A scalar singlet extension of the Standard Model

by

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under the supervision of

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Declaration of Authorship

I, Mr. Subhojit Sarkar, declare that this thesis titled, ‘A scalar singlet extension of the Standard Model’ and the work presented in it are mostly my own. I confirm that:

- This work was done mainly while I maintained my candidature for a research degree at this University.
- I have enlisted all published works that I have cited to complete my thesis.
- I have acknowledged all main sources of help.

Signed: Subhojit Sarkar

Date: 26-03-2015
“Nature uses only the longest threads to weave her patterns, so that each small piece of her fabric reveals the organization of the entire tapestry”

Richard P. Feynman

“No one undertakes research in physics with the intention of winning a prize. It is the joy of discovering something no one knew before”

Stephen Hawking

“We live in a Newtonian world of Einsteinian physics ruled by Frankenstein logic”

David Russell

“All of physics is either impossible or trivial. It is impossible until you understand it, and then it becomes trivial”

E. Rutherford
The SM, conceptually and phenomenologically fails to incorporate and explain few fundamental problems of particle physics and cosmology, such as a viable dark matter candidate, mechanism for inflation, neutrino masses, the hierarchy problem etc. In addition, the recent discovery of the 125 GeV Higgs boson and the top quark mass favor the metastability of the electroweak vacuum, implying the Higgs boson is trapped in a false vacuum. In this thesis we propose the simplest extension of the SM by adding an extra degree of freedom, a scalar singlet. The singlet can mix with the Higgs field via the Higgs portal, and as a result we obtain two scalar mass eigenstates (Higgs-like and singlet-like). We identify the lighter mass eigenstate with the 125 GeV SM Higgs boson. Due to the mixing, the SM Higgs quartic coupling receives a finite tree level correction which can make the electroweak vacuum completely stable. We then study the stability bounds on the tree level parameters and determine the allowed mass ($m_2$) region of the heavier mass eigenstate (or singlet-like) for range of mixing angles ($\sin \theta$) where all the bounds are satisfied. We also obtain regions of parameter space ($m_2 - \sin \theta$) for different signs of the Higgs portal coupling ($\lambda_{hs}$). In the allowed region, the singlet-like state can decay into two Higgs-like states. We find the corresponding decay rate to be substantial. Finally, we review various applications of the singlet extension, most notably, to the problem of dark matter and inflation.
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Dedicated to my family and friends...
Chapter 1

Introduction

The Standard Model (SM), consists of numerous fundamental particles (categorized into bosons and fermions) and physical parameters that have been determined in various experiments. One could claim it to be one of the most accurate theories ever developed by physicists, though not a complete one. Its inability to address the neutrino masses, the hierarchy problem, dark matter candidate etc. has invigorated physicists to explore its extensions. Perhaps, nature has chosen to exist in such an inexplicable way which requires diligence and careful study of its extensions. Few theories that have been suggested as extension to the SM include the Grand Unified Theories (GUT), Supersymmetry (SUSY) etc. Although these theories provide definitive solution to the famous ‘Hierarchy problem’, gauge unification etc., they incorporate plethora of parameters which invokes the problem of naturalness. In addition, SUSY associates superpartners corresponding to the SM particles and since these superpartners has not been observed experimentally (SUSY must be broken), it’s subjected to constructive criticism. There have been few proposals for the SUSY breaking mechanism such as, gauge mediated, gravity mediated and anomaly mediated breaking mechanism, but that simply begs the question.

Motivation to study physics doesn’t surface until one is well acquainted with the history behind the development of these theories, starting from Max planck’s quantization of radiation → P.A.M Dirac’s relativistic covariant equation of spin 1/2 particles → Yang-Mill’s gauge theories → the unification of electroweak interactions by S.L Glashow, S. Weinberg and A. Salam (often referred to as GWS theory) → till date. The GWS theory is a gauge theory and such (gauge) theories are usually accompanied with gauge bosons (or mediating particles) where they can be massive or massless depending on symmetry breaking. In the early 20th century, H. Yukawa proposed the same but he considered this mediating particles to be mesons. It was the pioneering works by the contemporary
physicists of the 20th century that we understand and exercise their ideas to build accurate models that explains most of the phenomenon observed in nature and the universe. To understand the fundamentals of interactions we first have to delve deeper in to the realm quantum field theory (QFT). To have a thorough understanding of the proposed model in this thesis, one has to be acquainted with the underlying symmetries associated with field theories. So what kind of symmetries can be realized with QFT? In QFT, a global symmetry that is manifest, leads to particle multiplets with restricted interactions. Another possibility is a global symmetry with spontaneous symmetry breaking (SSB) which results in unwanted massless goldstone bosons. Yet another possibility is a gauge (local) symmetry, where it requires the existence of massless vector fields corresponding to each generator (corresponding to the underlying group).

However, irrespective of the above three symmetry realizations the most interesting one that was realized and explored by S. Glashow, A. Salam and S. Weinberg was the spontaneously broken gauge symmetry. The GWS theory unifies the electromagnetic and weak interaction, often referred to as the electroweak (EW) interaction. The Lagrangian (or more generally the action) of the GWS theory is invariant under the symmetry transformation of $SU(2)_W \times U(1)_Y$ (gauge symmetry group), which then spontaneously breaks to $U(1)_{EM}$ giving massive gauge bosons ($W^\pm$ and $Z$ bosons) and a massless photon. Since direct Dirac mass term (such as $m_\nu \bar{\nu}_L \nu_R$) for the fermions in the SM Lagrangian are avoided (due to violation of gauge invariance), the fermions acquire their masses from Yukawa interaction via the spontaneous symmetry breaking mechanism. The scalar field that couples to the fermions breaks spontaneously and acquires a non-zero vacuum expectation value (vev), breaking the gauge symmetry and as a result giving massive fermions. This breaking mechanism was proposed by P. Higgs and the particle associated with the symmetry breaking is called the ‘Higgs boson’.

The Higgs boson was considered quite elusive and various collider experiments were conducted to shed light in its existence. However, after decades of hardwork and prolific engineering, the Large Hadron collider (LHC) of CERN successfully discovered the particle. Whether this scalar particle is the SM Higgs boson or an admixture of two or more scalar (spin-0) particles is a question that needs to be addressed and explored. In my personal opinion, I hope it’s the latter as it motivates physicists to explore theories beyond the SM.

Although the SM is an excellent low energy theory supported by the very precise experimental measurement of the physical parameters (such as fine structure constant, $\alpha$ etc.), its inability to incorporate dark matter candidate, inflation mechanism etc. motivates its extension. The aim of this thesis is to address such problems. The recent data from LHC insinuates that the observed 125 GeV Higgs boson can be the SM Higgs boson, favoring a metastable universe. We begin our thesis addressing the metastability problem by analyzing the Higgs sector in the SM. Chapter 2 is solely dedicated for this purpose.
In chapter 3, we provide a plausible solution to the metastable vacuum by introducing the simplest extension of the SM, a scalar singlet. We discuss the constraints imposed on the parameter space of the extension model by various experiments in Chapter 4. Chapter 5 will mainly be a review of the applicability of the simplest extension to the SM. We will find that it resolves the dark matter candidate problem encountered in the SM and plays a significant role in the Higgs-portal inflation (where it couples to gravity in a non-minimal way). Finally, I will conclude my thesis work in Chapter 6.
Chapter 2

The Higgs sector of the Standard Model

2.1 Introduction

To understand particle interactions, it is pivotal that we study the underlying physics that nests in the elusive sector of field theories. Often the mathematical rigor one encounters in field theories can at times seem demoralizing, at least for an amateur. However, behind these mathematical complexities lies the beauty of particle physics. To appreciate the contents in this chapter, it is enough that you are acquainted with quantum field theory. Recall in the previous section I mentioned the type of symmetries one can associate with a theory. In this chapter, I will discuss what these symmetries are? What phenomenon or mechanism causes fermions to acquire mass? We will study how this mechanism is incorporated in the GWS theory. We will see that this mechanism gives rise to a spin-0 particle called the Higgs boson. I will discuss briefly about the stability of the potential, the one-loop renormalized group (RG) equations of the Higgs self coupling and why is there a need to extend the SM?

Recall from your QFT course, particularly the Goldstone’s theorem which states that if the symmetry exhibited by a Lagrangian is spontaneously broken in quantum field theory, it gives rise to massless spin-0 bosons. Considering the last type of symmetry discussed in the previous chapter, for instance, the gauge theories which are associated with continuous group of local transformations (such as $SU(3), SU(2), U(1)$ etc) if are spontaneously broken then this broken symmetry results in particles receiving masses. In this type of symmetry breaking, the Goldstone mode provides the 3rd degree of freedom (or polarization) of a massive vector field (the mediating particle). Keeping this in mind I will proceed to discuss the SM and spontaneous symmetry breaking.
2.2 The Standard Model and Spontaneous symmetry breaking

The fermion sector of SM consists of quarks (anti-quarks) and leptons (anti-leptons) and the gauge sector is associated with vector bosons such as $W^\pm, Z^0, \text{gluons}$ etc. The experimental precession of a certain gauge theory, quantum electrodynamics (QED) is up to an unprecedented level where QED is invariant under the $U(1)$ internal symmetry group transformation whose generator, $Q$, is the electric charge. $U(1)$ being a continuous group, when gauged gives rise to a vector boson. Upon spontaneously breaking the symmetry, the associated broken generator corresponding to the broken symmetry gives rise to a massless vector boson, photon. We know photon is massless and any massless particle should have two transverse degree of freedom. So what role does the Goldstone boson play? This scalar particle must have exactly the right quantum numbers to appear as intermediate states. To state vaguely, it provides the right pole to make the vacuum polarization amplitude transverse. Note, that the Goldstone boson by itself doesn’t appear as an independent physical particle (because you can perform a gauge transformation to rid the theory of the Goldstone boson).

This lead to the exploration of this idea to other known particle interactions. The first step to constructing a gauge theory is by requiring that the Lagrangian be gauge and lorentz invariant (since systems cannot depend on the choice of inertial frame). Gauge symmetries are internal symmetry group under which we demand the Lagrangian to be invariant. The SM consists of a complete description of elementary particle interactions. The symmetry group that defines most of the particle interaction in the SM is $SU(3)_c \times SU(2)_L \times U(1)_Y$. In this thesis I will exclude the discussion of the QCD sector i.e the strong interaction between quarks via the exchange of the vector boson, gluons. We will only analyze the unification of electromagnetic and weak interaction. The motivation behind this is because of its simplicity compared to QCD.

2.2.1 Glashow-Weinberg-Salam theory

The GWS theory unifies the electromagnetic and weak interaction. They are best described by the gauge group $SU(2)_L \times U(1)_Y$, where the subscript $L$ represents lepton sector and $Y$ the hypercharge. The lepton sector turns a blind eye towards strong interaction hence we do not include the $SU(3)$ group. $SU(2)$ group has three hermitian generators, $\frac{\tau_i}{2}$, represented by pauli matrices and the unitary group $U(1)$ has one generator, $Y/2$. As the gauge group has four generator (three for $SU(2)$ and one from $U(1)$) we expect four vector bosons. The Lagrangian then should be invariant under these symmetry transformation. Let us pursue the kinetic term first. Recall that the kinetic
energy term for the Dirac fermions can be split into two separate pieces for the right handed and left handed fermions.

\[
\sum_{\psi} \bar{\psi}i\partial\psi = \sum_{\psi_L} \bar{\psi}_L i\partial\psi_L + \sum_{\psi_R} \bar{\psi}_R i\partial\psi_R \tag{2.1}
\]

Upon coupling \( \psi \) to gauge field, we can assign \( \psi_L \) and \( \psi_R \) to different representations of the gauge group. Then the two terms on the right hand side of equation (2.1) will contain two different types of covariant derivatives and since they lie in different representation, they would have independent gauge couplings. We assign the left handed fermions to transform as doublets under SU(2) representation while the right handed fermions as singlets. In the GWS model, the right handed fermions do not couple to the weak isospin \((T^3)\). For the left handed fields, the quark and lepton doublets are:

\[
Q^c_{i,L} = \left( \begin{array}{c}
{u^c_i} \\
{d^c_i}
\end{array} \right)_L, \quad E^c_{i,L} = \left( \begin{array}{c}
{\nu_i} \\
{e^-_i}
\end{array} \right)_L \quad \text{with,}
\]

\[
{u^c_i} = (u^c, c^c, t^c), \quad {d^c_i} = (d^c, s^c, b^c),
\]

\[
{e_i} = (e, \mu, \tau), \quad {\nu_i} = (\nu_e, \nu_\mu, \nu_\tau)
\tag{2.2}
\]

where ‘L’, denotes the left handed representation and ‘i’ denotes the flavour indices and the superscript ‘c’ denotes the color (RGB) indices. From here on, I will drop the superscript ‘c’ for color indices. Once we have specified \( T^3 \), the weak isospin quantum number, for each value of fermion field, the value of hypercharge \( Y \) must follow from the equation:

\[
Q = T^3 + \frac{Y}{2} \tag{2.3}
\]

Here \( Q \) is the charge quantum number. We find that the left handed quarks \( Q_{i,L} \), have hypercharge 1/3 and the left handed leptons \( E_{i,L} \), have hypercharge -1. Since the right handed fermionic fields live in a different representation of the gauge group, the hypercharge assignment is different for every particle. For instance, \( e_R \) (\( e^c_R \)) has hypercharge -2, \( u_{i,R} \) (\( u^c_{i,L} \)) and \( d_{i,R} \) (\( d^c_{i,L} \)) have hypercharge 4/3 and -2/3 respectively. Note the superscript ‘c’ here implies charge conjugation and has nothing to do with color indices. As right handed neutrinos are omitted in the SM, we exclude it’s hypercharge assignment. Let us discuss the interactions involved in the GWS theory. The kinetic term for the GWS theory takes the form:

\[
L_{K.E} = \bar{E}_L (i\partial \psi) E_L + \bar{Q}_{i,L} (i\partial \psi) Q_{i,L} + \bar{e}_R (i\partial \psi) e_R + \bar{u}_{i,R} (i\partial \psi) u_{i,R} + \bar{d}_{i,R} (i\partial \psi) d_{i,R} \tag{2.4}
\]
where,

\[ D_\mu Q_{i,L} = (\partial_\mu - ig_2 A_{i,\mu} - ig' B_\mu) Q_{i,L}, \]
\[ D_\mu E_{i,L} = (\partial_\mu - ig_2 A_{i,\mu} + ig' B_\mu) E_{i,L}, \]
\[ D_\mu u_{i,R} = (\partial_\mu - ig' B_\mu) u_{i,R}, \]
\[ D_\mu d_{i,R} = (\partial_\mu + ig' B_\mu) d_{i,R}, \]
\[ D_\mu e_{i,R} = (\partial_\mu + ig' B_\mu) e_{i,R} \]

Before proceeding any further let me enunciate a little further on the notations used above. \( D = \gamma_\mu D_\mu \), where \( \gamma_\mu \) is the 4×4 Dirac matrices and \( D_\mu \) is the covariant derivative. \( g' \) is the \( U(1)_Y \) coupling constant and \( g \) is the \( SU(2)_L \) gauge coupling constant. \( A_\mu \) is the \( SU(2)_L \) gauge boson (corresponding to the three generators) and \( B_\mu \) is the \( U(1)_Y \) gauge boson. \( \tau_i/2 \) represents the three pauli matrices.

The kinetic terms for gauge bosons can be written as:

\[ \mathcal{L}_{K.E-gauge-bosons} = -\frac{1}{4} F_{i\mu\nu} F^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \]

where:

\[ F^{A}_{i\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\epsilon^{ijk} A^j_\mu A^k_\nu \]
\[ B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \]

where \( F_{i\mu\nu} \) and \( B_{\mu\nu} \) are the antisymmetric field tensors corresponding to the vector bosons \( A_\mu \) and \( B_\mu \). We should then write equation (2.4) in terms of vector boson mass-eigenstates to determine the physical interpretation of the fermion-vector boson coupling:

\[ \mathcal{L} = \bar{E}_L (i\bar{\psi}) E_L + Q_{i,L} (i\bar{\psi}) Q_{i,L} + e_{i,R} (i\bar{\psi}) e_{i,R} + u_{i,R} (i\bar{\psi}) u_{i,R} + d_{i,R} (i\bar{\psi}) d_{i,R} + g(W^+ W^0 + W^- W^- + Z^0 Z^0) + eA_\mu J^{\mu}_{EM} \]

where the field currents \((J_{W^\pm, Z})\) description can be found in the Appendix [A.1]. As mentioned earlier, since both left and right handed fields live in different representation of the gauge group, direct Dirac mass terms of the form:

\[ \Delta \mathcal{L} = -m_e (\bar{e}_L e_R + \bar{e} R e_L) \]

is to be avoided in the Lagrangian as they break gauge invariance. Note in the equation (2.8) we do not find any fermionic mass term. So how do fermions get masses? Experimental evidence has showed the different mass spectrums for the fermions. Is our physics wrong? Before panicking, it is imperative that you have a good understanding of the
Yukawa sector. In brief, Yukawa terms contains coupling between a scalar doublet and a fermionic doublet along with a singlet (right handed fermions). This does not spoil the gauge invariance nor does it induce large divergence and it is renormalizable. The method to generate such a mass term that does not spoil the gauge invariance of the Lagrangian is called the spontaneous symmetry breaking. This is the topic we discuss next.

2.2.2 Spontaneous symmetry breaking (SSB) and Yukawa interaction

Detailed studies of spontaneous symmetry breaking can be found in many books but the book [4] is preferable over many. We will briefly discuss SSB and apply it to the Yukawa interactions. So what is spontaneous symmetry breaking (SSB)? In short, the SM Lagrangian and its equation of motion may possess certain symmetry, but the solution of the equation of motions may violate this symmetry. This is same as saying, a field acquiring a non-zero value at the minimum of the potential in a given direction of field. The vacuum no longer exhibits the above symmetry and is said to spontaneously broken. The most simplistic model is the Higgs mechanism which will be the primary discussion here.

In the Higgs mechanism, we introduce a set of scalar fields which transforms nontrivially under the symmetry group \((SU(2)_L \times U(1))\) mentioned in the previous section. We introduce only renormalizable terms in the Lagrangian to avoid large divergent terms. The Lagrangian of the simple self-interacting complex scalar field is:

\[
\mathcal{L}_\phi = (\partial_\mu \phi)\dagger(\partial^\mu \phi) - V(\phi)
\]

(2.10)

where,

\[
V(\phi) = m^2(\phi^\dagger \phi) + \lambda(\phi^\dagger \phi)^2
\]

(2.11)

Here we have excluded the odd terms (such as \(\phi\) and \(\phi^3\)) by imposing symmetry constraints. Then solving the equation of motion (EOM), we find for \(m^2 < 0\) the solution of equation of motion \((\phi^\dagger \phi = -m^2/2\lambda)\) is non-zero at a certain point. Of course if \(m^2 > 0\) then the only possible solution would be \(\phi = 0\) for which no SSB takes place. For the former case we construe that the field has acquired a non-zero vacuum expectation value (vev) \(< \phi^\dagger \phi > = -m^2/2\lambda\). From the figure [2.1], we can visualize the variation of the scalar potential w.r.t the complex scalar field. Usually we are free to choose the direction of symmetry breaking. Let us choose this direction around the real component of the field. Now consider a small perturbation around the minimum of the vacuum. Whether this minimum is an absolute or local minimum depends on the second derivative test.
The field is:

$$\phi(x) = (1/\sqrt{2})[v + \alpha(x) + i\eta(x)] \quad (2.12)$$

**Figure 2.1:** The figure imply the potential, $V$, varying as a function of complex scalar field. The two directions of field imply higgs boson towards real field direction and goldstone boson across the imaginary field direction. Note the circular lines indicate the U(1) symmetry rotation around the vertical axis. \[1\]

where $v$ is the vev, $\alpha(x)$ and $\eta(x)$ are the real and imaginary components of the complex scalar field. Substituting equation (2.12) into equation (2.10) we get:

$$L = \frac{1}{2}(\partial_\mu \alpha(x))^2 + \frac{1}{2}(\partial_\mu \eta(x))^2 - \lambda v \alpha(x) \eta(x) - (\lambda v^2)\alpha(x)^2 - \frac{1}{4} \lambda \alpha(x)^4 - \frac{1}{4} \lambda \eta(x)^4 - \frac{1}{2} \lambda \alpha(x)^2 \eta(x)^2 \quad (2.13)$$

Collecting the quadratic field terms (or mass terms), we see that the field $\alpha(x)$ has acquired a mass of $2\lambda v^2$ while the field $\eta(x)$ remains massless implying it’s a Goldstone boson. The Goldstone’s theorem states, that if a Lagrangian is invariant under a continuous symmetry transformation, then a massless particle exist upon spontaneously breaking the symmetry. In the SM, this particle will be of spin-0 and is termed as Goldstone boson (Note in Supersymmetry (SUSY), because of the existence of super-multiplets structure, one can encounter a Goldstone fermion rather than a Goldstone boson when SUSY is broken). We find that the number of massless particles equals the number of broken generators.

In gauge theories, we can however rid ourself of the massless particle from the equations by performing a gauge transformation. Consider for simplicity an abelian (i.e the groups whose generators commute) case, where a complex scalar field couples to both itself and to an electromagnetic field. In this case the Lagrangian takes the form:

$$L = -\frac{1}{4}(F_{\mu\nu})^2 + |D_\mu \phi|^2 - V(\phi) \quad (2.14)$$

where $V(\phi)$ is given by equation (2.11), $D_\mu = \partial_\mu + i e A_\mu$ is the covariant derivative and $F_{\mu\nu}$ is the antisymmetric field tensor. Clearly the Lagrangian in equation (2.14) is
invariant under a local \( U(1) \) transformation:
\[
\phi(x) \rightarrow \exp(i\theta(x))\phi(x), \\
A_\mu(x) \rightarrow A_\mu(x) - \frac{1}{e} \partial_\mu \theta(x)
\] (2.15)

Then using the perturbed field around the minimum of the potential as in equation (2.12), we find additional terms of the form:
\[
|D_\mu \phi|^2 = \frac{1}{2} \partial_\mu \alpha(x)^2 + \frac{1}{2} \partial_\mu \eta(x)^2 + \sqrt{2} e v A_\mu \partial^\mu \eta(x) + e^2 v^2 A_\mu A^\mu + \text{cubic terms} + \text{quartic terms}
\] (2.16)

However, we can perform a gauge transformation of the field \( \alpha(x) \) and \( \eta(x) \) such that, we get rid of \( \eta(x) \) completely from the equation. That is we can choose such a gauge, where the complex scalar field \( \phi(x) \) becomes real valued at every space-time point. Such a choice of gauge is often regarded as the ‘Unitarity gauge’ and the Lagrangian then becomes:
\[
\mathcal{L} = -\frac{1}{4} (F_{\mu\nu})^2 + (\partial_\mu \alpha)^2 + e^2 v^2 A_\mu A^\mu - V(\phi)
\] (2.17)

Thus we see that SSB of gauge theory leads to vector bosons receiving mass. This can be extended to non-abelian sector where the formalism is quite similar with the difference that generators no longer commute. We conclude that since the vector boson in the above formalism is massive, it has three degrees of freedom (d.o.f). Two corresponding to transverse and one to longitudinal d.o.f. Often we are tempted to say, that the vector boson consumed the massless goldstone boson to become massive.

Now that we have familiarized ourself with SSB and local gauge theory, we shall extend this idea to the Yukawa theory and see how SSB of the yukawa theory leads to massive fermions. As discussed earlier, a direct mass term in the Lagrangian is to be avoided to preserve gauge invariance. However, via the Higgs mechanism, fermions can attain mass without breaking gauge invariance through Yukawa terms. A Yukawa interaction term looks like:
\[
\lambda_y (\bar{\psi}_L \phi) \psi_R
\] (2.18)

where \( \lambda_y \) is the yukawa coupling. In the minimal standard model (MSM), one complex SU(2) doublet of scalar fields with \( Y=1 \) (Hypercharge), is introduced. Although models with two Higgs doublet and three Higgs doublet have been throughly studied in [5],[6],[7], we will focus only on the one Higgs doublet model. The Higgs doublet has two fields associated with it, a charged Higgs and a neutral Higgs : \( h^+ \) and \( h^0 \). Let us denote this
scalar doublet (or the Higgs doublet) as:

$$\Phi = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$$

(2.19)

The potential and the kinetic term takes the form:

$$\mathcal{L}(\Phi) = (D_\mu \Phi)^\dagger D^\mu \Phi - V(\Phi)$$

(2.20)

where,

$$D_\mu = \partial_\mu - ig_2 \tau^i A^i_\mu - ig'_2 B_\mu$$

(2.21)

Note we are still working in the GWS theory and not the SM. If I were to develop my theory in the SM then in the above equation (2.21) we would have a gluon vector boson term ($G_\mu$), with the strong gauge coupling ($g_s$) and the Gell-Mann matrices ($\lambda^i/2$). Following the GWS theory we could write the most general gauge invariant renormalizable potential as:

$$V(\Phi) = m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

(2.22)

Introduction of any other higher order terms in the potential (2.22), introduces divergences that cannot be regulated or cancelled in the theory, making the theory non-renormalizable. As always linear and cubic order terms are excluded by imposing a global $U(1)$ symmetry. We can perform a shift to the complex scalar doublet and write it in the unitary gauge where the terms are in real component fields and perturbed around the minimum of the potential.

$$\Phi = \frac{U(x)}{\sqrt{2}} \begin{pmatrix} 0 \\ h(x) + v \end{pmatrix}$$

(2.23)

where, $U(x)$ is the unitary matrix and $v$ is vev. We can now make a gauge transformation and rid ourself of the unitary matrix.

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h(x) + v \end{pmatrix}$$

(2.24)

Going back to the Yukawa interaction term (2.18), we can write them in terms of the SU(2) representation:

$$- \lambda^u_{ab} \bar{Q}_{a,L} i \tau_2 \Phi^* u_{b,R} + \lambda^d_{ab} (\bar{Q}_{a,L} \Phi) d_{b,R} - \lambda_e (\bar{E}_{L} \Phi) e_R + h.c$$

(2.25)

The $\lambda^u_{ab}$, $\lambda^d_{ab}$ and $\lambda_e$ are dimensionless coupling constant and are $3 \times 3$ matrices, $\phi^*$ is
the complex conjugate of the complex scalar field (or the Higgs doublet), \( Q_{a,L} \) are the quark doublets, \( E_L \) are the lepton doublets, \( u_{b,R} \), \( d_{b,R} \) are right handed Up and Down quark and \( e_R \) is the right handed electron. Notice that the hypercharge of each term sums up to zero. The complete Lagrangian consists of the fermion kinetic terms (2.4) and the gauge particles kinetic terms in equation (2.6), the Lagrangian of the scalar sector as in equation (2.20) and the Yukawa interactions as in equation (2.25). Upon minimizing the scalar potential and perturbing the field around minimum (as in equation (2.24)) we find the corresponding masses of the fermions:

\[
m_u = \frac{1}{\sqrt{2}} \lambda_u v, \quad m_d = \frac{1}{\sqrt{2}} \lambda_d v, \quad m_e = \frac{1}{\sqrt{2}} \lambda_e v
\]  

(2.26)

The vector boson masses arises from the kinetic term of the complex scalar (2.20) whose covariant derivative is given by equation (2.21):

\[
\left( \begin{array}{c} 0 \\ v \end{array} \right) \left( \frac{g}{2} \tau^a A^a_\mu + \frac{g'}{2} B_\mu \right)^2 \left( \begin{array}{c} 0 \\ v \end{array} \right)
\]  

(2.27)

which gives us the mass of vector bosons:

\[
M^2_W = \frac{1}{4} g^2 v^2, \quad M^2_Z = \frac{1}{4} (g^2 + g'^2) v^2, \quad M^2_A = 0
\]  

(2.28)

Thus we determined how the Higgs mechanism results in fermion masses. Our next step would be to analyze the scalar (or the Higgs) potential. The following discussion will shed some light towards the metastability issue encountered in the SM.

### 2.3 The Higgs sector and stability of scalar potential

Previously we showed how the Higgs mechanism helps us solve the fermionic mass issue. There is however a slight glitch in the SM as the Higgs mechanism cannot provide mass to the neutrinos in the SM. The issue is the fine-tuning problem of the Yukawa coupling associated with the neutrino mass term. I will not address that issue in the thesis as it lies beyond its scope. In this section, we will inspect the parameters involved in the Higgs (or scalar) potential, especially the Higgs quartic coupling. We will discuss its boundedness from below condition and how the quartic coupling can have an inflection point (where the coupling turns negative) before the Planck scale which can render the potential unstable at large scales. This gives rise to an existence of a global minimum at large field values implying the EW vacuum is a local minimum.
2.3.1 The Higgs quartic coupling

Recall from the potential mentioned in the previous section, as in equation (2.22), in this section we introspect the Higgs quartic coupling ($\lambda_h$). We can write the scalar potential as:

$$V(h) = m_h^2 (\Phi^\dagger \Phi) + \lambda_h (\Phi^\dagger \Phi)^2$$

(2.29)

where $\Phi$ being the scalar field, $m_h$ is a parameter with mass dimension one and $\lambda_h$ is the scalar field self coupling. Now if we expand the field around the minimum of the potential and impose the minimality condition ($v^2 = -\frac{m_h^2}{\lambda_h}$), we get in the unitary gauge:

$$V(h) = \frac{1}{4} m_h^2 v^2 + \frac{1}{2} (2\lambda_h v^2) h^2 + \lambda_h v h^3 + \frac{1}{4} \lambda_h h^4$$

(2.30)

Note that $m_h^2 < 0$. From the above equation (2.30), we see that the Higgs field has acquired a mass,

$$m_h^2 = 2\lambda_h v^2$$

(2.31)

In the SM, since the EW breaking scale is around 246 GeV (i.e $v = 246$ GeV) and if the recent discovery of the Higgs boson and its mass ($m_h \simeq 125$ GeV) from LHC implies that the particle is indeed the SM Higgs boson, we can fix the quartic coupling parameter at the EW breaking scale. This plays an essential role in understanding the stability of the potential from the SM perspective. The quartic coupling is a function of the energy scale (or field scale) hence we can study its behavior at different scales from its running i.e $\beta_h$. We study the beta-function of the quartic coupling which can be evaluated by either ‘Coleman-Weinberg’ approach or by solving the ‘Callan-Symanzik equation’. Usually the most interesting physics are expected at one-loop order since due to the smallness of the quartic coupling ($\lambda_h$) from perturbativity, higher order terms can be ignored.

The above discussion is based on tree level analysis which doesn’t include interesting physics as it doesn’t tell us much about the vacuum structure. Thus for consistency, we need to consider higher loop orders. If we were to include loop corrections (say at one-loop order), then we would need to redefine our potential as one loop order terms will contribute to the potential. We will call this modified version of the potential as quantum effective potential or simply effective potential. Once the loop contributions are included the effective potential takes the form:

$$V_{\text{eff}}(\phi) = V_{\text{tree}} + V_{\text{loop-contributions}}$$

(2.32)

This modified potential (or effective potential) up to one loop order was studied extensively by M. Sher in his paper [8] using ‘Coleman-Weinberg’ approach (2.32). The effective potential helps us understand the behavior of the scalar potential at large field
values, in particular, the behavior of the tree-level parameters such as, $\lambda_h$. So basically what we are computing are the variation of the tree-level parameters w.r.t some scale parameter (say $\Lambda$) by including higher order loop contributions. The above approach is called the ‘Coleman-Weinberg’ approach. Yet another approach mentioned earlier is the ‘Callan-Symanzik’ equation which can be found in [4] and will not be discussed here.

Consider for instance that the minimum we determined in the previous section is the absolute (or global) minimum? What does this imply on the boundedness of the tree-level potential? This would imply that the potential is always bounded from below and hence stable. However multiple local minimum could exist that lie above this global minimum, in which case we could study the inflation of the universe in the scalar-tensor framework [9]. If there exist yet another minimum at large field values, which is indeed the global minima, then quantum effects (such as tunneling) are to be studied. If the lifetime of tunneling from the EW vacuum to this absolute minimum is greater than that of the age of universe, we say our potential is only metastable. The next section is dedicated to address the stability of the EW vacuum.

### 2.3.2 Stability of scalar potential and RG equations in SM

For the potential to be stable we require the potential be bounded from below at all scales, $\Lambda$. This ensures that the minimum we determined and discussed comprehensively in the previous section, is indeed a global minimum. The boundedness from below condition implies $V(h) > 0$, for all field values. This implies that at any field value we require the tree-level potential to be positive definite (i.e. $\lambda_h > 0$). Note that we have implied strong stability condition i.e. there is no equality sign involved in our analysis of stability of potential. However, since we desire perturbation theory to work at all scales, we have to subject the quartic coupling to another constraint ($0 < \lambda_h < 1$) at all scales. Depending on the value $\lambda_h$ takes, radiative corrections plays a pivotal role. Thus it becomes imperative to study how $\lambda_h (\Lambda)$ varies w.r.t some field value or scale ($\Lambda$). To understand this scenario let us first write the beta function ($\beta_h$) at one-loop order for the Higgs quartic coupling ($\lambda_h$) in the SM:

$$16\pi^2\beta_{\lambda_h} = \left[ 24\lambda_h^2 - 6y_t^2 + \frac{3}{8}(2g^4 + (g^2 + g'^2)^2) + (-9g^2 - 3g'^2 + 12y_t^2)\lambda_h \right]$$ (2.33)
where
\[ g' : \text{U}(1) \text{ coupling constant}, \]
\[ g : \text{SU}(2) \text{ coupling constant}, \]
\[ y_t : \text{top quark yukawa coupling constant}, \]
\[ \beta_{\lambda_h} = \frac{d\lambda_h}{dt} \text{ where,} \]
\[ t = \ln \frac{\Lambda}{m_{\text{top}}} \quad m_{\text{top}} : \text{mass of top quark}. \] 

The terms on the right hand side of the equation (2.33) arise from one-loop order contribution to the Higgs quartic coupling \( \lambda_h \). Similary at one loop order the beta functions for the gauge and top-quark yukawa couplings in SM are,

\[ 16\pi^2 \beta_{y_t} = y_t \left[ \frac{9}{2} y_t^2 - \frac{9}{4} g^2 - \frac{17}{12} g'^2 - 8g_s^2 \right] \]

\[ 16\pi^2 \beta_{g_i} = b_i g_i^3, \] (2.35)

where for \( b_i \) the couplings follow the structure \( (\text{U}(1), \text{SU}(2), \text{SU}(3)) \). Below I have drawn two plots for \( \lambda_h \) and \( y_t \) adjacent to each other showing that for different initial values of the Higgs quartic coupling (taken at mass of top quark), the running of \( \lambda_h \) remains positive till planck’s scale at one and becomes negative at the other.

**Figure 2.2:** For initial value of \( \lambda_h [m_{\text{top}}] = 0.15 \), we find that the quartic coupling (yellow) remains positive till planck scale. The blue plot is the yukawa coupling of top quark.

**Figure 2.3:** For initial value of \( \lambda_h [m_{\text{top}}] = 0.12 \), the quartic coupling (in yellow) inflects and takes negative values before planck scale. The blue plot is yukawa coupling of top quark.

In the above figures [2.2] and [2.3], I have deliberately drawn the top yukawa coupling alongside the higgs quartic coupling. The reason behind this follows from the beta function of the quartic coupling \( \lambda_h \), as in equation (2.33), where the \( \lambda_h \) receives a large negative contribution from the top quark. The negative contribution arises due to fermionic loops. The top quark being so heavy tends to significantly decrease the Higgs quartic coupling as we increase the scale (the field scale or energy scale), \( \Lambda \). This phenomenon then causes a problematic scenario in the quartic potential. At higher field values, one can usually ignore the quadratic term keeping only the quartic term and
recalling the stability bounds imposed previously, we find that for the plot where $\lambda_h$ becomes negative, the quartic potential satisfies:

$$V_4 = \frac{\lambda_h}{4} h^4 < 0.$$  \hfill (2.36)

rendering the potential being unbounded from below. We can intellectually infer that under such circumstances, we expect another minimum at high field values which may be the true global minimum. As mentioned earlier, we can study the quantum tunnelling effect to this supposedly absolute minima and if the lifetime for tunnelling is greater than that to the age of the universe, we stipulate that the EW vacuum is metastable (or false vacuum). This opens up a wide spectrum of new ideas that extends and incorporates beyond the SM theory and helps us ameliorate the problem at hand. One such model is introduction of a scalar singlet, which can couple to the Higgs field and fine-tuning the parameters can help us achieve stability (i.e. make the potential bounded from below). This is the prime theme of the next chapter where we will see how fine-tuning the Higgs-singlet coupling ($\lambda_{hs}$) we can achieve the stability of the potential. It is to be noted here that at relatively low energies, say the EW breaking scale, we didn’t have to impose any additional constraints for $\lambda_h$ because at that scale the SM Higgs quartic coupling is positive and the potential is stable. In the next chapter, we will see that this is indeed the case.

2.4 Summary

Let us summarize what we have learned at this chapter. We have studied the unification of electromagnetic and weak interaction, the GWS theory. We understood how spontaneous symmetry breaking in the yukawa sector results in the fermions acquiring masses while conserving gauge invariance. We specifically discussed the SM Higgs sector where we argued that the Higgs quartic coupling ($\lambda_h$) need not remain positive till the Planck scale (or closer to Planck scale). This ensures that the low energy minimum (the EW minimum) is a local minimum and not a global one rendering our vacuum to be metastable. In the next chapter, where we include an additional degree of freedom - a scalar singlet, we will find that we can ameliorate the metastability issue due to a finite coupling between the singlet and the Higgs field.
Chapter 3

Scalar singlet extension of the SM

3.1 Introduction

In the previous chapter we discussed how fermions acquire masses from the Higgs mechanism. We studied the SM scalar sector (i.e. the Higgs-sector), in which we discussed the stability of the scalar potential. In particular we saw, how the Higgs quartic coupling ($\lambda_h$) can become negative at high field values, $\Lambda$, which forced us to consider the metastable EW vacuum. Along this direction, follows a long queue of questions that SM fails to answer. These loopholes in the SM, such as no neutrino mass, no potential dark matter candidates, the hierarchy problem etc insinuates the existence of theories beyond Standard Model.

In the recent scientific achievements, one that invigorated the entire physics community was the discovery of the Higgs boson at LHC. However there are uncertainties associated with it being the SM Higgs boson, which requires the probe of the Higgs quartic interaction. Thus one is motivated to analyze this problem by considering a bigger picture.

In this chapter I will consider the simplest extension of the SM Higgs sector by adding a scalar singlet which transforms trivially under gauge group of the SM [10]. The singlet couples to the SM fields only via the Higgs, which is often referred to as Higgs portal [11],[12] and plays a role in inflation as is studied in [13]. In section [3.3] I will discuss the scalar singlet extension of the SM, where we will find that there exists two scalar particles, one possessing the characteristics of the SM Higgs boson and there we will refer to it as the "Higgs-like" while the other as "singlet-like". Following in the next section [3.4], I will discuss the potential stability and the perturbative constraints. We will see, how for different signs of the Higgs-singlet mixing parameter ($\lambda_{hs}$), different restrictive bounds arise to ensure vacuum stability. I will follow this line of reasoning
with some numerical analysis in section [3.5], where I will discuss the new terms added to the SM beta function of the Higgs quartic coupling because of the mixing between the scalar singlet and the Higgs. In its subsection [3.5.1], I will determine the allowed region between the mass eigenstate of the "singlet-like" particle and its mixing angle (sin $\theta$) and verify the shape of the region by presenting plausible arguments. In the final section [3.6], I will derive the decay width of the heavier state ($H_2 \rightarrow H_1 + H_1$) to the lighter state as it is kinematically allowed. Phenomenologically, this decay width is observable in experiments given we increase the center of mass energy of LHC or like detectors.

### 3.2 Scalar singlet interactions with the SM fields

Before I move on to the discussion of the scalar theory it is intuitive to understand the singlet interactions with the SM fields. Let me begin by writing the Lagrangian density in this model and impose an additional $U'(1)$ symmetry. The Lagrangian density with an additional global $U'(1)$ symmetry is given by:

$$L = L_{SM} + \partial_{\mu}S^\dagger \partial^\mu S - V(S, H)$$

where,

$$V(S, H) = m_s^2 S^\dagger S + \lambda_{hs}(S^\dagger S)(H^\dagger H) + \lambda_s(S^\dagger S)^2$$

$L_{SM}$ is the SM Lagrangian written in Appendix [A.2], ‘$S$’ is the scalar singlet, $\lambda_s$ is its quartic coupling and $\lambda_{hs}$ is the Higgs-singlet coupling. Clearly the new terms added in the SM Lagrangian terms are indeed globally $U'(1)$ invariant. So from previous discussions on types of symmetries one would be tempted to ask the question, what happens if I gauge this extra $U'(1)$ symmetry? At first glance the obvious conclusion one will reach is that we would get an extra gauge boson with a new coupling and that the partial derivative in the singlet kinetic term would change to a covariant derivative incorporating the gauge field to make the Lagrangian a gauge invariant quantity. That is indeed the case but it is not yet a complete one. The only fields that are charged under the addition of this extra $U'(1)$ symmetry are the SM fields and the singlet field. The Higgs field however doesn’t receive any $U'(1)$ charge. After performing the mathematical rigor, I get new terms in the Lagrangian:

$$L = L_{SM} + \Delta L$$
where,
\[ \Delta \mathcal{L} = g_1' B'_\mu \sum_i Q_i \bar{\psi}_{SM,i} \gamma^\mu \psi_{SM,i} + (D'_\mu S)^\dagger (D'^\mu S) \]
\[ - \frac{1}{4} F'^{\mu\nu} F'^{\mu\nu} - \frac{\epsilon}{2} F''^{\mu\nu} + \sum_i \bar{\chi}(i \partial_\mu + g_1' Q'_i B'_\mu) \gamma^\mu \chi \]
\[ (3.4) \]

where,
\[ D'_\mu = \partial_\mu - i g_1' Q'_i B'_\mu \]
\[ (3.5) \]

and,
\[ F'_\mu = \partial_\mu B'_\nu - \partial_\nu B'_\mu \]
\[ (3.6) \]

where in equation (3.4), \( B' \) is the new gauge boson, \( g_1' \) is its corresponding coupling constant, \( \psi_{i,SM} \) is the SM fermion with charge ‘Q’, the second term is the singlet kinetic term with covariant derivative as in equation (3.5), the third term is the gauge boson kinetic term with \( F'^{\mu\nu} \) given by equation (3.6), the fourth term is mixing between \( U(1)_Y \) and the new \( U'(1) \) and \( \bar{\chi} \) is the SM fermions. What we observe from equation (3.4) is that with an additional \( U'(1) \) symmetry the SM fields and the scalar singlet becomes charged. This analysis has found its application in the type-I seesaw mechanism [14] and cold dark matter [15]. I will not discuss this any further as I do not wish to digress from the primary topic of discussion.

### 3.3 The scalar sector of the singlet extension model

Let us consider an additional complex scalar field, which is a singlet under the \( SU(2) \) gauge group of the SM. I will denote the singlet field as simply, ‘\( S' \)’. The complete tree level scalar potential then consists of: quadratic terms of the singlet \( (S'^2) \), quadratic terms of the Higgs field \( (H^2) \), quartic (or self-coupling) terms \( (H^1 H)^2 \) and \( (S'^\dagger S)^2 \) and mixing between the Higgs doublet and the singlet, \( (H^1 S)(S'^\dagger S) \). In addition to the SM symmetry group, I will impose a \( Z_2 \) symmetry i.e. \( S \rightarrow -S \) such that odd power terms are excluded. Then I can write the scalar potential that consists of the Higgs doublet and the scalar singlet as:

\[ V(H, S) = m_h^2 (H^\dagger H) + m_s^2 (S'^\dagger S) + \lambda_h (H^1 H)^2 + \lambda_{hs} (H^1 H)(S'^\dagger S) + \lambda_s (S'^\dagger S)^2 \]
\[ (3.7) \]

In the above equation (3.7), \( m_h, m_s \) are just parameters with mass dimension one while \( \lambda_h, \lambda_{hs} \) and \( \lambda_s \) are coupling constants with mass dimension zero. The above potential being \( Z_2 \) symmetric excludes linear and cubic terms. Higher order terms are excluded to avoid large divergences. Studies that includes the linear terms have been carried out in [16] where they provide explanations for the possibility of a dark matter candidate. I will discuss this in Chapter 4.
Rewriting the potential in unitary gauge:

\[ V(h, s) = \frac{1}{2} m_h^2 h^2 + \frac{1}{2} m_s^2 s^2 + \frac{1}{4} \lambda_h h^4 + \frac{1}{4} \lambda_{hs} h^2 s^2 + \frac{1}{4} \lambda_s s^4 \]  

(3.8)

where,

\[ H = \frac{1}{\sqrt{2}} U(x) \begin{pmatrix} 0 \\ h(x) \end{pmatrix} \]  

(3.9)

and,

\[ S = \frac{1}{\sqrt{2}} V(x)s, \]  

(3.10)

where both \( U(x) \) and \( V(x) \) are unitary matrices. Let us demand that the potential (3.8) has a minimum at \(< h >= v \) and \(< s >= u \) where the minimizing conditions involve:

\[
\frac{\partial V(h, s)}{\partial h} \bigg|_{<h>=v,<s>=u} = 0 \\
\frac{\partial V(h, s)}{\partial s} \bigg|_{<h>=v,<s>=u} = 0
\]  

(3.11)

From the minimizing condition we get:

\[
m_h^2 + \frac{\lambda_{hs}}{2} u^2 + \lambda_h v^2 = 0 \\
m_s^2 + \frac{\lambda_{hs}}{2} v^2 + \lambda_s u^2 = 0
\]  

(3.12)

I can further solve this to write an expression for \( v^2 \) and \( u^2 \) in terms of the parameters in the potential (3.8). I simply get:

\[
v^2 = 2 \frac{\lambda_{hs} m_s^2 - 2 \lambda_s m_h^2}{4 \lambda_h \lambda_s - \lambda_{hs}^2} \\
u^2 = 2 \frac{\lambda_{hs} m_h^2 - 2 \lambda_h m_s^2}{4 \lambda_h \lambda_s - \lambda_{hs}^2}
\]  

(3.13)

I can then evaluate the Hessian matrix (or squared mass matrix) around the vacuum expectation value of the Higgs and the singlet field to find:

\[
M_{h,s}^2 = \begin{pmatrix} \frac{\partial^2 V(h, s)}{\partial h^2} & \frac{\partial^2 V(h, s)}{\partial h \partial s} \\ \frac{\partial^2 V(h, s)}{\partial s \partial h} & \frac{\partial^2 V(h, s)}{\partial s^2} \end{pmatrix} = \begin{pmatrix} 2 \lambda_h v^2 & \lambda_{hs} uv \\ \lambda_{hs} uv & 2 \lambda_s u^2 \end{pmatrix}
\]  

(3.14)

The Hessian matrix describes the local curvature of a function of multiple variables. If the Hessian (its determinant) at the point \(< h >= v, < s >= u \) is positive definite, then \( M_{h,s}^2 \) has attained a local minima. The Hessian then introduces a new constraint at the EW breaking scale:

\[ 4 \lambda_h \lambda_s - \lambda_{hs}^2 > 0 \]  

(3.15)
Thus, from equation (3.13), we infer the extremum is a local minimum if the following conditions are met:

\[
\begin{align*}
\lambda_{hs}m_s^2 - 2\lambda_s m_h^2 &> 0 \\
\lambda_{hs}m_h^2 - 2\lambda_h m_s^2 &> 0 \\
4\lambda_h\lambda_s - \lambda_{hs}^2 &> 0
\end{align*}
\]

(3.16)

I can then perform the Jacobi method or a 2-by-2 symmetric Schur’s decomposition to determine the mass eigenvalues and the corresponding mass eigenstates from the mass squared matrix (3.14). Physical parameters or observables corresponds to mass eigenstates and eigenvalues which is why we performed the transformation. The orthogonal transformation corresponding to the diagonilization of \(M_{h,s}^2\) matrix is given by:

\[
O = \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\]

(3.17)

where \(O^TM_{h,s}^2O = \text{diag}(m_1^2, m_2^2)\). The mass squared eigenvalues are then given by:

\[
\begin{align*}
m_1^2 &= \lambda_h v^2 + \lambda_s u^2 - \sqrt{(\lambda_s u^2 - \lambda_h v^2)^2 + \lambda_{hs}^2 u^2 v^2} \\
m_2^2 &= \lambda_h v^2 + \lambda_s u^2 + \sqrt{(\lambda_s u^2 + \lambda_h v^2)^2 + \lambda_{hs}^2 u^2 v^2}
\end{align*}
\]

(3.18) (3.19)

where we will consider \(m_1^2 < m_2^2\). The mass eigenstates of these light and heavier Higgs particle are related to the fields ‘h’ and ‘s’ via:

\[
\begin{pmatrix}
H_1 \\
H_2
\end{pmatrix} = \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}\begin{pmatrix}
h \\
s
\end{pmatrix}
\]

(3.20)

where, \(\theta\) is the mixing angle and is determined by:

\[
\tan(2\theta) = \frac{\lambda_{hs} uv}{\lambda_h v^2 - \lambda_s u^2}
\]

(3.21)

Thereafter expanding equation (3.20) and writing the mass eigenstates in terms of the fields ‘h’ and ‘s’, I get:

\[
\begin{align*}
H_1 &= h \cos \theta - s \sin \theta \\
H_2 &= h \sin \theta + s \cos \theta
\end{align*}
\]

(3.22)

Where \(H_1\) is either ”Higgs-like” or ”singlet-like”. Among the two, which eigenstate is heavier is dependent on the choice of our parameters. For instance, \(H_1\) is the lighter eigenstate or Higgs-like if I choose \(\lambda_h v^2 < \lambda_s u^2\) or \(H_1\) will be the heavier eigenstate or singlet-like \(\lambda_h v^2 > \lambda_s u^2\). It is just a matter of convention. From here onwards I will choose \(H_1\) as the Higgs-like by constraining the mixing angle \(|\theta| > \pi/4\). Next, let us
analyze the parameter $\lambda_h$ in large singlet vev.

### 3.3.1 Large vev of singlet ($u >> v$)

Joan Elias-Miro and et al. have studied the stabilization of EW vacuum via scalar threshold effect in their paper [17]. In their work, they have shown how the interaction between the heavy scalar singlet and the lighter Higgs doublet leads to a positive tree level threshold correction for the Higgs quartic coupling ($\lambda_h$). Quite similar analysis has also been worked out by Oleg Lebedev in his paper [12]. I will briefly discuss the idea and motivation behind it as it will be useful in the later sections. Under the high singlet vev limit, I can perform a binomial expansion of equation (3.18) and (3.19):

$$m_1^2 \simeq \lambda_h v^2 + \lambda_u u^2 - \lambda_u u^2 \left[ 1 + \frac{\lambda^2}{2\lambda_s^2} v^2 - \lambda_h \frac{1}{\lambda_s} u^2 \right] \simeq 2\lambda_h v^2 - \frac{\lambda_h^2}{2\lambda_s} v^2$$  \hspace{1cm} (3.23)

Similarly for $m_2^2$, I find:

$$m_2^2 \simeq 2\lambda_u u^2 + \frac{\lambda^2}{2\lambda_s} v^2$$  \hspace{1cm} (3.24)

and the mixing angle becomes:

$$\tan 2\theta \simeq -\frac{\lambda_h v}{\lambda_u u}$$  \hspace{1cm} (3.25)

I have suppressed $O\left(\frac{v^2}{u^2}\right)$ and higher order terms. I can infer from equation (3.23) that the mass of the Higgs-like is no longer $\sqrt{2\lambda_h v}$, as is expected from the SM, instead it gets a finite negative contribution from the Higgs-scalar coupling ($\lambda_{hs}$). Therefore, for the mass of the Higgs boson discovered at LHC ($\simeq 125$ GeV), the Higgs quartic coupling receives a slight positive contribution at the breaking scale i.e:

$$\lambda_h - \frac{\lambda_{hs}^2}{4\lambda_s} \simeq 0.13$$  \hspace{1cm} (3.26)

Thus as long as we keep $\frac{\lambda_{hs}^2}{4\lambda_s}$ significant and within the perturbation range, the Higgs quartic coupling ($\lambda_h$) can be made positive at all scales and the potential can be kept bounded from below. I will put a pause on the discussion of this limit for now as I shall revert back to discuss the significance of this point later on.

### 3.4 Boundedness from below and parameter constraints

In this section, I will discuss how fine-tuning the Higgs-singlet coupling can lead to potential stability up to large scales. We will consider the scale to run up to the Planck’s scale ($O(10^{19})$). Near Planck’s scale, gravity plays a crucial role and can no longer be
ignored in the theory. However, in our formulation we will ignore the effect of gravity. A detailed study of the boundary constraints placed on the parameters ($\lambda_h$, $\lambda_s$ etc.) at Planck’s scale for the scalar singlet model can be found in [18] while subsequent research in light of the Higgs boson discovered at LHC where the particle is indeed the SM Higgs, then its boundary conditions at Planck’s scale has been comprehensively studied at [19]. Working at the unitary gauge, from the scalar potential equation (3.8) we determined previously as to how at the breaking scale we require our parameters to satisfy the conditions (3.16) for the extremum to be a global minimum or in simple terms for the potential to be stable. These constraints are very restrictive and we haven’t considered the different signs of the parameter $\lambda_{hs}$, which is required to span the complete parameter space. Only by considering the two possibilities can we reach a conclusion on the type of constraints we want to execute on our parameters. It is important to note that even at this point I have only considered and worked on tree-level potential. I have made no arguments that should suggest otherwise.

At large field values, I have the liberty to consider only the quartic terms in the potential as they outrun the quadratic terms in terms of exponential growth considerably. Then I could write the quartic potential in unitary gauge as:

$$V_4(h, s) \approx \frac{1}{4} \lambda_h h^4 + \frac{1}{4} \lambda_{hs} h^2 s^2 + \frac{1}{4} \lambda_s s^4$$  \hspace{1cm} (3.27)

I could simplify the above equation as:

$$V_4(h, s) = \frac{1}{4} \left[ (\sqrt{\lambda_h} h^2)^2 + (\sqrt{\lambda_s} s^2)^2 - 2\sqrt{\lambda_h} \lambda_s h^2 s^2 + 2\sqrt{\lambda_h} \lambda_{hs} h^2 s^2 + \lambda_{hs} h^2 s^2 \right]$$ \hspace{1cm} (3.28)

Using the above equation (3.28), I will consider the different possibilities of $\lambda_{hs}$.

**Case 1:** $\lambda_{hs} > 0$

In this case the right hand side (R.H.S) of equation (3.28) tells us that the first term can never be negative as it is a squared function. The second term cannot be negative either. Thus the quartic potential at large field values is always positive and the only constraints we need to implement to maintain the stability of the potential at large field values are:

$$\lambda_h(\Lambda) > 0 \quad and \quad \lambda_s(\Lambda) > 0$$ \hspace{1cm} (3.29)

where, $\Lambda$ is the some arbitrary scale (energy scale or field scale) for which we study the running of the couplings.

**Case 2:** $\lambda_{hs} < 0$

Looking back to equation (3.28), I can infer that the first term is always positive however this time there is no requirement for the second term to be always positive at large field...
scale. Thus for negative $\lambda_{hs}$ values we would observe a run-away direction at large field values. Hence for the stability of the potential we also require the second term to be positive and thus impose another new constraint:

$$4\lambda_h \lambda_s - \lambda_{hs}^2 > 0 \quad \text{at all scales} \quad (3.30)$$

Recall we obtained this additional constraint at low energy scale (or EW breaking scale) as per equation (3.15).

Thus we find that depending on the sign of $\lambda_{hs}$, the constraints on the parameters to ensure vacuum stability, differs. The next section will clarify its importance.

### 3.5 Numerical Analysis

This section mainly consists of plots obtained through numerical analysis on Mathematica, that supports the arguments mentioned in aforementioned sections. We will see, how the introduction of the scalar singlet rectifies the SM beta function of the Higgs quartic coupling. New terms are introduced on the RG equations at one-loop level due to singlet coupling to the Higgs. The modified version of the running of Higgs coupling ($\beta_h$) and the singlet coupling ($\beta_s$) at one loop level is given by:

$$16\pi^2 \beta_{\lambda_h} = 24\lambda_h^2 - 6y_t^2 + \frac{3}{8}(2g^4 + (g^2 + g'^2)^2) + (-9g^2 - 3y_t^2 + 12y_t^2)\lambda_h + \frac{1}{2}\lambda_{hs}^2$$

$$\quad (3.31)$$

$$16\pi^2 \beta_{\lambda_{hs}} = 4\lambda_{hs}^2 + 12\lambda_h \lambda_{hs} - \frac{3}{2}(3g^3 + g'^2)\lambda_{hs} + 6y_t^2 \lambda_{hs} + 6\lambda_s \lambda_{hs}$$

$$16\pi^2 \beta_{\lambda_s} = 2\lambda_{hs}^2 + 18\lambda_s^2$$

where $\beta_{\lambda_{h,s,s}} = \frac{d\lambda_{h,s,s}}{dt}$ and $t = \ln(\Lambda/m_{top})$. Again $\Lambda$, is the scale on which the couplings are dependent. In the figure [3.1] and [3.2] below.

**Figure 3.1:** For initial value of $\lambda_h[m_{top}] = 0.16, \lambda_s[m_{top}] = 0.01$ and $\lambda_{hs}[m_{top}] = 0.015$ we find that the quartic coupling remains positive till Planck scale.

**Figure 3.2:** For initial value of $\lambda_h[m_{top}] = 0.13, \lambda_s[m_{top}] = 0.01$ and $\lambda_{hs}[m_{top}] = 0.015$ the quartic coupling becomes negative before Planck scale.
I have shown a plot for the above three couplings up to planck’s scale. Initial values for the couplings were taken at the mass of top quark:

\[
g[m_{\text{top}}] = 0.64 \quad g'[m_{\text{top}}] = 0.35 \quad g_s[m_{\text{top}}] = 1.16 \quad g_t[m_{\text{top}}] = 0.93 \tag{3.34}
\]

We can infer from the above figure [3.1] and [3.2] that the Higgs quartic coupling remains positive for the figure to the left at all scales while it inflects and becomes negative for the figure to the right. From equation (3.26), this becomes clear because of the extra contribution to the Higgs coupling that comes the second term. Thus for values of \( \lambda_{hs}^2/4\lambda_s \) higher than approximately 0.015, the Higgs quartic coupling remains positive at all scales (at least up to the Planck’s scale). The singlet’s quartic coupling (\( \lambda_s \)) and the mixed coupling between the Higgs-singlet (\( \lambda_{hs} \)) also remains positive at all scales if they are positive at low energy scales. \( \lambda_{hs} \) remains positive at all scales because of the positive contribution from the top Yukawa coupling \( (y_t) \). Thus it all bows down to fine-tuning \( \lambda_{hs} \) and \( \lambda_s \). One would be obviously curious that for a fixed \( \lambda_{hs} \), for what particular values of \( m_2 \), the heavy mass eigenvalue and the mixing angle (\( \sin \theta \)) will the Higgs quartic coupling remain positive at all scales. Note, I have fixed the values of \( \lambda_{hs} \) because the parameter space \( (m_2 - \sin \theta) \) can be constrained from the data obtained from LHC and LEP. The constraint on \( \lambda_{hs} \) is very poor from the LHC data hence we keep it fixed at small values for our analysis. Why the smallness? Will become clear in the following sections. This is the focus point of the next section.

### 3.5.1 Allowed mass region of the singlet for different mixing angle

At the breaking scale, the parameters of the quartic coupling (\( \lambda_h \) and \( \lambda_s \)) can be expressed completely in terms of five independent parameters:

\[
\begin{align*}
\lambda_h &= \lambda_h(m_1, m_2, v, \lambda_{hs}, \theta) \\
\lambda_s &= \lambda_s(m_1, m_2, v, \lambda_{hs}, \theta).
\end{align*} \tag{3.35}
\]

Using equation (3.18), (3.19) and (3.21), I can easily form an equation that expresses \( \lambda_h \) and \( \lambda_s \) as in equation (3.35). I have performed the detailed calculation in Appendix [B.1]. After the mathematical rigor, I get the expressions:

\[
\begin{align*}
\lambda_h &= \frac{m_1^2}{2 v^2} + \sin^2 \theta \left( \frac{m_2^2 - m_1^2}{2 v^2} \right) \tag{3.36} \\
\lambda_s &= \frac{2 \lambda_{hs}^2}{\sin^2 2\theta} \left( \frac{v^2}{m_2^2 - m_1^2} \right) \left( \frac{m_2^2}{m_2^2 - m_1^2} - \sin^2 \theta \right).
\end{align*} \tag{3.37}
\]

Note the above two equations (3.36) and (3.37) are evaluated at the EW breaking scale which I have considered to be \( m_{\text{top}} \). The known parameters in the above equations are
$m_1 = 125.6 \text{ GeV}$ and $\nu = 246 \text{ GeV}$ (vacuum expectation value of the Higgs field),
while of the three unknowns I can fix $\lambda_{hs}$ at a particular value to determine the region
between $m_2$ and $\sin \theta$. The algorithm used for this analysis is:

- Fix the value of $\lambda_{hs}$ (can be positive or negative). For negative values an additional
  constraint (3.30) is to be imposed.
- Randomly generate values of $m_2$ and $\sin \theta$ simultaneously.
- Input these values in the equations (3.36) and (3.37) for which I get an initial value
  of $\lambda_h$ and $\lambda_s$
- Using the above initial value (IV) and the IV’s from equation (3.34) I solved the
coupled differential equation via Mathematica.
- Apply the stability conditions and if the quartic couplings are positive at all scales,
  register the corresponding value of $m_2$ and $\sin \theta$
- Finally plot the region for such values of $m_2$ and $\sin \theta$.

To simulate the above algorithm I have used Mathematica. Mathematica provides a
simple platform where one can solve ample of coupled differential equations with specified
initial values. I have used the format of Module to achieve this goal. In Module, I
deﬁned my coupled differential equations keeping the initial conditions (3.36) and (3.37)
as variables. To achieve the allowed region plot, I used another simulating function
‘RegionPlot’ available at Mathematica. In RegionPlot, I provided a range of values
for $m_2$ and $\sin \theta$ and imposed the stability conditions on the quartic couplings to get
the expected plot. The stability condition are imposed depending on the sign of $\lambda_{hs}$
as discussed in section [3.4] and equations (3.29) and (3.30). However, due to the
limitations in Mathematica, RegionPlot is only able to evaluate the imposed conditions
at a certain point hence I had to impose additional checks at intermediate scales of
order $O(10^{10})$, $O(10^{12})$ upto Planck’s scale ($O(10^{19})$) to ensure that the beta function
of quartic coupling stays positive throughout. These intermediate conditions takes the
form:

$$\lambda_{h,s}[10^{10}GeV] > 0 \quad \lambda_{h,s}[10^{12}GeV] > 0 \quad \lambda_{h,s}[10^{16}GeV] > 0 \quad \text{etc.} \quad (3.38)$$

Of course we want perturbation theory to hold at all scales, else the above conditions
hids the physics pertaining to the model. In short, the additional conditions imposed
on the quartic couplings to validate perturbation theory are:

**Case 1:** $\lambda_{hs} > 0$

$$0 < \lambda_h[m_{top}] < 1, \quad 0 < \lambda_s[m_{top}] < 1,$$
$$0 < \lambda_h[M_{planck}] < 1, \quad 0 < \lambda_s[M_{planck}] < 1 \quad (3.39)$$
Often you will notice the imposition of $4\pi$ [20] instead of 1 at the above equation. This arises from the coefficient of the feynman diagrams at one loop order. However, that is not of pivotal importance to our problem since we are concerned with the shape of the allowed region.

**Case 2:** $\lambda_{hs} < 0$ Under this scenario along with the above constraints in equation (3.39), we impose the additional condition (3.30). In the figure below I have shown two plots corresponding to the both cases discussed above:

![Figure 3.3: Left: Allowed region of $m_2$ and $\sin \theta$ for positive $\lambda_{hs}$](image1)

![Figure 3.4: Right: Allowed region of $m_2$ and $\sin \theta$ for negative values of $\lambda_{hs}$](image2)

**Plot Analysis:** In the above diagrams I have shown how for different signs of $\lambda_{hs}$, I obtain different allowed region between $m_2$ and $\sin \theta$. This is due to additional constraint (3.30) that needs to imposed for negative $\lambda_{hs}$ values. The plot analysis then follows:

- **Left[3.3]:** The plot’s behavior appears to be dominated for the initial condition of $\lambda_h$ from equation (3.36), since as discussed in the following publications [12],[13],[17],[20],[21], the values of $\lambda_s$ are required to be small to keep the Higgs quartic coupling positive at all scales. The perturbative constraint:

$$0 < \lambda_h |m_{top}| < 1$$

segments the allowed region at the upper convex surface rendering it to behave partly like a “hyperbolic” function. The partly “hyperbolic” behavior of the plot arises due to $\sin^2 \theta \times m_2^2$. A closer look tells us that for very high $m_2$ ($m_2 >> m_1$) the value of $\sin \theta$ has to be quite small for $\lambda_h$ to lie in the perturbative region and vice-versa but with $m_2 > m_1$. This condition strongly limits the range of values $\lambda_s[m_{top}]$ can take. For values of $m_2 < m_1$, it is observed that $\lambda_h |m_{top}|$ take values that renders the Higgs quartic coupling ($\lambda_h$) to become negative at high energy scales. This can be enlightened
from the section [3.3], where the SM Higgs coupling at the breaking scale is modified to:

\[
\lambda_h \bigg|_{SM+\text{singlet}} = \frac{m_{higgs}^2}{2v^2} + \frac{\lambda_s^2}{4\lambda_s} \tag{3.41}
\]

Once the perturbative conditions are met, the running of the coupling constants are further imposed conditions at Planck’s scale as in equation (3.39) along with conditions (3.38). This delimits the allowed region. The boundary at the upper convex surface of the plot [3.3] and [3.4] violates the condition:

\[
\lambda_h[M_{\text{planck}}] < 1 \tag{3.42}
\]

giving the upper region the shape of a “hyperbola”.

- **Right[3.4]**: The boundary of the lower convex surface of the ”hyperbolic-like” plot violates the condition:

\[
\lambda_h[M_{pl}] > 0 \tag{3.43}
\]

To understand the lower convex region, the behavior is best depicted when the coupling satisfies the conditions (3.41) and (3.26) since not for all positive values of \(\lambda_h[m_{top}]\) does the Higgs quartic coupling remain positive at all scales. The white (or empty) region below the lower convex surface corresponds to lower values of \(m_2\) and \(\sin \theta\). These values do not obey the equation (3.26) resulting in the violation of boundary condition (3.43). Oleg lebedev in his paper [12] has shown that for \(\lambda_s^2/(4\lambda_s) \gtrsim 0.015\), the Higgs quartic coupling can be kept positive at all scales. As one increases \(m_2\) and decreases \(\sin \theta\) (note: \(\lambda_s[m_{top}]\) can increase significantly) the coupling at the breaking scale are strictly constrained by (3.26),(3.41) and \(\lambda_s[m_{top}]\) because the possibility of the quartic couplings to become negative increases even though \(0 < \lambda_h[m_{top}] < 1\) is satisfied. Hence the region becomes narrower. At low values of \(m_2\) and high values of \(\sin \theta\), the regions starts contracting because \(\lambda_h[m_{top}]\) would be small and we would observe the violation of boundary condition (3.43). Thus the partly ”hyperbolic-like” behavior at both convex surfaces (upper and lower) is seen due to the dominating effect of \(\lambda_h[m_{top}]\) over \(\lambda_s[m_{top}]\) which are then further constrained by imposing conditions at Planck and intermediate scales giving us a region that looks like a ”boomerang”. Finally I have obtained the largest value of \(\lambda_{hs}\) for which we still get a minuscule region between \(m_2\) and \(\sin \theta\). The subsequent value of \(\lambda_{hs}\) is found out to be 0.23 [Note that this value includes the limitations of Mathematica]. Thus I have analyzed the parameter space \((m_2 \text{ vs } \sin \theta)\) of the singlet extension model which are then subjected to experimental constraints providing much better resolution.
3.6 Partial decay width of the heavier state to two lighter states

Since the singlet is quite heavy compared to the Higgs, perhaps we could study it’s possible decay channel, \( H_2 \rightarrow H_1 H_1 \). Let us begin by evaluating the partial decay width, \( \Gamma(H_2 \rightarrow H_1 H_1) \). I need to rewrite the scalar potential (3.8) around the minimum i.e replace \( h(x) \) with \( h(x)+v \) and \( s(x) \) with \( s(x)+u \) to obtain:

\[
V(h,s) = \frac{m_h^2}{2}(h(x)+v)^2 + \frac{m_s^2}{2}(s(x)+u)^2 + \frac{\lambda}{4}(h(x)+v)^4 \\
+ \frac{\lambda_{hs}}{4}(h(x)+v)^2(s(x)+u)^2 + \frac{\lambda_s}{4}(s(x)+u)^4
\] (3.44)

In the above equation (3.44) I need to apply the minimizing conditions (3.12) and simplify the potential. Thereafter, since I formulated the existence of two mass eigenstates \( (H_1 \text{ and } H_2) \) as in equation (3.22), I invert them and write them in terms of \( h(x) \) and \( s(x) \), i.e:

\[
h = H_1 \cos \theta + H_2 \sin \theta \\
s = -H_1 \sin \theta + H_2 \cos \theta
\] (3.45, 3.46)

Substituting them back in equation (3.44) and collecting only the terms that contribute to the decay process, I get:

\[
\Delta V_{H_2\rightarrow H_1 H_1} = \lambda' H_2 H_1 H_1
\] (3.47)

where \( \lambda' \) is the coupling associated with the decay process. In my notation:

\[
\lambda' = \cos^2 \theta \sin \theta v[\lambda_{hs} - 3\lambda_h] - \cos \theta \sin^2 \theta u(3\lambda_s - \lambda_{hs}) - \frac{\lambda_{hs}}{2} [u \cos^3 \theta - v \sin^3 \theta]
\] (3.48)

Since it’s a \( 1\rightarrow 2 \) decay process and the Feynman amplitude is simply a constant term, all I need to do is evaluate the Phase-space integral \( (R_2(p, m_1^2, m_2^2)) \) where,

\[
R_2(p, m_1^2, m_2^2) = \int \frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} \delta^4(p - (p_1 + p_2))
\] (3.49)

where, \( p \) is the momentum of initial particle and \( p_i \) are momentum of final state particles. After evaluating the phase-space integral I get:

\[
R_2(s) = \pi \sqrt{\frac{\lambda(s, m_1^2, m_2^2)}{2s}}
\] (3.50)
where, ‘s’ is the mandestam variable and $\lambda(s, m_1^2, m_2^2)$ is the kinematical function defined by:

$$\lambda(p^2, p_1^2, p_2^2) = [p^2 - (\sqrt{p_1^2} + \sqrt{p_2^2})^2][p^2 - (\sqrt{p_1^2} - \sqrt{p_2^2})^2]. \quad (3.51)$$

The condition for the decay process to be physical requires:

$$\lambda(p^2, p_1^2, p_2^2) \geq 0 \quad (3.52)$$

Assuming all four-vector are timelike, the above condition (3.52) becomes:

$$\sqrt{p^2} \geq m_1 + m_2 = \text{threshold} \quad (3.53)$$

which is a natural condition for decay. Then the partial decay width is:

$$\Gamma(H_2 \rightarrow H_1 H_1) = \frac{|\lambda'|^2}{8\pi m_2} \sqrt{1 - \frac{4m_1^2}{m_2^2}} \quad (3.54)$$

with $\lambda'$ given by equation (3.48). We could simplify the above equation and redefine our coupling constant ($\lambda'$) as in [2]. In that case the partial decay width is:

$$\Gamma(H_2 \rightarrow H_1 H_1) = \frac{\sin^2 \theta \lambda_{211}^2 v^2}{32\pi m_2^2} \sqrt{1 - \frac{4m_1^2}{m_2^2}} \quad (3.55)$$

where $\lambda_{211}$ is the new coupling corresponding to the above decay channel and is defined by:

$$\lambda_{211} = \frac{2m_1^2 + m_2^2}{v^2} \left( \cos^2 \theta + \frac{\lambda_{hh} v^2}{m_2^2 - m_1^2} \right) \quad (3.56)$$

### 3.7 Summary

Following the metastability problem discussed in the previous chapter I provided a plausible solution to the issue by introducing a scalar singlet field. The singlet can couple to the Higgs boson via the Higgs portal. I found that due to its coupling to the Higgs boson and non zero vev, there exists two mass eigenstates which corresponds to two scalars. I considered the lighter mass eigenstate to be the SM Higgs boson and analyzed the boundedness from below conditions that ensures stability of the tree level potential. Further, I analyzed the parameter space ($m_2 vs \sin \theta$) of the extension model. Through numerical analysis I determined the acceptable region of the parameter space ensuring EW vacuum stability. Lastly, I evaluated the partial decay width of the heavier mass eigenstate to two lighter eigenstates, i.e. $H_2 \rightarrow H_1 H_1$, as it is kinematically allowed. Next chapter I will discuss the experimental constraints on the parameter space.
Chapter 4

Experimental Constraints

A theory is only hypothetical until experimental measurements validate it’s existence. Any given theory has to comply with experiments for it to be considered viable, needless to say under acceptable error, where the errors may arise from statistical analysis or detectors etc. The same applies to the simplest extension of Standard Model where we introduce an extra degree of freedom, the scalar singlet. We will see in the next chapter how this singlet explains some of the delicate problems such as a dark matter candidate, unitarity issue, etc. that the SM fails to elucidate. Nonetheless it’s (the SM) is still a good theory at low energy scale as proven by the precision of fine structure constant, \(\alpha\). Thus it is imperative that the singlet extension parameters are subjected to the constraints obtained from experiments.

The scalar potential (5.11) has eight independent parameters as can be determined from the equation below:

\[
V(S, H) = \frac{m_h^2}{2} H^\dagger H + \frac{m_s^2}{2} S^2 + \frac{\lambda_s}{4} S^4 + \frac{\lambda_h}{4} (H^\dagger H)^2 \\
+ \frac{\lambda_{hs}}{4} S^2 H^\dagger H + \frac{\lambda_1}{2} H^\dagger H S + \frac{\lambda_2}{2} S + \frac{\lambda_3}{3} S^3
\]  

Upon applying the \(Z_2\) and \(U(1)\) symmetry we are left with five independent parameters \((m_h^2, m_s^2, \lambda_h, \lambda_{hs}, \lambda_s)\). However, few of the above parameters are unphysical hence we immediately switch to mass eigenstates after spontaneous symmetry breaking (SSB) to lie in the physical regime. Finally we are left with five free parameters:

\[
\lambda_h, \lambda_s, \lambda_{hs}, v, u.
\]  

At the EW breaking scale, the first two parameters of the above equation (4.2) are related to the masses of the scalar particles, \(\lambda_{hs}\) and their mixing angle \(\theta\) (B.16), (B.17). We will study the constraints imposed on these (4.2) parameters by:
• The Electroweak precision observable (EWPO)
• The Higgs quartic coupling measurement at LHC
• The Light Higgs signal at LHC
• The vacuum stability constraints

Being a review, I will closely follow the works [2], [20], [22]. The authors have used different methods to address the experimental constraints imposed on the free parameters from the collider experiments such as, LHC and LEP. We center this chapter around the work of Adam Falkowski and et al [2], whose work will be in this chapter.

4.1 Constraints imposed by EWPO

The parameters or observables that one associates with EWPO are namely:

\begin{align}
\alpha &\rightarrow \text{fine structure constant} \\
G_F &\rightarrow \text{Fermi Constant} \\
m_Z &\rightarrow \text{Mass of } Z \text{ boson} \\
m_W &\rightarrow \text{Mass of } W^\pm \text{ boson} \\
s_{\text{eff}}^2 &\rightarrow \text{effective mixing (Weinberg) angle} \\
\Gamma_{l^+l^-} &\rightarrow \text{leptonic partial decay width of the } Z \text{ boson}
\end{align}

We know that the Lagrangian of the GWS theory treats particles will different helicity states differently. In fact the coupling of the $Z^0$ boson to the left- and right-handed fermions differ quite significantly. This results in a polarization assymetry ($A_{LR}$) produced in the decay channel $Z^0 \rightarrow jj$ or vice-versa. The value of $s_{\text{eff}}^2$ (effective $\sin^2 \theta_W$) is then defined as per the observed asymmetry:

\begin{equation}
A_{LR} \equiv \frac{(1/2 - s_{\text{eff}}^2)^2 - s_{\text{eff}}^4}{(1/2 - s_{\text{eff}}^2)^2 + s_{\text{eff}}^4}
\end{equation}

It is straight forward to compute the values of the above parameters (4.3), at least at tree level. However, the resulting deviation observed from tree-level analysis for $m_W$, $s_{\text{eff}}^2$ and $\Gamma_{l^+l^-}$ are statistically unacceptable (with deviations as large as 15$\sigma$, 120$\sigma$ and 10$\sigma$). It then becomes imperative to include higher order corrections at least to one loop order. At one loop order the corrections arise from the self-energy diagrams of the $\gamma$, $W^\pm$ and $Z$ vector bosons. The non-oblique corrections (such as box-diagrams or vertex corrections) have only miniscule effect compared to the oblique correction (vacuum
polarization corrections) hence the former is usually excluded in the analysis. This can be elucidated by an appropriate reasoning where we understand that many charged particle couples to the vector bosons while only few particles (one or two) in the theory couples to a specific fermion.

In the singlet extension model, I will consider the oblique corrections and ignore the corrections from the non-oblique ones. The propagators of the vector bosons ($W$ and $Z$) receives two contributions from:

- An additional singlet ($H_2$) due to the mixing of the two scalars, and
- the modification of the scalar ($H_1$) coupling to the gauge bosons.

As we restrict ourselves to one loop contributions, the sum of all 1-particle irreducible (1PI) diagrams will be denoted by $\Pi^{\mu\nu}(q)$, with ‘$q$’ being the loop momentum. Then the one-loop corrections to the vector boson self-energies takes the form:

$$i\Pi_{VV}^{\mu\nu}(q) = i[\Pi_{VV}(q^2)g^{\mu\nu} - \Delta_{VV}(q^2)q^\mu q^\nu]$$ (4.5)

However, I shall constrict myself to the self-energy contribution ($\Pi_{VV}(q^2)$) only since the second term in the above equation (4.5) when dotted with the fermion current gives zero from the Dirac equation.

$$q^\mu J_\mu \rightarrow \bar{f}\gamma^\mu q_\mu f \rightarrow \bar{f}m f \rightarrow 0$$ (4.6)

An important point to note is that in QED while we study renormalization of electric charge, we encounter such vacuum polarization diagrams and we find that the 1PI looks quite similar to equation (4.5) where to avoid violation of Ward-identity we use dimensional regularization which then simplifies equation (4.5).

Similarly, using dimensional regularization, we can find the shift of the propagator function w.r.t the SM:

$$\delta\Pi_{VV}(q^2) = \frac{m_V^2 \sin^2 \theta}{4\pi^2 v^2} \left[ \frac{m_{H_2}^2 - m_{H_1}^2}{4} \left( \frac{1}{\epsilon} + 1 \right) + F(q^2, m_V^2, m_{H_1}^2) - F(p^2, m_V^2, m_{H_1}^2) \right]$$ (4.7)
where,

\[ F(q^2, m_V^2, m_{H_1,H_2}^2) = \int_0^1 dx \left[ m_V^2 - \frac{\Delta}{2} \right] \log \Delta, \quad \text{and} \]

\[ \Delta = x m_{H_1,H_2}^2 + (1-x)m_V^2 - q^2 x(1-x) \]

with ‘x’ resembling the parameter from feynman parametrization and \( \epsilon \) is the divergence term which arises after the expansion of Gamma function near the limit where space-time dimension closes to 4. \( F(q^2, m_V^2, m_{H_1,H_2}^2) \) is the loop integral contribution after feynman parametrization. The reason behind analyzing the propagator corrections is because they contribute to the vector bosons mass i.e \( m_V^2 \rightarrow m_V^2 + \Pi_{VV}(q^2 = m_V^2) \).

The observables used in the fit are the LEP-1 Z-pole observable [23], the W mass [24], the total width [25] and the hadronic width [26], as can be found in Table [C.1] mentioned in the Appendix [C]. We can determine the partial decay widths of the vector bosons (Z and W) that appear in the table:

\[ \Gamma(Z \rightarrow f\bar{f}) = \frac{N_f m_Z}{24\pi} g_{fZ; eff}^2 \]
\[ \Gamma(W \rightarrow f\bar{f}') = \frac{N_f m_w}{48\pi} g_{fW; eff}^2 \]

where \( N_f \) is the number of colors of the fermion ‘f’ and the effective couplings are defined as

\[ g_{fZ; eff} = \sqrt{\frac{g_L^2 + g_Y^2}{1 - \delta \Pi_{ZZ}(m_Z^2)}} \left[ T_f^3 - Q_f s_{eff}^2 \right] \]
\[ s_{eff}^2 = \frac{g_Y^2}{g_L^2 + g_Y^2} \left( 1 - \frac{g_L}{g_Y} \frac{\delta \Pi_{ZZ}(m_Z^2)}{m_Z^2} \right) \]
\[ g_{fW; eff} = g_{W; eff} = \frac{g_L}{\sqrt{1 - \delta \Pi_{WW}(m_W^2)}} \]

\( g_L \) and \( g_Y \) are the \( SU(2) \) and \( U(1) \) gauge couplings. In this model, \( \delta \Pi_{\gamma Z} \) and \( \delta \Pi_{\gamma\gamma} \) are both zero at one loop level. This is precisely because the photon is massless, \( \Pi_{\gamma\gamma}(0) = 0 \) and \( \Pi_{\gamma Z}(0) = 0 \). However it is to be noted that \( \Pi_{\gamma Z}(0) \) is non-zero in the case when the \( W^\pm \) is included in the loop calculations. The electroweak parameters \( g_L, g_Y, v \) which are evaluated from the input observables \( G_F, \alpha \) and \( m_Z \) receives finite correction at the one-loop order in the singlet-extension model given by:

\[ \frac{\delta g_L}{g_L} = \frac{1}{g_L^2 - g_Y^2} \left( \frac{2 \delta \Pi_{WW}(0)}{v^2} - 2 \cos^2 \theta_w \frac{\delta \Pi_{ZZ}(m_Z^2)}{v^2 + \frac{g_L^2}{2}} \delta \Pi_{\gamma\gamma}(0) \right), \]
\[ \frac{\delta g_Y}{g_Y} = \frac{1}{g_L^2 - g_Y^2} \left( -2 \frac{\delta \Pi_{WW}(0)}{g_L^2} \frac{v^2}{v^2} + 2 \sin^2 \theta_w \frac{\delta \Pi_{ZZ}(m_Z^2)}{v^2 - \frac{g_y^2}{2}} \delta \Pi_{\gamma\gamma}(0) \right), \]
\[ \frac{\delta v}{v} = -\frac{2 \delta \Pi_{WW}(0)}{g_L^2 v^2} \]
Using the above equation (4.11) and equation (4.10) and (4.9), we could evaluate the shift in the effective couplings due to the oblique corrections and thereafter compute the corrections to the precision observables. The simplest approach is to take the linear term of $\delta \Pi_{VV}$ and the values of EWPO from table [C.1] to construct a $\chi^2$ function that depends on $m_{H_2}$ and $\sin \theta$ and known SM parameters. The $\chi^2$ is the standard way to test the ability of a given theory to reproduce data. One can visualize this as, given a set of observables, say $O_i^{\text{expt}}$, which are determined from experiments are subjected to uncertainties arising due to inhomogeneity in the detector, data collection inefficiency etc. Let us call these uncertainties $\Delta O_i^{\text{expt}}$. Let us define the theoretical predictions for these observables as $O_i^{\text{th}}$ that depend on the parameters we define in our Lagrangian.

The best possible choices of the Lagrangian parameters ($\lambda_i$) is determined by fitting the data by minimizing the $\chi^2$ function:

$$\chi^2(\lambda_i) = \sum_i \frac{(O_i^{\text{expt}} - O_i^{\text{th}}(\lambda_i))^2}{(\Delta O_i^{\text{expt}})^2}$$  \hspace{1cm} (4.12)

Hence we minimize $\chi^2(m_{H_2}, \sin \theta)$ w.r.t $\sin \theta$ for each value of $m_{H_2}$ and determine the 95% confidence level (CL) limits by solving

$$\chi^2(m_{H_2}, \sin \theta) - \min_{\theta} \{\chi^2(m_{H_2}, \sin \theta)\} = 3.84$$  \hspace{1cm} (4.13)

In the figure [4.3], [4.4] we observe that for $m_{H_2} \lesssim 60$ GeV and $m_{H_2} \gtrsim 170$ GeV the limits are non-trivial and keeps becoming stronger as $H_2$ keeps increasing. For $m_{H_2} \gtrsim 450$ GeV, the electroweak precision constraints provide the strongest limits on the singlet model.

G.M Pruna and et al. have performed this analysis with a different approach. In their work [20] the constraints on the electroweak precession data are imposed using the S,T,U parameters [27], [28]. They found that the maximally allowed heavy Higgs mass are of the order $\mathcal{O}(35 \text{ TeV})$ for small mixing angles and using these mixing angles the EWPO gives additional constraints in the large mixing regions for $m_{H_2, \text{max}} \leq 1 \text{ TeV} \ (2 \text{ TeV})$, $|\sin \theta| \lesssim 0.6 \ (0.5)$. For $m_{H_2} \geq 2 \text{ TeV}$, the EWPO gave no additional constraint. However, Oleg Lebedev and et al’s formulation [2] works for any values of $m_{H_2}$ including the case for $m_{H_2} \ll m_{H_1}$. 
4.2 Constraints arising from the Higgs quartic coupling measurements at LHC

I have previously shown in chapter 3 that due to the mixing of the scalar singlet to the Higgs, the SM Higgs coupling receives finite correction \( (3.41) \) at the EW breaking scale. This correction later turns out to provide the necessary initial condition for the running \( (\beta_h) \) of the Higgs quartic coupling to remain positive ensuring vacuum stability. Note, the beta function of the Higgs quartic coupling also receives an additional contribution from \( \lambda_{hs} \) at one loop order due to its mixing with the singlet. The discovery of the Higgs boson at the LHC then provides additional constraint on the parameter space. We can determine these constraints from the Higgs mass obtained from two channels at LHC,

\[
\begin{align*}
H_1 & \rightarrow \gamma\gamma \\
H_1 & \rightarrow ZZ^* \rightarrow 4l
\end{align*}
\]  

The above two channels has provided the best mass resolution of the Higgs boson at LHC, hence it’s usually preferred over others. The above decay processes can be subjected to contamination from the heavier Higgs \( (H_2) \) decay. At this stage, it is plausible to consider \( m_{H_2} \) lying outside the region \([120,130]\) GeV for which we can ignore the contamination, as suggested from ATLAS and CMS Higgs searches in the above two channels. If \( m_{H_2} \in \[120,130]\) GeV interval is considered then the limits on the value of \( \sin \theta \) requires a much more elaborate analysis which incorporates different mass resolutions for various \( h \rightarrow \gamma\gamma \) and \( h \rightarrow 4l \). The signal strength of 125 GeV scalar boson observed at CMS and ATLAS is given in the table below:

<table>
<thead>
<tr>
<th>Channel</th>
<th>( \mu ) (ATLAS)</th>
<th>( \mu ) (CMS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_1 \rightarrow \gamma\gamma )</td>
<td>( 1.17^{+0.27}_{-0.27} ) [29]</td>
<td>( 1.12^{+0.24}_{-0.24} ) [30]</td>
</tr>
<tr>
<td>( H_1 \rightarrow ZZ^* \rightarrow 4l )</td>
<td>( 1.44^{+0.40}_{-0.33} ) [31]</td>
<td>( 1.00^{+0.29}_{-0.29} ) [30]</td>
</tr>
</tbody>
</table>

Table 4.1: The signal strength of the 125 GeV boson from LHC for the two decay channels [2]

Including the 15% threshold uncertainty in the Higgs production cross section which is the linear sum of the partial distribution function (PDF) and the QCD errors on the gluon fusion cross section [32] into a Gaussian-modeled nuisance parameter we get the combined Higgs signal strength

\[
\mu > 0.81, \text{ @95\% CL}
\]  

(4.15)
While working outside the range, i.e for $m_{H_2} \notin [120,130]$ GeV we get different bounds on $\sin \theta$.

For $m_{H_2} \geq m_{H_1}/2 \sim 62.5$ GeV, the bound on $\sin \theta$ is found to be:

$$\sin \theta < 0.44, \text{ @95\% CL}$$  \hspace{1cm} (4.16)

which is independent of $m_{H_2}$ while for $m_{H_2} < m_{H_1}/2$, the inclusion of the decay channel $H_1 \rightarrow H_2H_2$ makes the situation slightly complicated because the signal strength would tend to decrease. For the latter case stronger contraints are imposed on $\sin \theta$ due to its slight dependence on $m_{H_2}$ and larger dependence on $\lambda_{hs}$. We can understand the scenario from the figure below:

For large values of $|\lambda_{hs}|$ the limits are stronger and at some point almost the entire $m_{H_2} - \sin \theta$ plane gets excluded. From the above fig [4.1] and [4.2], we can infer that for low values of $m_{H_2}$ the values of $\lambda_{hs}$ are restricted to $\lambda_{hs} < 0.015$. However for given values of $m_{H_2}$ and $\sin \theta$, one can adjust a negative value of $\lambda_{hs}$ to get rid of the $H_1H_2^2$ coupling. In this case, the equation (4.16) gives a good limit.
4.3 Constraint from the Light Higgs signal at LHC

The 125 GeV Higgs signal from the LHC provides constraints to the parameter space. From what we understand, we should observe a reduction of the Higgs couplings to the SM fields due to the mixing. By narrow width analysis we can determine the reduction factor to be $\cos^2 \theta$. V.M. Lozano and et al. [22] derived the bound on the mixing: $\sin^2 \theta < 0.076$ (0.128) at 90% (95%) C.L. However, I will provide a detailed review of Adam Falkowski and et al. [2] who have worked out the constraints at both low and high mass regime. In their work they have used constraints imposed by LEP and LHC data on the Higgs-like scalar via:

- Searches for $H_2 \rightarrow \gamma\gamma$ in ATLAS [33] and CMS [34]
- Higgs boson production in four lepton channel ($H_2 \rightarrow ZZ$) in ATLAS [31] and CMS [35]
- Searches for $H_2 \rightarrow WW$ in $e\mu2\nu$ channel in ATLAS [36]
- $H_2 \rightarrow H_1H_1$ searches in CMS with the $2b2\gamma$ [37] and $4b$ [38] final states and in ATLAS with the $2b2\gamma$ final state [39]
- Search for the SM Higgs boson in LEP [40] dominated by the $b\bar{b}$ decay channel
- DELPHI search for a low mass Higgs in Z-decays [41]
- b-physics constraints on a low mass Higgs [42],[43],[44]

They determined that at low mass regime, i.e. for $m_{H_2} < 5$ GeV, the strongest constraint imposed on the parameter space is via the $B \rightarrow Kll$ decays [42], [43]. The bound on values of $\sin \theta$ were extracted from the analysis [45]. They found the values of the $\sin \theta$ to be less than $10^{-2} - 10^{-3}$, $\sin \theta < 10^{-2} - 10^{-3}$. Between the mass regime 5 GeV $< m_{H_2} < 10$ GeV, the corresponding bound on $\sin \theta$ was found to be $\sin \theta \lesssim 0.5$ which is imposed from the radiative $\Upsilon$ decays [44] and DELPHI searches for a light Higgs in Z-decays [41]. From 10 GeV up to 115 GeV the constraint on $\sin \theta$ values are imposed from the LEP Higgs searches [40]. They validated that depending on the exact mass of the $H_2$ (the second or heavier Higgs), the range of values for $\sin \theta$ is about $1 \times 10^{-1}$ to few $10^{-1}$. Between the energy range 120 GeV and 130 GeV, the constraints on the parameters are poor. This is mainly because of the presence of the SM-like Higgs. One can observe a large mixing between the two Higgses in this regime making it difficult to impose viable constraint on the parameter space. One could study the constraints on a minuscle region about the interval, above 130 GeV and below 120 GeV. In this strip region the constraints are slightly loosened on the parameter space as evaluated from
diphoton channels [33],[34]. Above 130 GeV to 450 GeV, the limits are dominated by the 4l channels [31],[35]. This values of $\sin \theta$ are found to be between 0.3-0.4. Above this range of mass parameter ($m_{H_2}$) indirect bounds are consistently stronger as studied in [20],[22]. We have largely talked about the constraints imposed on the parameter space ($m_{H_2}, \sin \theta$) from the various experiments however it’s also imperative to understand the role of $\lambda_{hs}$ on the exclusion limits. For $m_{H_2} \leq m_{H_1}/2$, the Higgs decay process $H_1 \rightarrow H_2 H_2$ would be important as it can mix with the Higgs signal strength. This is shown in figure [4.3], [4.4] where they have marginalised the mass region over $\lambda_{hs}$ values. For $m_{H_2} \geq m_{H_1}/2$, we find that for larger $\lambda_{hs}$ the $H_2 \rightarrow 4l$ channel is highly suppressed resulting in occupancy of large signal due to $H_1 \rightarrow H_2 H_2$ decay. However for $\lambda_{hs} \gtrsim 1$ the effect is non-negligible. We observe that for smaller values of $\lambda_{hs}$ the limits are loosely dependent on the values of $\lambda_{hs}$. Only for $\lambda_{hs} \gtrsim 2$, the constraint from $H_2 \rightarrow H_1 H_1$ becomes stronger than $H_2 \rightarrow 4l$.

4.4 Constraint imposed by vacuum stability vs. combined experimental constraint

To understand the constraint imposed from vacuum stability we need to impose the stability bound (3.29), (3.30) on the parameters. G.M Pruna and et al. [20] discussed in their paper the constraints imposed on the parameter space ($m_{H_2}, \sin \theta$) from the vacuum stability conditions. Using the first condition of equation (3.29) they found that for small mixing $\sin \theta \lesssim 0.0001$ the couplings become negative at high scales and increasing the Higgs mass to 1 TeV causes no significant change to the metastability problem. However for $0.001 \leq |\sin \theta| \leq 0.4$ and $\tan \beta (= v/u) \lesssim 0.4$ the condition (3.30) imposes the largest constraint on the Higgs mass.

Following O.Lebedev and et al. work [2], the combined bounds from direct searches, precision test and the $H_1$ coupling measurement favored by stability condition, they found that for $m_{H_2} \gtrsim 160$ GeV, all the constraints are compatible. This is depicted in the plot (shown in green) below where the entire stability favored region above 300 GeV is unconstrained whereas between 160 and 300 GeV there are pockets of allowed parameter space with $\sin \theta$ between 0.2 and 0.4.
4.5 Aspects of detecting the $H_2 \to H_1 H_1$ signal at LHC-13

In chapter 3 we discussed the possibility of the decay channel $H_2 \to H_2 H_1$, if the heavier Higgs ($H_2$) has mass above $m_2 > 250$ GeV. Given that such a scenario is plausible, perhaps it is important to explore this channel as it can allow us to further determine the properties of $H_2$. Following the notation as in [2], we will use $m_{H_2}$ instead of $m_2$ for the mass of the heavier state $H_2$. From equation (3.55), we find that the partial decay width is suppressed by a factor of $\sin^2 \theta$. The partial decay width is dependent on three unknown parameters $m_{H_2}$, $\sin \theta$ and $\lambda_{hs}$ where they hold the usual meaning used throughout the thesis. The parameters $m_{H_2}$ and $\sin \theta$ can be fixed using the SM-like decay modes of $H_2$, hence analyzing the decay mode $H_2 \to H_1 H_1$ can elucidate us to determine the parameter $\lambda_{hs}$.

The figure [4.5] to the left depicts the contours of equal $\sigma(pp \to H_2)BR(H_2 \to H_1 H_1)$ in the $m_{H_2} - \sin \theta$ plane. The figure [4.6] to the right shows the maximal production rate $\sigma(pp \to H_2)BR(H_2 \to H_1 H_1)$ at LHC-13 consistent with all the experimental constraints. From the plot [4.6], we infer that for fixed $\lambda_{hs}$, the maximal production rate is bound by the values of $\sin \theta$ which in turn is constrained for $m_{H_2} > 450$ GeV from LHC.
constraints and for $m_{H_2} < 450$ GeV from EW constraints. Notice that for increasing values of $\lambda_{hs}$, the production rate tends to increase, where $\sigma(pp \to H_2)BR(H_2 \to H_1 H_1)$ is found to be in picobarn range for values of $m_{H_2}$ up to 400 GeV. Imposing the extra stability/perturbativity constraint up to the Planck scale $M_{\text{planck}}$, we observe that the production rate decreases for $m_{H_2}$ above 350 GeV. The parameter space is then difficult to probe in experiments for such values of $m_{H_2}$ because the theoretical constraint provides the strongest bound on the model. However, for light enough $H_2$ the production rate is quite substantial leading the prospect for detecting the $H_2 \to H_1 H_1$ decay channel at LHC-13 quite good.

![Figure 4.5: Left: The maximal production rate $\sigma(pp \to H_2)BR(H_2 \to H_1 H_1)$ at LHC-13 for $\lambda_{hs} = 0.01$ in the $m_{H_2} - \sin \theta$ plane. [2]](image)

![Figure 4.6: Right: For different initial values of $\lambda_{hs}$, such as $\lambda_{hs} = 0.01$ (bottom), $\lambda_{hs} = 1$ (middle) and $\lambda_{hs} = 2$ (top), the plot depicts the rate $\sigma(pp \to H_2)BR(H_2 \to H_1 H_1)$ for maximal allowed values of $\sin \theta$. $m_{H_2}$ is in [GeV] [2]](image)

### 4.6 Summary

Finally, let us review what we have discussed in this chapter. Firstly we discussed the need for experimental constraints on the parameter space $(m_{H_2} - \sin \theta)$ of the singlet extension model for the theory to be validated. We were able to identify one of the scalar bosons with the 125 GeV Higgs-like boson observed at LHC. From the EWPO, we found that the limits on the singlet (or the heavier Higgs) are non-trivial for $m_{H_2} \lesssim 60$ GeV and $m_{H_2} \gtrsim 170$ GeV. For $m_{H_2} \gtrsim 450$ GeV, we observed the strongest constraint on the model. From the Higgs coupling measurements at the LHC, we used the $H_1$ signal strengths collected by ATLAS and CMS. We assumed that $m_{H_2}$ lies outside the interval [120,130] GeV to avoid contamination from the $H_2$ decay channels. We found that the constraint on the Higgs signal strength to be $\mu > 0.81$ at 95% CL. The following constraint on the mixing angle was found to be $\sin \theta < 0.44$ at 95% CL for $m_{H_2} \geq m_{H_1}/2 \sim 62.5$ GeV. For $m_{H_2} < m_{H_1}/2$ regime provides stronger constraints on $\sin \theta$ because of the strong dependence on $\lambda_{hs}$. At low $m_{H_2}$, we found the allowed $\lambda_{hs}$ range
to be $\lambda_{hs} < 0.015$. Next we discussed the limits from the direct Higgs-like scalar searches from LHC and LEP. We found the constraints on the $\sin \theta$ for different range of $m_{H_2}$, as low as 5 GeV. Apart from the region between 120 and 130 GeV which is poorly constrained, we found that above 130 GeV, the limits on $\sin \theta$ is, $\sin \theta < 0.3...0.4$. Then we included the vacuum stability conditions along with the above constraints and found that for $m_{H_2} \gtrsim 160$ GeV, all the above constraints are within acceptable limits. Finally we discussed in the previous section about the prospects for the $H_2 \to H_1 H_1$ decay channel at LHC-13. We observed that for values of $m_{H_2}$ approximately up to 400 GeV the production rate $\sigma(pp \to H_2)BR(H_2 \to H_1 H_1)$ is quite substantial (at picobarn level) and it increases with $\lambda_{hs}$. However we also observe that this rate quickly falls off above 500 GeV. Thus precise measurement of the Higgs quartic coupling can further help us efficiently constrain the parameter space of the singlet extension model.
Chapter 5

Application of the Scalar Singlet model

In chapter 3, I discussed the singlet extension model where I showed how electroweak vacuum stability can be achieved in a gauge-independent way. This extension model has further applications in the field of Cosmology and this is the prime focus of this chapter. It is a review delineating the bridge between Particle physics and Cosmology. Establishing a harmonical relationship between the physics at small scales governed by the laws of quantum mechanics to that of larger scales (in distance) governed by the laws of Gravity will be the sole purpose of this chapter. Of course by that I do not imply a solution to “Theory of Everything” but perhaps insinuate its existence.

5.1 Potential dark matter candidate

Although the Standard model has proven to be a remarkable low energy theory, however, its inability to incorporate a dark matter candidate has lead to physicists to explore its extensions. This section is a review of a possible dark matter candidate in the singlet extension of the SM. Pioneering work in this extension model were explored by many contemporary scholars such as, Vernon Barger and et al. in [46],[47] and by Matthew Gonderinger and et al. [16] where they considered a complex-scalar singlet and imposed $U(1)$ and $Z_2$ symmetry. The real gauge singlet scalar extensions were also explored in [48], [49], [50].

Before delving any further in to the above authors proposed models, it is perceptive to ask the question, what is “Dark Matter”? The phrase itself gives away the explanation: matter whose existence has been inferred only via gravitational effects. There
is substantial evidence that at least some of this dark matter are constituents of non-baryonic particles, i.e. composed of fundamental particles other than protons, electrons etc. These particles must have survived the Big Bang and are therefore either stable or have a lifetime longer than the age of the universe. Compelling evidence of its existence is found from the analysis of rotation curve of spiral galaxies. Other evidence of its presence comes from the observation of motion of galaxies and hot gas in clusters of galaxies [51], [52]. Let us begin by first considering the complex scalar singlet model and later by the real scalar singlet model.

5.1.1 DM candidate in complex scalar singlet extension model

I will review the work of Matthew Gonderinger and et al. [16] and Vernon Barger and et al. [46]. Consider a generalized renormalizable potential with the complex scalar singlet (like before we will consider the complex scalar field to be ‘S’ and the Higgs doublet to be ‘H’):

\[
V(H, S) = \frac{m_h^2}{2} H^{\dagger} H + \frac{\lambda_h}{4} (H^{\dagger} H)^2 + \left( \frac{|\delta_1| e^{i\phi_{b_1}}}{4} H^{\dagger} H S + c.c. \right) + \frac{\delta_2}{2} H^{\dagger} H \ | S |^2 \\
+ \left( \frac{|\delta_3| e^{i\phi_{b_3}}}{4} H^{\dagger} H S^2 \right) + \left( \frac{|a_1| e^{i\phi_{a_1}} S + c.c.}{4} \right) + \frac{|b_1| e^{i\phi_{b_1}} S^2 + c.c.}{4} \\
+ \frac{b_2}{2} S^4 + \left( \frac{|c_1| e^{i\phi_{a_1}} S^3 + c.c.}{6} \right) + \left( \frac{|c_2| e^{i\phi_{a_2}} S | S |^2 + c.c.}{6} \right) \\
+ \left( \frac{|d_1| e^{i\phi_{d_1}}}{8} S^4 + c.c. \right) + \left( \frac{|d_2| e^{i\phi_{d_2}} S^2 | S |^2 + c.c.}{6} \right) + \frac{d_2}{4} | S |^4
\]

(5.1)

where the Higgs doublet acquires a vev around the minimum of the potential:

\[
H = \begin{pmatrix}
0 \\
\nu \\
\end{pmatrix} \sqrt{2}
\]

(5.2)

At this stage the above potential (5.1) appears to be quite complicated but it is equivalent to the one that can be obtained by a single Higgs doublet and two real scalar singlet fields by performing a phase change. Nevertheless we attempt to simplify the above potential by applying certain symmetry constraints, (a) A \(Z_2\) symmetry constraint and (b) a global \(U(1)\) symmetry. Under the application of \(Z_2\) (It simply flips the sign of ‘S’) symmetry constraint, we notice that linear and cubic terms in ‘S’ should be eliminated from the potential. Thereafter, applying the global \(U(1)\) symmetry rids the potential of the complex parameters. We can then expand the complex scalar singlet in terms of real components, i.e

\[
S = (s + iA)/\sqrt{2}
\]

(5.3)
and assume it acquires a non-zero vev

$$< S > = u$$  \hspace{1cm} (5.4)$$

Imposing a global $U(1)$ symmetry then implies an existence of a Goldstone boson. We find that the Higgs field has acquired positive mass, the real part of the singlet field (‘s’) has acquired mass but its imaginary part (‘A’) remains massless. We associate A with a Goldstone boson that does not mix. Although it is stable, it’s simply a massless degree of freedom that has no phenomenological consequence. Thus in order to provide the pseudo scalar field ‘A’ with a viable mass, we can introduce a soft breaking term. Soft symmetry breaking do not create new divergent contribution to the mass of the scalars at high energies, hence the adverb “Soft”. One might encounter soft symmetry breaking terms repeatedly in Supersymmetry, however I shall refrain from such discussion as it is beyond the scope of this thesis. Typically we could identify the soft term from the Lagrangian whose coefficient has a positive power of mass. From equation (5.1), we identify that $b_1$-term satisfies our criteria. This results in A (Im (S)) receiving a finite mass. We note two essential features here: a) Terms in the Lagrangian proportional to $a_1$ and $b_1$ explicitly breaks $U(1)$ symmetry giving mass to the field ‘A’ and b) retaining $a_1$ terms results in the potential exhibiting $Z_2$ symmetry for Im(S) component of the singlet field, thereby making a stable dark matter candidate and avoiding the possibility of cosmological domain walls [53],[54],[55],[56],[57].

The potential then takes the form:

$$V(H, S) = \frac{m_h^2}{2} H^\dagger H + \frac{\lambda_h}{4} (H^\dagger H)^2 + \frac{\delta_2}{2} H^\dagger H | S |^2$$

$$+ (| a_1 | e^{i\phi_1} S + c.c) + \frac{b_2}{2} | S |^2 + \left( \frac{| b_1 | e^{i\phi_1}}{4} S^2 + c.c \right)$$

$$+ \frac{d_2}{4} | S |^4$$

(5.5)

Note that in the absence of linear $b_1$ and $a_1$ terms the potential is $U(1)$ symmetric and simply the scalar potential we discussed in the previous chapter. Upon expanding the potential (5.5) w.r.t its component fields, (5.3) we get:

$$V(h, s, A) = \frac{m_h^2}{4} h^2 + \frac{\lambda_h h^4}{16} + \frac{\delta_2}{8} h^2 (s^2 + A^2)$$

$$+ \frac{1}{4} (b_2 - b_1) s^2 + \frac{1}{4} (b_2 + b_1) A^2 - \sqrt{2} a_1 s + \frac{d_2}{8} s^2 A^2 + \frac{d_2}{16} (s^4 + A^4)$$

(5.6)

where we have expanded the Higgs field in unitary gauge. Then we can minimize the potential around the vev’s and obtain the minimization condition:

$$m_h^2 = -\frac{1}{2} \lambda_h v^2 - \frac{1}{2} \delta_2 u^2$$

$$b_2 = b_1 + 2 \sqrt{2} \frac{a_1}{u} - \frac{1}{2} d_2 v^2 - \frac{1}{2} \delta_2 v^2$$

(5.7)
Note, the minimization condition is obtained in a similar way from equation (3.11), with an additional minimizing condition:

$$\frac{\partial V(h, s, A)}{\partial A} = 0 \quad (5.8)$$

We can then determine the squared mass matrix in a similar way as equation (3.14):

$$M^2_{h, s, A} = \begin{pmatrix}
\frac{\partial^2 V(h, s, A)}{\partial h^2} & \frac{\partial^2 V(h, s, A)}{\partial h \partial s} & \frac{\partial^2 V(h, s, A)}{\partial h \partial A} \\
\frac{\partial^2 V(h, s, A)}{\partial s \partial h} & \frac{\partial^2 V(h, s, A)}{\partial s^2} & \frac{\partial^2 V(h, s, A)}{\partial s \partial A} \\
\frac{\partial^2 V(h, s, A)}{\partial A \partial h} & \frac{\partial^2 V(h, s, A)}{\partial A \partial s} & \frac{\partial^2 V(h, s, A)}{\partial A^2}
\end{pmatrix}$$

$$= \begin{pmatrix}
\frac{\lambda h^2 v^2}{2} & \frac{\delta_2 uv}{2} & 0 \\
\frac{\delta_2 uv}{2} & \frac{d_2 u^2 + \sqrt{2} a_1}{u} & 0 \\
0 & 0 & b_1 + \frac{\sqrt{2} a_1}{u}
\end{pmatrix} \quad (5.9)$$

From the above mass matrix, if we neglect the terms $b_1$ and $a_1$ we get the same squared mass matrix as in equation (3.14). We can then choose the limit on $a_1$. For the choice of $a_1 << u$, we find that, a) It reduces the number of unknown parameter by one and b) It ensures that the minimum is indeed a global minimum. The mass of the dark matter candidate in that case is $m_A \simeq \sqrt{b_1}$. We also obtain the mass eigenvalues of the two scalars that we denote as the “Higgs-like” and “Singlet-like” as per the constraints. This is similar to the discussion as in section [3.3] hence we will refrain from any further discussion, however due to non-zero terms the mass eigenvalues differ from our previous discussions. Thus we see, how in a complex scalar-singlet model we have an appropriate cold-dark matter candidate which receives a finite mass and is stable. However this came at a prize by introducing soft breaking terms. Is there another simpler approach where we do not feel the need to introduce soft breaking terms? The next model highlights this issue.

### 5.1.2 DM in real scalar singlet extension model

In this section I will review the dark matter candidate in real scalar singlet model. In addition to the application of the real singlet model to a potential dark matter candidate other similar models were proposed that encompasses the two Higgs doublet model [58],[59], introductions of multiple scalar singlets in [60],[61], including extra dimensional model [62] etc. The one real singlet scalar model has also found its application on dark matter candidate in [48],[63], [64],[65],[66], [67]. Let me begin by writting the complete
Lagrangian for a real singlet scalar extension for a viable dark matter candidate:

\[
\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2} \partial_{\mu} S \partial^{\mu} S - \frac{m_s^2}{2} S^2 - \frac{\lambda_s}{4} S^4 - \frac{\lambda_{hs}}{2} S H^\dagger H
\]

The scalar potential is simply:

\[
V(S, H) = \frac{m_h^2}{2} H^\dagger H + \frac{m_s^2}{2} S^2 + \frac{\lambda_s}{4} S^4 + \frac{\lambda_h}{4} (H^\dagger H)^2 + \frac{\lambda_{hs}}{4} S^2 H^\dagger H
\]

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\]

Here, $\mathcal{L}_{SM}$ is the SM Lagrangian as in Appendix [A.2], ‘$H$’ is the Higgs doublet and ‘$S$’ the real scalar singlet. From first view, the potential is not $\mathbb{Z}_2$ invariant. This would result in an unstable dark matter particle which is to be avoided at all cost. Hence we further impose a $\mathbb{Z}_2$ symmetry on the potential (5.11). A $\mathbb{Z}_2$ symmetry can be identified with:

\[
\begin{pmatrix}
H \\
S
\end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix}
H \\
S
\end{pmatrix}
\]

Note: $\mathbb{Z}_2$ symmetry and interchange symmetry ($\Pi_2$) are similar under a change in basis and since basis changes shouldn’t foil the phenomenology hence we often consider them to be equivalent. Of course one has to then subject different constraints to ensure stability of the potential. After applying the $\mathbb{Z}_2$ symmetry, the odd power terms in ‘$S$’ are removed and the potential simplifies:

\[
V(S, H) = \frac{m_h^2}{2} H^\dagger H + \frac{m_s^2}{2} S^2 + \frac{\lambda_s}{4} S^4 + \frac{\lambda_h}{4} (H^\dagger H)^2 + \frac{\lambda_{hs}}{4} S^2 H^\dagger H
\]

The potential looks exactly similar to the one in equation (3.8). However, we now demand that the singlet not acquire a vev at the breaking scale (or renormalization scale) unlike section [3.3]. Then we obtain a potential dark matter candidate, often referred to as ‘darkon’, whose mass is given by:

\[
m_D^2 = m_s^2 + \lambda_{hs} v_{EM}^2
\]

where ‘$v_{EM} = 246$’ GeV. Often the range of mass for the singlet (in here the darkon) is taken between, $5 \text{ GeV} \leq m_D \leq 1 \text{ TeV}$ for phenomenological purposes. This model has only three free parameters $m_D$, $m_h$ and $\lambda_h$. The DM candidate ‘$S$’, can annihilate into fermion-antifermion pairs, gauge boson pairs or the Higgs boson pairs. It’s annihilation crosssection has been studied in [48]. Here we label only the possible feynman diagrams:
5.2 The Higgs portal inflation and unitarity issues

Discovery of the Higgs boson at LHC has garnered the attention of contemporary physicists. Apart from its exquisite role in the SM as a particle that provides fermions with masses (with an exception to neutrinos), its role in Cosmic inflation [68],[69],[70] has been studied comprehensively in [71],[72]. This idea is driven from Cosmic inflations exemplary solution to the flatness, isotropy, homogeneity and relic problems and the model acting as bridge between the SM and Cosmology. The term ‘Inflation’ is usually ascribed to the accelerated expansion (or period of accelerated expansion) during the early universe. At the early universe, due to high energy densities, we are inclined to consider ‘fields’ rather than particles themselves. Inflation requires negative pressure, and the simplest form of matter which satisfies this condition is a scalar field (with spin-0). The corresponding scalar field responsible for inflation is called the ‘inflaton’. We will not delve in to the various solutions that the inflation model provides, instead we will briefly discuss the role of the Higgs field during inflation and later we will introduce our singlet in to the model and through mathematical rigor argue that the ‘inflaton’ can be a mixed state of the singlet and the Higgs and that this model alleviates the unitarity problem.

The Higgs field can play a crucial role as an ‘inflaton’ during inflation at large field values due to its large non-minimal coupling to gravity [73],[74],[75]. Essentially, the non-minimal coupling is of the order of $O(10^4)$. However it encounters a drawback to the unitarity problem [76],[77],[78] where even though the potential stability requires the Higgs quartic coupling to be positive at all scales, a RG analysis is veritable only up to the unitarity cutoff.
Oleg Lebedev and et al. [79] explored the implication of Inflation in a metastable electroweak vacuum. In their work they formulated that given the experimental evidence of the 125 GeV SM Higgs boson from LHC [29],[80] the data favors a metastable potential (existence of global minimum at large field value) implying that the Higgs field is trapped in a false vacuum. They considered the scenario of the large Higgs field limit where the Higgs quartic coupling turns negative at an “instability scale”, $\Lambda$, and argued through their formulation that for generic initial condition $h \lesssim M_{PL}$, the probability of the universe to evolve to its true ground state is overwhelming. They addressed this problem by proposing a simplistic model which is via addition of an inflaton (an extra scalar field $\phi'$). The inflaton couples to the Higgs field and it is the Higgs-Inflaton coupling that drives the Higgs field to small values during inflation. The constraint on the Higgs-Inflaton coupling ($\xi$) is that it should not give rise to large radiative corrections during the last 60 e-folds. Through their analysis they conjectured that during inflation, the Higgs field must have evolved to small values keeping the shape of the Higgs potential after inflation intact. They validated their proposal by constraining the values of $\xi$ between,

$$10^{-10} \lesssim \xi \lesssim 10^{-06} \quad (5.15)$$

However, as in the above case (5.15), the Higgs-inflaton coupling is very small making it difficult to probe at colliders. Instead if we consider a large non-minimal coupling of the inflaton to gravity, then $\xi$ can have substantially high values. In this case, the scalar potential along the $\phi-$direction is almost exponentially flat, while along the $h$-direction it is very steep. For $\xi \lesssim 0.1$, the Higgs field will quickly evolve to small values and such values of $\xi$ do not lead to significant quantum corrections to $\lambda_h$ and the inflaton potential.

In the following subsections we will take a step further and include the scalar singlet and study its role in the Higgs-portal inflation and its role in improving the unitarity issue. We will consider the scenario where the Higgs and the singlet couples to gravity in a non-minimal way. We will also consider that the inflaton, whose quantum fluctuations during inflation sets the initial condition for the large scale structure, is a combination of the Higgs with the singlet from the hidden sector. Depending on the sign of the coupling constant between the Higgs and the singlet, the inflaton can either be the Higgs or the singlet or their admixedtured state.

### 5.2.1 The Higgs portal inflation assisted by the singlet

In this section I will review the work of Oleg Lebedev and Hyun Min Lee in their paper [13]. Let us begin by considering a Lagrangian where the Higgs doublet is non-minimally
coupled to the gravity:

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{M^2}{2} R - \frac{\xi_h}{2} (H^\dagger H) R$$

(5.16)

where $\mathcal{L}_{SM}$ is the SM Lagrangian, $R$ the scalar curvature and $\xi_h$ is a constant and $M$ is some mass parameter. The coupling of the Higgs field to gravity is considered to be 'minimal' if $\xi_h = 0$. Under such a scenario, the parameter 'M' can be identified with Planck's scale $M_{\text{planck}}$. The third term of the above equation (5.16) is a lorentz invariant quantity and is required by the renormalization properties of the scalar field in a curved space-time background. F. Bezrukov and M. Shaposhnikov explored this Lagrangian in [71] however we previously discussed the inconsistency (the unitarity issue) that arise from this model.

Let us then introduce the possible singlet terms keeping the Lagrangian lorentz invariant and renormalizable. The real scalar singlet can couple to the scalar curvature, $R$. Then the Lagrangian in the Jordan frame in unitary gauge can then be written as:

$$\mathcal{L}/\sqrt{-g} = -\frac{M^2_{\text{pl}}}{2} R - \frac{\xi_h}{2} h^2 R - \frac{\xi_s}{2} s^2 R + \frac{1}{2} (\partial_\mu h)^2 + \frac{1}{2} (\partial_\mu s)^2 - V(h,s)$$

(5.17)

where,

$$V(h,s) = m_h^2 h^2 + m_s^2 s^2 + \lambda_h h^4 + \lambda_{hs} h^2 s^2 + \lambda_s s^4$$

(5.18)

The Higgs coupling $(\xi_h)$ to gravity and the singlet coupling $(\xi_s)$ to gravity are assumed to be large (i.e. non-minimal). We can then perform a transformation from Jordan frame to the Einstein frame (which is related to Jordan frame through a conformal transformation and a redefinition of the gravitational scalar field) where,

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega^2 = 1 + \frac{\xi_h h^2 + \xi_s s^2}{M^2_{\text{pl}}}$$

(5.19)

Although Jordan frame is the ‘real world’ frame, we are usually inclined to work in the Einstein frame as from the transformations (5.19) we can rid ourself of the non-minimal coupling to gravity that appear in equation (5.17). This makes the Lagrangian simpler, where the methods for calculating the physical quantities are well known, such as spectral index $n$ and tensor-to-scalar ration $r$. Now, consider the limit:

$$\xi_h h^2 + \xi_s s^2 >> M^2_{\text{pl}}$$

(5.20)

and for simplicity set $M_{\text{pl}}$ to 1. Then, we could ignore the 1 and simply have, $\Omega^2 \approx \xi_h h^2 + \xi_s s^2$. According to [75], the kinetic and potential terms in the Einstein frame can be
written as:

$$\mathcal{L}_{\text{kin}} = \frac{3}{4} \left( \partial_{\mu} \log(\xi_h \xi^2 + \xi_s^2) \right)^2 + \frac{1}{2(\xi_h \xi^2 + \xi_s^2)} \left( (\partial_{\mu} h)^2 + (\partial_{\mu} s)^2 \right)$$

$$U = \frac{1}{(\xi_h \xi^2 + \xi_s^2)^2} V$$

(5.21)

Upon performing a change of variable, the new kinetic terms read:

$$\chi = \sqrt{\frac{3}{2}} \log(\xi_h \xi^2 + \xi_s^2)$$

$$\tau = \frac{h}{s}$$

$$\mathcal{L}_{\text{kin}} = \frac{1}{2} \left( 1 + \frac{1}{6} \frac{(\xi_h \xi^2 + \xi_s^2)}{(\xi_h \xi^2 + \xi_s^2)} \right) (\partial_{\mu} \chi)^2 + \frac{1}{\sqrt{6} (\xi_h \xi^2 + \xi_s^2)^2} (\partial_{\mu} \chi)(\partial_{\mu} \tau)$$

$$+ \frac{\xi_h^2 \tau^2 + \xi_s^2 \tau^2}{2(\xi_h \xi^2 + \xi_s^2)^2} (\partial_{\mu} \tau)^2$$

(5.22)

Using the approximation $\xi \equiv \xi_h + \xi_s \gg 1$, we can ignore the second term and simplify the first term of $\mathcal{L}_{\text{kin}}$ (5.22) to get:

$$\mathcal{L}_{\text{kin}} = \frac{1}{2} (\partial_{\mu} \chi)^2 + \frac{\xi_h^2 \tau^2 + \xi_s^2 \tau^2}{2(\xi_h \xi^2 + \xi_s^2)^2} (\partial_{\mu} \tau)^2, \quad \text{and}$$

$$U = \frac{\lambda_h \tau^4 + \lambda_{hs} \tau^2 + \lambda_s}{4(\xi_h \xi^2 + \xi_s^2)^2}$$

(5.23)

Note that for large values of $\chi$, the potential is independent of it. We obtain the minima of $U$ for different values of $\tau$:

(a) $2\lambda_h \xi_s - \lambda_{hs} \xi_h > 0, 2\lambda_s \xi_h - \lambda_{hs} \xi_s > 0; \quad \tau = \sqrt{\frac{2\lambda_s \xi_h - \lambda_{hs} \xi_s}{2\lambda_h \xi_s - \lambda_{hs} \xi_h}}$

(b) $2\lambda_h \xi_s - \lambda_{hs} \xi_h > 0, 2\lambda_s \xi_h - \lambda_{hs} \xi_s < 0; \quad \tau = 0$

(c) $2\lambda_h \xi_s - \lambda_{hs} \xi_h < 0, 2\lambda_s \xi_h - \lambda_{hs} \xi_s > 0; \quad \tau = \infty$

(d) $2\lambda_h \xi_s - \lambda_{hs} \xi_h < 0, 2\lambda_s \xi_h - \lambda_{hs} \xi_s < 0; \quad \tau = 0, \infty$

(5.24)

In the first case, the inflation is a combination of the Higgs field and the singlet while in the last case we observe two local minima. Corresponding to the first three cases, we obtain the potential:

$$U \bigg|_{\text{min.}(a)} = \frac{1}{16} \frac{4\lambda_h \lambda_s - \lambda_{hs}^2}{\lambda_s \xi_h^2 + \lambda_{hs} \xi_s \xi_h}$$

$$U \bigg|_{\text{min.}(b)} = \frac{\lambda_s}{4 \xi_s^2}$$

$$U \bigg|_{\text{min.}(c)} = \frac{\lambda_h}{4 \xi_h^2}$$

(5.25)
As discussed in chapter 3, we again see the presence of the term $4\lambda_h \lambda_s - \lambda_{hs}^2 > 0$, which implies that at high field values, there is no run-away direction. There exist only one dynamical variable during inflation, $\chi$, as the $\tau$ field being heavy ($m \sim 1/\sqrt{\xi}$ in Planck units) can be integrated out. Keeping the subleading term $M_{pl}^2/(\xi_h h^2 + \xi_s s^2)$ in $\Omega^2$, the potential for the minima condition (a) in planck units becomes:

$$U(\chi) = \frac{\lambda_{eff}}{4\xi_h^2} \left(1 + \exp\left(-\frac{2\chi}{\sqrt{6}}\right)\right)^{-2}$$  \hspace{1cm} (5.26)

where,

$$\lambda_{eff} = \frac{1}{4} \frac{4\lambda_h \lambda_s - \lambda_{hs}^2}{\lambda_s + \lambda_h x^2 - \lambda_{hs} x}$$  \hspace{1cm} (5.27)

and,

$$x = \frac{\xi_s}{\xi_h}$$  \hspace{1cm} (5.28)

where, $\lambda_{eff}$ depends on the composition of the inflation. Thus we find that for the Higgs inflation [71], $\lambda_{eff} = \lambda_h$, for the singlet inflation, $\lambda_{eff} = \lambda_s/x^2$ and for inflation assisted by singlet mixing with the Higgs, $\lambda_{eff}$ is as in equation (5.27). From the above potential (5.26), we can determine the inflationary parameters. We notice that for large values of $\chi$, the exponential term is small and can be ignored, giving us a flat potential which facilitates inflation. As, $\chi$ takes smaller values, the $\epsilon$-parameter approaches 1 and inflation ends. The $\epsilon$-parameter is given by:

$$\epsilon = \frac{1}{2} \left(\frac{dU/d\chi}{U}\right)^2 \simeq \frac{4}{3\xi_h^2 h^4}, \text{ where}$$

$$\tilde{h} \simeq \frac{1}{\sqrt{\xi_h}} \exp(\chi/\sqrt{6})$$  \hspace{1cm} (5.29)

Thus we find that the initial value of the inflation for a given number of e-folds $N$ is $\tilde{h}_{in} \approx \sqrt{4N/3\xi_h}$ and for the end of inflation is $\tilde{h}_{end} = (4/3)^{\frac{1}{2}}/\sqrt{\xi_h}$. Using COBE normalization $U/\epsilon = (0.027)^4$, [81], we can fix $\xi_h$ in terms of $\lambda_{eff}$

$$\xi_h \simeq \sqrt{\frac{\lambda_{eff}}{3} \frac{N}{(0.027)^2}}$$  \hspace{1cm} (5.30)

In [82], it was shown that for $N=60$ and $\sqrt{\lambda_{eff}} \sim 1$, that the non-minimal gravity coupling was about $\xi_h \approx 50000$. The spectral index is predicted to be

$$n = 1 - 6\epsilon + 2\eta \simeq 1 - 2/N \simeq 0.97$$  \hspace{1cm} (5.31)

where $\epsilon$ (5.29) and $\eta$ are the slow roll parameters:

$$\eta = \frac{d^2U/d\chi^2}{U}$$  \hspace{1cm} (5.32)
while the tensor to scalar perturbation ratio is $r \simeq 12/N^2 \simeq 0.0033$

### 5.2.2 Ameliorating the unitarity issue via the singlet assistance

The Unitarity issue in the Higgs inflation stems from the non-minimal coupling of the Higgs doublet to gravity at large field values. The Unitarity cut-off during inflation can be found to be larger than the one in the vacuum [72],[83]. However, irrespective of its large cut-off value an inflationary plateau beyond the unitarity cut-off is not acceptable under the perturbative expansion. To understand this deep-rooted problem, consider a RG analysis of the model from low to high energy. In the second chapter I showed that for the SM vacuum to be stable, the Higgs quartic coupling needs to be positive at all scale. However, the dominant effect of top-quark prevents that and one can observe a scale where the Higgs quartic coupling ($\lambda_h$) turns negative. Thus the RG analysis in Higgs inflation is veritable only up to the unitarity cut-off which is way below the inflation scale ($M_{Pl}/\xi_h$). This implies we need to rectify our formalism by appending additional fields [84] or operators [85] at high energies. Let us see, if this unitarity issue can be solved by introducing a real scalar-singlet of sigma-model type proposed in [84]. I will use the notation as in [82]. The Lagrangian of the model in Jordan-frame can be written as:

$$L_J = \frac{1}{2} M^2 + \xi \sigma^2 + 2 \xi H^\dagger H \right) R - \frac{1}{2} (\partial_\mu \sigma)^2 - | D_\mu H |^2$$

$$- \frac{1}{4} \lambda_\sigma \left( \sigma^2 - u^2 + 2 \frac{\lambda_H \sigma}{\lambda_\sigma} H^\dagger H \right)^2 - \left( \lambda_H - \frac{\lambda_H^2}{\lambda_\sigma} \right) \left( H^\dagger H - \frac{v^2}{2} \right)^2$$

where $M,u$ and $v$ are mass parameters with $v \ll M,u$. This is so that the field, $\sigma$, is heavy. The couplings, $\xi, \zeta$ are positive non-minimal couplings with $\xi \gg \zeta$. We have used the effective quartic coupling introduced due to the addition of the real singlet and its mixing with the Higgs doublet, $\lambda \equiv \lambda_H - \frac{\lambda_H}{\lambda_\sigma}$. Since the basic idea is to make the unitarity cut-off, say $\Lambda_{UV}$ larger, we will work in the large non zero vev of $\sigma$, $\sigma \approx u$. It is straightforward to find that,

$$\Lambda_{UV} = \left( 1 + 6r \xi \right) \frac{M_{Pl}}{\xi}$$

where the Planck mass is now $M_{Pl}^2 = M^2 + \xi u^2$ and we measure the contribution of the $\sigma$ vev by the ratio $r = \xi u^2 / M_{Pl}^2$, which takes values between 0 and 1. In this scenario, we observe that for small vev i.e, $r \to 0$, the cut off is $M_{Pl}/\xi$ while for moderate values of $r \geq 1/\xi$, it is pushed up to $r M_{Pl}$. We observe that the sigma field dominates inflation due to its large non-minimal coupling to gravity (the Ricci scalar $R$), while the Higgs field simply follows the sigma field along a flat direction, provided $\lambda_{H\sigma} < 0$. The limit
on the mass of the $\sigma$ field in the vacuum can be obtained by simplifying the above Lagrangian. In this case we assume that tree-level Einstein term and the non-minimal coupling for the Higgs doublet is absent, $M = 0$ and $\zeta = 0$, in Jordan frame. Then under unitary gauge with $H^T = (0, h)/\sqrt{2}$, the action in Jordan frame is:

$$L_J \sqrt{-g_J} = \frac{\xi}{2} \sigma^2 R - \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{4} \lambda_{\sigma} \left( \sigma^2 - u^2 + 2\frac{\lambda H}{\lambda_{\sigma}} h^2 \right)^2 - \left( \lambda_H - \frac{\lambda^2 H}{\lambda_{\sigma}} \right) \left( h^2 - \frac{v^2}{2} \right)^2$$  \hspace{1cm} (5.35)

$u \equiv M_{Pl}/\sqrt{\xi}$ is chosen to reproduce the Jordan-frame action of the Higgs inflation with a positive non-minimal coupling $\xi_h = -\frac{\lambda_H}{\lambda_{\sigma}}$, for $\lambda_H < 0$. It is easy to determine the mass of $\sigma$ from the above equation (5.35):

$$M^2_{\sigma} = \lambda_{\sigma} 2r M^2_{Pl} \frac{R}{(1 + 6r\xi)} \simeq \lambda_{\sigma} \frac{M^2_{Pl}}{3\xi^2}.$$  \hspace{1cm} (5.36)

With COBE constraint, we obtain the mass sigma field to be $M_{\sigma} \approx 10^{13}$GeV. Below the sigma mass scale, the effective action in Jordan frame is:

$$L_J \sqrt{-g_J} = \frac{1}{2} \left( M^2_{Pl} + \xi_{eff} h^2 \right) R - \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{4} \lambda_{eff} (h^2 - v^2)^2$$  \hspace{1cm} (5.37)

where $\xi_{eff} = \frac{\lambda_H}{\lambda_{sigma}} \xi$ and $\lambda_{eff}$ the one mentioned earlier. We find that the Higgs coupling has attained a positive contribution rendering it a higher value than the SM quartic coupling. This forces us to consider the Higgs mass for which the instability scale is $\Lambda_I > 10^{13}$GeV. This requires us to set $m_h > 125$GeV which is marginally compatible from the experimental constraints obtained from ATLAS and CMS [86],[80]. Now lets us write the Lagrangian (5.35) in Einstein frame via a conformal transformation:

$$L_E \sqrt{-g_E} = \frac{M^2_{Pl}}{2} R - \frac{1}{2} \left( \frac{u}{\sigma} \right)^2 \left[ (1 + 6\xi)(\partial_\mu \sigma)^2 + (\partial_\mu h)^2 \right] - \frac{1}{4} \left( \frac{u}{\sigma} \right)^4$$

$$- \frac{1}{4} \lambda_{\sigma} \left( \sigma^2 - u^2 + 2\frac{\lambda H}{\lambda_{\sigma}} h^2 \right)^2 - \left( \lambda_H - \frac{\lambda^2 H}{\lambda_{\sigma}} \right) \left( h^2 - \frac{v^2}{2} \right)^2$$  \hspace{1cm} (5.38)

Then redefining the field as per:

$$\sigma \equiv u \exp(\chi/\sqrt{6} M_{Pl}) \text{ and } \tilde{h} \equiv uh/\sigma$$  \hspace{1cm} (5.39)
the above action (5.38) becomes:

\[
\mathcal{L}_E = \frac{M_{Pl}^2}{2} - \frac{1}{2} \left( 1 + \frac{1}{6\xi} + \frac{\tilde{h}^2}{6M_{Pl}^2} \right) (\partial_\mu \chi)^2 - \frac{1}{2} (\partial_\mu \tilde{h})^2 - \frac{1}{\sqrt{6}M_{Pl}} (\partial_\mu \chi)(\partial^\mu \tilde{h})
\]

\[
- \frac{1}{4} \frac{u^4}{4} \lambda_\sigma \left( 1 - \exp(-2\chi/\sqrt{6}M_{Pl}) + \frac{\lambda_{H\sigma}}{\lambda_\sigma} \frac{\tilde{h}^2}{u^2} \right)^2
\]

\[
+ \frac{1}{4} \left( \lambda_H - \frac{\lambda_{H\sigma}^2}{\lambda_\sigma} \right) \left( \tilde{h}^2 - v^2 \exp(-2\chi/\sqrt{6}M_{Pl}) \right)^2
\]

(5.40)

Note the second line of equation in the above equation (5.40) is the potential in Einstein frame. If we consider the sigma field to be very high i.e $|\sigma| \gg u$, the potential approximates for $\tilde{h}$:

\[
V_E \simeq \frac{1}{4} \left( \lambda_\sigma u^4 + 2\lambda_{H\sigma} u^2 \tilde{h}^2 + \lambda_H \tilde{h}^4 \right)
\]

(5.41)

Minimizing the above potential (5.41) we obtain two minima corresponding to $\lambda_{H\sigma} < 0$, at $\tilde{h} = \pm \sqrt{-\frac{2\lambda_{H\sigma}}{\lambda_H}} u$ and stabilizing the potential around one of the minima, we obtain:

\[
V_E = V_0 \left( 1 - \exp(-2\chi/\sqrt{6}M_{Pl}) \right)^2
\]

(5.42)

with,

\[
V_0 = \frac{u^4}{4} \left( \lambda_\sigma - \frac{\lambda_{H\sigma}^2}{\lambda_H} \right)
\]

(5.43)

Thus we see that the sigma-model drives a slow-roll inflation as the Higgs field is stabilized at large vev during inflation. We also note from the above equations that the vacuum stability condition for $\lambda_{H\sigma} < 0$ becomes the condition for positive inflaton vacuum energy. The stability condition is $\lambda_H - \frac{\lambda_{H\sigma}^2}{\lambda_\sigma} > 0$. We also know that tree level effects cannot be enough to stabilize the vacuum, hence we need to study the RG effects above the sigma mass. However quantum corrections arising due to gravity is subjected to fallacy as it is crude to talk about quantizing gravity using the field theory formalism.

### 5.3 Summary

In this chapter we discussed in details the dark matter candidate and the Higgs portal inflation using the singlet extension model. In the dark matter section, we argued the inability of the SM to incorporate a DM candidate leads to exploration of theories beyond the SM. In particular we saw that the singlet extension model has a plausible DM candidate which is stable. Next we discussed the role of the singlet in the Higgs portal inflation. Using the singlet extension of sigma model we were able to address the unitarity issue. It is remarkable how a simplest extension of the SM enunciates
and tackles difficult problems that is blind to the SM. We end this thesis by providing concluding remarks in the next chapter.
Chapter 6

Conclusion

The Standard model (SM) is an excellent low energy theory as it successfully describes most of the fundamental particle interactions that forms the matter around us. The corresponding scalar sector of the SM has been quite elusive until the recent discovery of the 125 GeV Higgs boson at LHC [86], [80] which if identified with the SM Higgs, favors metastability of the electroweak vacuum. This would imply the Higgs boson is trapped in a false vacuum. The Standard model also has some additional loop holes such as, it does not incorporate a dark matter candidate, mechanism for inflation, neutrino masses etc. Thus in order to account for the above voids in the theory it is necessary that we extend the SM.

In regard to the limitations in the SM mentioned above, in this thesis we proposed the simplest extension of the SM by adding a scalar singlet. The scalar singlet has the right gauge quantum numbers to couple to the Higgs field via the Higgs portal. However for the SM gauge group, the singlet turns a blind eye to the SM fields. Due to the mixing between the two scalar fields, we were able to obtain two spin-0 particles and we referred to them as, Higgs-like ($H_1$) and singlet-like ($H_2$). We identified the heavier mass eigenstate with the singlet-like and the lighter mass eigenstate with the 125 GeV SM Higgs boson. We found that the SM Higgs self coupling ($\lambda_h|_{SM}$) received a finite positive correction (3.26) at tree level which ameliorated the stability of the potential. This is primarily due to the tree level corrections to the Higgs mass-coupling relation (3.23) which survives in the heavy-singlet limit (3.3.1). Note that loop effects induced through the singlet plays a significant role in keeping the potential stable.

We then evaluated the stability bounds on the electroweak scale parameters (they include renormalization at the right scale) for different signs of the Higgs-singlet coupling ($\lambda_{hs}$). For a fixed value of $\lambda_{hs}$, we plotted the parameter space ($m_2 - \sin \theta$) by imposing the constraints obtained from the vacuum stability conditions. We observed that for different sign of $\lambda_{hs}$ we obtain distinct allowed regions of the parameter space because of
the additional constraint (3.30) imposed for negative $\lambda_{hs}$ values. We further determined the maximum value of the Higgs-singlet coupling, $\lambda_{hs} = 0.23$ for which there exist a finite region of parameter space.

In the allowed region (say $m_2 \gtrsim 250$ GeV), we find that the singlet-like state can further decay into two Higgs boson. Such a decay is kinematically allowed and we find that it is substantial, typically in picobarn range for $m_2$ approximately upto 400 GeV. Above 500 GeV we observe that the production rate begins to decrease rapidly.

Finally we discussed the application of the singlet extension of the SM in the dark matter sector and the Higgs portal inflation. By imposing $Z_2$ symmetry on the real singlet field, we found that the singlet is a possible cold dark matter candidate when it receives zero vacuum expectation value. For phenomenological purposes we usually consider the range of mass, $5$ GeV $\lesssim m_2 \lesssim 1$ TeV. The singlet also plays a crucial role in the Higgs portal inflation, where we considered the scenario in which the inflaton is a mixture between the Higgs and the singlet. We determined the scalar-tensor ratio of the model, $r \simeq 0.0033$ and the spectral index, $n \simeq 0.97$ which are in good agreement with experimental measurements. The singlet field of the sigma type model also plays a crucial role in restoring the unitarity in the unitarized Higgs inflation [84].

Thus the singlet extension model unravels a few remarkable problems that the SM fails to elucidate and motivates contemporary physicists to explore beyond the SM theories. What model describes the Universe as a whole (i.e. unify all fundamental interactions) is still an unplumbed sector that needs closer attention. Good candidates for the ‘Theory of Everything’ are String theory and Loop quantum gravity. Quantizing gravity appears to be one of the biggest hurdles and if resolved we can expect some ground breaking development in the field of theoretical and later experimental physics. We would garner ample information about the Universe, providing answers to deep-rooted problems such as, what caused the ‘Big Bang’? Source of Dark Energy? etc. that still appear aloof and under continuous scrutiny from contemporary physicists.
Appendix A

SM fields and Lagrangian

A.1 Vector Boson currents

\[ J_{W}^{\mu+} = \frac{1}{\sqrt{2}}(\bar{\nu}_L \gamma^\mu e_L + \bar{u}_L \gamma^\mu d_L) \]

\[ J_{W}^{\mu-} = \frac{1}{\sqrt{2}}(\bar{e}_L \gamma^\mu \nu_L + \bar{d}_L \gamma^\mu u_L) \]

\[ J_{Z}^\mu = \frac{1}{\cos \theta_w} \left[ \bar{\nu}_L \gamma^\mu \left( \frac{1}{2} \right) \nu_L + \bar{e}_L \gamma^\mu \left( -\frac{1}{2} + \sin^2 \theta_w \right) e_L ight. 
+ \bar{e}_R \gamma^\mu \left( \sin^2 \theta_w \right) e_R + \bar{u}_L \gamma^\mu \left( -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_w \right) u_L 
+ \bar{u}_R \gamma^\mu \left( \frac{2}{3} \sin^2 \theta_w \right) u_R + \bar{d}_L \gamma^\mu \left( -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_w \right) d_L 
+ \bar{d}_R \gamma^\mu \left( \frac{1}{3} \sin^2 \theta_w \right) d_R \right] \]

\[ J_{EM}^\mu = \bar{\nu} \gamma^\mu (-1) e + \bar{u} \gamma^\mu (+\frac{2}{3}) u + \bar{d} \gamma^\mu (-\frac{1}{3}) d \]
A.2 The standard model Lagrangian

\[ \mathcal{L}_{SM} = -\frac{1}{4} B^{\mu \nu} B_{\mu \nu} - \frac{1}{8} \text{tr}(W^{\mu \nu} W_{\mu \nu}) - \frac{1}{2} \text{tr}(G^{\mu \nu} G_{\mu \nu}) \quad (U(1), SU(2), SU(3)) \]

\[ + \left[ (\bar{\nu}_L, \bar{e}_L) \bar{\sigma}^\mu i D_\mu \left( \frac{\nu_L}{\bar{e}_L} \right) + \bar{e}_R \sigma^\mu i D_{mu} e_R + \bar{\nu}_R \sigma^\mu i D_\mu \nu_R + \text{h.c} \right] \quad \text{(lepton dynamical term)} \]

\[ - \frac{\sqrt{2}}{v} \left[ (\bar{\nu}_L, \bar{e}_L) \phi M^\nu e_R + \bar{e}_R \bar{M}^\nu \phi \left( \frac{\nu_L}{\bar{e}_L} \right) \right] \quad \text{(electron, muon, tauon mass term)} \]

\[ - \frac{\sqrt{2}}{v} \left[ (-\bar{\nu}_L, \bar{e}_L) \phi^* M^\nu \nu_R + \bar{\nu}_R \bar{M}^\nu \phi \left( \frac{-\nu_L}{\bar{e}_L} \right) \right] \quad \text{(neutrino mass term)} \]

\[ + \left[ (\bar{u}_L, \bar{d}_L) \bar{\sigma}^\mu i D_\mu \left( \frac{u_L}{d_L} \right) + \bar{u}_R \sigma^\mu i D_{mu} u_R + \bar{d}_R \sigma^\mu i D_\mu d_R + \text{h.c} \right] \quad \text{(quark dynamical term)} \]

\[ - \frac{\sqrt{2}}{v} \left[ (\bar{u}_L, \bar{d}_L) \phi M^d d_R + \bar{d}_R \bar{M}^d \phi \left( \frac{u_L}{d_L} \right) \right] \quad \text{(down, strange, bottom mass term)} \]

\[ - \frac{\sqrt{2}}{v} \left[ (-\bar{d}_L, \bar{u}_L) \phi^* M^u u_R + \bar{u}_R \bar{M}^u \phi \left( \frac{-d_L}{u_L} \right) \right] \quad \text{(up, charm, top mass term)} \]

\[ + D_\mu \phi D^\mu \phi - m^2_h [\bar{\phi}\phi - v^2/2]^2/2v^2 \quad \text{(Higgs kinematical and mass term)} \]

(A.2)

where h.c implies the hermitian conjugate. The derivative operators are:

\[ D_\mu \left( \frac{\nu_L}{\bar{e}_L} \right) = \left[ \partial_\mu - \frac{ig_1}{2} B_\mu + \frac{ig_2}{2} W_\mu \right] \left( \frac{\nu_L}{\bar{e}_L} \right) \quad \text{(A.3)} \]

\[ D_\mu \left( \frac{u_L}{d_L} \right) = \left[ \partial_\mu + \frac{ig_1}{6} B_\mu + \frac{ig_2}{2} W_\mu + ig G_\mu \right] \left( \frac{u_L}{d_L} \right) \quad \text{(A.4)} \]

\[ D_\mu \nu_R = \partial_\mu \nu_R \quad \text{(A.5)} \]

\[ D_\mu u_R = \left[ \partial_\mu + \frac{2ig_1}{3} B_\mu + ig G_\mu \right] u_R \quad \text{(A.6)} \]

\[ D_\mu d_R = \left[ \partial_\mu - \frac{ig_1}{3} B_\mu + ig G_\mu \right] d_R \quad \text{(A.7)} \]

\[ D_\mu \phi = \left[ \partial_\mu + \frac{ig_1}{2} B_\mu + \frac{ig_2}{2} W_\mu \right] \quad \text{(A.8)} \]

\[ B_\mu, W_\mu \text{ and } G_\mu \text{ are vector bosons and } \phi \text{ is a doublet under } SU(2) \text{ representation while} \]

\[ g_1, g_2, g_3 \text{ are the } U(1), SU(2) \text{ and } SU(3) \text{ gauge couplings.} \]
Appendix B

Parameters of the singlet model

Given the parameters $m_1, m_2, \lambda_h, \lambda_s, \lambda_{hs}, \theta, u, v$ I need to determine $\lambda_h[m_{top}], \lambda_s[m_{top}]$ in terms of $m_1, m_2, \theta, v, \lambda_{hs}$ i.e,

$$\lambda_h = \lambda_h(m_1, m_2, \theta, v, \lambda_{hs})$$

(B.1)

$$\lambda_s = \lambda_s(m_1, m_2, \theta, v, \lambda_{hs})$$

(B.2)

From now onwards I shall denote $\lambda_h[m_{top}]$ as simply $\lambda_h$. Given that we have three equations:

For $m_1^2$

$$m_1^2 = \lambda_h v^2 + \lambda_s u^2 - \sqrt{({\lambda_s u^2 - \lambda_h v^2})^2 + \lambda_{hs} u^2 v^2}$$

(B.3)

For $m_2^2$

$$m_2^2 = \lambda_h v^2 + \lambda_s u^2 + \sqrt{({\lambda_s u^2 - \lambda_h v^2})^2 + \lambda_{hs} u^2 v^2}$$

(B.4)

and finally I have an equation for $\theta$:

$$\tan(2\theta) = \frac{\lambda_{hs} u v}{\lambda_h v^2 - \lambda_s u^2}$$

(B.5)

B.1 Free parameters of the model

Using equation (B.5), I can express it in terms of $\lambda_h$ where I get:

$$\lambda_h = \lambda_{hs} \cot(2\theta) \frac{u}{v} + \lambda_s \frac{u^2}{v^2}$$

(B.6)
Then I simplify \( m_1^2 \) and \( m_2^2 \), as

\[
m_1^2 = (\lambda_h v^2 + \lambda_s u^2) - \sqrt{[\lambda_s u^2 - \lambda_h v^2]^2 + \lambda_h^2 u^2 v^2},
\]

(B.7)

Substituting equation (B.6) in equation (B.7) I get:

\[
m_1^2 = \lambda_h u v \cot(2\theta) + 2\lambda_s u^2 - \sqrt{\lambda_h^2 u^2 v^2 \cot^2(2\theta) + \lambda_h^2 u^2 v^2}
\]

(B.8)

Further simplifying, I get

\[
m_1^2 = \lambda_h u v \cot(2\theta) + 2\lambda_s u^2 - \lambda_h u v \csc(2\theta)
\]

(B.9)

Upon further simplification, I can write;

\[
m_1^2 = -\lambda_h u v \tan(\theta) + 2\lambda_s u^2
\]

(B.10)

The same can be derived for \( m_2^2 \) for which I get,

\[
m_2^2 = \lambda_h u v \cot(\theta) + 2\lambda_s u^2
\]

(B.11)

Now subtracting equation (B.10) with equation (B.11) I can determine ‘u’ by eliminating \( \lambda_s \) which gives me,

\[
u = \frac{(m_2^2 - m_1^2)^2 \sin(2\theta)}{2\lambda_h v}
\]

(B.12)

Then I can determine \( \lambda_s \) by substituting equation (B.12) in equation (B.10), which gives me

\[
\lambda_s = \frac{\lambda_h^2 v^2 [m_1^2 \csc^2(\theta) + m_2^2 \sec^2(\theta)]}{2[m_2^2 - m_1^2]v^2} = \frac{2\lambda_h^2 v^2 [m_1^2 \cos^2(\theta) + m_2^2 \sin^2(\theta)]}{[m_2^2 - m_1^2]^2} \times \sin^2(2\theta)
\]

(B.13)

Thus I have determined \( \lambda_s = \lambda_s(m_1,m_2,v,\theta,\lambda_h) \). All that is left to do is determine \( \lambda_h \). To do so, I will use the equations equation (B.13) for \( \lambda_s \) and equation (B.12) for ‘u’ and substitute them in equation (B.6) to determine \( \lambda_h \). I find,

\[
\lambda_h = \frac{[m_2^2 - m_1^2] \cos(2\theta)}{2v^2} + \frac{\sin^2(2\theta)}{8} \times \frac{[m_2^2 \sec^2(\theta) + m_1^2 \csc^2(\theta)]}{v^2}
\]

(B.14)

Upon further simplification of the above equation I get:

\[
\lambda_h = \frac{m_2^2 \cos^2(\theta) + m_1^2 \sin^2(\theta)}{2v^2}
\]

(B.15)
Simply rewriting $\lambda_h$ and $\lambda_s$ in terms of $\sin \theta$ instead of $\theta$

$$\lambda_h = \frac{m_1^2}{2v^2} + \sin^2 \theta \frac{m_2^2 - m_1^2}{2v^2} \tag{B.16}$$

$$\lambda_s = \frac{2 \lambda_{hs}^2}{\sin^2 2\theta} \frac{v^2}{m_2^2 - m_1^2} \left( \frac{m_2^2}{m_2^2 - m_1^2} - \sin^2 \theta \right). \tag{B.17}$$
Appendix C

Experimental constraints from different studies

C.1 Electroweak Precise Observable (EWPO)

<table>
<thead>
<tr>
<th>Observable</th>
<th>Definition</th>
<th>Experimental values</th>
<th>SM prediction</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_Z[GeV]$</td>
<td>$\sum_f \Gamma(Z \to ff')$</td>
<td>2.4952 ± 0.0023</td>
<td>2.4950</td>
<td>[23]</td>
</tr>
<tr>
<td>$\sigma_{had}[nb]$</td>
<td>$\frac{12\pi m_Z^2}{\Gamma_Z} \sum_f \Gamma(Z \to e^+e^-)\Gamma(Z \to q\bar{q})$</td>
<td>41.540 ± 0.037</td>
<td>41.484</td>
<td>[23]</td>
</tr>
<tr>
<td>$R_l$</td>
<td>$\sum_q \Gamma(Z \to q\bar{q})/\Gamma(Z \to e^+e^-)$</td>
<td>20.767 ± 0.025</td>
<td>20.743</td>
<td>[23]</td>
</tr>
<tr>
<td>$A_l$</td>
<td>$\Gamma(Z \to e^+e^-)\Gamma(Z \to q\bar{q})/\Gamma(Z \to e^+e^-)$</td>
<td>0.1499 ± 0.0018</td>
<td>0.1472</td>
<td>[87]</td>
</tr>
<tr>
<td>$A_{FB}^{l\bar{l}}$</td>
<td>$\frac{3}{4} A_{l\bar{l}}^2$</td>
<td>0.0171 ± 0.0010</td>
<td>0.0163</td>
<td>[23]</td>
</tr>
<tr>
<td>$\sin^2 \theta_{eff}(Q_{FB})$</td>
<td>$\frac{g_Y^2}{g_{l_1}^2 + g_Y^2} (1 - \frac{m_Z^2 \delta_{WW}(m_Z^2)}{g_Y m_Z^2})$</td>
<td>0.2324 ± 0.0012</td>
<td>0.23150</td>
<td>[23]</td>
</tr>
<tr>
<td>$R_b$</td>
<td>$\frac{\Gamma(Z \to dd)}{\sum_q \Gamma(Z \to q\bar{q})}$</td>
<td>0.21629 ± 0.00066</td>
<td>0.21578</td>
<td>[23]</td>
</tr>
<tr>
<td>$A_b$</td>
<td>$\Gamma(Z \to d\bar{d})\Gamma(Z \to d\bar{d})/\Gamma(Z \to q\bar{q})$</td>
<td>0.923 ± 0.020</td>
<td>0.935</td>
<td>[23]</td>
</tr>
<tr>
<td>$A_{FB}^{l\bar{l}}$</td>
<td>$\frac{3}{4} A_{l\bar{l}}^2 A_{l\bar{l}}$</td>
<td>0.0992 ± 0.0016</td>
<td>0.1032</td>
<td>[23]</td>
</tr>
<tr>
<td>$R_c$</td>
<td>$\frac{\Gamma(Z \to n\bar{n})}{\sum_q \Gamma(Z \to q\bar{q})}$</td>
<td>0.1721 ± 0.0030</td>
<td>0.17226</td>
<td>[23]</td>
</tr>
<tr>
<td>$A_c$</td>
<td>$\Gamma(Z \to u\bar{u})\Gamma(Z \to u\bar{u})/\Gamma(Z \to n\bar{n})$</td>
<td>0.670 ± 0.027</td>
<td>0.668</td>
<td>[23]</td>
</tr>
<tr>
<td>$A_{FB}^{l\bar{l}}$</td>
<td>$\frac{3}{4} A_{l\bar{l}}^2 A_{l\bar{l}}$</td>
<td>0.0707 ± 0.0035</td>
<td>0.0738</td>
<td>[23]</td>
</tr>
<tr>
<td>$m_W[GeV]$</td>
<td>$\sqrt{\frac{g_Y^2 v^2}{4} + \delta_{WW}(m_W^2)}$</td>
<td>80.385 ± 0.015</td>
<td>80.3602</td>
<td>[24]</td>
</tr>
<tr>
<td>$\Gamma_W[GeV]$</td>
<td>$\sum_f \Gamma(W \to ff')$</td>
<td>2.085 ± 0.042</td>
<td>2.091</td>
<td>[25]</td>
</tr>
<tr>
<td>$Br(W \to had)$</td>
<td>$\frac{\sum_f \Gamma(W \to q\bar{q})}{\sum_f \Gamma(W \to ff')}$</td>
<td>0.6741 ± 0.0027</td>
<td>0.6751</td>
<td>[26]</td>
</tr>
</tbody>
</table>

Table C.1: The above table represents the electroweak precision observables used in the analysis for determining the experimental constraints on the parameter space of the singlet extension model of the SM. We have used the experimental correlations between the LEP-1 Z-pole observables and the heavy flavour observables. Except for $Br(W \to had)$ [26], we use the best fit SM values from GFitter [87]. [2]
Appendix D

Dark Matter annihilation cross-sections

\[ \tilde{\sigma}_{ff} = \sum_f \frac{\lambda_f^2 m_f^2}{\pi} \frac{1}{(s - m_h^2)^2 + m_h^4 \Gamma_h^2} \frac{(s - 4m_f^2)^{1.5}}{\sqrt{s}} \quad (D.1) \]

\[ \tilde{\sigma}_{ZZ,WW} = \frac{\lambda_h^2}{4\pi} \frac{s^2}{(s - m_h^2)^2 + m_h^2 \Gamma_h^2} \sqrt{1 - \frac{4m_Z^2}{s}} \left( 1 - \frac{4m_W^2}{s} + \frac{12m_Z^2}{s^2} \right) \quad (D.2) \]

\[ \tilde{\sigma}_{hh} = \frac{\lambda_h^2}{4\pi} \sqrt{1 - \frac{4m_h^2}{s}} \left[ \left( \frac{s + 2m_h^2}{s - m_h^2} \right)^2 - \frac{16\lambda_h v_{EM}^2 s + 2m_h^2}{s - 2m_h^2} F(\xi) \right] + \frac{32\lambda_h^2 v_{EM}^4}{(s - 2m_e^2)^2} \left( \frac{1}{s^{1.5}} + F(\xi) \right) \quad (D.3) \]

where 's' is the invariant mass, \( v_{EM} \) is the EW breaking scale, \( F(\xi) \equiv \text{arctanh}(\xi) / \xi \) with \( \xi \equiv \sqrt{(s - 4m_h^2)(s - 4m_Z^2)/(s - 2m_h^2)} \). The Higgs decay width \( \Gamma_h \):

\[ \Gamma_h = \frac{\sum_f m_f^2 (m_f^2 - 4m_h^2)^{1.5}}{8\pi v_{EM}^2 m_h^2} + \frac{m_h^3}{32\pi v_{EM}^2} \sqrt{a - \frac{4m_Z^2}{m_h^2}} \left( 1 - \frac{4m_Z^2}{m_h^2} + \frac{12m_Z^4}{m_h^4} \right) \]

\[ + \frac{m_h^3}{16\pi v_{EM}^2} \sqrt{a - \frac{4m_Z^2}{m_h^2}} \left( 1 - \frac{4m_Z^2}{m_h^2} + \frac{12m_Z^4}{m_h^4} \right) + \frac{\lambda_h^2 v_{EM}^2}{8\pi} \sqrt{\frac{m_h^2 - 4m_D^2}{m_h^2}} \quad (D.4) \]
Bibliography


