In GARCH literature a symmetric conditional probability distribution is often assumed. As the financial data, such as stock market returns, often have both peaked and skewed distribution, this assumption can be too restrictive. We expand this by allowing for the conditional distribution to be asymmetric or skewed. Four distributions that have a normal variance-mean mixture representation are considered. Three of these distributions belong to the class of generalized hyperbolic (GH) distributions. The GH distribution is obtained by assuming that the (unobserved) mixing variable has the generalized inverse Gaussian (GIG) distribution.

As the GH distribution is regarded as too broad for GARCH modeling we employ its three special cases. These distributions are obtained by assuming that the mixing distribution has inverse Gaussian (IG), Gamma or reciprocal Gamma (RG) distribution. Resulting distributions are called normal inverse Gaussian (NIG), normal Gamma (NG) and normal reciprocal Gamma (NRG) distributions. In addition, we apply the so-called z distribution. This distribution is obtained by assuming that the mixing distribution has infinite convolution of exponentially distributed random variables.

In empirical applications employing two real stock market indices, it was first discovered that statistically significant estimates of the parameters of the skewed distributions were actually obtained. Especially, all the estimates implied negatively skewed conditional distributions. Most of the models based on skewed conditional distributions were also superior to the model based on the symmetric t distribution according to standard model selection criteria.

According to plots of the densities of the standardized residuals against the theoretical densities, skewed distributions have better ability to capture the tail behaviour of the financial data than the symmetric t distribution. Advantages of the skewed distributions were also observed through Value-at-Risk (VaR) applications, where the correct shape of the (left) tail of the distribution is of concern. We conclude that allowing for conditional skewness should be taken into consideration in GARCH modeling.

The GARCH-in-Mean model with z distribution was introduced by Lanne and Saikkonen (2007). Generalized hyperbolic distributions are overviewed for example in Eberlein and v. Hammerstein (2003). The calculation of the modified Bessel functions was enabled by Abramowitz and Stegun (1970) and especially Spanier and Oldham (1987).

Avainsanat-Nyckelord-Keywords
- generalized autoregressive conditional heteroskedasticity
- skewness
- normal variance-mean mixture
- generalized hyperbolic distributions
- stock markets
- stock market indexes

Säilytyspaikka-Förvaringsställe-Where deposited

Muita tietoja-Övriga uppgifter-Additional information