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On the form drag coefficient under ridged ice: Laboratory experiments and numerical simulations from ideal scaling to deep water

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Key Points:

- The local form drag coefficient C_{dw} under ridged ice is parameterized by both laboratory experiments and numerical simulations.
- In laboratory experiments with finite water depth, C_{dw} was found to be sensitive to the ridge keel depth and slope angle.
- A logarithmic function of keel slope angle is proposed for deep water C_{dw} applicable in sea-ice dynamic models.

Abstract

The bottom topography of ridged sea ice differs greatly from that of other sea-ice types. The form drag of ridge keels has an important influence on sea-ice drift and deformation. In this study, both laboratory experiment (**LabE**) and fluid dynamics numerical simulation (**FDS**) have been carried out for a physical ridge model in a tank to better understand the quantitative characteristics of the form drag. The **LabEs** covered both laminar and turbulent conditions. The local form drag coefficient of a keel, C_{dw} , varied with the keel depth h_w and the slope angle α_w in the turbulent regime. After validated by the **LabE** measurements, the **FDSs** were employed to extend the parameterization from the finite water depth to deep water. The results gave $C_{dw} = 0.68 \cdot \ln(\alpha_w/7.8^\circ)$, $R^2 = 0.998$, $10^\circ \leq \alpha_w \leq 90^\circ$, with C_{dw} ranging from 0.17 to 1.72, when the keel depth was much less than the water depth. For a large ridging intensity (keel depth/spacing ≥ 0.01), the variation of the local form drag coefficient and its contribution to total drag coefficient were sensitive to the keel slope angle. Assuming the log-normal distribution for this angle, the average value of the local form drag coefficient was 0.75, recommended for sea-ice dynamic models.

Plain Language Summary

Sea ice floats in the polar oceans. Wind and ocean currents generate drag forces for sea ice to drift. The form drag force, also known as the pressure drag force, on sea ice is closely associated with the shape and size of ice floes and ridges. Ridged sea ice has the most irregular geometry of all ice types of ice floes. The predicted thinning of Arctic

sea ice would lead to more easy deformation of ice floes, resulting in more abundant sea-ice ridge formation and enhancing the form drag forces on sea ice. The form drag coefficient needs to be accounted for in sea-ice models. The local form drag coefficient of ridges was investigated in this study by tank experiments and numerical simulations with an idealized ice ridge model. Based on the results, a logarithmic function was chosen to describe the relationship between the local form drag coefficient and the keel slope angle in deep water conditions. The new parameterization of the local form drag coefficient can be applied in sea-ice models.

1 Introduction

Arctic sea ice cover has been changing rapidly with global warming in the past several decades (Screen and Simmonds, 2010; Onarheim *et al.*, 2018; Notz and Stroeve, 2018). Arctic sea ice is not only shrinking in extent, but also in thickness (Rothrock *et al.*, 1999; Comiso, 2012; Stroeve *et al.*, 2012). The exchange of momentum between the atmosphere and the ocean is modified with the decline in Arctic sea ice (Martin *et al.*, 2016). A thinner ice cover is more easily broken under the actions of ocean waves and wind stress (Castellani *et al.*, 2014; Petty *et al.*, 2017). Consequently, the drift speed of Arctic pack ice has increased significantly, although the wind force has remained the same (Rampal *et al.*, 2009; Spreen *et al.*, 2011). In regions exposed to storm events, the sea-ice distribution can change quickly, with the drift speed having reached even 1 m/s (Itkin *et al.*, 2017). Thinner sea ice is more easily subject to deformation, resulting in sea-ice ridging or hummocking (Leppäranta, 2011). The drift patterns of ridged ice and

59 level ice differ considerably because of the complex topography of ridges, with a sail
60 on top and a keel at the bottom.

61 The atmospheric and oceanic drag forces on sea ice depend on the ice surface,
62 bottom roughness, and boundary-layer flows. Early methods of calculating these
63 momentum fluxes mostly regarded the drag coefficients as constants in non-stratified
64 boundary layers. Later on, based on the Monin-Obukhov and Rossby similarity theories
65 (Blackadar and Tennekes, 1968; Obukhov, 1971; Banke *et al.*, 1980), the revised drag
66 coefficients depended on the stability of the boundary-layers and the roughness length
67 of the sea-ice surface and bottom. The turbulent flow regime was the main consideration
68 in earlier studies (Overland, 1985; McPhee, 2012).

69 The drag force consists of the tangential shear stress or skin friction and the
70 form drag, which takes the drag force component normal to large roughness elements,
71 i.e., floe edges and ridges (Mai *et al.*, 1996; Garbrecht *et al.*, 2002; Lüpkes *et al.*, 2012).
72 With the decline of multiyear ice in the Arctic Ocean, a thinner ice cover is more easily
73 broken, and the variations in the surface topography are higher than previously
74 (Castellani *et al.*, 2014). Both the air-ice and ice-ocean drag coefficients, C_a and C_w ,
75 reach their maxima in the summer during their annual cycle, and the contributions from
76 the form drag are significant because the floe size and ridge spacing are reduced more
77 dramatically than the ice thickness during the melt season (Leppäranta *et al.*, 1995;
78 Tsamados *et al.*, 2014).

79 The drag force exerted on ice ridges obeys a quadratic drag law (Arya, 1973),
80 associated with wind and current velocity and the local form drag coefficients of
81 individual obstacles, denoted by C_{dw} for a keel and C_{da} for a sail, where the subscripts
82 w and a represent the ice-ocean and air-ice interface, respectively. With the increasing
83 spatial resolution of numerical sea-ice dynamic models, different types of ice have been
84 included in sea-ice dynamic models that reflect the ice thickness distribution, especially
85 the ridged ice contribution. The scheme by Arya is widely used in present models, for
86 instance, the Helsinki multicategory sea ice model (HELMi) (Haapala *et al.*, 2005;
87 Mårtensson *et al.*, 2012) and the Los Alamos sea-ice model (CICE) (Hunke *et al.*, 2013;
88 Tsamados *et al.*, 2014; Martin *et al.*, 2016). The results of the CICE model by Martin
89 *et al.* (2016) showed that the form drag on ice ridges contributed to more than half of
90 the total drag in the winter seasons from 1980 to 2010.

91 The local form drag coefficients, C_{da} and C_{dw} , are the basis of the large-scale
92 drag forces. Banke and Smith (1975) obtained C_{da} proportional to the sail height h_a and
93 the sail slope α_a . Garbrecht *et al.* (1999) continued to summarize a logarithmic approach
94 for the dependence of C_{da} on h_a based on the measurements of wind profiles on the
95 leeward side of a pressure ridge. Drag force studies have been mostly conducted in the
96 atmospheric boundary layer (ABL) over sea ice, and only a few cases have been
97 reported for ridge keels in the oceanic boundary layer (OBL). This is mainly because
98 field measurements are more difficult in underwater operations in the OBL than on the
99 ice surface. Laboratory experiments provide an effective way to investigate the

interaction between ice and water by directly measuring the drag force and altering the test parameters. For example, Pite *et al.* (1995) have investigated the drag force on the keel model by laboratory measurements. However, the depth of the turbulent boundary layer is limited by the fixed tank bottom in laboratory experiments. In deep, homogeneous water, the depth of the turbulent boundary-layer is an order of magnitude greater than the depth of keels.

This study is a continuation of the ice-ocean drag coefficient study by Lu *et al.* (2011). Here, we focus on the parameterization of the local form drag coefficient between ridged ice and ocean. This manuscript is organized as follows: Section 2 provides a theoretical parameterization of the ice-ocean drag coefficient based on Lu *et al.* (2011) followed by and the motivations of this study. In Section 3, two methods i.e. laboratory experiment (**LabE**) and fluid dynamics numerical simulation (**FDS**) used in this study are described. The results for the local form drag coefficient C_{dw} for a keel identified by both **LabE** and **FDS** are presented in Sections 4.1 and 4.2. Then **FDSs** are extended from finite depth water to deep water, and a parametrization scheme of C_{dw} is presented in Section 4.3. Applications of C_{dw} for a geometry-based ice-ocean drag coefficient and the results on the spatial distribution of keel angle are discussed in Section 5. Finally, conclusions are drawn in Section 6.

2 Parameterization of the Ice-ocean Drag Coefficient

Ice-ocean stress τ_w is commonly expressed by the quadratic bulk formula, in complex form as

$$\tau_w = \rho_w C_w |U - U_{wg}| (\cos \theta_w + i \sin \theta_w) (U - U_{wg}), \quad (1)$$

where ρ_w is the water density, C_w is the ice-ocean drag coefficient, U is the ice velocity, U_{wg} is the geostrophic flow beneath the Ekman layer, and θ_w is the boundary-layer turning angle or the Ekman angle. According to the drag partition theory, ice-ocean stress can be divided into two parts: the skin friction due to the surface shear stress, and the form drag on floe edges and ice ridges due to the lateral pressure difference across the obstacles (Arya, 1975). Further observations and model studies have shown that both ice-ocean and air-ice drag coefficients are related to ice concentration, floe size, keel and sail and other geometry parameters (Hassen-Bauer and Gjessing, 1988; Steele et al., 1989; Garbrecht et al., 2002; McPhee, 2002; Lüpkes and Birnbaum, 2005).

According to Eq. (1) and the drag partition theory, the geometry-based parametrizations of the drag coefficients have been proposed by Lu *et al.* (2011) and Tsamados *et al.* (2014), which are hereafter referred to as LU2011 and TS2014. LU2011 used the ice velocity relative to a fixed background geostrophic velocity U_{wg} , which is different from the reference velocity at a specific depth used in TS2014. Due to the different reference depths, a conversion factor C_h is needed to compare the keel drag coefficients between the two schemes. The details are introduced in the Supporting information of Text S1.

The parametrization of the total drag coefficient C_w accounts for the effects of sea-ice geometry, including sea-ice concentration, floe size, and ridging intensity, particularly in heavily ridged regions (e.g., offshore areas) and low ice concentration

regions (e.g., a marginal ice zone) (Martin *et al.*, 2016; Brenner *et al.*, 2020). As described by Lu *et al.* (2011), C_w can be calculated as follows:

$$C_w = C_{EW} + C_{RW} + C_{SW} = f_1 C_{ew} + f_2 C_{dw} + f_3 C_{sw} \quad (2)$$

$$f_1 = \frac{A d_w}{2 L_f} \left[1 - \left(\frac{A}{1-A} \frac{d_w}{L_f} \right)^{1/2} \right]^2 \quad (3)$$

$$f_2 = \frac{A h_w}{\pi L_r} \left[1 - \left(\frac{h_w}{L_r} \right)^{1/2} \right]^2 \quad (4)$$

$$f_3 = A \left(1 - m \frac{h_w}{L_r} \right) \quad (5)$$

where C_{EW} , C_{RW} , and C_{SW} represent the drag coefficients that contribute, respectively, to the form drag by floe edges, form drag on ice keels, and the skin friction on the ice surface. C_{ew} , C_{dw} , and C_{sw} are the local drag coefficients for the contributions of a single floe edge, a single ice ridge, and uniform ice surface, respectively. A is ice concentration, d_w/L_f is the ratio of floe draft to floe length, and h_w/L_r is the ridging intensity or the ratio of keel depth to keel spacing. The terms in the brackets for the functions f_1 and f_2 account for the shielding effects, and the factors in front of the brackets account for the vertical dimension of the drafts and adding up the drag spatially for a grid cell. f_3 is the roughness friction function, where m is a constant equal to 10 in the OBL (Lu *et al.*, 2011).

In the above parameterization, the form drag of individual keels, C_{dw} , varies over a wide range. It was measured as 0.51 – 0.62 in the laboratory experiment by Pite *et al.* (1995). In the calculation of the CICE model by TS2014 formulation, C_{dw} is set as a constant within the range of 0.13 – 0.39 (Note that their definition of C_{dw} needs a

conversion factor of 0.76 to compare with this study, for the details see Supporting information Text S1). In LU2011, C_{dw} was used as a constant equal to 0.5 in the sensitivity experiments.

Eq. (2) shows that variations of C_{dw} result in proportional variations of keel drag coefficient C_{Rw} . Since ridge keels in Arctic seas have a more complicated morphology than floe edges (Bitz *et al.*, 2001; Obert and Brown, 2011; Wadhams *et al.*, 2011), C_{dw} has a range of values corresponding to various keel shapes rather than being a constant. Using the simple triangular shape of ridge keel (Ekeberg *et al.*, 2015; Bonath *et al.*, 2018; Kharitonov *et al.*, 2020), the slope angle α_w and the keel depth h_w define the keel geometry. The goal of this study is to investigate how C_{dw} varies with the keel geometry, and a parameterization scheme of $C_{dw} = C_{dw}(\alpha_w, h_w)$ with its range of variation is presented, allowing an improved formulation of the ice-ocean drag coefficient in Eq.(2).

3 Methods

3.1 LabE Setup

The LabEs were performed in a rectangular water tank, which was 0.23 m wide, 4.5 m long, and 0.45 m deep, with glass panels fitted on the sides and the bottom, as shown in Fig. 1a. The ridge keel model was designed to be a wedge-shaped symmetric element using Perspex materials. When the keel model was driven along the water surface, the drag force on the ridge keel exerted by water was recorded. The force sensor was fixed between the motor and the keel model to measure the drag force in the

horizontal direction. The sensor range was 10 N, the accuracy was 0.001%, and the sampling frequency was 50 Hz. The experimental setup is shown in Fig. 1b.

Previous observations have shown that sea-ice ridges in the Arctic Ocean have a triangular sail, while their keel is triangular in first-year (FY) ridges and trapezoidal in multiyear (MY) ridges (Timco and Burden, 1997; Strub-Klein and Sudom, 2012). The present study focuses on the FY ice because of its increasing seasonality and prevalence in the Arctic Ocean nowadays (Kwok *et al.*, 2009; Onarheim *et al.*, 2018; Bi *et al.*, 2020). For a triangular keel moving in a homogenous fluid, an earlier dimensional analysis showed that the keel drag coefficient C_{dw} can be expressed as $C_{dw}=f(Re, D/h_w, \alpha_w)$ (Zu *et al.*, 2020), where Re is the Reynolds number, D/h_w is the ratio of water depth to keel depth, and α_w is the keel slope angle. The keel depth h_w and the slope angle α_w are the two control parameters of the keel model. Table 1 lists the observed keel depth and the slope angle in real conditions. Normally keel slope angles are within $20^\circ - 30^\circ$, but the maximum can be as high as 87.5° . The keel depth varies in a large range and acts as a factor of turbulence generation in the boundary-layer under the ice.

The **LabEs** covered the laminar and turbulent flow regimes and the transition regime between them. For different keel shapes, the transition flow regime was related to the Reynolds number, which reflects the flow pattern (Schlichting and Gersten, 2017):

$$Re = \frac{UL_w}{\nu}, \quad (6)$$

where $L_w = 2h_w/\sin(\alpha_w)$ is the keel wet length, and $\nu = 1.003 \times 10^{-6} \text{ m}^2/\text{s}$ is the kinematic viscosity coefficient of water viscosity.

Table 1. Ridge keel parameters of Arctic FY ice

Region	Slope angle /°	Keel depth/m	Reynolds number	References
Beaufort Sea and Central Arctic Ocean	26.6 (Mean)	-	-	Timco and Burden, 1997
East Coast Canada	24.92 (Mean)	2.94 (Mean)	$1.4 \times (10^5 - 10^7)$	Obert and Brown, 2011
Fram Strait	15–24	5.1–6.7	$2.5 \times 10^5 - 5.2 \times 10^7$	Sand <i>et al.</i> , 2015
Barents Sea	8–29	3.4–7.6	$1.4 \times 10^5 - 1.1 \times 10^8$	Sand <i>et al.</i> , 2015
Shokalsky Strait	11.2–87.5	4.36–4.97	$8.7 \times 10^4 - 5.1 \times 10^7$	Kharitonov, 2020

Note: The Reynolds number Re is calculated from keel wet length (Eq. 6), where ice velocity U takes on a characteristic value between 0.01 and 1 m/s (Thorndike and Colony, 1982; Kreyscher *et al.*, 2000; Leppäranta, 2011; Shu *et al.*, 2012).

In the present experiments, the keel slope angle α_w was designed at 10° , 20° , 30° , 45° , and 90° . The model in case $\alpha_w = 90^\circ$ is a 5 mm thick wall, with the depth one order of magnitude larger than the thickness. This extreme case improved the feasibility of the estimated function $C_{dw}(\alpha_w)$ over the entire range of the keel slope. The ratio of water depth to keel depth D/h_w was set into a range of 3.5 – 8.75, where the water depth is $D = 0.35 \text{ m}$ in the **LabEs**. The keel model velocity U was set within 0.03 – 0.3 m/s,

which included the turbulent flow conditions under the keel model with Re up to 3.4×10^5 . The experimental parameters are listed in Table 2.

Table 2. Parameters in **LabE** and **FDS**

Parameter	LabE	FDS
U	0.03, 0.06, 0.07, 0.08, 0.09, 0.1, 0.12, 0.15, 0.18, 0.24, 0.3 m/s	0.06, 0.07, 0.08, 0.09, 0.1, 0.12, 0.15, 0.18, 0.24, 0.3 m/s
h_w	0.04, 0.06, 0.08, 0.1 m	0.04, 0.06, 0.08, 0.1 m
α_w	20, 30, 45, 90 °	10, 20, 30, 45, 60, 90 °
D	0.35 m	0.35, 0.7, 1.05, 1.4 m
Re	8×10^2 - 1.7×10^5	1×10^4 - 3.4×10^5
D/h_w	3.5-8.75	3.5-35

Figure 1. (a) Tank and (b) sketch of **LabE** setup; U is the keel model velocity
simulating ice drift, h_w is the keel depth, and α_w is the keel slope angle.

3.2 FDS Setup

The **FDSs** were conducted for $Re > 10^4$, which represents the turbulent flow regime. The aim was to compare the simulations with the **LabE** results, and to perform deep water simulations to extend the research outcome, since the laboratory study was limited by the fixed, finite water depth. Provided that hydrodynamic similarity is satisfied, the relative motion of the flow in the **FDS** tank past the ridge keel at a uniform upstream velocity and the drag force should be the same as in the **LabE**, where the keel moves at the same speed in the stationary fluid.

Considering the uniformity of the flow field along the transverse section of the ridge model, the **FDS** was simplified into a 2-D flow problem. The flow structure here is 2-D rather than 3-D as in the real ocean, and the Coriolis effect is not included. The Coriolis force mainly alters the direction of the drag force but not the absolute value. Details about the Coriolis force are discussed in Supporting information Text S1. The **FDS** domain was the same as in the laboratory tank, as shown in Fig. 2. Typical cases with a slope angle of 45° were chosen to test the grid size independence of the simulations. The result stabilized at the grid size between 0.5 cm and 0.9 cm, and 0.5 cm was selected as the final grid size. To ensure a fully developed wake flow on the lee side of the keel, the ridge model was placed at one-third downstream in the tank (Fig. 2). The distance from the keel to the outlet was 30 times the maximum keel depth.

Figure 2. Sketch of the **FDS** domain. Unstructured grid cells occur near the keel.

The **FDSs** were based on the $k-\varepsilon$ turbulence model (Launder and Spalding, 1972). The momentum equation can be written as:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho_w} \frac{\partial p}{\partial x_i} + \frac{1}{\rho_w} \frac{\partial}{\partial x_j} \left(\mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \tau_{ij} \right), \quad (7)$$

where u_i denotes the velocity components in the Cartesian coordinates x_i , t is time, ρ_w

is water density, p is pressure, μ is the dynamic water viscosity coefficient, and τ_{ij} is the Reynolds stress, which is related to the mean velocity gradient according to the Boussinesq hypothesis of an isotropic eddy viscosity μ_t (Hinze, 1975):

$$\tau_{ij} = \mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \rho_w k \delta_{ij} \quad (8)$$

where k is the turbulent kinetic energy and δ_{ij} is the Kronecker delta. The numerical model is based on the Reynolds-averaged Navier-Stokes (RANS) equation (Eq. 7), which was numerically solved by the finite volume method, and the discretization scheme used the second-order upwind method in space and was stable over time. The solver adopted the SIMPLEC algorithm (Liu *et al.*, 2013; Karim *et al.*, 2014). For the k - ϵ two equation in renormalization group (RNG) method, $C_\mu = 0.0845$, $C_{1\epsilon} = 1.42$, $C_{2\epsilon} = 1.68$, and $\alpha_\epsilon = \alpha_k = 1.39$ (Yakhot and Orszag, 1986; Orszag *et al.*, 1993). Along the keel boundary, Reynolds stresses were zero due to the presence of a viscous sublayer that affected the distribution of the pressure p at the keel boundary. Normal stresses in the flow originated from the pressure p exerted on the keel. The total keel drag was obtained by integrating the frictional and normal stresses along the keel boundary, which lied along the keel bottom boundary $z = \eta(x)$ with given depth h_w and angle α_w :

$$F = \int_{x=-h_w/\tan\alpha_w}^{x=h_w/\tan\alpha_w} \left[\mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) n_z + \left(p + 2\mu \frac{\partial u}{\partial x} \right) n_x \right]_{z=\eta(x)} dx. \quad (9)$$

The fluctuation of the free surface was considered very small and ignored. Furthermore, a no-stress wall was used for the top boundary, where $z = D$:

$$w = 0, \partial u / \partial z = 0, p = 0, \quad (10)$$

At the bottom ($z=0$) and at the keel boundary ($z=\eta(x)$), the no-slip boundary conditions

were taken:

$$u = w = 0. \quad (11)$$

The boundary condition at the velocity inlet and outlet were:

$$x/h_w = -14: u = U \text{ and } w = 0, \quad (12)$$

$$x/h_w = 30: \frac{\partial u}{\partial x} = \frac{\partial w}{\partial x} = 0. \quad (13)$$

The outlet boundary was far from the flow separation region, and therefore the velocity gradient was zero. The velocity at the outlet was determined by the upstream flow, by continuity, and the streamwise velocity profile was still influenced by the wake effect of the keel topography. To test the influence of the outlet position on the force of the keel, three typical cases with the keel slope of 45° and the keel depth of 10 cm were selected. The keel drag force (Table 3) tended to a stable level with increasing x/h_w , where x is the horizontal coordinate. For $Re = 8.5 \times 10^4$, the force decreased slightly with x/h_w , but the averaged value with standard deviation was 2.27 ± 0.05 N, and the values were within 4%. When $x/h_w = 30$ was used as the computational domain length, the outlet was then far from the keel to ensure at most a minor effect on the results.

289

Table 3. Tests of the computational domain length x/h_w for the drag force $F(N)$.

x/h_w	30	60	90
$U=15\text{cm/s}, Re=4.2 \times 10^4$	0.57	0.57	0.57
$U=24\text{cm/s}, Re=6.8 \times 10^4$	1.44	1.44	1.44
$U=30\text{cm/s}, Re=8.5 \times 10^4$	2.35	2.24	2.23

291

In order to compare the **FDS** results with the **LabE** measurements, a no-slip condition was imposed on the bottom boundary. Since the bottom boundary-layer was always thin in these experiments, as shown in Fig. 5, no-slip and free-slip conditions made only a little difference in the results. Tests of the no-slip and free-slip conditions were conducted and the results are listed in Table 4.

Table 4. Tests of no-slip and free-slip conditions on the tank bottom.

$D/h_w=3.5(\alpha_w=45^\circ, h_w=10\text{cm})$	No-slip $F(\text{N})$	Free-slip $F(\text{N})$
$U=15\text{cm/s}, Re=4.2\times 10^4$	0.57	0.56
$U=24\text{cm/s}, Re=6.8\times 10^4$	1.44	1.45
$U=30\text{cm/s}, Re=8.5\times 10^4$	2.35	2.21

4 Results

We shall focus first on the local drag coefficient C_{dw} of an individual keel and its dependence on the keel depth h_w and slope angle α_w for a finite water depth. Then the results of the two methods are compared and the feasibility of **FDS** is validated by the **LabE** measurements. Finally, the **FDS** is extended to obtain results under deep water conditions ($D/h_w \rightarrow \infty$).

4.1 LabE Measurements

The local drag coefficient C_{dw} is associated with the measured drag force F by (Hoerner, 1965; Pite 1995; Schlichting and Gersten, 2017)

$$F = \frac{1}{2} C_{dw} \rho_w h_w U^2, \quad (14)$$

where U is the far field velocity of the keel model, and the water density is taken as ρ_w

311 $= 998.2 \text{ kg/m}^3$. As mentioned in Section 3.1, $C_{dw} = f(Re, D/h_w, \alpha_w)$. Figs. 3a-d show that
 312 C_{dw} varies with Re at different D/h_w and α_w (Note that open circles represent FDS
 313 results, see details in Section 4.2). According to the ridge parameters and ice velocity
 314 characteristics in Table 1 (Thorndike and Colony, 1982; Kreyscher *et al.*, 2000;
 315 Leppäranta, 2011; Shu *et al.*, 2012), Re ranges from 8.7×10^4 to 1.1×10^8 in the Arctic
 316 Ocean. Field experiments have been performed to study the turbulent boundary-layer
 317 under sea ice (McPhee, 2002 and 2008).

318 **LabEs** $\blacksquare$ $h_w = 4\text{cm}$ $\bullet$ $h_w = 6\text{cm}$ $\blacktriangle$ $h_w = 8\text{cm}$ $\blacklozenge$ $h_w = 10\text{cm}$ **FDSs** $\square$ $h_w = 4\text{cm}$ $\circ$ $h_w = 6\text{cm}$ $\triangle$ $h_w = 8\text{cm}$ $\diamond$ $h_w = 10\text{cm}$
 319 Figure 3. Plots of $\log_{10}(C_{dw})$ with $\log_{10}(Re)$ at keel depths $h = 4\text{cm}$, 6cm , 8cm , and
 320 10cm . Open circles represent **FDSs** (see details in Section 4.2), and solid circles
 321 represent **LabEs**. The keel slope angle α_w is (a) 20° , (b) 30° , (c) 45° , and (d) 90° . The
 322 part to the right of the vertical dashed lines is absolutely turbulent flow. The critical

Re are respectively (a) 5×10^4 for $\alpha_w=20^\circ$, (b) 2.5×10^4 for $\alpha_w=30^\circ$, (c) 1.6×10^4 for $\alpha_w=45^\circ$, and (d) 1×10^4 for $\alpha_w=90^\circ$.

The LabE laminar flow results can be described with a Re power-law. The relationship between C_{dw} and Re ($Re < 5 \times 10^4$) in the laminar regime was quantified by linear least-squares regression. The result was $C_{dw}(Re) = 4100 \cdot Re^{-0.5}$, with the coefficient of determination $R^2=0.79$ passing the significance test (F -test, $p<0.001$). Fig. 4 shows that C_{dw} depended only on Re , which was similar to previous studies, i.e., Hoerner (1965) and Pite (1995) found that the friction coefficient of a flat plate obeyed a $Re^{-0.5}$ power-law in the laminar region where the viscous shear stress dominated the drag force.

Figure 4. The laminar flow results for all keel shapes and linear least-squares regression fit.

4.2 FDS Results and Validation

FDSs consider the turbulent regime. The relative velocity u/U induced by a triangular geometry is presented in Fig. 5a. The streamlines around the keel were no longer smooth, and a vortex wake formed at the lee side (Fig. 5a). Behind the keel, a sheltered area formed (Garbrecht *et al.*, 1999), where the pressure was lower than the pressure at the same level at the front of the keel, and the form drag force created by the pressure difference was much greater than the skin friction drag force.

As the downstream distance from the keel increased, the influence of the keel geometry gradually weakened. Figs. 5b-f show a series of velocity profiles along the horizontal direction. $D/h_w = 3.5$ had a uniform upstream velocity profile with a very thin bottom boundary layer (when $u/U = 0.99$, $z/h_w \approx 0.2$). Directly below the keel, the flow accelerated because the keel narrowed the flume. At the same time, the velocity near the keel corner rapidly increased from 0 to the mean streamwise velocity of about $1.7U$. When $x/h_w = 5$, a wake area formed behind the keel with negative x -velocity on top. Due to the disturbance by the keel, a turbulent boundary-layer existed in the upper water of the flume. With increasing x/h_w , the thickness of the boundary-layer gradually developed downward.

Figure 5. (a) Example of velocity field with $\alpha_w = 45^\circ$, $D/h_w = 3.5$, and $U = 0.3 \text{ m/s}$ and a series of x -velocity u profiles along the horizontal direction: **(b)** $x/h_w = -10$, **(c)** $x/h_w = 0$, **(d)** $x/h_w = 5$, **(e)** $x/h_w = 15$, and **(f)** $x/h_w = 25$.

For the fully turbulent regime, the **FDS** results agreed with the **LabE** results, as shown in Fig. 3. At steep slope angles, the fit became worse. At $\alpha_w = 90^\circ$, the **FDS** results were slightly larger than the **LabE** outcome because of flow separation at the sharp wedge. We found that C_{dw} with constant α_w and h_w in the **FDS** was also independent on Re consistent with the LabE measurements. Fig. 3a-d show that C_{dw} in **FDSs** varied within 0.17 – 3.78 with α_w and h_w , which were two factors affecting C_{dw} .

The keel angle α_w is the key factor for C_{dw} . Fig. 6a shows that C_{dw} increased linearly with $\ln(\alpha_w)$ for $\alpha_w \geq 10^\circ$ when $U = 0.15 \text{ m/s}$ and $h_w = 10 \text{ cm}$. Fig. 6a also

presents the comparison between **FDS** results and **LabE** measurements, which are also consistent with a logarithmic variation.

The results in Fig. 3a-d were obtained for a finite water depth. The maximum value of the ratio of water depth D to keel depth h_w was 8.75. The keel depth affected the mean streamwise velocity and the pressure distribution around the keel. The variations in $C_{dw}(D/h_w)$ when $U = 0.15$ m/s and $\alpha_w = 45^\circ$ are shown in Fig. 6b as an example. As shown in Fig. 6b, C_{dw} decreases with D/h_w , being sensitive at small D/h_w . The increase in the keel depth obviously leads to an increase in the mean streamwise velocity at the keel. According to the Bernoulli equation, the sum of static and dynamic pressure, $p + \frac{1}{2}\rho U^2$, is a constant along streamlines. Thus, the drag force (also referred as the pressure difference Δp) is proportional to $\frac{1}{2}\rho \Delta U^2$. When the keel depth h_w increases, the channel section becomes narrower, and according to the law of mass conservation, $\frac{1}{2}\Delta U^2 = \frac{1}{2}(h_w/(D-h_w))^2 U^2$. Consequently, $C_{dw} \sim \Delta p/\rho U^2 = 1/(2(D/h_w - 1)^2)$. Based on this proportionality, we obtained the formula by the best linear fit to laboratory data between C_{dw} and $1/(D/h_w - 1)^2$, as shown in Fig. 6b. The coefficient of determination was $R^2 = 0.97$, and the significance level was less than 0.05. As D increased, the influence of h_w on C_{dw} weakened with $C_{dw} \rightarrow 1.37$ for $D \gg h_w$. The variation of C_{dw} with D/h_w in **FDS** was also validated by the **LabE** measurements in Fig. 6b. Thus, the **FDS** is feasible to examine the form drag coefficient C_{dw} .

Figure 6. Variation of C_{dw} with (a) α_w ($10^\circ \leq \alpha \leq 90^\circ$) when $U=0.15$ m/s and $h_w=10$ cm in **LabEs** and **FDSs** (note: logarithmic scale for α_w) for the turbulent regime, and (b) D/h_w ($h_w=4$ cm, 6 cm, 8 cm, and 10 cm, $D=35$ cm) when $U=0.15$ m/s and $\alpha=45^\circ$.

4.3 Deep Water Consideration for **FDS**

In the real ocean, the mixed layer formation has seasonal characteristics. In summer, the mixed-layer depth tends to be small, since meltwater creates a stable stratification. Then we have $D/h_w \sim 1$, and it is not appropriate to compare the tank water depth and the mixed layer depth in real ocean. In other seasons, $D/h_w \gg 1$ is mostly acceptable (e.g. McPhee, 2002; Strub-Klein and Sudom, 2012; Peralta-Ferriz and Woodgate, 2015). When the mixed layer depth is of the same magnitude as the keel depth, the mixed layer would be deepening along with the keel movement, which is beyond the scope of this study.

Additional FDS simulations for such conditions were therefore carried out. Assuming asymptotic conditions, we have turbulent flows where Re and D/h_w tend to

infinity. Then, the parametrization of $C_{dw}=f(Re, D/h_w, \alpha_w)$ would be independent of Re and D/h_w and has a simple functional form of $C_{dw}(\alpha_w)$.

We configured a variable water depth, $D = 35 - 140$ cm, in the FDS simulations. Fig. 7a shows that C_{dw} was more sensitive to h_w in shallow water than in deep water. In the shallow water conditions, the keel may induce disturbances around the halocline, and internal waves can also be generated in the pycnocline in the real ice-covered ocean (Fer *et al.*, 2014; McPhee, 2002; Pite *et al.*, 1995), which will be considered in further studies. With the increase of D/h_w , the influence of h_w on the drag force becomes small compared to variations arising from α_w , as shown in Figs. 7b and 9. When $D/h_w \geq 14$ ($D = 140$ cm), C_{dw} tends to be independent of the water depth at a fixed slope angle.

Figure 7. (a) Variations of C_{dw} with h_w and D for $U=0.3$ m/s (turbulent regime) and $\alpha=45^\circ$, and (b) the dimensionless relationship between C_{dw} and D/h_w .

Thus, the FDS results in section 4.2 can be extended to large D/h_w . The water depth D was changed from 35 cm to 140 cm, and the updated deep water C_{dw} is shown

in Fig. 8, where the bottom boundary had only a minor effect on flow past the keel. The velocity profiles are shown in the Supporting information as Figure S1. C_{dw} tended to a stable limit with Re at a constant keel angle in the turbulent regime. However, the variation of C_{dw} with h_w was much less than in the shallow depth case (Fig. 6b) with $C_{dw} \sim 0.5/(D/h_w - 1)^2$. When $D = 140$ cm and $D/h_w \geq 14$, the variation of h_w had only a minor effect on C_{dw} . For example, C_{dw} increased by less than 1% from $h_w = 4$ cm to $h_w = 10$ cm with $\alpha_w = 20^\circ$. For larger α_w , h_w still had some influence on C_{dw} (Figs. 8e-f), because the sheltered area behind a sharp keel increased more with h_w compared with a gently sloping keel. At $\alpha_w = 45^\circ$, C_{dw} decreased from 1.25 to 1.1 for D/h_w increasing from 14 to 35. But the keel angle α_w still had a significant influence on C_{dw} , in a way that resembles the variation law shown in Fig. 6a.

Figure 8. C_{dw} results for deep water ($D/h_w \geq 14$) obtained from extended **FDSs** when $h_w=4$ cm, 6 cm, 8 cm, and 10 cm ($D = 140$ cm): (a) $\alpha_w = 10^\circ$, (b) $\alpha_w = 20^\circ$, (c) $\alpha_w = 30^\circ$, (d) $\alpha_w = 45^\circ$, (e) $\alpha_w = 60^\circ$, and (f) $\alpha_w = 90^\circ$.

According to the above analysis, the influence of the Re and h_w on C_{dw} in deep water is weak in turbulent flow. Next, we pay attention to the relationship between C_{dw} and the keel angle α_w . Fig. 9 illustrates that C_{dw} increased from 0.17 to 1.66 for $10^\circ < \alpha_w < 90^\circ$. The results in Fig. 9 include all extended **FDS** cases, each point calculated as the mean value over all cases with a constant slope. A logarithmic fit was determined as:

$$C_{dw} = 0.68 \cdot \ln(\alpha_w / 7.8^\circ), \quad 7.8^\circ < \alpha_w < 90^\circ. \quad (15)$$

449 The coefficient of determination was $R^2=0.998$, which was above the significance level
 450 (F -test, $p < 0.001$). Note that Eq. (15) is only valid for angles greater than 7.8° ,
 451 suggesting that for smaller angles $C_{dw} \approx 0$.

452

453

454 Figure 9. Relationship between C_{dw} and α_w in deep water conditions.

455

456 Field observations and underwater sonar data have shown that keel slope angle
 457 has a log-normal distribution and more than 90% of the measured angles were greater
 458 than 8° , with the mean value of 23.2° (Davis and Wadhams, 1995) and 26.6° (FY ice,
 459 Timco and Burden, 1997). With the α_w of 23.2° – 26.6° , the coefficient C_{dw} is 0.74–0.83
 460 according to Eq. (15). This range is close to the upper limit of the form drag coefficient
 461 of ridge sails, C_{da} , summarized as 0.2 – 0.8 (Garbrecht *et al.*, 1999). Banke *et al.* (1976)
 462 made field observations over the air-ice interface and obtained a linear relationship
 463 between C_{da} and α_a : $C_{da} = 0.012 + 0.012\alpha_a$. The different forms between air-ice and ice-
 464 water interface may result from the boundary layer structure of air and water, or the

number of sample points and the way of approach in the fitting. Anyway, the results obtained here show that C_{dw} is larger than C_{da} .

5 Discussions

Dimensional analysis for **LabE** and **FDS** ensured a straightforward derivation of the impact of the keel geometry on the local drag coefficient, $C_{dw} = f(Re, D/h_w, \alpha_w)$. In the asymptotic conditions with Re and D/h_w assumed to be very large, C_{dw} depends only on α_w as shown in Eq. (15). It is interesting to understand this new effect of keel geometry on the parameterization formulae of the total ice-ocean drag coefficient as Eqs. (2-5), and its application in sea-ice dynamic models.

5.1 Sea Ice Topography and Keel Drag Coefficient

Except for α_w , three ice geometry parameters are also included in the C_w and C_{Rw} formulae (Eqs. 2–5): the ice concentration A ($0 \leq A \leq 1$), the floe aspect ratio of draft to length d_w/L_f , and the ridging intensity h_w/L_r . Due to the form drag, the total drag force depends on ice concentration (Andreas *et al.*, 2010) in a nonlinear manner for both air-ice and ice-ocean interface (Lu *et al.*, 2011; Lüpkes *et al.*, 2012). A previous sensitivity study (Lu *et al.*, 2011) has shown that when $A = 0.8$, the ice-ocean drag coefficient C_w reached the maximum. Therefore, we selected $A = 0.8$ as the focus of interest.

The floe size parameters covary with thickness and may be occasionally related to ice concentration (Zhang *et al.*, 2015; Roach *et al.*, 2017). A rough approximation based on Mai (1996) was used by Lüpkes and Birnbaum (2005):

$$L_f = d_a \frac{31}{1-A}, \quad (17)$$

where d_a is the floe freeboard. Eq. (17) resembles the parametrization of floe length and freeboard in the marginal ice zone by Lüpkes *et al.* (2012). For an ice density of 918 kg/m³ (Doble *et al.*, 2011), the ratio of floe draft to freeboard is $d_w/d_a \approx 8.9$, and thus $d_w/L_f \approx 0.29 (1 - A)$. Using this relation, we chose a typical ice concentration A to determine d_w/L_f in the sensitivity analysis. Ice floe size affects the sea-ice thickness distribution via mechanical deformation processes (Mårtensson *et al.*, 2012; Roach *et al.*, 2017), including the ridged or deformed ice thickness distribution.

To investigate the effect of the keel geometry on the form drag, the ice concentration and floe aspect ratio were set as constants, $A = 0.8$ and $d_w/L_f = 0.06$ (from Eq. 17), and the keel geometry parameters h_w/L_r and α_w are the variables in the sensitivity studies. The magnitude of h_w/L_r varied from 0.001 to 0.1 and α_w varied from 10°-90° according to field observations (Davis and Wadhams, 1995; Brenner *et al.*, 2020). The relevant ice geometry parameters are listed in Table 5. The sensitivity of the total ice-ocean drag coefficient C_{Rw} and the keel drag coefficient C_w was investigated by covarying the ice geometry parameters and the function $C_{dw}(\alpha_w)$.

Table 5. Geometry parameters of sea-ice bottom.

Ice parameters	Bottom value
Keel slope angle	$\alpha_w = 10^\circ\text{-}90^\circ$
Ratio of keel depth to spacing	$h_w/L_r = 0.001\text{-}0.1$
Ratio of ice draft to floe length	$d_w/L_f = 0.06$

Ice concentration	$A = 0.8$
Local ice lateral coefficient	$C_e = 1$
Local form coefficient of a keel	$C_{dw} = 0.68 \ln(\alpha/7.8^\circ)$
Local ice frictional coefficient	$C_s = 0.002$

Figure 10a shows the variation of C_{Rw} with α_w and h_w/L_r when $A = 0.8$. Due to the influence of α_w on C_{dw} , C_{Rw} varied greatly, increasing quickly with increasing α_w for $10^\circ \leq \alpha_w \leq 50^\circ$ and $h_w/L_r \geq 0.01$, from 0.4×10^{-3} to 15×10^{-3} . A similar trend of C_{Rw}/C_w with α_w and h_w/L_r is shown in Fig. 10b. With the increase of h_w/L_r , C_{Rw} gradually becomes the main part in C_w . When $h_w/L_r \geq 0.01$ and $\alpha_w \leq 50^\circ$, the variation of C_{Rw}/C_w was highly dramatic because C_{Rw} is proportional the geometry-based $C_{dw}(\alpha_w)$.

Figure 10. (a) Variations of C_{Rw} with α_w and h_w/L_r when $A=0.8$; (b) Variations of

C_{Rw}/C_w with α_w and h_w/L_r when $A=0.8$.

5.2 Distribution of keel slope angle

Since the variation of C_{Rw}/C_w is sensitive to the variation of α_w for large ridging intensity, how to select a representative α_w to calculate C_{dw} is important. In the Arctic Ocean, the keel slope angle has been shown to follow the log-normal distribution (Davis and Wadhams, 1995; Timco and Burden, 1997). Based on this distribution, the averaged value of the local drag coefficient can be calculated as

$$\begin{aligned} \overline{C_{dw}} &= \int_8^{90} C_{dw}(\alpha_w) p(\alpha_w) d\alpha_w \\ &= \int_{0.03}^{2.45} 0.68 * \ln\left(\frac{\alpha_w}{7.8^\circ}\right) \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2} \left(\ln\left(\frac{\alpha_w}{7.8^\circ}\right) - (\mu - \ln 7.8)\right)^2\right] d \ln\left(\frac{\alpha_w}{7.8^\circ}\right) \\ &= 0.75, \end{aligned} \quad (16)$$

where $\ln(\alpha_w) \sim N(3.17, 0.0028)$, $\mu = 3.17$ and $\sigma = 0.0531$. The integration in the second line of Eq. (16) is simplified by the normality of $\ln(\alpha_w/7.8^\circ)$ and considered as the product of the factor of 0.68 and the rest of the integral equal to 1.11. Note that the integration limits of $\ln(\alpha_w/7.8^\circ)$ should be $(-\infty, +\infty)$ in the normal distribution. However, we found that the interval (0.03, 2.45) covered the main part of the integral and only very little was left for the rest beyond. The averaged value $\overline{C_{dw}}$ is 0.75, corresponding to a mean keel slope angle of 23.5° (Fig. 9). This value was larger than $\overline{C_{dw}} = 0.5$ in LU2011 and 0.13 – 0.39 in TS2014 (the coefficient in TS2014 can be converted by a factor of $2C_h/\pi \approx 0.76$ compared to this study, for details see Supporting information Text S1).

6 Conclusions

Laboratory experiments (**LabE**) and numerical fluid dynamics simulations (**FDS**) were conducted to study the fluid flow and drag force on a sea ice ridge keel in a tank with homogeneous fluid. The momentum flux from the fluid to the keel was determined through the form drag coefficient C_{dw} . In the laminar flow regime, the shape of the keel had minor effects, and C_{dw} was proportional to $Re^{-0.5}$, which is consistent with previous laboratory research results (Hoerner, 1965; Pite *et al.*, 1995). In the turbulent regime, the **FDS** had agreement with the **LabE**. Unlike laminar flow, C_{dw} was sensitive to the shape of the keel, described by the depth h_w and the slope α_w , with an asymptotic limit at very large Re .

To eliminate the effect of the bottom boundary of the tank, the **FDSs** were used to obtain C_{dw} for deep water. It was shown that for the water depth $D \gg h_w$, C_{dw} is asymptotically independent of h_w , but sensitive to α_w . A logarithmic function was used to describe the relationship: $C_{dw} = 0.68 \cdot \ln(\alpha_w/7.8^\circ)$, $10^\circ < \alpha_w < 90^\circ$. The range of C_{dw} was from 0.17 to 1.72, larger than the range of C_{da} as summarized by Garbrecht *et al.* (1999).

Based on the dependence of C_{dw} on α_w , this new effect was investigated in the sensitivity study of the keel drag coefficient C_{Rw} and the total ice-ocean drag coefficient C_w . When the ridging intensity was $h_w/L_r \geq 0.01$, C_{Rw} increased monotonously and quickly for α_w within $10^\circ - 50^\circ$. Pressure ridges in Arctic seas show large variability in morphology, and the keel slope angles are mainly within $10^\circ - 90^\circ$ (Davis and Wadhams,

1995; Timco and Burden, 1997; Obert and Brown, 2011; Strub-Klein and Sudom, 2012). With the increase of α_w , C_{Rw} and C_{Rw}/C_w also dramatically increase for large ridging intensity ($h_w/L_r \geq 0.01$). Then the selection of α_w is a key problem since α_w has not been an input parameter in most sea-ice models. However, we can use the drag coefficient averaged over α_w , $\overline{C_{dw}}$, as an approximation. Assuming the log-normal distribution of the keel angle, the average is $\overline{C_{dw}} = 0.75$, which is larger than the empirical constant of 0.5 used earlier in LU2011 or the range of 0.13 – 0.39 used in TS2014. Thus, we may further question the effect of the keel geometry distributions on C_{dw} . Multiyear ridges tend to have different shapes than first-year ridges, and there may be seasonal variations in that summertime ridges are smoother than wintertime ridges, as well as variations between different seas.

With the decay of Arctic sea ice in recent decades, the geometry parameters of sea ice have also been changing. In sea-ice dynamic models, the ice thickness distribution is adjusted by dynamic and thermodynamic processes. Thinner sea ice is more easily subject to deformation, resulting in sea-ice ridging or hummocking (Leppäranta, 2011). The redistribution of deformed and undeformed ice has an influence on ice morphology including the keel slope angle, and also the amount of ridging further affects the drift of sea ice. Therefore, the effect of the keel shape on the form drag coefficient is crucial in short-term sea-ice forecasting, as well as in climatological sea-ice models. Further work is needed to model the keel drag coefficient

for more general conditions including shallow mixed-layers due to salinity stratification
in summer.

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Data Availability Statement

The **LabE** measurements and **FDS** dataset in this research can be accessed via the
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On the form drag coefficient under ridged ice: Laboratory experiments and numerical simulations from ideal scaling to deep water

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Contents of this file

Text S1

Figure S1

Text S1. The difference in parameterization schemes between LU2011 and TS2014

Introduction

There are three factors in the difference between the two parameterization schemes, LU2011 and TS2014.

- the reference velocity;
- the sheltering function;
- the dimension assumption;

Text S1 considers the impact of these differences and compares definitions of the local drag coefficient of an individual keel C_{dw} between two schemes.

The reference velocity

Two schemes are based on the physical process of ice drift. The formulæ in schemes are derived from the quadratic drag law for a single floe or keel and their distribution per unit area. The first difference between Tsamados *et al.* (2014) and Lu *et al.* (2011) is the expression of quadratic drag law.

TS2014 defines the integral

$$F = \frac{1}{2} C_{dwz} \rho \int_{z_{0l}}^{h_w} U(z)^2 dz \quad (\text{S.1})$$

where $U(z)$ is depth-dependent upstream velocity under ice, z_{0l} is the hydraulic roughness length of smooth ice, C_{dwz} is the local drag coefficient at reference depth z , since total drag coefficient $\underline{C}_w$ in TS2014 is defined as,

$$C_w = \frac{\tau}{\rho U(10)^2} \quad (S.2)$$

where τ is the form drag per unit horizontal area, $U(10)$ is the fluid velocity relative at 10m depth. Due to the difference in velocity, there is some difference in the local drag coefficient C_{dw} between the two schemes. TS2014 uses the law-of-the-wall to assume the upstream velocity profiles. The velocity profile is described by the law of the wall,

$$U(z) = u_* / \kappa \ln(z/z_{0l}) \quad (S.3)$$

where u_* is the friction velocity under the smooth ice, κ is the von Kármán constant, and z_{0l} is roughness length of the smooth ice. The formula of $U(z)$ is used in the Eq. (S.1).

LU2011 adopts the far-field velocity in the term $U^2 h_w$. The relationship can be expressed as

$$F = \frac{1}{2} C_{dw} \rho_w U^2 h_w \quad (S.4)$$

where F is the force on the frontal area of a keel or floe per unit length, C_{dw} is the local drag coefficient, U is the velocity at reference depth, and h_w is keel or floe depth. In this study, we also use the far-field velocity to calculate the local drag coefficient.

In real condition, far-field velocity is ice speed relative to geostrophic current. There is a similarity structure in the ice-ocean boundary layer (IOBL), but it is more complex than the ideal turbulent plate boundary layer due to the Ekman layer in which the famous spiral structure exists (McPhee, 2008). Shaw *et al.* (2008) and MCPhee (2012) suggested the IOBL fluid dynamic similarity to address several factors that affect the ice-ocean drag relationship, including ice velocity relative to geostrophic velocity. According to the Rossby similarity (Blackadar and Tennekes, 1968) and the law of the wall, the final expression of dimensionless form considers velocity rotating in the IOBL,

$$\frac{U}{u_*} = \frac{1}{\kappa} \left[\ln \left(\frac{u_*}{f z_0} \right) - A_i \mp i B_i \right] = \frac{1}{\kappa} [\ln(Ro_*) - A_i \mp i B_i] \quad (S.5)$$

where f is the Coriolis parameter, $Ro^* = u_*/f z_0$ is the surface friction Rossby number, and $\mp i B_i$ is the imaginary component depending on the hemisphere (- north). Parameters A_i and B_i are empirical values to be determined from stress statistics and velocity measurements or calculated from dimensionless depth-invariant eddy viscosity. According to the statistic by MCPhee (1979; 1994; 2008, Ch. 9) and Shaw *et al.* (2008), $A_i = 2.2 \pm 0.2$, $B_i = 2.2 \pm 0.1$ are constants for neutral condition. The effect of Coriolis force is evident in the velocity direction, however, the velocity value varies little with the Coriolis force, i.e., the value of real part of dimensionless velocity in Eq. (S.5) is 15.4 and the absolute value is 16.3 ($u_* = 0.031$ m/s, $z_0 = 0.05$ m, the selection basis of the parameters will be explained later). In our study, the analysis only considers the absolute value of the ice-water stress assuming that the water current below the frictional boundary layer is zero.

For the same ice velocity U , u_* and z_0 should obey the relationship as Eq. (S.5). MCPhee (2012) has found that the dimensionless velocity U/u_* tends to stable with the increase of U at constant z_0 . In the case, we select two typical roughness length values of z_0 and z_{0l} respectively representing rough ice (including pressure keels) and smooth ice. According to the relevant study by MCPhee (2002) and Shaw *et al.* (2008), $z_0 = 0.05$ m and

$z_{01}=0.001\text{m}$ are used in the calculation. When $U=0.1\text{-}0.5\text{m/s}$, we obtain that $u_*=0.008\text{-}0.031\text{m/s}$ and $u_{*1}=0.005\text{-}0.020\text{m/s}$ according to Eq. (S.5). Combined Eqs. (S.4) and (S.5), we get

$$U^2 h_w = h_w \frac{u_*^2}{\kappa^2} [(\ln(Ro_*) - A_i)^2 + B_i^2] = h_w \frac{u_*^2}{\kappa^2} \Phi(Ro_*), \quad (\text{S.6})$$

$$\Phi(Ro_*) = (\ln(Ro_*) - A_i)^2 + (B_i)^2; \quad (\text{S.7})$$

According to the law-of-the-wall (Eq. S.3) used in TS2014, the integration in Eq. (S.1) can be written as follow,

$$\int_{z_{01}}^{h_w} U(z)^2 dz = h_w \frac{u_{*1}^2}{\kappa^2} \left(\ln\left(\frac{h_w}{z_{01}}\right) \right)^2. \quad (\text{S.8})$$

If LU2011 adopts the Eq. (S.2), the total drag coefficient of keels at 10m reference depth is obtained,

$$C_{rw} = f_2^{LU} C_{dw} = \frac{1}{\pi} C_{dw} \frac{U^2}{U(10)^2} \frac{Ah_w}{L_r} S^2, \quad (\text{S.9})$$

where S is sheltering function, A is ice concentration, h_w is keel depth, and L_r is keel spacing. C_{rw} at 10m reference depth in TS2014 is as follow,

$$C_{rw} = f_2^{TS} C_{dwz} = \frac{1}{2} C_{dwz} \frac{Ah_w}{L_r} \frac{U(h_w)^2}{U(10)^2} S^2, \quad (\text{S.10})$$

where the parameters are same as that in Eq. (S.9). Combined with Eqs. (S.1, S.6, S.8, S.9 and S.10), the relationship between C_{dw} and C_{dwz} is got as follow,

$$C_{dwz} = C_{dw} \frac{U^2}{U(h_w)^2} \frac{2}{\pi} = C_{dw} \frac{u_*^2 \Phi(Ro_*)}{u_{*1}^2 \left(\ln\left(\frac{h_w}{z_{01}}\right) \right)^2} \frac{2}{\pi} = C_{dw} C_h \frac{2}{\pi} \quad (\text{S.11})$$

where C_h is the conversion coefficient between C_{dwz} at keel depth and C_{dw} . Note that the term of $(\ln(h_w/z_{01}))^2$ in C_h varies little with h_w and z_{01} (Tsamodas et al., 2014). With parameters analyzed in above text ($z_0=0.05\text{m}$, $u_*=0.020\text{m/s}$; $z_{01}=0.001\text{m}$, $u_{*1}=0.013\text{m/s}$), $C_h=1.1\text{-}1.3$ corresponding with $h_w=5\text{-}10\text{m}$ (C_h increases with the decrease of h_w). Note that the relationship between u_* and z_0 under ice in this study is still simplified by ignoring the effect of keel depth h_w on z_0 , and further studies are needed. The factor of $2/\pi$ will be discussed later.

The comparison between TS2014 and LU2011 is made in the oceanic environment. TS2014 is designed for that, and we take our lab result for the keel form drag and use it for a real sea ice ridge. Eqs. (S.1) and (S.4) hold true whether there is Coriolis present or not, but implicitly the Coriolis effect must be in the background when we take the lab results to the oceanic environment for the reference depth at the bottom of the Ekman layer. Then, Eq. (S.6) shows the ice velocity comes from Rossby Similarity Theory when u_* and z_0 are known. Using a non-rotational model for U would not be realistic in ocean environment. As a whole, taking our lab experiment to nature, it is clear that Coriolis comes into the picture and Eq. (S.6) is a proper formula for $U^2 h_w$.

The sheltering function

S in Eqs. (S.9-S.10) is the sheltering function and its form is different in the two schemes. LU2011 adopted the quadratic attenuation function by Steele *et al.* (1989), whereas TS2014 adopted the exponential attenuation function by Hassen-Bauer and Gjessing (1988). The details are beyond the scope of this paper.

The dimension assumption

In addition to these two factors (C_{dw} at reference velocity and the sheltering function), there is another difference between the two schemes: the dimension assumption. TS2014 accounts for keel orientation in both the orientation distribution function and the mean ridging density in 2D domain, while LU2011 only accounts for it in the orientation distribution function in 1D domain. When this parameterization is applied to keels, the constant factor of TS2014 is $\pi/2$ times the value in LU2011, as shown in Eq. (S.11).

Conclusion

In general, as shown in Eq. (S.11), the factors of C_h and $2/\pi$ should be considered when the two schemes are compared. We select the C_h value about 1.2 in our comparison. Then the total factors of $2C_h/\pi$ is about 0.76, in other words, C_{dw} in LU2011 should multiply 0.76 compared with C_{dwz} in TS2014.

Figure S1

As in Fig. 5 in the main article, Fig. S1 shows a series of velocity profiles along the horizontal direction when $D/h_w=14$.

Figure S1. A series of velocity profiles along the horizontal direction, with $\alpha_w = 45^\circ$, $D/h_w=14$ and $U=0.3\text{m/s}$. (a) $x/h_w=-10$, (b) $x/h_w=0$, (c) $x/h_w=5$, (d) $x/h_w=15$, (e) $x/h_w=25$.