On optimization of data assimilation in the HBM -circulation model

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The purpose of this study is to develop a method for optimising the data assimilation system of the HIROMB-BOOS -model at the Finnish Meteorological Institute by finding an optimal time interval and an optimal grid for the data assimilation. This is needed to balance the extra time the data assimilation adds to the runtime of the model and the improved accuracy it provides.

Data assimilation is the process of combining observations with a numerical model to improve the accuracy of the model. There are different ways of doing this, some of which are covered in this work.

The HIROMB-BOOS -circulation model is a 3D-forecast model for the Baltic Sea. The variables forecast are temperature, salinity, sea surface height, currents, ice thickness and ice coverage. Some of the most important model equations are explained here.

The HIROMB-BOOS -model at the Finnish Meteorological Institute has a preoperational data assimilation system that is based on the optimal interpolation method. In this study the model was run for a 2-month test period with different time intervals of data assimilation and different assimilation grids. The results were compared to data from five buoys in the Baltic Sea.

The model gives more accurate results when the time interval of the data assimilation is small. The thicker the data assimilation grid is, the better the results. An optimal time interval was determined taking into account the time the assimilation takes. An optimal grid was visually determined based on an optimal grid thickness, for which the added time had to be considered as well.

The optimized data assimilation scheme was tested by performing a 12-month test run and comparing the results to buoy data. The optimized data assimilation has a positive effect on the model results.
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Chapter 1

Introduction

The purpose of this study is to develop a method for optimising the data assimilation system of the HIROMB-BOOS model at the Finnish Meteorological Institute so that the added time to run the model and the improved quality are balanced as well as can be. Data assimilation is a technique used in improving numerical model results by combining observations with the model. There are different methods of data assimilation. In the Theory chapter the successive corrections methods, the optimal interpolation and the variational methods (3D-VAR and 4D-VAR) will be covered.

The HIROM-BOOS model, hereon referred to as HBM is a 3D general circulation model of the Baltic Sea and the North Sea. The version of HBM used at the Finnish Meteorological Institute, from now on referred to as HBM-FMI, forecasts temperature, salinity, sea surface height, currents, ice thickness and ice coverage for the Baltic Sea. Some of the most important model equations used in the HBM-model will be briefly explained in the Theory chapter.

The HBM-FMI model has a data assimilation system that is based on optimal interpolation, a method of data assimilation later explained in this work. The assimilation is done for sea surface temperature. The observational data is a satellite product. For clarity, it should be noted that the data assimilation scheme of the HBM-FMI model was not developed in this study, but rather is the object of optimization.

At the first stage of this study the optimal time interval for the assimilation
was tested by running the model without assimilation for reference and with assimilation twice a day, once a day, every second day and every third day. The test period was two months long and in the summer of 2013. The resulting surface temperatures were then compared to wave buoy data and Argo-buoy data from different parts of the Baltic Sea.

In the second stage the optimal thickness of the grid was tested by running the model with different grids for the same test period. The results of this stage were compared to same buoy data as in the first stage. After this the best grid was further tested and an optimal grid was visually defined based on the optimal grid spacing.

Finally a year-long run was done to see the difference the data assimilation with the optimal grid and optimal time interval makes compared to a model run without data assimilation. Also the effect the data assimilation of the sea surface temperature has on the sea surface salinity was studied by comparing the model results to salinity measurements.
Chapter 2

Theory

The first part of this chapter covers the most important physical equations used in the HBM-model. In the second part some of the most common methods of data assimilation will be explained.

2.1 Model equations

This section will briefly present the most essential equations used in the HBM-model.

2.1.1 Momentum Equations

Because the scale of flow is large in the model, the hydrostatic approximation applies and we only have to consider the horizontal momentum equations [1]

\[
\frac{\partial u}{\partial t} + \frac{u}{R \cos \phi} \frac{\partial u}{\partial \lambda} + \frac{v}{R \phi} \frac{\partial u}{\partial \phi} + \frac{w}{\partial z} \frac{\partial u}{\partial z} - \tan \phi \frac{u v}{R} = 2 \omega \sin \phi v - \frac{1}{\rho R \cos \phi} \frac{\partial p}{\partial \lambda} - \frac{1}{\rho R \cos \phi} \frac{\partial}{\partial \phi} (\cos \phi u' v') - \frac{\partial}{\partial z} (u' w') + \frac{\tan \phi}{R} u' v' \quad (2.1)
\]
\[
\frac{\partial v}{\partial t} + \frac{u}{R \cos \phi} \frac{\partial v}{\partial \lambda} + \frac{v}{R} \frac{\partial v}{\partial \phi} + \frac{w}{\partial z} - \frac{\tan \phi}{R} uu = -2\omega \sin \phi u - \frac{1}{\rho} \frac{\partial p}{\partial \phi} - \frac{1}{R \cos \phi} \frac{\partial}{\partial \lambda} (u'v') - \frac{1}{R \cos \phi} \frac{\partial}{\partial \phi} (\cos \phi v'v') - \frac{\partial}{\partial z} (v'w') - \frac{\tan \phi}{R} u'u' \tag{2.2}
\]

Here \( t \) is time, \( R \) is the radius of the earth, \( \phi \) and \( \lambda \) are the latitudes and the longitudes, \( z \) is depth and \( w \) is the mean upward velocity. The variables \( u \) and \( v \) are mean velocities due east and north and \( u', v' \) and \( w' \) are fluctuations according to Reynolds’s decomposition. \( \omega \) is the angular velocity of the rotating earth, \( p \) is pressure and \( \rho \) is the density of sea water.

The above equations are expressed in spherical coordinates. The velocities \( u, v \) and \( w \) are expressed as

\[
u = R \cos \phi \frac{d\lambda}{dt}, \quad v = R \frac{d\phi}{dt}, \quad w = R \frac{dz}{dt} \tag{2.3}\]

The momentum equations for horizontal currents in the sea include the Boussinesq approximation. It assumes that variations in density are small enough to be neglected [2].

The aim of the momentum equations is to calculate the mean flows. However, because the currents in the sea are turbulent, the closure problem presents itself. Here it means that the currents can not be calculated without assuming something about the statistics of the fluctuations [3]. This leads to the necessity of a turbulence model in which the parameterisations for the fluctuation components of the equation are defined.

### 2.1.2 The Continuity Equation

Because water is nearly incompressible, the conservation of mass can be seen as conservation of volume. The mass of water in a constant volume remains approximately constant, independent of pressure. In circular coordinates of the earth it is written as follows [1].

\[
\frac{1}{R \cos \phi} \frac{\partial u}{\partial \lambda} + \frac{1}{R \cos \phi} \frac{\partial(v \cos \phi)}{\partial \phi} + \frac{\partial w}{\partial z} = 0 \tag{2.4}
\]
In the HBM-model the continuity equation is calculated horizontally for squares in the $\phi-\lambda$ plane and vertically for segments bounded by the vertical layers of the model, on the top layer the surface and on the bottom layer the bottom. At the surface and the bottom the kinematic boundary conditions have to be accounted for. With these conditions the continuity equation becomes

$$\frac{1}{R \cos \phi} \frac{\partial}{\partial \lambda} \left( \int_{-H}^{\zeta} u dz \right) + \frac{1}{R \cos \phi} \frac{\partial}{\partial \phi} \left( \int_{-H}^{\zeta} \cos \phi v dz \right) + \frac{\partial \zeta}{\partial t} = 0 \quad (2.5)$$

where $\zeta$ is the surface elevation and $H$ is the depth of the water column. Here the equation has been summed over a water column. The sea surface height can be calculated from this equation.

### 2.1.3 The Hydrostatic balance

The hydrostatic balance, the balance between pressure and buoyancy, is expressed as [4]

$$\frac{\partial p}{\partial z} = -g \rho(z) \quad (2.6)$$

where $p$ is pressure, $g$ is acceleration due to gravity and $\rho(z)$ is the density of water as a function of depth. The hydrostatic pressure at the depth $z$ is determined by integrating the hydrostatic balance equation [1].

$$p(z) = p_{\text{air}} + g \int_{z}^{\zeta} \rho(z) dz \quad (2.7)$$

$p_{\text{air}}$ is the air pressure at the surface and the integration is done from the bottom to the surface. Another way to express the hydrostatic equation is

$$p(z) = p_{\text{air}} + g \rho_0 \zeta + g \int_{z}^{0} \rho(z) dz \quad (2.8)$$
The density $\rho_0$ is the density of surface water.

2.1.4 Shear stresses at the bottom and at the surface

Shear stresses at the surface of the water and at the sea floor affect the flow of water. Winds are the cause of shear stress at the sea surface [1].

$$\tau_{\lambda s} = \frac{\rho_{\text{air}}}{\rho_{\text{water}}} c_D W_\lambda \sqrt{W^2_\lambda + W^2_\phi}$$ (2.9)

$$\tau_{\phi s} = \frac{\rho_{\text{air}}}{\rho_{\text{water}}} c_D W_\phi \sqrt{W^2_\lambda + W^2_\phi}$$ (2.10)

$\tau_{\lambda s}$ and $\tau_{\phi s}$ are the longitudinal and latitudinal components of shear stress at the surface, $\rho_{\text{air}}$ and $\rho_{\text{water}}$ are the densities of air and water accordingly. $c_D$ is the wind drag coefficient. In the HBM-model the value used for it is $c_D = (0.63 + 0.066 W/(m/s))10^{-3}$. $W_\lambda$ and $W_\phi$ are the longitudinal and latitudinal components of wind at the sea surface.

At the sea bottom the cause of shear stress is the horizontal flow of water.

$$\tau_{\lambda b} = r u \sqrt{u^2 + v^2}$$ (2.11)

$$\tau_{\phi b} = r v \sqrt{u^2 + v^2}$$ (2.12)

$\tau_{\lambda b}$ and $\tau_{\phi b}$ are the longitudinal and latitudinal components of shear stress at the sea bottom and $r$ is the bottom friction coefficient. In the HBM-model the value of the bottom friction coefficient is $r = 0.0025$. 
2.1.5 Heat and salt budgets

The heat budget equation [1] is

\[
\frac{\partial T}{\partial t} + \frac{1}{R \cos \phi} \frac{\partial (uT)}{\partial \lambda} + \frac{1}{R \cos \phi} \frac{\partial (v \cos \phi T)}{\partial \phi} + \frac{\partial (wT)}{\partial z} = \\
\frac{1}{R \cos \phi} \frac{\partial}{\partial \lambda} \left( \frac{K_h}{R \cos \phi} \frac{\partial T}{\partial \lambda} \right) + \frac{1}{R \cos \phi} \frac{\partial}{\partial \phi} \left( \frac{\cos \phi K_h}{R} \frac{\partial T}{\partial \phi} \right) \frac{\partial}{\partial z} \left( K_v \frac{\partial T}{\partial z} \right)
\]  

(2.13)

\(K_h\) is the horizontal diffusion coefficient and \(K_v\) is the vertical diffusion coefficient. In the HBM-model they are defined as

\[
K_h = \frac{A_h}{2}, K_v = \frac{A_v}{Pr}
\]

(2.14)

Where \(A_h\) and \(A_v\) are horizontal and vertical eddy viscosities and \(Pr\) is the Prandtl number. The Prandtl number is the relation of kinematic viscosity and thermal diffusivity [3].

According to the heat budget equation the change in heat in time in a volume of water equals to the difference of the diffusion of heat and the advection of heat. On top of this the heat budget is dependent on the heat fluxes at the surface and at the bottom. The heat flux at the surface depends on the state of the atmosphere, namely the air temperature, cloudiness, surface winds and humidity.

The salt budget equation [1] is

\[
\frac{\partial S}{\partial t} + \frac{1}{R \cos \phi} \frac{\partial (uS)}{\partial \lambda} + \frac{1}{R \cos \phi} \frac{\partial (v \cos \phi S)}{\partial \phi} + \frac{\partial (wS)}{\partial z} = \\
\frac{1}{R \cos \phi} \frac{\partial}{\partial \lambda} \left( \frac{K_h}{R \cos \phi} \frac{\partial S}{\partial \lambda} \right) + \frac{1}{R \cos \phi} \frac{\partial}{\partial \phi} \left( \frac{\cos \phi K_h}{R} \frac{\partial S}{\partial \phi} \right) \frac{\partial}{\partial z} \left( K_v \frac{\partial S}{\partial z} \right)
\]  

(2.15)
2.1.6 Effect of surface waves on the large-scale flow

The trajectory of a water particle in a wave is almost circular. However, the circles are not fully closed so a displacement of water mass happens when there are waves. The water moves a little faster in the direction the wave is going than in the opposite direction. In other words, the wave is losing some of its momentum and the momentum becomes mass displacement[5]. This creates a large-scale mass flow called the Stokes drift. The momentum flux related to this is named radiation stress. The circular movement is not confined in the surface waters but reaches all the way to the bottom, becoming fainter as the depth increases.

The following equations [1] are for a wave moving in water of depth $H$ and they are calculated in spherical coordinates. The fluctuation of the surface level of the sea is obtained with the equation

$$
\zeta' = -\frac{a}{\omega} \frac{\partial (\sin(\alpha - \omega t))}{\partial t} = a \cos(\alpha - \omega t) \tag{2.16}
$$

The equation for the fluctuation of pressure is

$$
p' = \rho g a \frac{\cosh(k(H + z))}{\cosh(kH)} \cos(\alpha - \omega t) \tag{2.17}
$$

The fluctuations of the velocities $u$, $v$ and $w$ are obtained from the equations

$$
u' = a \omega \frac{\cosh(k(H + z))}{\sinh(kH)} \frac{1}{kR \cos \phi} \frac{\partial (\sin(\alpha - \omega t))}{\partial \lambda} = \frac{a \omega \cosh(k(H + z))}{\sinh(kH)} \cos(\alpha - \omega t) \frac{1}{kR \cos \phi} \frac{\partial \alpha}{\partial \lambda} \tag{2.18}
$$

$$
u' = a \omega \frac{\cosh(k(H + z))}{\sinh(kH)} \frac{1}{kR} \frac{\partial (\sin(\alpha - \omega t))}{\partial \phi} = \frac{a \omega \cosh(k(H + z))}{\sinh(kH)} \cos(\alpha - \omega t) \frac{1}{kR} \frac{\partial \alpha}{\partial \phi} \tag{2.19}
$$
\[ w' = a\omega \frac{\sinh(k(H + z))}{\sinh(kH)} \sin(\alpha - \omega t) \]  \hspace{1cm} (2.20)

Where \( a \) denotes the amplitude of the wave, \( \omega \) is the angular velocity, \( k \) is the wave number and \( \alpha \) is the phase of the wave. In a two-dimensional system \( \alpha \) is a two-dimensional function. The waves propagate in the direction of the gradient of this phase function \( \alpha \). This can be denoted as an angle theta:

\[
\cos \theta = \frac{1}{kR \cos \phi} \frac{\partial \alpha}{\partial \lambda} \hspace{1cm} (2.21)
\]

\[
\sin \theta = \frac{1}{kR} \frac{\partial \alpha}{\partial \phi} \hspace{1cm} (2.22)
\]

The trajectories of the propagating waves are always curving in spherical coordinates.

### 2.1.7 Sea ice thermodynamics

The sea ice variables calculated in the HBM-FMI are ice thickness \( h \) and ice coverage \( A \). Ice coverage is the percentage of a grid cell covered by thick ice. There is a threshold value between thin and thick ice and in the ice model thin ice is treated as open water. To determine the change in ice thickness and coverage in time the following equations are used [6]

\[
\frac{\partial h}{\partial t} = -\frac{\partial(uh)}{\partial x} - \frac{\partial(vh)}{\partial y} + S_h + \text{diffusion} \hspace{1cm} (2.23)
\]

\[
\frac{\partial A}{\partial t} = -\frac{\partial(uA)}{\partial x} - \frac{\partial(vA)}{\partial y} + S_A + \text{diffusion} \hspace{1cm} (2.24)
\]

Here \( A \) is smaller than or equal to 1. \( S_h \) and \( S_A \) are thermodynamic terms

\[
S_h = f(h/A)A + (1 - A)f(0) \hspace{1cm} (2.25)
\]
\[ S_A = \begin{cases} \frac{f(0)}{h_0} (1 - A) & \text{if } f(0) > 0 \\ 0 & \text{if } f(0) < 0 \\ 0 & \text{if } S_h > 0 \\ \frac{A}{2h} S_h & \text{if } S_h < 0 \end{cases} \] (2.26)

Here \( h_0 \) is the threshold thickness between thin and thick ice and \( F(h) \) is the growth rate of ice thickness.

### 2.2 Data assimilation

The idea of data assimilation is to make the model more accurate by merging observational data into the model state. In forecast models data assimilation is most often done in cycles so that the observational part is added to the model part and the analysis obtained in this way is used as the starting point for the next model run with assimilation [7]. An important question in data assimilation is how much weight is given to the observations and how much to the model. This question is handled in multiple ways in different data assimilation algorithms.

Data assimilation benefits the model by bringing it closer to reality. With an accurate model information about unobserved areas can be retrieved. Data assimilation can also give new information about errors in the model and in the observations based on which the observation instruments and the model can be further developed [8].

There are different ways of assimilating the observations into the model. Data assimilation can be done either sequentially or retrospectively and intermittently or continuously [9]. In sequential assimilation only observations from the past are used, whereas in retrospective assimilation future observations are used as well. In intermittent assimilation the observations are gathered from a small length of time. In continuous assimilation the observations are gathered from a longer time. This makes continuous assimilation smoother and more physically correct. However, intermittent assimilation is easier to do in practice.

In choosing the suitable algorithm for data assimilation many things have to be considered. There are definite downsides to the simpler data assimilation algorithms but also the difficulties in the implementation of the data
assimilation method and the numerical cost of using it have to be taken into consideration in practice [9].

Because weather forecast models tend to be made as accurate as possible with a best possible resolution, they are usually on the edge of current computational power and very heavy to run. This is why the data assimilation as an extra piece to the whole has to be made as low in numerical cost as possible. The result of these considerations is usually some sort of compromise between accuracy and efficiency.

The methods explained here include the successive corrections methods (the Cressman scheme and the Barnes scheme), optimal interpolation and variational methods (3D-VAR and 4D-VAR). These are some of the most commonly used ones.

2.2.1 Some basic terms

Some of the most basic terms [9] that will be repeatedly used are explained here.

The background field is the model field used in the assimilation process. The background model state vector \( x_b \) is the vector with the needed variables to describe the background field.

The analysis is the result of the assimilation process. The analysis model state vector \( x_a \) is the vector describing the analysis field.

Both the background model state vector and the analysis model state vector are attempts at describing reality. The true state \( x_t \) is the best description of reality in vector form. A perfect analysis model state vector \( x_a \) or a perfect background state vector \( x_b \) would be equal to the true state vector.

2.2.2 Successive Corrections Methods

The Cressman scheme and the Barnes scheme are two often used successive corrections methods [10]. The following part is common for both. The differences arise in the definition of the weighting function.
Modelled variables at each grid point are corrected with observations within the radius of influence from the grid point in the successive corrections methods [10]. This is done \(m\) times. The radius of influence is reduced with every iteration. By reducing the radius of influence the smaller scale features from the observations become visible against a background of larger scale observational features. The following formula [9, 10] is used to correct the variable \(x\) at grid point \(j\), for observation \(i\):

\[ x_a(j) = x_b(j) + \frac{\sum_{i=1}^{n} \omega(i, j)\{y(i) - x_b(i)\}}{\sum_{i=1}^{n} \omega(i, j) + \varepsilon^2} \tag{2.27} \]

Here \(y(i)\) is the \(i\):th observation from the gridpoint and \(\omega(i, j)\) is a weighting function that depends on the observation’s distance from the gridpoint. \(\varepsilon^2\) is the ratio of the observational error and the error of the background. For flawless observations \(\varepsilon^2\) would be zero.

In the Cressman scheme the weighting function [9, 10, 11] is:

\[ \omega(i, j) = \frac{R^2 - d_{i,j}^2}{R^2 + d_{i,j}^2}, \text{ for } d_{i,j}^2 \leq R^2 \tag{2.28} \]

\[ \omega(i, j) = 0, \text{ for } d_{i,j}^2 > R^2 \tag{2.29} \]

where \(R\) is the radius of influence and \(d_{i,j}\) is the distance between the gridpoint and the observation point. As can be seen from the equation above the observations outside of the radius of influence are excluded from the assimilation. In the Cressman scheme \(\varepsilon^2\) is zero.

According to the Barnes scheme [10] the weighting function is:

\[ \omega(i, j) = \exp \left( -\frac{d_{i,j}^2}{2R^2} \right) \tag{2.30} \]

In contrast to the Cressman scheme the Barnes scheme does not eliminate observations that are outside the radius of influence. It is often used in
analysing observational data without a background field. This being the case $\varepsilon^2$ is again zero.

Sometimes it can be problematic to define the weighting function [9]. The weighting functions of the aforementioned Cressman and Barnes schemes are just two ways of doing this. The weighting function can be defined so that the area of influence is for example shaped like an ellipse or a curved ellipse instead of a circle [11].

Because the errors are assumed to be zero, the weight of the observation is only dependent on the distance between the observation and the grid point [9]. In this case, if the observation is exactly at the gridpoint, the weight equals to one. This means that after the assimilation the value at the gridpoint is the same as the observed value. This is a disadvantage because the observed values have errors and this way all the information from the model at that gridpoint is lost. If the model is more accurate than the observations this just worsens the results compared to the background field. Usually a compromise between the model and the observations would be better.

In an analysis field assimilated with the successive corrections methods, if the errors are not taken into account, there are gradients that can be too steep and therefore physically unrealistic, especially if the observations are sparse [9]. These are some of the reasons why it is necessary to take the observational errors and the background errors into account when combining model results and observations.

2.2.3 Optimal Interpolation

Optimal interpolation is the data assimilation method used in the data assimilation system of HBM-FMI. It takes the background and observational errors into account and allows different weighting for observations from different sources [12]. The optimal interpolation analysis can be obtained with the following equation [13].

$$T_a = T_b + \sum_{i=1}^{n} \alpha_i (T_{oi} - T_b)$$  \hspace{1cm} (2.31)
Where $a$ refers to the analysis, $b$ to the background and $o$ to the observation. The letter $i$ notes the number of the observation.

This equation can be expressed in matrix form as follows [9, 14]. In it’s matrix form the optimal interpolation equation is valid for any number of variables and gridpoints.

$$x_a = x_b + K(y - H[x_b])$$

(2.32)

Here $y$ is the observation vector. $H$ is called the observation operator. It consists of the functions needed to compare the background state to the observed values. It has as many lines as there are elements in the observation vector. The functions in the observation operator make the conversions from the model grid points to the observation points. Thus, $(y - H(x_b))$ is the difference between the observations and the background field. The weight matrix $K$ can be expressed as follows [9].

$$K = BH^T (HBH^T + R)^{-1}$$

(2.33)

where $B$ is the covariance matrix of background errors and $R$ is the covariance matrix of observational errors. These will be explained more in the following section.

The basic goal is to reduce the total analysis error as much as possible. The weight matrix is small if the observational errors are bigger than the background errors. In other words the observations have only a small effect on the final analysis in this case. Respectively, if the model errors in the background field are larger than the observational errors, the weight matrix will be large and the final analysis will be strongly affected by the observations.

### 2.2.4 Errors

The background field, the observations and the analysis field all contain errors. These errors can be due to many reasons, for example instrumental errors in the observations, errors in the observation operator $H$ and modelling errors of the background field [9].
Probability density function

The background error vector is the difference of the background state vector and the true state vector [9, 13].

$$\varepsilon_b = x_b - x_t \quad (2.34)$$

The observational error vector is defined in a corresponding way:

$$\varepsilon_o = y - H(x_t) \quad (2.35)$$

The error vector is exactly known at the moment of comparing the background state vector and the true state vector. However, there are too many possible sources of error and it is impossible to know the exact proportions by which the different causes affect the background field. Thus, if we repeated the above equation a large amount of times with different errors in the background field, the error vector would get different values every time. The probability density function is the area limited by a histogram of the different errors. This is a good guess about the distribution of the error vector. Things like variances and average values can be calculated from the probability density function.

Covariance matrices

$B$ is the covariance matrix of the background errors [9]

$$B = (\varepsilon_b - \bar{\varepsilon}_b)(\varepsilon_b - \bar{\varepsilon}_b)^T \quad (2.36)$$

$\varepsilon_b$ is the background error and $\bar{\varepsilon}_b$ is the average background error.

$R$ is the covariance matrix of observational errors.

$$R = (\varepsilon_o - \bar{\varepsilon}_o)(\varepsilon_o - \bar{\varepsilon}_o)^T \quad (2.37)$$

$\varepsilon_o$ is the observational error and $\bar{\varepsilon}_o$ is the average observational error.
The background error covariance matrix is of a size $n \times n$ when the background vector is of a dimension $n$. On its diagonal the background error covariance matrix has the variances for every model variable and the other elements are cross-covariances of different pairs of model variables. For a three dimensional model

$$B = \begin{bmatrix} \text{var}(e_1) & \text{cov}(e_1, e_2) & \text{cov}(e_1, e_3) \\ \text{cov}(e_1, e_2) & \text{var}(e_2) & \text{cov}(e_2, e_3) \\ \text{cov}(e_1, e_3) & \text{cov}(e_2, e_3) & \text{var}(e_3) \end{bmatrix}$$

(2.38)

where $e_1$, $e_2$ and $e_3$ are background errors minus their means.

The observational error covariance matrix is defined with similar logic.

In weather prediction models of the atmosphere or the ocean, the background errors depend on the physical conditions of the real world and on the accuracy of the model equations. Background error variances can be estimated by looking at the climatological variances of the background field. If there are enough observations, the background error variances can be calculated more accurately from the variance of the discrepancies between the forecast field without analysis and the analysis field.

Observation error variances can be defined if the properties of the observation instruments are known. If there is bias in the observations or the background, it cannot be included in the observation error variances or the background error variances but has to be eliminated from the observed values or the background values accordingly.

### 2.2.5 Variational Methods

**3D-VAR**

The aim of variational methods is to find values for the vector $x$ that minimize the cost function [9, 15, 16, 17]:

$$J(x) = (x - x_b)^T B^{-1} (x - x_b) + (y - H[x])^T R^{-1} (y - H[x])$$

(2.39)

The minimizing of the cost function is done by iteratively calculating values for the cost function and its gradient [9, 17].
\[ \nabla J(x) = 2B^{-1}(x - x_b) - 2H^TR^{-1}(y - H[x]) \] (2.40)

The minimizing of the cost function is usually started from the background values. The point of starting the minimization is called the first guess. There are two common ways of ending the minimization [9]. The count of iterations can be limited or there can be a defined amount the norm of the gradient of the cost function should decrease. The change in the norm of the gradient mirrors how much closer to the ideal values the iteration process has taken the analysis.

4D-VAR

The 4D-VAR method is like the 3D-VAR method but it has a dependency on time. In the 4D-VAR method the data assimilation is done in a time window where there are many observations [7]. All the observations in the time window affect the analysis according to their distance from the point of analysis in time. In the case of 4D-VAR analysis the cost function becomes [9, 7, 18]

\[ J(x) = (x - x_b)^TB^{-1}(x - x_b) + \sum_{i=0}^{n} (y_i - H_i[x_i])^TR_i^{-1}(y_i - H_i[x_i]) \] (2.41)

where \( i \) denotes the moment in time. Thus, \( y_i \) is the observation at time \( i \), \( x_i \) is the background field at time \( i \) and \( H_i \) is the observation operator at time \( i \).

The model variables \( x_i \) are dependent on the model equations [9, 7, 18]

\[ x_i = M_{0\rightarrow i}(x) \] (2.42)

where \( M_{0\rightarrow i} \) is the model operator from the time 0 to the time \( i \). This makes it hard to solve the 4D-VAR equation in a general case. However, causality of the modelled phenomenon that translates to the forecast model in that it can be done in timesteps makes the determination of the background state at different times more simple. To further simplify the solution of the 4D-VAR
problem it can be assumed that the possibly non-linear $H_i$ and $M$ operators can be linearized [9].

Because of the aforementioned dependency on the model equations the 4D-VAR algorithm has the advantage of being totally coherent with the physical equations of the model whereas the 3D-VAR algorithm lacks this property [9]. This makes the 4D-VAR method in a way a better choice for assimilating observations into a forecast model.

However, there are also downsides to using the 4D-VAR algorithm in a real-time forecast model [9]. The assimilation process can be slower because the system has to wait for all the observations for the 4D-VAR period to start the assimilation. Also, the M operators have to be defined and this can be time consuming. Furthermore, the 4D-VAR method does not work well if the model errors are too large.

### 2.2.6 Analysis errors

Knowledge of the accuracy of the analysis is valuable. Especially in the case of a forecast model where the previous analysis is used as the initial background field it is important to know the analysis errors. In a forecast model the errors in the analysis consist of the errors in the previous analysis and the errors in the model that accumulate in time according to the accuracy of the model.

In optimal interpolation the analysis error covariance matrix [9] is

$$
A = (I - KH)B(I - KH)^T + KRK^T
$$

(2.43)

where $I$ is the unit matrix.

For variational methods, the analysis error covariance matrix [9, 18] is

$$
A = \left(\frac{1}{2}J''\right)^{-1}
$$

(2.44)

where $J''$ is called the Hessian of the cost function. It is the second derivative of the cost function with respect to $x$. 

18
Chapter 3

Methods

3.1 The HBM-model

The HBM-model is offspring of the German BSHcmod ocean circulation model from 1990s [19]. Since then the BSHcmod model was further developed in the MyOcean-project. Especially Sweden and Denmark developed their own branches from the BSHcmod model for operational use. HBM is an attempt to create a unified Baltic Sea model using model development experience from around the Baltic. HBM has been operational at FMI since 2012 running twice a day.

The grid of HBM is a staggered Arakawa C-grid [19]. In the Arakawa C-grid scalar variables are located at the grid points and velocities are located halfway between the gridpoints. Horizontally the grid is spaced evenly in degrees but in distance the grid spacing is sparser in the north-south direction than in the east-west direction. The area modelled in HBM-FMI is from 53.025 to 65.875°N and from 9.041667 to 30.29167°E (see Figure 3.1). In the optimization the focus is on the Baltic Sea leaving out the Danish Straits. Vertically the grid spacing depends on depth. Vertical layers until 100 meters are 5 meters thick. From there until 200 meters the layer thickness is 50 meters and after that 100 meters.

The HBM-FMI model is run twice a day with two different atmospheric forcings. Hirlam (High Resolution Limited Area Model) is the main model
used in FMI weather forecasts and ECMWF (European Centre for Medium-Range Weather Forecasts) produces another atmospheric forcing. At the moment of writing this the HBM-FMI model produces a forecast of 52 hours with the Hirlam forcing and a forecast of 69 hours with the ECMWF forcing, both twice a day. The modelled variables in HBM-FMI are temperature, salinity, sea surface height, currents, ice thickness and ice coverage.

### 3.2 Data assimilation in HBM-FMI

There is a preoperational data assimilation system in HBM-FMI. It is based on the optimal interpolation data assimilation scheme. The assimilated variable is sea surface temperature.

The data used for the assimilation is combined data from several satellites and it is a product of the MyOcean-project, an EU-funded ocean monitoring and forecasting project. The satellite data is renewed once a day. The assimilation grid points used in the preoperational assimilation system before optimization are presented in Figure 3.1.

The starting point of this study is a data assimilation system where the assimilation is performed twice daily, that is every time the model is run.

### 3.3 The method of optimization

The method of optimization developed in this study consists of three parts. First an optimal interval for the assimilation was tested. The model was run for a 2-month test period with assimilation twice a day, once a day, every second day, every third day and for reference without assimilation. The surface temperatures from the model runs were then compared to surface temperatures measured at four wave buoys and an Argo-buoy. The optimal time interval was defined from these comparisons and this time interval was used in the next stages of the optimization.

The second step in the study was to define an optimal thickness for the assimilation grid, the grid for the satellite observations. The model was run with three grids of different thicknesses. The starting grid is the grid
used in the first stage of the optimization process. Because the satellite measurements are done in a thicker grid, the starting grid is picking every 32nd observation in both latitudes and longitudes. The other two grids are twice as thick (every 16th point) and half as thick (every 64th point) as the original grid. The optimal grid spacing was determined by comparing the accuracies of the model results and the time the assimilation with different grids adds to the running time of the model. After this stage the thinner grids were further tested to ensure that they indeed are thick enough to affect the areas further away from the assimilation grid points. This was achieved by running the model for the test period with placing the thinner grids in different ways. This changed the distances between the assimilation grid points and the buoys and made it possible to study the differences caused by grid placing.

The third step was to visually improve the previously defined optimal grid by moving the assimilation grid points so that they were better placed according to the shape of the shores and bays. A 12-month test run was done with this improved grid and for reference a run without assimilation was done as well. These were again compared with the wave buoys to see the effects of the optimized data assimilation.

Figure 3.1: The preoperational assimilation grid before optimization for the full area. The dots denote the assimilation points.
Chapter 4

Results

The test runs were done for a two-month period from 1.7.2013 to 31.8.2013. The model results were compared to Argo-buoy measurements carried out in the Bothnian Sea and wave buoy measurements from four different buoys in the Baltic Sea, one in the Bothnian Bay, one in the Bothnian Sea, one in the Gulf of Finland and one in the Baltic Proper. The positions of the buoys are shown in the data assimilation grid figures. The results are visualized in two ways: as surface temperature curves for different model runs and observed values and as absolute differences between the observed temperature at the buoy and the modelled temperature. Figures for the mean differences of the four wave buoys and the different model runs are presented as well. Because all the model runs start from the same state, the model run curves in every figure start visibly separating from each other only after a few days.

4.1 Effects of the time interval of data assimilation

In the first part of the study different time intervals for the data assimilation were tested. Here are presented the results of model runs with assimilation intervals of 0.5, 1, 2 and 3 days. For reference also a model run without assimilation is presented. Here the grid for the assimilation is as seen in Figure 4.1. From now on this grid will be called Grid1.
All the buoy figures consist of two graphs. The upper one represents the observed sea surface temperatures at the buoy in question as well as the modelled ones and the satellite observations used in the data assimilation. The interruptions in the satellite temperature curves are due to local cloud coverage. The absolute differences of the modelled temperatures and the buoy temperatures are presented in the lower graph. In the legend of the Figures 4.2 to 4.7 noasm refers to the model run without assimilation, asm1 is the model run with assimilation twice a day, asm2 with assimilation once a day, asm4 with assimilation once in two days and asm6 with assimilation once in three days. Sat refers to the satellite observations and buoy to the temperatures measured at the buoy in question. Means of the absolute differences of buoy temperatures and model temperatures at every run are calculated in Table 4.1.

4.1.1 Argo-buoy

As can be seen in Figure 4.2, the observed Argo-buoy temperatures were mostly lower than the modelled ones. The effects of the data assimilation can clearly be seen from the absolute difference graph. It shows that the model results are mostly closer to the buoy measurements when data assimilation is done more often and that the quality decreases when it is done less, as expected. This feature is also visible in the mean values of Table 4.1.

The biggest differences between the model and the buoy measurements seem to be caused by sharp changes in the measured surface temperature. The model follows these changes more slowly. However, it has to be noted that at this particular time and place some of the biggest differences between the model and buoy measurements coincide not only with sharp temperature changes but also with the absence of satellite data due to clouds. This can be seen for example around the end of July when the difference between the model and the measurements is at its biggest.

Another notable feature is that the satellite measurements are for the most part closer to the buoy measurements than the model. This means that they are taking the model results mostly in the right direction. This is of course a most important feature of observational data used in data assimilation because if the observed data was of a worse quality than the model results without assimilation, the data assimilation would be of no use.
4.1.2 Wavebuoys

The general features of the wave buoys are visible in Figure 4.3. In July the model results seem to get better the shorter the interval between the runs with assimilation is. In August the differences between different runs get considerably smaller. The mean values for the mean curves Table 4.1) are smaller for the model runs with more frequent assimilation.

HELS-buoy

At the HELS-buoy, as opposed to the Argo-buoy, the surface temperatures observed at the buoy seem to be mostly higher than the modelled temperatures as can be seen from Figure 4.4. The effects of the data assimilation are not clear looking at these graphs. The mean values of Table 4.1 are very close to each other as well. Some similar features as in the Argo-buoy graphs are visible a few days before 21.7.2013 but only for a short while. This period of consistency with expectations does not coincide with the availability of satellite data as might be expected. There is another shorter period like this at about 27.7.2013. For this period there is satellite data available.

For the most part the model temperatures of different runs are close to each other. This might be explained with the distance of the buoy in question from the nearest assimilation grid point. The nearest grid point is not very close to the buoy as can be seen from Figure 4.1.

The biggest differences between the model and the buoy measurements are at times when the measured temperature changes sharply. The satellite measurements seem to follow the buoy measurements pretty well.

PERM-buoy

The modelled surface temperatures compared to the PERM-buoy measurements are consistent with the results at the Argo-buoy. For the most part the assimilated model runs show better results than the model run without data assimilation and the model runs with data assimilation done more often show smaller differences to the buoy temperatures than the model runs with
assimilation done with a longer interval. This can also be seen looking at the mean values of Table 4.1.

As can be seen in Figure 4.5, the surface temperatures measured at the buoy are for the most part lower than the modelled temperatures. There is a period in the first half of August when the buoy temperature is very close to the model temperatures. This accounts for the chaotic-looking part in the temperature difference graph. The effect of sharp temperature changes is again visible throughout the period of the study and the satellite measurements follow the curve of buoy measurements well.

**SELK-buoy**

Figure 4.6 shows similar results for the SELK-buoy as the results at the Argo-buoy and the PERM-buoy. This feature is visible in the mean values for the SELK-buoy in Table 4.1. The measured surface temperature at the buoy is mostly lower than the temperatures from the model. The aforementioned things about sharp temperature changes and the relation of the satellite observations and the buoy measurements go with this buoy as well.

In the summer of 2013 there was a problem with the temperature sensor of the SELK-wavebuoy. The temperatures measured by the buoy at that time are lower than they are supposed to be. The exact amount of error is hard to define since it was not constant but increased during the measuring period so that on 30.5.2013 the error was -0.8 °C and on 24.1.2014 it was -1.26 °C. Here the crude assumption has been made that the temperature was about 1 °C lower than supposed during the period 1.7.2013-31.8.2013 and it has been corrected accordingly for the purposes of this study.

**POHJ-buoy**

As can be seen in Figure 4.7, the measured temperatures at the buoy are mostly lower than the model temperatures. However, for approximately the second half of August the surface temperatures measured at the buoy are very close to the modelled temperatures. This is why the graphs at the end part of the difference figure are so close to each other and in a seemingly unexpected order.
In general, there are similar features at this buoy as at the previous ones excluding the HELS-buoy. The assimilation seems to have a positive effect on the quality of the model results. Table 4.1 shows that the mean difference values mostly decrease in the order of shorter assimilation intervals.

4.1.3 In conclusion

From these results we can conclude that model temperatures are closest to observations when the assimilation is done as often as possible. The model test period with assimilation twice a day seems to give the most realistic surface temperatures when compared to the buoy measurements. However, because the satellite data for the assimilation comes only once a day, it is more efficient to do the assimilation once a day. The following model test runs are done with this time interval of data assimilation.

Table 4.1: Means of temperature differences between the buoys and the model runs with different time intervals. In the MEAN column means of the HELS-, PERM-, SELK- and POHJ-buoy temperature differences. The temperature differences are in units of °C.

<table>
<thead>
<tr>
<th>Run</th>
<th>HELS</th>
<th>PERM</th>
<th>SELK</th>
<th>POHJ</th>
<th>MEAN</th>
<th>ARGO</th>
</tr>
</thead>
<tbody>
<tr>
<td>noasm</td>
<td>0.804001</td>
<td>0.664174</td>
<td>0.882108</td>
<td>0.588791</td>
<td>0.734769</td>
<td>0.910649</td>
</tr>
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<td>0.624988</td>
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<td>0.731195</td>
</tr>
<tr>
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<td>asm6</td>
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<td>0.804163</td>
</tr>
</tbody>
</table>
Figure 4.1: Grid1. Red dots denote the assimilation grid points.
Figure 4.2: Argo-buoy. Sea surface temperatures and temperature differences between the buoy and the model runs with different time intervals of assimilation and satellite observations.
Figure 4.3: Mean differences of the model runs with different time intervals of data assimilation at the wave buoys (HELS, PERM, SELK, POHJ).
Figure 4.4: HELS-buoy. Sea surface temperatures and temperature differences between the buoy and the model runs with different time intervals of assimilation and satellite observations.
Figure 4.5: PERM-buoy. Sea surface temperatures and temperature differences between the buoy and the model runs with different time intervals of assimilation and satellite observations.
Figure 4.6: SELK-buoy. Sea surface temperatures and temperature differences between the buoy and the model runs with different time intervals of assimilation and satellite observations.
Figure 4.7: POHJ-buoy. Sea surface temperatures and temperature differences between the buoy and the model runs with different time intervals of assimilation and satellite observations.
4.2 Effects of the data assimilation grid spacing on the results of the HBM-FMI model

The next step of the study was to determine an optimal grid spacing for the data assimilation. The starting point for this part was the aforementioned model run with assimilation once a day and with the assimilation grid Grid1. Two other model test runs were performed with Grid2 (Figure 4.8) and Grid3 (Figure 4.9). Grid2 is twice as thick as Grid1 and Grid3 is half as thick as Grid1. A notable feature of the spacing of the data assimilation grids is that the grid cells are longer in the north-south direction than in the east-west direction. The grids are evenly spaced in latitudinal and longitudinal degrees.

The interpretation of the following results is made difficult by the changing distance from the assimilation grid points to the buoys. Because we are looking only at a few points it is hard to draw conclusions about the whole sea. However, some things can be said about the whole, keeping in mind that, in the case of a single buoy, the results are strongly affected by the place of the buoy compared to the assimilation grid points.

4.2.1 Argo-buoy

At the Argo-buoy (Figure 4.10) the model run with Grid2 comes closest to the temperatures measured at the buoy. The second best is Grid1. The same features are visible in Table 4.2. Especially in the case of the drifting Argo-buoy the changing distances from the buoy to the nearest assimilation grid point make the results hard to evaluate. The buoy moves due north-east along the trajectory illustrated in the grid figures. At the end of the test period the buoy is very close to assimilation grid points in grids Grid1 and Grid2. There are two other earlier instances when the buoy comes very close to grid points in Grid2.
4.2.2 Wavebuoys

Looking at Figure 4.11, in July the model results seem to be best with the thickest grid and second best with the original grid. In August the differences between model runs are smaller. In Table 4.2 the means of the mean curves are smaller the thicker the assimilation grid is.

HELS-buoy

With the HELS-buoy (see Figure 4.12) the differences between different model runs, especially with the grids Grid 1 and Grid3, are very small. Grid2 is the best one again. The nearest grid point to the HELS-buoy is very near in the thickest Grid2. For the other grids it is further away. This can explain some of the difference between the run with the thickest grid and the other runs. According to Table 4.2, Grid2 is the only grid that provides any improvement at the HELS-buoy.

PERM-buoy

At the PERM-buoy (see Figure 4.13) the model run with Grid2 is the closest one to reality. For the most part Grid1 seems to be the second best option and Grid3 the worst. The mean values for the PERM-buoy in Table 4.2 confirm this. The closest point of assimilation in Grid2, like in Grid1, is closer to the PERM-buoy, whereas the closest point of assimilation in Grid3 is much further away.

SELK-buoy

At the SELK-buoy (see Figure 4.14) for half of July the model run with Grid2 seems again to work the best. After this period however, the different model runs are very close to each other. Also here the distance between the buoy and the closest assimilation grid point is smallest with Grid1 and Grid2 and bigger with Grid3. Table 4.2 shows a smaller differences between the model run with Grid1 and Grid2 than with the third grid.
POHJ-buoy

A similar feature as with the SELK-buoy in July is visible at the POHJ-buoy as well (see Figure 4.15). For the rest of the period of the study the results are very close to each other. The distance of the buoy from the nearest assimilation grid point is again considerably longer for Grid3 than the other grids. According to the mean values for the POHJ-buoy in Table 4.2, Grid1 is the most accurate one.

4.2.3 In conclusion

We can conclude that the spacing of the assimilation grid visibly affects the outcome of the model. Solely based on the accuracy of the model results, Grid2 would be the best choice. However, there is also the aspect of efficiency to consider. The assimilation added 10 minutes to the model run with Grid3, 13 minutes with Grid1 and 45 minutes with Grid2. Based on this Grid1 seems like a better choice for operational use although it is not quite as accurate as the thicker grid.

Table 4.2: Means of temperature differences between the buoys and the model runs with grids of different thicknesses. In the MEAN column means of the HELS-, PERM-, SELK- and POHJ-buoy temperature differences. The temperature differences are in units of °C.

<table>
<thead>
<tr>
<th>Run</th>
<th>HELS</th>
<th>PERM</th>
<th>SELK</th>
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<tr>
<td>noasm</td>
<td>0.804001</td>
<td>0.664174</td>
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</tr>
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Figure 4.8: Grid2. Red dots denote the assimilation grid points.
Figure 4.9: Grid3. Red dots denote the assimilation grid points.
Figure 4.10: Argo-buoy. Sea surface temperatures and temperature differences between the buoy and the model runs with different assimilation grids and satellite observations.
Figure 4.11: Mean differences of the model runs with different assimilation grids at the wave buoys (HELS, PERM, SELK, POHJ).
Figure 4.12: HELS-buoy. Sea surface temperatures and temperature differences between the buoy and the model runs with different assimilation grids and satellite observations.
Figure 4.13: PERM-buoy. Sea surface temperatures and temperature differences between the buoy and the model runs with different assimilation grids and satellite observations.
Figure 4.14: SELK-buoy. Sea surface temperatures and temperature differences between the buoy and the model runs with different assimilation grids and satellite observations.
Figure 4.15: POHJ-buoy. Sea surface temperatures and temperature differences between the buoy and the model runs with different assimilation grids and satellite observations.
4.3 Changing positioning of the grid in comparison to the observation locations

It is necessary to know the effect of the distance between the buoys and the data assimilation grid points. If there are large differences in the quality of the results depending on the distance between the point in consideration and the assimilation grid points surrounding it, the grid of assimilation is too thin. The effects were studied by moving Grid1 and Grid3 resulting in Grid1.2, Grid1.3, Grid3.2 and Grid3.3 (see Figures 4.16 and 4.17).

4.3.1 Argo-buoy

At the Argo-buoy (see Figures 4.18 and 4.19) the differences between the assimilation runs with the grids Grid1, Grid1.2 and Grid1.3 are relatively small. It seems like Grid1 is dense enough to be used in the assimilation. For the thinner Grid3 the differences were even smaller, as can be seen in Table 4.4. This is because all the different assimilation runs were closer to the reference run without assimilation. It just means that the assimilation generally does not make that much difference with a grid this thin.

4.3.2 Wavebuoys

In the mean difference figures (Figure 4.20) from the wavebuoys, similar features are visible as with the Argo-buoy. The differences for Grid3 are smaller than the differences for Grid1. This can be seen in the mean values in tables 4.3 and 4.4.

4.3.3 In conclusion

Grid1 seems to be thick enough to be used as the basis of the optimal grid. The differences according to the distance from the buoy with this grid are not pronounced.
Table 4.3: Means of temperature differences between the buoys and the model runs with variations of Grid1. In the MEAN column means of the HELS-, PERM-, SELK- and POHJ-buoy temperature differences. The temperature differences are in units of °C.

<table>
<thead>
<tr>
<th>Run</th>
<th>HELS</th>
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<th>SELK</th>
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Table 4.4: Means of temperature differences between the buoys and the model runs with variations of Grid3. In the MEAN column means of the HELS-, PERM-, SELK- and POHJ-buoy temperature differences. The temperature differences are in units of °C.

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<td>0.867068</td>
<td>0.551655</td>
<td>0.713697</td>
<td>0.849082</td>
</tr>
</tbody>
</table>
Figure 4.16: Grid1.2 and Grid1.3. Red dots denote the assimilation grid points.
Figure 4.17: Grid3.2 and Grid3.3. Red dots denote the assimilation grid points.
Figure 4.18: Argo-buoy. Sea surface temperatures and temperature differences between the buoy and the model runs with different Grid1 placings and satellite observations.
Figure 4.19: Argo-buoy. Sea surface temperatures and temperature differences between the buoy and the model runs with different Grid3 placings and satellite observations.
Figure 4.20: Mean differences of the model runs with variations of Grid1 and Grid3 at the wave buoys (HELS, PERM, SELK, POHJ).
4.4 Impact of the improved assimilation

A final grid Grid4 (see Figure 4.21) was visually defined based on Grid1. 12-month test runs, from 1.7.2013 to 30.6.2014, were done to get a better sense of the impact of the improved data assimilation system with Grid4 compared to a HBM-FMI model run without assimilation. In Figures 4.22 to 4.25 a moving average of 20 surrounding values has been calculated for the buoy. This was done because there were buoy measurements for every half hour and as a consequence a lot of noise was present in the buoy-graphs.

4.4.1 HELS-buoy

At the HELS-buoy (Figure 4.22) the assimilated model run is almost all the time closer to the buoy temperatures. The difference between the model runs is especially clear from 15.11.2013 to 11.1.2014. During this period the model results were further away from the buoy measurements than at other times.

There is no buoy data available from 22.1.2014 to 1.4.2014 because the buoy was lifted from the sea for the winter. The model shows ice from the end of January 2014 until the end part of February. The model run with assimilation shows the temperatures rising more than a week earlier than the run without assimilation.

The measured buoy temperatures are higher than the model temperatures almost all year. Only from the end of May 2014 to June 2014 there is a clearer period when the measured temperatures are lower than the modelled temperatures.

4.4.2 PERM-buoy

The results from the PERM-buoy are presented in Figure 4.23. Also at the PERM-buoy the assimilation has taken the model closer to the measured temperatures.

There is a longer period during which buoy temperatures are not available, from 27.12.2013 to 26.5.2014. This is because the buoy was lifted from the sea.
well before there was ice and put back only after the ice had fully melted. The model shows ice from the middle of January to the beginning of March.

The buoy temperatures are lower than the model temperatures mostly through the summer of 2013 and the beginning of autumn. For the end of autumn and for December 2013 the buoy temperatures are higher than the model. The model is mostly warmer for the period of 2014 for which there is buoy data.

4.4.3 SELK-buoy

For the SELK-buoy (Figure 4.24) the assimilated run is mostly better than the model run without assimilation. The buoy temperatures seem to be higher and lower than the model at approximately the same times for the SELK-buoy as for the PERM-buoy.

The model run without assimilation shows ice from the middle of January until the end of February. The ice period is only half of this for the model run with assimilation.

There is a break in the buoy observations from 24.1.2014 to 6.4.2014. During this time the temperature sensor of the buoy was calibrated and the temperatures should be right for the spring and summer of 2014.

4.4.4 POHJ-buoy

At the POHJ-buoy (Figure 4.25) the measured temperatures fluctuate a lot in the scale of days. The Assimilation brings the model closer to the measurements most of the time.

The POHJ-buoy is the only one with a full year of measurement. The model temperatures stay above zero for the whole year. In the winter the gap between the buoy measurements and the modelled temperatures is especially big.

There is a similar feature to be observed for the POHJ-buoy as for the PERM and SELK-buoys. The buoy temperatures seem to be lower than the model during summer and higher than the model during winter.
Figure 4.21: Grid4. The optimized grid. Red dots denote the assimilation grid points.
Figure 4.22: HELS-buoy, the 12-month test run.
Figure 4.23: PERM-buoy, the 12-month test run.
Figure 4.24: SELK-buoy, the 12-month test run.
Figure 4.25: POHJ-buoy, the 12-month test run.
4.5 Effects of the improved data assimilation scheme on modelled sea surface salinity

The assimilation of sea surface temperature has an impact on salinity in a model where variables are connected. The modelled sea surface salinities are compared to measurements from the Utö measuring station (Figure 4.26). The period of comparison is from 1.7.2013 to 13.8.2013. A longer period would have been preferable. However, the measuring buoy started floating free after this time so there is no more buoy data for the model run period.

The model seems to give lower salinities than the buoy for the whole period of comparison (Figure 4.27). The model run with optimized data assimilation does not differ much from the reference run without assimilation. For this period the assimilated model run seems to be more accurate for the most part.

Figure 4.26: Location of the Utö station
Figure 4.27: Modelled salinities with and without assimilation and measured salinities at the Utö station.
Chapter 5

Conclusions

The optimization method developed during this study worked. It made it possible to compare different assimilation intervals and grids and thus define an optimal way to merge the satellite observations into the model.

The results of the data assimilation are better if it is done more often. This is why the optimal interval for the assimilation of the HBM-FMI model is once a day. This is as often as there is new satellite data to be used in the assimilation.

The assimilation grid for the satellite data should be as thick as possible. However, a thicker assimilation grid slows the running of the model. This is why a compromise had to be made and the medium thick grid out of the three grids tested was determined the best. The visually defined optimal assimilation grid was based on the medium thick grid.

In conclusion, the improved data assimilation has a positive impact on the HBM-FMI model results. In the 12-month test period of the improved assimilation system the assimilated model run was visibly better than the model run without assimilation.
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Bibliography


