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Entrepreneurship and Financial Markets with Adverse Selection

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ABSTRACT

We study an adverse selection model where all agents are endowed with initial wealth, are nonetheless capital constrained, and choose to invest as entrepreneurs or financiers, or not to invest. We show that entrepreneurship and financial markets can arise in many cases where opening the markets to outside investors (i.e., financial market liberalization) would lead to them being eliminated. We find that without outside investors i) there exist Pareto-efficient and inefficient equilibria; ii) adverse selection has severer consequences in poorer economies; iii) increasing initial wealth may lead from a Pareto-efficient to an inefficient equilibrium; iv) increasing the proportion of agents with positive NPV projects leads from an inefficient to an efficient equilibrium; v) agents with negative (positive) NPV projects only earn rents in (non)-wealth constrained economies; and vi) removing the storage technology destroys the only Pareto-efficient equilibrium in non-wealth constrained economies. Our model allows us to analyze various policies concerning financial market integration and liberalization, financial stability, the need for sophisticated financial institutions, development aid, and the promotion of entrepreneurship.

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I. INTRODUCTION

The aim of this paper is to explore entrepreneurship and the functioning of financial markets under asymmetric information when the roles of agents are determined within the model. In a departure from most of the existing literature, in our model all agents are capital constrained and have an investment project whose quality is their private information. There is an occupational choice in the sense that agents choose whether to participate in financial markets and, if they participate, whether to become entrepreneurs or financiers. The set-up creates a natural environment to study whether entrepreneurship, and by implication, a market for financial claims emerge in equilibrium and whether the eventual markets are efficient. Our model allows us to analyze the effects of financial market integration and liberalization, various shocks to the economy, the need for more sophisticated financial institutions, and the usefulness of various policies such as development aid and the promotion of entrepreneurship.

We find that the occupational choice of agents mitigates the adverse effects of asymmetric information. As usual, with outside investors the financial market can collapse to autarky, i.e., not open at all, and if it opens, it will yield a Pareto inefficient outcome. The equilibria where the financial market does not open or operates inefficiently do not vanish without outside investors. For a wide range of parameter values, however, the market does open and yields Pareto or interim efficient outcomes. This finding suggests that financial market liberalization can reduce the efficiency of financial markets. In an extreme case, opening financial markets to outside investors has a devastating effect in our model, leading to a collapse of the market.

Because our model includes both economies where the total initial wealth is sufficient to implement all (positive NPV) projects, and wealth constrained
economies, we can study the effects of an aggregate shortage of liquidity as in Holmström and Tirole (1998). In turns out that Pareto efficient and inefficient equilibria exist both in wealth constrained and unconstrained economies, as does autarky. Contrary to what one might expect, the economy level wealth constraint does not necessarily dilute the performance of the financial market: Relaxing the wealth constraint can lower interest rates to the extent that it induces agents with low quality projects to seek funding. This means that even small shocks may change the equilibrium: for example, increasing initial wealth - in the form of development aid, for example - may cause a shift from a Pareto efficient equilibrium into a Pareto-inefficient one. The wealth constraint also affects the distribution of economic rents: Agents with good projects may earn rents in non-wealth constrained economies, whereas agents with bad projects may earn rents in wealth constrained economies.

Regarding modeling of financial markets, we build on the influential contributions by de Meza and Webb (1987) and Holmström and Tirole (1997). In their models there are borrowers with investment projects and some initial wealth, and lenders with funds but without projects. We study a model where borrowers have private information about the value of their projects as in de Meza and Webb (1987) but we assume, like Holmström and Tirole (1997), that successful entrepreneurs’ returns differ and both entrepreneurs and financiers are wealth constrained. In Section II we replicate some of de Meza and Webb’s (1987) and Holmström and Tirole’s (1997) results, and argue that the interaction between the entrepreneurs’ pledgeable income and their returns in case of successes have been overlooked in the literature.

In Section III we allow all agents a choice between entrepreneuship, financing others’ ventures, and remaining outside financial markets. There our paper ties with the literature on occupational choice in the presence of credit markets constraints.
This literature studies the effects of credit rationing on the rest of the economy (e.g., Banerjee and Newman, 1993), or two-way interaction between these markets (Ghatak, Morelli, Sjöström, 2002). In contrast to the current paper, the occupational choice is always between becoming an entrepreneur or a worker. To the best of our knowledge, Boyd and Prescott (1986) and Shleifer and Wolfenzon (2002) are the only studies besides ours where there is a choice between investing as an entrepreneur and a financier. In Boyd and Prescott (1986) agents i) have investment projects whose quality is their private information and ii) can choose whether to invest in their project or evaluate the quality of a project. Our model is simpler than theirs in that we do not allow information acquisition. In contemporaneous and independent work Shleifer and Wolfenzon (2002) also develop an equilibrium model of financial markets where entrepreneurs have some initial wealth and different project returns. Otherwise their model and focus, however, substantially differ from ours.

In Section IV we consider how financial market performance depends on the efficiency of the storage technology. It transpires that the efficiency of storage technology has only minor effects, except that the scope of the Pareto-efficient equilibrium in non-wealth constrained economies is increasing in the efficiency of the storage technology. At the limit where there is no storage technology, the Pareto-efficient equilibrium disappears.

Most of our analysis through Sections II-IV rests on a simple graphical representation of the results: the analytical details are deferred into the Appendix. Policy implications are collected into Section V, and conclusions into Section VI.

II. THE MODEL WITH OUTSIDE INVESTORS

To clarify the role of borrowers’ pledgeable income we build on the ideas in de Meza and Webb (1987) and Holmström and Tirole (1997). There is a unit mass of
entrepreneurs who have a potential project, and unlimited entry by outside investors without a project of their own. Each projects needs financing of $I$ to be implemented. All agents are risk-neutral and have some assets $A$, $0 < A < I$. For the moment we also assume that there is a storage technology that converts the assets to a consumption good at a zero rate of return. In Section IV we consider an imperfect storage technology under which $A$ depreciates at rate $1 - \delta$, $\delta \in [0,1]$.

Entrepreneurs' projects have different success probabilities and conditional returns. A project that fails yields zero to all agents. A proportion $h$ ($0 < h < 1$) of agents are High (H) types who are endowed with a positive NPV project, the rest are Low (L) types with a negative NPV project. We assume that $p_H R_H > I > p_L R_L$ and $R_L > R_H$, where $p_i$ is the success probability and $R_i$ the return (conditional on success) of an entrepreneur of type $i$, $i \in \{H, L\}$. Project success and wealth are verifiable, but project type is private information following, e.g., Bolton and Scharfstein (1990). In what follows, we present most of our analysis using a graph in the $(A, h)$-space. Natural parameter boundaries are given by $h \leq 1$, and $A < I$.

The financial market works as follows. In the first stage, potential entrepreneurs decide whether to approach outside investors or to resort to the storage technology.$^1$ As in Stiglitz and Weiss (1981) and de Meza and Webb (1987), it is cheaper for H-type entrepreneurs to use their own rather than outside funds: they will therefore invest all their initial wealth ($A$) to their own projects and only raise the rest ($I - A$) from outside. In order not to reveal their type, L-type entrepreneurs have no other option but to follow. Contract terms stipulate the conditional payment from the entrepreneur to outside investors in case of success. Once financing needs have been

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$^1$ When there are outside investors, agents who do not become entrepreneurs are indifferent between using the storage technology and financing entrepreneurs’ projects. Saying that all of them resort to the storage technology is an expositional decision without implications.
settled, entrepreneurs execute their projects in the second stage. Project success is verified, successful entrepreneurs compensate outside investors according to contract terms, and consumption takes place.

The choice of potential entrepreneurs can be formalized as an individual rationality condition. Denoting expected profits for a type \( i \) entrepreneur by \( \pi_i^e \), the IR constraint is

\[
(1) \quad \pi_i^e \equiv p_i (R_i - R_B) \geq A \quad \forall i, \quad i \in \{H, L\},
\]

where superscript \( e \) denotes entrepreneurship and \( R_B \) is the (fixed) payment that a successful entrepreneur pays to her investors. Because of competitive supply of finance, the cost of financing is determined by the outside investors’ zero-profit condition, i.e.,

\[
(2) \quad R_B = \frac{I - A}{p},
\]

where \( p \) is the average success probability of those who become entrepreneurs. From (1) and (2) we can observe that there are potentially three equilibrium outcomes. A Pareto-efficient separating equilibrium where \( p = p_H \), i.e., only H-type agents become entrepreneurs, occurs if \( \pi_H^e > A > \pi_L^e \), and a Pareto-inefficient pooling equilibrium where \( p = h p_H + (1-h) p_L \), i.e., all agents become entrepreneurs, occurs if both \( \pi_H^e \) and \( \pi_L^e \) are larger than \( A \). If both \( \pi_H^e \) and \( \pi_L^e \) are smaller than \( A \), the financial market does not open and agents invest in the storage technology. We term this outcome autarky.

After substituting (2) for (1) we see that autarky and a pooling equilibrium exist as in the standard model, but that a separating equilibrium also exists. In Figure 1 we have depicted the \((A, h)\)-values for which each of the three different equilibria exist. The two IR constraints defined by (1) are shown in Figure 1: the L-type IR
constraint is an upward sloping curve, whereas the H-type IR constraint slopes downwards. There are also two vertical lines, the \( \hat{A} \equiv p_L p_H (R_L - R_H) / (p_H - p_L) \)-line and the \( \bar{A} \equiv \hat{A} + p_L (p_H R_H - I) / (p_H - p_L) = p_L (p_H R_L - I) / (p_H - p_L) \)-line, whose meaning we clarify below. In the upper left-hand corner to the left of \( \bar{A} \) and above the IR constraints, the pooling equilibrium exists, and autarky can be found below it. The Pareto efficient equilibrium exists to the right of \( \bar{A} \).

**FIGURE 1 HERE**

By building on the intuition of Holmström and Tirole (1997) it is easy to explain why the efficient equilibrium emerges when the agents’ initial wealth is large enough. The agents’ expected return on L-type projects trivially becomes smaller than their initial endowment when \( A \) exceeds \( p_L R_L \) but, given the equilibrium \( R_b > 0 \), the expected return on the L-type projects is less than \( A \) in the whole region to the right of \( \bar{A} < p_L R_L \). At \( A = \bar{A} \) the competitive interest rate is such that the L-type agents are indifferent between investing in their projects and consuming their initial wealth. To the left of \( \bar{A} \) competitive outside investors drive the interest rate so low that there can be no separation between different types of entrepreneurs. This suggests that if the supply of finance was restricted, e.g., by a monopoly supplier, the economy could also reach Pareto-efficiency to the left of \( \bar{A} \). The suggestion will be confirmed in the next section.

The meaning of the vertical \( \hat{A} \)-line can also be explained by using the insights from Holmström and Tirole (1997). The pledgeable income of type \( i \) agent, i.e., the maximum amount of an \( i \)-type entrepreneur can credibly promise to pay back to a financier by \( R_i A / p_i \). The pledgeable incomes of H- and L-type agents are equal when \( A = \hat{A} \), but L-type agents’ pledgeable income exceeds that of H-type agents’ when \( A < \hat{A} \).
and vice versa when $A > \hat{A}$. As a result, no separating equilibrium is possible when $A < \hat{A}$.

The preceding discussion suggests that the level of agents’ pledgeable income matters. To explain why we compare our findings with Stiglitz and Weiss (1981) and de Meza and Webb (1987). In Stiglitz and Weiss (1981) there is a mean-preserving spread between projects. Here it would mean that $P_H R_H = P_L R_L$ and, consequently, L-type agents’ pledgeable income would always be larger than H-type agents’. This corresponds the area to the left of $\hat{A}$ in Figure 1. De Meza and Webb (1987) consider first-order stochastic dominance as we do. However, they also assume that the project returns conditional on success are the same. Here it would mean that $R_H = R_L$ and, consequently, $\hat{A} = 0$. The pledgeable income of H-type agents would exceed that of L-type agents, corresponding the area to the right of $\hat{A}$.

As against this background, it is not surprising that the predictions by Stiglitz and Weiss (1981) hold to the left of $\hat{A}$ whereas the predictions by de Meza and Webb (1987) hold to the right of $\hat{A}$ (but to the left of $\bar{A}$). For example, a tax on the interest income could implement a Pareto-efficient outcome to the right of $\hat{A}$ whereas an interest rate subsidy could be efficient to the left of $\hat{A}$. One can also show that a credit-rationing equilibrium cannot exist to the right of $\hat{A}$.

### III. THE MODEL WITHOUT OUTSIDE INVESTORS

In the absence of outside investors the amount of funds available for investment is limited. A natural consequence is that all agents face a choice between becoming entrepreneurs or financiers. We think of an economy as being wealth constrained if the total wealth of all agents is not sufficient to finance all H-types’ projects. Identically, an economy is not wealth constrained if the total initial wealth exceeds the financing needs of all H-type projects. The diagonal divides economies
into wealth constrained (above the \( h=A/I \)-line), and the non-wealth constrained ones (below).

The financial market works as before. Since we do not allow for financial institutions that gather and process information, the financial market in our model could be interpreted as a frictionless (stock) market. In other words, after the projects have been implemented, the total payments from all entrepreneurs are divided evenly among all financiers. Thus it is as if a financier buys a stake in the average implemented project, instead of implementing her private project. Loosely speaking, it makes no difference whether one envisions a financial market where some potential financiers come together to finance one or a few projects (to equate demand and supply within the “coalition”), or a market where all financiers buy a similar stake in every implemented project. Both result in the same expected payment to financiers.

An advantage of such a simple financial market is that it is easy to evaluate its performance. The market collapses to autarky when all agents resort to the storage technology and there are neither entrepreneurs nor financiers. In a Pareto-efficient allocation all or as many H-type projects as possible are financed whilst no L-type projects are financed. Accordingly, in a Pareto-inefficient equilibrium at least some L-type projects are carried out.

Let us denoted the proportion of type \( i \) agents that become entrepreneurs by \( \mu_i \). As \( \mu_i \in [0,1] \), we have a 3x3 matrix of equilibria as shown in Table 1.\(^2\) It is immediately clear that three out of the nine cannot exist. If no H-type agent becomes an entrepreneur, the potential financiers’ individual rationality constraint is violated. Similarly, due to our assumption that \( A<I \), it is not possible that all agents become entrepreneurs. The remaining six configurations cannot be ruled out a priori. They

\(^2\) These nine categories can be split further according to whether all type \( i \) agents participate or not.
consist of autarky and five cases where financial markets emerge as an equilibrium outcome. We name the five potential equilibria with financial markets according to what occupations \((e = \text{entrepreneur}, f = \text{financier}, s = \text{user of storage technology})\) agents of type \(i\) choose. For example, \(H^e L^f\) (column one, row one in Table 1) is the equilibrium where all H-type agents become entrepreneurs, and L-types split between becoming financiers, and using the storage technology.

**TABLE 1 HERE**

Both Pareto-efficient equilibria are in the first column of Table 1. Of these, the one in the last row is strictly better than the one in the middle row. Similarly, in the middle column, the equilibrium in the last row is more desirable than the one in the middle row. The equilibrium in the last column is the worst of the five equilibria with economic activity.

Removing the outside investors changes the adverse selection problem. Any equilibrium is now constrained by four conditions. The first are the individual rationality (IR) constraints we saw in the previous section, with a slight modification. Now agents compare the expected profits from becoming active, either as an entrepreneur or as a financier, to resorting to the storage technology. Second are the incentive compatibility (IC) constraints of both types’ of agents. By IC constraints we mean the choice between entrepreneurship and being a financier. The third relationship equalizes the supply of funds from financiers with the demand of funds by entrepreneurs. Finally, the contracts used in the financial market are determined by equating the payments by successful entrepreneurs to the expected compensation for financiers.

Denoting expected profits for a type \(i\) agent from activity \(j\) by \(\pi^j_i\), the IR constraints are
where superscript \( e \) denotes entrepreneurship and \( f \) financiership. The IC constraints are written as

\[
\pi^i_j \geq A \forall i, j, i \in \{H, L\}, j \in \{e, f\},
\]

Depending on the equilibrium (see Table 1) and agent type, the IC or IR constraint or both may bind, and the IC constraint may hold strictly one way (e.g., all H-type agents become entrepreneurs) or the other (e.g., all L-type agents become financiers).

The equality of supply (left hand side) and demand (right hand side) of funds is given by

\[
A[(1 - \mu_H - \chi_H)h + (1 - \mu_L - \chi_L)(1 - h)] = (1 - A)[\mu_H h + \mu_L (1 - h)],
\]

where \( \mu_i \) and \( \chi_i \in [0,1] \) denote the proportion of type \( i \) agents who become entrepreneurs and who employ the storage technology.

Finally, the expected payment made by entrepreneurs must equal the expected payment received by financiers:

\[
[\mu_H h \bar{p}_H + \mu_L (1 - h) \bar{p}_L]R_F = R_f[(1 - \mu_L - \chi_L)(1 - h) + (1 - \mu_H - \chi_H)h].
\]

The term in the square brackets on the left hand side is the expected (equilibrium) number of successful entrepreneurs while the term in the square brackets on the right hand side is the (equilibrium) number of financiers. In (6) \( R_F \) is the expected payment received by a financier that is independent of agent type. Substituting \( \pi^H_L = \pi^f_L = R_F \) for the IC constraints (4) shows that \( R_F \) also captures the opportunity cost of entrepreneurship.

Since solving the range of parameters where conditions (3)-(6) hold for all five equilibria with economic activity (see Table 1) is a straightforward but tedious exercise, we relegate the details of the calculations to the Appendix. Here we next...
consider the equilibrium $H^eL^f$ as an example and then graphically describe the remaining equilibria.

A. Example: $H^eL^f$

In $H^eL^f$, $\mu_H = 1$ and $0 < \mu_L < 1$, i.e., all H-type agents are entrepreneurs and L-type agents become either entrepreneurs or financiers (i.e., nobody chooses the storage technology, $\chi_i = 0$, $i \in \{L, H\}$). Since all agents are active, both types’ IR constraints are satisfied in equilibrium. L-type agents’ IC constraint holds with equality, whereas H-type agents (weakly) prefer entrepreneurship to being financiers.

To guarantee that all agents participate, we require that

$$R_F \geq A.$$  \hspace{1cm} (7)

The L-type agents’ IC constraint is given by

$$p_L (R_L - R_H) = R_F.$$  \hspace{1cm} (8)

The left hand side gives the expected return for an L-type agent from becoming an entrepreneur and the right hand side gives the expected return from becoming a financier. As L-type agents split between the two choices, they must be indifferent between them.

Since all H-type agents become entrepreneurs, their expected return from that choice must (weakly) exceed that from becoming a financier, i.e.,

$$p_H (R_H - R_H) \geq R_F.$$  \hspace{1cm} (9)

The aggregate supply and demand for finance is balanced when

$$(1 - \mu_L)(1 - h)A = [h + \mu_L (1 - h)](I - A)$$

holds. Finally, our assumptions on financial market transactions yield the following equilibrium relationship between the payment by a successful entrepreneur, and the expected payment received by a financier:
(11) \[ [h_{p_H} + \mu_L (1-h) p_L] R_B = R_F (1-\mu_L)(1-h) \].

Conditions (8), (10) and (11) determine the endogenous variables \( \mu_L \), \( R_B \), and \( R_F \). Solving first for \( \mu_L \) from (10) gives

(12) \[ \mu_L^* = \frac{A - h I}{(1-h)I} \].

The proportion of L-types who become entrepreneurs has to be less than unity. This is guaranteed by our assumption \( I > A \). As \( \mu_L^* \) also has to be nonnegative in \( H^e L^f \), (12) immediately reveals that \( H^e L^f \) can only exist if \( A/I \geq 1 \). In other words, \( H^e L^f \) cannot exist in a wealth constrained economy (see Figure 2).

Our next step is to use (11), (10) and (7) to solve for the equilibrium payments \( R_B^* \) and \( R_F^* \). They are given by

(13) \[ R_B^* = \frac{(I-A)}{I[h_{p_H} + (1-h) p_L]} p_L R_L \] and

(14) \[ R_F^* = \left[ 1 - \frac{p_L (I-A)}{I[h_{p_H} + (1-h) p_L]} \right] p_L R_L \].

Equations (13) and (14) suggest for H-types, the payments \( R_B^* \) and \( R_F^* \) are independent of project outcome, whereas for L-types the payments are functions of project outcome. One might tempted to think that that the equilibrium financial contract can be interpreted as a debt contract for H-type agents, but as an equity contract for L-type agents. The problem with this interpretation is that the contracts cannot be written contingent on project returns ex ante as the returns, conditional on success, are not verifiable. Thus, the payment from a successful L-type entrepreneur to her financiers is fixed, although it is a function of her project return.

After solving for the endogenous variables, we still need to find the parameter values satisfying the agents' IR and IC constraints (7)-(9). The L-type IC constraint
(8) binds as they split in their occupational choices. As H-types strictly prefer entrepreneurship to becoming financiers, and being a financier is at least as rewarding as resorting to the storage technology, their IR constraint (7) does not bind. This means that the relevant constraints are the L-type IR constraint (7) and the H-type IC constraint (9). Substituting (14) into (7) shows that the L-type IR constraint is satisfied (guaranteeing that no L-type agent stores her initial wealth) if

\begin{equation}
A \leq \frac{p_L R_L I (p_H - p_L)}{lh(p_H - p_L) + p_L (I - p_L R_L)}.
\end{equation}

The H-type IC constraint (guaranteeing that no H-type agent prefers becoming a financier over entrepreneurship) is satisfied if

\begin{equation}
A \geq I - \frac{(p_H R_H - p_L R_L)I [bp_H + (1-h)p_L]}{(p_H - p_L) p_L R_L}.
\end{equation}

In Figure 2 we use the \((A, h)\)-space to represent the set of parameter values for which \(H^eL^f\) exists.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{FIGURE2.png}
\caption{FIGURE 2 HERE}
\end{figure}

\(H^eL^f\) only exists in non-wealth constrained economies as suggested by (12). There L-types’ IC constraint binds, whereas H-types’ IR constraint does not. H-types’ IC constraint (16) is a downward sloping line in the \((A, h)\)-space: it cuts out the vertical and horizontal axes close to the origin, i.e., poor economies with a small number of good projects do not satisfy this constraint. L-types’ IR constraint (15) is a monotonically upward sloping curve that starts at the origin and cuts the \(h=AI\) diagonal once. Below the curve, some L-types would prefer not to participate.

**B. Existence of Equilibria**

Following a similar procedure as for the case of \(H^eL^f\) in Section III.A., we derive the values of the endogenous variables and determine the necessary and sufficient conditions for the existence of the six candidate equilibria (see the
Appendix). We present graphically the equilibria and discuss them. At the end of the Section, we summarize our main results.

In Figure 3, we employ the labeling of Table 1 to indicate the areas in which particular equilibria exist in the \((A, h)\)-space. There are two key lines: the \(h = A/h\) diagonal and the vertical \(\hat{A} \equiv p_L p_H (R_L - R_H)/(p_H - p_L)\)-line, whose meaning was explained in Section II. The diagonal not only divides the economies into wealth and non-wealth constrained ones, but is also a border of various equilibria in many cases. To the right of the vertical \(\hat{A}\)-line the equilibria are unique.

**FIGURE 3 HERE**

Let us first examine wealth constrained economies, i.e., the area above the diagonal, starting from the right hand side of the Figure. In wealth constrained economies all available wealth is not enough to finance all available H-type projects. In the upper right hand corner we find economies having a high proportion of H-types, and relatively high initial wealth. There, an equilibrium exists where all L-types become financiers, and H-types mix in their occupations between entrepreneurship and finance \((H^\ell L^f)\). All funds are directed into H-type projects, and therefore the equilibrium is Pareto-efficient.

Moving to the left, entrepreneurship becomes an option to L-types when we reach the vertical \(\hat{A}\)-line. Between it and another vertical line, \(\hat{A}(I_H R_H)\), we have two or three equilibria in the upper part of the Figure. One is the same \(H^\ell L^f\) as on the right hand side of that line. Another is \(H^\ell L^\ell\) where both types mix their occupational choices, i.e., both L- and H-types become entrepreneurs and financiers. The third equilibrium is \(H^\ell L^f\) where all L-types are entrepreneurs and H-types mix.

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\(^3\) In the Appendix, we present a separate figure for each equilibrium.
their occupations between entrepreneurship and finance. Since there are L-type agents among entrepreneurs in $H^f L^f$ and $H^f L'$, they are Pareto-inferior to $H^f L^f$.

Multiple equilibria can be explained by recalling from Section II the necessary and sufficient conditions for the existence of a separating equilibrium. The necessary condition is that the opportunity cost of entrepreneurship ($R_F^* \geq A$) exceeds $\hat{A}$, because then the pledgeable income of H-type agents exceeds that of L-type agents. The sufficient condition is that the opportunity cost of entrepreneurship exceeds the L-type agents’ expected profit from entrepreneurship, i.e., that equilibrium payments $R_F^*$ and $R_B^*$ are sufficiently high. In Section II competition between outside financiers drove down the equilibrium payments and the separating equilibrium emerges only to the right of the vertical $\bar{A} \equiv p_L(p_H R_L - 1)/(p_H - p_L)$-line. Compared with outside financing, the aggregate wealth constraint restricts the supply of finance and hence generates higher equilibrium payments. In particular, in $H^f L'$ the equilibrium payments $R_F^*$ and $R_B^*$ are so high that the opportunity cost exceeds both $\hat{A}$ and the L-types’ expected profits from entrepreneurship to the right of $\hat{A}(p_H R_H)$-line and therefore a separating equilibrium also exists between it and $\hat{A}$. To the left of $\hat{A}$ it is also possible to construct the pooling equilibria $H^f L^f$ and $H^f L'$ where the opportunity cost of entrepreneurship remains below $\hat{A}$ and, accordingly, the H-type agents’ pledgeable income below the L-type agents’.

Between the vertical lines, $\hat{A}(p_H R_H)$ and $\hat{A}$, a triangle exists close to the $h=1$ border, where only the Pareto-efficient $H^f L'$ exists. In this region there are sufficiently many H-types to raise the returns on financing high enough to prevent L-types from becoming entrepreneurs. Proceeding to left, once we cross $\hat{A}(p_H R_H)$ financial markets cease to operate except for a small area close to the $h=1$ border. Quite naturally, when $h$ is sufficiently high, the financial market emerges as an
equilibrium institution. The lower is the agents’ initial wealth, the higher the needed proportion of H-type agents that prevents the collapse of the financial market.

We then turn to the non-wealth constrained economies – those below the diagonal, and again start from the right. Because there is no aggregate shortage of liquidity, the model initially behaves as in the case of outside finance of the previous Section. The first equilibrium, $H^*L^*$, is Pareto-efficient. All H-type projects are financed, and L-types are indifferent between financing the H-types and using the storage technology. On the right hand side of the vertical $p_1R_L$-line, the storage technology is by assumption more lucrative than entrepreneurship for all L-type agents. Because of costly financing ($R_B>0$), the storage technology initially remains superior to entrepreneurship for L-type agents on the left of the $p_1R_L$-line.

Going further to left, once we hit the vertical $\bar{A}$ -line, the equilibrium changes. The area to the left of the line supports $H^*L^{\phi}$ which is Pareto-inefficient. Although all H-type agents become entrepreneurs, so do some L-type agents, too. Below the L-type IR-curve (equation (15)), the L-type participation constraint is satisfied with equality through some L-types opting for the storage technology. This means that below the curve, the equilibrium is $H^*L^{\phi}$ where all financiers (L-types) earn only $\bar{A}$ in expectation, and hence all L-type entrepreneurs also earn only $\bar{A}$ in expectation. H-type entrepreneurs’ IC (and IR) constraint is given by (16).

Above the L-type IR-curve, all L-type agents participate and thus nobody uses the storage technology even though available assets exceed the financing needs of H-type agents. Demand and supply of funds is equated through some L-type agents becoming entrepreneurs. H-types’ IC constraint (16) and $\hat{A}(I/l_HR_H)$ cross the diagonal at the same point. To the left these lines, autarky is the only equilibrium in the lower triangle.
To conclude the discussion on the efficiency of equilibria, we investigate whether the Pareto-inefficient equilibria are interim (incentive) efficient in the sense of Holmström and Myerson (1983).4 In the wealth-constrained economies the pooling equilibria $H^iL^f$ and $H^iL^f$ cannot be interim efficient to the right of $\hat{A}(l/p_HR_H)$ because the Pareto-efficient $H^iL^f$ exists there. To the left of $\hat{A}(l/p_HR_H)$, a social planner cannot induce separation as the pledgeable income of L-type agents exceeds that of H-type agents. In the non-wealth constrained economies the pooling equilibria $H^eL^f$ and $H^eL^f$ are not interim efficient, because the social planner could achieve Pareto efficiency by imposing a high enough $R_H$ ($R_H = (\kappa R_L - A)/\kappa L$). This would work because in this region the pledgeable income of H-type agents is higher than that of L-type agents.5

The above results regarding equilibria in the model without outside investors form our first Proposition.

**PROPOSITION 1**: Without outside investors,

a. All logically possible six equilibria exist for some set of parameters.

b. Autarky is the unique equilibrium in both wealth constrained and non-wealth constrained economies.

1) To the left of $\hat{A}(l/p_HR_H)$ and below (D.19) (see the Appendix for (D.19)) and

2) Below (16) and to the left of $\hat{A}$.

---

4 Loosely, in an interim incentive efficient equilibrium a benevolent social planner encountering the same informational imperfections as the individual agents cannot improve upon the market outcome without violating the agents’ individual rationality and incentive compatibility constraints.

5 If the social planner were allowed to dictate the occupational choices of agents, efficiency could in some cases be further improved. In the region to the left of $\hat{A}(l/p_HR_H)$, the planner could raise efficiency by randomly allocating agents into entrepreneurship. This would be feasible when $h \geq (I - \kappa R_H)/(\kappa R_H - \kappa R_L)$. Improvements on autarky would also be possible between $\hat{A}(l/p_HR_H)$ and $\hat{A}$ and below (15), if the social planner were allowed to randomly select agents to use the storage technology. With positive probability, a high-enough proportion of L-type agents would be forced to use the storage technology, pushing the proportion of H-type agents in the remaining population above the threshold $(A/I)$ needed to obtain economic activity.
c. Pareto-efficient and inefficient equilibria exist both in wealth constrained and non-constrained economies.

d. Except for autarky the Pareto-inefficient equilibria to the right of $\hat{A}(l/p_H R_H)$ are not interim efficient.

e. Multiple equilibria can only exist between $\hat{A}(l/p_H R_H)$ and $\hat{A}$, and above (16).

The relevance of Proposition 1 can be clarified by comparing the results to the standard model with outside investors. A look at Figures 1 and 3 reveals that to the right of $\overline{A}$ the only difference between outside finance and endogenous finance is in wealth constrained economies; there, outside finance allows the execution of all positive NPV projects. To the left of $\overline{A}$ the difference is stark: apart from the upper left hand corner where the equilibrium $H^L$ prevails, financial markets without outside investors yields a better outcome than with them. With outside finance, autarky is the equilibrium for a large range of parameter values for which in our model, a financial market emerges in equilibrium. It is even possible that endogenous finance yields Pareto-efficiency where the markets with outside financiers collapse to autarky. We summarize our comparison of the models with and without outside investors into the following proposition:

**PROPOSITION 2:**

a. To the right of $\overline{A}$, the outcome with outside investors Pareto-dominates the outcome without outside investors in wealth constrained economies. In non-wealth constrained economies, the equilibria coincide.

b. Between $\hat{A}$ and $\overline{A}$, the outcomes without outside investors weakly Pareto-dominate the outcomes with outside investors.
c. To the left of $\hat{A}$, the outcomes without outside investors weakly Pareto-dominates the outcomes with outside investors, except for the region where the equilibrium $H^eL^e$ exists.

C. Rents

We define economic rents as profits in excess of those that an agent would have earned in her next best occupation; we call informational rents profits to agents of type $i$ that are in excess of the profits that agents of type $j$ earn from the same occupation.\textsuperscript{6}

A moment of reflection and a look at Figure 3 reveal that H-type agents may only earn economic rents as entrepreneurs in non-wealth constrained economies. There, except in equilibrium $H^eL^e$, they strictly prefer entrepreneurship to other occupations, whilst L-type agents are indifferent between at least two occupations. The rents are also informational, as H-type agents earn more from entrepreneurship than L-types.

Analogously, it turns out that only L-types may earn rents in wealth-constrained economies. In $H^eL'$, L-type agents earn economic but not informational rents as their profits from becoming financiers strictly exceed profits from becoming entrepreneurs, or of not participating. In $H^eL'$, L-types earn both economic and informational rents since they strictly prefer entrepreneurship to financiership whereas H-types are indifferent between them.

There is also a link between the equilibrium payments and rents: agents only earn rents when their payments are independent of the project return even in equilibrium. H-type entrepreneurs’ payments depend on their project return in wealth-constrained economies where they do not earn rents but are independent of the return.
in non-wealth constrained economies when they do earn rents. L-types earn rents as financiers in $H^fL^f$ and as entrepreneurs in $H^fL^e$ where their payment is independent of their project return, but H-types’ payments are a function of the project returns. There is, however, no causal relationship between rents and payments, since they are simultaneously determined as part of equilibrium.

We summarize our findings regarding rents in the following proposition:

PROPOSITION 3:

a. H-type agents may earn rents only in non-wealth constrained economies; L-type agents may earn rents only in wealth-constrained economies.

b. H-type agents earn rents only as entrepreneurs; L-type agents may earn rents as entrepreneurs and as financiers.

c. When agents earn rents as entrepreneurs, economic and informational rents coincide.

d. Agents only earn rents when their equilibrium payments are independent of the project return.

IV. IMPERFECT STORAGE TECHNOLOGY

So far we have maintained the standard assumption of an exogenous storage technology that fully converts the initial assets to consumption goods. The existence of such a storage technology is vital in the literature that studies the stability of the banking system. For instance, it is known that removing the storage technology eliminates bank runs in Diamond and Dybvig’s (1983) model and its variations. To verify whether our findings are sensitive to the efficiency of the storage technology, we now assume that there is an imperfect storage technology under which $A$ depreciates at rate $1-\delta$, $\delta \in [0,1]$.

These are close, but not necessarily the same as the standard definitions, because in our model, agents
The only difference to the previous model is that the agents’ IR constraints (3) should be rewritten as

(17) \( \pi'_i \geq \delta A \quad \forall i, j, i \in \{H, L\}, j \in \{e, f\}. \)

When \( \delta \) is close to unity, our previous analysis is robust to the introduction of imperfect storage technology by continuity. As one might expect, however, the equilibria will change if \( \delta \) becomes small, because all agents are willing to invest either as financiers or as entrepreneurs even if their returns are small. To get an idea of the changes, let us reconsider the example of Section III.A \((H'\bar{L}'\bar{f})\) under an imperfect storage technology. To guarantee that all agents participate, we require that

(18) \( R_e \geq \delta A. \)

All other equations remain unchanged except that the L-type IR constraint (15) now takes the form

(19) \( A \leq \frac{p_L R_L I h(p_H - p_L)}{I h(p_H - p_L) + p_L (\delta I - p_L R_L)}. \)

When \( \delta \) is close to unity, (19) remains a monotonically upward sloping curve in the \((A, h)\)-space. Decreasing \( \delta \) shifts the curve to the right, increasing the range of parameters where \( H'\bar{L}'\bar{f} \) exists. It can be shown that when \( \delta \) approaches zero, \( H'\bar{L}'\bar{f} \) exists for all parameter values in the non-wealth constrained region in so far as H-types' IC constraint (16) holds. This is quite natural, since without a storage technology, the L-type IR constraint is trivially satisfied.

We now report the changes for the other equilibria when \( \delta = 0 \). Besides shortening the discussion, letting \( \delta = 0 \) further generalizes our model. There is no longer an exogenous storage technology, but the only way to transfer wealth over time, and to transform it from initial wealth to a consumption good, is to invest either as an

can choose their occupation, i.e., change from the supply side of a resource to the demand side.
entrepreneur or as a financier. Although our model lacks the second investment period
of their model, this is similar in spirit to Holmström and Tirole (1998) who evaluate
whether financial markets alone are able to supply enough liquidity.

FIGURE 4 HERE

The results of this exercise are summarized in Figure 4 (the calculations are
available from the authors upon request). Obviously, $H^s L^f$ and $H^s L^{e,f}$, the equilibria
where the storage technology was a viable option, cease to exist. Instead, $H^s L^f$, $H^{e,f} L^{e,f}$
and $H^{e,f} L^r$ exist for larger parameter value ranges. $H^s L^f$ remains unchanged. Thus,
removing the storage technology causes only modest changes to financial market
performance, suggesting that financial markets alone can take care of transformation
of wealth over time. The largest effect is an efficiency effect: the inefficient $H^f L^{e,f}$
exists where the efficient $H^f L^{e,s}$ used to exist. As a result, only Pareto-inefficient
equilibria exist in non-wealth constrained economies.

V. POLICY IMPLICATIONS

Though there are several limitations to our simple model, we boldly offer some
policy recommendations. The first deals with financial market integration and
liberalization. If financial market liberalization means the introduction of foreign
investors without projects of their own, the predictions are rather clear. Liberalization
can help a very poor country from autarky and generate a Pareto-efficient separating
equilibrium in a rich country. But liberalization is likely to be devastating for a
country that has medium initial wealth (compare the middle sections of Figures 1 and
3). This is reminiscent of Aghion, Bacchetta, and Banerjee (2003) where financial
market liberalization destabilizes an economy at an intermediate level of financial

---

7 For instance, future work should consider more than two types of agents, allow variation in agents’
initial wealth, and render the model more dynamic.
development which, in their model, is directly related to the initial wealth of the economy.

The consequences of financial market integration meaning the merger of financial markets of several countries are more convoluted. If the countries are identical, the integration has no effects. In the case of dissimilar countries, the outcome of integration crucially depends on the distribution of size, wealth and the proportion of good entrepreneurs across the countries. Let us consider as an example the integration of a rich country where there is scarcity of good projects with a poor country of the same size but with a lot of good projects. If there are so many good projects after integration that the integrated markets are wealth constrained, the outcome is likely to be Pareto efficient. In contrast, if the aggregate wealth of the integrated market satisfies the needs of all good entrepreneurs, it is likely to operate in a Pareto inefficient equilibrium.

One can also study widely adopted policies that seek to promote entrepreneurship (see, e.g., European Commission, 2001). Increases in the proportion of good entrepreneurs always have a beneficial effect on the economy, even if they lead to an aggregate wealth constraint. Keeping initial wealth constant, an increase in \( h \) either keeps the economy in the initial equilibrium category, or moves it from the existing equilibrium into a better one. Another straightforward “policy experiment” is to move the vertical \( \bar{A} \equiv p_L(p_H R_L - I)/(p_H - p_L) \)-line to left. This increases the set of parameter values for which a Pareto-efficient equilibrium exists in non-wealth constrained economies. The worse is either the return on the successful projects of L-type agents or the smaller their probability of success, or the higher is the success probability of H-type agents’ projects, the smaller are the problems created by adverse selection. In the limit, the expected value of the L-type agents’ projects is zero; this is
practically equivalent to assuming that agent type is observable ex ante. Our experiment thus suggests that if the policies promoting entrepreneurship raise the returns on the successful L-type projects without succeeding in making them positive NPV projects, they may be misguided. This also means that a tax on firm profits above $R_{H} - R_{L}$ may help restore efficiency (for a similar finding, see de Meza and Webb, 1987).

Shocks to model parameters change the values of the endogenous variables even if the equilibrium type remains the same as before the shock. When the parameters initially are close to a border, even small shocks may change the type of equilibrium. It is clear that in our model decreases in initial wealth may shift the economy from a Pareto-efficient equilibrium to an inefficient one (e.g., from $H^cL^s$ to $H^cL^d$), or even to autarky (e.g., from $H^dL^f$ to autarky). But it is also possible that an increase in initial wealth may have the same effect. Think of an economy that is in a Pareto-efficient $H^dL^f$ equilibrium (point 1 in Figure 5). Increasing initial wealth may move such an economy to the Pareto-inefficient $H^cL^f$ (point 2), if the increase is sufficient to turn a wealth constrained economy into an unconstrained one. Even substantial increases in initial wealth may result in such an adverse shift. One could interpret such policies as development aid, as the increase in initial wealth comes from outside the economy in question. Such policies will result in an increase of aggregate production, but possibly at the cost of inefficiency. However, increasing initial wealth is an effective tool in raising an economy out of autarky.

FIGURE 5 HERE

Finally, let us consider the role for financial intermediaries that collect and analyze information. There is no need for complicated financial institutions in the Pareto efficient equilibria. However, financial intermediaries could improve the
functioning of the economy when we have low initial wealth (autarky), moderate initial wealth ($H^fL^f$), or moderate to high initial wealth and a non-wealth constrained economy ($H^L^f$). This supports the findings in the literature on financial development suggesting that banks are important in less developed economies, while financial markets become important in developed economics (see Levine, 1997, and Allen and Gale, 2001, for surveys). The welfare-improving prospects of financial intermediaries also increase when financial markets alone have to perform transformation of wealth over time (i.e., when $\delta = 0$, or small).

Perhaps the most surprising rationale for financial intermediaries comes from the observation that competitive financial markets can drive interest rates too low from an efficiency point of view. As in de Meza and Webb (1987), we show in Section II how competition between outside financiers results in the oversupply of funds for a wide range of parameter values. Such oversupply of funds also occurs with endogenous finance in non-wealth constrained economies where competition between agents who become financiers generates the pooling equilibria $H^L^f$ and $H^L^{ef}$.  

VI. CONCLUSIONS

In this paper we study whether entrepreneurship and financial markets can endogenously emerge in equilibrium and how efficient are the eventual markets. We take the basic building blocks of our model from the literature and supplement them by subjecting all agents to capital constraints and allowing them to choose their occupations. In the usual partial equilibrium setting, the only equilibria are autarky without entrepreneurs since the financial markets do not open up, and a Pareto-inefficient equilibrium where all agents with a project become entrepreneurs. We first show that merely allowing potential entrepreneurs to have initial wealth may lead to a Pareto-efficient equilibrium if the initial wealth is high enough. We then exclude
outside investors and find that Pareto-efficient equilibria exist both in wealth constrained and non-wealth constrained economies, as do Pareto-inefficient equilibria. Autarky is also an equilibrium, but only for the poorest economies where agents’ initial wealth is low. In this respect, it makes only a modest difference whether there are “many” or “few” good projects in the population. We show that opening up the market to outside investors – i.e., financial market liberalization - often leads to a deterioration of entrepreneurial occupational choices and the performance of financial markets, even to the extent that where lack of outside finance yields to Pareto-efficiency, introducing it leads to autarky. The effects of financial market integration are ambiguous, and greatly depend on the type of economies that integrate.

Our model shows that, in the face of asymmetric information, the simplest type of financial markets may perform their role in resource allocation and asset transformation surprisingly well, and that while increasing the proportion of high-quality entrepreneurs is a remedy for removing inefficiency, increasing the initial wealth of an economy may not be.
REFERENCES


<table>
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<td>Not possible</td>
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<td>$H^eL^{fs}$</td>
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<td>Not possible</td>
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Notes:
$\mu_i$ = the proportion of $i$ type agents that become entrepreneurs in equilibrium.
Figure 1

- L-type IR
- H-type IR
- Pareto inefficient pooling equilibrium
- Pareto efficient separating equilibrium
- Autarky
Figure 2  \((H^e L^{ef})\)
Superscript $e$ = entrepreneurship

$f$ = financiers

$s$ = storage technology
Figure 4

Pareto efficient equilibria

Pareto inefficient equilibria

AUTARKY

\[ \frac{I^A}{P_{H}R_{H}} \quad \frac{I^A}{P_{L}R_{L}} \]
In this Appendix, we go through all possible equilibria besides autarky. For each equilibrium, we present
- the constraints,
- the equilibrium values of endogenous variables, and
- the equilibrium existence conditions.

We shorten the exposition by using the following notation: \(\Delta p = p_H - p_L\), \(\Delta R = R_H - R_L\), \(\gamma = p_H R_H - I\), \(\lambda = I - p_L R_L\), and \(\Delta W = \gamma \lambda = p_H R_H - p_L R_L\). The definitions have obvious interpretations. Since our approach to solve the model is rather mechanical, we explain the solution for the first equilibrium in more detail than for the subsequent equilibria. We also omit intermediate steps as these are straightforward (albeit sometimes tedious).

A. \(H^e L^f\) and \(H^e L^{fH}\)

\(H^e L^f\) is described in the example in Section III.A, so we first define \(H^e L^{fH}\) and then merely characterize its relation to \(H^e L^f\). In \(H^e L^{fH}\), all H-types are entrepreneurs and L-types are indifferent among entrepreneurship, financing, and using the storage technology, i.e., \(\mu_H = 1\), \(\mu_L \in (0,1)\), \(\chi_H = 0\), and \(\chi_L \in (0,1)\). The situation here is otherwise similar to \(H^e L^f\) described in Section III.A except that \(\chi_L\) is strictly positive. This means that (7) must hold as an equality, i.e.,

\[
R_F = A. \text{ "L- and H-type IR" (A.1)}
\]

The agents’ IC constraints are as before in (7) and (9), i.e.,

\[
p_L (R_L - R_H) = R_F. \text{ "L-type IC" (A.2)}
\]

\[
p_H (R_H - R_H) \geq R_F. \text{ "H-type IC" (A.3)}
\]

The economy level “budget constraint” (10) becomes

\[
(1 - \mu_L - \chi_L)(1 - h)A = \left[ h + \mu_L (1 - h) \right] (I - A). \text{ "Equality of supply and demand for funds" (A.4)}
\]

and, similarly, the financial market equilibrium condition (11) is

\[
[h p_H + \mu_L (1 - h) p_L] R_B = R_F (1 - \mu_L - \chi_L)(1 - h). \text{ (A.5)}
\]
"Financial market transactions"

Conditions (A.1)-(A.5) constrains $H^{\phi}L^{\phi}$. Equation (A.3) restricts the range of parameters and an equation system consisting of (A.1), (A.2), (A.4) and (A.5) determines the values of the endogenous variables $R_h, R_f, \chi_L$, and $\mu_L$. The equilibrium value of the expected payment received by financier, $R^*_f$, equals $A$ by (A.1). Then, solving (A.1) and (A.2) for $R_B$ gives.

$$R^*_F = \frac{p_L R_L - A}{p_L}.$$  \hspace{1cm} (A.6)

Upon substituting (A.1) and (A.6) into (A.5) we have two equations, (A.4) and (A.5), that determine the remaining two endogenous variables, $\chi_L$, and $\mu_L$. After somewhat involved algebra they can be written as

$$\mu^*_L = \frac{h}{1-h} \left[ \frac{(p_L R_L - A) \Delta p}{p_L \hat{\lambda}} - 1 \right] = \frac{h}{1-h} \left( \frac{R^*_B \Delta p}{\hat{\lambda}} - 1 \right)$$  \hspace{1cm} (A.7)

and

$$\chi^*_L = \frac{1}{1-h} \left[ 1 - \frac{(p_L R_L - A) \Delta p h}{p_L \hat{\lambda} A} \right] = \frac{1}{1-h} \left[ 1 - \frac{R^*_B \Delta p h}{\hat{\lambda} A} \right],$$  \hspace{1cm} (A.8)

where the last equalities come from (A.6).

The equilibrium exists if $\chi^*_L$ and $\mu^*_L$ given by (A.7) and (A.8) satisfy our initial assumptions that $\mu_L \in (0,1)$ and $\chi_L \in (0,1)$, and if the agents' IC and IR constraints are satisfied with $R^*_h$ given by (A.6). The first four existence conditions are

$$\mu^*_L < 1 \iff A > p_L \left( R_L - \frac{\hat{\lambda}}{h \Delta p} \right),$$  \hspace{1cm} (A.9)

$$\mu^*_L > 0 \iff A < p_L \left( R_L - \frac{\hat{\lambda}}{\Delta p} \right) = \hat{A} + \frac{p_L \gamma}{\Delta p} = \bar{A},$$  \hspace{1cm} (A.10)

$$\chi^*_L < 1 \iff A < \frac{p_L R_L I \Delta p}{I \Delta p + p_L \hat{\lambda}} = \frac{p_L R_L I \Delta p}{p_N I - p_L^{\hat{\lambda} R_L}},$$  \hspace{1cm} (A.11)

and

$$\chi^*_L > 0 \iff A > \frac{p_L R_L I h \Delta p}{I h \Delta p + p_L \hat{\lambda}} = \frac{p_L R_L I h \Delta p}{I(p_N + h \Delta p) - p_L^{\hat{\lambda} R_L}}.$$  \hspace{1cm} (A.12)
Since L-type IC and IR bind by (A.1) and (A.2), the fifth existence condition comes from H-type IC (A.3). If it is satisfied, H-type IR (A.1) also trivially holds. Inserting (A.1) and (A.6) into (A.3) shows that H-type IC holds if

\[ A \geq \hat{A}. \]  

(A.13)

Equations (A.9)-(A.13) define the range of parameters for which \( H^f L^{ds} \) exists. Since the critical values of \( A \) in (A.11) and (A.12) are strictly larger than the respective critical values in (A.10) and (A.9), the binding critical values are given by (A.10) and (A.12). They in turn cross each other at the diagonal \( h=\lambda / I \). This means that \( H^f L^{ds} \) only exists in non-wealth constrained economies. In terms of the \((A, h)\)-space, \( H^f L^{ds} \) exists in the area between the vertical lines (A.13) and (A.10), and below the curve (A.12), as depicted in Figure A.1.

**FIGURE A.1 HERE**

When (A.12) (which is identical to equation (15)) is violated, the H-type IC changes from (A.13) to (16), i.e., to \( A \geq I - I \Delta W \left( p_L + h\Delta p \right) / \Delta p p_L R_L \). Thus, \( H^f L^{ds} \) exists in the range of parameters described in Section III.A., i.e., in the area shaped by curve (A.12), the downward sloping line (16) and \( h=\lambda / I \) diagonal. Note also that curve (A.12), the vertical \( \lambda / I \) line and the downward sloping line cross at the same point where \( h = h_l \equiv \hat{\lambda} \). \n
\[ \frac{I \Delta W}{\lambda} \]

**B. \( H^f L^f \) and \( H^{ds} L^{ds} \)**

We first prove that \( H^{ds} L^{ds} \) cannot exist. In this equilibrium \( \mu_H \in (0,1) \), \( \mu_L = 0 \) and both \( \chi_H \) and \( \chi_L \in (0,1) \). The equilibrium is constrained by the following five conditions:

\[ R_F \geq A. \) "L- and H-type IR"

(B.1)

\[ p_L (R_L - R_B) \leq R_F, \) "L-type IC"

(B.2)

\[ p_H (R_H - R_B) = R_F, \) "H-type IC"

(B.3)

\[ \left[ (1 - \mu_H - \chi_H)h + (1 - \chi_L)(1 - h) \right] A = \mu_H h (I - A), \]

(B.4)

"Equality of supply and demand for funds."

and

\[ h \mu_H p_H R_B = R_F \left[ (1 - \mu_H - \chi_H)h + (1 - \chi_L)(1 - h) \right], \]

(B.5)
"Financial market transactions"

In \( H^f \hat{L}^f \) (B.1) holds with equality. Solving (B.4) for \( \mu_H \) yields

\[
\mu_a = \frac{A}{hl} \left[ 1 - \chi_H h - \chi_L (1 - h) \right]
\]  

(B.6)

Using (B.3) and (B.6) in (B.5) yields \( R_s \) as

\[
R_s^* = \frac{R_H (I - A)}{A}.
\]  

(B.7)

Inserting (B.7) back into (B.3) gives

\[
R_r^* = p_H R_H \frac{A}{I}.
\]  

(B.8)

Since \( R_r^* \) in (B.8) is strictly larger than \( A \), the initial assumption (B.1) that the agents’ IR constraints bind is invalid. This means that the equilibrium cannot exist for positive \( \chi_H \) and \( \chi_L \).

\( H^f \hat{L}^f \) can be characterized by setting \( \chi_H = \chi_L = 0 \) in (B.6). This means that

\[
\mu_H^* = \frac{A}{hl}.
\]  

(B.9)

Equation (B.9) gives two equilibrium existence conditions:

\[
\mu_H^* < 1 \iff A < hl,
\]  

(B.10)

and

\[
\mu_H^* > 0 \iff A > 0.
\]  

(B.11)

By means of (B.7) and (B.8) the third existence condition, the L-type IC constraint (B.2), can be written as

\[
A \geq \frac{I}{p_H R_H} \hat{A}.
\]  

(B.12)

Equations (B.10)-(B.12) define the range of parameters for which \( H^f \hat{L}^f \) exists. As shown in Figure A.2, the equilibrium exists in wealth constrained economies for \( A \in [\hat{A} I/p_H R_H, I] \).

FIGURE A.2 HERE

C. \( H^c \hat{L}^c \)
In this equilibrium, $\mu_H = 1$, $\mu_L = 0$, $\chi L = 0$ and $\chi L \in (0,1)$. In words, all H-types are entrepreneurs and L-types are either financiers or use the storage technology. The five basic conditions constraining the equilibrium are

$$R_F = A, \ "L\text{-type IR}"$$  \hspace{1cm} (C.1)

$$p_L (R_L - R_H) \leq R_F, \ "L\text{-type IC}"$$  \hspace{1cm} (C.2)

$$p_H (R_H - R_B) \geq R_F, \ "H\text{-type IC and IR}"$$  \hspace{1cm} (C.3)

$$(1 - \chi_L) (1 - h) A = h (I - A),$$  \hspace{1cm} (C.4)

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and

$$h p_H R_B = R_F (1 - \chi_L) (1 - h).$$  \hspace{1cm} (C.5)

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The equilibrium value of $R_F$ trivially equals $A$ by (C.1). By substituting (C.1) into (C.5), the other endogenous variables, $\chi_L$ and $R_B$, can be solved from (C.4) and (C.5). They are given by

$$\chi_L^* = \frac{A - h I}{A (1 - h)}$$  \hspace{1cm} (C.6)

and

$$R_B^* = \frac{I - A}{p_H}.$$  \hspace{1cm} (C.7)

From (C.6) we see that $\chi_L^* < 1$ by assumption $A < I$. Similarly, inserting (C.1) and (C.7) into (C.3) shows that H-types' IC and IR constraints are equivalent to $p_H R_B > I$ which holds by assumption. Thus, $H^c L^p$ is defined by two existence conditions. First,

$$\chi_L^* \geq 0 \ L \Rightarrow A \geq h I$$  \hspace{1cm} (C.9)

must hold. Second, the L-type IC constraint (C.2) must hold. Employing (C.1) and (C.7), it can be rewritten as

$$A \geq \hat{A} + \frac{p_L Y}{\Delta P} = \bar{A},$$  \hspace{1cm} (C.10)

where the right hand side equals (A.10). Equations (C.9) and (C.10) show that $H^c L^p$ only exists in non-wealth constrained economies for $A \in [\hat{A} + p_L \chi L \Delta P, I]$ (see Figure A.3).
We first prove that $H^eL^e$ cannot exist. The set-up of $H^eL^e$ practically mirrors $H^eL^{e^*}$ of Section A of the Appendix, because here $\mu_H \in (0,1)$, $\mu_L = 1$, $\chi_H \in (0,1)$ and $\chi_L = 0$. In words, all L-types are entrepreneurs, and H-types are indifferent between entrepreneurship, financing, and using the storage technology. The five basic constraints in $H^eL^e$ are

$$R_F = A, \quad \text{"L- and H-type IR"} \quad (D.1)$$

$$p_L(R_L - R_B) \geq R_F, \quad \text{"L-type IC"} \quad (D.2)$$

$$p_H(R_H - R_B) = R_F, \quad \text{"H-type IC"} \quad (D.3)$$

$$(1 - \mu_H - \chi_H)hA = [1 - h + \mu_H h](I - A). \quad (D.4)$$

"Equality of supply and demand for funds",

and

$$[h\mu_H p_H + (1 - h)p_L]R_B = R_F (1 - \mu_H - \chi_H)h. \quad (D.5)$$

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An equation system consisting of (D.1) and (D.3)-(D.5) determines the values of the endogenous variables, $R_B, \chi_H$, and $\mu_H$. Solving (D.1) and (D.3) for $R_B$ gives

$$R_B^* = \frac{p_H R_H - A}{p_H}. \quad (D.6)$$

Upon substituting (D.1) and (D.6) into (D.5) we have two equations (D.4) and (D.5) that determine the remaining two endogenous variables, $\chi_H$, and $\mu_H$. After somewhat involved algebra they can be written as

$$\mu_H^* = \frac{1 - h}{h} \left[ 1 - \frac{\Delta p(p_H R_H - A)}{p_H \gamma} \right] = \frac{1 - h}{h} \left( 1 - \frac{R_B^* \Delta p}{\gamma} \right) \quad (D.8)$$

and

$$\chi_H^* = \frac{1}{h} \left[ 1 - \frac{\Delta p(p_H R_H - A)I(1 - h)}{p_H \gamma A} \right] = \frac{1}{h} \left[ 1 - \frac{R_B^* \Delta p I(1 - h)}{\gamma A} \right] \quad (D.9)$$

Equations (D.8) and (D.9) provide four equilibrium existence conditions:
\[ \mu_H^* \leq 1 \iff A \leq \frac{p_H [h \gamma + (1-h)\lambda]}{\Delta p(1-h)}, \quad \text{(D.10)} \]

\[ \mu_H^* \geq 0 \iff A \geq \frac{p_H (I - p_L R_H)}{\Delta p}, \quad \text{(D.11)} \]

\[ \chi_H^* \leq 1 \iff A \leq \frac{p_H R_H I \Delta p}{I \Delta p + p_H \gamma} = \frac{p_H R_H I \Delta p}{p_H^2 R_H - p_L I}, \quad \text{(D.12)} \]

and

\[ \chi_H^* \geq 0 \iff A \geq \frac{p_H R_H I (1-h) \Delta p}{I (1-h) \Delta p + p_H \gamma} = \frac{p_H R_H I (1-h) \Delta p}{p_H^2 R_H - I (p_L + h \Delta p)}, \quad \text{(D.13)} \]

Since H-types' IC and IR bind, and L-types' IR is satisfied through their IC, the L-type IC is the fifth equilibrium existence condition. It is satisfied if

\[ A \leq \hat{A}. \quad \text{(D.14)} \]

The equilibrium may exist between the vertical lines (D.11) and (D.12), which is a nonempty set. However, the vertical line (D.14) is smaller in value than the vertical line (D.11). This means that the equilibrium cannot exist for positive \( \chi_H \)

In contrast, \( H^f L^e \) does exist.\(^8\) To see this, let \( \chi_H = 0 \) in (D.4) to get

\[ \mu_H^* = \frac{A - (1-h)I}{hI}. \quad \text{(D.15)} \]

Substituting (D.15) and (D.3) for (D.5) and letting \( \chi_H = 0 \) yields

\[ R^* = \frac{p_H R_H (I - A)}{I (p_L + h \Delta p)}. \quad \text{(D.16)} \]

Inserting (D.16) back into (D.3) gives

\[ R^*_f = \frac{p_H R_H [p_H A - \Delta p I (1-h)]}{I (p_L + h \Delta p)}. \quad \text{(D.17)} \]

From (D.15) we see that \( \mu_H^* < 1 \) holds by our assumption that \( A < I \). An equilibrium existence condition is thus

\[ \mu_H^* \geq 0 \iff A \geq (1-h)I. \quad \text{(D.18)} \]

The H-type IR is now \( R_f \geq A \), which - using (D.17) - can be expressed as

\[^8\text{Note that in } H^f L^e, \text{ (D.1) holds with a weak inequality.}\]
Similarly, by means of (D.16) and (D.17) the L-type IC (D.2) is given by

\[
A \geq \frac{p_H R_H I \Delta p (1-h)}{I \Delta p (1-h) + p_H \gamma} = \frac{p_H R_H I \Delta p (1-h)}{p_H^* R_H - I [p_L + h \Delta p]} \quad \text{(D.19)}
\]

Conditions (D.18)-(D.20) define the range of parameters for which \( H^e L^e \) exists. This is shown in the \((A, h)\)-space in Figure A.4.

**FIGURE A.4 HERE**

Conditions (D.18)-(D.20) practically mirror those of \( H^e L^e \) described in Section III.A. Equation (D.18) defines the downward sloping \( h=1-A/l \) diagonal that starts from the \((A=0, h=1)\) corner and ends in the \((A=l, h=0)\) corner. The L-type IC constraint (D.20) is a downward sloping line that cuts the \( h=1-A/l \) diagonal at the same point as the vertical \( \hat{A}l/p_l R_l \) line. H-types’ IR constraint (D.19) is a monotonically downward sloping curve that starts from the \((A=0, h=1)\) corner and cuts the \( h=1-A/l \) diagonal once. H-types’ IR and L-types’ IC constraints and the vertical \( \hat{A} \) line cross at the same point at

\[ h = h_2 \equiv 1 - \hat{A} \frac{\gamma}{\Delta WI} \].

In sum, \( H^e L^e \) exists above the H-type IR curve (D.19) and below the L-type IC line (D.20). This area exists in the upper-left corner of the \((A, h)\)-space where \( A \in [0, \hat{A}] \) and \( h \in [h_2, 1] \).

E. \( H^e L^f \) and \( H^f L^e \)

We first prove that \( H^e L^f \) cannot exist for a non-trivial set of parameters. In this equilibrium

\[
\mu_H \in (0,1), \quad \mu_L \in (0,1), \quad \chi_H \in (0,1) \quad \chi_L \in (0,1).
\]

In words, all agents are indifferent between entrepreneurship, financing, and using the storage technology. The agents’ IR and IC constraints bind, i.e., it must hold that

\[
R_F = A, "L- \text{ and H-type IR}" \quad \text{(E.1)}
\]

\[
p_L (R_L - R_H) = R_F, "L-type IC" \quad \text{(E.2)}
\]

and

\[
p_H (R_H - R_L) = R_F, "H-type IC" \quad \text{(E.3)}
\]
Solving (E.2)-(E.3) for \( R_B \) gives

\[
R_B^* = \frac{\Delta W}{\Delta p}.
\] (E.6)

Thus, there is a unique value of

\[
A = p_H (R_H - \frac{\Delta W}{\Delta p}) = p_L (R_L - \frac{\Delta W}{\Delta p}) = \hat{A}.
\] (E.7)

for which this equilibrium can exist. This means that only \( H^{L} L^{ef} \) (where \( \mu_H \in (0,1) \), \( \mu_L \in (0,1) \),

and \( X_H = X_L = 0 \)) may exist for a non-trivial range of parameters.

\( H^{L} L^{ef} \) is constrained by the following five basic conditions:

\[
R_f \geq A, \ "L- and H-type IR"
\] (E.8)

\[
p_L (R_L - R_H) = R_f, \ "L-type IC"
\] (E.9)

\[
p_H (R_H - R_H) = R_f, \ "H-type IC"
\] (E.10)

\[
[(1 - \mu_H)h + (1 - \mu_L)(1 - h)]A = \mu_L (1 - h) + \mu_H h (1 - A),
\] (E.11)

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and

\[
[(1 - \mu_H)h + (1 - \mu_L)(1 - h)]R_f = [p_L \mu_L (1 - h) + p_H \mu_H h]R_H.
\] (E.12)

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Equation system (E.9)-(E.12) determines the values of the endogenous variables, \( R_f, R_B, \mu_L, \)

and \( \mu_H. \) Solving (E.9) and (E.10) for \( R_B \) and \( R_f \) gives

\[
R_B^* = \frac{\Delta W}{\Delta p}
\] (E.13)

and

\[
R_f^* = p_H (R_H - \frac{\Delta W}{\Delta p}) = p_L (R_L - \frac{\Delta W}{\Delta p}) = \hat{A}.
\] (E.14)

Substituting (E.13) and (E.14) into (E.12) and solving (E.11) and (E.12) for \( \mu_L \) and \( \mu_H \) yields

\[
\mu_H^* = \frac{1}{h\Delta W} \left( \hat{A} - \frac{A_P_L R_L}{I} \right)
\] (E.15)

and

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\[ \mu_L^* = \frac{1}{(1-h)\Delta W} \left( \frac{Ap_H R_H}{I} - \hat{A} \right). \]  
(E.16)

Equations (E.15) and (E.16) yield four equilibrium existence conditions:

\[ \mu_H^* < 1 \quad \Leftrightarrow \quad A > \frac{I}{p_L R_L} (\hat{A} - h\Delta W), \]  
(E.17)

\[ \mu_H^* > 0 \quad \Leftrightarrow \quad A < \frac{I}{p_L R_L} \hat{A}, \]  
(E.18)

\[ \mu_L^* < 1 \quad \Leftrightarrow \quad A < \frac{I}{p_H R_H} [\hat{A} + (1-h)\Delta W], \]  
(E.19)

and

\[ \mu_L^* > 0 \quad \Leftrightarrow \quad A > \frac{I}{p_H R_H} \hat{A}. \]  
(E.20)

From equations (E.8)-(E.10) we see that agents’ IC constraints bind and IR constraints are satisfied if

\[ A \leq \hat{A}. \]  
(E.21)

This is the fifth equilibrium existence condition. However, we see that if condition (E.21) holds, (E.18) also holds. The equilibrium is thus defined by equations (E.17), and (E.19)-(E.21). Since (E.19) is identical to (D.20) we know that it cuts the vertical \( \hat{A} \)-line at \( h = h_2 \) where \( h_2 \equiv 1 - \frac{\hat{A}y}{\Delta WI} \) as defined in Section D of the appendix. This means that when \( h \) is large, i.e., \( h \in [h_2,1] \), the downward sloping line (E.19) and the vertical line (E.20) are the binding constraints. For \( h \in [h_3, h_2] \) where \( h_3 \equiv \frac{\hat{A}}{p_H R_H} \), the binding constraints are the vertical lines (E.20) and (E.21). For \( h \in [h_1, h_3] \), where \( h_1 \equiv \frac{\hat{A}^\lambda}{I\Delta W} \) as defined in Section A of the appendix, the binding constraints are (E.17) (which is identical to (15)) and (E.21). For \( h < h_1 \), the equilibrium does not exists, since (E.17) is violated.

**FIGURE A.5 HERE**

In Figure A.5 we illustrate how in terms of the \((A, h)\)-space, \( H \cap L\) exists in a parallelogram between the vertical lines (E.20) and (E.21) and the downward sloping lines (E.17) and (E.19). This parallelogram exists for \( A \in (\hat{A}l/p_H R_H, \hat{A}) \) and \( h \in [h_1,1] \).
Figure A.1 \((H^e L^{efs})\)

\[
\begin{align*}
\chi_{L^*} &> 0 \\
\mu_{L^*} &> 0
\end{align*}
\]
Figure A.2 ($H_{ref}^f L^f$)

L-type IC (B.12)

\[ \frac{I_A}{P_H R_H} \]

\[ \mu_H^* < 1 \] (B.10)

\[ h=1 \]
Figure A.3 \( (H^e L^{f_s}) \)

\[
\chi_{L} > 0 \quad \text{(C.6)}
\]

\[
\chi_{L}^{*} > 0 \quad \text{(C.10)}
\]
Figure A.4 \((H^e L^e)\)

\[ \mu_{\text{LL}} > 0 \quad \text{(D.18)} \]

\[ \text{h=1} \]

L-type IC
\[(\text{D.20})\]

H-type IR
\[(\text{D.19})\]
Figure A.5 ($H^fL^g$)

- Pareto efficient equilibria
  - $H^fL^f$
  - $H^fL^{fs}$

- Pareto inefficient equilibria
  - $H^fL^{gf}$
  - $H^fL^{gs}$

\[ \mu_L^* < 1 \quad \text{(E.19)} \]
\[ \mu_L^* > 0 \quad \text{(E.20)} \]
\[ \mu_H^* > 0 \quad \text{(E.18)} \]
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