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LIQUIDITY PROVISION AND OPTIMAL BANK REGULATION
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JEL Classification: G21, G28

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http://www.hanken.fi/hanken/eng/page1579.php

SHS intressebyrå IB (Oy Casa Security Ab), Helsingfors 2004

ISBN 951-555-816-6
Liquidity Provision and Optimal Bank Regulation

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February 23, 2004

Abstract

We extend the set of regulatory instruments for banks’ liquidity provision by adding the fraction of perfectly-liquid accounts. We demonstrate how this instrument induces self-selection on behalf of the depositors who are differentiated according to their probability of facing a liquidity shock. This self-selection leads to a market segmentation, which can break the bundling of deposits with risk and thereby enhance social welfare. The optimal regulatory policy is explicitly characterized as a function of banks’ investment return, the return of depositors realizing a liquidity shock, and the costs borne by depositors who lack a sufficient amount of liquidity.

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(Draft = narrowsimpl4.tex 2004/02/23 19:03)
1. Introduction

In recent years we have witnessed a rapid worldwide consolidation process of the banking industry. The securitization of markets has broken the traditional link between maintaining deposits and making loans. Nowadays, large corporations have direct access to international capital markets at terms which might often outperform bank funding. Thus, banks have to operate in a world where the markets for financial services – banking, brokerage as well as insurance – have become increasingly competitive. This development has serious consequences from the point of view of traditional banking, since, as Kay (1998) wrote, “The rational for the traditional association of functions that we call a bank has simply disappeared, and most of these specific functions – retail marketing of financial services, financial advice to companies, monitoring the creditworthiness of large companies – are better done by some specialist institution that is not necessarily a bank.” Also, with the vast development of money market funds, insurance investment, and investment via brokerage firms in the United States, Europe and Asia, and with the fast globalization of investment opportunities, consumers do not suffer from any lack of investment opportunities and startup firms can raise capital in a wide variety of markets without approaching banks. In fact, given the large number of money market funds and financial instruments existing today, consumers can always pick such a ratio of risk and return that exactly match their degree of risk aversion.

Faced with intensified competition from other types of financial markets, banks presently make use of their retail deposit base as a collateral for risky investment activities in securities markets such as trading, market making and placing as well as for traditional lending to illiquid long-term investment projects. In this way the banks effectively bundle their deposit base with their risky investment activities. This bundling of deposits with risk generates a market failure, which is supported, and not corrected, by the regulatory environment where the banks’ risks are effectively underwritten by the governments, and ultimately by the taxpayers. This market failure applies in particular to the banks which are large enough to enjoy full protection under the prevailing “too-big-to-fail”-doctrine [see, for example, Feldman and Rolnick (1998)].
Basically, the banks' bundling of deposits with illiquid and risky investment activities generates a welfare distortion, because those depositors who wish to utilize bank services associated with deposit accounts may be willing to pay an extra fee to the bank in return for perfectly liquid and risk-free accounts. This market failure cannot be corrected within a regulatory framework with the control over the reserve requirement as the sole instrument for restricting the banks’ risk exposure.

In the present article we extend the set of instruments for the regulation of banks by adding the fraction of perfectly-liquid accounts as a new regulatory instrument. By adding this instrument within the framework of a model with depositors differentiated according to their probability of facing a liquidity shock, the market failure generated by the bundling of deposits with risk can be reduced or eliminated. Based on depositor-specific private information each depositor can decide which type of account to use. Thus, self-selection on behalf of the depositors will induce a market segmentation, which can break the bundling of deposits with risk. Our analysis will delineate those circumstances under which the addition of perfectly-liquid deposit accounts will represent a Pareto improvement relative to a world where risky accounts represent the only channel whereby consumers can enjoy the deposit services offered by the banking industry. Furthermore, we characterize the optimal regulatory policy when the set of policy instruments includes the fraction of perfectly-liquid accounts in addition to the traditional reserve requirements applied on risky accounts. In particular, we outline the conditions under which the currently applied policy of applying only reserve requirements is optimal.

The existing literature views depository institutions as “pools of liquidity” providing households with insurance against idiosyncratic shocks that affect their consumption needs. In an influential model by Diamond and Dybvig (1983) banks provide liquidity to depositors, who are, ex ante, uncertain about their intertemporal preferences over consumption sequences [see also Bryant (1980) and Villamill (1991)]. In the Diamond-Dybvig model demand deposits are needed because liquidity shocks are not publicly observed and therefore cannot be insured. They demonstrate how deposit contracts offer insurance to households
and how such contracts can potentially induce a Pareto efficient allocation of risk. Subsequently, Diamond and Rajan (2001) have designed a model in which they defend the bundling of banks’ deposits with illiquid and risky investment activities. The argument of Diamond and Rajan builds on the view that banks have specific collection skills with respect to the illiquid projects in their outstanding loan portfolio. Demand deposits represent a commitment mechanism making sure that the banks have incentives to create liquidity when a sufficiently large fraction of the depositors face liquidity needs.

It is well known the interaction between pessimistic depositor expectations might generate bank runs as an inefficient Nash equilibrium within the framework of the Diamond and Dybvig (1983) model. As Diamond and Dybvig originally pointed out, deposit insurance systems can eliminate such inefficient Nash equilibria. Despite the indisputable insurance benefits, empirical observations as well as theoretical research convincingly demonstrate how federal deposit insurance will encourage banks to engage in excessive risk taking [see, for example, Cooper and Ross (1998)]. For that reason researchers have systematically investigated mechanisms other than deposit insurance as instruments for reducing the instability of the banking system. In line with Freixas and Rochet (1997), the adoption of narrow banking seems to be the most natural mechanism to eliminate this instability.\footnote{Narrow banking refers to regulatory systems where the banks are required to back demand deposits entirely by safe and liquid short-term assets.}

The research contributions evaluating the consequences of narrow banking have typically conducted comparisons between the polar cases of complete narrow banking and risky banking operating under reserve requirements, which determine the boundary conditions of the banking activities.\footnote{See, for example, Friedman (1960, Chap.3) or Wallace (1996).} In the present analysis we contribute to this fundamental debate by introducing the possibility of designing banking systems where one fraction of the banking activities are required to obey the principles of narrow banking, whereas the complementary fraction operates as risky banking controlled by an optimally-determined reserve requirement. We ask the following question: What is the optimal combination of narrow banking and risky banking supported by reserve requirements? We also explore the consequences of...
deposit insurance systems on the optimal regulation of the banking industry.

Our study is organized as follows. Section 2 presents the model. In Section 3 we calculate the equilibrium interest rates and fees. Section 4 characterizes the optimal regulatory policy for supporting a socially optimal system of liquidity provision by banks. In Section 5 we explore the consequences of deposit insurance systems for optimal regulation. Finally Section 6 offers some concluding comments.

2. The Model

Consider an imperfectly-competitive banking industry with two banks labeled bank A and bank B. There are 3 periods. In period 1 banks determine their fees, and depositors determine which bank to make a deposit with. In period 2 some depositors realize a liquidity need and attempt to withdraw their entire deposit. In period 3 banks collect the return on their investments, liquidate all accounts, and pay interest on interest-bearing accounts.

2.1 Depositors

There is a continuum of uniformly distributed depositors indexed by the pair \((\lambda, x)\) on the unit square \([0, 1] \times [0, 1]\). This captures depositors who are differentiated along two dimensions. The banks are horizontally differentiated. Bank A located at \(x = 0\), whereas bank B is located at \(x = 1\). Each depositor deposits $1 either in bank A or bank B. The index \(x\) measures the disutility (transportation cost) associated with making a deposit with bank A, whereas \((1 - x)\) measures the disutility associated with banking at B. The index \(\lambda\) measures depositors’ probability of realizing a liquidity need in period 2.

Let \(\beta\) denote a depositor’s basic utility derived from the services obtained by opening a bank account and making the $1 deposit. The variables \(i_A, i_B, f_A, f_B\) denote the deposit rates and the fees applied on each $1 deposit, for banks A and B, respectively. Let \(\theta_A\) and \(\theta_B\), where \(0 \leq \theta_i \leq 1\), denote the endogenously-determined per-depositor amount of money available for withdrawal in period 2 upon realizing a liquidity need. Finally, let \(v\) denote the
value of the opportunity faced by a depositor realizing a liquidity need in period 2. That is, higher value of $v$ makes an earlier withdrawal more beneficial to depositors. Altogether, the expected utility of a depositor indexed by $(\lambda, x)$ is given by

$$U_{\lambda,x} \overset{\text{def}}{=} \begin{cases} \beta - \tau x + \lambda(\theta_A)^\gamma v + i_A - f_A & \text{Deposits with bank } A \\ \beta - \tau (1 - x) + \lambda(\theta_B)^\gamma v + i_B - f_B & \text{Deposits with bank } B. \end{cases}$$

Thus, if banks maintain 100% reserves then $\theta_i = 1$. In this case depositors who realize a liquidity need (probability $\lambda$) can fully gain a utility of $v$ by making an early withdrawal of their $1$ investment. In contrast, if banks lend out some or all of the amount deposited, depositors who face liquidity needs can withdraw only $\theta_i < 1$ and thus gain a utility of $(\theta_i)^\gamma v$ from early withdrawal. The parameter $\gamma \geq 1$ reflects an increased marginal benefit for more funds available for withdrawal upon realizing a liquidity need. Higher values of $\gamma$ indicate that if a liquidity need is realized, withdrawing a small amount yields low benefits relative to withdrawing the full amount of $1$. In other words, the parameter $\gamma \geq 1$ captures that there are increasing returns to scale on projects for which a liquidity constraint means that the project can be implemented only partially. Finally, $\tau > 0$ is the standard Hotelling’s differentiation (transportation cost) parameter. Low values of $\tau$ capture situations with intense competition between the banks.

2.2 The regulator

The regulator of the banking industry imposes two restrictions on banks. The first, denoted by $\rho$ ($0 \leq \rho \leq 1$), is the commonly practiced reserve requirement. That is, $\rho$ is the fraction of risky deposits which the bank must keep liquid, whereas $(1 - \rho)$ is the fraction of risky deposits that banks use for lending activities towards projects which are illiquid in the short run.

The second policy instrument, denoted by $\delta$ ($0 \leq \delta \leq 1$) is the maximal fraction of accounts on which banks can maintain the minimal reserve requirement. Thus, $(1 - \delta)$ is the

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$^3$This could capture the deposit-specific opportunity of buying an urgent durable consumption good, of making an investment at favorable terms or of exploiting a good business opportunity. Models characterizing optimal bank regulation frequently do not take this feature into account. As we will see later on, the presence of this feature has important consequences for the optimal design of bank regulation.
fraction of accounts that must be kept 100% liquid. Accordingly, we will make use of the following terminology.

**Definition 1**

We say that the regulator allows/mandates

- **Risky banking** if \( \delta = 1 \);
- **Narrow banking** if \( \delta = 0 \), and
- **Mixed banking** if \( 0 < \delta < 1 \).

Risky accounts (subjected to a minimum reserve requirement \( \rho < 1 \)) constitute what is widely observed in today’s private banking. Current banking regulation does not operate with the fraction \( \delta \) as a policy instrument. However, the academic literature on banking regulation has to a large extent focused on comparisons of narrow banking systems with banking systems operating with a minimal reserve requirement as the only policy instrument. In this respect our analysis represents a more ambitious research task insofar as our goal is to characterize the socially optimal combination of the instruments \( \rho \) and \( \delta \) as the basis for the design of banking regulation. In particular, we will specify circumstances under which mixed banking Pareto dominates risky banking for all possible levels of mandated minimum reserve requirements.

2.3 Banks

Banks A and B set their interest rates, \( i_A \) and \( i_B \), and fees, \( f_A \) and \( f_B \), subject to the regulator’s imposition of the fraction of risky accounts, \( \delta \), and the minimum reserve requirement, \( \rho \), for risky accounts. Let \( r \) denote a bank’s return on an outside investment project.

The utility function (1) implies that the interest paid on a $1 deposit, \( i_A \) and \( i_B \), is partially offset by the fees levied by the banks, \( f_A \) and \( f_B \). Therefore, it is sufficient to assume that banks will pay interest on risky accounts without having to levy any fee, but will not pay any interest on for-fee perfectly-liquid accounts. More precisely, let \( q_j^R \) and \( q_j^F \)
denote the number of risky and liquid accounts opened with bank $j$, respectively. Then, each bank $j$ chooses interest paid on risky and fee levied on liquid accounts to solve

$$\max_{i_j, f_j} \pi_j = [(1 - \rho) r - i_j] q_j^R + f_j q_j^L \quad \text{for} \quad j = A, B.$$  (2)

The first term measures the profit bank $j$ earns from risky accounts. On each risky account the bank’s return is determined by the difference $(1 - \rho) r - i_j$, which reflects that the fraction $\rho$ has to be held as liquid reserve. The second term in (2) measures the revenue collected from the fees on perfectly-liquid accounts.

3. Equilibrium Interest Rates and Fees

Figure 1 illustrates a possible out-of-equilibrium allocation of depositors between the banks and their choice of whether to open a risky account or a liquid account.\(^4\)

In view of Figure 1 and the utility function (1), depositors who choose to open a risky account and are indifferent between banks $A$ and $B$ are indexed by

$$x^R = \frac{1}{2} + \frac{\lambda v (\theta_A^* - \theta_B^*) + i_A - i_B}{2\tau}.$$  (3)

In Figure 1, $x^R$ is drawn as a function of the depositors’ probability of facing liquidity shocks for an out-of-equilibrium case where $i_A > i_B$ and $\theta_A < \theta_B$, just for the sake of illustration. Clearly, depositors indexed close to $\lambda = 0$ have a low probability of realizing liquidity needs and therefore compare mainly the interest rates, $i_A$ and $i_B$, paid by the banks. Therefore, for $\lambda = 0$, more depositors will choose bank $A$ than $B$ if and only if $i_A > i_B$.

However, as $\lambda$ increases towards $\delta$, depositors also compare the available funds for withdrawal upon realizing liquidity needs as given by $\theta_A$ and $\theta_B$. Figure 1 assumes that $\theta_A < \theta_B$ so depositors with high probability of realizing a liquidity need find bank $A$ less attractive, hence, $x^R$ is declining with $\lambda$.

Substituting $\theta_A = \theta_B = 1$ into the utility function (1) implies that depositors who choose

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\(^4\)Figure 1 is out-of-equilibrium also in the sense that $\delta$ is not the proportion of risky accounts as mandated by the regulator.
3.1 Banks’ optimization

In view of Figure 1, the number of depositors opening risky and liquid accounts with each bank are

\[
q^R_A = \int_0^\delta x^R d\lambda, \quad q^L_A = (1 - \delta)x^L, \quad q^R_B = \int_0^{1 - x^R} d\lambda, \quad \text{and} \quad q^L_B = (1 - \delta)(1 - x^L),
\]

where \(x^R\) and \(x^L\) are given in (3) and (4), respectively.

\[x^L = \frac{1}{2} + \frac{f_B - f_A}{2\tau}.
\]
Now, the imposed regulation that no more than $\delta$ accounts can be risky, implies that depositors indexed by $\lambda = \delta$ must be indifferent between opening a risky and liquid accounts. Therefore, substituting $\lambda = \delta$ into the utility function (1), we have $\beta - \tau x + i_A + v\theta_A^\gamma \delta = \beta - \tau x - f_A + v\delta$ for bank $A$, and analogously for bank $B$. Hence, the fee levied on liquid accounts in bank $i$ can be expressed as

$$f_j = \delta v(1 - \theta_j^\gamma) - i_j, \quad \text{set by bank } j = A, B.$$  \hfill (6)

Substituting (6) into (4), we obtain

$$x^L = \frac{1}{2} + \frac{\delta v(\theta_A^\gamma - \theta_B^\gamma) + i_A - i_B}{2\tau} \tag{7}$$

Substituting (3), (6), and (7) into (2), each bank $j$ chooses a single variable, the interest rate paid on risky accounts, to solve,

$$\max_{i_j} \pi_j = [(1 - \rho)r - i_j] \int_0^\delta \left[ \frac{1}{2} + \frac{\lambda v(\theta_j^\gamma - \theta_k^\gamma) + i_j - i_k}{2\tau} \right] d\lambda$$

$$+ [\delta v(1 - \theta_j^\gamma) - i_j] \left[ \frac{1}{2} + \frac{\delta v(\theta_j^\gamma - \theta_k^\gamma) + i_j - i_k}{2\tau} \right], \tag{8}$$

where $j, k = A, B$ and $j \neq k$. The profit function (8) is strictly concave in $i_j$ since $\frac{\partial^2 \pi_j}{\partial (i_j)^2} = -1/\tau < 0$. The first-order conditions are given by

$$0 = \frac{d\pi_j}{di_j} = \frac{\delta v[\theta_A^\gamma (3\delta - 4) + \theta_B^\gamma (2 - \delta)]}{4\tau} - \frac{\delta^2 v + \delta[\rho(\rho - 1)] + 2i_j - i_k + \tau}{2\tau}. \tag{9}$$

3.2 Expectation formation for withdrawable funds

In order to solve for the equilibrium interest rates, $i_A$ and $i_B$, we must first compute $\theta_A$ and $\theta_B$. Therefore, we make the following assumption.

**Assumption 1**

*Depositors have perfect foresight for $\theta_A$ and $\theta_B$. That is, depositors are able to compute the equilibrium amount of money each depositor in a risky account can withdraw upon realizing a liquidity need.*
Figure 2: Equilibrium allocation of depositors, and expected withdrawals from risky accounts.

Figure 2 illustrates the allocation of depositors among banks and accounts in a symmetric equilibrium.

Denote by $w^R_j$ the expected number of depositors with bank $j$’s risky accounts who realize a liquidity need. Formally, in a symmetric equilibrium we have

$$w^R_A = \delta \int_0^\lambda x^R \, d\lambda, \quad w^R_B = \delta \int_0^\lambda (1 - x^R) \, d\lambda, \quad \text{hence} \quad w^R_A = w^R_B = \delta \int_0^\lambda \frac{\lambda}{2} \, d\lambda = \frac{\delta^2}{4}. \quad (10)$$

We start by investigating bank $A$. The expected number of withdrawals from bank $A$’s risky accounts, $w^R_A$, is drawn in Figure 2 as a triangle (which is the area under the line $x = \lambda x^R$). Next, the number of risky accounts (also equals to the amount of money deposited into risky accounts) is $q^R_A = 0.5\delta$. However, since the banks maintain only a fraction $\rho$ of the funds deposited into risky accounts, only $\rho q^R_A$ dollars are available for withdrawal from risky accounts. Therefore, the expected amount of money that can be withdrawn by each depositor
who deposits each $1 in bank A’s (similarly, in bank B) risky account is

\[ \theta_A = \theta_B = \begin{cases} \rho q_R^A w_R^A = \rho 0.5 \delta = \frac{2 \rho}{\delta} & \text{for } \rho \leq \frac{\delta}{2} \\ 1 & \text{for } \rho > \frac{\delta}{2} \end{cases} \]  

Equation (11) and the utility function (1) imply that there is no benefit from establishing a reserve requirement \( \rho > \delta/2 \). This means that the regulator’s choice of socially optimal reserve ratio analyzed in Section 4 can be restricted to the interval \([0, \delta/2]\), instead of \([0, 1]\).

3.3 Equilibrium interest rates and fees

Substituting (11) into (9), the equilibrium interest rates paid on risky accounts are

\[ i_A = i_B = \delta \left[ v + (1 - \rho)r \right] - \delta^2 v - \delta v (1 - \delta) \left( \frac{2 \rho}{\delta} \right)^\gamma - \tau. \]  

Observe that we do not rule out negative interest rates, in which case the interest turns into a fee paid on risky accounts. Next, substituting (12) into (6) and into (8) yields the banks’ fee levied on each liquid account, and the equilibrium profit levels. Thus,

\[ f_A = f_B = \delta^2 v + \tau - (1 - \rho) \delta r - \delta^2 v \left( \frac{2 \rho}{\delta} \right)^\gamma \quad \text{and} \quad \pi_A = \pi_B = \frac{\tau}{2}. \]  

Therefore, the profits made by the banks are independent of the regulation parameters \( \rho \) and \( \delta \) (the reserve requirement and the maximum fraction of risky accounts). This invariance holds true because as all depositors are served (i.e., there is no reservation utility) banks manage to readjust their interest and fees so that their profit is maximized at a constant level (equals to the profit level obtained in the one-dimensional Hotelling model).

The following proposition demonstrates how the equilibrium interest rates and fees given in (12) and (13) are affected by regulation and the parameters of the model.

**Proposition 1**

In a duopoly banking industry where banks compete on interest rates and fees, for every bank \( j = A, B \),

(a) An increase in the degree of competition increases equilibrium interest rate, and reduces equilibrium fees. Formally, \( di_j/d\tau < 0 \) and \( df_j/d\tau > 0 \).
(b) An increase in the reserve requirement of risky accounts reduces the equilibrium interest rates paid on risky accounts. However, the effect on the fees levied on liquid accounts is ambiguous. Formally, \( \frac{d_i}{d\rho} < 0 \) and \( \frac{d_f}{d\rho} \geq 0 \), if and only if \( r \geq 2\gamma v(\delta/\rho)^{1-\gamma} \).

(c) Interest rates increase whereas fees decrease with the return on banks’ investment. Formally, \( \frac{d_i}{dr} > 0 \) and \( \frac{d_f}{dr} < 0 \).

Proposition 1(a) is rather intuitive, since competition is reduced when \( \tau \) increases, thus equilibrium fees are raised and interest rates are lowered. Part (b) demonstrates that those who deposit into a risky account are willing to trade higher reserves for lower interest. In other words, higher reserves weaken competition between the banks. Part (c) implies that banks raise interest and lower fees when they earn higher returns on their investments, thus, sharing the extra gains with depositors. In other words, a higher return on banks’ investments intensifies the competition between the banks.

4. Regulation and Social Welfare

In this section we approach the climax of our analysis. We characterize the socially optimal combination of the two regulatory instruments: (a) The reserve requirement on risky accounts, \( \rho^* \); and (b) the mandated maximum fraction of risky accounts, \( \delta^* \). In particular, we characterize the conditions under which it is socially optimal to require that banks maintain a certain fraction of accounts one-hundred percent liquid.

Since banks’ profits given in (13) are invariant to the regulator’s policy control variables, \( \rho \) and \( \delta \), maximizing social welfare is equivalent to maximizing aggregate surplus of depositors. We define aggregate depositor surplus as the sum of depositors’ utility levels. From (1) we have

\[
DS \overset{\text{def}}{=} 2 \int_0^{\delta} \int_0^{0.5} [\beta - \tau x + i_A + v(\theta_A)^{\gamma} \lambda] \, dx \, d\lambda + 2 \int_0^{0.5} \int_0^{1} [\beta - \tau x - f_A + v\lambda] \, dx \, d\lambda. \tag{14}
\]

The first term in (14) is the aggregate surplus of depositors with risky accounts (we multiply by “2” to add the utility of depositors with bank \( B \)). The second term is the aggregate surplus from liquid accounts (where \( \theta_A = \theta_B = 1 \) since all deposits are available for withdrawal).
Substituting (13) and (12) and then (11) into (14), the social planner chooses $\rho^*$ and $\delta^*$ to solve

$$\max_{\rho, \delta} DS = 2^{\gamma - 1}\delta^2 v \left(\frac{\rho}{\delta}\right)^\gamma + \frac{4\beta - 2\delta^2 v + 4\delta r (1 - \rho) - \tau + 2v}{4}.$$ (15)

### 4.1 A simple case: $\gamma = 1$

As it turns out, most of our intuition concerning optimal regulation of deposit accounts can be derived from the simple case where depositors derive a constant marginal utility from the funds available for withdrawal from risky accounts during a realization of a liquidity need. This case corresponds to having $\gamma = 1$ in the utility function (1). Substituting $\gamma = 1$ into (15), the social planner chooses $\rho^*$ and $\delta^*$ to solve

$$\max_{\rho, \delta} DS(\rho, \delta) = 4\beta - 2\delta^2 v + 4\delta [\rho v + (1 - \rho)] - 5\tau + 2v.$$ (16)

The first-order conditions are given by

$$\frac{\partial DS}{\partial \rho} = \delta (v - r), \quad \text{and} \quad 0 = \frac{\partial DS}{\partial \delta} = v(\rho - \delta) + r(1 - \rho).$$ (17)

Therefore, for a given $\delta$, the reserve requirement maximizing depositors’ surplus (and also social welfare) is

$$\rho^* = \begin{cases} \frac{4}{3} & \text{if } v > r \\ 0 & \text{if } v < r, \end{cases}$$ (18)

where the value for the maximal $\rho$ is taken from (11). Substituting (18) into (17) yields

$$\delta^* = \begin{cases} \frac{2r}{r + v} & \text{if } v > r \\ \max \left\{ \frac{r}{v}, 1 \right\} = 1 & \text{if } v < r \end{cases} \quad \text{hence} \quad \rho^* = \begin{cases} \frac{r}{r + v} & \text{if } v > r \\ 0 & \text{if } v < r. \end{cases}$$ (19)

Now, we are ready to state our main proposition.

**Proposition 2**

(a) If the return on depositors’ early withdrawal exceeds banks’ investment return, formally if $v > r$, then the reserve requirement and the fraction of risky accounts that maximize depositor surplus and social welfare are given by

$$\rho^* = \frac{r}{r + v} \quad \text{and} \quad \delta^* = \frac{2r}{r + v}.$$
(b) In contrast, if \( v < r \) then \( \rho^* = 0 \) and \( \delta^* = 1 \).

Proposition 2(b) can be seen as characterizing the circumstances under which the commonly observed risky banking system is socially optimal. In light of Proposition 2(b) the current system with very low reserve requirements seems justified if it holds true that the returns of the investment projects funded by banks exceed those of depositors realizing a liquidity need.

The particular novelty of this paper is the characterization of the opposite case given in Proposition 2(a). Namely, if the returns of the investment projects funded by banks fall short of those of depositors realizing a liquidity need, the socially optimal policy is for the regulator to require that banks maintain a fraction of liquid accounts in addition to the imposition of a reserve requirement on risky accounts. Proposition 2(a) demonstrates the condition under which the current banking system where banks do not offer any perfectly-liquid accounts to depositors is inefficient. Under this condition, depositor surplus and social welfare are maximized when commercial banks segment depositors according to their probability of realizing liquidity needs. Under this regulation, Figure 2 illustrates that depositors with high probabilities (\( \lambda > \delta^* \)) of realizing liquidity needs will choose to pay higher fees (or, give up the interest paid on risky accounts) and have their banks maintain 100% reserves on their funds. In addition, depositors with low probabilities of realizing liquidity needs (\( \lambda \leq \delta^* \)) will choose to open the now commonly observed risky accounts subjected to a \( \rho^* \) reserve requirement.

4.2 The general case: \( \gamma > 1 \)

The utility function (1) implies that larger values of \( \gamma \) make each additional fraction of funds available for withdrawal increasingly beneficial to depositors who realize a liquidity need. This is in contrast to our analysis in Section 4.1, conducted under the assumption that \( \gamma = 1 \), where depositors who realized a liquidity need had a constant marginal utility from the funds available for immediate withdrawals.

The general solution to the problem of maximizing depositor surplus, given in (16) but
now solved for $\gamma > 1$, is given by

$$\rho^* = \frac{r \left( \frac{r}{\gamma v} \right)^{\frac{1}{\gamma - 1}}}{v(\gamma - 2) \left( \frac{r}{\gamma v} \right)^{\frac{1}{\gamma - 1}} + r \left( \frac{r}{\gamma v} \right)^{\frac{1}{\gamma - 1}} + 2v},$$

(20)

and

$$\delta^* = \frac{2r}{v(\gamma - 2) \left( \frac{r}{\gamma v} \right)^{\frac{1}{\gamma - 1}} + r \left( \frac{r}{\gamma v} \right)^{\frac{1}{\gamma - 1}} + 2v}.$$

(21)

Therefore, (20) and (21) imply

$$\frac{\rho^*}{\delta^*} = \frac{1}{2} \left( \frac{r}{\gamma v} \right)^{\frac{1}{\gamma - 1}} \quad \text{and in particular, if } r = \gamma v, \quad \rho^* = \frac{1}{2} \quad \text{and } \delta^* = 1. \quad (22)$$

Equation (22) yields the following proposition.

**Proposition 3**

The ratio of the optimal reserve requirement and the optimal fraction of risky accounts, $(\rho^*/\delta^*)$, increases with the banks’ investment return, $(r)$, whereas it decreases with the rate of return of depositors realizing a liquidity shock $(v)$ and with the parameter $\gamma$, which captures the costs of insufficient liquidity.

In what follows, we will refer to the case where $r = \gamma v$ as the benchmark case. In this case, (22) implies that social welfare is maximized when there are only risky accounts, but banks must maintain 50% of deposited money as reserves. Table 1 displays numerical simulations around the benchmark case in order to examine how varying the parameters of the model $(r, v, \text{and } \gamma)$ affect optimal regulation of $\rho$ and $\delta$, given in (20) and (21).

To better understand how the policy variables $\rho$ and $\delta$ are affected by the parameters of the model, we plot numerical simulations for these values. Figure 3 plots the policy variables as a function of depositors’ value of early withdrawal parameter, $v$. Our benchmark case (see also Table 1) occurs when $v = r/\gamma$, where it is optimal to have only risky accounts (currently observed), with a 50% reserve ratio on all accounts. From that point, an increase in $v$ implies that it is socially optimal to provide more perfectly-liquid accounts and to reduce the reserve requirement on risky accounts. This means that market segmentation (between
Table 1: Optimal reserve requirement and fraction of risky accounts. Bold figures represent the benchmark case where $r = \gamma v$.

Figure 3: Optimal $\rho$ and $\delta$ as functions of $v$.

high-probability and low probability of early withdrawal) is necessary when $v$ increases (value of an early withdrawal).

Figure 4 shows that an increase in the bank's investment return $r$, from the benchmark value $r = \gamma v$ results in a smaller optimal reserve requirement. Clearly, a higher investment return means that the banks should lend more funds by lowering their reserve requirement. However, since a lower reserve ratio calls for lower reserves on risky accounts, depositors with high probability of liquidity needs are "compensated" by offering them liquid accounts. This explains why both curves drawn in Figure 4 are downward sloping as $r$ increases.

The same intuition applies to Figure 5 which demonstrates how an increase in the parameter affecting the marginal utility of an early withdrawal affects the optimal policy in-
Instruments. Here, an increase in the parameter $\gamma$ results in further segmentation of the market into risky and liquid accounts where risky accounts are subjected to a lower reserve requirement.

5. Deposit insurance

Our analysis has so far ignored the widely observed deposit insurance for the simple reason that the benchmark for optimal regulation should always be characterized without any additional distortions in the banks’ incentives for maintaining liquidity to meet the potential needs of depositors. Once deposit insurance is introduced, banks have no incentive to compete with respect to liquidity provision, since liquidity provision is then guaranteed within the framework of this insurance system. In this section we introduce deposit insurance into the model and conduct a welfare analysis and compare it to a banking industry with no deposit insurance analyzed in Section 4.
There are various ways in which a deposit insurance system can be implemented. As it turns out, most forms of deposit insurance eventually lead to the same welfare conclusions since the burden of financing this system falls on deposits either in the form of higher fees (lower interest), or in the form of direct financing by the tax payers. In this section we view deposit insurance as any other insurance industry, where the insurer levies a premium on each dollar deposited to be paid by the banks. We will assume that the insurer is a non-profit organization, which means that it charges a competitive premium set at the level that covers depositors’ expected withdrawals upon realizing liquidity needs. Of course, we ignore another form a deposit insurance which is imbedded into the political system where in practice governments tend to bail out failing banks regardless of whether there is a run on the bank or not.

We look at a non-profit deposit insurance system that sets a premium of \( m \geq 0 \) for each dollar deposited. From depositors’ point of view, deposit insurance means that banks can always generate a sufficient amount of cash to meet any level of withdrawals. Therefore, from depositors’ perspective bank accounts are 100% liquid despite the fact that banks maintain only the fraction \( \rho \) as reserves. Formally, substituting \( \theta_A = \theta_B = 1 \) into (1), depositors’ utility functions under deposit insurance can be written as

\[
U_{\lambda,x} = \begin{cases} 
\beta - \tau x + \lambda v + i_A - f_A & \text{Deposits with bank } A \\
\beta - \tau (1 - x) + \lambda v + i_B - f_B & \text{Deposits with bank } B.
\end{cases}
\]

(23)

With no loss of generality we set \( f_A = f_B = 0 \) and then interpret \( i_A \) and \( i_B \) as the net-of-fee interest rates (which obviously can take negative values).

The utility function (23) implies that the number of depositors with each bank are given by

\[
\hat{x} = \frac{1}{2} + \frac{i_A - i_B}{2\tau} = q_A = 1 - q_B.
\]

(24)

Each bank \( j \) takes the deposit insurance premium \( m \) as given, and chooses the net-of-fee interest rate to maximize \( \pi_j = [(1 - \rho)r - i_j - m]q_j, \ j = A, B \). The equilibrium net-of-fee interest rate paid on deposits and the resulting profit levels are given by

\[
i_A = i_B = (1 - \rho)r - \tau - m \quad \text{and} \quad \pi_A = \pi_B = \frac{\tau}{2}.
\]

(25)
Thus, banks’ profit levels are unaffected by the deposit insurance premium since this premium is fully rolled over on depositors in the forms of lower interest rates or higher fees. This result confirms the widespread view that deposit insurance serves as subsidy from depositors to banks.

The utility functions (23) imply that aggregate depositors’ surplus is given by

\[
DS = 2 \int_0^{\frac{1}{2}} \int_0^{0.5} (\beta - \tau x + i_j + \lambda v) \, dx \, d\lambda = \beta - m + (1 - \rho)r + \frac{2v - 5\tau}{4},
\]

(26)

where \(i_j = i_A = i_B\) is substituted from (25).

In order to complete our welfare investigation of deposit insurance, we must compute the “fair” insurance premium \(m\) levied on the banks. From (10), we know that the expected early withdrawals from each bank \(j\) sum up to \(w_j = 1/4\), \(j = A, B\). However, with no deposit insurance, the available amount for withdrawal from each bank is \(\rho q_j = \rho/2\). Hence, deposit insurance is expected to “rescue” each bank by funding an amount of \(w_j - \rho q_j\). With two banks, and a continuum of depositors distributed on \([0, 1] \times [0, 1]\), the fair insurance premium is

\[
m = 2(w_j - \rho q_j) = \begin{cases} \frac{1}{2} - \rho & \text{if } \rho \leq \frac{1}{2} \\ 0 & \text{if } \rho > \frac{1}{2} \end{cases}.
\]

(27)

Figure 6 illustrates how the deposit insurance premium \(m\) associated with a zero-profit deposit insurance system is affected by a change in the mandated reserve requirement \(\rho\). Figure 6 and (27) imply that an increase in the mandated reserve requirement decreases the premium on deposit insurance, because higher reserves decrease the expected deficit from
the realization of depositors’ liquidity need. When \( \rho = 1/2 \) the expected withdrawals equal the reserves served by banks, in which case insurance is not needed.

Substituting (27) into (26) yields

\[
DS(\rho) = \beta + (1-r)\rho + r + \frac{2v - 2 - 5\tau}{4}, \quad \text{or} \quad DS(m) = \beta - (1-r)m + \frac{2(r + v) - 5\tau}{4}. \tag{28}
\]

Therefore, aggregate depositors’ surplus increases with \( \rho \) (hence, decreases with \( m \)) if and only if \( r \leq 1 \). Now, (25) implies that the banks’ profit levels are not affected by the reserve requirement and the deposit premium. Hence, the policy which maximizes aggregate depositors’ surplus also maximizes social welfare. Therefore, we now state the following proposition.

**Proposition 4**

(a) If the net return on banks’ investment is below 100% (i.e., \( r \leq 1 \)), social welfare and aggregate depositor surplus are maximized when \( \rho^* = 1/2 \) and \( m^* = 0 \).

(b) If the net return on banks’ investment exceeds 100% (i.e., \( r > 1 \)), then \( \rho^* = 0 \) and \( m^* = 1/2 \).

In other words, unless banks earn a return above 100% on their external investments, deposit insurance reduces social welfare and the optimal regulatory policy would be to resort to mandating a reserve requirement that eliminates the need for a deposit insurance. In contrast, if the net return exceeds 100%, deposit insurance is needed since this high return is partially rolled over to depositors. In this case, a subsidy from depositors to banks (in the form of a deposit insurance) improves social welfare and aggregate depositor surplus.

To summarize this section, we have shown that unless \( r > 1 \), deposit insurance is inferior to mandating a reserve requirement. Recall that Proposition 2(a) provides the condition under which the introduction of liquid accounts socially dominates a simple reserve requirement regulation. Altogether, in this section we have demonstrated that deposit insurance cannot Pareto dominate the introduction of liquid accounts.
6. Conclusion

Our paper points out that the present regulatory design of the banking system rules out perfectly liquid and safe banking as a financial product. In the absence of this financial product, banks have an incentive to bundle the deposit activities with their risk taking. This bundling, however, generates distortions and these distortions seem to have become more important as banks are threatened by increasingly severe competition from other types of financial intermediaries as well as capital markets.

In this article we have extended the set of instruments for regulating banks’ liquidity provision by adding the fraction of perfectly-liquid accounts as a new regulatory instrument. We have demonstrated how the presence of this additional instrument will induce self-selection on behalf of the depositors who are differentiated according to their probability of facing a liquidity shock. This self-selection will lead to a market segmentation, which can break the bundling of deposits with risk and thereby enhance social welfare. In this respect our analysis characterized those circumstances under which the addition of perfectly-liquid deposit accounts will represent a Pareto improvement relative to a banking industry where risky accounts are the only channel whereby consumers can enjoy the financial services offered by the banks.

We derived the optimal regulatory policy when the set of policy instruments includes the fraction of perfectly-liquid accounts in addition to the traditional reserve requirements applied on risky accounts. The optimal policy was explicitly characterized as a function of banks’ investment return, the return of depositors realizing a liquidity shock, and of the costs borne by depositors with an insufficient amount of liquidity. In particular, we outlined the conditions under which the current policy of applying only reserve requirements or under which a system with extreme narrow banking would be optimal. Finally, we also explored the effects of deposit insurance systems within the framework of the extended set of regulatory policy instruments.
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The Yrjö Jahnsson Working Paper Series in Industrial Economics is funded by The Yrjö Jahnsson Foundation.

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