

# Calculation of current loop lines for the magnetic knot

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## Abstract

Rañada *et al.* (1998) proposed a magnetic-knot model for ball lightning. Their current-density vector will be represented in terms of field lines, whose equations are solved in a closed form under a very accurate approximation. This allows easy visualization of the current loops and, more important, their classification according to how many turns a given line requires to become closed. Relatively few lines of the infinity of possibilities will close after one or a few turns; for them, the available magnetic energy can be sufficient to maintain the current high enough to keep the loop conductive. Most loops never close and cannot be excited by the finite magnetic field. Each line is labeled by a number (of a continuum of real numbers), which is related in a simple way to the number of turns to close it. It is also noted that the concept of aerogel, proposed in earlier models, may offer a structural basis for the formation of the current lines. An aerogel filament, if it is formed of carbon or metal, may provide good enough conductivity at much lower temperatures than ionized air.

## 1. Introduction

Rañada *et al.* (1998) constructed a solution to Maxwell's equations which is represented by a magnetic field  $\mathbf{B}$  consisting of loops inside a sphere. The field is coupled to a current density  $\mathbf{j} = \nabla \times \mathbf{B} / \mu_0$ , which forms linked loops. In the ball lightning model, the current is assumed not to be continuously distributed but to be confined within very thin filaments (streamers) so that the total energy of the current system remains reasonably low. The purpose of this paper is to classify, or enumerate, the current lines according to how many turns a given line needs to close into a loop. Most of the lines will require an infinite number of loops, that is, they never close, and cannot sustain a current by the finite magnetic field. Some lines will close after one or a few turns, and these should carry the main current and provide the structure of the ball.

## 2. The magnetic-field and current-density vectors

Equations (1) and (2) reproduce here those of Rañada *et al.* (1998), apart from a normalizing constant:

$$\mathbf{B} = -(\sin^2(\pi R)/(\pi L^2 R^2)) [n \cos(\theta) \mathbf{e}_r - n\pi R \cot(\pi R) \sin(\theta) \mathbf{e}_\theta + \pi R \sin(\theta) \mathbf{e}_\phi] \quad (1)$$

$$\mathbf{j} = (\sin^2(\pi R)/(\pi \mu_0 L^3 R^3)) [-2\pi R \cos(\theta) \mathbf{e}_r + 2\pi^2 R^2 \cot(\pi R) \sin(\theta) \mathbf{e}_\theta + n \sin(\theta) (\pi^2 R^2 \cot^2(\pi R) - \pi^2 R^2 - 1) \mathbf{e}_\phi] \quad (2)$$

Here  $\mathbf{e}_k$  are the unit vectors of the spherical coordinates  $(r, \theta, \phi)$ ,  $R = r/L$  is the radius vector normalized by the sphere radius  $L$ , and  $n$  is called the linking number. The simplest case  $n = 1$  will

be assumed here.

The flow line of a vector field is defined by a set of equations which involves the ratios of coordinate differentials to the corresponding vector components. For the spherical-coordinate components of the current density, this is

$$dr / j_r = r d\theta / j_\theta = r \sin(\theta) d\phi / j_\phi \quad (3)$$

Denoting  $s = \pi R$ , which varies between 0 and  $\pi$  within the sphere, this becomes

$$- ds / (2s \cos(\theta)) = d\theta / (2s \cot(s) \sin(\theta)) = s d\phi / (s^2 \cot^2(s) - s^2 - 1) \quad (4)$$

The left equation integrates immediately into  $\sin(\theta) = k/\sin(s)$ , where  $k$  is an integration constant ( $0 < k < 1$ ). A value of this constant identifies a current line. The  $s$ - $\phi$  equation can now be written as

$$d\phi/ds = (1/2)[1 + 1/s^2 - \cot^2(s)] \sin(s) / \sqrt{[\sin^2(s) - k^2]} \quad (5)$$

It appears that the  $1/s^2$  part of the numerator cannot be integrated in a closed form. A closed solution for  $\phi$  is desirable because it greatly facilitates the determination of the role of  $k$ . It turns out that in the whole range of  $0 < s < \pi$  (but not outside it) the following approximation is extremely accurate:

$$[1 + 1/s^2 - \cot^2(s)] \sin(s) \approx (3/2 + 4/\pi^2) \sin(s) - (1/2)[1/\sin(s) - 1/\tan(s)] \quad (6)$$

Now each term of the right-hand side, multiplied by the square-root factor  $D(s, k) = 1/\sqrt{[\sin^2(s) - k^2]}$ , can be integrated in closed form, and the integral function  $\phi$  (apart from an additional constant), denoted by  $F(s, k)$ , becomes

$$F(s, k) = - (1/2)(3/2 + 4/\pi^2) \arctan(D(s, k) \cos(s)) \\ + (1/(4k)) \arctan(kD(s, k) \cos(s)) - (1/(4k)) \arctan(kD(s, k)) \quad (7)$$

### 3. Construction of current lines

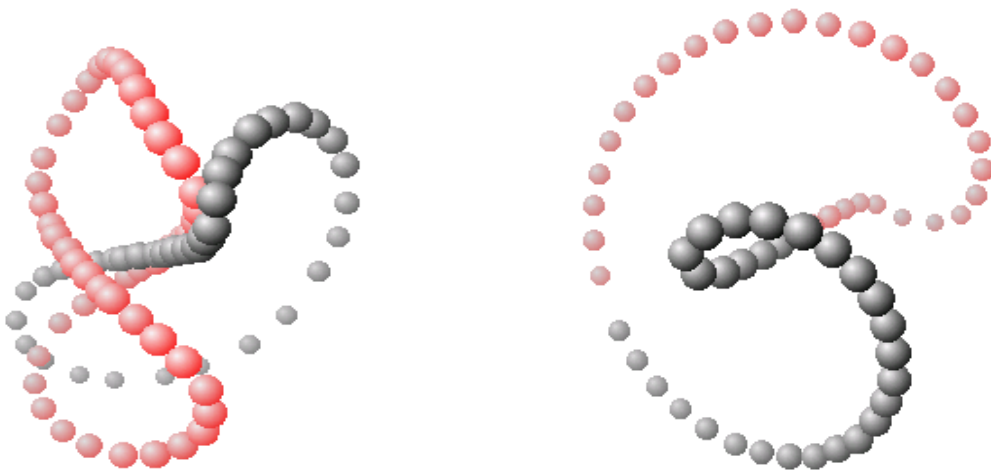
The smallest "radius"  $s$  for line  $k$  is given by  $\sin(s) = k$ , that is,  $\theta$  has the "equator" value  $\pi/2$ . The integrated  $\phi$  value is independent of  $k$  and is  $F(s_{\min}) = -(\pi/4)(3/2 + 4/\pi^2) = -87.5^\circ$ . In this case,  $\cos(s) > 0$  in Eq. (7). For  $s_{\max}$ ,  $\cos(s) < 0$  and the two first arctan functions are  $-\pi/2$ , so  $F(s_{\max}) = (\pi/4)(3/2 + 4/\pi^2) - (\pi/4k)$ . This angle varies widely with  $k$ , and if all lines are set continuous at  $s_{\min}$ , that is,  $\phi = F(s) - F(s_{\min})$  for  $\theta < \pi/2$  ("north") and  $\phi = -F(s) + F(s_{\min})$  for  $\theta > \pi/2$  ("south"), they do not generally meet at  $s_{\max}$ . To obtain a continuous loop, a north line is continued at the equator by a south line of the same  $k$  but a different  $\phi$  offset. It is obvious that the offset must be a rational fraction of  $360^\circ$  if the line is to close after a finite number of turns.

For lines with the same phase at  $s_{\min}$ , the phase shift (offset) at  $s_{\max}$  is  $2\pi m$  when

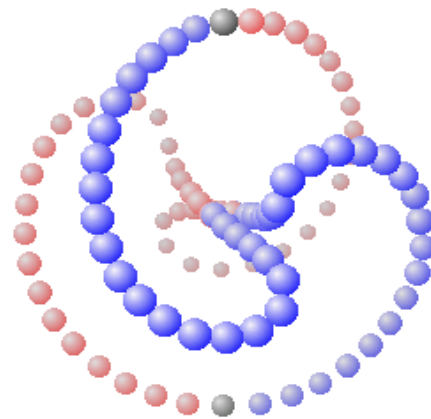
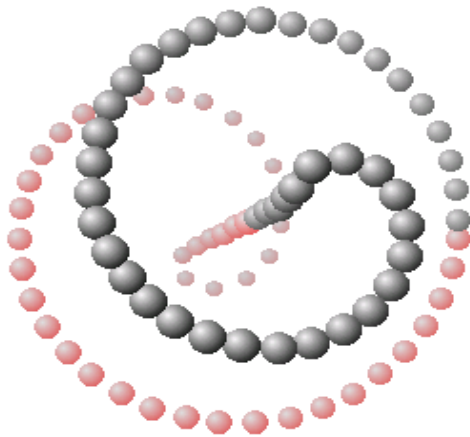
$$k = 1/[2(3/2 + 4/\pi^2) - 4m] \approx 1/(3.811 - 4m) \quad (8)$$

Note that  $m$  cannot be a positive integer because  $k$  is positive. All non-positive integers  $m = 0, -1, -2, \dots$  give a single closing loop, with  $k \approx 0.262, 0.128, 0.085, \dots$ . The small values of  $k$  mean that, when penetrating the equator, these loops accumulate near the center and near the surface of the sphere. Curve  $m = 0$  forms a simple deformed ring with an axial south-north part and an outer closing part which remains relatively meridional. Curve  $m = -1$  has also an axial part but is closed after a  $360^\circ$  zonal turn near the surface. Curve  $m = -2$  is similar to this but the zonal part makes two  $360^\circ$  turns. For larger negative integers, the current near the surface becomes more and more zonal

(parallel to the equator) and the number of full turns increases.

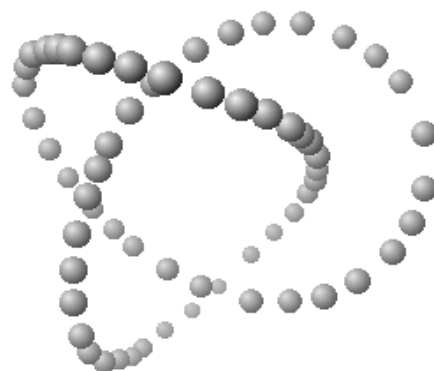
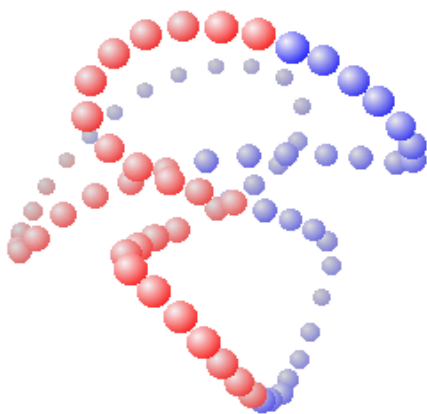


$m = 0, k = 0.262$  (black);  $m = 1/2, k = 0.552$  (red).  $m = -1, k = 0.128$  (color change: equator)



$m = -2, k = 0.085$

$m = -1/2, k = 0.172$



$m = 1/3, k = 0.404$

$m = 2/3, k = 0.874$

Fractional values of  $m$  require combination of more than one  $\phi$  solution to give closed loops; for half-integers such as  $1/2$  and  $-1/2$ , a solution is completed by a solution with a  $180^\circ$  offset. For  $1/3$ ,  $2/3$  etc., two offsets,  $120^\circ$  and  $240^\circ$ , are needed. Curves with  $m$  a half-integer contain two axial parts with corresponding parts closer to the surface. For  $m$  a third-integer there are three axial currents, and the connecting close-to-surface parts are correspondingly more complicated. The seven cases mentioned are illustrated in the figure. The continuous current flow lines have been computed at discrete points, and a point is represented by a sphere, the size of which varies with the distance from the observer to enhance the three-dimensional impression. Case  $m = -1/3$  (not shown) has three axial lines like  $m = 1/3$ , but it has a longer zonal part like the other cases with negative  $m$ .

Any pair of loops with different  $k$  values are linked, as exemplified in the top left part of the figure. This is partly guaranteed by the fact that for smaller  $k$ , the curve crosses the equator closer to the axis, and the crossing angle also varies with  $k$ .

The loops in the figure are the simplest ones. For values of  $m$  which are not small integers or simple fractions, the current loops are more complicated and involve longer paths which are more unlikely to be excited or maintained by the magnetic field. Even the simplest loops form together a very tangled current system. The initial conditions may also have an effect on which loops are generated most easily: if the ball is excited by a linked pair of orthogonal circles, a pair like  $m = 0$  and  $m = 1/2$  (the first in the figure) might be created most easily.

#### 4. Magnetic field lines

The field lines of the magnetic field  $\mathbf{B}$  in Eq. (1) can be obtained in a similar way as the flow lines of  $\mathbf{j}$ , and the equations are integrated readily, without need for approximations, to give  $\sin(\theta) = k/\sin(s)$  and  $F(s, k) = \ln(\sin(s)) + \text{const}$ . These lines are simple loops near a meridional plane, and it appears that the larger  $k$ , the smaller the loop, and loops with different  $k$  are not linked. All possible loops form a toroidal tube structure whose central circle lies at the equatorial plane and encloses the sphere's axis. It seems that the relatively small deviations of each field line from a meridional plane suffice for the current density to be linked.

In a previous article, Rañada and Trueba (1996) discuss a different magnetic field given in  $xyz$  coordinates; in spherical coordinates, it is (apart from a normalizing constant)

$$\mathbf{B} = (4/(2\pi L(1 + R^2)^3)) [-(1 + R^2)\cos(\theta)\mathbf{e}_r + (1 - R^2)\sin(\theta)\mathbf{e}_\theta - 2R \sin(\theta)\mathbf{e}_\phi] \quad (9)$$

The field-line equations for this field are easily solved, and it turns out that for each value of the integration constant, a pair of linked, nearly perpendicular simple loops are obtained. For two different constants, all four loops are linked (Fig. 1 in Rañada and Trueba (1996) shows three of these). These lines are, however, not confined within a sphere; for some values of the integration constants, the loops become arbitrarily large. The flow lines of the corresponding current density are more difficult to integrate and are not attempted here.

#### 5. Aerogel as a current conductor?

Based on earlier Russian studies, Smirnov (1987) suggests that during a discharge which would give rise to a ball lightning, small metal (or carbon) particles may be joined into thin filaments or *aerogel* by the strong current. Such a discharge could be provided by an upward positive leader which attempts to reach the negative stepped leader of a negative flash but misses the contact in favor of another upward leader. The head of the unlucky positive upward leader seems to be a good candidate for a ball lightning. If the leader originates from, say, a cable loop of a telephone pole, or from a tangle of tree branches, it may provide vaporized metal or carbon and a favorable initial geometry.

An aerogel fiber would grow along the generating current, thus providing a natural conductor for the current which remains after the formation. In addition, burning of the aerogel may release

energy which helps keeping the ball hot for a long enough time.

## 6. Suggestion for experiments

If ball lightning is a real physical phenomenon, it should be possible, sooner or later, to reproduce it in the laboratory. It is unlikely to involve any exotic features or extreme conditions which could not be provided in a laboratory. The magnetic-knot model suggests that the linked-current structure is mathematically sound and easily understandable, but the initial conditions to excite it may occur rarely.

A straightforward method would be to try to feed strong current pulses into a circuit which consists of linked wire loops. A pulse should be strong enough to vaporize the wire at least partly; perhaps water should also be present as suggested by Turner (1994). There must be enough space for the ball to develop without contact to walls: the upward leader may require several meters to advance until its head reaches sufficient size for a stable looped aerogel structure to form. If the ball remains too small, the loops may disturb each other and their linking breaks down.

## 7. Discussion

No theory should be taken for granted before it can be verified by experiments. This is especially true for ball lightning, for which mere eyewitness observations provide insufficient basis for theory. The only hope is a repeatable production of ball lightning in the laboratory, and once successful – if ever – it can be measured to test a theory. The magnetic-knot model offers a conceptually simple case worth attempting to study in well-arranged experiments.

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