect.

Consider a complete graph, a circle and a star of four agents with valuations $\theta_1 \leq \theta_2 \leq \theta_3 \leq \theta_4$. We assume $c = 0$ for expositional reasons. Table 1 gives firm's profits in the maximal NE for different social networks. Social networks are given in the rows, columns correspond to the technical networks (activity level). Tables 2 and 3 in the appendix present consumer surplus and total surplus (consumer surplus plus profits). The observations from the comparison are summarised in the propositions 7-12. Since they are tendencies derived from a specific case, we do not give any formal proofs of the propositions but discuss each one in the main text.

Table 1: Profits, $\theta_1 \leq \theta_2 \leq \theta_3 \leq \theta_4$

	Complete network	3-buyer network	2-buyer network
Complete graph	$4 (3\theta_1)$	$3 (2\theta_2)$	$2 (\theta_3)$
Circle A	$4 (2\theta_1)$	$3 (\theta_2)$	$2 (\theta_3)$
Circle B	$4 (2\theta_1)$	$3 (\min \{2\theta_2, \theta_3\})$	Dominated
Star, 2 as centre	$4 (\theta_1)$	$3 (\min \{2\theta_2, \theta_3\})$	Dominated
Star, 3 as centre	$4 (\theta_1)$	$3 (\theta_2)$	$2 (\theta_3)$

From the above table we can infer that alternative social structures support different optimal monopoly prices. Optimal monopoly price is affected by the number of links, the topology of the social network, and agent configuration (which types are connected, e.g. circle A and B have the

same topology, but are different configurations).

Proposition 7 *Monopoly price is (weakly) increasing in the number of links in the social network.*

When a link is added to the network, there are not consumers whose utility would be negatively affected by the addition prior any price modifications. The addition may increase utility of some consumers, which is the reason why the firm can potentially increase its price. A price increase is feasible if the added link is not redundant so that the link effectively eases pricing constraint. This result is universal.

Proposition 8 *(i) Complete graph generates the highest total surplus. (ii) Consumer surplus and total surplus are maximised in a complete network in all social networks.*

The more links there are in the network, the higher is the generated value in the network. Part (ii) of proposition 8 is an implication of supermodularity. Since profits are just transfers from consumer surplus, total surplus is maximised when the maximal number of links is activated. Consumer surplus is maximised in the complete network because the price is the lowest in the complete network.

Propositions 7 and 8 comprise the size effect. A more complex issue is how the network topology and agent configuration affect the price level. Propositions 9-12 summarise these effects.

Proposition 9 *Agent configuration is irrelevant in pricing if the social network is a complete graph. In other social network topologies, configuration matters.*

Compare the circles A and B, and assume that a 3-buyer network maximises profits. The networks differ only in the way who is connected to whom. Still, the monopolist makes higher profits in B. Consumer θ_2 benefits from the links with high types θ_3 and θ_4 , and the firm is able to capture some (or all) of this rent. In the circle A, the fact that low types are neighbours leads to lower profits.

The role of social relations becomes more drastic when we compare 2-buyer networks. We see that in the circle B and the star with θ_2 at the centre, 2-buyer networks are always dominated (in terms of profits) by 3-buyer networks. On the contrary, in the circle A and the star with θ_3 at the

centre, they do not have to be because high types are clustered. If a low valuation consumer is in a focal position (θ_2 in the circle B or in star's centre), the firm may be forced to sell at a lower price in order to guarantee his participation.

There are two types of critical consumers who have connections that are important from all network members' perspective. One type are focal topology-wise, e.g. the centre in a star. The second, more subtle, type is focalised by high heterogeneity between the critical consumer's valuation and his neighbours' valuations. Consumer θ_2 in the circle B is an example of this type. His position constrains pricing only if his type is sufficiently low ($2\theta_2 < \theta_3$), otherwise the important connections with high types θ_3 and θ_4 are redundant in the sense that his participation is guaranteed. If $\theta_3 < 2\theta_2$, price is not constrained due to him, and his position is actually beneficial to the firm.

Proposition 10 *Critical agents have (i) topologically central positions (e.g. centre in star) (ii) connections with important network members (low types with high type neighbours). Critical agents can increase profits or constrain the optimal price depending on the network topology and agent configuration.*

Under perfect information, the optimal monopoly price is determined by the combination of consumer heterogeneity and social network structure. Consider the circles A and B again. Let the complete network be optimal in A. This means that $8\theta_1 > 3\theta_2$ and $8\theta_1 > 2\theta_3$. Now, if we also have $8\theta_1 < 6\theta_2$ and $2\theta_2 > \theta_3$ it is optimal for the firm to choose the 3-buyer network in B. Why? The firm finds it profitable to increase the price so that θ_1 opts out. At the same time, the high types θ_3 and θ_4 induce their common neighbour θ_2 to purchase. Hence, in some graphs the monopolist limits supply whereas in other graphs that are identical save the configuration of agents, it covers the whole market even if it cannot price discriminate. Full coverage is more likely when consumers' relative valuations are close together. In more heterogeneous markets, the monopolist is better off by excluding the lowest types from the market by setting a sufficiently high price. This trade-off corresponds to the textbook case of monopoly mark-up.

Proposition 11 *The firm excludes low types from the service in (relatively) heterogeneous markets. Homogeneous markets are completely covered.*

Proposition 11 leads us to a more interesting result. When consumers are homogeneous, the firm prefers to have high types dispersed in the network. Dispersed high types support the purchases of lower types. On the other hand, if the valuations are highly heterogeneous, so that the firm prefers to exclude low types from the service, dispersion of high types hurts the firm. It would be better for the firm that high types are neighbours.

Proposition 12 *In homogeneous markets, the dispersion of high types is good for the firm. In heterogeneous markets, the dispersion of high types constrains the firm.*

Figure (1) illustrates how monopoly's choice affects the total surplus created in the network. We have used valuations $\theta_1 = 1$, $\theta_2 = 2$, $\theta_3 = 3$, and $\theta_4 = 4$ to illustrate. The activity level that maximises profits is reported on the top line. The bottom line identifies the underlying social network. Total surplus is optimal profits plus consumer surplus. The maximum value equals total surplus in the complete network. The network size effect is clearly visible, as maximal value increases in the number of links. Topological effects come through in two ways. First, the star with consumer θ_3 at the centre generates higher maximum value than star with consumer θ_2 at the centre. What drives the difference is supermodularity of the payoff function. Topological effects show up also in the total surplus. The total surplus in the circle B is only 55% of the total surplus in the circle A. Respectively, total surplus in the star 2 is 92% of total surplus in the star 3. In the circle B and the star 2, total surplus is only 37% of total surplus in a complete graph.

The role of critical agents is evident when comparing surpluses of the circles A and B. In A, the firm chooses complete network, which maximises total surplus, but in B, the firm excludes θ_1 from the network. This happens because the critical consumer θ_2 is the common neighbour to high types θ_3 and θ_4 in B. The high types θ_3 and θ_4 require only one neighbour who buys. Consumer θ_2 is of medium type, who needs both neighbours to buy before he buys. As a result, θ_1 is rendered redundant in the circle B and it pays off to exclude him. The drop in the total surplus is due to reduction in consumer surplus. In the circle B, the firm makes only 13% higher profits compared with the circle A, but consumer surplus is reduced by 83%.

Figure 1: Total surplus and maximal generated value.

A change in the opposite direction can be observed in the comparison between star 2 and star 3. There is a slight increase in total surplus in moving from star 2 to star 3. As the consumer θ_2 loses his preferential position as the centre, the firm lowers its price (from 3 to 2) in order to include θ_2 in the network. The increase in consumer surplus offsets the decrease in profits.

Next, let us increase the valuation θ_3 from 3 to 3.5, while maintaining everything else. What this apparently positive change does, is that it increases the maximal value in all networks. Total surplus is increased in all networks except in the star with θ_3 as the centre. The total surplus in star 3 is significantly lowered (by 38%). Why? In the new situation the top two consumers have valuations sufficiently higher than the two bottom ones. The new optimal network structure for the firm is a 2-buyer network in star 3 (when with $\theta_3 = 3$ it was a 3-buyer network). Exclusion of both θ_1 and θ_2 increases firm's profits by 17%. At the same time, consumer surplus is reduced by 92%, which dominates the increase in profits. Due to θ_2 's critical position, he is not excluded in the star 2 or the circle B. The complete graph and the circle A remain fully covered.

The comparison of profits and consumer surplus has illustrated how they crucially depend on

Figure 2: Insiders - Outsider

the underlying social relations. The comparison has revealed how the strength of network externalities can be overestimated. An assumption on a complete graph as the prevailing social structure, when the true social structure is something less connected, produces significantly exaggerated estimates for consumer surplus and monopoly rents.

We close the analysis on perfect information with a counter-example which illustrates the complexity perfect information creates. Consider a modified star network: "insiders-outsider" illustrated in figure (2) with valuations $\theta_1 < \theta_2 < \theta_3 < \theta_4$. The consumer θ_1 has obviously a preferential position. Let the firm prefer a 2-buyer network over a 3-buyer network, i.e. $V_2 = 2(\theta_2) > V_3 = 3(2\theta_1)$. But, if the outsider θ_4 has a very high valuation $3\theta_1 < \theta_4$, the firm may prefer the complete network over the 2-buyer network, even if buyers' valuations are very heterogeneous. Let $3\theta_1 < \theta_2 < 6\theta_1 \Rightarrow V_4 = 4(3\theta_1) > V_2 = 2(\theta_2) > V_3 = 3(2\theta_1)$. When this holds, types θ_3 and θ_4 can differ significantly from θ_1 and θ_2 (high heterogeneity), and the firm still covers the whole market. Why is this possible? It is possible because of two factors. One, θ_3 and θ_4 are not neighbours, so the firm cannot sell only to them. Two, θ_1 has many links which compensate his low valuation.

4 Asymmetric information

In this section, we limit the players' ability to observe their opponents' valuations. The valuations θ are now pure private information. Because θ 's are i.i.d., the buyers are ex ante symmetric but ex post heterogeneous. The social network structure $\mathcal{G}$ and distribution $F(\theta)$ remain common

knowledge.

Write π_{ij} as the probability consumer i puts on the event that his neighbour j buys the device. The expected payoff from the link between i and j is independent from any other link i has. Consequently, the expected payoff from link $\{i, j\}$ to i is just $\pi_{ij}\theta_i$, and $\pi_{ji}\theta_j$ to his neighbour j . The consumer i 's expected utility from $a_i = B$ is the sum over all his links

$$\mathbb{E}[u_i(\theta_i, B)] = \sum_{j \in \mathcal{N}_i} \pi_{ij}\theta_i - p. \quad (3)$$

If the expected payoff from buying the product exceeds the reservation value of zero, the agent makes the purchase. Buyers obviously benefit from lower uncertainty over the purchasing decisions of their neighbours, $\frac{\partial \mathbb{E}[u_i(\theta_i, B)]}{\partial \pi_{ij}} > 0 \forall i, j \in \mathcal{N}_i$. At the margin, the consumer's valuation is

$$\tilde{\theta}_i(\pi_{\mathcal{N}_i}) = \frac{p}{\sum_{j \in \mathcal{N}_i} \pi_{ij}}. \quad (4)$$

Pure strategy for consumer i is $a_i : [\theta^-, \theta^+] \rightarrow \{B, N\}$, and his best response is the switching strategy $a_i^* = B$, if $\theta_i \geq \tilde{\theta}_i(\pi_{\mathcal{N}_i})$ and $a_i^* = N$, if $\theta_i < \tilde{\theta}_i(\pi_{\mathcal{N}_i})$. The probability that consumer i buys, given his beliefs over his neighbours' actions π_{ij} and price p , is

$$\pi_i = 1 - F\left(\min\left\{\theta^+, \tilde{\theta}_i(\pi_{\mathcal{N}_i})\right\}\right).$$

The coordination game with asymmetric information Γ_{AI} consists of consumers $\mathcal{I}$ arranged on graph $\mathcal{G}$, pure actions $a = \{B, N\}$, types $(\theta_i)_{i=1}^I$ with prior distribution $F(\theta)$, and payoffs $u_i(N) = 0$ and $\mathbb{E}[u_i(B)]$ given by equation (3). The game Γ is parameterised by price p . Bayesian Nash equilibrium (BNE) of Γ_{AI} is characterised in definition 13.

Definition 13 *Bayesian Nash equilibrium of the asymmetric information game Γ_{AI} is the pure strategy profile*

$$\begin{aligned} a_i^* &= B \Leftrightarrow \theta_i \geq \tilde{\theta}_i(\pi_{\mathcal{N}_i}^*) \\ a_i^* &= N \Leftrightarrow \theta_i < \tilde{\theta}_i(\pi_{\mathcal{N}_i}^*) \end{aligned},$$

where $\tilde{\theta}_i(\pi_{\mathcal{N}_i}^*) = \frac{p}{\sum_{j \in \mathcal{N}_i} \pi_{ij}^*}$ and $\pi_i^* = 1 - F\left(\min\left\{\theta^+, \tilde{\theta}_i(\pi_{\mathcal{N}_i}^*)\right\}\right)$ for all $i \in \mathcal{G}$ and θ_i .

Supermodularity carries over to the asymmetric information regime.

Lemma 14 *The coordination game with asymmetric information Γ_{AI} is supermodular with positive spillovers.*

Proof.

- (i) The set $\pi_i \in [0, 1]$ is a compact subset of $\mathbb{R}$.
- (ii) The payoffs exhibit increasing differences. Write the expected utility of action $a = B$ versus $a = N$ as $\mathbb{E}[v_i(\theta_i, \pi_{ij})] = \mathbb{E}[u_i(\theta_i, B)] - \mathbb{E}[u_i(\theta_i, N)] = \mathbb{E}[u_i(\theta_i, B)]$ for all $i \in \mathcal{G}$ and $j \in \mathcal{N}_i$, where $\mathbb{E}[u_i(\theta_i, B)]$ is given by equation (3). We have $\mathbb{E}[v_i(\theta'_i, \pi_{ij})] \geq \mathbb{E}[v_i(\theta_i, \pi_{ij})]$ for all $\theta'_i > \theta_i$.
- (iii) The payoff function $\mathbb{E}(u_i) : \{B, N\} \times \theta \rightarrow \mathbb{R}$ is continuous.

We conclude from (i) - (iii) that the game Γ_{AI} is supermodular. Positive spillovers arise because the payoff gain is strictly increasing in neighbours' strategies, $\frac{\partial \mathbb{E}[v_i(\theta_i, \pi_{ij})]}{\partial \pi_{ij}} > 0$ for all $j \in \mathcal{N}_i$. ■

The implications of supermodularity are familiar. It guarantees that there exists a largest and a smallest equilibrium element. The smallest BNE is the empty network where $\pi_i^* = 0$ for all $i \in \mathcal{G}$. On the other hand, the structure of any non-empty BNE depends on the price. Positive spillovers mean that the largest BNE is Pareto-dominating, which, we argue, focalises the efficient equilibrium.

In contrast to the perfect information case, there may be agents whose ex post utility is negative. Ex post, these agents would choose to play N as well. Similarly, all agents for whom $\frac{p}{\sum_{j \in \mathcal{N}_i} \pi_{ij}^*} > \theta_i > \frac{p}{\sum_{j \in \mathcal{N}_i} |e_{ij}|}$, where $|e_{ij}|$ gives the ex post number of neighbours who bought the device, holds and who played N , would like to have played B . These ex post inefficiencies easily arise with uncertainty over neighbours' valuations, whereas in the case of perfect information, ex post inefficiencies can arise only due to systematic irrational behaviour.

The firm maximises expected profits $\mathbb{E}(V) = \sum_{i \in \mathcal{G}} \pi_i^* [p(\pi^*) - c]$. The firm cannot choose the activity level directly, as it could with perfect information. Instead, we let the firm maximise profits by choosing quantity, i.e. the probability π_i^* . The inverse demand $p(\pi^*)$ is derived from the BNE of the coordination game Γ_{AI} .

We have now characterised the asymmetric information case in general terms. Next we study the equilibria and pricing choices in a complete graph, a circle, and a star.

4.1 Symmetric networks

Asymmetric information makes all symmetric networks (where each consumer has equal number of neighbours) analytically the same. It only needs to understand that a symmetric social network coupled with private information on θ makes all agents ex ante homogeneous.

Lemma 15 *With asymmetric information, each consumer $i \in \mathcal{G}$ has an identical probability to buy in a symmetric network.*

Proof. Let $n \in [1, I - 1]$ be the number of neighbours for consumer i in a population of I that is arranged on a symmetric graph $\mathcal{G}^{sym}$. By symmetry n is the number of neighbours for all consumers. Assume first that the probabilities are different so that for all other consumers except i , the probability to buy is π and for i it is $\pi_i < \pi$. We can write consumer i 's expected utility from $a_i = B$ as

$$\begin{aligned}\mathbb{E}[u_i(\theta_i, B)] &= \sum_{k=0}^n \binom{n}{k} \pi^k (1 - \pi)^{n-k} k \theta_i - p \\ &= \pi n \theta_i - p.\end{aligned}$$

Similarly, the expected payoff for consumer $j \neq i$ is

$$\begin{aligned}\mathbb{E}[u_j(\theta_j, B)] &= \sum_{k=0}^{n-1} \binom{n-1}{k} \pi^k (1 - \pi)^{(n-1)-k} k \theta_j + \pi_i \theta_j - p \\ &= [(n-1)\pi + \pi_i] \theta_j - p.\end{aligned}$$

The equilibrium condition that consumer i buys is $z_i(B) = 1 - F\left(\min\left\{\theta^+, \frac{p}{n\pi}\right\}\right)$, and for all other consumers except i it is $z_{-i}(B) = 1 - F\left(\min\left\{\theta^+, \frac{p}{(n-1)\pi + \pi_i}\right\}\right)$. The functions $z_i(\pi)$ and $z_{-i}(\pi_i, \pi)$ are increasing in π and in (π, π_i) respectively. If $\pi_i < \pi$ holds, then it must also be that $z_i(\pi) > z_{-i}(\pi_i, \pi)$ which leads to a contradiction with the initial assumption. The case $\pi_i > \pi$ leads to a corresponding contradiction. Hence, in the equilibrium it must be that $\pi_i = \pi$ for all $i \in \mathcal{G}^{sym}$. ■

Both the complete graph and the circle are symmetric networks. We work through a generalised version of symmetric networks where all agents have n neighbours. For the complete graph $n = I - 1$ and for the circle $n = 2$. Note that some configurations are impossible. For example, it is impossible to construct a symmetric network with five agents each having three neighbours. The generalised version does apply to complete graphs and circles of any number of consumers though.

The expected payoff from $a_i = B$ can be written as $\mathbb{E}[u_i(\theta_i, B)] = n\pi\theta_i - p$. The common system of probabilities satisfies

$$\pi = 1 - F\left(\min\left\{\theta^+, \frac{p}{n\pi}\right\}\right). \quad (5)$$

Introduction of incomplete information in the model has reduced the number of equilibria. However, the coordination problem is not entirely solved. There can be maximum of three different equilibria. Firstly, the empty network is BNE. To see that the empty network is a BNE, substitute $\pi = 0$ in equation (5). It is seen directly that all agents play $a = N$ with probability one. In addition, there can be at most two positive equilibria. In the interval $\pi \in]0, \frac{p}{n\theta^+}]$, there are no equilibrium values. To check the existence of positive equilibria, we solve the equation $\pi = 1 - F\left(\frac{p}{n\pi}\right)$ for π . Real roots exist when $(\theta^+ n)^2 - 4(\theta^+ - \theta^-)np \geq 0$. In the cases where there are two positive equilibria, the higher value is the maximal BNE. Lower equilibria are associated with coordination failure. Supermodularity with positive spillovers guarantees that the maximal BNE Pareto dominates.

The equilibrium condition (5) gives the inverse quasi-demand $p(\pi)$. In the area where $\theta^+ \leq \frac{p}{n\pi}$ price is indeterminate, and the probability to buy is zero $\pi = 0$. The firm operates in the region where price is determinate. The inverse demand is

$$p = n\pi [\theta^+ - (\theta^+ - \theta^-)\pi].$$

The firm maximises profits by choosing the optimal level of π . Firm's expected profits are $\mathbb{E}(V) = I\pi [p(\pi) - c]$. The first order condition gives the standard monopoly mark-up rule

$$\frac{p(\pi^*) - c}{p(\pi^*)} = \frac{1}{\eta}, \quad (6)$$

where π^* is the optimal value and $\eta = -\frac{\partial \pi^*}{\partial p} \frac{p(\pi^*)}{\pi^*}$ the elasticity of the quasi-demand.

Consider the special case of zero unit costs, $c = 0$. Equation (6) gives $\pi^* = \frac{2\theta^+}{3(\theta^+ - \theta^-)}$, and $p(\pi^*) = \frac{2(\theta^+)^2}{9(\theta^+ - \theta^-)}n$, which satisfy second order conditions⁹. The derived values represent the desired equilibrium for the firm. When the obtained equilibrium price $p(\pi^*)$ is plugged back into equation (5), we can solve again for the corresponding equilibrium probabilities. As suggested,

⁹ $\frac{\partial^2 \mathbb{E}(V)}{\partial \pi^2} \Big|_{\pi = \frac{2\theta^+}{3(\theta^+ - \theta^-)}} = -2\theta^+ In < 0$.

there exists two positive equilibria

$$\pi = \frac{\theta^+ \pm \frac{1}{3}\theta^+}{2(\theta^+ - \theta^-)}.$$

Denote the larger value, associated with the maximal BNE, as π_+^* . Firm's expected profits are in that case

$$\mathbb{E}(V_+^*) = \frac{4}{27} \left(\frac{\theta^+}{\theta^+ - \theta^-} \right)^2 \theta^+ I n.$$

The difference in realised profits between the maximal BNE and the lower (positive) equilibrium is $(\pi_+^* - \pi_-^*) p(\pi^*) = \frac{1}{2} \mathbb{E}(V_+^*)$. Empty network yields zero profits of course.

The maximal and empty network are Cournot tâtonnement stable BNE, whereas the lower positive equilibrium is an unstable one. The checks for stability are provided in the appendix. Because the low equilibrium is an unstable one, convergence occurs towards zero or the maximal BNE, unless the tâtonnement process begins exactly at the lower equilibrium.

Expected total consumer surplus in the maximal BNE is given by

$$\begin{aligned} \mathbb{E}(CS) &= I \int_{\bar{\theta}}^{\theta^+} f(\theta) [n\pi_+^* \theta - p^*] d\theta \\ &= \frac{4}{27} \left(\frac{\theta^+}{\theta^+ - \theta^-} \right)^2 \theta^+ I n \end{aligned}$$

Expected consumer surplus equals profits, $\mathbb{E}(CS) = \mathbb{E}(V_+^*)$.

We are ready to compare the asymmetric information model (with $c = 0$) with the results from the perfect information case (propositions 7-12). We see from $p(\pi^*) = \frac{2(\theta^+)^2}{9(\theta^+ - \theta^-)} n$ that the monopoly price is increasing in the number of links (agrees with proposition 7). For a given number of consumers, the complete graph supports the highest price. However, symmetry has removed the preferential roles that existed under perfect information. Agent configuration is irrelevant since consumers are ex ante homogeneous (disagrees with propositions 9 and 10(ii)). In the circle B, the consumer θ_2 was in a critical position under perfect information. The firm had to guarantee his participation in order to evade the empty network. How the highest types are positioned in the network does not affect profits under asymmetric information (disagrees with propositions 11 and 12).

Proposition 16 *Monopoly price increases as the number of neighbours increases. Agent configuration does not affect monopoly price under asymmetric information.*

It is obvious now that the complete graph corresponds to the conventional network externalities model where the underlying social structure is abstracted away. When we take the probability π as the fraction of the total population who buy, we arrive at a basic membership externality model where the agent's utility increases as the number of people join the network.

Complete graph generates highest total surplus ($\mathbb{E}(V_+^*) + \mathbb{E}(CS)$), which agrees with results from the perfect information case (proposition 8). Both equilibrium profits and expected consumer surplus increase also as n is increased. This brings up the problem of overestimation of network externalities. If we use the complete graph, when the true social network is something less connected, we end up exaggerating the surplus generated in the network.

To see what the impact of heterogeneity is, we apply a mean-preserving spread $[\theta^- - x, \theta^+ + x]$ on the uniform distribution of types $F(\theta)$. We can write the expected consumer surplus, which equals maximal BNE profits, as

$$\mathbb{E}(CS) = \frac{4}{27} \left(\frac{\theta^+ + x}{(\theta^+ + x) - (\theta^- - x)} \right)^2 (\theta^+ + x) In. \quad (7)$$

To see the effect of increased heterogeneity, differentiate (7) with respect to x . We get that marginal increase in heterogeneity hurts both the consumers and the firm. The reason is that increased heterogeneity increases uncertainty in the model. Consumers' valuations are pure private information, so the uncertainty over neighbours increases. The firm, on the other hand, cannot distinguish between networks where high types are neighbours and networks where high types are dispersed. Hence, it is incapable of taking advantage of clusters of high types, as it could with perfect information (disagrees with propositions 11 and 12).

Increased heterogeneity leads the monopoly to reduce its price in general, $\frac{\partial p^*}{\partial x} = -\frac{4(\theta^+ + x)(\theta^- - x)}{9(\theta^+ - \theta^- + 2x)^2} < 0$ which is negative when $\theta^- > x > 0$, but positive with $\theta^- = 0$. Increased uncertainty reduces the probability to buy $\frac{\partial \pi_+^*}{\partial x} = -\frac{2(\theta^+ + \theta^-)}{3(\theta^+ - \theta^- + 2x)^2} < 0$.

Proposition 17 *Surplus effects:*

- (i) *Complete graph supports the highest expected consumer surplus and maximal BNE profits.*
- (ii) *Increased heterogeneity decreases expected consumer surplus and maximal BNE profits.*

Proof.

- (i) *Both $\mathbb{E}(CS)$ and V_+^* are strictly increasing in n . So the maximum is reached at complete graph $n = I - 1$.*
- (ii) $\frac{\partial \mathbb{E}(CS)}{\partial x} = \frac{\partial \mathbb{E}(V_+^*)}{\partial x} = \frac{4}{27} In \frac{(\theta^+ + x)^2}{(\theta^+ - \theta^- + 2x)^3} [-(\theta^+ + x) - 3(\theta^- - x)]$, *which is negative when $\theta^- \geq 0$ and x is small.*

■

The asymmetric information case is analytically easier to handle than perfect information case, because agent configuration plays no role. Some of the predictions of the perfect information case hold, but some are invalidated. The asymmetric information case is more suitable for large social networks, where each agent has many connections. The adverse possibility to overestimate generated surplus is more serious in larger networks however.

4.2 Star

For the star, we obtain an equilibrium system that comprises two distinct probabilities for buying. One is for the centre and the other for peripheral agents. The firm has to choose a price that applies to all consumers. This assumption creates a price bias in favour of the centre. We allow price discrimination in section 4.3.

Consumers' utilities are $\mathbb{E}[u_C(\theta_C, B)] = \sum_{j \in \mathcal{N}_C} \pi_{Cj} \theta_C - p$ for the centre, and $\mathbb{E}[u_i(\theta_i, B)] = \pi_{iC} \theta_i - p$ for peripheral agent $i \in \{\mathcal{G} \setminus C\}$. Since peripheral consumers are a priori symmetric, by lemma 15, their behaviour is characterised by a common probability. The centre places probability $\pi_{Ci} = \pi$ on the event that a peripheral consumer $i \in \{\mathcal{G} \setminus C\}$ buys. Each peripheral consumer $i \in \{\mathcal{G} \setminus C\}$ places probability $\pi_{iC} = \pi_C$ that the centre buys.

Lemma 18 *BNE in a star is characterised by (π_C, π) , where π_C is the probability that the centre buys and π is the probability that a peripheral agent buys. The equilibrium satisfies*

$$\begin{aligned} \pi_C &= 1 - F\left(\min\left\{\theta^+, \frac{p}{(I-1)\pi}\right\}\right) \\ \pi &= 1 - F\left(\min\left\{\theta^+, \frac{p}{\pi_C}\right\}\right) \end{aligned} \tag{8}$$

Proof. Proof is obtained directly from lemma 15 and uses the symmetry property. By symmetry, the centre must apply identical probability π to all peripheral agents, and all peripheral agents hold identical beliefs over the centre's behaviour π_C when information is asymmetric. ■

The firm solves the profits maximisation problem by choosing the probability π . As in symmetric networks, asymmetric information eliminates the role of agent (type) configuration in pricing. From system (8) we get the market clearing price and the centre's probability to buy as a function of π

$$\begin{aligned} p(\pi) &= \pi_C(\pi) [\theta^+ - (\theta^+ - \theta^-) \pi] \\ \pi_C(\pi) &= \frac{\theta^+ (I-1) \pi}{\theta^+ + (I-2) (\theta^+ - \theta^-) \pi} \end{aligned}$$

Periphery's and centre's strategies are complements in the sense $\frac{\partial \pi_C}{\partial \pi} > 0$. The difference between probabilities is $\pi_C - \pi = \frac{(\theta^+ - (\theta^+ - \theta^-) \pi) \pi}{\theta^+ + (\theta^+ - \theta^-) \pi}$ which is always non-negative, saying that the probability that the centre buys is higher than the probability that a peripheral agent buys.

The firm maximises expected profits $\mathbb{E}(V) = [\pi_C(\pi) + (I-1) \pi] [p(\pi) - c]$. The FOC gives a modified inverse elasticity rule

$$\frac{p(\pi^*) - c}{p(\pi^*)} = \frac{1}{\eta} \left\{ \frac{[2\theta^+ + (I-2)(\theta^+ - \theta^-) \pi^*] [\theta^+ + (I-2)(\theta^+ - \theta^-) \pi^*]}{(\theta^+)^2 + [\theta^+ + (I-2)(\theta^+ - \theta^-) \pi^*]^2} \right\}, \quad (9)$$

where $\eta = -\frac{\partial \pi^*}{\partial p} \frac{p(\pi^*)}{\pi^*}$ is the price elasticity of the quasi-demand of a peripheral agent.

Because the result (9) is difficult to use analytically, let us consider the specific case with zero unit costs and a uniform distribution $\theta \sim Unif[0, 1]$ with a non-degenerate star ($I \geq 3$). In this case, there is only one real root to the equation (9) in the range $\pi \in (0, 1)$, which yields positive profits¹⁰, and the corners $\pi = \{0, 1\}$ yield zero profits. Hence, the only real root in the range $\pi \in (0, 1)$ is in fact the global maximum. Because the derivative $\frac{\partial \mathbb{E}(V)}{\partial \pi}$ at point $\pi = \frac{2}{3}$ ($\pi = \frac{1}{3}$) is positive (negative), the optimal π must be in range $\frac{1}{3} < \pi^* < \frac{2}{3}$. So, the probability to buy for a peripheral consumer is less than the probability to buy in symmetric graphs. The mark-up

¹⁰ Second order conditions for maximal profits are satisfied for the non-zero equilibrium. This can be checked numerically for the particular case $c = 0$, $\theta^+ = 1$, $\theta^- = 0$. We have $\frac{\partial^2 \mathbb{E}(V)}{\partial \pi^2} < 0$ for $I \geq 3$. A stability check for the equilibrium is in the appendix.

associated with the periphery is therefore higher than the standard monopoly mark-up associated with symmetric graphs.

Proposition 19 *A consumer in the periphery has a lower probability to buy, and the centre has a higher probability to buy, compared with a consumer in a symmetric network.*

Proposition 19 states that the topological effect on the monopoly price is never latent under asymmetric information. The firm's pricing strategy resembles those cases of perfect information where the centre is a binding constraint to pricing (because the centre has sufficiently low valuation). In the case of asymmetric information, the firm always takes into account the topologically focal centre by guaranteeing him a higher probability to buy.

Next it can be verified that the optimal π^* is decreasing in I , whereas the optimal $\pi_C^* = \pi_C(\pi^*)$ is growing in I . The centre benefits the more people join his neighbourhood. More interestingly, a peripheral consumer is negatively affected by an additional peripheral consumer, even though the additional agent does not affect his neighbourhood directly. Why? The centre's probability to buy increases as a peripheral agent is added. The firm can compensate this addition by increasing the price. Numerical runs show that as I grows very large, the optimal π^* approaches $\frac{1}{2}$, and the optimal π_C^* approaches $\frac{I-1}{I} \approx 1$. In the minimal case where $I = 3$, the optimal values are $\pi^* \approx 0.5971$ and $\pi_C^* \approx 0.7478$. The larger the periphery is, the higher is the centre's market power thus larger the surplus he captures. This happens whether the centre is actually a pricing constraint in the respective perfect information game or not. The monopoly price is the lowest at $I = 3$, where it equals $p(\pi^*) \approx 0.3012$. As the periphery becomes very large, the optimal price approaches $\frac{1}{2}$. Profits and the total expected consumer surplus (centre's surplus plus periphery's surplus) increase in the number of peripheral consumers.

Proposition 20 *The effect of changes in the size of periphery:*

- (i) *The centre benefits the larger the periphery is.*
- (ii) *A peripheral consumer is adversely affected by an addition of a new peripheral consumer.*
- (iii) *Profits maximising price increases as the number of peripheral agents increases, and the firm's profits increase. Total consumer surplus increases as the periphery grows.*

Figure 3: Uncompensated total surplus (log scale), $\theta \sim Unif[0, 1]$, $c = 0$.

Proposition 20 agrees with the size effects of perfect information regime, as well as, with the symmetric networks case under asymmetric information.

Consider a spread $\theta \sim Unif[-x, 1+x]$, $x > 0$ and small. Although we now give the lowest type a negative valuation, it does not affect results as long as the spread we consider is small. The lowest type already had a dominant strategy $a = N$ for any given positive price p . A numerical run shows that the firm increases the optimal price for a small x . However, the increase in uncertainty has a negative effect on profits and consumer surplus.

Proposition 21 *Small increase in uncertainty decreases equilibrium profits, and total consumer surplus associated with the periphery and the centre.*

4.3 Comparison and price discrimination

We close the analysis of the asymmetric information variant with a comparison of the symmetric networks and the star. We ignore the integer problem in order to get results easily illustrated, thus $I \geq 3$ and continuous. We first confirm the results about the size factor.

Figure (3) illustrates how the complete graph (dotted line) generates far higher total surplus (profits plus consumer surplus) than the *uncompensated* circle (dashed line) or star (solid line)

do. This is because each additional consumer induces $2(I - 1)$ new links in the complete graph whereas only two in the circle and the star.

For small numbers of consumers, the circle produces higher total surplus compared to the star, but for large networks, the star generates higher total surplus. The solid line crosses the dashed line just before the number of consumer reaches $I = 30$. The circle always has 2 links (one two-directional link) more than the star, which returns higher consumer surplus for small networks. The star, however, supports inherently lower price than symmetric networks, thus for large networks consumer surplus is higher in the star. Since the firm maintains its strategy constant with respect to the number of consumers in the circle, but adjusts its price in the star as the number of consumers is increased, the relation between the two surpluses changes. In small star networks, the firm is more pressed to set a low price in order to attract the centre. As the periphery grows in number, the firm increases its price as it compensates (negatively) for larger periphery.

Because the firm maintains a lower price in the star than in a symmetric network, and because there are less links in the star network, firm's profits are the lowest in the star for a given number of consumers.

We can isolate the topological effect by comparing *compensated* networks. The comparison of compensated networks shows how the monopoly price changes when network topology changes, while the maximal value of the network is kept constant. Let us fix the maximal value generated in the complete graph of size I . A compensated circle has $I_C = \frac{I(I-1)}{2}$ consumers and a compensated star $I_s = \frac{I(I-1)+2}{2}$ consumers.

From picture (4) we can read that the firm is the worst off in the star network of compensated size. The asymmetric network structure constrains the firm. As a result, consumer surplus is higher in the compensated star than in the circle or the complete graph. Total surplus is higher in the compensated star network due to higher consumer surplus. This is seen in figure (5).

In the star, the centre's market power reduces firm's profits. When price discrimination is

Figure 4: Compensated profits, $\theta \sim Unif[0, 1]$, $c = 0$.

Figure 5: Compensated total surplus, $\theta \sim Unif[0, 1]$, $c = 0$.

forbidden, the firm always sets a price such that the centre is more likely to purchase than a peripheral agent. Hence, there arises the question, if the firm could benefit from price discriminating with respect to the network position. Under perfect information, both symmetric and asymmetric networks hold incentives to price discriminate. Basically, the firm could capture all surplus from each consumer. When we introduce asymmetric information, all heterogeneity that prevails a priori in the symmetric networks is eliminated, so that there are no means to price discriminate. In asymmetric networks, network location-based price discrimination is feasible.

With price discrimination, the equilibrium probability system in the star is

$$\begin{aligned}\pi_C &= 1 - F\left(\min\left\{\theta^+, \frac{p_C}{(I-1)\pi}\right\}\right) \\ \pi &= 1 - F\left(\min\left\{\theta^+, \frac{p}{\pi_C}\right\}\right)\end{aligned}.$$

Prices p_C for the centre and p for the periphery are determined separately. The firm maximises expected profits $\mathbb{E}(V) = \pi_C [p_C (\pi_C, \pi) - c] + (I-1) \pi [p (\pi_C, \pi) - c]$ by choosing (π^*, π_C^*) .

Let unit costs be zero, $c = 0$. Solving the profits maximisation problem gives the optimal probabilities

$$\pi_C^* = \pi^* = \frac{2\theta^+}{3(\theta^+ - \theta^-)}.$$

Proposition 22 *Price discrimination removes the bias that was in favour of the centre.*

By price discriminating, the firm of course captures a larger share of maximal value generated in the network. If we consider the case $\theta \sim \text{Unif}[0, 1]$, $c = 0$, and compensated networks. It is straightforward to calculate that price discrimination increases the firm's profits to the same level as in the compensated symmetric networks. Respectively, total consumer surplus falls to the level of symmetric networks.

Two important insights can be drawn from this section. One, the complete graph generates the highest surplus for a given number of consumers. This means that use of a complete graph as a mapping of social relations calls for caution. There is a possibility to overestimate the value of the network, when the true network is something less-connected. When the size of the population

is large, the estimate of the value can be seriously exaggerated. Two, network topology matters for price strategy, and consequently for how the surplus is split between players. Considering the compensated networks, the social optimum is the star network. The monopoly power of the firm is reduced due to the asymmetric network topology. The centre captures a large part of the rents, which happens at periphery's cost. The monopoly, on the other hand, prefers a symmetric network. If the firm is able to price discriminate, it can increase its profits, and in the special case $\theta \sim Unif[0, 1]$, $c = 0$, the firm reaches the same profits level as in the symmetric networks. However, price discrimination leads to an efficiency loss as the consumer surplus is reduced more than the firm gains.

5 Extensions

We conclude the analysis with few extrapolations on the basic model.

5.1 Investments in entry under asymmetric information

Symmetric networks produce multiple equilibria under asymmetric information. If the firm was certain that the maximal equilibrium is the correct one, it would be willing to invest up to $\mathbb{E}(V_+^*)$ to enter the market. More generally, we could assume that the firm holds beliefs $\beta_k \in [0, 1]$ on the possible equilibrium k , with $\sum_k \beta_k = 1$. Maximum acceptable sunk cost to enter the market is then $\kappa = \sum_k \beta_k \mathbb{E}(V_k^*)$, where $\mathbb{E}(V_k^*)$ are the expected profits from equilibrium k .

In growth industries, in the early development stages, forecasting future states of the world involve highly qualitative and subjective metrics that make it difficult to estimate β_k . Therefore, it should not come as a surprise that many (most) of the dotcoms that founded their business models on increasing returns found themselves insolvent in a short time. Secondly, dotcoms' business models were often "eyeball game" strategies where network externalities were assumed to generate demand automatically. Such business models did not take into account consumers' local and asymmetric social relations, which, as we have shown, reduce the strength of network effects.

5.2 Interaction propagation and intrinsic utility

The main model presents a very specific utility function, in which each link generates value θ for the consumer. In other words, utility is independent of the counter party of the social relation, and it only depends on the number of neighbours. A consumer induces an indirect externality to his neighbourhood. His own high probability to buy increases neighbours' probabilities to buy. The motivation for this specification is that high types enjoy more of the novel device from each social relation they have. For example, high type can be a synonym for a technically able person. Such a person is likely to get more out of new technology than those people who find new gadgets difficult to use.

However, it would not be unrealistic that the value of the new device also depended on the social relation. Some contacts could be more important than others. Ideally, we could impose a value distribution from which the value of each link is drawn. Allowing this kind of heterogeneity between links, inevitably necessitates expanding the model to discuss also usage decisions in addition to the plain buying decision. This is an area that calls for further research. For a model with a exogenous *symmetric* social network structure and buying and usage decisions, see Sääskilähti (2005).

Alternatively, utility could be a function of both parties' valuations. This way, high types induce a direct positive externality to their neighbours, in addition to the indirect externality. For example, a sharing rule of the following kind could capture the desired "propagation" dynamics. Consider the link between consumers i and j . Consumers contribute agent-specific values θ_i and θ_j to an active link. The active link generates total utility of $v_{ij}(\theta_i, \theta_j)$. In the simplest form, generated utility presents constant returns when $v_{ij}(\theta_i, \theta_j) = \theta_i + \theta_j$. Utility generation of course can present decreasing ($\frac{\partial v(\theta_i, \theta_j)}{\partial \theta_k} < 1, k = i, j$) or increasing returns ($\frac{\partial v(\theta_i, \theta_j)}{\partial \theta_k} > 1, k = i, j$). Total utility is shared by the consumers according to a rule, by which a share $r_{ij} = r(v_{ij})$ goes to consumer i and $1 - r_{ij}$ goes to j . Basically, the sharing rule could be anything. The

sharing rule complicates the analysis a great deal as it does away with symmetry properties. The consumer must now think about what types his neighbours' neighbours are, what their neighbours' neighbours' neighbours are, potentially ad infinitum. The solution to this problem might require a limitation on consumer rationality (e.g. myopic consumers), which in general is undesirable.

There is an interesting connection between increasing returns interaction and telecommunications models. Cambini & Valletti (2005) present a conventional telecommunications model with call propagation features. Their model builds on the Armstrong (1998) - Laffont et al. (1998) paradigm. Interaction network is assumed a complete graph, where every consumer has a social relation with all other consumers. When consumers interact pair-wise, interaction propagates in the sense that calling induces calls back. The more one consumer calls the other, the more the other consumer calls back. Hence, their model incorporates the direct externality, but abstracts away heterogeneity in social relations.

In the main model, we have assumed that there is no intrinsic utility associated with the good. Inclusion of intrinsic utility would not change the results qualitatively. It could facilitate analysis, by removing some possible equilibria. In fact, equilibrium uniqueness is reachable if we impose sufficient heterogeneity between consumers with respect to intrinsic utility (see Herrendorf et al. 2000). The key to uniqueness is that we have one group of consumers who buy as a strictly dominant strategy, independent of other people's strategies, at the same time as another group does not buy as a strictly dominant strategy. Under perfect information, this results to equilibrium uniqueness. Under asymmetric information, the model is easily turned into a game of correlated private values. Such a set-up allows to use global games techniques to derive a unique equilibrium. Our related paper Sääskilahti (2005) does a comprehensive analysis on equilibrium uniqueness in a model of monopoly pricing of network goods under perfect and incomplete information.

5.3 Random and scale-free networks

Our selection of social networks has been limited. Complete graph, circle, and star obviously do not characterise fully any real social network. Our choice, of course, corresponds to the most primitive network configurations that induce different pricing strategies. The limited number of cases is not different from the general literature on social networks. It is typical that any large-scale network turns out to be analytically cumbersome. However, the results that come out of the literature have clear predictions on more complex networks. Our results fare no worse.

There are two classes of networks that are of particular interest. Random networks (Erdős-Rényi model) and scale-free networks (see Albert & Barabási (2002) for technical review and Barabási & Bonabeau (2003) for informal discussion).

Random networks theory associates graphs with some specified probabilistic characteristics. The nice feature about random networks is that despite randomness, they present a large degree of regularity. Regularity is captured in the probabilistic characteristics of the network, in particular, in the average number of links a member has. The number of links, or the degree n_i , of a node i is given by a binomial distribution $\Pr\{n_i = n\} = \binom{I-1}{n} \pi^n (1 - \pi)^{I-1-n}$ for a population of I individuals. π is the connection probability between two (randomly chosen) nodes. The apparent a priori regularity of random graphs, makes them potentially very applicable. The players would then take expectations on the valuation as well as on the number of links. It is straightforward to see that the results of our model carry on to random network settings. Because consumers are a priori symmetric, the firm sets prices in similar fashion to the symmetric networks under asymmetric information. Sundararajan (2005) constructs a model of local interaction where the underlying social network is a random graph, but focuses on the equilibria of the network members' game without the monopoly pricing problem.

Scale-free networks lack the regularity of random networks. They cannot be characterised by an average number of links. The unifying characteristic of scale-free networks is that the degree of the

network follows a power distribution $P(\lambda) \sim \lambda^{-\beta}$, where the degree λ captures the connectedness of a network member. The power law tells that in a scale-free network there are only a handful of members who have very many links, and a very large number of members with only very few links. Scale-free networks are common, as they characterise certain social interaction networks (sexual relationships, academic collaboration), the Internet, even protein interaction networks in the human body. The star we have analysed approximates scale-free networks. Our insights from the star extend to general scale-free networks.

A scale-free network is very tolerant towards random elimination of links, but very vulnerable towards targeted removal of the topologically focal hubs. Ballester et al. (2004) show how crime is best prevented by a "key player removal" policy. Elimination of the hub criminals destroys the crime network in the most efficient way. Reversely, diffusion of diseases or innovations occurs very rapidly in scale-free networks, because the hubs spread information very efficiently. This can be compared with random networks, where information travels with far less speed. In our model, the firm can remove the empty network equilibrium by providing free goods to some consumers. Provision of free goods corresponds to "piloting" where the firm tests the novel device with a selected group of consumers before the commercial launch. Piloting has two functions. One, pilot users spread information about the goods (create latent demand). Two, they help in product development by testing the product in real life situations. In the star network, the firm should target the centre for the most rapid deployment of information about a new device. On the other hand, the centre is a potential source for large rents, so it may be more profitable to provide free goods to a few peripheral agents instead. The firm could then rely on indirect information transmission through the centre to other peripheral agents.

Chwe (2000) presents a model where agents coordinate their actions with the help of a communication network. Chwe's (2000) primary interest is in political action, but the model has important implications on diffusion of new products as well. His analysis on how agents time their actions is particularly valuable. The analysis allows to categorise agents into social roles like

"early adopters" and "followers". He analyses how the network structure affects the diffusion of an action¹¹. Chwe (2000) also discusses how initial seeding of agents who are biased towards revolting affects the diffusion speed. This is an analogy to how to choose pilot customers: whether pilot users should be dispersed in the network or clustered.

Granovetter (1973) introduced the concept of weak and strong links. A strong link network means that most of one consumer's neighbours are also his neighbours' neighbours: "My friends' friends are also my friends". News or innovations travel faster in weak link networks where neighbourhoods overlap little. Differentiation between weak and strong link networks has two implications to our model. If the consumer has not bought the device and his neighbours have not bought it either, then having weak links is better, since the impact of someone outside the neighbourhood who has bought the good travels greater distance. On the other hand, if the consumer has bought the device, then it is better to have strong links. Why? Strong links are good because neighbours observe that the consumer has bought the device, plus they know that their neighbours observe also that the consumer has bought the device. Since neighbours' neighbours tend to be the buyer's neighbours also, the purchase induces an effective positive externality in a strong link neighbourhood.

6 Conclusions

We have analysed monopoly pricing of social goods when the market is characterised by social relations. The model presents a stylised version of coordination goods such as mobile phones, e-mail clients, or online game consoles. We have showed that in markets where social relations are important the conventional models of network externalities fall short and need to be refined. In particular, the implicit assumption of a completely connected graph that does away all topological asymmetries can result in serious overestimation of network effects. Consequently, both achievable monopoly rents and total surplus generated in the network become exaggerated. Our model is

¹¹ Gladwell (2000) discusses social roles in networks. He reports how political revolt, crime, or product penetration hinges on the information reaching the critical agents (i.e. social roles) at a proper time.

an improved treatment of markets of communication goods, where social relations determine the patterns of interaction, and consequently determine the demand for the good.

Key findings of the paper are that social relations play a role in firm's pricing strategy. The optimal monopoly price is (weakly) increasing in the number of links in the network. More interestingly, the optimal price is affected by the network topology. Agent configuration also matters under perfect information.

Agents who have important connections capture a higher surplus compared to more peripheral agents. Under perfect information, focal positions can arise in symmetric and asymmetric networks. A preferential position is either due to central network position (network topology) or due to important neighbours (agent configuration). Once agents' valuations are private information, the preferential positions in symmetric networks are eliminated. However, topologically central agents in asymmetric networks capture always higher surplus than peripheral agents. This contrasts the results from perfect information regime where a preferential position is always dependent on other agents. Especially, a topologically central position can be redundant if the agent occupying that location does not constrain the firm by having a relatively low valuation.

Total surplus is maximised in the complete network (full activity). However, private (monopoly's) incentives to cover the market tend to differ from social optimum. In the case of perfect information, the firm chooses to cover the whole market in one social structure, and in another, in all aspects identical network except in terms of who is connected to whom, it chooses to limit supply. Its price strategy depends on the network topology, and on the configuration and heterogeneity of agents. The firm chooses a partial (technical) network in cases where consumer heterogeneity is high and high types form tight clusters.

Heterogeneity between consumers can benefit the firm under perfect information. Profits increase as heterogeneity increases if the high type consumers are clustered. When the high types are each others' neighbours, the firm can charge a high price from them and exclude low types from the service. If the high types are dispersed in the network, low types' participation is needed

to have a non-empty network, and therefore the firm does not benefit from increased heterogeneity.

Asymmetric information turned out to be analytically more straightforward. Much of the complexity of perfect information was eliminated as the role of agent configuration lost all importance. The number of neighbours (each member has) is the only network-specific parameter which affects the optimal price. Monopoly pricing induces always an efficiency loss. The lowest types are *ex ante* excluded. As the number of links grows, monopoly increases its price. Higher heterogeneity equals higher uncertainty that reduces profits and consumer surplus.

In asymmetric networks, the firm chooses a price that guarantees a higher probability to buy for the central agents under asymmetric information. Monopoly price is increasing in the size of the periphery, and the centre and the periphery get opposite surplus effects as the number of peripheral agents is increased. An additional peripheral consumer increases the expected utility of the centre. A peripheral agent is not directly affected by the additional consumer, however, increased price decreases his expected utility.

When we compare the compensated networks under asymmetric information, we see that the star is the social optimum, but the firm prefers a symmetric network. If the firm is allowed to price discriminate, its profits rise to the level it obtains in the symmetric networks. Price discrimination reduces total surplus as consumer surplus drops more than profits increase.

We have focused on the static properties of social networks. An obvious extension would be to expand the model in time, as a multi-period model would shed light on optimal price paths. It would be interesting to see what is the order of purchases in asymmetric networks. More complexity could be added by allowing direct externalities in link activation. Finally, allowing transfers or communication between consumers would introduce signalling aspects in the coordination game under asymmetric information. The interesting question is then how the firm could benefit from high types' preferences to signal their types to neighbours.

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8 Appendix

8.1 Perfect information examples

8.1.1 Complete graph

Example 23 Consider a complete graph with four consumers. Let the valuations be $\theta_1 < \theta_2 < \theta_3 < \theta_4$, and $c < 3\theta_1$ so that costs do not constrain firm's decisions. Complete network is feasible only if $p \leq 3\theta_1$. Partial network with three buyers is feasible if $\max\{3\theta_1, \theta_3\} < p \leq 2\theta_2$, and with

two buyers if $\max\{3\theta_1, 2\theta_2\} < p \leq \theta_3$. We omit the uninteresting case of the empty network¹². Firm's profits are $V_4 = 4(3\theta_1 - c)$, $V_3 = 3(2\theta_2 - c)$, and $V_2 = 2(\theta_3 - c)$ respectively. Depending on the relative values of θ_1, θ_2 and θ_3 (the highest type does not matter), the firm chooses between a complete network and a partial network of either 2 or 3 buyers. A comparison of profits suggests that complete network is chosen if valuations are close together.

- Complete network: $V_4 > V_3$ and $V_4 > V_2 \Leftrightarrow \theta_1 > \max\{\frac{1}{2}\theta_2 + \frac{1}{12}c, \frac{1}{6}\theta_3 + \frac{1}{6}c\}$.

Partial 3-buyer network is chosen when middle valuations θ_2 and θ_3 are close together, but significantly higher than θ_1 .

- 3-buyer network: $V_3 > V_4$ and $V_3 > V_2 \Leftrightarrow \theta_2 > \max\{2\theta_1 - \frac{1}{6}c, \frac{1}{3}\theta_3 + \frac{1}{6}c\}$.

Partial 2-buyer network is chosen when there is a large difference between the two lowest and the two highest valuations.

- 2-buyer network: $V_2 > V_4$ and $V_2 > V_3 \Leftrightarrow \theta_3 > \max\{6\theta_1 - c, 3\theta_2 - \frac{1}{2}c\}$.

When the market is relatively homogenous in terms of consumers' valuations, the firm will choose a complete network in the equilibrium. It benefits from high sales volumes. Even if the monopolist is unable to price discriminate, it may choose to cover the whole market. On the other hand, if agents are heterogenous, then it pays off to exclude low types from the market by charging a high price.

8.1.2 Circle

Example 24 Consider a circular network with four consumers with valuations $\theta_1 < \theta_2 < \theta_3 < \theta_4$, and $c < 2\theta_1$ so that costs do not interfere pricing, and focus on the maximal equilibrium. Immediately, we can see that there are two cases that yield different results. In the case A, the high valuation types θ_3 and θ_4 are neighbours (a circle where θ_1 has neighbours θ_2 and θ_3 , and where his neighbours are θ_2 and θ_4 yield identical results). In the case B, they are not. The network structure sets limits to the firm's choices in the case B. Consumer θ_2 located between θ_3 and θ_4 , holds a critical position. Any non-empty equilibrium must include him. In both cases, complete network occurs if $2\theta_1 \geq p$, and firm's profits are $V_4 = 4(2\theta_1 - c)$. Partial network with three buyers is feasible in the case A if $2\theta_1 < p \leq \theta_2$, and in the case B if $2\theta_1 < p \leq \min\{2\theta_2, \theta_3\}$. Partial network with two buyers is feasible in the case A if $\max\{2\theta_1, \theta_2\} < p \leq \theta_3$. Two buyer network is always dominated by other structures in the case B. We skip the uninteresting case of the empty network¹³. The firm chooses the complete network only when consumers' valuations are sufficiently close together.

- Complete network in the case A: $V_4 > V_3^A$ and $V_4 > V_2^A \Leftrightarrow \theta_1 > \max\{\frac{3}{8}\theta_2 + \frac{1}{8}c, \frac{1}{4}\theta_3 + \frac{1}{4}c\}$.
- Complete network in the case B: $V_4 > V_3^B \Leftrightarrow \theta_1 > \frac{3}{8}\min\{2\theta_2, \theta_3\} + \frac{1}{8}c$.

The firm chooses a three buyer network in both cases if the lowest type has a significantly lower valuation, and the other consumers have valuations not too different from each other.

- 3-buyer network in the case A: $V_3^A > V_4$ and $V_3^A > V_2^A \Leftrightarrow \theta_2 > \max\{\frac{8}{3}\theta_1 - \frac{1}{3}c, \frac{2}{3}\theta_3 + \frac{1}{3}c\}$.
- 3-buyer network in the case B: $V_3^B > V_4 \Leftrightarrow \min\{2\theta_2, \theta_3\} > \frac{8}{3}\theta_1 - \frac{1}{3}c$.

¹² Price $p > \max\{3\theta_1, 2\theta_2, \theta_3\}$ guarantees an empty network in the maximal NE. Firm's profits are $V_0 = 0$ in this case.

¹³ In the case A, price $p > \max\{2\theta_1, \theta_3\}$ yields an empty network. In the case B, empty network is produced with price $p > \max\{2\theta_1, \theta_M\}$, where $\theta_M = \min\{2\theta_2, \theta_3\}$. Profits are zero in both cases.

In three buyer networks, the firm benefits if the high types (θ_3 and θ_4) are dispersed in the network. We have $V_3^A < V_3^B$ always. High types support the purchases of their common neighbour θ_2 , so that network configuration relaxes firm's pricing constraint.

The firm chooses a two buyer network in the case A when the two highest types have significantly higher valuations compared with the two lowest types. In the case B, two buyer network is always dominated either by the complete network or the three buyer network.

- 2-buyer network in the case A: $V_2^A > V_4$ and $V_2^A > V_3^A \Leftrightarrow \theta_3 > \max \{4\theta_1 - c, \frac{3}{2}\theta_2 - \frac{1}{2}c\}$.

In general when the agents have valuations close together, the firm prefers the complete network, and if the high types have significantly higher valuations than the low types, the firm chooses a partial network. This relation is disturbed by the way consumers are arranged. Segregation between two highest types and two lowest types may be blocked by the agent configuration, as it happens in the case B.

When the difference in valuations of two highest and two lowest types grow large, so that we have $\theta_3 > \max \{4\theta_1 - c, \frac{3}{2}\theta_2 - \frac{1}{2}c\}$, the firm strictly prefers two buyer network. In the case A, this causes no problems to the firm as it can exclude θ_1 and θ_2 from the market. However, in the case B, the network structure may constrain the firm. It is forced to sell to three consumers, which yields lower profits if $\theta_2 < \frac{1}{3}\theta_3 + \frac{1}{6}c$. In this case, the firm prefers the case where θ_3 and θ_4 are neighbours. Respectively, if we have $\theta_3 < \max \{4\theta_1 - c, \frac{3}{2}\theta_2 - \frac{1}{2}c\}$, then the firm is better off if high types are dispersed in the network.

8.1.3 Star

Example 25 Consider the following four consumer example with a centre and three peripheral agents. Let the peripheral consumers' valuations be $c < \theta_1 < \theta_2 < \theta_3$.

- (i) Complete network is optimal if $\begin{cases} \min \{\theta_1, 3\theta_C\} > \frac{3}{4}(\min \{\theta_2, 2\theta_C\}) + \frac{1}{4}c \\ \min \{\theta_1, 3\theta_C\} > \frac{1}{2}(\min \{\theta_3, \theta_C\}) + \frac{1}{2}c \end{cases}$.
- (ii) 3-buyer network is optimal if $\begin{cases} \min \{\theta_2, 2\theta_C\} > \frac{4}{3}(\min \{\theta_1, 3\theta_C\}) - \frac{1}{3}c \\ \min \{\theta_2, 2\theta_C\} > \frac{2}{3}(\min \{\theta_3, \theta_C\}) + \frac{1}{3}c \end{cases}$.
- (iii) 2-buyer network is optimal if $\begin{cases} \min \{\theta_3, \theta_C\} > 2(\min \{\theta_1, 3\theta_C\}) - c \\ \min \{\theta_3, \theta_C\} > \frac{3}{2}(\min \{\theta_2, 2\theta_C\}) - \frac{1}{2}c \end{cases}$.

From (i)-(iii) we see that higher heterogeneity in θ supports partial networks, whereas if consumers are sufficiently homogeneous in terms of θ , the firm chooses a complete network. The topologically important position of the centre is illustrated. The firm must guarantee his participation, thus its price may be constrained.

8.2 Consumer surplus and total surplus under perfect information

Table 2: Consumer surplus, $\theta_1 \leq \theta_2 \leq \theta_3 \leq \theta_4$

	Complete network	3-buyer network	2-buyer network
Complete graph	$3(\theta_2 + \theta_3 + \theta_4) - 9\theta_1$	$2(\theta_3 + \theta_4) - 4\theta_2$	$\theta_4 - \theta_3$
Circle A	$2(\theta_2 + \theta_3 + \theta_4) - 6\theta_1$	$(2\theta_3 + \theta_4) - 2\theta_2$	$\theta_4 - \theta_3$
Circle B	$2(\theta_2 + \theta_3 + \theta_4) - 6\theta_1$	$\max\{(\theta_3 + \theta_4) - 4\theta_2, (2\theta_2 + \theta_4) - 2\theta_3\}$	Dominated
Star, 2 as centre	$(3\theta_2 + \theta_3 + \theta_4) - 3\theta_1$	$\max\{(\theta_3 + \theta_4) - 4\theta_2, (2\theta_2 + \theta_4) - 2\theta_3\}$	Dominated
Star, 3 as centre	$(\theta_2 + 3\theta_3 + \theta_4) - 3\theta_1$	$(2\theta_3 + \theta_4) - 2\theta_2$	$\theta_4 - \theta_3$

Table 3: Total surplus, $\theta_1 \leq \theta_2 \leq \theta_3 \leq \theta_4$

	Complete network	3-buyer network	2-buyer network
Complete graph	$3(\theta_1 + \theta_2 + \theta_3 + \theta_4)$	$2(\theta_2 + \theta_3 + \theta_4)$	$\theta_3 + \theta_4$
Circle A	$2(\theta_1 + \theta_2 + \theta_3 + \theta_4)$	$\theta_2 + 2\theta_3 + \theta_4$	$\theta_3 + \theta_4$
Circle B	$2(\theta_1 + \theta_2 + \theta_3 + \theta_4)$	$2\theta_2 + \theta_3 + \theta_4$	Dominated
Star, 2 as centre	$\theta_1 + 3\theta_2 + \theta_3 + \theta_4$	$2\theta_2 + \theta_3 + \theta_4$	Dominated
Star, 3 as centre	$\theta_1 + \theta_2 + 3\theta_3 + \theta_4$	$\theta_2 + 2\theta_3 + \theta_4$	$\theta_3 + \theta_4$

8.3 Stability of equilibria under asymmetric information

We provide checks for equilibria stability along the line of a Nash tâtonnement process (see e.g. Fudenberg & Tirole 1991). This process checks equilibrium stability against small perturbations.

8.3.1 Symmetric networks

The equilibrium condition can be deconstructed into two equations $\pi = z_i$ (the 45-degree line) and $z_i = 1 - F\left(\min\left\{\theta^+, \tilde{\theta}_i\right\}\right)$, which must be equal in the equilibrium for all $i \in \mathcal{G}$. The condition for asymptotic stability is $\left|\frac{\partial \pi}{\partial z_i}\right| \left|\frac{\partial z_i}{\partial \pi}\right| < 1 \ \forall i \in \mathcal{G}$. The deconstructed equilibrium condition is

$$\begin{cases} \pi = z \\ z = 1 - F\left(\frac{p}{n\pi}\right) \end{cases} \quad \text{for positive equilibria}$$

$$\begin{cases} \pi = z \\ z = 0 \end{cases} \quad \text{for the empty network.}$$

We have for the maximal BNE $\left|\frac{\partial \pi}{\partial z}\right| \left|\frac{\partial z}{\partial \pi}\right|_{\pi=\pi_+^*} = \frac{1}{2}$, and the equilibrium is stable. For the lower positive BNE the same check returns $\left|\frac{\partial \pi}{\partial z}\right| \left|\frac{\partial z}{\partial \pi}\right|_{\pi=\pi_-^*} = 2$, which indicates that the equilibrium is unstable. The empty network is also stable since $\left|\frac{\partial \pi}{\partial z}\right| \left|\frac{\partial z}{\partial \pi}\right|_{\pi=0} = 0$.

8.3.2 Star

In the region where a non-zero positive equilibrium can exist, the equilibrium conditions are

$$\pi_C = 1 - F\left(\frac{p}{(I-1)\pi}\right)$$

$$\pi = 1 - F\left(\frac{p}{\pi_C}\right)$$

We study only the case $c = 0$, $\theta^+ = 1$, and $\theta^- = 0$, which we discuss in the main text. Since the model does not give out explicit equilibrium values that would be easily applied to the stability check $\left|\frac{\partial \pi}{\partial \pi_C}\right| \left|\frac{\partial \pi_C}{\partial \pi}\right| < 1$, we resort to a numerical test. When the equilibrium values π_C^* , π^* and $p(\pi_C^*, \pi^*)$ are substituted in the stability equation, we can plot the curve $s = \left|\frac{\partial \pi}{\partial \pi_C}\right| \left|\frac{\partial \pi_C}{\partial \pi}\right|$ for different values of I . It turns out that s remains below one for $I \geq 3$, and it approaches zero as I grows very large. Hence, the equilibrium is a stable one. The other equilibrium, namely the empty network, is obviously a stable one as well.