RESERVATIONS, REFUNDS, AND PRICE COMPETITION
Reservations, Refunds, and Price Competition

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Abstract

We analyze the incentives of service providers to utilize advance reservation systems allowing for refunds in an imperfectly competitive service industry with price competition. We investigate how the refund option affects equilibrium prices, and characterize the conditions under which the refund option is utilized. In contrast to the monopoly models where a single provider can capture a higher share of consumer surplus by utilizing the refundability option, competition reduces this surplus. Thus under price competition the nonrefundable booking strategy maximizes industry profit.

Keywords: Advance booking, reservation systems, refunds, imperfect competition

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1. Introduction

Most the literature on advance booking, reservations, and refunds analyzes a single-service provider. That is, the results are confined to a monopoly market structure. In this paper we analyze an imperfectly competitive service industry where service providers compete on consumers by utilizing advance reservation systems allowing for refunds, in addition to competing in prices.

Theories of Industrial Organization tend to associate the time of purchase with the time of delivery of goods and services. In practice, there are many markets for services and goods where buyers and sellers maintain contacts long before the service or the good are scheduled to be delivered. This pre-delivery contact is usually called *advance booking*, or simply a *reservation*. Reservation systems are observed in almost all privately-provided services. The most noticeable ones are transportation services such as the airline industry, railroad, car rentals and bus travel. Reservations procedures are also utilized in small businesses such as restaurants, fancy barber shops, and law offices. However, service industries do vary with respect to their refund policies. Whereas car rentals and hotels generally offer refundable bookings, entertainment places, such as movie theaters, offer nonrefundable tickets.

In the Economics literature there are a few papers analyzing the refundability option as a means for segmenting the market or the demand. Most studies, so far, focused on a single seller. Those studies that analyze industries with multiple sellers generally assume that prices are fixed, thus leaving firms to compete on capacity allocation only. Contributions by Gale and Holmes (1992, 1993) compare a monopolist’s advance bookings with socially-optimal ones. Gale (1993) analyzes consumers who learn their preferences only after they are offered an advance purchase option. On this line, Miravete (1996) and more recently Courty and Li (2000) further investigate how consumers who learn their valuation over time can be screened via the introduction of refunds in their advance booking mechanism. Dana (1998) also investigates market segmentation under advance booking made by price-taking firms. In a recent study, Netessine and Shumsky (2001) incorporate leftover consumers from competitors into revenue management in the airline industry. Finally, Ringbom and Shy
(2003) study the effects of capacity constraints on the advance reservation strategy of a single service provider.

In the present paper, we formally introduce competition into an industry utilizing advance booking systems.\footnote{Gale (1993) also analyzes a duopoly setting, with consumers who learn their preferences over time.} We investigate an industry providing services like travel arrangements (airline, train, bus, hotel, car rental), or repair, maintenance, education, and so on. These industries are characterized by services that are time dependent and non-storable. This means that both buyers and sellers must commit to a certain predetermined time at which the service is set to be delivered. Therefore, service providers tend to utilize advance reservation system as part of their business and marketing strategies. For this reason, our model allows for multiple strategies where service providers can choose their refund policy in addition to setting prices. We utilize this model to investigate how the degree of competition (manifested by the degree of service differentiation) affects equilibrium choices of refundability options given to consumers.

The present study is organized as follows. Section 2 develops a model of a duopoly service industry utilizing advance booking systems. Section 3 solves for equilibrium prices, profits, and customer participation rates under the refundable and the nonrefundable booking strategies. Section 4 solves for the equilibrium booking strategies used by service providers. Section 5 extends our investigation by integrating dual refundability options into advance reservation systems. Section 6 offers concluding comments.

2. A Model of Competition and Advance Booking

Consider a service industry with two imperfectly-competitive service providers, selling two differentiated services.

2.1 Service providers

There are two service providers, labeled $A$ and $B$. Let $p_A$ and $p_B$ be the prices they charge for providing the service. In addition to setting prices, each service provider utilizes an advance
reservation procedure in which they must inform consumers whether the reservation price is refundable \( (R) \) or nonrefundable \( (N) \). By offering a refundable booking, the service provider collects the price only if the consumer shows up at the time of delivery. In contrast, offering a non-refundable booking means that the consumers pay at the time when the reservation is made and is therefore independent of whether the consumers show up or not at the pre-agreed delivery time.

Service providers bear two types of per-customer costs. Let \( k \geq 0 \) denote the service provider’s cost of making a reservation for one customer. Note that this cost could be significant if the provider does not have any alternative use (no salvage value) for an unused capacity. Alternatively, it may not exist if capacity has an immediate alternative use upon no shows of consumers. In addition, service providers bear a per-customer cost of operation which we denote by \( c \geq 0 \). The difference between the reservation cost and the operation cost is that the latter is borne only if the customer shows up for the service, whereas the reservation cost is borne regardless of whether the customer shows up. Finally, we assume that both service providers have a sufficient amount of capacity to accommodate all reservations without having to overbook consumers.

### 2.2 Consumers

Consumers are differentiated in two dimensions: Location preference and probability of showing up to collect a reserved service. We assume that consumers are uniformly distributed on the unit square indexed by \((\sigma, x)\). The index \( x \) \((0 \leq x \leq 1)\) measures the distance (disutility) from service provider \( A \), whereas \((1 - x)\) measures the distance from \( B \). Thus, \( x \) serves as the standard Hotelling index of differentiation. The index \( \sigma \) \((0 \leq \sigma \leq 1)\) measures the probability that this customer will show up for the delivery of the prebooked service.

Let \( \beta \) denote a consumer basic utility from actually consuming this service. We assume that the utility of a consumer indexed by \((\sigma, x)\) is given by
\[ U(\sigma, x) \overset{\text{def}}{=} \begin{cases} 
\sigma(\beta - p_A) - \tau x & \text{Book with } A & \text{ticket is refundable} \\
\sigma\beta - p_A - \tau x & \text{Book with } A & \text{ticket is nonrefundable} \\
\sigma(\beta - p_B) - \tau (1 - x) & \text{Book with } B & \text{ticket is refundable} \\
\sigma\beta - p_B - \tau (1 - x) & \text{Book with } B & \text{ticket is nonrefundable},
\end{cases} \]

where the parameter \( \tau > 0 \) measures the degree of service differentiation, thus captures the degree of competition between the two service providers. That is, competition becomes more intense when \( \tau \) takes lower values. The utility function (1) reveals that the benefit \( \beta \) is collected only if the consumer shows up (with probability \( \sigma \)). The major difference between refundable and nonrefundable bookings is that a refundable ticket is paid only if the consumers shows up (with probability \( \sigma \)), whereas a nonrefundable ticket is paid upfront (with probability 1).

3. **Equilibrium Prices and Profits Under the Different Booking Strategies**

The next three subsections analyze price equilibria under three refund policies: Both service providers choose to provide full refunds for no shows; both choose to make non-refundable bookings; and lastly service provider \( A \) offers refundable bookings whereas provider \( B \) non-refundable bookings. As commonly assumed in “location” models, we restrict the consumer differentiation parameter in the following way.\(^2\)

**Assumption 1**

*The service brands are sufficiently differentiated. Formally, \( \tau \geq 3c + 5.5k \).*

3.1 **Both providers offer refundable bookings**

The utility function (1) implies that the consumers who are indifferent between making reservations with \( A \) and \( B \) are implicitly solved from \( \sigma(\beta - p_A) - \tau x = \sigma(\beta - p_B) - \tau (1 - x) \).\(^2\)

\(^2\)This assumption could be relaxed somehow at the “expense” of dealing with some additional cases. In terms of Figure 2 below, when \( \tau \) falls below this value, the left intercept exceeds 1, a case which not analyzed in this paper.
Hence,

\[ \hat{x}^R(\sigma) = \frac{1}{2} + \frac{\sigma(p_B - p_A)}{2\tau}. \]  

(2)

Figure 1 illustrates how consumers are divided between making reservations with A and B, according to (2).

![Diagram](https://via.placeholder.com/150)

**Figure 1:** Reservations allocation between service providers A and B. *Note:* Figure drawn assuming out-of-equilibrium prices $p_A > p_B$.

Figure 1 illustrates under the arbitrary assumption $p_A > p_B$, that consumers of type $\sigma = 0$ are equally divided between the service providers, since these consumers will never show up and hence will collect the (full) refund with probability 1. In contrast, for consumer types with $\sigma > 0$, more of them will prefer to book with B for the simple reason that B is cheaper. Clearly, “most” high $\sigma$ types prefer the cheaper provider since they tend to show up (and pay) with a higher probability.

The main feature of the present model is that the number of reservations is always higher than the expected number of actual show-ups. Denote by $q_A$ and $q_B$ the number of reservations made at each provider, and by $s_A$ and $s_B$ the expected number of show-ups for the service at the preagreed delivery time. In view of (2) and Figure 1, the number of reservations are given by
\[ q_A = \int_0^1 \hat{x}^R(\sigma) \, d\sigma = \int_0^1 \left[ \frac{1}{2} + \frac{\sigma(p_B - p_A)}{2\tau} \right] \, d\sigma \quad (3a) \]

\[ q_B = \int_0^1 \left[ 1 - \hat{x}^R(\sigma) \right] \, d\sigma = \int_0^1 \left[ \frac{1}{2} + \frac{\sigma(p_A - p_B)}{2\tau} \right] \, d\sigma. \quad (3b) \]

Therefore, the expected number of show-ups are given by

\[ s_A = \int_0^1 \sigma \hat{x}^R(\sigma) \, d\sigma = \int_0^1 \sigma \left[ \frac{1}{2} + \frac{\sigma(p_B - p_A)}{2\tau} \right] \, d\sigma \quad (4a) \]

\[ s_B = \int_0^1 \sigma \left[ 1 - \hat{x}^R(\sigma) \right] \, d\sigma = \int_0^1 \sigma \left[ \frac{1}{2} + \frac{\sigma(p_A - p_B)}{2\tau} \right] \, d\sigma. \quad (4b) \]

Service provider \( i \) \((i = A, B)\) takes \( p_j \) \((j \neq i)\) as given, and chooses \( p_i \) to solve

\[ \max_{p_i} \pi_i(p_A, p_B) = (p_i - c)s_i - kq_i \begin{array}{c}
= \frac{1}{12\tau} \left[ (p_i - c)(3\tau - 2p_i + 2p_j) + 3k(p_i - p_i - 2\tau) \right] 
\end{array} \quad (5) \]

Thus, the profit of service provider \( i \) is composed of expected operation profits \((p_i - c)s_i\) which is conditional on \( s_i \) show-ups, minus the cost of making \( q_i \) reservations. The profit maximization problem (5) is concave in \( p_i \), yielding a unique best response function given by \( p_i(p_j) = (2c + 3k + 2p_j + 3\tau)/4 \). Therefore, the unique equilibrium prices and profit levels when both providers allow for full refunds are

\[ p^R_A = p^R_B = c + \frac{3(k + \tau)}{2} \quad \text{and} \quad \pi^R_A = \pi^R_B = \frac{3\tau - k}{8}. \quad (6) \]

Substituting the equilibrium prices (6) into the participation rates (3a)–(4b) yields the number of reservation make with each provider and the expected show-ups

\[ q^R_A = q^R_B = \frac{1}{2} \quad \text{and} \quad s^R_A = s^R_B = \frac{1}{4}. \quad (7) \]
3.2 Both providers offer nonrefundable bookings

The utility function (1) implies that the consumers who are indifferent between making reservations with A and B are implicitly solved from \( \sigma \beta - p_A - \tau x = \sigma \beta - p_B - \tau (1 - x) \).

Hence,

\[
\hat{x}^N = \frac{1}{2} + \frac{p_B - p_A}{2\tau},
\]

which is independent of \( \sigma \) and therefore forms a horizontal line in Figure 1. Hence, the number of reservations made at each provider are

\[
q_A = \hat{x}^N = \frac{1}{2} + \frac{p_B - p_A}{2\tau} \quad \text{and} \quad q_B = 1 - \hat{x}^N = \frac{1}{2} + \frac{p_A - p_B}{2\tau}.
\]

The expected number of show-ups are then given by

\[
s_A = \int_0^1 \sigma \left[ \frac{1}{2} + \frac{p_B - p_A}{2\tau} \right] \, d\sigma \quad \text{and} \quad s_B = \int_0^1 \sigma \left[ \frac{1}{2} + \frac{\sigma (p_A - p_B)}{2\tau} \right] \, d\sigma
\]

(10)

Service provider \( i \) (\( i = A, B \)) takes \( p_j \) (\( j \neq i \)) as given, and chooses \( p_i \) to solve

\[
\max_{p_i} \pi_i(p_A, p_B) = (p_A - k)q_i - cs_i = \frac{1}{4\tau} [(2p_i - 2k - c)(p_j - p_i + \tau)]
\]

yielding a best-response function given by \( p_i = \frac{[2(p_j + k + \tau) + c]}{4} \). Therefore, the equilibrium prices and profit levels when both providers use nonrefundable booking are given by

\[
p_A^N = p_B^N = \frac{c + 2(k + \tau)}{2} \quad \text{and} \quad \pi_A^N = \pi_B^N = \frac{\tau}{2}.
\]

(12)

Substituting the equilibrium prices (12) into the participation rates (9) and (10) yields the number of reservation make with each provider and the expected show-ups

\[
q_A^N = q_B^N = \frac{1}{2} \quad \text{and} \quad s_A^N = s_B^N = \frac{1}{4}.
\]

(13)

3.3 A utilizes refundable bookings, B non-refundable bookings

The utility function (1) implies that the consumers who are indifferent between making reservations with A and B are implicitly solved from \( \sigma(\beta - p_A) - \tau x = \sigma \beta - p_B - \tau (1 - x) \).

Hence,

\[
\hat{x}^{RN} = \frac{1}{2} + \frac{p_B - \sigma p_A}{2\tau},
\]

(14)
which forms a downward sloping linear division line as in Figure 1, but with a higher intercept given by $1/2 + p_B/(2\tau)$. Hence, the number of reservations made at each provider is

$$q_A = \int_0^1 \left[ \frac{1}{2} + \frac{p_B - \sigma p_A}{2\tau} \right] d\sigma \quad \text{and} \quad q_B = \int_0^1 \left[ \frac{1}{2} + \frac{\sigma p_A - p_B}{2\tau} \right] d\sigma.$$  \hspace{1cm} (15)

The expected number of show-ups are then given by

$$s_A = \int_0^1 \sigma \left[ \frac{1}{2} + \frac{p_B - \sigma p_A}{2\tau} \right] d\sigma \quad \text{and} \quad s_B = \int_0^1 \sigma \left[ \frac{1}{2} + \frac{\sigma p_A - p_B}{2\tau} \right] d\sigma.$$  \hspace{1cm} (16)

Each service provider chooses her price to solve

$$\max_{p_A} \pi_A(p_A, p_B) = (p_A - c)s_A - kq_A \quad \text{and} \quad \max_{p_B} \pi_B(p_A, p_B) = (p_B - k)q_B - cs_B$$  \hspace{1cm} (17)

yielding best-response functions given by $p_A = [3(p_B + k + \tau) + 2c]/4$ and $p_B = [p_A + 2(k + \tau) + c]/4$. Therefore, the equilibrium prices and profit levels when service providers use different booking strategies are

$$p_A^R = \frac{11c + 18(k + \tau)}{13} \quad \text{and} \quad p_B^N = \frac{6c + 11(k + \tau)}{13}.$$  \hspace{1cm} (18)

Substituting the equilibrium prices (18) into the profit functions (17) yields

$$\pi_A^R = \frac{8c^2 - 3c(35k + 48\tau) - 6(35k^2 + 96k\tau - 108\tau^2)}{2028\tau}$$  \hspace{1cm} (19a)

$$\pi_B^N = \frac{6(2k - 11\tau)^2 - 70c^2 - 3c(35k + 61\tau)}{2028\tau}.$$  \hspace{1cm} (19b)

Equation (18) as well as subtracting (19b) from (19a) imply that

$$p_A^R - p_B^N = \frac{5c + 7(k + \tau)}{13} > 0 \quad \text{and} \quad \pi_A^R - \pi_B^N = \frac{2c^2 + c\tau - 2(3k^2 + 4k\tau + \tau^2)}{52\tau}.$$  \hspace{1cm} (20)

Equation (20) implies the following proposition, which is proved in Appendix A.

**Proposition 1**

*In a price competition between the service providers,*
(a) The provider utilizing refundable bookings charges a higher price than the provider utilizing nonrefundable booking. Formally, \( p_A^R > p_B^N \). However,

(b) the provider utilizing refundable bookings earns a lower profit than the provider who does not refund.

To conclude our analysis of asymmetric booking strategies, we turn to investigating consumers’ participation rates. Substituting the equilibrium prices (18) into (15), the number of reservations made with \( A \) and \( B \) are

\[
q_A^R = \frac{c + 2(2k + 15\tau)}{52\tau} \quad \text{and} \quad q_B^N = \frac{2(11\tau - 2k) - c}{52\tau}.
\]

Substituting (18) into (16), the expected number of show-ups are given by

\[
s_A^R = \frac{3(12\tau - k) - 4c}{156\tau} \quad \text{and} \quad s_B^N = \frac{4c + 3(k + 14\tau)}{156\tau}.
\]

We can therefore state the following proposition.

**Proposition 2**

(a) The service provider utilizing the refund booking strategy makes more reservations compared to the provider utilizing a non-refundable booking strategy. Formally, \( q_A^R > q_B^N \). However,

(b) the expected number of show-ups is lower for the provider utilizing the refundable booking strategy. Formally, \( s_A^R < s_B^N \).

Proposition 2(a) is illustrated in Figure 2.

Figure 2 demonstrates that despite the high price, the refund option makes provider \( A \) more attractive to most consumers. Proposition 2(b) demonstrates that despite the higher number of reservations, service provider \( A \) has a lower expected number of show-ups. This stems from the fact that \( A \) is more attractive to “most” consumers with a low probability of showing up.
4. Equilibrium Booking Strategies

In this section we solve for the equilibrium booking strategies utilized by service providers. Our interpretation for this double-strategy market game is that both service providers first announce whether they allow for refunds, and only then both providers set their prices according to the computations described in Section 3.

Comparing (6) with (19b), we see that there exists an equilibrium where both providers select the refundable booking strategy if

$$\frac{3\tau - k}{8} > \frac{6(2k - 11\tau)^2 - 70c^2 - 3c(35k + 61\tau)}{2028\tau}.$$  \hspace{1cm} (23)

Comparing (12) with (19a), there exists an equilibrium where both use the non-refundable booking strategy if

$$\frac{\tau}{2} > \frac{8c^2 - 3c(35k + 48\tau) - 6(35k^2 + 96k\tau - 108\tau^2)}{2028\tau}.$$  \hspace{1cm} (24)

Appendix B provides the proof for the following proposition.
Proposition 3
(a) There exist exactly two equilibrium booking strategies: Both providers choose refundable bookings, and both choose nonrefundable bookings.

(b) Industry profit is higher when both providers choose nonrefundable bookings than refundable bookings.

Proposition 3 rules out asymmetric equilibria where one provider utilizes refundable bookings whereas the other only non-refundable bookings. An immediate implication of Proposition 3(b) is that if service providers would collude on the nature of their booking strategies, they would choose nonrefundable bookings. Therefore,

Corollary 1
Industries with observed refundable bookings are not colluding on their booking strategy.

5. Dual Reservation Systems

In this section we study the incentives of competing service providers to offer customers a menu of two price options at the time reservations are made. Formally, we assume that at the time reservations are made, service provider \(i\) \((i = A, B)\) offers customers to choose whether pay \(p_i^N\) which is not refundable, or instead to pay \(p_i^R\) and obtain a full-refund upon no-show.

Let \(\hat{\sigma}_A\) and \(\hat{\sigma}_B\) denote customers who make a reservation with \(A\) and \(B\), respectively, and are indifferent between choosing the non-refundable option and the full-refund option. Formally, the utility function (1) implies that \(\hat{\sigma}_i\) is implicitly solved from \(\sigma(\beta - p_i) - \tau x = \sigma\beta - p_i - \tau x\). Hence,

\[
\hat{\sigma}_A = \frac{p_A^N}{p_A^R} \quad \text{and} \quad \hat{\sigma}_B = \frac{p_B^N}{p_B^R}.
\]

(25)

Note that \(\hat{\sigma}_i < 1\) since in equilibrium the price of a non-refundable ticket can never exceed the price of a refundable ticket.

Next, recall from (2) that \(\hat{x}^R(\sigma)\) index consumers who book a refundable ticket and are indifferent between booking with \(A\) and \(B\). Similarly, \(\hat{x}^N\) defined in (8) index consumers
who book a nonrefundable ticket and are also indifferent between booking with A and B. Finally, from (14) we can derive the index of consumers who are indifferent between making a nonrefundable booking with A and a refundable booking with B. Therefore,

$$\hat{x}^R(\sigma) = \frac{1}{2} + \frac{\sigma (p_B - p_A)}{2\tau}, \quad \hat{x}^N = \frac{1}{2} + \frac{p_B - p_A}{2\tau}, \quad \text{and} \quad \hat{x}^{NR}(\sigma) = \frac{1}{2} + \frac{\sigma p_B^R - p_A^N}{2\tau}. \quad (26)$$

Figure 3 illustrates, in the $\sigma \times x$ space of consumers, how consumers are divided among the four booking options (two by each provider).

**Figure 3:** Customers’ choices of booking options under dual booking strategy.

Figure 3, (25) and (26) imply that the number of consumers who book with A (according to the two booking classes) are

$$q_A^R = \int_0^{\hat{x}^R} \hat{x}^R(\sigma) \, d\sigma \quad \text{and} \quad q_A^N = \int_{\hat{x}^N}^{\hat{x}^{NR}(\sigma)} \hat{x}^N \, d\sigma + 1 \int_{\hat{x}^N}^{\hat{x}^N} \frac{1}{\sigma} \, d\sigma. \quad (27)$$

It follows that the expected show-ups to A’s services are

$$s_A^R = \int_0^{\hat{x}^R} \hat{x}^R(\sigma) \, d\sigma \quad \text{and} \quad s_A^N = \int_{\hat{x}^N}^{\hat{x}^{NR}(\sigma)} \hat{x}^N \, d\sigma + \int_{\hat{x}^N}^{\hat{x}^N} \frac{1}{\sigma} \, d\sigma. \quad (28)$$
Similarly, the number of reservations made with $B$ are

$$q^R_B = \int_0^{\hat{x}_B} [1 - \hat{x}^R(\sigma)] \ d\sigma + \int_{\hat{x}_B}^{\hat{x}_A} [1 - \hat{x}^{NR}(\sigma)] \ d\sigma \quad \text{and} \quad q^N_B = \int_{\hat{x}_B}^{1} [1 - \hat{x}^N] \ d\sigma. \quad (29)$$

It follows that the expected number of show-ups for $B$’s services are

$$s^R_B = \int_0^{\hat{x}_B} \sigma [1 - \hat{x}^R(\sigma)] \ d\sigma + \int_{\hat{x}_B}^{\hat{x}_A} \sigma [1 - \hat{x}^{NR}(\sigma)] \ d\sigma \quad \text{and} \quad s^N_B = \int_{\hat{x}_B}^{1} \sigma [1 - \hat{x}^N] \ d\sigma. \quad (30)$$

The problem of each service provider $i$ ($i = A, B$) is the choose a pair of prices $(p^R_i, p^N_i)$ to solve

$$\max_{p^R_i, p^N_i} \pi_i = p^R_i s^R_i + p^N_i q^N_i - c(s^R_i + s^N_i) - k \left( q^R_i + q^N_i \right). \quad (31)$$

The first two terms are the revenue generated from consumers who book refundable and nonrefundable tickets. The last two terms are the expected costs of delivering the service and the cost of making the reservations.

Unfortunately, the profit functions (31) are not globally concave with respect to their own prices. Therefore, we are unable to provide a general characterization of the equilibrium prices. However, we can still state the following proposition.

**Proposition 4**

There does not exist a symmetric equilibrium where both service providers charge the same (finite) prices for both refundable and nonrefundable bookings, and where some consumers book refundable tickets.

**Proof.** Solving the four first-order conditions of the two maximization problems (31) yields,

$$p^R_A = p^R_B = p^N_A = p^N_B = c + \frac{3(k + \tau)}{2}. \quad (32)$$

However, substituting these prices into a second-condition we obtain $d^2 \pi_i / dp^2_i = (k + \tau) / \{2\sigma[2c + 3(k + \tau)]\} > 0$.

One immediate conclusion drawn from Proposition 4 is given in the following corollary.

**Corollary 2**

There does not exist an equilibrium where both providers utilize only refundable bookings.
Proof. The prices listed in (32) imply that $\hat{\sigma}_A = \hat{\sigma}_B = 1$, meaning that all consumers book only refundable tickets. By Proposition 4, these prices do not constitute an equilibrium. □

Finally, the following proposition (proved in Appendix C) characterizes one equilibrium.

**Proposition 5**

The outcome $p_{NB}^N = p_{NA}^N = \tau + k + c/2$ and $\hat{\sigma}_A = \hat{\sigma}_B = 0$ is an equilibrium.

Comparing Propositions 4 and 5, as well as Corollary 2 with Proposition 3(a) reveals that the outcome where both providers choose only refundable bookings is no longer an equilibrium when we enlarge the strategy space of service providers via the introduction of the menu of dual bookings.

6. Conclusion

The novelty of the present paper is the explicit introduction of competition into service industry utilizing advance bookings. Most studies, so far, focused on a single seller, or multiple sellers under fixed prices.

From a policy point of view, our model demonstrates a clear tradeoff between the division of surplus amongst service providers and consumers when refunds become available. That is, under competition, a higher refund rate reduces firms’ profits in favor of an increase in consumer surplus. This finding differs from the results obtained in the monopoly advance booking models where a single service provider can extract a higher surplus when it offers full refunds. Here, due to competition, service providers cannot raise prices to the level where the entire surplus from the refundability option can be captured.

We demonstrate that aggregate industry profit is maximized when both service providers utilize nonrefundable bookings. The latter result may explain why refundable tickets are rarely observed in the various entertainment industries. For example, movie theaters, and sports events such as soccer, basketball, and hockey games rarely offer refunds on no-shows.

We conjecture that the reason why car-rental companies offer full refunds instead of no refunds stems from the fact that their customers are generally screened by providers of complementary services who select themselves to serve those customers with a high probability.
of showing up. For example, consumers who make car-rental reservations at major airports fly in by airline companies who also screen consumers via price discrimination mechanisms. Moreover, in this example, no shows in the car-rental industry are often caused by the airline companies (e.g., traffic delays) and not by customer behavior. Thus, no-shows in the car rental industry cannot be compared with no-shows in the entertainment industries for which only the consumers themselves are to be accounted for no-shows.

Appendix A. Proof of Proposition 1

We begin by normalizing the operation cost such that $c = \gamma k$, where $\gamma \geq 0$. Now, by (20), $\tau (\pi^R_A - \pi^N_B) = 2\gamma^2 k^2 + \gamma k \tau - 2(3k^2 + 4k\tau + \tau^2)$. Therefore, $\pi^R_A - \pi^N_B < 0 \iff -2\tau^2 + \gamma k \tau - 8k\tau + 2\gamma^2 k^2 - 6k^2 < 0 \iff \frac{\tau}{k} > 2 + \frac{2\gamma}{k}$. By Assumption 1, $4\tau/k > 12\gamma + 22$. Finally, $12\gamma + 22 = \gamma - 8 + \sqrt{121\gamma^2 + 660\gamma + 900} > \gamma - 8 + \sqrt{17\gamma^2 - 16\gamma + 16}$. But by Assumption 1, $4\tau/k > 12\gamma + 22$. Finally, $12\gamma + 22 = \gamma - 8 + \sqrt{121\gamma^2 + 660\gamma + 900} > \gamma - 8 + \sqrt{17\gamma^2 - 16\gamma + 16}$.

Appendix B. Proof of Proposition 3

Part (b) follows from a direct comparison of the profit levels given in (6) and (12). To prove part (a) we demonstrate that under Assumption 1 the relations (23) and (24) hold.

Substitute $c = \gamma k$, where $\gamma \geq 0$, into the condition (23) and cross multiply to obtain $69\tau^2 + (21 + 366\gamma)\tau k + (140\gamma^2 + 210\gamma - 48)k^2 > 0$. Obviously, the threshold where the strategy pairs RR and RN yield the same profit levels is characterized by the roots of this equation. Here, we extract the root of the competition parameter, $\tau$. As only the larger root can be positive, condition (23) becomes equivalent with the following condition

$$\frac{\tau}{k} > \frac{\tau^R}{k} = \frac{-(366\gamma + 21) + 13\sqrt{3 \cdot (188\gamma^2 - 84\gamma + 27)}}{138}. \quad (33)$$

Similarly, condition (24) can be rewritten as $366\tau^2 + (576 + 144\gamma)\tau k - (8\gamma^2 - 105\gamma + 210)k^2 > 0$. Consequently, condition (24) can be expressed as

$$\frac{\tau}{k} > \frac{\tau^N}{k} = \frac{-(72\gamma + 288) + 13\sqrt{6 \cdot (8\gamma^2 + 3\gamma + 6)}}{366}. \quad (34)$$
Since by Assumption 1 we must have it that $\tau/k > 3\gamma + 5.5$, we observe that for all $\gamma \geq 0$:

$$3\gamma + 5.5 - \frac{\tau_R}{k} = \frac{414\gamma + 759 + 366\gamma + 21 - 13\sqrt{3 \cdot (188\gamma^2 - 84\gamma + 27)}}{138}$$

$$= \frac{13}{138} \left[ 60(\gamma + 1) - \sqrt{3 \cdot (188\gamma^2 - 84\gamma + 27)} \right]$$

$$= \frac{13}{138} \left[ \sqrt{3600(\gamma^2 + 2\gamma + 1)} - \sqrt{564\gamma^2 - 252\gamma + 81} \right] > 0 \quad (35)$$

and

$$3\gamma + 5.5 - \frac{\tau_N}{k} = \frac{1098\gamma + 2013 + 72\gamma + 288 - 13\sqrt{3 \cdot (16\gamma^2 + 6\gamma + 12)}}{366}$$

$$> \frac{13}{366} \cdot \frac{90\gamma + 177 - \sqrt{3 \cdot (4\gamma + 4)^2}}{366} = \frac{13}{366} \cdot \frac{78\gamma + 165}{366} > 0 \quad (36)$$

Appendix C. Proof of Proposition 5

Assume that firm $B$ offers only non-refundable tickets at the price $p_B^N = \tau + k + c/2$. Then we calculate firm $A$’s best response. In this case firm $A$ is free to set the nonrefundable price $p_A^N$ and the refundable price $p_A^N/\hat{\sigma}_A$. Now, $q_A^R = \int_{\hat{\sigma}_A}^{\hat{\sigma}_A} \hat{x}^R(\sigma) d\sigma$ and $q_A^N = \int_{\hat{\sigma}_A}^{1} \hat{x}^N(\sigma) d\sigma$, become

$$q_A^R = \frac{1}{2\tau} \int_{0}^{\hat{\sigma}_A} (2\tau + k + c/2 - \sigma p_A^N) d\sigma = \frac{(4\tau + 2k + c - p_A^N)}{4\tau} \hat{\sigma}_A,$$

and

$$q_A^N = \frac{1}{2\tau} \int_{\hat{\sigma}_A}^{1} (2\tau + k + c/2 - p_A^N) d\sigma = \frac{(4\tau + 2k + c - 2p_A^N)}{4\tau} (1 - \hat{\sigma}_A), \quad (37)$$

and the expected showups become

$$s_A^R = \frac{1}{2\tau} \int_{0}^{\hat{\sigma}_A} [(2\tau + k + c/2) \sigma - \sigma^2 p_A^R] d\sigma = \frac{3(4\tau + 2k + c - 4p_A^N)}{24\tau} \hat{\sigma}_A^2,$$

and

$$s_A^N = \frac{1}{2\tau} \int_{\hat{\sigma}_A}^{1} (2\tau + k + c/2 - p_A^N) \sigma d\sigma = \frac{(4\tau + 2k + c - 2p_A^N)}{8\tau} (1 - \hat{\sigma}_A^2). \quad (38)$$

When we substitute $p_A^R = p_A^N/\hat{\sigma}_A$, the optimization problem is reduced to

$$\max_{\hat{\sigma}_A, p_A^N} \pi_A = p_A^N (s_A^R + q_A^N) - c(s_A^R + s_A^N) - k(\hat{q}_A^R + q_A^N). \quad (39)$$
The first-order condition with respect to the price $p^N_A$ yields

$$
p^N_A(\hat{\sigma}_A) = \frac{24\tau + 24k + 12c - \hat{\sigma}_A(12\tau + 12k + (3 + 2\hat{\sigma}_A)c)}{24 - 16\hat{\sigma}_A}.
$$

(40)

Since $\frac{\partial^2 \pi^N_A}{\partial (p^N_A)^2} = (2\hat{\sigma}_A - 3)/(3\tau) < 0$, the profit function is concave in $p^N_A$. Next, we observe that

$$\left. \frac{\partial \pi^N_A}{\partial \hat{\sigma}_A} \right|_{p^N_A = \tau + k + c/2} = \frac{c^2}{24\tau} < 0.$$

As $\frac{\partial^2 \pi^N_A}{\partial \hat{\sigma}_A^2} = -cp^N_A/(6\tau) < 0$, $\hat{\sigma}_A = 0$ and $p^N_A = \tau + k + c/2$ is a potential maximum.

The cross-derivative $\frac{\partial^2 \pi^N_A}{\partial \hat{\sigma}_A \partial p^N_A} = -((4\hat{\sigma}_A + 3)c + 4(3k - 4p^N_B + 3\tau)$, indicates that the profit function need not be universally concave. (Technically, the sign of $\text{det}(\nabla^2 \pi_A)$ can switch). However, the only stationary point satisfying $\hat{\sigma}_A > 0$ and $p^N_A > 0$ is at $\hat{\sigma}_A = 1$ and $p^N_A = (5c + 4(k + \tau))/8$. (For any $0 < \hat{\sigma}_A < 1$ and $p^N_A$ satisfying (40), $\frac{\partial \pi^N_A}{\partial \hat{\sigma}_A}$ is negative).

Therefore, there cannot exist any optimal deviation to the “interior” ($0 < \hat{\sigma}_A < 1$). The maximal profit at $\hat{\sigma}_A = 1$ is

$$\pi^N_A \bigg|_{\hat{\sigma}_A = 1, p^N_A = \frac{5c + 4(k + \tau)}{8}} = \frac{c^2 - 24c(k + \tau) - 48k^2 - 96k\tau + 144\tau^2}{48} < \frac{1225\tau}{9 \cdot 384}.$$

The last inequality follows from Assumption 1, as $k \geq 0$ and $3c < \tau$. Since $1225\tau/(9 \cdot 384) < \tau/2$ then $A$’s optimal response is $p^N_A = \tau + k + c/2$ and $\hat{\sigma}_A = 0$. 

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