Fiscal Multipliers in a Structural VEC Model with Mixed Normal Errors

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Abstract

This paper estimates the effects of fiscal policy shocks on GDP in the United States with a vector error correction (VEC) model where shocks are identified by exploiting the nonnormal distribution of the model residuals. Unlike previous research, the model used here takes into account cointegration between the variables and identifies fiscal policy shocks without imposing any restrictions. The approach also allows statistical testing of previous identification strategies, which may help discriminate between them and hence also explain differences between various fiscal multiplier estimates. According to the results, a deficit financed government spending shock has a weak negative effect on output, whereas a tax raise to finance government spending has a positive impact on GDP.

JEL Classification: C32, E62

Keywords: fiscal policy, vector error correction model, mixed normal distribution

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1 Introduction

After the recent financial crisis many central banks have had to come to terms with the limits of conventional monetary policy. Because of the zero lower bound on one hand and the prolongation of economic downturn on the other, policymakers and economists alike have again turned their attention to fiscal policy. Concerning the countries of the euro area, common monetary policy, which is not necessarily optimal from the point of view of a single member country emphasizes the role of fiscal policy.

Compared to monetary policy, fiscal policy has been viewed as a less agile policy instrument mainly due to implementation lags but also because of its multi-faceted nature. Fiscal policy consists of the allocation of government expenditure between different categories of consumption and investment as well as decisions about its finance with a particular tax-debt mix. These political decisions are taken at different levels of government administration (eg. federal, state, provincial, municipal). Unlike monetary policy, the stance of which can be summarized by an interest rate announced by the central bank, fiscal policy regime cannot be described by a single variable.

Nonetheless there has been an upsurge of academic research on the macro-economic effects of government expenditure and tax changes in recent years. Broadly speaking the key question of interest is whether government spending can stimulate the economy, and what is the size (and sign!) of this fiscal, or government spending multiplier.

Given the variety of theoretical and empirical results, recently many researchers have asked whether the multiplier depends on the state of the economy, i.e. whether government fiscal stimulus is more effective when it is used to supplement scant private demand in an economic downturn than in an upturn (Auerbach & Gorodnichenko 2012, Caggiano, Castelnuovo, Colombo & Nodari, 2015). Interestingly Caggiano et al. (2015) show that this indeed is the case with deep recessions and extreme economic peaks in the US, while no statistically significant differences between normal times, i.e. normal economic downturns and upturns are found. Also Owyang, Ramey and Zubairy (2013), and Ramey and Zubairy (2014) find no evidence of larger fiscal multipliers during downturns. This means that research based on linear models is informative about the effectiveness of the fiscal policy instrument in normal times. Given the relative rarity of events like the recent Great Recession\footnote{During the 32-year period studied by Caggiano \textit{et al.} (2015), the authors identified only two deep recessions in the U.S., whereas according to the NBER based Recession Indicator the total number of recessions amounted to five.}, knowledge about the effectiveness of fiscal stimulus during an ordinary business cycle is admittedly valuable. Therefore the focus in this paper is on
linear models.

Vector autoregressive (VAR) models seem to have become the main econometric tool to study the macroeconomic effects of both monetary and fiscal policy (Ramey 2012, Caldara & Kamps 2008). Both strands of empirical literature need to tackle the inherent shock identification problem. Fiscal policy research has relied on four identification strategies: 1) the recursive approach due to Sims (1980) applied to fiscal policy by e.g. Auerbach and Gorodnichenko (2012), 2) the frequently applied structural VAR proposed by Blanchard and Perotti (2002), 3) the sign restrictions developed by Uhlig (2005) and applied by Mountford and Uhlig (2009) and 4) the narrative approach introduced by Ramey and Shapiro (1998), where unexpected increases in military spending are exploited.

Studies using different VAR model specifications and identification schemes have come to diverging conclusions about the size and sometimes even the sign of the multiplier. Unlike with monetary policy, the fifth available strategy has not yet been applied to the study of fiscal policy, namely statistical identification methods. Statistical methods that yield additional data-based information may be helpful in shock identification, and/or possibly help choose the most suitable among the proposed identification strategies.

Therefore this paper applies the statistical method due to Lanne and Lütkepohl (2010), where nonnormality of the errors is exploited to identify the structural shocks. More precisely, the errors are assumed to follow a mixture of two normal distributions. The identification strategy of Lanne and Lütkepohl (2010) allows not only to identify structural shocks without any additional identifying restrictions, but also to statistically test whether any of the previously used identification strategies are compatible with the properties of the data. Obtaining results that are not dependent on the chosen identification strategy may be seen as a robustness check of previous empirical research.

Unlike any of the previous studies using VARs – linear and non-linear – the vector error correction (VEC) model used in this paper also takes into account the cointegration properties of the variables. The usual practice in the literature is to include the log levels of variables such as GDP, government spending and taxes (Ramey & Zubairy 2014), even though they are likely to contain a unit root. Phillips (1998) demonstrates that impulse responses are not consistently estimated in structural VARs (SVARs) with variables in levels in the case of unit roots, whereas the VEC specification significantly improves them even for short horizons when the cointegration relations are either known or consistently estimated. Phillips (1998) points out that differing treatment of nonstationarity in models such as unrestricted VAR, Bayesian VAR with unit root priors and reduced rank regression has
substantial effects on policy analysis. An additional advantage of the VEC specification is that the cointegration relations provide identification restrictions and allow to distinguish shocks that have either permanent or transitory effects.

As it has not yet been done for fiscal VARs, this paper 1) expands the set of identification strategies with increasingly popular statistical methods and 2) takes into consideration the cointegration properties of the time series. Both extensions – dealing with the nonstationarity of the data, and combining statistical and theoretical information for identification – are expected to increase the accuracy of results (Phillips 1998, Herwartz & Lütkepohl 2014).

Quarterly data for the United States are used. The data cover the period 1981Q3 to 2012Q4 and were previously used by Caggiano et al. (2015), as well as Auerbach and Gorodnichenko (2012). Similarly to Caggiano et al. (2015), fiscal policy anticipation effects, or foresight are addressed by including the fiscal news variable proposed by Gambetti (2012).

The analysis highlights differences between the different VAR specifications used to analyze the effects of fiscal policy. The impulse responses based on the VEC model with mixed normal errors are quite different from those typically obtained from SVAR models, as the latter mostly coincide with theoretical models in the Keynesian tradition. According to our results, a government spending shock has a weak but negative effect on GDP, while the response of taxes is not statistically different from zero even if no restrictions on taxes are placed. As government revenue does not change, this can be interpreted as a fiscal policy shock financed with deficit as in Mountford and Uhlig (2009). Also quite surprisingly, a government revenue shock triggers a positive response in both government expenditure and GDP. In line with the interpretation of the spending shock, this can be interpreted as a tax raise to finance government spending, which has a positive impact on GDP.

Our results indicate that a positive 1% government spending shock decreases output at most by 0.2% and leaves taxes unaffected, whereas a positive 1% government revenue shock increases output by 0.7% on impact, and the effects are statistically significant.

Compared to Caggiano et al. (2015), who have previously used the same dataset, the major difference lies in the impact responses to the fiscal news shock, which the authors interpret as an anticipated fiscal expenditure shock. This is not surprising given their recursive identification strategy, where zero impact effect of the fiscal news shock on all variables is imposed. On the other hand, the shapes of the impulse responses are similar. When in the mixture VEC model the responses to the fiscal news shock are left unrestricted, we observe that the impact responses of government expenditure and output are negative but increasing, whereas government revenue reacts positively before
it starts to decrease. Of these, only the response of government expenditure is statistically significant.

The rest of the paper is organized as follows. In Section 2 an overview of some crucial issues in studying fiscal policy with VARs is given. Technical details of the empirical method are put forward in Section 3. Section 4 covers the empirical analysis and Section 5 concludes.

2 Fiscal VARs

2.1 Identification Schemes

Ramey (2011) provides a review of both theoretical and empirical research on the government spending multiplier. Theoretically defined multipliers get a wide range of values depending on the type of model used, the assumptions about the behaviour of monetary policy, the type and persistence of government spending, and how it is financed (Ramey 2011). Consequently, the size of the multiplier is first and foremost an empirical issue.

A complicating aspect of empirical research has been data availability. For most countries, until very recently, national accounts were provided on annual basis only and therefore quarterly time series are quite short. Some researchers have overcome the problem by studying panels of states or countries (e.g. Ilzetzki, Mendoza & Végh 2013, Suárez Serrato & Wingender 2011). For the United States, quarterly data for the main variables exists since 1947 and therefore it is probably the most popular country that has been studied. \(^2\)

Besides the trouble of data availability, to study fiscal multipliers with VARs it is important to identify a fiscal shock that is not only exogenous to the state of the economy but is also unanticipated (Ramey & Zubairy 2014). These issues will be discussed in more detail next.

Fiscal policy, and consequently a fiscal policy shock, is not captured by one variable only. Fiscal policy consists of the allocation of expenditure between investment and consumption, which in turn can be financed with taxes, debt or both. A wide variety of policies implies different fiscal policy shocks, and the researcher needs to choose which one to analyze. Alternatively, Mountford and Uhlig (2009) show how to combine different fiscal shocks to analyze the effects of various fiscal policies.

\(^2\)In fact, all of the empirical studies on the fiscal multiplier reviewed by Ramey (2011) use data for the United States.
When fiscal policy is studied with VARs, although a VAR-process
\[ y_t = c + A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t \]
where \( y_t \) is a \((K \times 1)\) vector of observable time series variables, \( c \) is a \((K \times 1)\) vector of constants, the \( A_j \)'s \((j = 1, \ldots, p)\) are \((K \times K)\) coefficient matrices and the error term \( u_t \) is \( K \)-dimensional white noise with \( u_t \sim (0, \Sigma_u) \) is estimated, interest lies in structural shocks \( \varepsilon_t \sim (0, \mathbf{I}_K) \), which are related to the reduced form errors as
\[ u_t = B \varepsilon_t \]
and
\[ E(u_t u_t') = \Sigma_u = BB' = BB'. \]

Shock identification then involves defining a transformation matrix \( B \), which allows to recover the structural shocks of interest \( \varepsilon_t \) from the reduced form errors \( u_t \).\(^3\)

Even though the covariance matrix (2) can always be consistently estimated, these relations are not enough to identify all \( K^2 \) elements in \( B \). Due to the symmetry of the covariance matrix, these equations only define \( \frac{K(K+1)}{2} \) equations. Hence shock identification within the standard VAR framework requires \( K^2 - \frac{K(K+1)}{2} = \frac{K(K-1)}{2} \) additional restrictions on \( B \).

Four different ways to identify a fiscal policy shock have been considered in the literature: a recursive approach, restrictions based on institutional knowledge, sign restrictions and narrative approach.

Many researchers have identified fiscal shocks with a Cholesky decomposition of the covariance matrix of the VAR residuals. Fatás and Mihov (2001) and Caldara and Kamps (2008) include five variables in their baseline specification but both order the key variables government expenditure \((G_t)\), GDP \((Y_t)\) and government revenue, or taxes net of transfers \((T_t)\) in the vector \( y_t \) as \( y_t = (G_t, Y_t, T_t) \), while Auerbach and Gorodnichenko (2012) and Caggiano et al. (2015) use the ordering \( y_t = (G_t, T_t, Y_t) \). The Fatás and Mihov (2001) ordering implies the relations
\[ \begin{bmatrix} u_t^G \\ u_t^Y \\ u_t^T \end{bmatrix} = \begin{bmatrix} * & 0 & 0 \\ * & * & 0 \\ * & * & * \end{bmatrix} \begin{bmatrix} \varepsilon_t^G \\ \varepsilon_t^Y \\ \varepsilon_t^T \end{bmatrix} \]
where the asterisks denote unrestricted elements and zeros indicate the elements that are restricted to be zero. Both of these recursive orderings amount

\(^3\)This specification corresponds to the B-model in Lütkepohl (2007, Ch 9).
to assuming that tax decisions are taken after spending is determined. According to Fatás and Mihov (2001), this is a plausible but unfortunately untestable hypothesis under normality. This is an example of the kind of hypothesis that can be statistically tested within the framework applied in this paper.

As is well known, applying a Cholesky decomposition results in a recursive structural model, which may or may not be justified on economic grounds. Sometimes researchers do sensitivity analysis with alternative orderings of the $K$ variables. Kilian (2013) criticizes this approach for two reasons. First, even with a relatively small number of variables (say, $K = 4$), there is a large number of permutations of the ordering $(4 \cdot 3 \cdot 2 \cdot 1 = 24)$ and it is not credible that all of these have been tried out. Second, as he illustrates with an example, even if there were no difference across these specifications it does not prove that the model is recursive in the first place.

Non-recursively identified models are explicitly structural models, which exploit external information to derive short-run restrictions. Blanchard and Perotti (2002) introduce a model of US fiscal policy where unexpected movements in taxes ($u^T_t$), government expenditure ($u^G_t$) and output ($u^Y_t$) are due to their responses to unexpected movements in other variables and to structural shocks. They write the vector of reduced form errors $u_t = (u^T_t, u^G_t, u^Y_t)$ as the following system

\begin{align}
  u^T_t &= a_1 u^Y_t + a_2 \varepsilon^G_t + \varepsilon^T_t \\
  u^G_t &= b_1 u^Y_t + b_2 \varepsilon^T_t + \varepsilon^G_t \\
  u^Y_t &= c_1 u^T_t + c_2 u^G_t + \varepsilon^Y_t \tag{4}
\end{align}

To identify the above system of equations, they derive restrictions from institutional knowledge about tax, transfer and spending programs in the United States. The identification scheme is a three-step procedure.

As a first step they recognize that fiscal policy is implemented with lags so that it takes longer than a quarter for discretionary fiscal policy to respond to economic activity. This means that when quarterly data is used, the coefficients $a_1$ and $b_1$ reflect only the automatic responses of fiscal variables to changes in output. The authors then use external information on elasticity of taxes and spending to GDP to compute the coefficients. They find no automatic feedback from economic activity to government expenditure, implying $b_1 = 0$. Their estimate of the aggregate net tax elasticity to output results in $a_1 = 2.08$ but other estimates have also been obtained in the literature.

In the second step they construct cyclically adjusted reduced form residuals $u^{T,CA}_t \equiv u^T_t - a_1 u^Y_t = a_2 \varepsilon^G_t + \varepsilon^T_t$ and $u^{G,CA}_t \equiv u^G_t - b_1 u^Y_t = b_2 \varepsilon^T_t + \varepsilon^G_t$. Assuming that tax decisions are taken before spending decisions, they then
set $a_2 = 0$ and estimate $b_2$. Alternatively, one could set $b_2 = 0$ and estimate $a_2$, implying the opposite interpretation.

The final step consists of using the cyclically adjusted residuals $u_t^{T,CA}$ and $u_t^{G,CA}$ as instruments for $u_t^T$ and $u_t^G$ in the last equation, which allows to estimate $c_1$ and $c_2$.

Following these steps all the coefficients are estimated and impulse responses to fiscal shocks can be computed using the matrices\(^4\)

\[
\begin{bmatrix}
1 & 0 & a_1 \\
0 & 1 & b_1 \\
-c_1 & -c_2 & 1
\end{bmatrix}
\begin{bmatrix}
u_t^T \\
u_t^G \\
u_t^Y
\end{bmatrix}
= 
\begin{bmatrix}
1 & 0 & 0 \\
b_2 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\varepsilon_t^T \\
\varepsilon_t^G \\
\varepsilon_t^Y
\end{bmatrix}
\]

This identification approach has been criticized by Caldara and Kamps (2012) who show that small changes in the assumed elasticities of taxes and of government expenditure result in large differences in the estimated multipliers. The example also illustrates that non-recursively identified VAR models are very similar to traditional simultaneous equation models, which suffer from the difficulty of finding strong instruments needed for identification (Kilian 2013).

Mountford and Uhlig (2009) impose restrictions on the shape of the impulse responses to identify four shocks from a 10-variable VAR: a business cycle shock ($\varepsilon_t^Y$), a monetary policy shock ($\varepsilon_t^i$), a government expenditure shock ($\varepsilon_t^G$) and a government revenue shock ($\varepsilon_t^T$). Unlike the previous approaches, the sign-restrictions approach does not require the number of shocks to equal the number of variables.

Applied to a system of three variables and disregarding the monetary shock, the Mountford and Uhlig (2009) identification scheme is the following. A business cycle shock is defined as a shock that jointly moves output ($Y_t$) and government revenue ($T_t$) in the same direction for at least four quarters. It is thus assumed that when taxes and output move in the same direction it must be that an improvement in the business cycle is generating more revenues to the government and not the other way around. Two fiscal policy shocks, a government expenditure and a revenue shock, result in government spending ($G_t$) and revenue ($T_t$) to rise for a year after the shock, respectively. The impact of a government spending shock on revenue need not be restricted, and vice versa. Alternatively, their respective initial effects could be restricted to zero. The sign restrictions are summarized in Table 1.

Unlike short-run restrictions, sign restrictions represent inequality restrictions. The problem with sign-identified models is that they are only set

\(^4\)This specification corresponds to the AB-model in Lütkepohl (2007, Ch 9) where the structural shocks are defined as $Au_t = Be_t$. 

7
identified, which means that a wide range of structural shocks may be compatible with the data and without further assumptions, there is no way to discriminate between the models (Kilian 2013).

Part of the literature has relied on the event-study, or narrative approach, which was introduced by Ramey and Shapiro (1998). Their approach circumvents the identification problem by focusing on episodes of military buildups, as this type of government spending is believed to be exogenous to the state of the economy. However, Ramey (2011) points out that the events leading to buildups such as the start of World War II and the Cold War could have other effects on the economy, and thus indirectly influence the multiplier.

If statistical identification of shocks (see Section 3) is obtained following Lanne and Lütkepohl (2010), then the restrictions on the contemporaneous relationships between the variables imposed in the previous identification schemes can be statistically tested. As suggested by Lütkepohl and Netšunajev (2014), the compatibility of the sign restrictions with the data can also be checked by comparing the shapes of the resulting impulse responses with the restrictions in Table 1. These approaches may also be helpful in labeling the statistically identified shocks, which is always based on outside information (Lanne, Meitz & Saikkonen 2015, Lütkepohl & Netšunajev 2014).

2.2 Fiscal Foresight

The announcement and implementation of changes in fiscal policy are known to have different timings. There is a time lag between proposing and passing a law, and between signing the legislation into law and implementing it. This means that changes in government finances may be predictable by the time the law takes effect and the surprise of a change in fiscal policy takes place earlier (Leeper, Walker & Yang 2013, Mountford & Uhlig 2009.) For example Ramey (2011) has shown that increases in military spending, that have been widely used for identification (e.g. Ramey & Shapiro 1998, Barro & Redlick 2011), and other non-defense government spending changes are anticipated several quarters before they actually occur. The anticipation of fiscal shocks by economic agents is referred to as fiscal foresight.

Fiscal foresight creates problems with structural VAR analysis. If economic agents adjust their behavior based on anticipated future shocks, or news shocks, while standard VARs take into account current and past shocks only, analysis based on these may be misleading. Leeper et al. (2013) show that foresight about changes in future variables leads to non-invertible moving average representations. Instead of the standard (causal) VAR representation, in this case the process has a noncausal representation.

Using data for the United States, Lanne and Saikkonen (2009) provide
evidence of noncausality in a VAR model with fiscal foresight. This finding invalidates analyses based on conventional causal VARs, as the errors from a standard VAR cannot be used to exactly uncover the true fiscal shocks.

Unfortunately even if noncausality is detected, methods for e.g. impulse response analysis from noncausal VAR models are not yet readily available (Lanne & Saikkonen 2013). As the foresight problem arises because the econometrician does not have all the information that economic agents may have, an alternative approach is to solve the inherent missing variable problem by adding variables to the VAR (see Lütkepohl 2014 and the references therein).

This paper follows Caggiano et al. (2015) who apply the expectations revisions, or news variable approach proposed by Gambetti (2012). A news variable $n^g_{it}$ is constructed from forecast revisions of the growth rate of real government expenditure and added to the VAR. In other words the VAR is augmented with information about the anticipated fiscal spending shock, which should bring the econometrician’s information set closer to that of economic agents.

3 Vector Error Correction (VEC) Model with Nonnormal Error Distribution

Unlike what is typically done in the existing fiscal policy literature, in this paper a vector error correction model (VECM) is specified and estimated to take into account the cointegration properties of the variables. If some or all of the variables are $I(1)$ and some of the variables are cointegrated, instead of the vector autoregressive (VAR) representation, there are advantages in using the VEC representation of the process. Utilizing the cointegration properties of the variables provides identification restrictions and allows to distinguish between permanent and transitory shocks.

A reduced form VEC($p$) model with cointegration rank $r < K$ is (deterministic terms omitted for simplicity)

$$\Delta y_t = \alpha \beta' y_{t-1} + \Gamma_1 \Delta y_{t-1} + \cdots + \Gamma_{p-1} \Delta y_{t-p+1} + u_t$$

where $y_t$ is a $K \times 1$ vector of time series, $\alpha$ is a $K \times r$ matrix of loading coefficients, $\beta$ is a $K \times r$ cointegration matrix, $\Gamma_j$ is a $K \times K$ short run coefficient matrix for $j = 1, \ldots, p - 1$, and $u_t \sim (0, \Sigma_u)$ is a white noise error vector. The process has a vector moving average (VMA) representation

$$y_t = \Xi \Sigma_{i=1}^\infty u_i + \Xi_j \Sigma_{j=0}^\infty u_{t-j} + y_0^*$$
where the $\Xi_j^*$ are absolutely summable and $\chi^*_i$ contains the initial values (see e.g. Lütkepohl 2007, Chapter 9).

Therefore the long-run effects of the shocks are captured by the common trends term

$$\Xi \Sigma_{i=1}^{\infty} \chi_i$$

and the matrix

$$\Xi = \beta_{\perp} \left[ \alpha'_{\perp} \left( I_K - \Sigma_{i=1}^{\infty} \Gamma_i \right) \beta_{\perp} \right]^{-1} \alpha'_{\perp}$$

has rank $K - r$. The symbols $\alpha_{\perp}$ and $\beta_{\perp}$ denote the orthogonal complements of $\alpha$ and $\beta$, respectively. Substituting the relation $\chi_i = B \epsilon_i$ in the common trends term (5) gives $\Xi B \Sigma_{i=1}^{\infty} \epsilon_i$. $\Xi B$ contains the long-run effects of the structural shocks and has rank $K - r$. At most $r$ of the shocks can have transitory effects only, and they are associated with zero columns in the long run matrix $\Xi B$.

To obtain additional information for identification, Lanne and Lütkepohl (2010) assume that the $K$-dimensional error term $\chi_t$ is a mixture of two serially independent normal random vectors

$$\chi_t = \begin{cases} e_{1t} \sim N(0, \Sigma_1) & \text{with probability } \gamma \\ e_{2t} \sim N(0, \Sigma_2) & \text{with probability } 1 - \gamma \end{cases}$$

where $N(0, \Sigma)$ denotes a multivariate normal distribution with mean 0 and covariance matrix $\Sigma$. In the model $\Sigma_1$ and $\Sigma_2$ are $(K \times K)$ covariance matrices that are assumed to be distinct, $\gamma$ is the mixture probability, $0 < \gamma < 1$, a parameter of the model. $\gamma$ is only identified if $\Sigma_1 \neq \Sigma_2$, hence this is assumed to hold. If some parts of $\Sigma_1$ and $\Sigma_2$ are identical then some components of $\chi_t$ may be normally distributed. In any case there only needs to be one nonnormal component in $\chi_t$. The distribution of the reduced form error term now becomes

$$\chi_t \sim (0, \gamma \Sigma_1 + (1 - \gamma) \Sigma_2)$$

The distributional assumption for $\chi_t$ allows to define a locally unique $B$ matrix in the following way. As shown in the Appendix A by Lanne and Lütkepohl (2010), a diagonal matrix $\Psi = \text{diag}(\psi_1, \ldots, \psi_K)$, $\psi_i > 0$ ($i = 1, \ldots, K$) and a $(K \times K)$ matrix $W$ exist such that $\Sigma_1 = WW'$ and $\Sigma_2 = W \Psi W'$ and $W$ is locally unique except for a change in sign of a column, as long as all $\psi_i$’s are distinct. Now we can rewrite the covariance matrix of the reduced form error vector $\chi_t$ as

$$\Sigma_\chi = \gamma WW' + (1 - \gamma)W \Psi W' = W(\gamma I_K + (1 - \gamma) \Psi)W'$$

10
Then following equations (1) and (2), a locally unique $B$ matrix is given by

$$B = W(\gamma I_n + (1 - \gamma)\Psi)^{1/2}$$  \hspace{1cm} (8)

This is sufficient for identification.

This choice of $B$ also means that the orthogonality of shocks is independent of regimes. This can be seen by applying (2) to the covariance matrices as

$$B^{-1}\Sigma_u B^{-1} = I_k$$
$$B^{-1}\Sigma_1 B^{-1} = (\gamma I_k + (1 - \gamma)\Psi)^{-1}$$
$$B^{-1}\Sigma_2 B^{-1} = (\gamma I_k + (1 - \gamma)\Psi)^{-1}\Psi$$  \hspace{1cm} (9)

As the equations in (9) are all diagonal matrices, the choice of $B$ as in (8) yields shocks that are orthogonal in both regimes. The model is estimated with maximum likelihood (ML) method.

A number of other statistical identification procedures for SVAR models have been proposed in the literature recently, and they have already been applied to monetary policy (see eg. Lanne and Lütkepohl 2014). Rigobon (2003) and Lanne and Lütkepohl (2008) have developed methods based on regimes with different covariance structures. Heteroskedasticity may arise, for example, as a result of financial crises. These methods further assume that changes in the covariance occur at fixed points during the sample period. This may be a problematic assumption if no such break points are known to exist.

In contrast Lanne, Lütkepohl and Maciejowska (2010) as well as Lütkepohl and Netsunajev (2014) model the volatility shifts as a Markov regime switching process, where changes in volatility are endogenously determined.

All of these methods are based on either conditional or unconditional heteroskedasticity. More recently Lanne, Meitz and Saikkonen (2015) have introduced a yet more general approach, which encompasses most of the previously introduced methods. Similarly to the method employed in this paper, in their approach identification is based on non-Gaussianity of the error terms but more wide-ranging specifications for the error distribution are allowed.

The choice of the identification method based on mixed normality used in this paper is largely dictated by the data. There is no known break in the sample as required by Rigobon (20013) and Lanne and Lütkepohl (2008). On the other hand, modeling volatility regimes as a Markov switching process as in Lanne et al. (2010) is numerically demanding, especially if short time series
are used. Finally, Lanne et al. (2015) only discuss a stationary VAR process, the use of which is not feasible given that our data are cointegrated. Further evidence in support of the specific distributional assumption is presented in Section 4.2.

4 Empirical Analysis of the Fiscal Multiplier in the US

4.1 Data

In the analysis quarterly US data in a four variable VECM $y_t = (G_t, T_t, Y_t, \eta_{13})'$ is used, where $G$ is log real government (federal, state, local) expenditure on consumption and investment, $T$ is log real government receipts of direct and indirect taxes net of transfers to businesses and individuals, and $Y$ is log real gross domestic product (GDP) in chained 2009 dollars. The variables are constructed using the Bureau of Economic Analysis’ NIPA Tables. These data are available since 1947Q1 and were previously used by Auerbach and Gorodnichenko (2012), Mountford and Uhlig (2009) and Caggiano et al. (2015), among others.

To deal with fiscal foresight, we follow Caggiano et al. (2015) and include a fourth variable in the VAR, a so-called public expenditure news variable $\eta_{13}^g$. As the variable $\eta_{13}^g$ is constructed from the forecast revisions of the growth rate of real government expenditure, collected by the Survey of Professional Forecasters (SPF) since 1981Q3, the whole sample is restricted to the period 1981Q3-2013Q1.

The cumulated fiscal news variable is constructed by summing up revisions of expectations as follows (Caggiano et al. 2015, Gambetti 2012):

$$\eta_{13}^g = \sum_{j=1}^{J} (E_t g_{t+j} - E_{t-1} g_{t+j})$$

where $E_t g_{t+j}$ is the forecast of the growth rate of real federal government expenditure from period $t + j - 1$ to period $t + j$ based on the information available at time $t$. Therefore $E_t g_{t+j} - E_{t-1} g_{t+j}$ represents the news that

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5 Government expenditure is the sum of consumption expenditure and gross investment minus the consumption of fixed capital. Government revenue is computed as the difference between current receipts and government social benefits. The implicit GDP deflator is used to transform nominal series into real terms.

6 The public expenditure news variable was provided by Giovanni Caggiano. All other variables were constructed by the author.
becomes available to private agents between time $t - 1$ and $t$ about the growth rate of government expenditure $j$ periods ahead. As the SPF collects forecasts conditional on time $t - 1$ up to time $t + 3$, to exploit the largest number of news available, $J = 3$ is selected (Caggiano et al. 2015).

Caggiano et al. (2015) show that residuals typically employed in a standard trivariate VAR are partly predictable by the components of $\eta_{13}$ and cannot be interpreted as fiscal shocks. According to the authors, the forecast revisions included in the variable $\eta_{13}$, which they interpret as a measure of anticipated fiscal shocks, can augment the information content of the VAR system. They further show that changes in the news variable $\eta_{13}$ either anticipate changes in Ramey’s (2011) military spending news variable, or at least are not anticipated by it.

### 4.2 Model Setup

The empirical analysis starts with checking the orders of integration of the four times series, which are depicted in Figure 1. The results of the augmented Dickey-Fuller (ADF) unit root tests for the series are in Table 2. A trend was included in the test for all series and autoregressive lags were chosen according to the Akaike information criterion. The tests show that all the variables included in the analysis are $I(1)$, although $T$ only at the 5% but not at the 10% significance level.

The next step is to investigate the cointegration rank of the four dimensional VECM for $y_t = (G_t, T_t, Y_t, \eta_{13})'$. This requires determining the number of lagged differences in the system first. Here we use the fact that if a VAR($p$) process contains cointegrated variables, the process has a VEC($p-1$) representation. In other words the order $p$ is chosen so that no residual autocorrelation is left in the corresponding VAR model. For a reduced form Gaussian VAR, AIC, HQ and BIC select VAR(6), VAR(2) and VAR(1) models, respectively. According to the adjusted portmanteau test there is autocorrelation left in the VAR(1) model ($p$-value < 0.001), while a $p$-value of 0.082 for VAR(2) suggests that a second order model is sufficient.

The results of the Johansen Trace test with an unrestricted constant are reported in Table 3. The cointegration rank $r = 0$ is rejected at all significance levels, while $r = 1$ clearly cannot be rejected at the 5% level and is barely rejected at the 10% level. The Saikkonen and Lütkepohl (2000) cointegration test – also reported in Table 3 –, which is less dependent on the deterministic terms included provides further support for $r = 1$.

---

7 Due to the low power of the test, the rank is often selected according to the 10% significance level (Brüggemann & Lütkepohl 2005).

8 As a robustness check, the mixture VECM was estimated with $r = 2$ as well and the
To conclude the initial analysis, diagnostic tests are performed to assess the suitability of the VEC(1) model with $r = 1$. There appears to be no remaining autocorrelation (adjusted portmanteau test $p$-value 0.18). There is however evidence of nonnormality in the errors. This is evident from the quantile-quantile (QQ) plots of the model residuals, plotted in Figure 2. Normality is also rejected by formal normality tests, of which the Doornik and Hansen test for joint normality gets a $p$-value < 0.001, and the $p$-values of univariate Jarque-Bera tests are reported in Table 4.

Figure 2 illustrates that most discrepancies from a normal distribution occur at the tails. The curved pattern of the QQ plots for government expenditure, government revenue and GDP can arise because of a left skewed data distribution compared to the normal, while the QQ plot of the fiscal news variable shows heavy tails at both ends of the distribution. These observations are confirmed by the figures in Table 4. In fact, government expenditure, government revenue and GDP feature negative/left skewness, whereas the fiscal news variable is positively/right skewed. Moreover, the kurtosis shows values greater than 3 for all variables, indicating heavier tails and higher peaks than in a normal distribution.

Heavy tails and skewness are typical features of financial time series such as asset returns. To accommodate these characteristics, mixtures of normal distributions have been used to analyze financial data. According to Tsay (2005), studies of stock returns have started to use a mixed normal distribution because it can capture the skewness and excess kurtosis of the time series. By using a mixture distribution, one can obtain densities with higher peaks and heavier tails than in the normal distribution. Kon (1984), for example used a mixed normal model to explain the observed significant kurtosis and significant positive skewness in the distribution of daily rates of stock returns. Overall, because of their flexibility, mixture models are increasingly exploited to model unknown distributions (McLachlan & Peel 2000).

In the present VECM setup with mixed normal errors, normal distribution is obtained if $\Sigma_1 = \Sigma_2$ in (6). Therefore the normality tests may be seen as a test of $H_0: \Sigma_1 = \Sigma_2$, the rejection of which supports the assumption that $\Sigma_1 \neq \Sigma_2$, and hence a mixed normal error distribution (Lanne and Lütkepohl 2010).

Given these properties of the data, explicitly modeling the error distribution as a mixed normal distribution is well grounded. The considerable advantage of the specific distributional assumption is that it yields additional databased information, which allows to identify the model without any restrictions. As a result, identification restrictions derived from other sources test results are qualitatively the same as the ones reported in Section 4.3.
(such as those presented in Section 2.1) become over-identifying and their validity can be statistically tested.

4.3 Estimation Results and Structural Identification

The estimation of the mixture VEC model proceeds in two steps (Lanne and Lütkepohl 2010). As the cointegration relations are not known beforehand, they are first estimated with the Johansen reduced rank regression, which yields $\beta = (1, -0.447, -0.171, -0.007)$.\(^9\) In the second step the log-likelihood function is maximized with respect to the other parameters, conditioning on the estimated cointegration relations.\(^10\)

In the ML estimation, VECM coefficients from a linear model are used as starting values to estimate the parameters of an unrestricted VEC model with a mixed normal distribution. Estimation results of the unrestricted model are reported in the left column of Table 5 and in Table 7. If the $\psi_i's$ are distinct, the model has been identified. As shown in Table 5, the estimation results are quite precise and the $\psi_i's$ get approximate values 0.11, 0.26, 0.06 and 0.76, while the mixture probability $\gamma$ is estimated to be 0.24.

Statistical identification delivers orthogonal shocks but their labeling has to be based on outside information (Lanne, Meitz & Saikkonen 2015, Lütkepohl & Netšunajev 2014). One option is to test the validity of a recursive identification scheme that has been used before (such as (3) in a 3 variable case). If the previously used restrictions cannot be rejected, the recursive structure provides a straightforward interpretation of the resulting impulse response functions. Statistical testing of a recursive identification scheme is therefore an important part of the economic interpretation of the results.

To this end, another VEC model is estimated where lower triangularity is imposed on the $B$ matrix as in Caggiano et al. (2015)\(^11\). In estimating the restricted model, the ML estimates of the unrestricted model are used as starting values. In both cases, to ensure nonsingularity of the covariance matrices, their determinants are bounded away from zero. Also the diagonal elements of the $\Psi$ matrix are bounded away from zero, as required.

The results of the key parameters are reported in the middle column of Table 5 together with the outcome of the likelihood ratio test, and the rest

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\(^9\)The first step computations were performed with JMulTi.

\(^10\)These computations were done with GAUSS programs, where the CMLMT library was used. To avoid numerical problems in estimation, the fiscal news variable is scaled to match the magnitude of the other variables.

\(^11\)In the present mixture model this is done in practice by restricting the $W$ matrix in $B = W(\gamma I_n + (1 - \gamma)\Psi)^{1/2}$ to be lower triangular.
of the results in Table 8. The LR test has the asymptotic $\chi^2$ -distribution with 6 degrees of freedom given by the number of restrictions. The recursive structure is clearly rejected ($p$-value < 0.001) and hence it is not helpful in labelling the shocks.

In the next section it will be explored whether the sign restrictions reported in Table 1 are in line with the statistically identified shocks. If so, then the shocks can be labelled accordingly.

The VECM specification allows one more option based on long run relations between the variables, as shown in Lütkepohl (2007, Chapter 9). Suppose the cointegration rank is known to be $r$. Then according to Section 4, there are at most $r$ transitory shocks, $\varepsilon'_t$ and at least $K - r$ permanent shocks, $\varepsilon''_t$. Arranging them such that $\varepsilon'_t = (\varepsilon''_t, \varepsilon''_t)$, it follows that $\Xi B = [\Phi_{K \times (K - r)} : 0_{K \times r}]$ where $\Phi_{K \times (K - r)}$ is an $K \times (K - r)$ matrix. In a VEC model with $r < K$, all shocks can in principle be permanent shocks and $\Xi B$ may not have zero columns even if it has reduced rank.

In Section 4.2., $r = 1$ was found for the data at hand. This translates into the following set of long run restrictions\(^\text{12}\)

\[
\Xi B = \begin{bmatrix}
* & * & * & 0 \\
* & * & * & 0 \\
* & * & * & 0 \\
* & * & * & 0
\end{bmatrix}
\] (10)

which can also be tested with a LR-test. Therefore another restricted VEC model with mixed normal errors is estimated. In addition to the long run restriction in (10), the following matrix of impact effects is assumed

\[
B = \begin{bmatrix}
* & 0 & 0 & * \\
* & * & 0 & * \\
* & * & * & * \\
* & * & * & *
\end{bmatrix}
\] (11)

In other words, the often used recursive structure for the key variables $y_t = (G_t, T_t, Y_t)$ is imposed as well (see Section 2.1). This implies that government expenditure does not respond contemporaneously to shocks to other variables, while government revenue does not react contemporaneously to output shocks.

Note that the restrictions imposed here (10 and 11) differ from the ones required for identification in a standard VECM framework (see e.g. Lütkepohl 2007, Chapter 9). Because the matrix $\Xi B$ has reduced rank $K - r$,

\[^{12}\text{Again the asterisks denote unrestricted elements and zeros indicate the elements that are restricted to be zero.}\]
each column of zeros stands for $K - r$ independent restrictions only. In other words the $r$ transitory shocks represent $r(K - r)$ independent restrictions, i.e. 3 in the present case. As just-identification in the standard VECM requires a total of $\frac{K(K-1)}{2}$ restrictions, additional restrictions based on theoretical considerations are needed. To identify both transitory and permanent shocks, it is not sufficient to impose arbitrary restrictions on $B$ and $\Xi B$, though. The advantage of the VEC specification in the standard setting is that the $r(K - r)$ restrictions are based on the cointegration rank, which can be determined by statistical tests.

In the current framework, assuming that structural shocks are in fact identified with the mixed normality of errors, any restrictions become overidentifying and can be statistically tested. Testing the exclusion restrictions in (11) is of interest because they are commonly assumed to obtain just-identification with standard three variable VARs. Obviously, the three restrictions in (11) alone are not enough to identify a four variable VAR.

The estimation results of the second restricted VEC model are reported in the right column of Table 5 and in Table 9. The $p$-value of the LR test (0.698) based on the $\chi^2(6)$-distribution indicates that restrictions (10) and (11) are well supported by the data (see the right column of Table 5).

Pairwise equality of the $\psi_i$s is also tested with Wald tests, which are reported in Table 6. Since the estimators have the usual normal limiting distributions, the Wald tests have asymptotic $\chi^2$-distributions (Lanne & Lütkepohl 2010). It turns out that the equality of $\psi_1$ and $\psi_2$, or $\psi_1$ and $\psi_3$ in the unrestricted model cannot be rejected at conventional levels, which implies that the $B$ matrix may not be unique. The nonuniqueness of $B$ may imply that the actual number of degrees of freedom of the $\chi^2$-distribution in the LR-test is less than 6 (see e.g. Lütkepohl and Velinov 2014). Given the rejection of the first restricted model at 6 degrees of freedom, the same test statistic leads to rejection with a lower number of degrees of freedom as well.

Therefore even though the $B$ matrix may not be unique, by assuming mixed normality of the errors, the restrictions imposed are sufficient to reject the recursive identification scheme. On the other hand, given the small value of the LR test statistic related to the second restricted model, even with less than 6 degrees of freedom there is still no strong evidence against the imposed restrictions.

4.4 Impulse Response Analysis

Given the previous results, two sets of impulse responses are computed: those based on the unrestricted VEC model with mixed normal errors and those
based on the restricted model, where both contemporaneous and long-run restrictions (10 and 11) not rejected by the data are imposed. In both cases the 90% Hall’s percentile confidence bands are obtained from 1000 replications of bootstrap impulse responses. Following Herwartz and Lütkepohl (2014), to ensure that only bootstrap replications around the parameter space of the original estimation step are considered, bootstrap parameter estimates of $c$, $W$, $\alpha$ and $\Gamma_1$ are determined conditionally on the initially estimated $\Sigma$ and $\gamma$. Bootstrap estimates are obtained by nonlinear optimization of the log-likelihood with ML estimates as starting values.

The impulse responses based on the unrestricted model and their confidence bands are displayed in Figure 3. Each row contains the responses of all variables to one shock. The size of each shock is set to unity. However for economic interpretation outside information is needed. Hence we try to exploit the sign restrictions in Table 1.

Taking the 90% confidence bands as the possible range of impulse responses supported by the data, from Figure 3 it is clear that based on the signs in Table 1 the shocks cannot be unequivocally labelled. The first shock is the only candidate for a (positive) business cycle shock, to which both taxes ($T_t$) and output ($Y_t$) respond positively for at least 4 quarters. However, the first shock is also the only possible (positive) government revenue shock, as taxes ($T_t$) increase on impact and the effect lasts for more than 4 quarters. If the first row is labelled as a (positive) government revenue shock, then there is no shock that fulfills the sign restrictions required for a business cycle shock and vice versa. There are also two candidates for a (positive) government spending shock, namely the first and the third, if taxes ($T_t$) are allowed to react on impact. The responses of the other variables to these two shocks are very similar; especially output responds positively in both cases.

Since there are no statistically significant impulse responses that clearly satisfy the restrictions, the sign restrictions used by Mountford and Uhlig (2009) do not provide sufficient information for unique labelling of the statistically identified shocks.

Next we turn to the impulse responses based on the second restricted model, which are shown in Figure 4. In this case the long and short run restrictions provide interpretation so that the sign restrictions are no longer needed. As the impulse responses are computed by restricting the impact effects as in (11), the following contemporaneous effects are ruled out: a government revenue shock has no contemporaneous impact on government expenditure ($G_t$), and an output shock cannot have a contemporaneous effect on government expenditure ($G_t$) and revenue ($T_t$). From the long run restriction (10) we also know that the effect of the last shock - fiscal news ($\eta_{13}^g$) - is transitory. Based on these we are able to uniquely label the shocks as
a government spending, government revenue, output and fiscal news shock. In other words they appear in the same order as the variables in the vector $y_t = (G_t, T_t, Y_t, g_{13})'$. 

The first row of Figure 4 depicts impulse responses to a positive government spending shock. Interestingly, the response of output is negative although very weak, while the response of taxes is not statistically different from zero, even if no restrictions on taxes are placed. As government revenue does not change, this can be interpreted as a fiscal policy shock financed with deficit as in Mountford and Uhlig (2009). From a practical point of view, this is very much of interest since fiscal stimulus packages are mostly financed with deficit.

The second row reports impulse responses to a positive government revenue shock. The impact response of government expenditure is restricted to zero, but it becomes positive and significant after 6 quarters, and so follows the shape of GDP. In other words, surprisingly, a government revenue shock is found to trigger a positive response in both government expenditure and GDP. In line with the interpretation of the spending shock, one could interpret government spending financed with a tax raise to have a positive impact on GDP.

The third row displays impulse responses to a positive output shock. Although the impact response of government revenue is restricted to zero here, the output shock behaves like a business cycle shock in Mountford and Uhlig (2009): both output and government revenue increase, whereas the response of government expenditure is not countercyclical, but it also increases although with a lag. The reason given by Mountford and Uhlig (2009) also applies here, namely the government expenditure variable is defined as consumption plus investment but does not include transfer payments, which automatically vary countercyclically.

Finally, the last row shows impulse responses to a positive fiscal news shock, which Caggiano et al. (2015) interpret as an anticipated fiscal expenditure shock. The shapes of the impulse responses are similar to Caggiano et al.’s (2015) but there are differences in the impact effects. This is not unexpected given their identification strategy, which imposes zero impact effects of the fiscal news shock on all variables. When in the mixture VEC model the responses to the fiscal news shock are left unrestricted, we see that the impact responses of government expenditure and output are negative but increasing, while the government revenue reacts first positively and then starts to decrease. Of these, the responses of government revenue and output are insignificant, however.

As impulse responses to a unit shock are analyzed here, multiplied by one hundred they can be interpreted as a percentage change. This implies that
a positive 1% government expenditure shock decreases output at most by 0.2% and leaves taxes unaffected, whereas a positive 1% government revenue shock increases output by 0.7% on impact, and the effects are statistically significant.

A comparison with previous empirical studies reveals that the effects of fiscal policy obtained from SVARs are typically of the opposite sign, in accordance with theoretical models in the Keynesian tradition. There is however a lot of variation in the size of the multiplier, both within and across studies (see e.g. Ramey 2011).

Similarities also exist. Perotti (2004) finds evidence of large differences in the effects of fiscal policy in the pre- and post-1980 periods. His results for the whole US sample (1960Q1-2001Q4) are similar to those obtained by others using the same sample, whereas a negative spending multiplier emerges for the post-1980 period. He concludes that there has been a drastic reduction in the effects of government spending shocks on GDP after 1980. His results are therefore in line with the ones obtained in this paper, which also considers the post-1980 period.

Mountford and Uhlig (2009) analyze a government spending shock financed with deficit by not allowing taxes to change for 4 quarters. They find that deficit spending only weakly stimulates the economy on impact and has a negative effect on output in the long run. Their basic government spending shock resembles the deficit spending shock: although no restrictions on government revenue are placed, it does not change significantly. The same result is obtained here, hence we interpret our government spending shock as deficit financed.

The results of Ilzetzki et al. (2010) obtained with a panel VAR indicate that during periods of high public debt (more than 60% of GDP), the fiscal multiplier has a negative, statistically significant long run effect, but it is not statistically different from zero on impact. They conclude that fiscal stimuli may even become weaker and yield negative multipliers in the near future as public debt ratios are high.

When Auerbach and Gorodnichenko (2012) control for expectations in their nonlinear VAR, the sample is restricted to the post-1980 period, and yields a negative multiplier in expansions. In their framework the regime is not allowed to change, though. Caggiano et al. (2015) allow the regime to change but only discuss the effects of the fiscal news shock, which they interpret as an anticipated government spending shock. Multipliers, in turn, are computed from shocks to (unanticipated) government spending shocks. Given that the same data is used here, it is interesting to note that impulse response functions of similar shape but with different impact effects are generated by the mixture VEC model. In their nonlinear framework, multipliers
in expansion get negative values for long horizon.

Contrary to Caggiano et al. (2010) and similarly to the results presented here, Ramey and Zubairy’s (2013) military spending news shock triggers a negative output response in recessions, and government spending becomes negative after 2-3 years. Their multiplier in a recessionary state gets both negative and positive values.

5 Conclusions

In the fiscal policy literature using structural vector autoregressions (SVARs) fiscal policy shocks are identified in several ways. Fiscal multipliers, i.e. estimates of the impact of fiscal stimulus on output, are then defined either as the peak of the impulse response or as an accumulated response. As is well known, the VAR identification strategy matters for the impulse responses, and hence may be one reason for the differing results.

Moreover, as the usual practice in the literature is to use the log of variables, the estimated elasticities are converted to dollar equivalents with an ex post conversion factor, a practice which has also been criticized (Ramey & Zubairy 2014). Using log levels of variables such as real GDP, government revenue and expenditure also introduces another potential source of uncertainty in the analysis, namely nonstationarity. Phillips (1998) demonstrates that impulse responses are not consistently estimated in the SVARs with variables in levels in the case of unit roots, whereas the vector error correction (VEC) specification significantly improves them even for short horizons. Phillips (1998) found that differing treatment of nonstationarity in various models has substantial effects on policy analysis.

This paper contributes to the existing fiscal policy literature in two ways. First, unlike any of the previous studies using VARs – linear and non-linear – the vector error correction (VEC) model used in this paper also takes into account the cointegration properties of the variables. Second, statistical properties of the data are exploited to identify the model, and to test the validity of two popular identification strategies in the fiscal VAR literature.

As proposed by Lanne and Lütkepohl (2010), the nonnormality found in the VAR residuals is explicitly modelled, which allows to identify structural shocks without any restrictions. In the Lanne and Lütkepohl (2010) method a mixed normal error distribution is used because of its suitability for the features often found in the residuals. Any restrictions from other sources used for identification then become over-identifying and can be statistically tested.

The test results indicate that the commonly used recursive structure for
all four variables is too restrictive from a statistical point of view. However, a long run restriction together with a recursive structure for the key variables government expenditure \((G_t)\) government revenue \((T_t)\) and GDP \((Y_t)\) is not rejected by the data. As Caggiano et al. (2015) point out, ordering the fiscal news variable last in a recursive model may be seen as inconsistent with expectational effects.

Two sets of impulse responses based on the VEC model with a mixed normal distribution are computed together with the 90% Hall’s percentile confidence bands. Compatibility of the sign restrictions used by Mountford and Uhlig (2009) is assessed based on a model where no restrictions are imposed. If the confidence bands are taken as the possible range of impulse responses supported by the data, the sign restrictions do not provide enough information to uniquely pin down the statistically identified shocks. Therefore, albeit the mixture VEC model provides no clear evidence against the sign restrictions, their compatibility with the data cannot be without doubt confirmed either.

In the next step fiscal policy shocks are analyzed based on a model with restrictions not rejected by statistical tests. The resulting impulse responses are quite different from those typically obtained from SVAR models. The latter mostly coincide with theoretical models in the Keynesian tradition. According to our results, government spending shock has a weak but negative effect on GDP, while the response of taxes is not statistically different from zero even if no restrictions on taxes are placed. As government revenue does not change, this can be interpreted as a fiscal policy shock financed with deficit as in Mountford and Uhlig (2009). Also quite surprisingly, a government revenue shock triggers a positive response in both government expenditure and GDP. In line with the interpretation of the spending shock, this can be interpreted as a tax raise to finance government spending, which has a positive impact on GDP.

Compared to Caggiano et al. (2015), who have previously used the same dataset, the major difference lies in the impact responses to the fiscal news shock. This is not surprising given their recursive identification strategy, where zero impact effect of the fiscal news shock on all variables is imposed. On the other hand, the shapes of the impulse responses are similar. When in the mixture VEC model the responses to the fiscal news shock are left unrestricted, we observe that the impact responses of government expenditure and output are negative but increasing, while government revenue reacts positively before it starts to decrease. Of these, only the response of government expenditure is statistically significant.

Our results indicate that a positive 1% government expenditure shock decreases output at most by 0.2% and leaves taxes unaffected, whereas a
positive 1% government revenue shock increases output by 0.7% on impact.

References


Table 1: Sign restrictions on impulse response functions.

<table>
<thead>
<tr>
<th></th>
<th>( T_t )</th>
<th>( G_t )</th>
<th>( Y_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Business cycle shock ((\varepsilon^i_t))</td>
<td>+</td>
<td></td>
<td>+</td>
</tr>
<tr>
<td>Government revenue shock ((\varepsilon^r_t))</td>
<td>+ (0)</td>
<td></td>
<td></td>
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<tr>
<td>Government expenditure shock ((\varepsilon^e_t))</td>
<td>(0)</td>
<td>+</td>
<td></td>
</tr>
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</table>

Notes: A ‘+’ means that the impulse response of the variable in question is restricted to be positive for quarters \( k = 0,\ldots,3 \) following the shock. A blank entry indicates that no restrictions are imposed, while a ‘0’ means no response on impact.

Table 2: Augmented Dickey Fuller Unit Root Tests

<table>
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<th>Included lags</th>
<th>Test statistic</th>
<th>Critical values</th>
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<td></td>
<td></td>
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<td>10%</td>
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<tr>
<td>G</td>
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<td>-3.13</td>
</tr>
<tr>
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Table 3: Cointegration tests

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<tr>
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<th>$p$-value</th>
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Saikkonen and Lütkepohl test

<table>
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Table 4: Summary Statistics

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<th>$p$-value</th>
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</table>

Table 5: Estimation results of the VECM with mixture distribution, restricted and unrestricted B matrix. Standard errors in parenthesis.

$$y_t = (G_t, T_t, Y_t, \eta_{13}^T)^T$$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unrestricted B</th>
<th>Restricted B</th>
<th>Restricted B and $\Xi B$</th>
</tr>
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<td>0.1241 (0.0251)</td>
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<td>0.0613 (0.0242)</td>
<td>0.1415 (0.0578)</td>
<td>0.1893 (0.0830)</td>
</tr>
<tr>
<td>$\gamma_5$</td>
<td>0.7617 (0.3066)</td>
<td>0.7248 (0.0001)</td>
<td>0.7483 (0.2768)</td>
</tr>
<tr>
<td>max $l_T(\theta)$</td>
<td>1620.56</td>
<td>1605.15</td>
<td>1618.64</td>
</tr>
<tr>
<td>LR</td>
<td>30.82</td>
<td>3.84</td>
<td></td>
</tr>
<tr>
<td>$p$-value</td>
<td>$2.743 \times 10^{-5}$</td>
<td>0.698</td>
<td></td>
</tr>
</tbody>
</table>

28
Table 6: p-values of Wald tests for equality of psi’s for models from Table 4.

\[ y_t = (G_t, T_t, Y_t, \eta_{t13}) \]

<table>
<thead>
<tr>
<th>( H_0 )</th>
<th>Unrestricted B</th>
<th>Restricted B</th>
<th>Restricted B and ( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\psi}_1 = \hat{\psi}_2 )</td>
<td>0.250</td>
<td>0.164</td>
<td>0.753</td>
</tr>
<tr>
<td>( \hat{\psi}_1 = \hat{\psi}_3 )</td>
<td>0.237</td>
<td>0.803</td>
<td>0.297</td>
</tr>
<tr>
<td>( \hat{\psi}_1 = \hat{\psi}_4 )</td>
<td>0.040</td>
<td>0.019</td>
<td>0.021</td>
</tr>
<tr>
<td>( \hat{\psi}_2 = \hat{\psi}_3 )</td>
<td>0.096</td>
<td>0.211</td>
<td>0.243</td>
</tr>
<tr>
<td>( \hat{\psi}_2 = \hat{\psi}_4 )</td>
<td>0.011</td>
<td>1.22 \times 10^{-10}</td>
<td>0.016</td>
</tr>
<tr>
<td>( \hat{\psi}_3 = \hat{\psi}_4 )</td>
<td>0.025</td>
<td>0.006</td>
<td>0.058</td>
</tr>
</tbody>
</table>

Table 7: Estimated parameters of the unrestricted mixture VEC model.
Standard errors in parenthesis.

<table>
<thead>
<tr>
<th>Elements of each vector</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.1929 (0.0327)</td>
<td>-0.0872 (0.0955)</td>
<td>0.0299 (0.0210)</td>
<td>-0.1841 (0.9192)</td>
</tr>
<tr>
<td>alpha</td>
<td>-0.0764 (0.0134)</td>
<td>0.0372 (0.0391)</td>
<td>-0.0097 (0.0086)</td>
<td>0.0901 (0.3858)</td>
</tr>
<tr>
<td>( \Gamma_1[1, \cdot] )</td>
<td>0.0572 (0.0785)</td>
<td>-0.0654 (0.0250)</td>
<td>-0.2005 (0.1625)</td>
<td>0.0010 (0.0030)</td>
</tr>
<tr>
<td>( \Gamma_1[2, \cdot] )</td>
<td>-0.3292 (0.2395)</td>
<td>-0.0545 (0.1043)</td>
<td>1.1938 (0.5267)</td>
<td>0.0042 (0.0080)</td>
</tr>
<tr>
<td>( \Gamma_1[3, \cdot] )</td>
<td>-0.0423 (0.0511)</td>
<td>0.0093 (0.0174)</td>
<td>0.1986 (0.1132)</td>
<td>0.0001 (0.0019)</td>
</tr>
<tr>
<td>( \Gamma_1[4, \cdot] )</td>
<td>0.2517 (1.8379)</td>
<td>-0.3674 (0.7459)</td>
<td>0.2943 (6.649)</td>
<td>-0.1713 (0.0660)</td>
</tr>
<tr>
<td>( W[1, \cdot] )</td>
<td>0.0007 (0.0026)</td>
<td>-0.0175 (0.0161)</td>
<td>0.0011 (0.0018)</td>
<td>-0.1817 (0.0945)</td>
</tr>
<tr>
<td>( W[2, \cdot] )</td>
<td>0.0040 (0.0020)</td>
<td>-0.0099 (0.0063)</td>
<td>-0.0043 (0.011)</td>
<td>-0.0344 (0.0615)</td>
</tr>
<tr>
<td>( W[3, \cdot] )</td>
<td>0.0028 (0.0015)</td>
<td>0.0300 (0.0113)</td>
<td>0.0020 (0.0015)</td>
<td>-0.1553 (0.1205)</td>
</tr>
<tr>
<td>( W[4, \cdot] )</td>
<td>0.0072 (0.0011)</td>
<td>0.0003 (0.0040)</td>
<td>0.0033 (0.0010)</td>
<td>0.0776 (0.0253)</td>
</tr>
</tbody>
</table>

Notes: \( \Gamma[i, \cdot] \) and \( W[i, \cdot] \) indicate the \( i \)th row of matrices \( \Gamma \) and \( W \), respectively.
Table 8: Estimated parameters of the mixture VEC model with B restricted to lower triangular. Standard errors in parenthesis.

<table>
<thead>
<tr>
<th>Elements of each vector</th>
<th>constant</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.2080 (0.0302)</td>
<td>-0.0719 (0.0870)</td>
<td>0.0370 (0.0189)</td>
<td>-0.1870 (0.9346)</td>
<td></td>
</tr>
<tr>
<td>alpha</td>
<td>-0.0826 (0.0123)</td>
<td>0.0309 (0.0355)</td>
<td>-0.0127 (0.0077)</td>
<td>0.0924 (0.3814)</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_1[1,\cdot]$</td>
<td>0.0692 (0.0781)</td>
<td>-0.0659 (0.0270)</td>
<td>-0.1867 (0.1436)</td>
<td>0.0012 (0.0029)</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_1[2,\cdot]$</td>
<td>-0.3232 (0.2207)</td>
<td>-0.0707 (0.0751)</td>
<td>1.2120 (0.4242)</td>
<td>0.0023 (0.0080)</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_1[3,\cdot]$</td>
<td>-0.0464 (0.0485)</td>
<td>0.0146 (0.0154)</td>
<td>0.2025 (0.0881)</td>
<td>0.0006 (0.0018)</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_1[4,\cdot]$</td>
<td>0.2517 (1.7378)</td>
<td>-0.3756 (0.6624)</td>
<td>0.2933 (3.9988)</td>
<td>-0.1487 (0.0855)</td>
<td></td>
</tr>
<tr>
<td>$W[1,\cdot]$</td>
<td>0.0091 (0.0010)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W[2,\cdot]$</td>
<td>-0.0012 (0.0011)</td>
<td>0.0323 (0.0050)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W[3,\cdot]$</td>
<td>0.0032 (0.0010)</td>
<td>0.0116 (0.0029)</td>
<td>0.0062 (0.0007)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W[4,\cdot]$</td>
<td>0.0005 (0.0008)</td>
<td>0.0002 (0.0023)</td>
<td>0.0001 (0.0005)</td>
<td>0.2491 (0.0142)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: $\Gamma[i,\cdot]$ and $W[i,\cdot]$ indicate the $i$th row of matrices $\Gamma$ and $W$, respectively.

Table 9: Estimated parameters of the mixture VEC model with contemporaneous and long run restrictions. Standard errors in parenthesis.

<table>
<thead>
<tr>
<th>Elements of each vector</th>
<th>constant</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.1935 (0.0319)</td>
<td>-0.0144 (0.0243)</td>
<td>0.0358 (0.0162)</td>
<td>-0.0184 (0.0366)</td>
<td></td>
</tr>
<tr>
<td>alpha</td>
<td>-0.0767 (0.0130)</td>
<td>0.0072 (0.0099)</td>
<td>-0.0121 (0.0066)</td>
<td>0.0238 (0.0086)</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_1[1,\cdot]$</td>
<td>0.0578 (0.0780)</td>
<td>-0.0651 (0.0247)</td>
<td>-0.1798 (0.1487)</td>
<td>0.0009 (0.0030)</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_1[2,\cdot]$</td>
<td>-0.3426 (0.2333)</td>
<td>-0.0762 (0.0868)</td>
<td>1.2355 (0.4414)</td>
<td>0.0026 (0.0080)</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_1[3,\cdot]$</td>
<td>-0.0437 (0.0507)</td>
<td>0.0070 (0.0166)</td>
<td>0.2082 (0.1024)</td>
<td>-0.0002 (0.0019)</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_1[4,\cdot]$</td>
<td>0.2564 (1.8405)</td>
<td>-0.4177 (0.6657)</td>
<td>0.2954 (4.3939)</td>
<td>-0.1749 (0.0654)</td>
<td></td>
</tr>
<tr>
<td>$W[1,\cdot]$</td>
<td>0.0035 (0.0012)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W[2,\cdot]$</td>
<td>0.0016 (0.0014)</td>
<td>0.0322 (0.0042)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W[3,\cdot]$</td>
<td>-0.0030 (0.0017)</td>
<td>0.0131 (0.0037)</td>
<td>0.0049 (0.0008)</td>
<td>-0.0373 (0.0183)</td>
<td></td>
</tr>
<tr>
<td>$W[4,\cdot]$</td>
<td>0.0073 (0.0008)</td>
<td>0.0004 (0.0034)</td>
<td>0.0034 (0.0008)</td>
<td>0.0730 (0.0219)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: $\Gamma[i,\cdot]$ and $W[i,\cdot]$ indicate the $i$th row of matrices $\Gamma$ and $W$, respectively.
Figure 1: Plot of logarithmic time series 1981Q3-2012Q4. G = government expenditure, T = government revenue, Y = GDP, news = cumulated fiscal news.
Figure 2: Residuals of VEC(1) - model with cointegration rank r=1, QQ plots
Figure 3: Impulse response functions with 90% Hall’s percentile confidence bands of the unrestricted VEC model with mixed normal residuals.
Figure 4: Impulse response functions with 90% Hall’s percentile confidence bands of the restricted VEC model with mixed normal residuals.