The evolution of the adverse selection problem: a review of theoretical and empirical developments

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Adverse selection is a core part of our understanding of market failure, and that understanding has changed significantly since Akerlof's seminal paper. The impact of adverse selection on a market represents a large loss of efficiency, and thus there are two questions economists should seek to answer - the question of empirical existence, and the question of rectification.

I review some theoretical models of adverse selection, which suggest certain kinds of intervention to recover market surplus lost due to adverse selection issues. I also review empirical literature to do with the existence of adverse selection in practice, and a randomized experiment which confirms the existence of adverse selection and the viability of a potential intervention.

I find that the empirical evidence, while not unanimous, suggests that adverse selection is a real problem that causes lost market surplus in a number of markets, including a broad spectrum of insurance markets and some financial markets. I find that there is little reason to believe a single kind of intervention will work - rather, I conclude that interventions should be tailored to specific instances of adverse selection, making use of empirical and theoretical research directed at similarly specific markets or models. More research is needed to improve the empirical methodology and to further refine theoretical models of adverse selection before the economics profession can produce reliable policy advice for dealing with adverse selection.
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1 Introduction

In 2001, George Akerlof won (with two others) the Nobel Memorial Prize in Economic Sciences for his 1970 paper “The Market for Lemons: Quality Uncertainty and the Market Mechanism”. It was a paper that was rejected by the American Economic Review and the Review of Economic Studies as trivial, and the Journal of Political Economy referees argued that the paper’s argument was incorrect (Gans & Shepherd, 1994).

Akerlof’s paper is now one of the most cited papers in economic writing, with more than 18,000 citations\(^1\). It asked what would happen if certain markets had an informational distribution such that buyers were uncertain as to the quality of the good they sought to purchase - in this case, the example was the market for used automobiles, in which sellers naturally have a better understanding of the quality of their cars than potential buyers do. In the paper, Akerlof showed that it was possible for equilibria to involve mutually beneficial trades not taking place - even to the extent of no trade at all taking place, if the market characteristics were set up a certain way.

Akerlof’s work inspired others to consider the question for different markets - health insurance markets (Rothschild & Stiglitz, 1976), labor markets (Spence, 1973), and online auctions (Hou et al., 2009) are a few examples. According to Levin (2001), adverse selection is now viewed as a fundamental cause of market failure in the economics profession.

Two questions are thus of obvious importance. The first is the question of existence - while Akerlof showed that a theoretical basis for this kind of market breakdown existed, the empirical question of whether it exists or not in practice was unanswered. Empirical studies followed - in particular, early positive evidence came from Dahlby (1983, 1992), as well as Puelz & Snow (1994).

The second important question is that of redress, or rectification. Akerlof had identified a problem, a cause of market failure - could the economics profession find solutions that could recapture lost surplus caused by adverse selection? Contributions to this study included Leland (1979), who formalized minimum quality standards as a rather obvious option (although one of limited use, as we shall see). Other authors investigated leasing (Guha & Waldman, 1997), non-price signaling mechanisms (Taylor, 1999), and refined our understanding of how features of the market impacted the equilibria that developed (in

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\(^1\)Based on a Google Scholar search as of 01/28/2014
particular, Hendel & Lizzeri (1999) studied the impact of changes to the durability of the good or asset).

In this thesis I review the evolution of the theoretical adverse selection literature to include the passage of time, paying particular attention to proposals for recovering lost surplus. I also briefly review empirical evidence for the existence of adverse selection as a real issue in markets.

Empirical studies of adverse selection have produced mixed results, so one might question if it is worth considering surplus recovery mechanisms. I concur, however, with the conclusions of Cohen & Siegelman (2009) in a recent review of empirical literature on adverse selection in insurance markets:

Rather, our assessment of the work in this area is that there is a good basis for expecting the existence of adverse selection and a coverage-risk correlation to vary across markets and, indeed, even across segments of the same market. What we should expect empirical work to provide in this area, then, is not a once-and-for-all answer to the question about whether adverse selection exists but rather an ever-improving understanding of the circumstances under which a coverage-risk correlation should and should not be expected to arise.

Given this, then, I focus my thesis on surplus recovery mechanisms so as to understand which mechanisms would work best in the various markets where we can expect adverse selection to rear it’s head. We should not expect to solve adverse selection in different markets via an identical intervention - for instance, the used automobile market can reasonably be expected to respond differently to minimum quality standards than the health insurance market. Some of the models I will review would not even make sense as a description of the health insurance market while making perfect sense as a description of equity markets. If the empirical evidence suggests that adverse selection arises in different markets for different reasons, and in different ways, we should therefore intervene in different markets in different ways, targeting different features of the market.

The next section of this thesis, Section 2, reviews the basic model of adverse selection as identified by Akerlof - the static model. Models of this type make no allowance for the existence of time, and study the static equilibrium that develops. I then discuss Leland’s (1979) paper proposing minimum quality standards as a surplus recovery mechanism, and Levin’s (2001) paper which considers how adverse selection reacts to changes in the degree
of informational asymmetry.

In Section 3, I consider the alternative - models in which the passage of time is considered. I find that the passage of time is itself a surplus recovery mechanism (in some sense), and consider two very similar models of dynamic adverse selection (Fuchs & Skrzypacz, 2013; Janssen & Roy, 2002) and the interventions they suggest.

In Section 4, I briefly review the empirical literature on adverse selection, leaning heavily on the 2009 review of insurance markets from Cohen & Siegelman. As indicated before, I echo their conclusions.

Finally, in Section 5, I consider the issues facing a central authority that seeks to intervene in a market suffering from adverse selection losses, and I conclude by suggesting avenues of research I think will prove most fruitful in the future.

Readers who are not familiar with the basics, or who seek full proofs of some of my claims, will find useful information in the Appendix.
2 A Basic Model of Adverse Selection

Consider an agent who wishes to sell an asset she owns. In a normal competitive market, we can analyze this situation with little effort. The seller will have some reservation price \( p_R \). If the price she receives for selling the asset is greater than her reservation price, she will sell, otherwise she will not. Buyers will not offer more than \( p_R \) unless their valuation of the asset exceeds \( p_R \), so if trade occurs, it will be mutually beneficial.

This analysis relies on an implicit assumption of the fundamental welfare theorems that the characteristics of all assets are common knowledge to market participants (Mas-Colell, Whinston, & Green, 1995). Another way of thinking about this is to view a market as existing for all assets which are identical, and for different markets to exist for assets which are different. If we allow violation of this complete markets assumption, the analysis becomes more difficult, and more interesting.

The most common examples used are used-car markets (owing to Akerlof), insurance companies (owing mostly to Rothschild & Stiglitz), and labor markets. In the first case, the owner of a car knows more about that car’s quality than potential buyers. In the second, potential buyers of insurance know more about their risk profile than the insurance company. In the final case, potential workers know more about their productivity than the firms that wish to hire them. This informational asymmetry on its own causes some inefficiencies, and is a necessary condition for adverse selection problems to arise.

In what follows we will consider a general case of a market for a durable asset. This will most strongly resemble examples based on a used-car market, but the analysis proceeds along the same lines for all the examples above\(^2\).

Let there be an agent who wishes to sell an asset she owns. The asset has some quality \( \theta \). Let \( [\theta, \bar{\theta}] \subset \mathbb{R} \) denote the set of possible qualities, with \( 0 \leq \theta < \bar{\theta} < \infty \). The proportion of assets with quality \( \theta \) is given by the distribution \( F(\theta) \), which we assume to be non-degenerate (i.e. there are at least two possible qualities). Note that the density \( f(\theta) \) is analogous to the supply curve for the asset being sold (Leland, 1979). For simplicity, we will assume that the seller’s reservation price for an asset of quality \( \theta \) is \( \theta \).

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\(^2\)Fundamentally, adverse selection concerns a situation where the sellers have an informational advantage compared to the buyers. In a labor market model, the employees are the sellers (their asset is productivity). Note that with some mental gymnastics, one could re-define the sellers and buyers in an adverse selection problem and produce an informed principal problem, and vice-versa.
The seller faces a competitive market of potential buyers. Because the agent is the only seller, the buyers are price takers. Buyers have a reservation price for an asset of quality $\theta$ equal to $r(\theta)$.

If quality were common knowledge, in this case, trade would occur as long as one buyer had $r(\theta) > \theta$, at a price of $r(\theta)$ (because the buyers are price takers). Let the price at which the asset trades be $p^*$. Aggregate surplus would be given by:

$$ (p^* - \theta) + (p^* - r(\theta)) $$

The left-hand term represents the seller’s surplus and the right-hand term the buyer’s surplus. Since $p^* = r(\theta)$ in a competitive market, aggregate surplus simplifies to $(r(\theta) - \theta)$. This clearly maximizes aggregate surplus, and is thus a Pareto optimal outcome. But we are concerned with a case where quality is not known beforehand to the buyers. In this case, $p^*$ must be independent of $\theta$.

For simplicity, at this point we will introduce an assumption that all buyers have the same reservation price for an asset of given quality (that is, $r(\theta)$ is the same for all buyers). This is not a realistic assumption for obvious reasons. However, relaxing the assumption is equivalent to introducing an informed principal problem to this analysis, where we are concerned only with adverse selection. The analysis above does not require this assumption, but remains unchanged by it. If this assumption were relaxed, we would encounter trivial equilibria where no trade occurred. We will further assume that $r(\theta) \geq \theta$ for all $\theta$.

Buyers will thus form a rational expectation of quality. They must have some information with which to form this, so we allow them to know the form of the distribution function $F(\theta)$, and its density $f(\theta)$. The price that any buyer is willing to offer (equivalently, the equilibrium price in the market) is thus:

$$ p^* = r(E[\theta|F(\theta)]) $$

As in Leland (1979), we can without loss of generality re-scale so that $\theta$ is distributed uniformly on the interval $[0, 1]^3$. After re-scaling, $p^*$ becomes $r(\frac{1}{2})$.

\[3\text{We do this by setting } \theta = F(\theta)/F(\bar{\theta})\]
If our seller has an asset with $\theta > \frac{1}{2}$, she will not put the asset on the market. Thus, $p^*$ determines the highest quality asset that will be brought to market. However, this changes the buyer’s expectations - they know that if an asset is on the market, its quality is at most $p^*$. Thus, for any equilibrium price $p > 0$, buyers will believe that average quality is:

$$\theta_{avg}(p) = E[\theta | \theta \in [0, p]] \quad (3)$$

For the uniform distribution, this means that $\theta_{avg} = \frac{p}{2}$. Thus, the necessary condition for any trade to occur is that $r(\frac{p}{2}) \geq p$. If $r(\theta) = \frac{3}{2} \theta$, for instance, $r(\frac{p}{2}) = \frac{3}{4}p$, and no trade will occur at any price. If $r(\theta) = \theta + \frac{1}{4}$, $r(\frac{p}{2}) = \frac{p}{2} + \frac{1}{4}$, so trade will occur when $p < \frac{1}{2}$ - and thus no assets with $\theta > \frac{1}{2}$ will trade\(^4\), but those with lower qualities will.

These outcomes are obviously not Pareto optimal - we have assumed that $r(\theta) \geq \theta$, so aggregate surplus cannot be maximized unless all assets trade.

A different way of understanding adverse selection is to view it as the outcome of a game-theoretic model in which there are two buyers, who maintain all the common knowledge outlined above. The behavior is evident in a two-stage game: In the first stage, the buyers simultaneously announce a price offer for the asset. In the second stage, the potential seller decides whether to sell and which buyer to sell to.

The subgame perfect Nash Equilibria (henceforth SPNE) of this game are characterized primarily by the requirement that the buyers receive zero expected surplus - if there were an SPNE in which a buyer received an expected surplus greater than zero, the other buyer would be strictly better off if they were to offer $r(E[\theta | F(\theta)]) - \alpha$ for some $\alpha > 0$. This logic obtains for all successful offers of the above form until $\alpha = 0$, which means that an SPNE requires that either no trade occurs in stage 2, or trade occurs at a price of $r(E[\theta | F(\theta)])$.

One obvious SPNE is thus for both buyers to offer $r(E[\theta | F(\theta)])$. As is conventional, in this case we assume that the seller is indifferent between the buyers, and chooses between them randomly (if the offer is high enough for trade to occur). Neither buyer can unilaterally deviate to earn a greater surplus - if they deviate to a lower offer, they will never acquire the asset, and if they deviate to a higher offer they will earn a negative expected surplus. If no trade occurs at the above offer, there are trivially no unilateral

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\(^4\)The fringe case, where $\theta = \frac{1}{2}$, depends on our assumption about what happens when a seller is indifferent. Conventionally it is assumed that they sell.
deviations that are beneficial.

Having established that in this basic model adverse selection tends to produce outcomes that are not Pareto optimal, it trivially follows that “a central authority who knows all agents’ private information [...] and can engage in lump-sum transfers among agents in the economy, can achieve a Pareto improvement over these outcomes” (Mas-Colell, Whinston, & Green, 1995). As they immediately note, however, this is an unrealistic conception of the central authority. It would be laughable to propose that the government has perfect information about the quality of all used cars, all equities, worker productivities, or insurance risks. Without perfect information, the central authority faces additional constraints, and thus an outcome which cannot be improved by such a constrained authority is called a constrained Pareto optimum.

One of the constraints that applies is that the central authority cannot use lump-sum transfers between agents of different type (or, in our model, sellers of different $\theta$), because determining type is impossible. Lump-sum transfers as a method of improving an outcome must rely on something that is known to the market participants as well. Thus, we can consider the problem without loss of generality by imagining that the central authority replaces the buyers in our model. In the appendix we reproduce the explanation of constrained Pareto optima (with reference to a labor market model) from Mas-Colell, Whinston, & Green (1995).

In response to a market with adverse selection, it is clear that both signaling and screening mechanisms familiar from game theory have at least the potential to improve the efficiency of outcomes. A thorough explanation of basic signaling and screening versions of the adverse selection model can be found in the appendix. As we are first handling static models, we will proceed to discuss Leland’s (1979) model of minimum-standards or licensing, which is in some ways similar to a screening system.

Leland introduces a minimum quality standard to the static formulation of the adverse selection problem in a very simple manner - he sets some level $L \in [\bar{\theta}, \bar{\theta}]$ and eliminates all supply for which $\theta < L$. Thus, the interval over which supply is available becomes $[L, \bar{\theta}]$. Maintaining the re-scaling to a uniform distribution over the interval $[0, 1]$, we can see that (3) in the presence of a minimum quality standard becomes:

$$\theta_{avg} = E[\theta | \theta \in [L, p]]$$ (4)
Thus, $\theta_{\text{avg}} = \frac{p+L}{2}$. This changes the necessary condition for any trade to occur to $r(\frac{p+L}{2}) \geq p$. In the case of the example given previously ($r(\theta) = \frac{3}{2}\theta$), this reduces to $L > \frac{p}{2}$. Since the introduction of a quality limit reduces the supply of lower-quality assets, but increases the average quality (and therefore the probability of market entry), there are competing effects and the outcome cannot be known \textit{a priori} (Leland, 1979).

The degree to which a minimum quality standard will be beneficial depends on the exact structure of the market. Leland concludes that a minimum quality standard will provide larger benefits if the market has:

- Greater sensitivity to quality variations
- Low elasticity of demand
- Low marginal production (of quality) cost
- Low values for low-quality service

This list pertains to Leland’s chosen example of a service industry - in the case of the asset market example we use here, this list corresponds roughly to $r(\theta)$ having a large range, low elasticity of demand, a low discount rate, and low values of $r(\theta)$ for low values of $\theta$ respectively. Particularly, Leland finds that if opportunity cost decreases with quality (which would correspond to a discount rate changing with quality)\footnote{Which is of course a nonsensical statement in this model. However, in Leland’s service industry model the opportunity cost makes sense and motivates interesting and important parts of the analysis}, minimum quality standards will always be desirable. Although we have not covered it here, Leland also finds that “if a professional group or industry is allowed to set minimum quality standards (self-regulation), these standards may be set too high or too low. On balance, however, there is some reason to expect too-high standards to be the more likely case.”

Another of the more obvious interventions possible is to somehow reduce the degree of informational asymmetry. Ignoring the question of practicality, Levin (2001) shows us that this is not as straightforward as it sounds. Levin identifies two effects with opposite influences. As a seller’s informational advantage increases, demand is reduced by the “buyers curse” effect\footnote{That is, buyers are less likely to buy because the only reason a seller would enter the market is if their asset was low quality}. However, Levin also finds a supply shift as well.

Levin’s model is broadly similar to the model we have already enumerated. However,
in order to accommodate the concept of more or less informational advantage, Levin does not require that the potential seller knows her asset’s quality with certainty. Thus, instead of the seller having an asset of quality $\theta \in [\theta, \bar{\theta}]$, the seller observes a private signal $X$. The distribution of quality is commonly known, as before.

Thus, the buyer side of the model is unchanged from before. Buyers believe that the asset is of quality $E[\theta | F(\theta)]$. The seller, however, refines her belief about the distribution to arrive at a belief that her asset is of quality $E[\theta | F(\theta | X = x)]$ for some signal $x$.

Levin assumes, as we did earlier, that gains from trade always exist ($r(\theta) \geq \theta$ for all $\theta$). He also assumes that $F(\theta | x')$ first-order stochastically dominates $F(\theta | x)$ for any $x' > x$. This assumption means that signals are “stochastically ordered so that observing a higher signal is favorable news about quality” (Levin, 2001). Without this assumption Levin would need to specify functional forms for the valuation functions.

As in the simpler form of the model we are familiar with, Levin simply considers the feasibility of trade at some equilibrium price $p$. A seller will only trade at price $p$ if $E[\theta | F(\theta | X = x)] \leq p$. The buyer, who knows this, will only buy at $p$ if $E[r(\theta) | E[\theta | F(\theta | X = x)] \leq p] \geq p$.

Levin provides something to approximate a supply curve - he lets $\alpha(p)$ denote the ex ante probability that a seller enters the market at $p$:

$$\alpha(p) = \sup\{\alpha : E[\theta | F_\alpha(X) = \alpha] \leq p\}$$  \hfill (5)

Where $F_\alpha(X)$ is the marginal distribution of the signal $X$. $E[\theta | F_\alpha(X) = \alpha]$ is the seller’s expected value conditioned on receiving a signal $X = F_\alpha^{-1}(\alpha)$. Due to the assumption of first-order stochastic domination, $\alpha(p)$ is increasing in $p$. Levin uses this to establish a market performance metric - noting that the maximum possible extent of trade for a given information structure is $\alpha^* = \sup_{p \in P} \alpha(p)$, where $P$ is the set of prices at which a buyer will enter the market. The price at which this extent of trade is realized is the constrained Pareto-efficient equilibrium (Levin, 2001).

To illustrate the that greater informational advantages to the seller can produce oppo-
site movements, we will use Levin’s contrasting examples, adapted from Akerlof’s original example, which limits $\theta$ to taking one of three values:

$$r(\theta) = \begin{cases} 
14 & \text{if } \theta = 0 \\
28 & \text{if } \theta = 20 \\
42 & \text{if } \theta = 40 
\end{cases} \quad (6)$$

This case fulfills the requirement that there are always gains from trade, so in a market without informational advantages we would expect all types to trade. This is true whether the buyers and sellers are equally informed or equally uninformed.

Allow, then, the seller to determine if her asset has $\theta = 0$, but no more. We have given the seller an informational advantage, and the end result is that only assets with $\theta = 0$ will trade. Buyers are unwilling to pay anything over $E[r(\theta)] = 28$, while sellers are unwilling to part with an asset for which $\theta = \{20, 40\}$ for a price under 30. Trade for assets with zero quality will occur at some price between 0 and 14.

If we now allow the seller to determine the exact quality of her asset, it becomes possible to trade assets for which $\theta = \{0, 20\}$, despite the fact that we have increased the informational advantage for the seller (Levin, 2001).

Levin argues that this is because the original 3-case Akerlof example is a “noiseless partition-information structure” (Levin, 2001). The advantage we are giving the seller in each example above is an advantage that allows her to determine that her asset is in a subset of the set known to buyers - it allows her to partition her assets more finely. In the first example, the finer partition is only relevant if the seller is on the margin - it drives her out of the market if she discovers that her asset is worth more than the market price. In the second example, the finer partition is only relevant if the seller is out of the market - it brings her back into the market if her asset is revealed to be worth trading at the market price. Levin generalizes this result - readers are directed to Levin’s paper for the full details of this generalization, as it is far too complex for this review.

Levin also provides an example of nonmonotonicity in the opposite direction (as the seller
becomes more informed). Again, he uses the Akerlof 3-type framework:

$$r(\theta) = \begin{cases} 
10 & \text{if } \theta = 0 \\
28 & \text{if } \theta = 20 \\
85 & \text{if } \theta = 40 
\end{cases} \quad (7)$$

In this case, if the seller can perfectly distinguish her asset type while buyers have no information, all types will trade at $E[r(\theta)] = 41$. If we allow the buyer to partition such that he can distinguish between $\theta = \{0, 20\}$ and $\theta = 40$, trade for the highest quality assets will occur at a price between 40 and 85 - but the low and middle quality assets cannot trade, since the equilibrium price for them is 19. A reduction in informational asymmetry has reduced the amount of trade (Levin, 2001). As before, Levin’s generalization of this result is too complex for a full explanation here, and readers are directed to his paper.

Levin also considers how this analysis works with more general ideas of the information content of the private signals. While the explanation of these informativeness models is too complex for us here, Levin concludes that “the expected value of the object to the buyer conditional on an ex ante probability $q$ that the seller will tender is decreasing in the quality of private information” - the buyer’s curse effect.

However, the competing effect Levin identified is also present. Improving the seller’s information “may cause the seller’s value for the marginal quality to fall”. This is a selection effect that increases supply. Levin concludes with an effort to determine which effect dominates under certain conditions, and provides the conditions under which the buyers curse effect is dominant and dominated.

As noted in the introduction, an idea which bridges the gap between static and dynamic models (to some extent) is the idea of a reputation system. Yamagishi & Matsuda (2002) study the idea with reference to the systems prevalent in online auctions at the time (primarily eBay and Yahoo).

Reputation systems bridge the gap in the sense that a reputation system requires the passage of time, and multiple trades, in order to work. A seller offering her first good for auction has no reputation, because she has no known history to the buyers. Thus, in this sense, reputation systems do not fit into static models. However, they do not fit into dynamic models either because the passage of time is not directly linked to the prices offered
or accepted. One could theoretically model reputation systems by randomly assigning an endowment of reputation to sellers - while this bears no relation to the actual process by which reputations are acquired, it allows us to study a static moment of the market with reputation systems without loss of generality, so as to understand the impact reputation systems have.

According to Yamagishi & Matsuda, the progenitor of reputation systems are systems which force a commitment before trade can occur. The example chosen is raw rubber traders in South East Asia, where it is common for rubber traders to enter into long commitments with suppliers. Thus, traders are unlikely to trade with an unknown supplier unless the price offered is extremely low (in this sense, the market faced by a new supplier is analogous to the basic setting outlined by Akerlof).

If traders share information about their suppliers, the cost they face in keeping their suppliers from behaving dishonestly is reduced - dishonest behavior is shared, and thus the cost of behaving dishonestly and being caught is significantly higher (as the supplier will be almost unable to trade except at the lemons price).

Yamagishi & Matsuda conducted an experiment where volunteers took on both the roles of buyers and sellers. As sellers, they expended some amount of money to produce an abstract commodity, where the quality of the commodity was a function of the expenditure. They then put the commodity on a market for a set price, and announce the quality of the commodity. Sellers are allowed to lie about the quality in the announcement. Acting as buyers, the participants can buy for the set price. After buying, the commodity is immediately resold to the experimenter for a 50% premium over the true quality, not the advertised quality. Thus, buyers have an incentive to purchase commodities that they believe to be accurately advertised.

The control version of the experiment verified that adverse selection existed and posed a problem. Rational sellers were expected to expend only the minimum amount on production, and thus rational sellers to buy for only 150% of the minimum amount. As one would expect, in the control condition the difference between advertised quality and true quality was very large. The average true quality decreased over time as participants learned. Both results suggest the existence of a strong adverse selection effect (Yamagishi & Matsuda, 2002).
A second version of the experiment assigned a unique, permanent identity to each player. The average quality in this case was much higher and did not trend upwards or downwards during the longer session. Interestingly, dishonesty was still found to be profitable, although significantly less so, and adverse selection effects were still identified (although at a lesser degree of severity).

A final experiment allowed buyers to evaluate the seller they had just traded with, and provided a reputation score (of inexact nature, to prevent exact identification) to future buyers. The average quality of goods produced was not far removed from the average quality when players had unique identities. Dishonest behavior was not profitable with the reputation system, although neither was honesty.

The results from Yamagishi & Matsuda’s experiment suggest that reputation systems have some alleviating effect on the adverse selection problem, but are limited by the ability of some central authority to prevent market participants from creating new identities (and thus, resetting their reputations to the default). Online auctions, where such systems are prevalent, face the issue that participants can assume a practically unlimited number of identities. In other markets, where it would perhaps be possible to attach a reputation to an identity which is costly to replace (for instance, drivers licenses or passports), it is worth wondering if reputation systems would be politically tractable.

Static models of adverse selection are characterized by the result that only a subset of potential sellers actually trade. Market failure can be measured in a quantifiable way by comparing the size of the non-traded subset of qualities, and the loss of surplus realized by adverse selection is related to the potential gains from trade which are not realized.
3 Dynamic Adverse Selection

In very simple terms, dynamic models of adverse selection differ only in that they introduce and model the effect of passing time. However, this is not a trivial thing - it must be specified what effect the passing of time has, and how the market operates as time passes. Thus, it is not possible to simply “add” some time parameter to the static model explained earlier.

The simplest dynamic model arises when you introduce a continuous time parameter and allow trade to occur at any time. The key difference between a static and dynamic model is that in the dynamic model the potential seller is faced with a sequence of “trade/do not trade” decisions, rather than a single such decision as in the static model. According to Fuchs & Skrzypacz (2013), this option to delay trade can be used by the market as a screening mechanism. As with changes to informational asymmetry, introducing the time component produces both new costs and new benefits.

The screening provided by the option of costly delay tends to increase the overall liquidity of the market, or the maximum number of types that trade, compared to a static model (Fuchs & Skrzypacz, 2013). In a similar vein to Levin (2001), however, for sellers on the margin of a static market, the option to delay trade is unequivocally beneficial, so dynamic trading models invariably have less trade in the first period - this makes the adverse selection problem even worse.

The model used in Fuchs & Skrzypacz (2013) is a relatively simple adaptation of the static model to the addition of a time component. Other dynamic models (for instance, Daley & Green, 2011) are significantly more complicated. In this section we will concentrate on the simple model from Fuchs & Skrzypacz, and a closely related model by Janssen & Roy (2002).

As in our static model, there is an agent who wishes to sell an asset she owns. It seems traditional in the dynamic literature to assume that the seller faces a liquidity shock, which provides a theoretical justification for future assumptions about gains from trade. The asset has some quality $\theta$, drawn from a commonly known distribution $F(\theta)$ and strictly positive density $f(\theta)$. She faces a competitive market of potential buyers, who value an asset of given quality at $b(\theta)$. Again, we make an assumption to establish gains from trade. However, following Fuchs & Skrzypacz (2013), we now make a slightly stronger
restriction - \( b(\theta) > \theta \) for all \( \theta < 1 \) and \( b(1) = 1 \). We have changed from \( r() \) to \( b() \) because we must now explicitly account for discounting. Following Fuchs & Skrzypacz, we do this by assuming all players discount at rate \( r \) and that \( \delta = e^{-rT} \).

We introduce time as \( t \in [0, \infty] \). At some time \( t = T, T \leq \infty \), we assume that the seller’s knowledge becomes common knowledge - that is, that the market reverts to one without an adverse selection problem. The handling of the passage of time is the main difference between Fuchs & Skrzypacz and Janssen & Roy - the remainder of the model is very similar.

Fuchs & Skrzypacz define a set \( \Omega \subseteq [0, T] \) to contain the values of \( t \) for which the market is open. The smallest possible \( \Omega \) is \( \{0, T\} \).

The notion of an equilibrium in a dynamic model is not as simple as it was before. Fuchs & Skrzypacz define an equilibrium as a pair of functions \( \{p_t, k_t\} \) where the former is the market price at \( t \) and the latter is the “cutoff type” at \( t \) - that is, \( k_t \) is the highest quality asset that trades at \( t \). This equilibrium pair must satisfy the following conditions:

\[
p_t = E[b(\theta) \mid \theta \in [k_{t-}, k_t]], \text{ where } k_{t-} \text{ is the cutoff type at the previous market opening time.}
\]

Sellers maximize profits according to the rule given by \( k_t \)

In any market opening period, the price is at least \( p_t \geq b(k_{t-}) \)

The first two conditions are standard and do not require explanation. The last condition is necessary to avoid trivial equilibria where the market does not clear (Fuchs & Skrzypacz, 2013). Finally, it is assumed that all trades are observed and remembered - thus, a buyer cannot purchase an asset and then place it on the market in a later period without that asset’s history being recalled, and its price being properly updated.

The static model in the previous section corresponds to a dynamic model with trading allowed only at \( t = \{0, T\} \) (Fuchs & Skrzypacz, 2013). The interesting part of dynamic models becomes obvious if we instead allow trade to occur at any \( t \in [0, T] \). Following Fuchs & Skrzypacz, we will denote this set of opening times as \( \Omega_C \).

Intuitively, it is obvious that the continuous trading market will produce different equilibria. A seller who is on the margin in a static market will obviously be better off if she
delays trade by some small amount. The price in the next period would by definition be 
$b(k_0)$. All dynamic equilibria will have less trade in period 0 than the static model (Fuchs
& Skrzypacz, 2013). The equilibrium with $\Omega_C$ is characterized by the constraints:

$$p_t = b(k_t) \quad (8)$$

$$r(p_t - k_t) = \dot{p} \quad (9)$$

(8) is equivalent to the zero-profit condition - the price must be equal to the value of 
the cutoff type for the current period ($k_t$). (9) states that the current cutoff type $k_t$ “is 
indifferent between trading in period $t$ and waiting for a $dt$ and trading at a higher price” 
(Fuchs & Skrzypacz, 2013). The notation $\dot{x}$ refers to the derivative of $x$ with respect to 
time, or the rate of change. Thus, the cutoff type can be described by the differential 
equation:

$$r(b(k_t) - k_t) = b'(k_t)\dot{k}_t \quad (10)$$

Thus the total surplus from trading in a continuously open market is given by:

$$S_C = \int_0^T e^{-rt} (b(k_t) - k_t)\dot{k}_t dt + e^{-rT} \int_1^0 (b(\theta) - \theta) d\theta \quad (11)$$

Or, in other words, the unique equilibrium in a continuously open market is the solution 
to:

$$p_t = b(k_t) \quad (12)$$

$$k_0 = 0 \quad (13)$$

$$r(b(k_t) - k_t) = b'(k_t)\dot{k}_t \quad (14)$$

Fuchs & Skrzypacz find that the loss of trade surplus (compared to a market without 
adverse selection) with continuous trading approaches 3 times as much\(^8\) loss as the static 
model (or the analogous $\Omega = \{0, T\}$), as $\delta$ approaches 0 (that is, as private information 
becomes infinitely long-lived). If, instead, $\delta$ approaches 1, (extremely short-lived private 
information), waiting until $T$ becomes an almost optimal choice for all players, so market 
analysis can have very little impact.

While the conclusion that less-frequent trading can outperform continuous trading is sur-
prising, the intuition behind it is relatively straightforward. Forcing potential sellers to

\(^8\)For a specific case of $\theta \in [0, 1]$, $b(\theta) = \frac{1 + \delta}{2}$, and $T = \infty$
commit to a single opportunity to trade (while retaining an informational advantage) causes a large loss of surplus if a trade agreement is not realized - so a lot of surplus is lost in a static or infrequent model. However, this loss of surplus provides a significant incentive to the players to reach an agreement. If the market is open continuously, sellers have very little incentive to sell in any specific period instead of waiting for a small time, although with $T$ large enough, all types will trade (Fuchs & Skrzypacz, 2013).

It should be noted that this is not a general result - for certain constructions of the market, continuous trading will outperform the infrequent/static model as long as $T$ is large enough. We reproduce the proof from Fuchs & Skrzypacz (2013) here:

Let $\theta$ be distributed in such a way that with probability $\varepsilon$ it is uniformly distributed on $[0, 1]$, with probability $\alpha(1 - \varepsilon)$ it is uniformly distributed on $[0, \varepsilon]$, and with probability $(1 - \alpha)(1 - \varepsilon)$ it is uniformly distributed on $[\theta_1, \theta_1 + \varepsilon]$ for some $\theta_1 > b(0)$. This means that $\theta$ is distributed with large concentrations around 0 and $\theta_1$. $b(\theta) = \frac{1 + \theta}{2}$ as in the original example.

For small values of $\varepsilon$ there exists an $\alpha < 1$ such that:

$$E[b(\theta)|\theta \leq \theta_1 + \varepsilon] < \theta_1$$  \hspace{1cm} (15)

Thus, in the infrequent trading or static model, trade will only happen for low values of $\theta$. (15) is roughly analogous to (3) from our explanation of the static model. If $\alpha$ is such that:

$$\alpha b(0) + (1 - \alpha)b(\theta_1) < \theta_1$$  \hspace{1cm} (16)

then the infrequent/static equilibrium price converges to $b(0)$ as $\varepsilon \to 0$ and $T \to \infty$. Thus, the total surplus converges to:

$$\lim_{\varepsilon \to 0, T \to \infty} S_I = \alpha b(0) + (1 - \alpha)\theta_1$$  \hspace{1cm} (17)

Meanwhile, the total surplus of the continuous trading model converges to:

$$\lim_{\varepsilon \to 0, T \to \infty} S_C = \alpha b(0) + (1 - \alpha)[e^{-r\tau(\theta_1)}b(\theta_1) + (1 - e^{-r\tau(\theta_1)})\theta_1]$$  \hspace{1cm} (18)

(18) simplifies to:

$$\lim_{\varepsilon \to 0, T \to \infty} S_I + (1 - \alpha)(e^{-r\tau(\theta_1)}(b(\theta_1) - \theta_1))$$  \hspace{1cm} (19)
\( \tau \) corresponds to the inverse of the function \( k_t \). For any \( \theta_1 < b(\theta) \), the last term in (19) is strictly positive, which proves the result - for a market constructed like this one, infrequent trading is outperformed by continuous trading.

The main result of Fuchs & Skrzypacz, though, is that there exists another form of market timing that will always outperform continuous trading. Using their notation, this is \( \Omega^{EC} = \{0\} \cup (\Delta, T) \) - that is, the market opens at time 0, closes immediately afterwards, and then opens continuously from time \( \Delta \) until \( T \) (for \( \Delta > 0 \)). Fuchs & Skrzypacz show that there will always be a \( \Delta \) that causes \( \Omega^{EC} \) to outperform \( \Omega^C \).

Under \( \Omega^C \) opening rules, for a given \( \Delta \), there is some \( k_\Delta \). To differentiate it from \( k_\Delta \) in other opening schemes, let it be \( k^C_\Delta \). Under \( \Omega^{EC} \), then, the cutoff is \( k^{EC}_\Delta \).

Under \( \Omega^{EC} \), the market reopens at \( t = \Delta \) and is continuously open from that point on. The equilibrium in this part of the game is the same as the equilibrium under \( \Omega^C \), except that the starting (lowest) type is different. For \( t \geq \Delta \):

\[
p_t = b(k_t) \tag{20}
\]

\[
r(b(k_t) - k_t) = b'(k_t)k_t \tag{21}
\]

These should look familiar, as they are (12) and (14). We must add a boundary condition to account for the different starting type - that condition is \( k_\Delta = k^{EC}_\Delta \). Since buyers make zero profits, it follows that:

\[
p_0 = E[b(k)|k \in [0, k^{EC}_\Delta]] \tag{22}
\]

Additionally, it follows that type \( k^{EC}_\Delta \) is indifferent between trading at \( t = 0 \) and price \( p_0 \) or at \( t = \Delta \) and price \( p_\Delta = b(k^{EC}_\Delta) \). Equivalently:

\[
b(k^{EC}_\Delta) - p_0 = (1 - e^{-r^{\Delta}})(b(k^{EC}_\Delta) - k^{EC}_\Delta) \tag{23}
\]

This enables the solution for \( k^{EC}_\Delta \) to be found from:

\[
b(k^{EC}_\Delta) - E[b(k)|k \in [0, k^{EC}_\Delta]] = (1 - e^{-r^{\Delta}})(b(k^{EC}_\Delta) - k^{EC}_\Delta) \tag{24}
\]

It can then be shown that (Fuchs & Skrzypacz, 2013):

\[
\lim_{\Delta \to 0} \frac{\delta k^{EC}_\Delta}{\delta \Delta} = \frac{2rb(0)}{b'(0)} \tag{25}
\]
The left-hand side of (24) represents the benefit of waiting to sell, and is approximately \( \frac{b(k^{EC}E) - v(0)}{2} \), while the right-hand side (the cost of waiting) is approximately \( r \Delta b(0) \) (Fuchs & Skrzypacz, 2013). For small \( \Delta \), it follows (approximately) that as \( \Delta \to 0 \):

\[
\delta k_{EC}^{\Delta} = 2 rb(0) b'(0) \tag{26}
\]

Thus, we have a characterization of \( \lim_{\Delta \to 0} \frac{\delta k_{EC}^{\Delta}}{\delta \Delta} \). We need only a characterization of \( \lim_{\Delta \to 0} \frac{\delta k_C^{\Delta}}{\delta \Delta} \) to establish the result. This is more straightforward.

\( k_t \) is defined by (14), and for small \( \Delta \):

\[
k_{\Delta}^C \approx r \Delta \frac{b(0)}{b'(0)} \tag{27}
\]

In exact terms:

\[
\lim_{\Delta \to 0} \frac{\delta k_{\Delta}^C}{\delta \Delta} = \frac{rb(0)}{b'(0)} \tag{28}
\]

The right-hand side of (28) is the characterization we sought, and it clearly follows that:

\[
\lim_{\Delta \to 0} \frac{\delta k_{EC}^{\Delta}}{\delta \Delta} = 2 \lim_{\Delta \to 0} \frac{\delta k_{\Delta}^C}{\delta \Delta} \tag{29}
\]

Since both the left and right-hand sides of (29) converge to 0, Fuchs & Skrzypacz are able to conclude that “approximately twice as many types trade before \( \Delta \) if the market is closed than if it is opened in \((0, \Delta)\)”. Since all types trade eventually, and the equilibrium after the early closure is a continuous equilibrium, it immediately follows that \( \Omega^{EC} \) always has a \( \Delta \) that outperforms \( \Omega_C \).

The reason for this is that the early closure creates a fixed cost of delay for sellers considering trading in the interval \((0, \Delta)\). Even at \( p_0^C \) (to follow our cutoff notation before), some would prefer to trade than to wait until \( \Delta \). This set of desperate sellers grows approximately as fast as \( k_{\Delta}^C \) with \( \Delta \) (Fuchs & Skrzypacz, 2013).

The price, however, does change in period 0. \( p_0^{EC} \) is necessarily greater than \( p_0^C \) due to the pooling of trade in period 0, because the pooling of trade reduces the adverse selection problem faced by buyers. Fuchs & Skrzypacz find that, for small \( \Delta \):

\[
p_0^{EC} \approx \frac{b(0) + b(k_{EC}^{\Delta})}{2} \tag{30}
\]

20
Since $p_0^{EC}$ grows at half the speed of $b(k_\Delta^C)$, $k_\Delta^{EC} \approx 2k_\Delta^C$.

Fuchs & Skrzypacz also show that for a relatively large class of market designs, $\Omega = \{0, T\}$ outperforms all other $\Omega$, including $\Omega^{EC}$. This class of market designs is defined by both of the following functions being decreasing:

$$
\frac{f(c)}{F(c)} \left( \frac{b(c) - c}{1 - \delta + \delta b'(c)} \right) \quad (31)
$$

$$
\frac{f(c)}{F(c)}(b(c) - c) \quad (32)
$$

When (31) and (32) are decreasing, it follows that the marginal profit of a monopsonist buyer making a single offer to buy crosses zero only once. A formal proof can be found in the appendix of Fuchs & Skrzypacz (2013).

Fuchs & Skrzypacz note that $\Omega^{EC}$ is a problematic idea in practice - how does one define $t = 0$ for a real market? While not necessarily intractable, this question appears to be extremely difficult to answer, and without a definition of $t = 0$, $\Omega^{EC}$ is impracticable. Instead, Fuchs & Skrzypacz propose and consider another design - that of late closure, which we shall call $\Omega^{LC} = (0, T - \Delta) \cup \{T\}$ - the market is open continuously until some time before $T$, and then closed until $T$. This makes a bit more sense, as we could consider $T$ to be, for instance, the date when the results of a bank audit are released, or something similar. Comparison of $\Omega^{LC}$ with other $\Omega$ is more complicated because there will be a period of no trades prior to $T - \Delta$, which causes the equilibrium outcome to diverge from $\Omega_C$ after this quiet period begins, and further causes some types to trade earlier under $\Omega_C$ than $\Omega^{LC}$. These complications require a direct comparison of total surplus during certain periods of time (Fuchs & Skrzypacz, 2013).

Fuchs & Skrzypacz find that the equilibrium in $\Omega^{LC}$ is characterized by $p_{T-\Delta}^*, k_{T-\Delta}^*$, and $t^*$ which solve the following system:

$$
E[b(c)|c \in [k_t^*, k_{T-\Delta}^*]] = p_{T-\Delta} \quad (33)
$$

$$
(1 - e^{-r\Delta})k_{T-\Delta} + e^{-r\Delta}b(k_{T-\Delta}) = p_{T-\Delta} \quad (34)
$$

$$
(1 - e^{-r(T-\Delta-t^*)})k_{t^*} + e^{-r(T-\Delta-t^*)}p_{T-\Delta} = b(k_t^*) \quad (35)
$$
(33) is the zero-profit condition at \( t = T - \Delta \), (34) is the indifference condition for the cutoff type at \( t = T - \Delta \), and (35) is the indifference condition of the lowest type that reaches \( t = T - \Delta \), who is choosing between trading at \( t^* \) and waiting until \( T - \Delta \).

For \( t \in [0, t^*] \), the equilibrium is the same as under \( \Omega_C \). For \( t \in (t^*, T - \Delta) \), \((p_t, k_t) = (b(k_t^*), (k_t^*)\), and at \( t = T - \Delta \), \((p_t, k_t) = (p_{T - \Delta}^*, k_{T - \Delta}^*)\) (Fuchs & Skrzypacz, 2013).

(35) is the condition that produces the period of no trade for \( t \in (t^*, T - \Delta) \). Fuchs & Skrzypacz nonetheless find that \( \Omega_{LC} \) outperforms \( \Omega_C \) for \( b(c) = 1 + c^2 \) and \( F(c) = c \), their usual example. Due to the complexity of the necessary comparisons outlined above, Fuchs & Skrzypacz are not able to generalize the result to a certain class of market designs - they note, for instance, that if \( f(c) = 2 - 2c \) and \( b(c) = c + 1 \) it can be directly shown that \( \Omega_{LC} \) will lower gains from trade.

Janssen & Roy (2002) produce an analysis of the dynamic version of the adverse selection problem which is useful to us because it does very little except show how the problem changes when time is introduced. Unlike Fuchs & Skrzypacz (2013), no mechanism design questions are considered, and the behavior of the time parameter is slightly more restricted.

As in the static model, we have a Walrasian market for a durable good whose quality is \( \theta \in [\theta, \bar{\theta}] \). Janssen & Roy consider a continuum of potential sellers, rather than a single seller (although the analysis remains essentially unchanged), and set the set of potential sellers to be the unit interval, or \( I \). Each seller \( i \) knows her asset’s quality \( \theta(i) \), and her reservation price is the “infinite horizon discounted sum of gross surplus derived from ownership of the good” (Janssen & Roy, 2002), which is assumed to be \( \theta(i) \) as in Fuchs & Skrzypacz (2013). Sellers are distributed \( ex \ ante \) over the quality interval according to a probability measure \( \mu \), associated with the distribution function \( F \).

There is a continuum of potential buyers with measure greater than 1, where all buyers are identical and have unit demand. A buyer’s valuation of a good with quality \( \theta \) is \( b\theta \) with \( b > 1 \). Thus, there always exist gains from trade. Buyers, as in the static model, know the \( ex \ ante \) distribution of quality, but not the quality of the asset offered by any particular seller. Janssen & Roy allow for no re-entry by buyers after they make a trade, and no entry by sellers after the initial time period.

Finally, time is a discrete series indexed by \( t = 1, 2, ..., \infty \). All agents discount their
future returns using a common discount factor \( \delta, 0 \leq \delta < 1 \). Thus, sellers earn a per-period gross surplus equal to \((1 - \delta)\theta(i)\).

Janssen & Roy assume that the distribution of quality is continuous with no mass point, and that the support of \( \mu \) is an interval \([\bar{\theta}, \bar{\theta}], 0 < \bar{\theta}, \bar{\theta} < +\infty \). The distribution function \( F \) is assumed to be continuous and strictly increasing on this support. Further, since if \( bE[\theta] \geq \bar{\theta} \) there is no adverse selection problem, attention is restricted to cases where \( bE[\theta] < \bar{\theta} \).

Following Janssen & Roy in their choice of notation, let \( \eta(x, y) \) be the conditional expectation of quality \( \theta \) given that \( \theta \in [x, y] \) and \( \theta \leq x < y \leq \bar{\theta} \). \( \eta(x, y) \) is continuous on intervals that satisfy this requirement and strictly increasing in \( x \) and \( y \) when \( x \) and \( y \) are ordered correctly (Janssen & Roy, 2002).

Sellers form beliefs as to a series of expected prices \( p = \{p_t\}_{t=1, 2, \ldots, \infty} \), choose whether or not to sell, and if they choose to sell, they choose which period to sell in. Sellers earn a gross surplus of \( \theta(i) \) by not selling, and earn a net surplus if they sell in period \( t \) of:

\[
\sum_{i=0, \ldots, t-1} [(1 - \delta)\theta]^{\delta^i} + \delta^t p_t - \theta = \delta^t (p_t - \theta) \tag{36}
\]

Thus we can denote the set of time periods in which a particular seller finds it optimal to sell as \( T(\theta, p) \):

\[
T(\theta, p) = \{ t \geq 1 : \theta \leq p_t \text{ and } \delta^{t-1}(\theta - p_t) \geq \delta^{t'-1}(\theta - p_{t'}) \text{ for all } t' \geq 1 \} \tag{37}
\]

If a seller with quality \( \theta \) facing prices \( p \) finds it optimal never to sell, \( T(\theta, p) = \{ \infty \} \). Each seller chooses a time period \( \tau \) in \( T(\theta(i), p) \) in which they offer their asset for sale, which results in a distribution of assets available in any given time period. Given \( p \), the average quality of asset available at \( t \) is \( E[\theta(i) | i \in I, \tau(i, p) = t] \) (Janssen & Roy, 2002).

Janssen & Roy follow convention by assuming that all buyers are identical (and thus have identical, symmetric, beliefs about quality). In addition, since the market setup assumes that there are strictly more buyers than there are sellers, it follows that in any period where trade occurs buyers earn zero expected net surplus (since they are price takers, essentially). A slightly less obvious implication is that buyers are thus indifferent between trading in any period where trade occurs (Janssen & Roy, 2002).
Using this model, an equilibrium is characterized by a price sequence $p = \{p_t\}_{t=1,2,\ldots,\infty}$, a set of selling decisions $\tau(i, p)$ for $i \in I$, and a sequence $\{E_t(p)\}_{t=1,2,\ldots,\infty}$, where $E_t(p)$ is the symmetric expectation of quality in period $t$ held by all the buyers (Janssen & Roy, 2002). As in Fuchs & Skrzypacz (2013), these equilibria are constrained by some common conditions and one novel condition:

- Sellers maximize according to $\tau(i, p)$
- Buyers maximize and markets clear
- Expectations are fulfilled
- Minimal consistency of beliefs

A full explanation of these conditions in this model can be found in Janssen & Roy (2002). The first three correspond roughly to the first two conditions in Fuchs & Skrzypacz (2013), while the final conditions in each paper roughly correspond to each other. The purpose of the minimal consistency of beliefs condition is to ensure that certain trivial equilibria are not feasible - particularly the equilibrium where prices are 0 for all $t$ and no trade occurs.

The primary conclusions reached by Janssen & Roy (2002) are unsurprising after seeing Fuchs & Skrzypacz (2013) - prices and quality of goods traded increase over time, sellers have a strictly increasing incentive (in $\theta$) to wait to sell, and so forth. Janssen & Roy characterize the sequence of qualities sold in a way that immediately appears similar to the use of cutoffs in Fuchs & Skrzypacz (2013).

Where Fuchs & Skrzypacz rely on previous results, however, Janssen & Roy show directly that “in any dynamic equilibrium all goods (no matter how high the quality of such goods) must be traded in finite time”. The reasoning is as follows:

If time were allowed to advance for infinitely many periods, there would be a sub-sequence of periods in which trade actually occurred. The highest and lowest qualities traded on this subsequence must be increasing and convergent, since they are bounded above by $\bar{\theta}$. If we assume they converge to $\theta^*$, prices must converge to $b\theta^*$. Since $b > 1$, seller surplus converges to a strictly positive amount.

However, our definition of an equilibrium requires that the marginal seller in each period be indifferent between selling in the current period and selling in the next period of
trading, which amounts to saying that the ratio of surplus earned in the current period to surplus earned in the next period cannot exceed \( \delta < 1 \). This is a contradiction to the convergence of both surpluses to a number greater than 1. Thus, trade can only occur in finite periods.

The equilibrium constraint of minimal consistency of beliefs ensures that there can be no subset of \([\bar{\theta}, \bar{\theta}]\) for which trade never occurs. Thus all qualities will trade in finite time. If \( T \) is defined as the period in which the final asset is sold, we can also reasonably believe that \( T \) is increasing with \( \delta \). Higher values of \( \delta \) correspond to less seller impatience, and thus constrains the rate at which equilibrium prices increase over time (Janssen & Roy, 2002).

As in Fuchs & Skrzypacz (2013), Janssen & Roy proceed to consider certain kinds of equilibria, beginning with an equilibrium path in which all trade occurs in successive periods and there is no “break” in trading (analogous in concept to the continuous trading model in Fuchs & Skrzypacz (2013)). They find that such an equilibrium exists when either \( \delta \) is small, or when \( \delta \) is large and \( \theta \) is distributed in a specific way - for instance, if \( \theta \) is uniformly distributed, then for \( \delta \in [0, 1) \), there can be an equilibrium path with no break in trading.

Janssen & Roy also identify one case where the dynamic equilibrium necessarily contains an intermediate break period (or periods) in which no trade occurs. In short, this case arise if the density function of \( \theta \) is such that for some interval it is steeply increasing. In this case, average price must increase too rapidly for sellers to be indifferent between selling in consecutive periods - the necessary condition for the continuous equilibrium path.

Both of the preceding models consider a dynamic Walrasian market. Recently, some authors have departed from the Walrasian market space. In particular, Kultti et al. (2012) set up a dynamic matching model. According to the authors, this creates a model with a decentralized market (as trades occur in private meetings) and has the potential (through the matching model used) to avoid the cyclicality noticed in Walrasian markets by, among others, Janssen & Roy (2004)\(^9\)

Kultti chooses to use an urn-ball matching model, with the result being that “the number of competing buyers a seller confronts is determined randomly, resulting in variation in

\(^9\)See also Janssen & Karamychev (2002)
the local conditions among traders.” Thus, the primary cause of the cyclicality noticed by Janssen and his co-authors competes against the randomness. This creates the possibility of a steady state equilibrium in which high quality goods are traded in every period.

In addition to setting up a decentralized market, the authors explicitly define the price formation process, and do so using auctions. This would not be feasible in the centralized market models such as those covered previously, and has the added benefit of reducing the problem of multiple equilibria (Kultti et al., 2012).

The model is set up with an equal measure of buyers and sellers, with some proportion \( q \) of sellers producing high-quality goods at cost \( c_H \) and the remainder producing low-quality goods at cost \( c_L \). Buyers receive \( u_H \) from owning a high-quality good and \( u_L \) from owning a low-quality good, with \( u_H > c_H > u_L > c_L \). Thus, gains from trade exist for all potential trades but high-quality goods will not be traded for the price of low-quality goods. In the conventional static, Walrasian setting, this results in a relatively simple condition for market failure.

The authors find that the pooling equilibrium is viable “within a narrower range of quality distributions than in the competitive setting” for the interval \([0, \delta_1]\), where \( \delta_1 \) is some allowed discount rate. For the remainder of the allowed interval \([\delta_1, 1]\) the pooling equilibrium is viable in exactly as large a range of quality distributions as in the competitive setting. The problem of adverse selection is strongest when \( \delta \) approaches 0. This echoes the result from Fuchs & Skrzypacz, where the relative loss of efficiency increases with decreasing \( \delta \).

If, however, the proportion \( q \) of high-quality sellers is large enough (but not too large)\(^{10}\), Kultti et al. find that another possibility is a partial pooling equilibrium, in which the necessary condition for a trade of low-quality goods is that a single buyer meets a low-quality seller, while the necessary condition for a trade of high-quality goods is that more than one buyer meets a single high-quality seller. This partial pooling equilibrium remains steady even in cases that would lead to market failure in the competitive setting, but disappears as a potential equilibrium if \( \delta \) is high enough.

Moreno & Wooders (2010) produce a very similar model, with similar conclusions. The pri-

\(^{10}\)If the proportion were too large, the initial offer made by single buyers would be sufficient to cause a trade
mary differences are the matching model, which does not guarantee a meeting as in Kultti et al. (2012), and the price formation mechanism, which remains a bargaining mechanism (albeit with all bargaining power attached to the buyer) rather than auctioning. The results are similar - high-quality sellers are expected to trade with lower probability than low-quality sellers, and thus leave the market at a slower rate. This leads to the roots of a cycle - as high-quality sellers become a higher proportion of the sellers in the market, the adverse selection problem weakens, and eventually the price offered by rational buyers allows all types to trade, clearing the market and resetting the cycle.

As in Kultti et al., Moreno & Wooders find that when “frictions are small but non-negligible, the surplus generated under decentralized trade is greater than the competitive surplus, and it decreases as frictions become smaller.”

All the models we have covered in this section support the primary point - dynamic markets for durable goods with asymmetric information retain the capability of trading assets of all qualities, in contrast to the static model in which only a subset of qualities is traded. The metric of market failure in the dynamic market is thus not represented by a set of non-trading qualities, but by the necessity for sellers of higher quality to delay their trade in order to realize gains from trade. This delay is costly, and is the source of the lost surplus due to adverse selection. This metric can, of course, be applied to the static formulation as well, with slightly more work, allowing for the conclusions drawn with regard to relative losses of surplus between dynamic and static markets.
4 Empirical Evidence and Discussion

4.1 Empirical Evidence

In the empirical literature on adverse selection, almost every single paper handles insurance markets of some kind. Cohen & Siegelman (2009) believe that this is due to insurance being one of the first markets people thought of as being home to adverse selection, as well as being empirically suitable due to quality of data. Insurance records allow a researcher to have all the information about a policyholder that the insurance company had in the first place. This allows for a relatively simple (in intuitive terms) test for the presence of adverse selection.

Fundamentally, the theory of adverse selection predicts that in an insurance market, where potential policyholders have access to privileged information regarding their own risk, they will use that information to make decisions about which insurance policy to purchase. Buyers who know themselves to be higher-risk (without the insurance company being privy to this knowledge) will tend to purchase higher levels of insurance coverage with lower deductibles. In short, coverage and risk are expected to be positively correlated.

There are two problems that make this significantly more complicated in practice, however. The first is that an market in which adverse selection is present and has an effect in line with theory can still, under certain circumstances, produce no correlation between risk and coverage. There are a large number of possible reasons for this, including:

- Absence of useful private information
- Private Information for some but not all buyers
- Failure of buyers to utilize private information
- Superior information or predictive ability by sellers
- Offsetting factors
- Institutional and Regulatory factors

A rigorous explanation of these reasons and how they might stop a risk-coverage correlation from developing even in the presence of adverse selection can be found in Cohen & Siegelman (2009).
The second problem is the issue of moral hazard, which can produce a coverage-risk correlation even in the absence of an adverse selection problem. Moral hazard has to do with hidden actions rather than hidden information. In simple terms, a consumer who purchases, for instance, automobile insurance, may then alter their behavior to take into account the changes to their costs that the purchasing decision caused - they may become more risky drivers as a result of their coverage, rather than choosing their coverage as a result of their risk. Disentangling moral hazard effects from adverse selection is an important issue, as the theory behind both effects is well understood and largely a consensus. We briefly survey some methods in the appendix.

Despite these issues, a number of researchers have made efforts to identify adverse selection in insurance markets. What follows is a relatively short survey of some of the more interesting results.

Cardon & Hendel (2001) look at data from the 1987 National Medical Expenditure Survey from the United States Department of Health and Human Services, which recorded consumption data for employed Americans related to their healthcare, including policies offered, policies chosen, and final consumption of healthcare. They test for a link between insurance demand and final consumption of healthcare - that is, they seek to determine if those consumers who consumed more healthcare after the fact were more likely to choose healthcare policies which provided more coverage. While Cardon & Hendel do find that those who are healthier tend to choose lower coverage policies, they are unable to attribute this link to what they call “unobservables” - rather, they find that much of the link is explained by observables, including demographics, particulars of employment status, geographic location, race, and income. Alternatively, Cardon & Hendel posit that instead there might be an issue of contract length - in Cochrane (1995), a model is proposed which suggests that individuals and small firms are unable to commit to long-term health insurance contracts, and thus cannot find affordable insurance, while large firms are able to commit to long-term contracts and reap the benefits. This then has an impact on risk pools facing insurers.

Somwaru & Makki (1998) examine crop insurance instead of health insurance. This is a particularly interesting idea, because the traditional target market of healthcare is arguably problematic. What an agent stands to lose if she cuts just slightly too many costs in her healthcare spending is her entire life, while what she stands to lose if she cuts too much on crop insurance (or indeed automobile insurance) is simply some surplus. One
could make the argument that healthcare markets are not ideally suited to adverse selection because there are incentives in place to avoid strict expected value maximization in them.

The data set chosen for this examination is from the United States Department of Agriculture’s Risk Management Agency. This agency keeps records on individual farmers who buy federally-backed crop insurance instruments. Somwaru & Makki restrict themselves to looking at data from 1997 in the states of Iowa and Nebraska, a data set in which farmers were offered four different insurance products. Due to data limitations, Somwaru & Makki restrict the comparison to only two instruments - Crop Revenue Coverage (CRC) and Multiple Peril Crop Insurance (MPCI), the latter of which has a longer history of use. In simple terms, MPCI delivers a payout to farmers if their crop yield falls below a guaranteed level, while CRC delivers a payout if the farmer’s revenue falls below a certain level. CRC represents a higher liability per acre for the insurance company, which results in higher premiums being paid for CRC coverage.

In examining the data set initially, Somwaru & Makki find that farmers who chose CRC coverage saw higher loss frequencies in both crops studied (2.8% higher loss frequency for corn and 2.1% higher loss frequency for soybeans). In addition, farmers who chose CRC coverage were much more likely to choose higher-coverage policies than farmers who chose MPCI coverage.

Somwaru & Makki use a binomial LOGIT model to test for the presence of adverse selection in this data set. The observables they consider are acreage covered, farm practice, relative expected yield-span category, coverage type, and premium rates (Somwaru & Makki, 1998). They find that high-risk farmers are more significantly more likely to select CRC coverage and higher coverage levels, although no effort is made to disentangle moral hazard from adverse selection.

Klonner & Rai (2007) break from the convention of studying insurance, instead looking at a data set concerned with borrowing. Their data set is from an Indian financial institution which sets interest rates via competitive bidding. Due to the government imposing an interest rate ceiling and then relaxing it (in 1993 and then 2002, respectively), Klonner & Rai are able to compare default patterns before and after each policy change.

It is a prediction established by Stiglitz & Weiss (1981) that riskier borrowers should
be willing to pay higher interest rates than safer borrowers in the presence of adverse selection. The key contribution made by Klonner & Rai is the use of their particular data set (particularly, the introduction and removal of the interest rate ceiling) to isolate the effect of adverse selection from moral hazard.

The instrument studied is a Rosca (Rotating Savings and Credit Association). According to Klonner & Rai, these are popular in developing countries. A Rosca consists of a group of people who meet regularly, contributing a fixed amount of money to a pot. At each meeting, the collected pot is given to one of the participants in the Rosca. The choice of recipient can be either randomized or based on competitive bidding, the latter being Klonner & Rai’s focus. Once a participant has received a pot he is ineligible to bid for another. The winning bid is distributed as a dividend to all members of the Rosca. Thus, higher winning bids translate to higher interest payouts for later recipients of the pot. As one would expect, the winning bid tends to fall as the duration for the loan is reduced.

Klonner & Rai provide the example of a 3-person Rosca which meets once a month, with a monthly contribution of $10. If the winning bid in the first month is $12, each participant receives a dividend of $4, and the winner of the first pot gains (net) $12. In the second month the winning bid is $6 (with only 2 eligible bidders). In the final month, since there is only one eligible bidder, the winning bid is $0. This implies an interest rate of 43% for the first winner, and a 25% interest rate for the final winner, who is essentially a pure saver.

In 1993, however, the government imposed a ceiling on the bid size to 30% of the total pot. In Klonner & Rai’s example, this corresponds to $9. If this restriction is binding in early rounds, as Klonner & Rai’s data set suggests, the winner in early rounds is determined randomly among all those willing to pay $9. The winner of this first round bid receives (net) $14. This substantially lowers the interest rate on borrowing early, and the rate for those saving.

Klonner & Rai essentially argue that, if adverse selection is present, riskier borrowers should win more early pots without the ceiling than with it, as the ceiling hampers the ability for members of a Rosca to self-select. This, however, does not isolate moral hazard. Isolation of moral hazard is accomplished by conditioning on the winning bid - essentially, controlling for identical loan terms and comparing default rates before and after the bid ceiling when loan terms faced by the early borrower are identical.
Klonner & Rai find that while the policy changes did not impact overall default rates significantly, the difference in default rates between early and late borrowers was significantly affected, indicating a strong adverse selection effect. They find the expected reversal of the effect in 2002 when the bid ceiling is lifted.

In a recent paper, Cohen & Siegelman (2009) review the empirical literature on adverse selection in insurance markets, focusing on work which tests for the basic link between coverage and risk, which leads higher risk individuals to purchase higher coverage.

Cohen & Siegelman divide their review into various sections, covering automobile insurance, annuities, life insurance, and reverse mortgages. In the case of automobile insurance, early empirical studies by Dahlby (1983 and 1992) and Puelz & Snow (1994) found evidence of adverse selection, but were criticized for methodological issues (Dionne, Gouriéroux, & Vanasse (2001)). Chiappori & Salanié (2000) found no correlation between risk and coverage, in a study focusing on beginning drivers with limited experience, using a robust methodology. However, Cohen (2005) suggests that Chiappori & Salanié’s results may have been due to the focus on beginning drivers, who could be expected not to know their own risk profile. Cohen (2005) used the same methodologies as Chiappori & Salanié, finding similar results for beginning drivers but a significant correlation between coverage levels and accident risk for drivers with more than three years experience (Cohen & Siegelman, 2009; Cohen, 2005). Cohen also found that insurers with bad records had a higher likelihood of switching insurers, possibly to flee their record and the high premiums associated with it.

Richaudeau (1999) studied the choice of insurance, focusing on the choice between basic coverage and comprehensive insurance (where the latter offers significantly more coverage). Richaudeau found that when he controlled for total miles driven, a piece of information not known by insurers, coverage choice did not significantly predict the number of accidents. Without that control, however, the coefficient comes very close to being statistically significant. Richaudeau concluded that adverse selection was present in the form of drivers who drive more total miles choosing higher coverage even if they are not riskier per mile driven.

Saito (2006) studied the Japanese automobile insurance market and found only a weak and insignificant correlation between crash risk and the purchase of coverage for the policyholder’s own vehicle. Surprisingly, Saito also found a negative and significant relationship between crash risk and the purchase of a zero-deductible policy (Cohen & Siegelman, 2009;
Saito, 2006). Saito concluded that policyholders had little informational advantage.

In insurance markets concerned primarily with mortality risk, Friedman & Warshawsky (1990) provide one of the first investigations of adverse selection, and found that those who purchased annuities tended to outlive similar people who did not purchase annuities, although this seems to suggest moral hazard more than it does adverse selection. Finkelstein & Poterba performed two studies (in 2002 and 2004), one of which we discuss at the end of this section in more detail, finding evidence that annuity purchasers had an informational advantage.

Davidoff & Welke (2007) study reverse mortgages, which allow policyholders to borrow against the proceeds from the future sale of their homes, which typically occurs due to the death of the policyholder (hence, this is concerned with mortality risk, although not entirely). They reject the possibility of adverse selection in the market, although they do find evidence that suggests moral hazard may be a concern.

Life insurance markets, in contrast with annuities, have tended to produce negative conclusions with regard to the presence of adverse selection, despite being concerned with the same risk profile. Cawley & Philipson (1999) find that death rates for life insurance policyholders is lower than for non-policyholders, that life insurance premiums tend to fall with increasing coverage levels, and find that those who self-report as having higher mortality risk are less likely to hold life insurance policies.

Health insurance is concerned with a different risk profile, and the body of literature tends to find evidence that those with poorer health choose higher coverage plans. Cutler & Zeckhauser (1998) review fourteen studies of adverse selection in the health insurance market and all find some type of adverse selection. However, some studies (for instance, Buchmeller et al., 2004; Ettner, 1997) find no evidence of a correlation between risk and coverage. Cutler & Reber (1998) study the changes in policyholder choices for employees of Harvard, after Harvard switched from subsidizing only high coverage plans to offering a fixed dollar subsidy, increasing the cost of the highest coverage plan by approximately $500 (Cohen & Siegelman (2009)). Those policyholders with the best risk profiles abandoned the highest coverage plan and chose lower coverage levels, and those who made the switch to lower coverage plans tended to have lower medical expenses than those who continued with the higher coverage (Cutler & Reber, 1998).
Finkelstein & Poterba (2004) study a data set concerning annuities in the United Kingdom. The key contribution they make, aside from adding their study to the large list of studies dealing with annuities and adverse selection, is that it tests for two predictions of adverse selection in an annuity market, where most studies only test for one. Finkelstein & Poterba test both for a correlation between higher risk and features attractive to high risk annuitants, and find that “equilibrium pricing of insurance policies reflects variation in the risk pool across different policies”. This allows them to consider adverse selection separately on different contract features rather than in some aggregated sense.

Finkelstein & Poterba find, as in most of the studies discussed above, that there is “little evidence of adverse selection on the amount of payment in the event that the insured risk occurs” - the classic kind of adverse selection that has been tested for. However, they also find that there is strong evidence of adverse selection on other contract features. In particular, they find significant explanatory power in mortality rates and policy pricing - those who purchase annuities self-select to ensure that the annuity will continue to pay out after death (and, to a lesser extent, to get more attractive payout patterns)\textsuperscript{11}. Back-loaded annuities are found to be priced higher, which is consistent with separating equilibrium theory in markets with asymmetric information.

\textsuperscript{11}In simple terms, this means that after controlling for other variables, longer-lived annuitants tended to select annuities with back-loaded payment streams, while shorter-lived annuitants tended to select annuities that made payments to the estate in the event of the annuitant’s early death.
5 Concluding Discussion

As we noted in the introduction, part of the motivation for this thesis was to produce information that would be useful to a central authority seeking to intervene in cases of market failure due to adverse selection. Fuchs & Skrzypacz argue, for instance, that their model is a natural fit for studying the market failures that were prevalent during the 2007-2009 financial crisis, when a number of asset markets became almost entirely illiquid. They suggest as an example the market design $b(c) = \gamma c$ for $2 > \gamma > 1$ and $F(c) = c$, a model which arises if sellers have higher discount rates than buyers. In this model, the (unique) SPNE has no trade occur until $T$, and no gains from trade being realized (Fuchs & Skrzypacz, 2013). Thus, one government intervention would be to offer a price $p_g > 0$ to any seller who wishes to sell.

The average quality of assets purchased would be $\frac{p_g}{\gamma}$, which would lose the government money on each purchase. However, these purchases would remove “toxic assets” from the market, and truncate the distribution of quality in the market to $c \in \left[\frac{p_g}{\gamma}, 1\right]$ (Fuchs & Skrzypacz, 2013). After the distribution is truncated, even though we have proceeded in time to some $\Omega = [0, T]$, the market would become liquid again.

Post-intervention, the market would be characterized by:

$$rk_t \frac{\gamma - 1}{\gamma} = k_t$$

(38)

Thus, larger initial interventions speed up the trade in the post-intervention market (Fuchs & Skrzypacz, 2013). In addition to this, the intervention also benefits sellers who do not sell directly to the government by restoring gains from trade in the market, due to the reduction in the adverse selection problem.

Philippon & Skreta (2012) is clearly motivated by the interventions of the Federal Reserve in the 2007-2009 financial crisis. They determine optimal (i.e. cost-minimizing) interventions with the goal of restoring market activity after a market fails due to adverse selection. Like Fuchs & Skrzypacz, however, their model is of limited use in practice, as it relies on too many assumptions and simplifications. The most interesting result is that the optimal form of intervention is a debt contract (either directly lending funds or issuing debt guarantees).
Further complicating the issue of policy advice is the issue of moral hazard. Farhi & Tirole (2012) show that optimal behavior by private lenders depends on the anticipated policy response to problems that arise as a result of leverage decisions, because interventions cannot be perfectly targeted. An intervention such as that proposed by Fuchs & Skrzypacz might reduce adverse selection but increase total social costs via moral hazard.

It is worth wondering why one would ever use a static characterization of the adverse selection problem, given that in practice no market exists that does not have to deal with the passage of time. However, using Leland’s paper as an example, we can identify at least one good reason - certain markets have other features that make a dynamic characterization difficult to imagine and complicated to work with. Healthcare provision, Leland’s chosen example, illustrates this rather well.

Healthcare providers are not potential sellers of assets. They sell a service, which is limited in quantity by the passage of time. It makes little sense to imagine a doctor choosing to delay entry to the healthcare market in order to signal a higher quality. The impact of passing time in certain markets is largely irrelevant to the determination of equilibrium price in that market, and in such markets it makes more sense to characterize them as static models.

The same reasoning leads to the conclusion that potential interventions in a market best characterized as static would not be interventions identified for dynamic models. Restricting the ability to provide healthcare to certain times makes little sense, and it is difficult to see how this could alleviate the adverse selection problem. Likewise, minimum quality standards as proposed by Leland would be ill-suited as an intervention to an asset market, such as the used car market, because enforcement would require the regulating authority to acquire information it could not acquire in practice.

That said, the results from Yamagishi & Matsuda (2002) suggest that reputation systems might not be impacted in terms of effectiveness. I am not aware of any attempts to theoretically model the impact and mechanics of a reputation system in the presence of asymmetric information - such a model would, I think, be very useful in an evaluation of reputation systems more generally. Preliminary thought on what such a model would entail, however, suggests the task would be extremely complex and possibly intractable.

As the preceding section makes clear, however, the empirical evidence for adverse se-
lection is mixed at best. There are numerous reasons that an asymmetric information structure might not result in adverse selection inefficiencies (Cohen & Siegelman, 2009). There is reason to believe that adverse selection may impact markets features other than price, which requires a modified theoretical approach (Finkelstein & Poterba, 2004). In those markets where adverse selection does have the theory’s predicted impacts, there is little to explain why those impacts are absent in other, similar markets - why is there a strong link between coverage levels and risk in health insurance markets, yet not in life insurance markets? More detailed research is necessary to answer this question.

Cohen & Siegelman do not believe that future empirical studies of adverse selection will yield more consistent or conclusive results. Instead, their conclusion is that “there is a good basis for expecting the existence of adverse selection and a coverage-risk correlation to vary across markets” (Cohen & Siegelman, 2009). The simplifications necessary to create tractable models of adverse selection (and moral hazard) change the role of the empirical studies - instead of providing solid answers as to the existence of adverse selection, empirical studies refine our understanding of the circumstances which lend themselves to coverage-risk correlations.

It is also worth noting that the assumption of identical buyers held in common through most of the models looked at in this thesis could provide an explanation of mixed empirical evidence. Hemenway (1990) (and, to some extent, De Meza & Webb (2001)) provide the foundations of an argument that adverse selection is muted in reality due to a correlation between risk aversion and levels of risk - in other words, riskier consumers tend to be less risk averse while less risky consumers pursue risk-reducing behaviors. Hemenway provides a model of this, showing that it is even possible to reverse the adverse selection effect and create “propitious” selection. De Meza & Webb refine this model for an insurance market, finding a pure strategy, partial pooling SPNE.

Returning to consideration of Fuchs & Skrzypacz (2013) and similar models, a large obstacle presents itself to empirical studies with the aim outlined above - the question of what $t = 0$ actually means. In many papers which add time to study of adverse selection, sellers are assumed to enter the market in response to a liquidity shock which leaves them lacking liquidity. This is a theoretically attractive explanation, but requires an almost impossibly detailed data set to be of use in an empirical study - how is a researcher supposed to determine when an asset holder realizes they lack liquidity? One option would be to restrict attention to initial public offerings, but IPOs can be delayed, and it is possible
that these delays are used as a signal to the market - thus, \( t = 0 \) cannot be set at the
time of the IPO without missing on potential signaling periods. Janssen & Karamychev
(2002) show that in dynamic markets with dynamic entry, equilibria can be qualitatively
different to markets without dynamic entry as long as time on the market is not public
knowledge. Roy (2012) provides evidence that the homogeneous buyer assumption made
by Fuchs & Skrzypacz simplifies away many interesting trade-offs.

In light of the models studied, it is difficult to argue that there is a clear policy inter-
vention for asset markets in general to reduce adverse selection. The argument made by
Cohen & Siegelman - that interventions should be tailored to specific dimensions of the
market targeted, guided by relevant empirical studies - seems uncontroversial. Certainly,
without a great deal more work, the models given to us by Fuchs & Skrzypacz, or Janssen
& Roy, have too many gaps, and too many simplifying assumptions, to be used for policy
guidance. The most promising result is the late closure idea from Fuchs & Skrzypacz, but
implementing such a policy seems impracticable without central control of news agencies.

Further research, both empirical and theoretical, will be beneficial. We would benefit from
empirical research which identifies particular features of a market which lend themselves
to producing empirically visible adverse selection effects, and then pursuing theoretical
research to explain how such a link works. That research might even produce exciting av-
ennues for further empirical research. It is relatively uncontroversial that adverse selection
is a real problem in real markets, but our models and understanding of the problem is still
too simplistic to be of much use in guiding policy interventions.
6 Appendix

6.1 Constrained Pareto Efficiency

We present here a detailed explanation of the concept of a Constrained Pareto-Efficient equilibrium, as first mentioned in Section 2. A Constrained Pareto-Efficient equilibrium is an equilibrium that cannot be improved by a central authority’s intervention when that authority is unable to observe the private information in the model. Thus, for instance, if the central authority is unable to observe the quality of a potential seller’s asset, and thus cannot improve the efficiency of the market equilibrium, that equilibrium is a constrained Pareto-Efficient equilibrium even if the outcome is not fully Pareto efficient. Note that a true Pareto optimum is necessarily a constrained Pareto efficient outcome as well.

As in the following sections, this explanation owes itself to Mas-Colell, Whinston, & Green (1995). It is intuitively difficult to comprehend an illustration of this concept in the kind of asset-sale model we set up in Section 2, so we will follow Mas-Colell, Whinston, & Green in illustrating this via their labor market model of adverse selection.

In the labor market version of the adverse selection model, we consider the problem of two firms choosing to offer wages for labor. They do not know the productivity of a worker until they hire them. In the labor market version as set up in Mas-Colell, there is the possibility for multiple equilibria, when the equilibrium wage matches the opportunity cost of working.

However, it is still possible to show that the highest-wage SPNE\textsuperscript{12} of any adverse selection game is a constrained Pareto efficient outcome.

To begin, it is tautological that if all workers are employed in the highest-wage SPNE, that SPNE must be fully (and thus necessarily constrained) Pareto optimal. It would be impossible to improve the outcome for any player without making another player worse off.

We can without loss of generality prove constrained Pareto efficiency by imagining that the central authority simply takes control of both firms and seeks to maximize worker surplus while maintaining zero profits for the firms (Mas-Colell, Whinston, & Green, 1995). Thus, the central authority will offer a wage $w_c$ and an unemployment benefit $u_c$. The set

\textsuperscript{12}This would be analogous to the highest total surplus SPNE in the Fuchs & Skrzypacz or Leland models
of workers who accept employment will then be \([\tilde{\theta}, \hat{\theta}]\). We first must determine whether a central authority could reproduce the outcome of the highest-wage SPNE. Doing this is equivalent to causing all workers with productivity \(\theta \leq \hat{\theta}\) to accept employment, for \(\hat{\theta} \in [\tilde{\theta}, \bar{\theta}]\). The authority can choose only \(w_c\) and \(u_c\), and must do so satisfying these two conditions:

\[
\begin{align*}
\text{(39)} & \quad u_c + r(\hat{\theta}) = w_c \\
\text{(40)} & \quad w_c F(\hat{\theta}) + u_c (1 - F(\hat{\theta})) = \int_{\tilde{\theta}}^{\hat{\theta}} \theta f(\theta) d\theta
\end{align*}
\]

(39) ensures that no worker with \(\theta \leq \hat{\theta}\) chooses to reject employment. (40) is the central authority’s balanced budget constraint - note that it is clearly true that the central authority will never run a surplus, as eliminating that surplus by increasing \(u_c\) and \(w_c\) by some arbitrary amount would be budget feasible and achieve a Pareto improvement. These constraints give:

\[
\begin{align*}
\text{(41)} & \quad u_c(\hat{\theta}) = F(\hat{\theta})(E[\theta | \theta \leq \hat{\theta}] - r(\hat{\theta})) \\
\text{(42)} & \quad w_c(\hat{\theta}) = F(\hat{\theta})(E[\theta | \theta \leq \hat{\theta}] + r(\hat{\theta}))
\end{align*}
\]

To show that this is identical to the highest-wage SPNE, let \(\theta^*\) be the highest productivity worker who accepts employment in that SPNE. From the basic analysis of a labor market adverse selection model, \(r(\theta^*) = E[\theta | \theta \leq \theta^*]\) (Mas-Colell, Whinston, & Green, 1995). Thus, if we were to set \(\hat{\theta} = \theta^*\), (41) would result in \(u_c = 0\) and (42) in \(w_c = r(\theta^*)\). This is the same as the highest-wage SPNE in the competitive market.

To achieve a Pareto improvement on this outcome, the central authority clearly must set \(\hat{\theta} \neq \theta^*\). If \(\hat{\theta} < \theta^*\), a different form of (42) results in:

\[
\begin{align*}
\text{(43)} & \quad w_c(\hat{\theta}) \leq \int_{\tilde{\theta}}^{\hat{\theta}} \theta f(\theta) d\theta + r(\theta^*)(1 - F(\hat{\theta})) \\
\text{(44)} & \quad w_c(\hat{\theta}) - r(\theta^*) \leq F(\hat{\theta})(E[\theta | \theta \leq \hat{\theta}] - r(\theta^*))
\end{align*}
\]

The right hand side of (44) is equal to \(F(\hat{\theta})(E[\theta | \theta \leq \hat{\theta}] - E[\theta | \theta \leq \theta^*])\), which is strictly negative. This means that \(w_c < r(\theta^*)\), which is the highest-wage SPNE equilibrium - thus, in the case where \(\hat{\theta} < \theta^*\), workers with productivity \(\tilde{\theta}\) are strictly worse-off than in the highest-wage SPNE.

\[13\]It can clearly never be the case that an adverse selection problem results in the set of workers accepting employment being truncated on the low productivity end
The only alternative outcome is the opposite - $\dot{\theta} > \theta^*$. From the analysis of the basic market, it is clear\textsuperscript{14} that $E[\theta|r(\theta) \leq w] < w$ for all $w > w^*$. Since $r(\theta^*) = w^*$ and $r(.)$ is strictly increasing, it follows that:

$$E[\theta|r(\dot{\theta})] < r(\dot{\theta}), \dot{\theta} > \theta^* \tag{45}$$

$$E[\theta|r(\theta)] = E[\theta|\theta \leq \dot{\theta}] \tag{46}$$

This means that $E[\theta|\theta \leq \dot{\theta}] - r(\dot{\theta}) < 0$ for all $\dot{\theta} > \theta^*$. In this case, we look to (41) to determine that if $\dot{\theta} > \theta^*$, it is workers with productivity $\theta$ who are made worse off than in the highest-wage SPNE. Thus, the highest-wage SPNE is a constrained pareto optimum.

### 6.2 Signaling Mechanisms

This explanation of a signaling mechanism in a marketplace suffering from adverse selection is a slightly modified version of the explanation found in Mas-Colell, Whinston, & Green (1995). We follow Mas-Colell and start with signaling mechanisms.

The most trivial example of a signaling mechanism in the context of an adverse selection model would be a costless procedure that allows the informed party to publicly reveal their type - in the example from Section 2, this would be a way for sellers to credibly show that their asset is of quality $\theta$. If such a mechanism existed, the only agents who would not make use of said mechanism would be those with $\theta$ at the minimum possible value in the distribution $F(\theta)$. Any agent who did not make use of the mechanism would be believed by the market of buyers to be selling an asset of minimal quality. Since the mechanism is assumed to be costless, the market would revert to an efficient competitive equilibrium.

Even without such a perfect signaling mechanism, however, the potential for efficiency gains via signaling can exist. For the purposes of intuitive clarity, we will set up the models in this section using a labor market example. The conclusions reached will hold for all markets with adverse selection, and the reasoning will be analogous.

Let there be a number of identical firms which can hire workers. These firms produce

\textsuperscript{14}Most explanations move quickly through this portion of the argument, so it may be helpful to realize that what this means is simply that, given a worker sees a net payoff from accepting employment, his productivity cannot be higher than the wage in the highest-wage SPNE, in which he is paid $w^*$
the same output using a constant returns to scale technology, with labor as the only input. Firms are risk-neutral, seek to maximize their expected profits, and are price takers\(^{15}\).

Workers have a productivity \(\theta\). \(\theta\) is distributed according to a nondegenerate function \(F(\theta)\). Workers seek to maximize their earnings from labor. They can acquire earnings either by working on their own (earning \(r(\theta)\)) or at a firm (earning some wage \(w\)). For the purposes of illustrating the signaling potential, we will restrict this model to a case where there are only two types of worker. Thus, \(\theta \in \{\theta_L, \theta_H\}\), with \(\theta_H > \theta_L > 0\) and some nonzero probability that \(\theta = \theta_H\).

We depart from the standard model here by allowing workers to get some level of education, which is observable. We further assume that education does not actually have an impact on productivity. However, we do assume that the cost associated with acquiring some level of education is less (in both total and marginal terms) for workers with high \(\theta\). For simplicity, we also assume that \(r(\theta_H) = r(\theta_L) = 0\) - that is, the opportunity cost of accepting employment is zero for all workers.

This model needs to be approached as a game. The worker is assigned either \(\theta_H\) or \(\theta_L\) by nature, and then (conditional on which \(\theta\) they were assigned) chooses a level of education \(e\). After obtaining that level of education, the worker enters the job market. Firms, conditional on the observed education level \(e\), simultaneously make wage offers. Finally, the worker decides whether to work for a firm and which firm to work for.

As is conventional, we begin by analyzing the final stage of the game. Firms are assumed to have some function \(\mu(e)\), such that if they observe an education level \(e\) they believe the worker is of productivity \(\theta\mu(e)\). Thus, after observing the worker’s level of education, a firm’s expectation of that worker’s productivity is:

\[
\mu(e)\theta\mu + (1 - \mu(e))\theta_L
\] (47)

The pure strategy Nash equilibrium with 2 firms (which is a simplification without loss of generality) is to offer a wage equal to (20).

On the worker’s side, both high and low productivity workers have decreasing (in \(\theta\))

\(^{15}\)In the asset sale model, this corresponds to buyers having identical valuation functions, being risk-neutral, maximizing their expected payoff, and being price takers
marginal rates of substitution between wages and education. Thus, the indifference curves for low and high productivity workers will cross only once, and high productivity workers will have a smaller slope at the crossing point.

There are two kinds of pure-strategy equilibria that might arise - separating equilibria and pooling equilibria. In the former, worker’s choices of education are different, while in the latter their choices are the same.

In separating equilibria, the worker’s choice of education is a function of her productivity. Let $e^*(\theta)$ be the equilibrium level of education for a given worker, and let $w^*(e)$ be the equilibrium wage offered as a function of observed education level.

By definition, a perfect Bayesian equilibrium requires that firms who observe $e^*(\theta_L)$ assign a probability of 1 to the worker’s productivity being $\theta_L$, and upon observing $e^*(\theta_H)$ assign probability 1 to productivity being $\theta_H$. It follows that resultant wage offers are $\theta_L$ and $\theta_H$ respectively. It further follows that for a separating equilibrium to be stable, $e^*(\theta_L) = 0$ - a low-productivity worker chooses to acquire no education. If this were not the case, the worker would be paying a cost to acquire her education, but receiving no increase in wage. Thus, the maximum payoff would be seen only with a choice of zero education. Thus, $w^*(0) = \theta_L$.

We cannot precisely determine $e^*(\theta_H)$, but if we denote it as $\bar{e}$ it is obvious that $w^*(\bar{e}) = \theta_H$. The equilibrium wage offers for $e \in (0, \bar{e})$ are free to vary quite wildly, possibly supporting multiple perfect Bayesian equilibria.

It is clear in this case that lower productivity workers are worse off than when no signaling is possible - they receive a wage of $\theta_L$ rather than $E[\theta|F(\theta)]$, which is strictly less if $F(\theta)$ is nondegenerate (as we assumed it is). Surprisingly, however, it is possible for higher productivity workers to also be worse off (Mas-Colell, Whinston, & Green, 1995). This is because the existence of a signaling mechanism removes the option for higher-productivity workers to choose the wage $E[\theta|F(\theta)]$ while paying zero education cost. If education is sufficiently costly, then, signaling mechanisms can produce an efficiency loss across the entire market. Note that education becomes costlier (in relative terms) as the proportion of high productivity workers grows - with very high proportions of high productivity workers, $E[\theta|F(\theta)]$ is quite high, but every high productivity worker has to acquire a costly education simply to show they are not one of the minority of low productivity workers.
Pooling equilibria are the only other alternative in the presence of signaling mechanisms. A pooling equilibria obviously requires that \( e^*(\theta_L) = e^*(\theta_H) = e^* \). In this case, perfect Bayesian equilibria require that firms, upon observing \( e^* \), assign a probability \( \lambda \) to the worker being high productivity, where \( \lambda \) is the proportion of high productivity workers. The equilibrium offered wage reverts in this case to \( E[\theta|F(\theta)] \). It is, somewhat surprisingly, possible to maintain certain pooling equilibria where all workers choose some level of education, even though the equilibrium where all workers choose zero education pareto-dominates equilibria with nonzero education.

### 6.3 Screening Mechanisms

As before, this explanation is largely taken from Mas-Colell, Whinston, & Green (1995).

Signaling mechanisms are mechanisms by which the informationally advantaged party makes a decision with the intent of signaling information about their private knowledge. By contrast, screening mechanisms are mechanisms by which the informationally disadvantaged party makes an effort to distinguish the types of agents they face.

As in the previous section, we restrict our attention to a case of two worker types, \( \theta_H > \theta_L > 0 \), with some fraction \( \lambda \in (0, 1) \) of those workers being of \( \theta_H \) productivity. The opportunity cost of accepting employment is still assumed to be zero (alternatively, productivity working at home is zero). The explanation differs from the previous section first in that jobs are allowed to differ in some respect analogous to difficulty - for instance, hours worked per week, or the level of focus required of the worker.

However, this task level adds nothing to the output of the worker. The only impact of increased difficulty or hours worked is to lower the worker’s utility. A worker of type \( \theta \) still outputs \( \theta \) independent of the difficulty or hours worked. As we are now allowing a non-wage factor to impact utility, we must define a utility function more carefully. Thus, a worker of type \( \theta \) who receives a wage \( w \) and faces a difficulty metric of \( t \geq 0 \) receives utility:

\[
 u(w, t|\theta) = w - c(t, \theta) \tag{48}
\]

To ensure that this behaves well, it is necessary that \( c(0, \theta) = 0 \), \( c() \) is increasing in \( \theta \), convex in \( t \), and has negative slope.
The game we study is a two-stage game. The first stage has two (again, a simplification without loss of generality) firms announce sets of offered contracts, where a contract is a pair \((w,t)\). There is no limit on the number of offered contracts except that it be finite. In the second stage, workers of each type choose whether to accept a contract and which contract to accept. If a worker is indifferent between two contracts, she chooses the least difficult, and if a worker is indifferent about accepting employment, she accepts employment. If a worker is indifferent between the contracts from two firms, she chooses randomly with equal probability.

Mas-Colell, Whinston, & Green (1995) prove that if worker types are observable and firms are allowed to make offers conditional on worker type, the subgame perfect Nash equilibrium requires that a worker of type \(\theta_i\) receives the offer \((\theta_i,0)\) and firms earn zero profits.

If worker types are not observable, that outcome cannot arise, because every worker with productivity \(\theta_L\) will prefer contracts \((\theta_H,0)\) and firms will make a loss by offering the higher wage contract.

To determine what an equilibrium would look like such a situation, it is first noted that in any equilibrium firms earn zero profits - the proof for this follows the standard logic that if any firm makes a non-zero profit, the other firm can profitably deviate by offering a slightly different contract pair to capture all the profit, and this process can be repeated to bring total profits arbitrarily close to zero. Since losses are unsustainable, the only remaining candidate for an equilibrium situation is that neither firm makes any profit.

Mas-Colell, Whinston, & Green (1995) then go on to prove the less intuitive claim that no pooling equilibria exist with screening mechanisms. There exists only one candidate for a pooling equilibrium (an equilibrium in which workers of both types will choose the same contract), which is a contract pair \((w_p,t_p)\) where \(w_p = E[\theta] = t_p\). If this pair were offered by both firms, each firm would have a profitable deviation open to it - offer a single contract \((\bar{w},\bar{t})\) where \(w_p < \bar{w} < \theta_H\). This pair will capture all workers of type \(\theta_H\) and none of type \(\theta_L\). Since we have assumed \(\bar{w} < \theta_L\), the firm that makes this deviation will make strictly positive profits, and thus from the previous paragraph it is clear that this deviation cannot yield an equilibrium. Thus, no pooling equilibria exist.

Finally, Mas-Colell, Whinston, & Green (1995) prove that in any separating equilibria with contract pair \((w_L,t_L)\) and \((w_H,t_H)\), the profits from each individual contract must
be zero - in other words, \( w_L = \theta_L \) and \( w_H = \theta_H \). The zero-profit condition establishes that for the lower-value contract, \( w_L \geq \theta_L \) - if this were not the case, the lower contract would produce strictly positive profits and thus enable a profitable deviation. If \( w_H < \theta_H \), the single-crossing nature of the indifference curves for each worker type result in a profitable deviation again being available. The only case without profitable deviations is the case where each contract offers zero profits.

Since we have assumed that \( t \) plays no part in the productivity (it merely makes jobs less appealing as it grows), we can easily see that the contract offered to workers of type \( \theta_L \) in a separating equilibrium is \((\theta_L, 0)\) - a firm could offer a contract \((\bar{w}, \bar{i})\) with \( \bar{w} < \theta_L \), which would still be accepted due to a lower value of \( t \), earning strictly positive profits.

This, finally, allows us to derive the contract offered to workers of type \( \theta_H \). There is some contract \((\theta_H, i_H)\) which will attract low-quality workers due to the higher wage offered, even with the higher difficulty \( i_H \). \( i_H \) is determined by reference to the indifference curves of the workers, so the contract that will separate high quality workers in equilibrium is \((\theta_H, \bar{i}_H)\) where \( \bar{i}_H \) satisfies \( \theta_H - c(\bar{i}_H, \theta_L) = \theta_L - c(0, \theta_L) \).

It is, however, not necessary that an equilibrium exist at all. It is possible that from the above contract pair a profitable deviation exists whereby a firm offers a contract or pair of contracts with the intent of attracting all workers, not simply one group. As we have already established that pooling equilibria cannot exist, this deviation cannot result in an equilibrium - but it is nonetheless possible for a profitable such deviation to exist. Thus, in certain cases screening mechanisms can result in a market with no equilibria.

When equilibria do exist, they are constrained Pareto-optimal outcomes (Mas-Colell, Whinston, & Green, 1995).

### 6.4 Separating Adverse Selection & Moral Hazard

This explanation of the problem faced in empirical adverse selection studies and some of the methods used to handle it owes much to Cohen & Siegelman (2009).

Moral hazard concerns are one of the primary reasons that insurance contracts include deductibles (Winter, 2000; Harris & Raviv, 1978), which restrict insurance payouts to cover only losses above some fixed amount. For instance, a health insurance policy with a
deductible of $1500 would only pay out if the policyholders total healthcare spending was greater than $1500, and would only cover the difference. This preserves an incentive for the policyholder, although the incentive is still somewhat warped.

If moral hazard is present, it can be expected to produce a correlation between coverage and risk, which will look the same to researchers as adverse selection - the difference between the two theories is an unobservable source of the risk.

Thus researchers who seek to identify the presence of moral hazard or adverse selection exclusively must determine a way of isolating their effects. This is by far the most difficult part of any empirical study on adverse selection. There are three general approaches to this isolation - randomized and natural experimentation, looking at the dynamics of insurance contracts, and looking at the interaction between the coverage-risk correlation and policyholder characteristics (Cohen & Siegelman, 2009).

Randomized experiments are quite rare in economics, due to the difficulty and often exorbitant costs associated. However, at least one example exists - Manning et al. (1987) used data from the RAND Health Insurance Experiment, looking for behavior changes in those who were assigned higher levels of spending (the assignment was random). Their results were consistent with moral hazard. Because of the nature of randomized experiments, it is generally assumed that exogenous changes to the coverage does not have any effect on policyholder’s underlying risk.

Natural experiments are more common - in section 4 we looked at Klonner & Rai (2007), which used a policy change in India as a natural experiment basis. Chiappori, Durand, & Geoffard (1998) utilized an exogenous change in French health insurance (the introduction of a 10% copayment) to look for moral hazard, and found evidence suggesting it was present only in some dimensions of health insurance (for instance, the introduction of a copayment had no impact on the number of doctor office visits, but did have an impact on the number of doctor visits to the patient’s home). In section 4 we also looked at Cardon & Hendel (2001), which used data from the National Medical Expenditure Survey, which was essentially a natural experiment.

Focusing on the dynamics of insurance companies is a more complicated option, but obviously benefits from not needing an experiment. For instance, it is common for insurers to set premiums based on the policyholder’s past claim history (in simple terms, a policy-
holder who has many losses can expect higher premiums than one with no losses). Moral hazard should thus be expected to produce a negative correlation between prior claims and accidents in following years. Adverse selection should produce a positive correlation between those two statistics.

Finally, researchers can look at the interaction between the coverage-risk correlation and policyholder characteristics. For instance, Cohen (2005) identifies a coverage-risk correlation, but also shows that this correlation only exists for policyholders with a 3 or more years of driving experience. This is explained by adverse selection - drivers obtain private information about their riskiness as they gain experience - while a moral hazard explanation is more problematic.

While these methods work reasonably well, they are limited - they require specific kinds of data which are not always available, and under certain circumstances may be of no use at all (for instance, if moral hazard and adverse selection both have a valid explanation for an interaction between the correlation and policyholder characteristics). More work is necessary on methods of disentangling moral hazard and adverse selection effects.
7 References


