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Competition in store complexity takes us halfway between Diamond and Bertrand*

Abstract

We consider a new, dynamic price search model with fixed or random deadlines to study in detail how consumers search within and across stores during a single search spell. To endogenize the intensity of competition, we allow firms to adjust freely the level of frictions in their online stores. Interestingly, this pins down uniquely the numbers of informed and uninformed consumers. We show that there exist two similar inefficient equilibria, both with a prominent firm and a non-prominent firm, where these numbers are exactly the same. The outcome is thereby precisely halfway between Diamond equilibrium and Bertrand equilibrium.

JEL Classification: D43, D83

Keywords: deadlines, endogenous search costs, Diamond equilibrium, Bertrand equilibrium

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1 Introduction

Managing the traffic of incoming and outgoing consumers is an important part of running an online store. As consumers are typically busy, it is not irrelevant in which order they sample the stores and how long a time they tend to stay in. Since firms can affect the related consumer turnover rates in many ways by the organization of their online store, it is also natural to regard them as endogenous. In this paper we analyze the effects of such firm induced search frictions in a new way that, as a rather special novelty, allows us put a single model based number, among other things, on the size of persistent price dispersion in markets for homogenous commodities and on the relative numbers of informed consumers and uninformed consumers. This ratio is one of the main parameters in the literature following the seminal articles by Varian (1980) and Stahl (1989).[1]

Stressing out the importance of website design as a part of the marketing mix, there is ample evidence that consumers are very sensitive to the navigation experience. Indeed, it is claimed (Walker, 2013) that a large fraction of shopping carts is abandoned without checking out for reasons like "website navigation too complicated", "website crashed", "process was taking too long", or "website timed out".[2] Moreover, as customers use shopping carts for entertainment and as an organizational tool and quite often need several touch points with the website before buying (Kukar-Kinney and Close, 2010), when they are finally looking for something to buy, they generally have quite a clear idea about the website design. This implies in turn that developing a reputation that its website is fast and easy to navigate could give the firm a significant advantage.

In this paper we attempt to capture a flavor of this real online search experience in a simple theoretical model in a way that relates our results to those in the previous literature on (i) equilibrium price dispersion (e.g., Burdett and Judd (1983), Butters (1977), Baye et al. (2006a), Baye et al. (2006b), and Morgan et al. (2004)) and (ii) endogenous search frictions (e.g., Wilson (2010), Carlin and Manso (2011), Ellison and Wolitzky (2012), Piccione and Spiegler (2012), and Chioveanu and Zhou (2013), to name some).

We develop a dynamic, deadline based search model, closely related to Varian (1980) and Stahl (1989) and to the extension by Wilson (2010), where we specify in more detail how consumers actually search *within and across* competing stores *during a single search spell.* To endogenize the intensity of competition, we let each of our firms publicly commit to a level of frictions within its store - say, figuratively, by putting some sand or oil in the wheels in terms of how its website content unravels to browsing consumers. These

[1] Interestingly, while our model is aimed to shed new light on the origins of search frictions, our theory based numbers do not seem to fare too badly in comparison to empirical work. For instance, Baye et al. (2006b) find that in a well-established online market for consumer electronics the difference between the highest price and the lowest price is on average 57 %. Our simple model would predict it should be 50 %.

[2] Other, often mentioned reasons were related either to prices, other costs such as handling and delivery, or payment process (hardship, security), and not having buying intent.

frictions then basically determine the expected time costs of search in each store.

As our key result, the deadline and frictions let us pin down uniquely the usually exogenous numbers of informed and uninformed consumers: we show that there exist two mirror image equilibria where these numbers are exactly the same. The outcome is thereby, arguably, precisely halfway between Diamond equilibrium (the monopoly case, only uninformed consumers) and Bertrand equilibrium (the competitive case, only informed consumers). One could say that the market endogenously settles in a "compromise".

Specifically, both equilibria feature a faster, prominent firm and a slower, non-prominent firm. All consumers search in an efficient manner from the former to the latter. Each firm has however its own, strategic incentive to generate positive frictions, which delay the consumer search process and render the equilibria inefficient; altogether, in a Poisson setting, the related surplus loss is 6 %. The prominent firm, the non-prominent firm, and consumers divide the remaining surplus in proportions 2:1:1, respectively.[3]

Our finding thus suggest that, while competition in search frictions may not eliminate them entirely, thanks to it, the negative welfare effects can still be quite restricted; surplus sharing is, obviously, somewhat more strongly affected because consumers optimally start from the more expensive, faster store. As the cherry on top, we find it also quite remarkable that these intriguingly sharp predictions arise in a model where there are essentially no parameters: our setup does have a (fixed or random) deadline for consumers but this is only a normalization. The equilibrium set is invariant to deadline changes.[4]

There is a large empirically motivated literature to develop theory models to explain persistent price dispersion for homogenous commodities. The seminal article by Varian (1980) observes that a natural way to generate price variance is to assume that consumers are heterogenously informed about market prices. In another classic work Stahl (1989) demonstrates that it is actually possible to span continuously from Diamond equilibrium to Bertrand equilibrium by varying the share of informed consumers to uninformed consumers. This share determines the intensity of competition in the market. However, despite the obvious interest, to our current reading, there has been no work to analyze what exactly these shares should be from an equilibrium perspective.

To bridge the gap, we consider this model where two firms choose the frictions in their stores (in public) and their prices (in private) and, then, consumers search without costs for a unit of time, until their deadline. Frictions refer to rates of the Poisson process for which a consumer discovers the price in a store. At each moment in time, a consumer decides in which firm's store to search; it is costless to switch.

[3]The surplus loss comes from the prominent firm's problem and the surplus sharing and the half-and-half division of consumers originates from the non-prominent firm's problem.

[4]In a Poisson setting for example, the firms set in equilibrium $(\theta^1, \theta^2) = (2.76, 1.03)$. Now if the deadline is scaled from $t = 1$ up to $t = 2$, these rates just have to be halved to make them $(\theta^1, \theta^2) = (2.76/2, 1.03/2)$. Clear prominence order and the exact surplus loss come from the Poisson structure but, otherwise, it can be dispensed with; we could let the firms set directly the numbers of informed and uninformed consumers.

This makes our model well suited to analyze online search, where switching costs are low. Our findings then resonate with the general idea that, although search technology keeps improving, due to endogenous countervailing adjustments by firms, total search costs may not converge to zero. The point has been made, for instance, in a symmetric model by Ellison and Wolitzky (2012) and in an asymmetric one by Wilson (2010).[5,6] Both start with a setup as in Stahl (1989) but let the firms readjust a consumer's time cost of search by various actions coined with the term obfuscation. Ellison and Wolitzky (2012) show that obfuscation can be beneficial to all firms even after the consumers have learned to expect it. Since consumers' search costs are convex in search time, any time delay in the first store makes the second search more costly, with a locking effect. Wilson (2010) observes that duopolies have generally a well-known non-obfuscating firm and a less-known obfuscating firm. By making its price hard to find, the obfuscating firm induces it competitor to specialize in a less elastic part of demand, which relaxes price competition. A resembling mechanism is at work also here; in particular the equilibrium price distributions look much alike. Our paper can thus quite reasonably be considered a modification of Wilson (2010) to analyze the effects of obfuscation on the numbers of informed and uninformed consumers. While these numbers seem like one of the most natural adjustment margins to changes in the difficulty of acquiring information, Ellison and Wolitzky (2012) and Wilson (2010) keep them fixed.

Both papers find a multiplicity of equilibria for cases when there is no obfuscation cost. In our case we can avoid this and pin down instead an essentially unique equilibrium pattern because the fact that consumers have limited time for search creates an implicit obfuscation cost. Namely, the higher the frictions the firms set, the larger the number of consumers who fail to find anything and are thus basically driven out of the market. This externality arising from obfuscation has not been taken into account in related models, which have mainly looked at the intensive margin (prices), mostly ignoring the extensive margin (demand).[7]

Another obvious difference is related to modeling approach. To endogenize the numbers of informed and uninformed consumers, we use a new kind of search model, based on deadlines and gradual arrival of price information within stores, that abstracts from the typical hold-up problem in sequential search models where search costs are paid up-front. The advantage of our approach is that we need not assume fixed search costs or fixed

[5]Our findings can also be juxtaposed with those in papers about market prominence. We find that a strict prominence order will arise. The first prominent store is faster and has, therefore, also higher prices and profit. There is no literature consensus about this: in Armstrong et al. (2009) and Rhodes (2011) the first store has a higher price whereas in Arbatskaya (2007) and Wilson (2010) it is the other way.

[6]Our model has also some connections with competitive search models à la Peters (1991), Moen (1997), and Burdett et al. (2001): While we analyze a market where the firms commit to frictions that indirectly advertize the price, competitive search models explore a market where the firms commit to prices that indirectly advertize the frictions. The frictions are modeled by a Poisson process in both cases.

[7]However, see Taylor (2015) who studies a case where obfuscation has positive welfare effects.

switching costs: these arise endogenously from the interplay of frictions and deadlines. Our model is thus more self-contained. It has also a more detailed account of how consumers search in stores.[8]

This paper is organized in the following way: The model is given in Section 2 followed by a consumer's search problem and a firm's pricing problem. Section 3 contains the main part of analysis: a firm's problem of choosing the frictions, separately for the prominent firm and the non-prominent firm. We offer some closing remarks in Section 5. Most proofs appear in Appendix.

2 Model

There is a unit mass of consumers $B = [0, 1]$, each with a unit of time $t \in [0, 1]$ to find a certain product and better prices, and two firms $i \in \{1, 2\}$ selling these products in their online stores for typically different prices. The unit production cost is normalized to zero and the consumer valuation to one. As a novelty, the firms have the full control over the "frictions" in their store $\theta^i \in [0, \infty]$. The firms also choose their prices $p^i \in [0, 1]$ or, as equilibria are in randomized pricing strategies, their price distributions $F^i \in \Delta [0, 1]$.

Search is a random gradual process, which takes place in one of the stores at a time. A consumer's search cost is zero for $t < 1$ (before the deadline $d = 1$) and infinite for $t > 1$ (after the deadline $d = 1$).[9] For every point in time $t \in (0, 1)$, the consumers can thus decide afresh weather to search in store $i = 1$ or in store $i = 2$. In store $i = 1$, the price, p^1, can be found at Poisson rate θ^1 whereas, in store $i = 2$, the price, p^2, can be found at Poisson rate θ^2. The consumers can switch freely on the go and recall earlier prices.[10,11]

The precise timing is:

1. Firms commit to rates $\boldsymbol{\theta} = (\theta^1, \theta^2)$ in public.
2. Firms fix the prices $\mathbf{p} = (p^1, p^2)$ to be found.
3. Consumers search dynamically from $t = 0$ to $t = 1$ and buy the product with the best price in the end.

[8] Usually a store is treated as a black box; but see Petrikaite (2015) and Hämäläinen (2016).

[9] As explained in the related article by Hämäläinen (2016) the deadline can either be fixed or random (the consumer will lose her patience at some random time point, modeled by another Poisson process).

[10] This gives our model a slight flavor of a Poisson bandit problem (see Bergemann and Välimäki (2006) for a compact review) where each store represents an "arm". We operate however, exceptionally, without discounting in finite continuous time. There is also no tradeoff between exploitation and exploration because the arms have a known expected value and they break up after the firm's price is found.

[11] Depending how one views the idea that consumers have a fixed or random deadline, a kind of rule of thumb to ration their search time, our model could be considered either behavioral or rational. Due to the flat search cost, it has also some common traits with both sequential and non-sequential search models (Baye et al., 2006a). Yet, all our consumers search dynamically rationally until their deadline.

Thus, we have a three stage extensive game with a dynamic program embedded in the final stage. Or, equivalently for this case, a two stage game where, first, the firms publicly commit to the frictions and, then, the firms choose their randomized pricing strategies and the consumers select their sequential search strategies.[12,13]

2.1 Search

The game next is solved by *backwards induction*. Without loss of generality we assume that $\theta^1 \geq \theta^2$. Thereby, the expected price in the *faster* store is denoted by $E(p|F^1)$, the expected price at the *slower* store is denoted by $E(p|F^2)$, and the expected minimum of the two prices by $E(p|F_{min})$. A consumer's problem can then be captured by the following Bellman equation, which gives the value of searching at time t for a consumer who has not yet found a price:

$$V_t := \max_{i=1,2} V_t^i = \max_{i=1,2} \Big(\theta^i dt \Big((1 - e^{-\theta^{-i}(1-t)})(1 - E(p|F_{min})) + e^{-\theta^{-i}(1-t)}(1 - E(p|F^i)) \Big) + (1 - \theta^i dt) V_{t+dt} \Big). \tag{1}$$

Before a price is found, the consumer chooses store i over store $-i$ at time t only if the associated value V_t^i is at least as large as the comparable value V_t^{-i}. These consumer values capture the following ideas: If a consumer searches in firm i's store during a small length of time $dt > 0$, she finds its price p^i with probability $\theta^i dt$. When that happens, the consumer obviously switches immediately to find also the other firm's price. If the first price is observed at time t, the probability of discovering also the other price is thus $1 - e^{-\theta^{-i}(1-t)}$ (in that case, the consumers the minimum of p^i and p^{-i} but, otherwise, she buys for the only price she has found, p^i). To simplify the following analysis we assume here next that, if the stores initially look the same to the consumers, i.e., if $V_t^1 = V_t^2$ for $t = 0$, half the consumers start their search from each and, if no reason for switching arises thereafter, i.e., if $V_t^1 = V_t^2$ for $t > 0$, they continue with their first store. To characterize consumer search behavior, it thus only remains to determine how many consumers start from each firm and whether they have an incentive switch the store at an some intermediary time

[12] Observe that like in Wilson (2010) it is important for the firms to commit to the frictions (they represent here a firm's long-term investment in a particular search technology within its store). Namely, if it was feasible to change the frictions after the first stage, the non-prominent firm would like to serve immediately all the consumers who visit its store. If the consumers knew this, they would first visit the non-prominent firm. For that particular case, there might hence not exist an equilibrium in pure strategies for frictions.

[13] This game is *not* equivalent with a strategic game in which the firms choose a distribution of prices $\Delta[0,1]$ and a distribution of rates $\Delta[0,\infty]$ and the consumers choose a search plan. Indeed, the reason why Bertrand equilibrium is eliminated is that we let the firms set the rates first and only then choose the prices. Note however that, even in this modification with simultaneous moves, Bertrand equilibrium $(\boldsymbol{\theta}, \mathbf{p}) = (\infty, \infty; 0, 0)$ would not be robust to a slight tremble in p^i or θ^i: both would make $p^{-i} > 0$ a profitable deviation.

point $t \in (0,1)$ before their first price discovery.

Conveniently, we find that the optimal consumer strategy is stationary:

Lemma 1 *The consumers switch the firm only when a price is found.*
(i) If $\theta^i \left(1 - E(p|F^i)\right) > \theta^{-i} \left(1 - E(p|F^{-i})\right)$*, the consumers start from firm* $i = 1, 2$ *and search there until they find its price.*
(ii) If $\theta^1 \left(1 - E(p|F^1)\right) = \theta^2 \left(1 - E(p|F^2)\right)$*, the consumers could start from either firm and search there until they find its price.*

Thus, consumer strategy can be represented by the fraction of consumers, t^1, who start from firm $i = 1$. The rest of them, $t^2 = 1 - t^1$, start of course from firm $i = 2$. The two of these are captured together by $\mathbf{t} = \left(t^1, t^2\right)$.

Note that in contrast to the usual exogenous partition as in Varian (1980) and Stahl (1989), the interplay of frictions $\boldsymbol{\theta}$ and consumer strategy $\mathbf{t}$ now partitions the set of consumers *endogenously* into four disjoint sets

$$B_\emptyset + B_1 + B_2 + B_{1,2} = 1,$$

where consumers $B_\emptyset$ fail to find any price, consumers B_i ("captives" or "uninformed consumers") find just one of the two prices, p^i, and consumers $B_{1,2}$ ("shoppers" or "informed consumers") manage to find both.

The number of consumers observing no price is

$$B_\emptyset = t^1 e^{-\theta^1} + t^2 e^{-\theta^2},$$

and the number of trades is hence equal to

$$1 - B_\emptyset = 1 - t^1 e^{-\theta^1} - t^2 e^{-\theta^2}.$$

The numbers of captives to each firm are

$$B_1 = t^1 \theta e^{-\theta} \text{ and } B_2 = t^2 \theta e^{-\theta}, \text{ for } \theta = \theta^1 = \theta^2, \tag{2}$$

and

$$\begin{aligned} B_1 &= t^1 \int_0^1 e^{-\theta^2(1-\tau)} \theta^1 e^{-\theta^1 \tau} d\tau = t^1 \frac{\theta^1}{\theta^2 - \theta^1} \left(e^{-\theta^1} - e^{-\theta^2}\right), \\ &= \frac{\theta^1}{\theta^1 - \theta^2} \left(e^{-\theta^2} - B_\emptyset\right), \text{ for } \theta^1 \neq \theta^2, \end{aligned} \tag{3}$$

and

$$
\begin{aligned}
B_2 &= t^2 \int_0^1 e^{-\theta^1(1-\tau)}\theta^2 e^{-\theta^2\tau}d\tau = t^2 \frac{\theta^2}{\theta^1-\theta^2}\left(e^{-\theta^2}-e^{-\theta^1}\right),\\
&= \frac{\theta^2}{\theta^1-\theta^2}\left(B_\emptyset - e^{-\theta^1}\right), \text{ for } \theta^2 \neq \theta^1,
\end{aligned} \tag{4}
$$

where, in the integrands, $e^{-\theta^i\tau}$ is the probability that the consumer does not find store i's price during time interval $t \in [0,\tau]$, θ^i is the probability that the consumer succeeds to discover this price exactly at moment $t=\tau$, and $e^{-\theta^{-i}(1-\tau)}$ is the probability that the consumer does not find store $-i$'s price during time interval $t \in [\tau, 1]$. The shoppers are just the residual

$$
B_{1,2} = 1 - B_\emptyset - B_1 - B_2. \tag{5}
$$

These notions will be used repeatedly in the firm's pricing problem. It is clear from above that $\frac{\partial B_{1,2}}{\partial t^1} = 0$, $\frac{\partial B_\emptyset}{\partial t^1} < 0$, $\frac{\partial B_1}{\partial t^1} > 0$ and $\frac{\partial B_2}{\partial t^1} < 0$. In consequence, if consumer search becomes more efficient, the number of shoppers does not change but the number of trades increases and the faster (slower) firm gains more (less) captives.

To maximize the number of trades, the consumers should thus search in the faster store at least until they have found one price; otherwise, more fail to trade. That is, efficient search requires that, if store $i=1$ has strictly lower frictions than store $i=2$, i.e., $\theta^1 > \theta^2$, all the consumers must start from store $i=1$, i.e., $t^1 = 1 - t^2 = 1$.

2.2 Prices

Now, for any partition of consumers $\{B_\emptyset, B_1, B_2, B_{1,2}\}$, the profit Π^i to firm i has, as is standard, a price-sensitive part (shoppers) and a price-insensitive part (captives):

$$
\Pi^i(p^i) = \left(B_i + B_{1,2}(1 - F^{-i}(p^i))\right) p^i.
$$

The equilibrium price distribution can thus be calculated much like in Varian (1980) and Stahl (1989) for symmetric cases and Wilson (2010) for asymmetric cases:

Lemma 2 *Consider $\boldsymbol{\theta} = \left(\theta^i, \theta^{-i}\right)$ and $\boldsymbol{t} = \left(t^1, t^2\right)$ such that $B_1 \geq B_2$, $B_1 > 0$, and $B_{1,2} > 0$. Then, there exists a unique equilibrium price distribution $\boldsymbol{F} = \left(F^1, F^2\right)$ where*

$$
F^1(p) = \frac{B_2 + B_{1,2}}{B_{1,2}} - \frac{\Pi^2}{B_{1,2}}\frac{1}{p} \text{ for all } p \in \left[\underline{p}, 1\right),
$$

with an atom $\alpha := \frac{B_1 - B_2}{B_1 + B_{1,2}} \leq \underline{p}$ at the highest price $\overline{p} = 1$, and

$$
F^2(p) = \frac{B_1 + B_{1,2}}{B_{1,2}} - \frac{\Pi^1}{B_{1,2}}\frac{1}{p}, \text{ for all } p \in \left[\underline{p}, 1\right].
$$

The lowest price is given by $\underline{p} = \frac{B_1}{B_1+B_{1,2}}$ *and the firms' profits by*

$$\Pi^1 = B_1 \text{ and } \Pi^2 = \underline{p}B_2 + (1-\underline{p})B_1 \leq B_1$$

Observe also that both Diamond equilibrium and Bertrand equilibrium could arise in our model for suitably chosen frictions $\boldsymbol{\theta}$: if $B_{1,2} = 0$ (no shoppers; this would arise under $\boldsymbol{\theta} = (0,0)$, $\boldsymbol{\theta} = (a,0)$ and $\boldsymbol{\theta} = (0,a)$ for any $a > 0$), the firms use a pure strategy $p^i = 1$ and, if $B_{1,2} > 0$ but $B_1 = B_2 = 0$ (no captives; this would arise under $\boldsymbol{\theta} = (\infty,\infty)$), the firms use a pure strategy $p^i = 0$.

Generally, the store with more captives has higher prices and profit. It mixes between using random discount prices $p^1 < 1$, to compete for shoppers, and the monopoly price $p^1 = 1$, to tax its numerous captives. There could hence be an atom at one. The other store who has fewer captives would instead never use the monopoly price and, thus, randomizes only the size of the discount, $p^2 < 1$.

In other words, the stores' equilibrium pricing strategies are wired so as to let them specialize in different groups of consumers. This aligns the firms' payoffs and helps to relax the price competition. The profit to the high-profit firm, Π^1, equals the number of captives it attracts, B_1, whereas the profit to the low-profit firm, Π^2, is a weighted average of its own captives, B_2, and the other firm's captives, B_1. Note that the weights, $\underline{p} = \frac{B_1}{B_1+B_{1,2}}$ and $1-\underline{p} = \frac{B_{1,2}}{B_1+B_{1,2}}$ could be taken as a measure of how close the market is to Bertrand equilibrium (arises with $B_{1,2} > 0, B_1 = B_2 = 0$) or to Diamond equilibrium (arises with $B_{1,2} = 0, B_1 > 0, B_2 \geq 0$). Specifically, if the consumers have high "bargaining power", captured by a low $\underline{p}$, the firms have more closely aligned preferences but, if the firms have high "bargaining power", captured by a high $\underline{p}$, they compete more fiercely over their share of the cake. As it later turns out, the outcome that obtains can thus be regarded as a compromise of some kind between the two firms and the consumers. In particular, we find that in equilibrium $\underline{p} = 1/2$, $B_1 = B_{1,2}$, and $B_2 = 0$.

It is now straightforward to calculate the expected prices for later use:

$$\begin{aligned}
E(p|F^1) &= \int_{\underline{p}}^1 pf^1(p)dp + \alpha = \frac{\Pi^2}{B_{1,2}}\ln\left(\frac{1}{\underline{p}}\right) + \alpha \\
\geq E(p|F^2) &= \int_{\underline{p}}^1 pf^2(p)dp = \frac{\Pi^1}{B_{1,2}}\ln\left(\frac{1}{\underline{p}}\right) \\
\geq E(p|F_{min}) &= \int_{\underline{p}}^1 p\left(f^2(p)\left(1-F^1(p)\right) + f^1(p)\left(1-F^2(p)\right)\right)dp \\
= &\frac{B_1\Pi^1 + B_2\Pi^2}{B_{1,2}^2}\ln\left(\frac{1}{\underline{p}}\right) + \frac{\Pi^1\Pi^2}{B_{1,2}^2}\left(\frac{1-\underline{p}}{\underline{p}}\right).
\end{aligned}$$

3 Equilibria

3.1 Fixed point in search and prices

We move on to analyze equilibrium frictions. Based on the earlier analysis, we find importantly that any pair of frictions induces a unique fixed point in search and prices:

Proposition 1 *For any $\boldsymbol{\theta}$, there exists a unique fixed point in search and prices $(\boldsymbol{t}, \boldsymbol{F})$ where $\boldsymbol{F} = \boldsymbol{F}(\boldsymbol{\theta}, \boldsymbol{t})$ and $\boldsymbol{t} = \boldsymbol{t}(\boldsymbol{\theta}, \boldsymbol{F})$. In particular,*

1. *if $\theta^1 \left(1 - E(p|F^1(\boldsymbol{\theta}, (1,0)))\right) \geq \theta^2 \left(1 - E(p|F^2(\boldsymbol{\theta}, (1,0)))\right)$, then $t^1 = 1 - t^2 = 1$, $B_1 > B_2 = 0$ and $E(p|F^1) > E(p|F^2)$, whereas*

2. *if $\theta^1 \left(1 - E(p|F^1(\boldsymbol{\theta}, (1,0)))\right) < \theta^2 \left(1 - E(p|F^2(\boldsymbol{\theta}, (1,0)))\right)$, then $t^1 = 1 - t^2 < 1$, $B_1 \geq B_2 > 0$, where $\boldsymbol{t}$ is the unique solution to*

$$\frac{\theta^2}{\theta^1} = \frac{1 - E(p|F^1(\boldsymbol{\theta}, \boldsymbol{t}))}{1 - E(p|F^2(\boldsymbol{\theta}, \boldsymbol{t}))} = 1 - \alpha(\boldsymbol{\theta}, \boldsymbol{t}).$$

Concerning Proposition 1 note that, by Equations (2), (3), (4) and (5), B_1, B_2 and $B_{1,2}$ are determined by $\boldsymbol{\theta}$ and $\mathbf{t}$ uniquely whereas, by Lemma 2, $\mathbf{F}$ is dependent on $\boldsymbol{\theta}$ and $\mathbf{t}$ only through B_1, B_2, and $B_{1,2}$. This feature enables us to construct a hypothetical price distribution $\mathbf{F}(\boldsymbol{\theta}, \mathbf{t})$ based on each pair $(\boldsymbol{\theta}, \mathbf{t})$ by first calculating the associated $B_1(\boldsymbol{\theta}, \mathbf{t}), B_2(\boldsymbol{\theta}, \mathbf{t})$, and $B_{1,2}(\boldsymbol{\theta}, \mathbf{t})$ and thereafter the induced $\mathbf{F}(B_1, B_2, B_{1,2})$. Proposition 1 has also two noteworthy corollaries:

Corollary 1 (Effects of frictions on search efficiency) *The consumers search efficiently if the firms are either exactly similar in terms of their frictions, $\theta^1 = \theta^2$, or distinctly different for efficient search based prices, $\theta^1 \left(1 - E(p|F^1(\boldsymbol{\theta}, (1,0)))\right) \geq \theta^2 \left(1 - E(p|F^2(\boldsymbol{\theta}, (1,0)))\right)$.*

Corollary 2 (Effects of frictions on market prominence) *Lower frictions grant a firm more prominent market position and thus higher prices and profit: if $\theta^i \geq \theta^{-i}$, then more consumers start from the firm and $B_i \geq B_{-i}$ implying $\Pi^i \geq \Pi^{-i}$ and $E(p|F^i) \geq E(p|F^{-i})$.*

To sum up, we have now both symmetric and asymmetric candidate equilibria: If the firms are equally fast, half the consumers start from each firm and firms use symmetric pricing strategies and make the same profit whereas, if firm $i = 1$ is faster than firm $i = 2$, it wins a more prominent position in the market and has higher prices and profit.

We concentrate next on pure strategies in frictions, although it is clear that there also exist equilibria where the firms mix in frictions.[14] Pure strategies seem more natural, however, because we consider a game in which frictions become common knowledge for the following subgame where the firms set their prices and the consumers search.

[14] An extension to a larger market also involves randomized strategies; a memo available upon request.

3.2 Frictions: analytical results

This part contains our main results. First, we rule out the existence of symmetric equilibria in general and, further, the existence of asymmetric equilibria where the consumers are indifferent to which firm they start from. This demonstrates particularly that Diamond equilibrium and Bertrand equilibrium cannot arise in this game. Our next result also implies that we can later focus on cases in which there is a prominent firm and a non-prominent firm – and where all consumers start their search from the first one.

Lemma 3 *There exists no equilibrium (a) where the firms use pure strategies for frictions, $\theta^2 \leq \theta^1 < \infty$, and (b) a positive fraction of consumers start from firm $i = 1$ and a positive fraction of consumers start from firm $i = 2$, i.e., $t^1 = 1 - t^2 < 1$. Furthermore, the consumers are never indifferent to which store they start from; their preference order is always strict.*

The non-existence of Diamond equilibrium and Bertrand equilibium can be observed also more simply:

Remark 1 *There exist no Bertrand equilibrium, where either of the two firms generates no frictions and the market price equals zero.*

Proof. Bertrand equilibrium requires that both firms choose zero frictions $\boldsymbol{\theta} = (\infty, \infty)$. Yet, both firms gain if one of them deviates to some finite rate θ because it raises their profit up from zero to $\frac{B_i B_{1,2}}{B_i + B_{1,2}} = \left(1 - e^{-\theta}\right) e^{-\theta}$ (to the deviator, who has $t^{-i} = 0$ due to its positive frictions $\theta^{-i} < \infty$) and $B_i = 1 - e^{-\theta}$ (to the non-deviator, who gains $t^i = 1$ thanks to its markedly lower frictions $\theta^i = \infty$). ■

Remark 2 *There exist no Diamond equilibrium, where at least one firm generates infinite frictions and the market price equals one.*

Proof. As the consumers always search, Diamond equilibrium requires that at least one of the firms is practically out of the market due to its infinite frictions, $\boldsymbol{\theta} = \left(\theta^i, 0\right), \left(0, \theta^{-i}\right)$. Its profit then equals zero because it serves nobody. However, for any lower level of frictions, the firm's profit is positive, $\Pi^i = B_i > 0$ or $\Pi^{-i} = \underline{p} B_{-i} + \left(1 - \underline{p}\right) B_i > 0$. There is hence a profitable deviation to higher $\theta' > 0$. ■

By Lemma 3, we now know that any equilibrium where firms use pure strategies for frictions must have a faster, prominent firm and a slower, non-prominent firm. This asymmetry of frictions arises as a natural way to relax price competition because the firms can then specialize in different consumer segments: the prominent firm especially to its uninformed captive consumers with totally inelastic demand and the non-prominent firm to the price sensitive informed consumers. The idea is essentially the same as in Wilson (2010). Nevertheless, as the consumers trade off higher prices for stronger frictions, by choosing sufficiently different levels of frictions the firms can here, additionally, also

guarantee that the consumers search in an efficient manner and that there is not much waste. Therefore, a strict prominence order arises in an equilibrium. The strictness of the prominence order means that all consumers strictly prefer to start from the faster firm $i = 1$; they switch to the slower firm $i = 2$ only when they have found the price p^1. As a result, the non-prominent firm does not attract any captives $B_2 = 0$. The profits for the prominent firm and the non-prominent firm are hence given by $\Pi^1 = B_1$ and $\Pi^2 = (1 - \underline{p})B_1 = (1 - \alpha)B_1 = B_1 B_{1,2}/(B_1 + B_{1,2})$, respectively. We next describe both firms' best responses in terms of their frictions.

3.2.1 Prominent firm's problem

The prominent firm maximizes the following expression:

$$\max_{\theta^1} B_1(\boldsymbol{\theta}) = \max_{\theta^1} \frac{\theta^1}{\theta^1 - \theta^2}\left(e^{-\theta^2} - B_{\emptyset}(\theta^1)\right)$$

The prominent firm's profit is given by the number of uninformed consumers, who are its captives. Since consumers switch the store once they find a price, the prominent firm has a tradeoff between maximizing the number of consumers who find its own price (by decreasing the frictions, increasing the inflow) and minimizing the number of consumers who find the other firm' price (by increasing the frictions, decreasing the outflow). It is hence optimal for it to avoid extremes and generate intermediate frictions. Unfortunately, this implies that the number of trades is suboptimal.

Proposition 2 *There exists no efficient equilibria, where the prominent firm generates no frictions.*

The intuition for this is that, since the prominent firm cannot reap (bear) the full positive (negative) externality that faster (slower) search has on the consumers, it has no incentive to serve every consumer instantaneously. Therefore, any equilibrium is inefficient: though consumer search behavior is efficient, frictions are too high.

To put it another way, while the consumers are free to switch the store at any point, we know that in equilibrium they do so only after they have found a price. This entails that the rates at which price information arrives play a role of an implicit endogenous switching cost. If the frictions are weaker in the first store, there is more time to discover the price in the second one. That intensifies price competition. Therefore, although one store could serve the entire market if it chose to play down its frictions, it has no incentive to do so because that would also eliminate the switching cost.

It is noteworthy that both firms have thus a strategic incentive to generate intermediate frictions, which does not arise, say, from a built-it cost saving motive. We analyze the welfare consequence of this more in the following numerical part, where describe equilibria. For the Poisson case we find that the surplus loss amounts to 6%.

3.2.2 Non-prominent firm's problem

The non-prominent firm maximizes the following expression:

$$\max_{\theta^2} \frac{B_1(\boldsymbol{\theta})B_{1,2}(\boldsymbol{\theta})}{B_1(\boldsymbol{\theta})+B_{1,2}(\boldsymbol{\theta})} = \max_{\theta^2} \frac{\frac{\theta^1}{\theta^1-\theta^2}\left(e^{-\theta^2}-B_\emptyset(\theta^1)\right)\left(1-B_\emptyset(\theta^1)-\frac{\theta^1}{\theta^1-\theta^2}\left(e^{-\theta^2}-B_\emptyset(\theta^1)\right)\right)}{1-B_\emptyset(\theta^1)},$$

or, equivalently, the product of the other firm's captives and shoppers

$$\max_{\theta^2} B_1(\boldsymbol{\theta})B_{1,2}(\boldsymbol{\theta}) = \max_{\theta^2} \frac{\theta^1}{\theta^1-\theta^2}\left(e^{-\theta^2}-B_\emptyset(\theta^1)\right)\left(1-B_\emptyset(\theta^1)-\frac{\theta^1}{\theta^1-\theta^2}\left(e^{-\theta^2}-B_\emptyset(\theta^1)\right)\right).$$

This formulation demonstrates clearly that in the neighborhood of the equilibrium, the non-prominent firm has here an unprecedented incentive to equalize the numbers of informed consumers and uninformed consumers. Its demand is coming only from shoppers but due to their intensifying effect on competition it wins them over more frequently if the prominent firm has more captives, which raises its prices.

The non-prominent firm has thus mixed incentives in choosing the frictions: if it elevates θ^2, the number of informed consumers does go up (its has more potential demand) but then the number of uninformed consumers goes down (competition becomes stronger); the opposite happens if it lowers θ^2. This clear tradeoff makes it profitable for the firm to avoid extremes and choose instead a intermediate level of θ^2.

Proposition 3 *There are equally many informed consumers and uninformed consumers in an equilibrium.*

More specifically said, the non-prominent firm has an incentive to make sure that the outcome is exactly in between Diamond equilibrium and Bertrand equilibrium, as measured by the relative numbers of informed consumers $\frac{B_1}{B_1+B_{1,2}} = \underline{p}$ and uninformed consumers $\frac{B_{1,2}}{B_1+B_{1,2}} = 1-\underline{p}$. This entails that any equilibrium must have $\underline{p} = \alpha = 1/2$. This is then reflected also in the surplus sharing: 50 % - 25 % - 25 %. We discuss the idea more in the subsequent numerical part.

3.3 Frictions: numerical results

In this part we present numerical results to illustrate our findings and to support our claim that there are just two equilibria in this game. The firm's reaction curves are presented by Figure 1.[15] They have a discontinuity at $\left(\theta^1,\theta^2\right) \approx (2.33, 2.33)$ and they cross each other at $\left(\theta^1,\theta^2\right)^\star \approx (2.76, 1.03)$ when $\theta^1 \geq \theta^2$ (the assumed case) and at $\left(\theta^1,\theta^2\right)^\star \approx (1.03, 2.76)$

[15] Grey color near the 45-degree line marks areas where we have only approximate results (no knowledge about the exact best response, only upper bounds on profit).

when $\theta^2 \geq \theta^1$ (the inverse case). This pins down our two equilibrium points and suggests that there exists a unique cutoff level for frictions $\theta' \approx 2.33$ such that: if the other firm if faster than this cutoff, $\theta^{-i} < \theta'$, firm i's best response is to become the prominent firm, i.e., $BR_i(\theta^{-i}) > \theta^{-i}$, and, if the other firm is slower than it, $\theta^{-i} > \theta'$, firm i's best response is to become the non prominent firm, i.e., $BR_i(\theta^{-i}) < \theta^{-i}$.

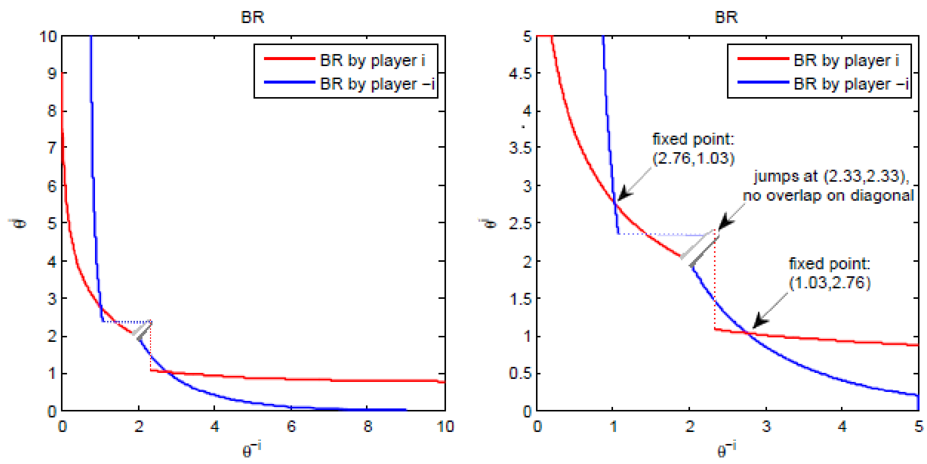

Figure 1: Best response functions: zoom-out (left), zoom-in (right).

Claim 1 *There exist two equilibria in pure strategies for frictions, with the same unique form:* $\boldsymbol{\theta}^\star \approx (1.03, 2.76)$ *and* $\boldsymbol{\theta}^\star \approx (2.76, 1.03)$.

Proof. It is easy to ascertain that the first-order conditions of the prominent firm's problem and the non-prominent firm's problem (Conditions (9), (10) and (11) in Appendix) are fulfilled uniquely by $(\theta^1, \theta^2) \approx (2.76, 1.03)$ if we assume that $\theta^1 \geq \theta^2$. Otherwise, we rely on Figure 1 and what we have in Appendix. ■

Observation 1 *Both equilibria have the same unique form:*

1. *Frictions: there is a prominent firm who sets frictions* $\theta^i = 2.76$ *and a non-prominent firm who sets frictions* $\theta^{-i} = 1.03$. *Thereby, the expected wait time in the former is about* 36% *of the total time and the expected wait time in the latter is about* 97% *of the total time. Note that these times can be regarded as endogenous search costs or switching costs.*

2. *Search: The consumers search in the prominent firm until they find their first price quote,* $t^i = 1$ *and* $t^{-i} = 0$. *Thus, 47 per cent of the consumers find both prices,* $B_{1,2} \approx 0.47$, *and 47 per cent of the consumers find a price from the prominent firm but not from the non-prominent firm,* $B_i \approx 0.47$*; 6 per cent of the consumers fail to find a price,* $B_\emptyset \approx 0.06$.

3. *Prices: The prominent firm offers the monopoly price* $(p = 1)$ *and a random discount price* $(p < 1)$ *equally often,* $\alpha = 0.5$*; the non-prominent firm always offers a random discount price. Given that a firm offers a discount, the expected discount size is 31 per cent of the monopoly price at either firm; the largest such regularly used discount is 50 per cent,* $\underline{p} = 0.5$*.*

4. *Surplus sharing: The prominent firm is making the double of what the non-prominent firm is making,* $\Pi^i = B_i \approx 0.47, \Pi^{-i} = \alpha B_{1,2} \approx 0.5 \cdot 0.47$*. The prominent firm also gets half the surplus, the non-prominent firm gets a quarter and the consumers get a quarter; 6 per cent of the cake is wasted.*

Proof. An elementary calculation that uses the fact that $\boldsymbol{\theta} \approx (2.76, 1.03)$ and the expressions that we have provided above for $B_i(\boldsymbol{\theta})$, $B_{1,2}(\boldsymbol{\theta})$, $B_{\emptyset}(\boldsymbol{\theta})$, and $E(p|\mathbf{F})$. ■

This friction pattern is the unique one even if we extend or shorten the deadline. In other words, the outcome is just the same (except for a possible renaming of firms) in terms of search, prices, and profit whether the consumers can search for a decade or a minute. In particular, for all choices of deadline, the firms have an incentive to adjust the frictions such that the numbers of informed consumers and uninformed consumers are the same.

Remark 3 *An identical equilibrium outcome arises whatever the deadline* $d < \infty$ *is as long as it is finite: if* (θ^i, θ^{-i}) *is an equilibrium when the search horizon is* $t \in [0, 1]$*, then* $(\frac{\theta^i}{d}, \frac{\theta^{-i}}{d})$ *is an equilibrium when the search horizon is* $t \in [0, d]$*.*

Observe, however, Bertrand equilibrium would be the unique equilibrium that if there were no deadline and Diamond equilibrium would be another equilibrium if there was no time at all.

Remark 4 *There is a discontinuity in the equilibrium set as* $d \to \infty$ *because, at* $d = \infty$*, Bertrand equilibrium with* $p^i \equiv 0$ *is the unique equilibrium.*

Remark 5 *There is a discontinuity in the equilibrium set as* $d \to 0$ *because, at* $d = 0$*, Diamond equilibrium with* $p^i \equiv 1$ *is another equilibrium.*

To summarize, the set of equilibria is invariant to finite translations in the deadline, which is the only exogenous parameter in our model. Bertrand equilibrium is possible only if the consumers are extremely patient and Diamond equilibrium only if the consumers are extremely impatient. Otherwise, the outcome is precisely in between these extremes in the sense that there are exactly as many informed consumers as there are uninformed consumers.

4 Closing remarks

We introduce a new price search model that features endogenous frictions, modeled by the gradual arrival of price information within stores and deadlines. Assuming that frictions represent a firm's long-term investment in a particular search technology, we find that there exists a unique inefficient equilibrium pattern. There is a prominent firm, a non-prominent firm, and, as our key finding, exactly equally many informed and uninformed consumers in the market. In the Poisson setting, which we study for concreteness, welfare loss amounts to approximately 6 per cent of the cake.

We observe that an identical result arises as long as there is a deadline by which a consumer must stop. It could be two seconds or two decades; that does not matter. It is because of this deadline that both firms have a strategic incentive to slow down searching consumers slightly – but not in extreme amounts: If they keep frictions very high, the consumers fail to find anything but, if the frictions are very low, the consumers become perfectly informed, which drives the firms into a price war. Interestingly, as the deadline approaches infinity, this Bertrand equilibrium reappears.

Appendix

PROOF OF LEMMA 1

Step 1: Optimal search

For a starter, note that a consumer can find either zero prices, only firm $i = 1$'s price, only firm $i = 2$'s price, or both prices. In the first case her payoff of course equals zero but in three latter cases her payoffs can be denoted more shortly as follows

$$CS^1 := 1 - E(p|F^1), CS^2 := 1 - E(p|F^2), \text{ and } CS_{min} := 1 - E(p|F_{min}).$$

It is clear that the probability of finding zero prices in minimized and the probability of finding two prices maximized by searching in the faster store until a price is found. If the faster store is also the cheaper one, it is also clearly optimal to start from there.

Now the only unresolved case is thus the one where the faster store has higher prices, i.e., where $\theta^1 > \theta^2$ and $CS^1 > CS^2$. This is also the relevant case here because, as we prove later, in equilibrium this kind of tradeoff between frictions and prices arises.

Note that, as the consumers can switch freely any moment t, their continuation value V_{t+dt} in equation (1) is the same whether the consumer is currently at firm $i = 1$ or at firm $i = 2$. This implies that, to maximize the consumer value, V_t, the consumer should search in the store who is offering the largest marginal descent in consumer value, $\dot{V}_t$:

$$\text{argmax}_i V_t^i = \text{argmin}_i \dot{V}_t^i.$$

Now provided the consumer stays in store i during the next short time interval $[t, t + dt]$, this

time derivative of the consumer value can be written as follows:[16]

$$
\begin{aligned}
\frac{V_{t+dt} - V_t^i}{dt} &= -\theta^i \Big(e^{-\theta^{-i}(1-t-dt)} (1 - E(p|F^i) - V_{t+dt}) \\
&\quad + (1 - e^{-\theta^{-i}(1-t-dt)})(1 - E(p|F_{min}) - V_{t+dt}) \Big) \rightarrow \\
\dot{V}_t^i &= -\theta^i \left(e^{-\theta^{-i}(1-t)} (E(p|F_{min}) - E(p|F^i)) + (1 - E(p|F_{min}) - V_t) \right).
\end{aligned}
$$

Obviously, the consumer value is positive, $V_t^i \geq 0$, and the change in consumer value is negative, $\dot{V}_t^i \leq 0$, for any t and i. Otherwise, it would pay off to stay idle.

To sum up what we have, this entails that for any point in time $t \in [0, 1]$ a consumer who has not yet discovered a price chooses store $i = 1$ over store $i = 2$ *iff*

$$
\begin{aligned}
&\theta^1 e^{-\theta^2(1-t)} (CS^1 - V_t) + \theta^1 (1 - e^{-\theta^2(1-t)})(CS_{min} - V_t) \geq \\
&\theta^2 e^{-\theta^1(1-t)} (CS^2 - V_t) + \theta^2 (1 - e^{-\theta^2(1-t)})(CS_{min} - V_t),
\end{aligned} \tag{6}
$$

or, *iff*

$$
\begin{aligned}
&\theta^1 e^{-\theta^2(1-t)} (CS^1 - CS_{min}) + \theta^1 (CS_{min} - V_t) \geq \\
&\theta^2 e^{-\theta^1(1-t)} (CS^2 - CS_{min}) + \theta^2 (CS_{min} - V_t).
\end{aligned} \tag{7}
$$

Using these expressions, we proceed by showing that, if a consumer prefers one store over the other at a given point in time, t', this is her preference order also later, for any $t > t'$; the stores are thus *absorbing*.

For the first case, suppose that a consumer prefers firm $i = 1$'s store over firm $i = 2$'s store at time t. That would give us:

$$
\begin{aligned}
&\theta^1 e^{-\theta^2(1-t)} \left(CS^1 - CS_{min} \right) + \theta^1 \left(CS_{min} - V_t \right) \\
&-\theta^2 e^{-\theta^1(1-t)} \left(CS^2 - CS_{min} \right) - \theta^2 \left(CS_{min} - V_t \right) \geq 0
\end{aligned}
$$

and

$$
\dot{V}_t = -\theta^1 e^{-\theta^2(1-t)} \left(CS^1 - CS_{min} \right) - \theta^1 \left(CS_{min} - V_t \right).
$$

To see now whether the consumer's preference for store $i = 1$ over store $i = 2$ becomes stronger or weaker over time, we differentiate (7) with respect to time to obtain

[16] Observe that the time derivative is well defined as long as the consumer does not change the store at t. Furthermore, even if the consumer does switch the store at t, as long as the consumer does not switch stores infinitely often, we can still use these same expressions which then only refer to the right derivative of consumer's value. It is the right derivative that matters for search incentives.

$$
\begin{aligned}
&\theta^1\theta^2 e^{-\theta^2(1-t)}\left(CS^1 - CS_{min}\right) - \theta^1\dot{V}_t \\
&- \theta^1\theta^2 e^{-\theta^1(1-t)}\left(CS^2 - CS_{min}\right) + \theta^2\dot{V}_t \\
&= \theta^1\theta^2 e^{-\theta^2(1-t)}\left(CS^1 - CS_{min}\right) - \theta^1\theta^2 e^{-\theta^1(1-t)}\left(CS^2 - CS_{min}\right) \\
&+ \left(\theta^1 - \theta^2\right)\left(\theta^1 e^{-\theta^2(1-t)}\left(CS^1 - CS_{min}\right) + \theta^1\left(CS_{min} - V_t\right)\right) \\
&+ \theta^1\theta^2\left(CS_{min} - V_t\right) - \theta^1\theta^2\left(CS_{min} - V_t\right) \\
&= \theta^1\left(\theta^1 e^{-\theta^2(1-t)}\left(CS^1 - CS_{min}\right) + \theta^1\left(CS_{min} - V_t\right)\right) \\
&- \theta^1\left(\theta^2 e^{-\theta^1(1-t)}\left(CS^2 - CS_{min}\right) + \theta^2\left(CS_{min} - V_t\right)\right) \geq 0.
\end{aligned}
$$

For the other case, suppose that a consumer prefers firm $i = 2$'s store over firm $i = 1$'s store at time t. That we have:

$$
\begin{aligned}
&\theta^1 e^{-\theta^2(1-t)}\left(CS^1 - CS_{min}\right) + \theta^1\left(CS_{min} - V_t\right) \\
&-\theta^2 e^{-\theta^1(1-t)}\left(CS^2 - CS_{min}\right) - \theta^2\left(CS_{min} - V_t\right) \leq 0
\end{aligned}
$$

and

$$
\dot{V}_t = -\theta^2 e^{-\theta^1(1-t)}\left(CS^2 - CS_{min}\right) - \theta^2\left(CS_{min} - V_t\right).
$$

Again, to see whether the consumer's preference for store $i = 1$ over store $i = 2$ becomes stronger or weaker over time, we differentiate (7) with respect to time

$$
\begin{aligned}
&\theta^1\theta^2 e^{-\theta^2(1-t)}\left(CS^1 - CS_{min}\right) - \theta^1\dot{V}_t \\
&- \theta^1\theta^2 e^{-\theta^1(1-t)}\left(CS^2 - CS_{min}\right) + \theta^2\dot{V}_t \\
&= \theta^1\theta^2 e^{-\theta^2(1-t)}\left(CS^1 - CS_{min}\right) - \theta^1\theta^2 e^{-\theta^1(1-t)}\left(CS^2 - CS_{min}\right) \\
&+ \left(\theta^1 - \theta^2\right)\left(\theta^2 e^{-\theta^1(1-t)}\left(CS^2 - CS_{min}\right) + \theta^2\left(CS_{min} - V_t\right)\right) \\
&+ \theta^1\theta^2\left(CS_{min} - V_t\right) - \theta^1\theta^2\left(CS_{min} - V_t\right) \\
&= \theta^2\left(\theta^1 e^{-\theta^2(1-t)}\left(CS^1 - CS_{min}\right) + \theta^1\left(CS_{min} - V_t\right)\right) \\
&- \theta^2\left(\theta^2 e^{-\theta^1(1-t)}\left(CS^2 - CS_{min}\right) + \theta^2\left(CS_{min} - V_t\right)\right) \leq 0.
\end{aligned}
$$

Altogether, this implies that the consumers have no incentive to switch before they find a price.[17] In other words, they always prefer to search in the same firm's store from the beginning $t = 0$ up to the deadline $t = 1$ given that no price is found in the meantime. To identify which store this is, note that, at the deadline $t = 1$, consumers prefer firm $i = 1$'s store over firm $i = 2$'s store *iff* the following condition holds

$$
\theta^1 CS^1 \geq \theta^2 CS^2. \quad \blacksquare
$$

Step 2: Value function

We can now also show how to derive the consumer value function V_t. Based on what we just found in Step 1, it is without loss to assume that all consumers start from store i and switch to

[17]Note that the derivative $\dot{V}_t$ is well defined in both cases since the consumer has no incentive to switch the firm: by continuity of (1), there exist no kink in V_t unless the consumer changes the store.

store $-i$ only when they find a price. Note first that

$$\dot{V}_t^i = -\theta^i \left(e^{-\theta^{-i}(1-t)}(CS^1 - CS_{min}) + CS_{min} - V_t \right)$$

defines a linear first order differential equation

$$\dot{V}_t^i - \theta^i V_t = -\theta^i \left(e^{-\theta^{-i}(1-t)}(CS^1 - CS_{min}) + CS_{min}. \right).$$

A solution to the related homogenous equation is

$$V_t = ce^{\theta^i t},$$

where c is a constant. To solve the non-homogenous equation, we can use the variation of the constants method in which we let the constants $c(t)$ be dependent on time such that

$$V_t = c(t)e^{\theta^i t}, \dot{V}_t = c(t)\theta^i e^{\theta^i t} + c'(t)e^{\theta^i t}.$$

This implies that

$$\dot{V}_t^i + \theta^i V_t = c'(t)e^{\theta^i t} = -\theta^i \left(e^{-\theta^{-i}(1-t)}(CS^i - CS_{min}) + CS_{min} \right)$$

and

$$\begin{aligned} c(t) &= -\int \theta^i e^{-\theta^i t} e^{-\theta^{-i}(1-t)}(CS^i - CS_{min})dt - \int \theta^i e^{-\theta^i t} CS_{min} dt + d, \\ &= \frac{\theta^i}{\theta^i - \theta^{-i}} e^{-\theta^i t} e^{-\theta^{-i}(1-t)}(CS^i - CS_{min}) + e^{-\theta^i t} CS_{min} + d, \end{aligned}$$

where d is a constant. The consumer value is thereby given as

$$V_t = \left(\frac{\theta^i}{\theta^i - \theta^{-i}} e^{-\theta^i t} e^{-\theta^{-i}(1-t)}(CS^i - CS_{min}) + e^{-\theta^i t} CS_{min} + d \right) e^{\theta^i t},$$

where the constant d is determined by the terminal condition

$$V_1 = \frac{\theta^i}{\theta^i - \theta^{-i}}(CS^i - CS_{min}) + CS_{min} + de^{\theta^i} = 0$$

implying

$$de^{\theta^i} = -\frac{\theta^i}{\theta^i - \theta^{-i}}(CS^i - CS_{min}) - CS_{min}.$$

The general solution to the terminal value problem is given by

$$\begin{aligned} V_t = V_t^i &= \frac{\theta^i}{\theta^i - \theta^{-i}} \left(e^{-\theta^{-i}(1-t)} - e^{-\theta^i (1-t)} \right) (CS^i - CS_{min}) + \left(1 - e^{-\theta^i (1-t)} \right) CS_{min} \\ &= B_i^t \left(CS^i - CS_{min} \right) + \left(1 - B_\emptyset^t \right) CS_{min} = B_i^t CS^i + B_{1,2}^t CS_{min}, \end{aligned}$$

where

$$B_i^t = \frac{\theta^i}{\theta^i - \theta^{-i}} \left(e^{-\theta^{-i}(1-t)} - e^{-\theta^i(1-t)}\right), \text{ for } \theta^i \neq \theta^{-i},$$
$$B_i^t = e^{\theta^i(1-t)}, \text{ for } \theta^i = \theta^{-i},$$
$$B_{1,2}^t = \left(1 - e^{-\theta^i(1-t)}\right). \quad \blacksquare$$

PROOF OF LEMMA 2

Step 1: General form of price distributions

Lemma 4 *Assume $B_i > 0$, either for firm $i = 1$ or firm $i = 2$, and $B_{1,2} > 0$. Then, the following hold true in any equilibrium:*

1. *The firms use randomized pricing strategies: F^1 and F^2.*
2. *Both F^1 and F^2 have the same interval support $supp(F) = \left[\underline{p}, \bar{p}\right]$, where $0 < \underline{p} < \bar{p} = 1$.*
3. *Neither has an atom at $p \in \left[\underline{p}, 1\right)$: $lim_{x \to p-} F^i(x) = F^i(p)$ for all $p < 1$ and $i = 1, 2$.*
4. *If F^1 has an atom at $p = 1$, F^2 has not and, if F^2 has an atom at $p = 1$, F^1 has not.*

We assume in this proof that $B_{1,2} > 0$ (there are shoppers) and $B_1 > 0$ or $B_2 > 0$ (there are captives). We also take $\varepsilon > 0$ to represent some tiny (infinitesimal) number.

First, we analyze three cases to prove by contradiction that both firms mix in equilibrium. In doing so, we make the assumption that one of the firms uses a pure strategy p^i. *Case* 1: $p^i < \min supp(F^{-i})$. As the demand $B_i + B_{1,2} > 0$ is unchanged as long as p^i stays below $\min supp(F^{-i})$, there is a profitable deviation for firm i from price p^i to price $p^i + \varepsilon$. *Case* 2: $p^i > \max supp(F^{-i})$. As the demand $B_{-i} + B_{1,2} > 0$ is unchanged as long as p^i stays above $\max supp(F^{-i})$, there is a profitable deviation for firm $-i$ from a price $p \in supp(F^{-i})$ to a price $p + \varepsilon$. *Case* 3a: $p^i > 0$ and $p^i \in supp(F^{-i})$. As the demand $B_{-i} + B_{1,2}(1 - F^i(p))$ jumps up at $p = p^i$, there is a profitable deviation for firm $-i$ from price p^{-i} to price $p^{-i} - \varepsilon$. *Case* 3b: $p^i = 0$ and $0 \in supp(F^{-i})$. Note that there are some captive consumers but, as both of the firms use the price zero, both of them are making zero profit. Thus, the firm who has captive consumers has a profitable deviation up from zero to extract some profit from the captive consumers. Altogether, Cases 1, 2, 3a and 3b demonstrate that (i) both stores use randomized pricing strategies and that (ii) both stores' profit and prices are bounded away from zero.

Next, we consider the supports $supp(F^i)$ and $supp(F^{-i})$ of the firm's randomized strategies F^i and F^{-i}. Suppose that $supp(F^i) \neq supp(F^{-i})$. This implies that there is some open set $U \neq \emptyset$ such that, with no loss of generality, $U \subset supp(F^i)$ and $U \cap supp(F^{-i}) = \emptyset$. But now, as the demand is unchanged for all $p^i \in U$ there is a profitable deviation up from the lower prices in U to the higher prices in U. This shows that the firm mix over the same set of prices $supp(F) := supp(F^i) = supp(F^{-i})$.

Last, we examine the support for possible gaps and jumps/atoms and delineate its boundaries. *Gaps*: Suppose the support is not connected but has a gap $\left[\underline{g}, \overline{g}\right] \cup supp(F) = \emptyset$ but for some $\left[\underline{g} - \varepsilon, \underline{g}\right] \cup supp(F) \neq \emptyset$ and $[\overline{g}, \overline{g} + \varepsilon] \cup supp(F) \neq \emptyset$. Then, as the demand is unchanged for all $p \in \left[\underline{g}, \overline{g}\right]$, there is a profitable deviation from some price $p \in \left[\underline{g} - \varepsilon, \underline{g}\right]$ to some price $p \in \left[\underline{g}, \underline{g} + \varepsilon\right]$. *Atoms*: Suppose the strategy F^i is not continuous but has an atom $\alpha^i > 0$ at $p^i_\alpha \in supp(F)$. Then, as the demand from shoppers, $\left(1 - F^i(p)\right) B_{1,2}$, is reduced by α^i at p^i_α, there is a profitable deviation for firm $-i$ from a price p^i_α or some price $p^i_\alpha + \varepsilon$ to some price $p^i_\alpha - \varepsilon$. This implies that there can be an atom at the upper bound only and used by a single firm only; this makes sure the other firm does not use p_α or any $p^i_\alpha + \varepsilon$, from which it would have a profitable deviation. *Bounds*: (i) Consider the highest price $\overline{p}$ the firms use. Note that the firm who has that price is only selling

to its captive consumers $B_i > 0$. Hence, there is a profitable deviation up in $\bar{p}$ unless it equals 1. (ii) Consider the lowest price $\underline{p}$ the firms use. As both of the stores make some profit, there is a profitable deviation up in price $\underline{p}$ unless it is bounded away from 0. ■

Step 2: Closed form of price distributions

Based on above, we only need to determine the firms' profits Π^i, the lower bound $\underline{p} > 0$ of the support, whether we need an atom $\alpha^i > 0$ at the upper bound $\bar{p} = 1$ of the support for firm $i = 1$ or $i = 2$, and the cumulative distribution functions F^1 and F^2.

Note first that, if firm i uses a price $p = 1 - \epsilon$, which lies just below the upper bound, it sells to its captives with probability one and to the shoppers with probability α^{-i}, which gives the likelihood that firm $-i$ has the price $p = 1$. Evaluated at the upper bound the firm's profit is thus given by $\Pi^i = B_i + \alpha^{-i} B_{1,2}$, for $i = 1, 2$.

Instead, by setting the lowest price $\underline{p}$, the firm can attract both its captives and the shoppers with probability one. Evaluated at the lower bound the firm's profit hence becomes $\Pi^i = (B_i + B_{1,2})\, \underline{p}$, for $i = 1, 2$. As the profit has to be the same over the whole support to sustain randomized pricing strategies, equating

$$\Pi^i = B_i + \alpha^{-i} B_{1,2} = (B_i + B_{1,2})\, \underline{p}$$

for $i = 1$ and $i = 2$ gives us the lower bound

$$\underline{p} = \frac{B_i + \alpha^{-i} B_{1,2}}{B_i + B_{1,2}} = \frac{B_{-i} + \alpha^{i} B_{1,2}}{B_{-i} + B_{1,2}}.$$

Assuming $B_i \geq B_{-i}$, this is solvable only if $\alpha^i = \frac{B_i - B_{-i}}{B_i + B_{1,2}} \geq 0$ implying $\alpha^{-i} = 0$. To simplify, we hence refer to α^i by the shorter notion α. The profits can thus be written as $\Pi^i = B_i$ and $\Pi^{-i} = B_{-i} + \alpha B_{1,2}$ and the lower bound is $\underline{p} = \frac{B_i}{B_i + B_{1,2}}$.

The cumulative distribution functions F^1 and F^2 can now be obtained in closed-form by observing that the profit has to be invariant everywhere in the support. In particular, if a firm $i = 1, 2$ sets price p, its profit is expressed as follows

$$\Pi^i = \left(B_i + (1 - F^{-i}(p)) B_{1,2}\right) p, \text{ for } i = 1, 2,$$

which gives

$$F^i(p) = \frac{B_{-i} + B_{1,2}}{B_{1,2}} - \frac{\Pi^{-i}}{B_{1,2}} \frac{1}{p}, \text{ for } p \leq 1,$$

as required.

Observe also that the profit $\Pi^{-i} \leq \Pi^i$ can be rewritten as a convex combination of firm i's captives and firm $-i$'s captives

$$\Pi^{-i} = B_{-i} + \alpha B_{1,2} = B_{-i} + \frac{B_i - B_{-i}}{B_i + B_{1,2}} B_{1,2}$$

$$\Pi^{-i} = \left(1 - \frac{B_{1,2}}{B_i + B_{1,2}}\right) B_{-i} + \frac{B_{1,2}}{B_i + B_{1,2}} B_i$$

$$\Pi^{-i} = \underline{p} B_{-i} + (1 - \underline{p}) B_i,$$

Also, if we continue still with that last expression we get,

$$\begin{aligned}\Pi^{-i} &= -\underline{p}\,(B_i - B_{-i}) + B_i \\ &= -\frac{B_i}{B_i + B_{1,2}}\,(B_i - B_{-i}) + B_i \\ &= \left(1 - \frac{B_i - B_{-i}}{B_i + B_{1,2}}\right) B_i \\ &= (1-\alpha)\,\Pi^i.\end{aligned}$$

This expression will be needed a bit later in the paper. ■

PROOF OF PROPOSITION 1.

Note first that by Lemma 1, if

$$\theta^1 \left(1 - E(p|F^1\right) > \theta^2 \left(1 - E(p|F^2)\right).$$

then all consumers start from firm $i = 1$, i.e., $t^1 = 1 - t^2 = 1$, whereas, if

$$\theta^1 \left(1 - E(p|F^1\right) < \theta^2 \left(1 - E(p|F^2)\right).$$

then all consumers start from firm $i = 2$, i.e., $t^1 = 1 - t^2 = 0$. Otherwise, if

$$\theta^1 \left(1 - E(p|F^1\right) = \theta^2 \left(1 - E(p|F^2)\right).$$

then any $t^1 = 1 - t^2 \in [0, 1]$ and $t^2 \in [0, 1]$ such that $t^1 = 1 - t^2$ would do.

As discussed in the main text, remember that we can always assign a unique joint price distribution $\mathbf{F} := (F^1, F^2)$ to any frictions inside stores, $\boldsymbol{\theta}$, and fractions of consumers starting from each firm, $\mathbf{t}$. Namely, together $\boldsymbol{\theta}$ and $\mathbf{t}$ generate a unique partition of consumers $\{B_\emptyset, B_1, B_2, B_{1,2}\}$, which then in turn gives us a unique joint price distribution $\mathbf{F}$ characterized by Lemma 2; the marginals can be denoted by $F^i(\boldsymbol{\theta}, \mathbf{t}) = F^i(\theta^1, \theta^2, t^1, t^2)$. This notation will be helpful in describing the relationship between frictions $\boldsymbol{\theta}$, search $\mathbf{t}$, and prices $\mathbf{F}$.

Note first that, if $\theta^1 \left(1 - E(p|F^1(\boldsymbol{\theta}, (1, 0)))\right) \geq \theta^2 \left(1 - E(p|F^2(\boldsymbol{\theta}, (1, 0))))\right)$, then the pair $\mathbf{F}(\boldsymbol{\theta}, (1, 0))$ and $\mathbf{t} = (1, 0)$ is clearly a fixed point. In other words, the price ratio which would arise if all consumers began from store $i = 1$, $E(p|F^1(\boldsymbol{\theta}, (1, 0)))/E(p|F^2(\boldsymbol{\theta}, (1, 0)))$, is not too high to discourage consumers from actually starting from that more expensive firm.

It is also clear that, if we started to increase the fraction t^1, starting from the level $t^\star$ where the firms have equally many captives $B_1 = B_2$ for the given level of frictions and raising t^1 gradually up to one, by continuity of $E(p|F^1(\boldsymbol{\theta}, \mathbf{t}))/E(p|F^2(\boldsymbol{\theta}, \mathbf{t}))$ in $\mathbf{t}$, we must span all the values of $E(p|F^1)/E(p|F^2)$ between one and $E(p|F^1(\boldsymbol{\theta}, (1, 0)))/E(p|F^2(\boldsymbol{\theta}, (1, 0)))$.

Hence, if we concentrate on cases where

$$\theta^1 \left(1 - E(p|F^1(\boldsymbol{\theta}, (1, 0)))\right) < \theta^2 \left(1 - E(p|F^2(\boldsymbol{\theta}, (1, 0))))\right)$$

and $\theta^1 > \theta^2$ for which we would have

$$\theta^1 \left(1 - E(p|F^1(\boldsymbol{\theta}, (t^\star, 1 - t^\star)))\right) > \theta^2 \left(1 - E(p|F^2(\boldsymbol{\theta}, (t^\star, 1 - t^\star))))\right),$$

by continuity there necessarily exist a fixed point in search and prices, where $\mathbf{t}$ in between $\mathbf{t} = (1, 0)$ and $\mathbf{t} = (t^\star, 1 - t^\star)$ and the following equality holds

$$\frac{\theta^1}{\theta^2} = \frac{1 - E(p|F^2(\boldsymbol{\theta}, \mathbf{t}))}{1 - E(p|F^1(\boldsymbol{\theta}, \mathbf{t}))}. \tag{8}$$

To elaborate on this, if (8) is satisfied, all consumers are indifferent between starting from either firm. Each of them can hence be assigned to any start store. If they are assigned according

to $\mathbf{t}$, the firms are willing to price in accordance with $\mathbf{F}(\boldsymbol{\theta}, \mathbf{t})$: we have a fixed point.

Observe also that in the symmetric case with $\theta^1 = \theta^2$ we have a symmetric fixed point

$$\theta^1 \left(1 - E(p|F^1(\boldsymbol{\theta}, (t^\star, 1 - t^\star)))\right) = \theta^2 \left(1 - E(p|F^2(\boldsymbol{\theta}, (t^\star, 1 - t^\star)))\right).$$

For uniqueness, we can rely on the monotonicity of $E(p|F^1(\boldsymbol{\theta}, \mathbf{t}))/E(p|F^2(\boldsymbol{\theta}, \mathbf{t}))$ in $\mathbf{t}$:

$$\begin{aligned}
\frac{1 - E(p|F^2(\boldsymbol{\theta}, \mathbf{t}))}{1 - E(p|F^1(\boldsymbol{\theta}, \mathbf{t}))} =& \frac{1 - \frac{\Pi^2}{B_{1,2}} ln\left(\frac{1}{\underline{p}}\right) - \alpha}{1 - \frac{\Pi^1}{B_{1,2}} ln\left(\frac{1}{\underline{p}}\right)} \\
=& \frac{1 - \frac{\underline{p}B_2 + (1-\underline{p})B_1}{B_{1,2}} ln\left(\frac{1}{\underline{p}}\right) - \alpha}{1 - \frac{\Pi^1}{B_{1,2}} ln\left(\frac{1}{\underline{p}}\right)} \\
=& \frac{1 + \frac{\underline{p}B_1 - \underline{p}B_2}{B_{1,2}} ln\left(\frac{1}{\underline{p}}\right) - \frac{B_1}{B_{1,2}} ln\left(\frac{1}{\underline{p}}\right) - \alpha}{1 - \frac{\Pi^1}{B_{1,2}} ln\left(\frac{1}{\underline{p}}\right)} \\
=& \frac{1 + \alpha \frac{B_1}{B_{1,2}} ln\left(\frac{1}{\underline{p}}\right) - \frac{B_1}{B_{1,2}} ln\left(\frac{1}{\underline{p}}\right) - \alpha}{1 - \frac{\Pi^1}{B_{1,2}} ln\left(\frac{1}{\underline{p}}\right)} \\
=& 1 - \alpha,
\end{aligned}$$

where

$$\frac{\partial \alpha}{\partial t^1} = \frac{\partial}{\partial t^1} \frac{B_1 - B_2}{B_1 + B_{1,2}} > 0,$$

because $\frac{\partial B_1}{\partial t^1} > 0$, $\frac{\partial B_2}{\partial t^1} < 0$ and $\frac{\partial B_{1,2}}{\partial t^1} = 0$; these partials are easy to sign based on 2, 3, 4 and 5. As a result, as we increase t^1, starting from the point $t^\star$ where $B_1 = B_2$ holds, all the way up until unity, $E(p|F^1(\boldsymbol{\theta}, \mathbf{t}))/E(p|F^2(\boldsymbol{\theta}, \mathbf{t}))$ decreases: the fixed point is unique. ■

PROOF OF LEMMA 3.

We just proved that for $t^1 < 1$,

$$\begin{aligned}
\frac{\theta^2}{\theta^1} &= 1 - \alpha \\
\frac{\theta^2}{\theta^1} &= \frac{B_2 + B_{1,2}}{B_1 + B_{1,2}} \\
\frac{\theta^2}{\theta^1} &= \frac{1 - B_\emptyset - B_1}{1 - B_\emptyset - B_2} \\
\frac{\theta^2}{\theta^1} &= \frac{1 - B_\emptyset\left(1 - \frac{\theta^1}{\theta^1 - \theta^2}\right) - \frac{\theta^1}{\theta^1 - \theta^2} e^{-\theta^2}}{1 - B_\emptyset\left(1 + \frac{\theta^2}{\theta^1 - \theta^2}\right) + \frac{\theta^1}{\theta^2 - \theta^2} e^{-\theta^1}} \\
\frac{\theta^2}{\theta^1} &= \frac{\theta^1 - \theta^2 + \theta^2 B_\emptyset - \theta^1 e^{-\theta^2}}{\theta^1 - \theta^2 + \theta^2 e^{-\theta^1} - \theta^1 B_\emptyset}.
\end{aligned}$$

We can hence solve for $B_\emptyset$ as

$$B_\emptyset = -\frac{1}{2}\frac{\theta^2}{\theta^1}\left(1-e^{-\theta^1}\right) - \frac{1}{2}\frac{\theta^1}{\theta^2}\left(1-e^{-\theta^2}\right) + 1.$$

From here on, it is useful to work with the reparametrization $\rho = \frac{\theta^2}{\theta^1}$, which gives

$$B_\emptyset = -\frac{1}{2}\rho\left(1-e^{-\theta^1}\right) - \frac{1}{2}\rho^{-1}\left(1-e^{-\theta^2}\right) + 1.$$

Now, since $\frac{\partial\theta^1}{\partial\rho} = -\frac{\theta^1}{\rho}$ and $\frac{\partial\theta^2}{\partial\rho} = \frac{\theta^2}{\rho}$,

$$\frac{\partial B_\emptyset}{\partial\rho} = -\frac{1}{2}\left(1-e^{-\theta^1}\right) + \frac{1}{2}\theta^1 e^{-\theta^1} + \frac{1}{2}\rho^{-1}\left(1-e^{-\theta^2}\right) + \frac{1}{2}\rho^{-2}\theta^2 e^{-\theta^2}$$

or, returning to the original variables,

$$\frac{\partial B_\emptyset}{\partial\rho} = \frac{1}{2}\left(-\left(1-e^{-\theta^1}-\theta^1 e^{-\theta^1}\right) + \frac{\theta^1}{\theta^2}\left(1-e^{-\theta^2}+\theta^1 e^{-\theta^2}\right)\right).$$

This is positive for all $\theta^1 \geq \theta^2 > 0$ because

$$\frac{1-e^{-\theta^1}-\theta^1 e^{-\theta^1}}{\theta^1} < \frac{1-e^{-\theta^1}}{\theta^1} < \frac{1-e^{-\theta^2}}{\theta^2} < \frac{1-e^{-\theta^2}+\theta^1 e^{-\theta^2}}{\theta^2}$$

and the function $\frac{1-e^{-x}}{x}$ is decreasing in x.

We can now revert to $\rho = \frac{\theta^2}{\theta^1} = \frac{1-B_\emptyset-B_1}{1-B_\emptyset-B_2}$ to solve it for B_1 and B_2 as a function of ρ

$$B_1 = (1-\rho)(1-B_\emptyset) + \rho B_2,$$

$$B_2 = \left(1-\rho^{-1}\right)(1-B_\emptyset) + \rho^{-1}B_1.$$

Their partials with respect to ρ are given by

$$\frac{\partial B_1}{\partial\rho} = -(1-B_\emptyset-B_2) - (1-\rho)\frac{\partial B_\emptyset}{\partial\rho} + \rho\frac{\partial B_2}{\partial\rho},$$

$$\frac{\partial B_2}{\partial\rho} = \rho^{-1}(1-B_\emptyset-B_1) - \left(1-\rho^{-1}\right)\frac{\partial B_\emptyset}{\partial\rho} + \rho^{-1}\frac{\partial B_1}{\partial\rho}.$$

As we can now take $\rho = \rho(\theta^1, \theta^2)$ as a firm's choice variable, the first order conditions are

$$\frac{\partial\Pi^1}{\partial\rho} = 0 \Longleftrightarrow \frac{\partial B_1}{\partial\rho} = 0 \Longleftrightarrow \rho\frac{\partial B_2}{\partial\rho} = 1 - B_\emptyset - B_2 + (1-\rho)\frac{\partial B_\emptyset}{\partial\rho} > 0$$

and

$$\frac{\partial\Pi^2}{\partial\rho} = 0 \Longleftrightarrow \frac{(1-\alpha)\Pi^1}{\partial\rho} = 0 \Longleftrightarrow \frac{\partial\rho B_1}{\partial\rho} = 0 \Longleftrightarrow B_1 + \rho\frac{\partial B_1}{\partial\rho} = 0.$$

Both or them cannot be satisfied for the same ρ because a firm's profit is positive, $\Pi^1 = B_1 > 0$. This implies that it cannot be optimal for both firms to use such rates θ^1 and θ^2 that $t^1 < 1$.

Observe that while this shows that $\frac{\theta^2}{\theta^1} = 1-\alpha$ and $t^1 < 1$ never hold in equilibrium, it does not yet show that $\frac{\theta^2}{\theta^1} = 1-\alpha$ and $t^1 = 1$ could not arise. There are kinks in the first derivatives of Π^1 and Π^2 at the boundary values of frictions where $\frac{\theta^2}{\theta^1} = 1-\alpha$ and $t^1 = 1$ both hold. To rule them out we need to get a bit ahead of our calculations so far.

Namely, our later analysis shows that the existence of this kind of a corner solution requires that the prominent firm has a weak incentive to decrease θ^1, the non-prominent firm has a weak incentive to increase θ^2 and the boundary condition holds. Together these three conditions can be stated as (see the proofs for Propositions 2 and 3 and Claim 1)

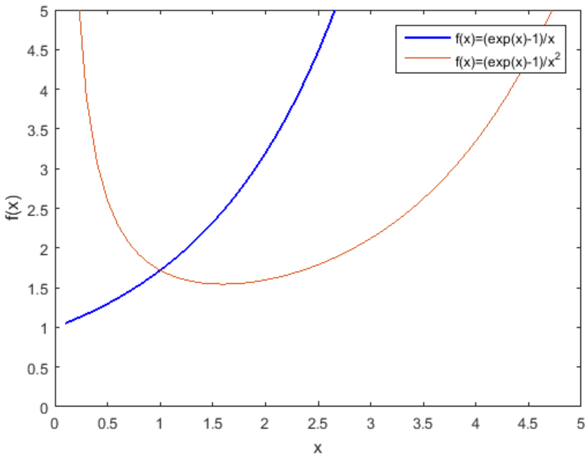

Figure 2: Graphs of two functions appearing repeatedly in the proofs.

$$\frac{e^{\delta}-1}{\delta} \geq \rho^{-1} \tag{9}$$

$$\frac{e^{\delta}-1}{\delta} \geq \frac{1}{2}\frac{e^{\theta^1}-1}{\theta^1} \tag{10}$$

$$\frac{e^{\delta}-1}{\delta^2} = \frac{e^{\theta^1}-1}{(\theta^1)^2} \tag{11}$$

where $\delta = \theta^1 - \theta^2 \geq 0$ and $\rho = \theta^2/\theta^1 \leq 1$. Now, note that 10 and 11 cannot hold unless $\theta^2 \leq \theta^1/2$ and $\rho^{-1} \geq 2$. However, for that high values of $\rho^{-1} \geq 2$, Condition (9) is satisfied only when $\delta > x'$, where $x' \approx 1.6$ marks the value that minimizes the function $\frac{e^x-1}{x^2}$. Condition (11) gives a one for one correspondence between δ and θ^1 and cannot be satisfied unless $\delta \leq x'$ and $\theta^1 \geq x'$.[18] This contradiction thus rules out the case in which $\frac{\theta^2}{\theta^1} = 1 - \alpha$ and $t^1 = 1$ both hold. In words, consumers are never indifferent to their start store in equilibrium. ■

PROOF OF PROPOSITION 2.

We consider case by case firm i's best response, θ^i, to firm $-i$'s frictions, θ^{-i}.

Case 1: $\theta^{-i} = 0$.

If firm $-i$ is out of the market, firm i acts like a monopolist and serves its consumers instantaneously: $\theta^i = \infty$.

Case 2: $\theta^{-i} \in (0, \infty)$.

First, if the firm chooses an extremely slow rate $\theta^i = 0$ it serves nobody and extracts no profits.[19]

[18] Expect for cases where $\theta^1 = \delta$ and $\theta^2 = 0$ which we deal with in the proof of Proposition 2.

[19] For $\theta^i = 0$, $B_{-i} = 1 - e^{-\theta^{-i}}$ and $B_{\emptyset} = e^{-\theta^{-i}}$ while $B_i = B_{1,2} = 0$.

Second, if the firm chooses an extremely fast rate $\theta^i = \infty$ such that $t^i = 1$, the firm's profit is given as[20]

$$\Pi^i = e^{-\theta^{-i}}.$$

Third, if the firm chooses a finite but sufficiently fast rate $\theta^i >> \theta^{-i}$ such that $t^i = 1$, the firm's profit can be written as

$$\Pi^i = \frac{\theta^i}{\theta^i - \theta^{-i}} \left(1 - e^{-(\theta^i - \theta^{-i})}\right) e^{-\theta^{-i}}.$$

It is now easy to show that

$$\frac{\theta^i}{\theta^i - \theta^{-i}} \left(1 - e^{-(\theta^i - \theta^{-i})}\right) > 1$$

as long as $\left|\theta^i - 1\right| > \left|\theta^{-i} - 1\right|$.

This implies that, by choosing a large enough finite θ^i, the firm is guaranteed to extract more revenue than by choosing $\theta^i = 0$ or $\theta^i = \infty$.

Case 3: $\theta^{-i} = \infty$.

Note first that, if both firms have an infinite rate, $\theta^i = \infty$, all consumers find all prices and both firms' profits go to zero.

Instead, if firm $-i$ has an infinite rate and firm i has a finite rate, $\theta^{-i} = \infty$ and $\theta^i < \infty$ such that $t^{-i} = 1$, firm i's profit is[21]

$$\Pi^i = \frac{B_{-i} - B_i}{B_{-i} + B_{1,2}} B_{1,2} = e^{-\theta^i} \left(1 - e^{-\theta^i}\right),$$

It is maximized by $\theta^i = ln(2) < \infty$.

It is thus clear from Cases 1, 2 and 3 that $\theta^i = \infty$ cannot arise in equilibrium. ■

PROOF OF PROPOSITION 3.

For values outside of the boundary where $\frac{\theta^2}{\theta^1} = 1 - \alpha$, the first order condition is

$$\frac{\partial B_1}{\partial \theta^2}(1 - B_\emptyset - B_1) - \frac{\partial B_1}{\partial \theta^2} B_1 = 0$$

where $B_{1,2} = 1 - B_\emptyset - B_1$. The unique solution is thereby given by $B_1 = B_{1,2}$.

Note also that $\frac{\partial B_1}{\partial \theta^2} \leq 0$ because

$$B_1 = \theta^1 e^{-\theta^1} \frac{e^\delta - 1}{\delta}$$

and $\frac{\partial}{\partial \delta} \frac{e^\delta - 1}{\delta} \geq 0$ and $\frac{\partial}{\partial \theta^2} \delta = -1$. ■

PROOF OF CLAIM 1.

Here we delineate some properties of the best response functions to support our analysis. We first state the prominent firm's and the non-prominent firm's problems. We use the following reparametrizations: $\delta = \theta^1 - \theta^2 \geq 0$ and $\rho = \theta^2/\theta^1 \leq 1$ (as in the proof of Lemma 3).

(A) The prominent seller maximizes its profit B_1

[20] For $\theta^i = \infty$, $B_i = e^{-\theta^{-i}}$ and $B_{1,2} = 1 - e^{-\theta^{-i}}$ while $B_{-i} = B_\emptyset = 0$.

[21] Here, $B_{-i} = e^{-\theta^i}$ and $B_{1,2} = 1 - e^{-\theta^i}$ whereas $B_i = B_\emptyset = 0$.

$$\max_{\theta^1} \theta^1 e^{-\theta^1} \frac{e^\delta - 1}{\delta}$$

such that the prominence order stays the same:

$$\rho \leq 1 - \alpha(\theta^1, \theta^2).$$

(B) The non prominent seller maximizes its profit $(1-\alpha)B_1$

$$\max_{\theta^2} \Pi^2 \frac{B_1 B_{1,2}}{B_1 + B_{1,2}} = \max_{\theta^2} \Pi^2 \frac{B_1 B_{1,2}}{1 - B_\emptyset}$$

such that the prominence order stays the same:

$$\rho \leq 1 - \alpha(\theta^1, \theta^2).$$

It is clear from the proof of Proposition 3 that there exists a unique solution to the non-prominent firm's problem. We next show that there exist a unique solution also to the prominent firm's problem.

Step 1: *Proof that there exists a unique solution $\theta^1(\theta^2)$ to the prominent firm's problem*

Start by considering a relaxed unconstrained problem

$$\max_{\theta^1 \geq \theta^2} \Pi^1,$$

where the prominent firm's profit is represented by

$$\Pi^1(\theta^1, \theta^2) := \frac{\theta^1}{\theta^1 - \theta^2} \left(e^{-\theta^2} - e^{-\theta^1} \right).$$

The first partial with respect to θ^1 is

$$\frac{\partial \Pi^1(\theta^1, \theta^2)}{\partial \theta^1} = \frac{\theta^1}{\theta^1 - \theta^2} e^{-\theta^1} - \frac{\theta^2}{(\theta^1 - \theta^2)^2} \left(e^{-\theta^2} - e^{-\theta^1} \right).$$

Thus, an increase in θ^1 increases Π^1 *iff*

$$f(\theta^1, \theta^2) := -\frac{e^\delta - 1}{\delta} + \rho^{-1} \geq 0.$$

Therefore, the existence and uniqueness of the solution to this relaxed problem basically come from on the fact that the growth rate of $\frac{e^\delta - 1}{\delta} = \frac{e^{(\theta^1 - \theta^2)} - 1}{\theta^1 - \theta^2}$ is exponential in θ^1 whereas the growth of $\rho^{-1} = \frac{\theta^1}{\theta^2}$ is linear in θ^1. Let us consider this in more detail.

We can now differentiate this function f we just defined with respect to θ^1 to obtain

$$f'(\theta^1, \theta^2) := -\frac{\delta e^\delta - e^\delta + 1}{\delta^2} + \frac{1}{\theta^2} < -e^\delta + \frac{e^\delta - 1}{\delta} + \rho^{-1}.$$

This implies that, if $f = -\frac{e^\delta - 1}{\delta} + \rho^{-1}$ is negative, the change in $f' = -\frac{e^\delta - 1}{\delta} + \rho^{-1}$ is negative (strictly negative for $\delta > 0$ and zero for $\delta = 0$) because

$$e^{\delta} \to 2\frac{e^{\delta}-1}{\delta^2}, \text{ as } \delta \to 0,$$
$$e^{\delta} > 2\frac{e^{\delta}-1}{\delta^2}, \text{ for } \delta > 0.$$

In consequence, $\frac{e^{\delta}-1}{\delta}$ and ρ^{-1} cannot cross more than once. The solution to the first order condition for an interior optimum $\frac{e^{\delta}-1}{\delta} = \rho^{-1}$ is thereby unique.

Regarding existence, note that, if we start with θ^1 just above θ^2 and, thus, with $\rho \approx 1$ and $\delta \approx 0$,

$$\lim_{\theta^1 \to \theta^2+} f(\theta^1, \theta^2) = 0 \text{ and } \lim_{\theta^1 \to \theta^2+} f'(\theta^1, \theta^2) = \frac{1}{\theta^2} - \frac{1}{2}.$$

The existence of interior solution $\theta^1 > \theta^2$ to this relaxed problem thus hinges on the condition that the other store has strong enough frictions, i.e., $\theta^2 < 2$. Otherwise, our firm would prefer to raise its own frictions by setting $\theta^1 \leq \theta^2$.

Returning back to the original problem, it is hence clear that, if the constraint $\theta^1 \geq \theta^2 \frac{1}{1-\alpha}$ binds,

$$\frac{e^{\delta}-1}{\delta} > \rho^{-1}$$

(without the constraint, the firm would choose a lower θ^1) but, if the constraint $\theta^1 \geq \theta^2 \frac{1}{1-\alpha}$ is slack,

$$\frac{e^{\delta}-1}{\delta} = \rho^{-1}$$

(without the constraint, the firm would choose the same θ^1). ■

Step 2: *Proof that the solution $\theta^1(\theta^2)$ is decreasing if the constraint is slack*

Differentiating totally the first order condition

$$\frac{e^{\delta}-1}{\delta} = \frac{\theta^1}{\theta^1}$$

gives

$$\frac{d\theta^1}{d\theta^2} = \frac{\xi(\delta) - \frac{\theta^1}{(\theta^2)^2}}{\xi(\delta) - \frac{1}{\theta^2}},$$

where

$$\xi(x) := \frac{e^x}{x} - \frac{e^x - 1}{x^2} \geq 0.$$

This is negative if $\xi(\delta) \in \left[\frac{1}{\theta^2}, \frac{\theta^1}{(\theta^2)^2}\right]$.

Part 1: We first prove that $\xi(\delta) \geq \frac{1}{\theta^2}$, when $\frac{e^{\delta}-1}{\delta} = \rho^{-1}$ holds:

$$
\begin{aligned}
\frac{e^\delta}{\delta} - \frac{e^\delta - 1}{\delta^2} &\geq \frac{1}{\theta^2} \\
\frac{e^\delta}{\delta} - \rho^{-1}\frac{1}{\delta} &\geq \frac{1}{\theta^2} \\
e^\delta - 2\rho^{-1} &\geq -1
\end{aligned}
$$

which holds because,

$$
e^\delta - 2\rho^{-1} = \frac{e^\delta(\delta - 2) + 2}{\delta} \geq 0 \geq -1.
$$

Part 2: We then prove that $\xi(\delta) \leq \frac{\theta^1}{(\theta^2)^2}$, when $\frac{e^\delta - 1}{\delta} = \rho^{-1}$ holds:

$$
\begin{aligned}
\frac{e^\delta}{\delta} - \frac{e^\delta - 1}{\delta^2} &\leq \frac{\theta^1}{(\theta^2)^2} \\
\frac{e^\delta}{\delta} - \frac{1}{\delta}\rho^{-1} &\leq \frac{1}{\theta^2}\rho^{-1} \\
e^\delta &\leq \rho^{-2}
\end{aligned}
$$

which holds because,

$$
e^\delta \leq \left(\frac{e^\delta - 1}{\delta}\right)^2.
$$

Altogether, this implies that $\frac{d\theta^1}{d\theta^2} \leq 0$ when $\frac{e^\delta - 1}{\delta} = \rho^{-1}$ binds. ■

Step 3: *Proof that the solution $\theta^2(\theta^1)$ is decreasing if the constraint is slack*

When $t^1 = 1 - t^2 = 1$, $B_1 = B_{1,2}$ is equivalent to

$$
2\frac{e^\delta - 1}{\delta} = \frac{e^{\theta^1} - 1}{\theta^1}.
$$

Differentiating it totally results in

$$
\frac{d\theta^1}{d\theta^2} = \frac{2\xi(\delta) - \xi(\theta^1)}{2\xi(\delta)},
$$

where

$$
\xi(x) := \frac{e^x}{x} - \frac{e^x - 1}{x^2} \geq 0.
$$

To see when this is negative, observe

$$
\begin{aligned}
2\xi(\delta) - \xi(\theta^1) &\le 0 \\
2\left(\frac{e^\delta}{\delta} - \frac{e^\delta - 1}{\delta^2}\right) - \left(\frac{e^{\theta^1}}{\theta^1} - \frac{e^{\theta^1} - 1}{(\theta^1)^2}\right) &\le 0 \\
2\frac{e^\delta - 1 + 1}{\delta} - 2\frac{e^\delta - 1}{\delta^2} - \frac{e^{\theta^1} - 1 + 1}{\theta^1} + \frac{e^{\theta^1} - 1}{(\theta^1)^2} &\le 0 \\
2\frac{1}{\delta} - 2\frac{1}{\delta}\frac{e^\delta - 1}{\delta} - \frac{1}{\theta^1} + \frac{1}{\theta^1}\frac{e^{\theta^1} - 1}{\theta^1} &\le 0 \\
\left(2\frac{1}{\delta} - \frac{1}{\theta^1}\right) - \left(\frac{1}{\delta} - \frac{1}{\theta^1}\right) 2\frac{e^\delta - 1}{\delta} &\le 0 \\
\frac{1}{\delta} + \left(\frac{1}{\delta} - \frac{1}{\theta^1}\right)\left(1 - 2\frac{e^\delta - 1}{\delta}\right) &\le 0 \\
1 + \rho\left(1 - 2\frac{e^\delta - 1}{\delta}\right) &\le 0.
\end{aligned}
$$

We can now solve ρ from $2\frac{e^\delta - 1}{\delta} = \frac{e^{\theta^1} - 1}{\theta^1}$,

$$
\rho = 1 - 2\frac{e^\delta - 1}{e^{\theta^1} - 1},
$$

and then continue with the calculation,

$$
\begin{aligned}
\left(1 - 2\frac{e^\delta - 1}{e^{\theta^1} - 1}\right)\left(1 - 2\frac{e^\delta - 1}{\delta}\right) &\le -1 \\
-2\frac{e^\delta - 1}{e^{\theta^1} - 1} - 2\frac{e^\delta - 1}{\delta} + 4\frac{(e^\delta - 1)^2}{\delta(e^{\theta^1} - 1)} &\le -2\frac{e^{\theta^1} - 1}{e^{\theta^1} - 1} \\
-\frac{(e^\delta - 1)\delta}{(e^{\theta^1} - 1)\delta} - \frac{(e^\delta - 1)(e^{\theta^1} - 1)}{(e^{\theta^1} - 1)\delta} + 2\frac{(e^\delta - 1)^2}{(e^{\theta^1} - 1)\delta} + \frac{(e^{\theta^1} - 1)\delta}{(e^{\theta^1} - 1)\delta} &\le 0 \\
-\frac{(e^\delta - 1)\delta}{\delta} - \frac{(e^\delta - 1)(e^{\theta^1} - 1)}{\delta} + 2\frac{(e^\delta - 1)^2}{\delta} + \frac{(e^{\theta^1} - 1)\delta}{\delta} &\le 0 \\
-(e^\delta - 1) - \frac{(e^\delta - 1)(e^{\theta^1} - 1)}{\delta} + \frac{(e^\delta - 1)(e^{\theta^1} - 1)}{\theta^1} + (e^{\theta^1} - 1) &\le 0 \\
(e^{\theta^1} - 1)\left(\frac{e^\delta - 1}{\delta} - 1\right) &\ge (e^\delta - 1)\left(\frac{e^{\theta^1} - 1}{\theta^1} - 1\right) \\
\frac{(e^{\theta^1} - 1)}{\frac{e^{\theta^1} - 1}{\theta^1} - 1} &\ge \frac{(e^\delta - 1)}{\frac{e^\delta - 1}{\delta} - 1}.
\end{aligned}
$$

This is true because $\theta^1 \ge \delta$ and $\frac{(e^x - 1)}{\frac{e^x - 1}{x} - 1}$ is increasing in x. We have thus demonstrated that $\frac{d\theta^1}{d\theta^2} \le 0$ as long as $2\frac{e^\delta - 1}{\delta} = \frac{e^{\theta^1} - 1}{\theta^1}$ binds. ■

Step 4: *Proof that the solutions $\theta^1(\theta^2)$ and $\theta^2(\theta^1)$ are monotone if the constraint binds:*

When $t^1 = 1 - t^2 = 1$, $1 - \alpha = \rho$ is equivalent to

$$
\frac{e^\delta - 1}{\delta^2} = \frac{e^{\theta^1} - 1}{(\theta^1)^2}.
$$

Since $\frac{e^x-1}{x^2}$ is strictly decreasing for $x < x'$ and strictly increasing for $x > x'$ where $x' \approx 1.6 > 0$, this can be satisfied only if $\delta = \theta^1 - \theta^2 < x'$ and $\theta^1 > x'$. Moreover, if θ^1 increases, then $\delta = \theta^1 - \theta^2$ decreases. Therefore, if $1 - \alpha = \rho$ binds, then $\frac{d\theta^1}{d\theta^2} \geq 0$ holds. ■

Step 5: *Proof that any solution to both the prominent firm's problem and the non-prominent firm's problem is unique*

Any pair (θ^1, θ^2) that solves both the prominent firm's problem and the non-prominent firm's problem satisfies the following first order conditions

$$\frac{e^\delta - 1}{\delta} = \rho^{-1}, \tag{12}$$

$$\frac{e^\delta - 1}{\delta} = \frac{1}{2}\frac{e^{\theta^1} - 1}{\theta^1}. \tag{13}$$

It is also clear based on our previous analysis that, if we start from some given θ^2, there can be only one $\theta^1 = \theta'$ that satisfies (12) for this θ^2 and only one $\theta^1 = \theta''$ that satisfies (13) for that θ^2. Moreover, if we join the two conditions by equating the right hand sides, the resulting condition,

$$\frac{1}{2}\frac{e^{\theta^1} - 1}{(\theta^1)^2} = \frac{1}{\theta^2}, \tag{14}$$

is satisfied by a unique θ^1 for which it holds that $\theta^1 > x' \approx 1.6$; the last one is a condition that should hold in an equilibrium according to Step 4. ■

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