Helsinki Center of Economic Research

Discussion Papers

Are worst quality goods always traded fastest in stationary equilibrium?

Saara Hämäläinen
University of Helsinki and HECER

Discussion Paper No. 400
March 2016

ISSN 1795-0562

HECER – Helsinki Center of Economic Research, P.O. Box 17 (Arkadiankatu 7), FI-00014 University of Helsinki, FINLAND, Tel +358-2941-28780, E-mail info-hecer@helsinki.fi, Internet www.hecer.fi

HECER
Discussion Paper No. 400

Are worst quality goods always traded fastest in stationary equilibrium?*

Abstract

We investigate welfare and equilibrium trading in a decentralized search market with asymmetric information and bilateral communication opportunities. Buyers and sellers meet randomly and pairwise and view a shared signal of the seller's quality. In the following signaling game, the sellers can thus either rely on this costless signal (pool) or costly signaling (separate). We characterize the full set of equilibria for exogenous market quality and outside options.

We study what kinds of dynamics can be sustained when average quality in the market and outside options arise endogenously in a stationary Markovian equilibrium. We show that in the lemons market (low entrant quality) only standard dynamics can arise (low quality is more liquid) but in a better market (high entrant quality) also inverse dynamics are possible (high quality is more liquid). We argue that both cases are relevant because neither market is efficient.

JEL Classification: D82, D83

Keywords: dynamic trading, random search, asymmetric information, learning, signaling

Saara Hämäläinen

Department of Political and Economic Studies
University of Helsinki
P.O. Box 17 (Arkadiankatu 7)
FI-00014 University of Helsinki
FINLAND

e-mail: saara.hamalainen@helsinki.fi

* An earlier version of this paper has been circulated under different names. I thank in particular my supervisor Hannu Vartiainen and the excellent opponent in my defense Diego Moreno, and Takuro Yamashita, Juuso Välimäki, Marko Terviö, Tuomas Takalo, Shouyong Shi, Klaus Kultti, Pauli Murto, Cyril Monnet, Lucas Maestri, Aviad Heifetz, Mats Godenhielm, Lassi Ahlvik, and the audiences in HECER applied micro and lunch seminars, TSE student workshop and FEA, SING, CoED and EARIE meetings for comments. I am grateful to Finnish Academy of Science and Letters, OP-Pohjola Group Research Foundation, Yrjö Jahnsson Foundation, Emil Aaltonen Foundation, and Finnish Cultural Foundation for financial aid. Any shortcomings are my own.

1 Introduction

> [In dynamic markets], the inefficiencies caused by asymmetric information manifest in the fact that sellers of higher qualities need to wait longer than sellers of lower quality in order to sell. The cost of waiting is an important factor that must be considered in any assessment of the loss in welfare caused by adverse selection.
>
> [Nevertheless], observed market dynamics may differ considerably from our predictions. This is because in the real world, the price mechanism is augmented by other non-market institutions and technologies that enable agents to signal or screen information and they alter the behavior of agents as well as the pattern of trade.
>
> Janssen and Roy (2004, pp. 567-568)

We investigate welfare and equilibrium trading in a decentralized search market with asymmetric information and additional bilateral communication opportunities. Buyers and sellers match in pairs and view a shared signal of the seller's quality. We show that in the lemons market low quality is always more liquid in a stationary Markovian equilibrium but in a better market, where the entering average quality is so high that the static Walrasian market need not fail, the opposite case is also possible.

Models or adverse selection are applied to study problems that arise when different qualities are traded in a single market. The basic idea is that all goods are blended together so that buyers cannot be sure of what quality they obtain from a random seller. This implies that, in a static model without any information percolation from the selling side to the buying side, the market can fail to trade highest quality goods as the maximum price a buyer is willing to pay for average quality could be below the highest quality seller's reservation value (Akerlof, 1970). In a dynamic setting, the general finding is that this "lemons problem" is slightly less stringent because, if the sellers can wait and delay trade, it becomes possible to sort different qualities *temporarily* into different submarkets; yet, this means that low quality should be traded at a faster rate than high quality (see, e.g., Moreno and Wooders (2010, 2016); Inderst and Muller (2002); Inderst (2002); Janssen and Roy (2002, 2004); Blouin (2003); Guerrieri et al. (2010), Kim (2012), Camargo and

Lester (2014), and Cho and Matsui (2015)).[1]

The utilized sorting mechanism is essentially the following: sellers of low quality goods can be induced to trade for lower prices than sellers of high quality goods if the chance to trade for higher prices arrives too far ahead in the future or so infrequently that it does not pay for the low quality sellers to wait that long; high quality sellers are instead willing to do so because they have no other possibility to trade profitably.

While these *standard trading dynamics* are now a robust result in the literature, in this paper we show that the usual way of formulating the dynamic lemons problem has limitations, which can however be overcome with a slightly more general interpretation. With this less restricted approach, we find that different equally natural *inverse trading dynamics* can arise, where highest quality is systematically traded fastest. Our discovery therefore helps to reconcile the lemons model better with the commonly held idea that, if there is any chance of guessing the quality, better goods are bought first. As some additional information is typically exchanged before signing the contract, we do not see for example the best new doctors idling in the market for extended periods of time until some institution is finally willing to accept their salary proposal. Indeed, in these kinds of markets, the costs of a long time delay or hiring the wrong person could be so high that both sides have an incentive to make the most of whatever information they have at the moment of trading, reliable or only remotely relevant.[2]

Motivated by these ideas, our paper thus asks a specific question: Is low quality always traded faster than high quality – in a stationary Markovian equilibrium of a large anonymous market, where buyers and sellers use only pure strategies, so that nothing very complicated is going on? We find that the answer is yes and no. We use a model which allows us to consider not only the "lemons market" where the exogenous entrant quality is low but also a "better market" where the exogenous entrant quality is high.

To provide an answer to the conundrum, we are now able to show definitively

[1] We do not attempt to review this extensive literature here but these papers serve as excellent pointers to others. A fuller literature review is offered for example by the important recent paper of Moreno and Wooders (2016), which studies non-stationary trading dynamics and potential for performance improving intervention in a decentralized lemons market, and by the uncompromising and admirably rigorous contribution of Cho and Matsui (2015), which only looks at the lemons market but shares our aim of concentrating on simple, stationary equilibria in a decentralized dynamic trading model.

[2] Though not central to performance in many jobs, things like looks, dress, acting confident etc. might still pay some role in evaluations (see Hamermesh et al. (2002) and Mobius and Rosenblat (2006)) in addition to the hard information in the vita.

that it is impossible to have inverse dynamics in the lemons market but not it the better market. Notice that both of these cases are relevant as examples of markets where adverse selection is a problem because, as it turns out, the market is always inefficient in both.

We look for the (im)possibility of maintaining inverse dynamics in quite a large set of candidate equilibria. To give us maximum leeway, we let a matched buyer and seller first obtain a shared signal that is informative of the seller's quality and then, after both have seen this, we allow the seller make the buyer a take-it-or-leave it price offer. This will turn our game into a signaling game where the level of the shared signal may affect whether different sellers make a pooling price offer or separating price offers.

To cast this back into the language of job market search, our formulation attempts to capture the idea that, when an applicant and the agents of a hiring institution meet face-to-face to negotiate a contract, some telltale, quality relevant information is quite often transmitted under both parties' eyes. The purpose of a job talk, an interview or an internship is to give the hiring agents a first impression of the applicant. Interestingly, in all these cases, also the applicant typically knows if it is (not) good.[3]

This shared signal gives the matched pair a piece of correlated information, that could take their prospects of reaching an agreement either up or down. As an intermediate contribution we first characterize the set of perfect Bayesian equilibria in the trading game between a buyer and a seller given some fixed shared signal, fixed continuation values for the buyer and sellers of different qualities, and fixed average quality in the market.[4] To see what kind of trading patterns are sustainable in the long run, we later endogenize these outside options and the average market quality. In doing so, we assume in the same vein as in seminal article by Moreno and Wooders (2010) that (i) entry flows to the market are exogenous, (ii) buyers and sellers in the market are matched randomly with a new potential trading partner each time period, and (iii) they exit the market when they have traded.

We find that the availability of the signal always raises either the buyer's or the high quality seller's continuation values over zero. This implies that, when the signal is very low, either the buyer or a high quality seller will refuse to trade with their current trading partner and will instead wait until they get a chance to trade under

[3] In our model, the buyer observes a signal s^b of the seller's quality and the seller observes a signal s^i of the buyer's signal; for simplicity, we assume they are the same, i.e., $s^b = s^i =: s$.

[4] To make sure we cover all the equilibria, we consider maximal punishments off the equilibrium path. All the equilibria are consistent with the intuitive criterion (Cho and Kreps, 1987).

a higher signal. As a result, all equilibria are characterized by a cutoff signal s' such that, if the signal is higher than the cutoff, the sellers can rely on this *costless signal* to support a pooling price offer whereas, if the signal is lower than the cutoff, the sellers must resort to *costly signaling*, which here means that, if the low quality seller offers a low (revealing) price, it is accepted but, if the high quality seller offers a high (revealing) price, is rejected.[5] Some mutually beneficial trading opportunities can thus not be commensurated: there is waste for the lowest signals.

Basically, the cases of standard dynamics and inverse dynamics in our model now simply differ in what happens when the signal is lower than the cutoff. Namely, in a better market, the rents that come from trading for the same higher prices as high quality sellers can make low quality sellers so picky that they no longer have an incentive to trade for their low revealing price if the signal is low. High quality then trades faster than low quality because it is more likely to prompt favorable signals that are higher than the cutoff.

In the lemons market, however, it is impossible to sustain this because the price that the buyer is willing to pay at the cutoff signal is necessarily lower than the buyer value of entry level average quality; high quality sellers are not willing to trade for so low prices. There is hence an inconsistency embedded in this idea in the lemons market. That derives from the standard monotone likelihood ratio property and from the fact that inverse dynamics here entail that average market quality is below average quality at entry.

To the best of our knowledge, there are only a few earlier papers which find inverse trading dynamics. First, in the paper that has now a classic status in the literature, Taylor (1999) studies a housing market where the buyer with the highest bid for the house can inspect the property before buying. If the house is in good condition, nothing negative is found and the buyer thus proceeds to purchase the property but, if the house is in bad condition, some buyers find it out and might hence refuse to buy it. As a result, better houses are more liquid than houses that have some problems, which might surface in a sufficiently careful examination. Long time on the market can thus act as a negative quality signal to testify of this. Second, in a very elegant and well written contemporary contribution, Kaya and Kim (2015)

[5]This is the only possibility when we concentrate on equilibria with pure strategies; with mixed strategies, the low quality seller's offer is still accepted with probability one but the high quality seller's offer could be accepted with non zero probability, as long as it is low enough so that a low quality seller has no incentive to mimic a high quality seller. We fully characterize mixed equilibrium strategies in Section 4 and consider standard and inverse dynamics with pure strategies in Section 5. Our welfare results are in Section 3. We discuss the organization of the paper further after we have set up the model in Section 2.

explore a setup where a seller of an asset meets a sequence of randomly arriving buyers, who make the seller a price offer after (i) receiving a private quality signal and (ii) observing how long the asset has been for sale. They let the buyers' prior beliefs be exogenous in their model and find that, depending on their starting level, either quality could be traded faster. Nevertheless, inverse dynamics can arise only if the buyers have initially sufficiently inflated beliefs about asset quality. Otherwise, standard dynamics prevail. Third, contributing to the study of over-the-counter trading, Zhu (2012) studies thin markets where there is only a finite number of potential trading partners. To find a good price in the opaque market, the seller of an asset can visit multiple potential buyers who make the seller exploding offers one by one. In contrast to Taylor (1999) and Kaya and Kim (2015), the buyers do not see the seller's time on the market but they can recognize if they have been contacted by the same seller before. In this kind of a non-anonymous market, a repeat contact by the seller reveals his poor outside options and, thus, makes the buyer lower the price offer.

It is noteworthy that unlike us none of these papers which find inverse dynamics considers a stationary memoryfree equilibrium and that all of them have an information linkage between earlier and subsequent encounters with the seller and some buyers. Moreover, additional degrees of freedom have been gained by not requiring that buyers' expectations are disciplined by average market quality. This underscores the difficulty in our objective of finding inverse dynamics in a stationary Markovian equilibrium for a large anonymous market, where expectations of seller quality depend on endogenous market conditions. Under inverse trading dynamics, average market quality is reduced from its entry level, such that the prior can never be inflated the way it is in Kaya and Kim (2015). That is the reason why we only find inverse dynamics after shifting our attention to better markets where average entrant quality is not too low. Our main conceptual innovation is thereby to prove that also a better market always suffers from illiquidity, that arises from the interplay of dynamics and information percolation; the focus on the lemons market only seems thus *artificial*. Previously, Kultti et al. (2012) have shown that under dynamic trade markets can fail under a wider set of parameters than typically considered. With a public Brownian motion news flow, Daley and Green (2011) have observed that an endogenous lemons problem arises both in, what we call, the lemons market and a better market. However, Kultti et al. (2012) and Daley and Green (2011) both find standard trading dynamics. Our paper is hence the first to tie these ideas together. Acknowledging that bilaterally arranged meetings often endow the parties

with some correlated information, we mark that (ii) a large class of dynamic markets with common values uncertainty becomes inefficient and then show that (ii) inverse dynamics are possible in a better market but not in the lemons market, without assumed non-stationarities.

This demonstrates that it may not be an anomaly in data or an outcome of complicated history dependencies for prices if sellers of better quality appear to have an easy time of finding a willing trading partner in a market with asymmetric information. Our paper shows that it could indeed be a steady state phenomenon in a large decentralized market, although the opposite case is also possible. This is in accordance with the mixed empirical evidence about the relationship between unemployment duration and the worker's hiring probability and the time on the market and house price (e.g., Heckman and Borjas (1980), Heckman and Singer (1984), and Asabere and Huffman (1993)).[6]

2 Model

We consider dynamic trading in a large decentralized market in infinite, discrete time t. In each time period a unit mass of buyers and equally many sellers enter the market. All buyers and sellers in the market are then randomly matched in order to trade. Every seller has one indivisible and imperishable good and every buyer wants to buy exactly one such good. Even so, it is still possible that due to the asymmetric information about the value of a seller's good and the option to delay trade, only some of the matched buyers and sellers manage to trade with their current trading partner; they exit the market. The others however remain in the market in hope to have better luck next time, with some other later trading partner. The next time period a new generation of buyers and sellers indeed enters the market with the ones who stayed and all are again randomly matched.[7] This periodic trading process continues *ad infinitum*. Possible waiting naturally reduces the gains from

[6] Unrealistic positive or negative expectations and alternating trading patterns might still have a crucial role to play in real markets with quality uncertainty.

[7] The matching function from the set of buyers in the market at each time point to the set of sellers in the market at that time point is *one-to-one* and *onto*.

trade. Buyers and sellers discount future payoffs by common factor $\delta \in (0, 1)$.[8,9]

Lemons market and better market

We use a model which allows us to capture the essence of the adverse selection problem in two parameters λ ("the relative gains from trade") and g ("the gap") whose relative sizes determine how severe the problem is. Half of the entering sellers has a high quality good, $\theta = h$, and half of them a low quality good, $\theta = l$. The quality is the seller's private information. It affects the buyer's value of the good, u_θ and the seller's production costs or reservation value, c_θ. Namely, if a buyer and a seller trade for price p, the buyer's payoff is $u_\theta - p$ and the seller's payoff is $p - c_\theta$; they are risk neutral. We assume the following relations for the payoffs:

$$\begin{aligned} u_h &= 1 + g \\ c_h &= \lambda + g \\ u_l &= \lambda \\ c_l &= 0 \end{aligned}$$

As standard, the buyer appreciates the good more than the seller, $u_h > c_h$ and $u_l > c_l$, and both the buyer and the seller value a high quality good more than a low quality good, $u_h > u_l$ and $c_h > c_l$. The gains from trade in low quality are denoted by $\lambda = u_l - c_l > 0$ and the gains from trade in high quality by $1 - \lambda = u_h - c_h > 0$; their total if fixed to unity. The gap in between is denoted by $g = c_h - u_l \geq 0$. Together with our simplifying assumption that half of the entering sellers has a high quality good, this implies that we can indeed use λ to adjust the relative gains from

[8] Our basic model of a large decentralized market is much like in Moreno and Wooders (2010). The main differences come in the bilateral meetings: we add the shared signal and let the seller make the take-it-or-leave-it price offer whereas they have no signal and they let the buyer make the take-it-or-leave-it price offer; we also let the buyers and sellers be matched every time period. They only study the lemons market and find the standard dynamics as a unique stationary equilibrium outcome of their model.

[9] Another somewhat resembling model is Lauermann and Wolinsky (2016) which, however, analyzes a static market and different research questions. The common traits are that the uninformed party views a signal (which might be either seen by both parties or observed by only this party) and that also the informed party can affect the price (they bargain over it). The key difference is the payoff structure because they only consider cases where the buyer's value is known; this ensures efficient equilibria.

trade and the gap g for the severeness of the adverse selection problem.[10]

We distinguish between the lemons market and what we call a better market.

Definition 1 *The lemons market has $g > 1 - \lambda$.*

The lemons market is characterized by a large gap between the high cost, c_h, and the low utility, u_l, and low gains from trade in high quality relative to those in low quality.

Definition 2 *A better market has $g \leq 1 - \lambda$.*

In the lemons market, the buyer's expected utility of trading with a random seller is thus below high quality seller's reservation value

$$E(u) = \frac{1}{2}(1 + g) + \frac{1}{2}\lambda < \lambda + g = c_h,$$

whereas, in a better market, the buyer's expected utility of average quality in the market is above high quality seller's reservation value

$$E(u) = \frac{1}{2}(1 + g) + \frac{1}{2}\lambda \geq \lambda + g = c_h.$$

In other words, in the static Walrasian market where all the goods must be traded for one price without delay, the lemons market fails for certain but a better market may not. This is also depicted in Figure 1. It displays the total supply and total demand in a static Walrasian market with one generation of buyers and sellers, half with a high quality good and half with a low quality good. In the lemons market (left panel),[11] $\lambda + g$ is always larger than $E(u)$. Hence, there is no price that equates the supply and demand. High quality does not trade as the lowest price for which high quality sellers are willing to trade, $\lambda + g$, is above the highest price for which buyers are willing to trade, $E(u)$. Note additionally that, if those better goods are now withdrawn from the market, as the story often proceeds, the average quality unravels and the problem becomes even worse. Low quality could still be traded for prices $p \in [0, \lambda]$. In a better market (right panel),[12] any of the prices $p \in [E(u), \lambda]$ however equates the supply and demand.

Remark 1 *In a static Walrasian market, the total trade surplus is $S = \frac{1}{2}\lambda$ in the*

[10]It is clear that none of our qualitative results would change if the numbers of entering low quality sellers and high quality sellers would not be the same; this choice just allows us the simple parametrization with only λ and g.

[11]For the picture we used $\lambda = 0.67$ and $g = 0.5$.

[12]For the picture we used $\lambda = 0.33$ and $g = 0.5$.

lemons case and $S = \frac{1}{2}\lambda + \frac{1}{2}(1-\lambda) = 1$ *in a better case.*

Generally, our dynamic market differs from a static Walrasian market in that different qualities can be traded (i) at different expected rates and (ii) for different expected prices. In particular, if a buyer or a seller fails to trade in a certain period, there is still a chance to trade later on. This implies that both the average quality in the market is endogenous (it could deviate either up or down from $E(u)$ in the static Walrasian market) and the buyers and sellers' outside options are endogenous (they might raise from zero in a static Walrasian market). It is therefore not immediate from the outset whether our market should work better or worse than a static Walrasian market. Intuitively, a rise in the average market quality would boost trade because the adverse selection problem would then be more relaxed whereas a rise in the outside options slows down trade as the trading partners become more selective. We also give the matched buyer and seller additional opportunities for information transmission prior to trade.

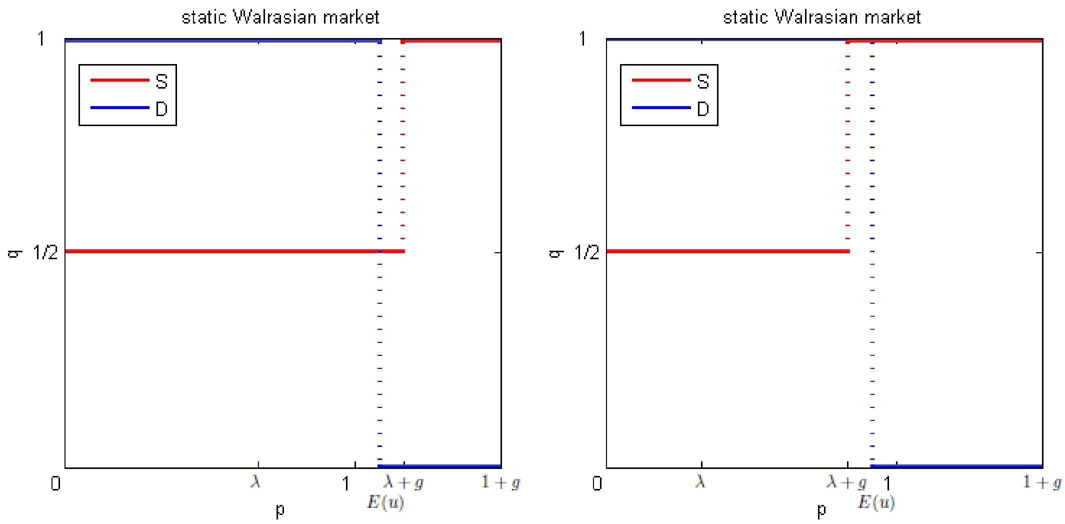


Figure 1: Supply (S) and demand (D) in a static Walrasian market: examples of the lemons market (left) and a better market (right).

Meeting between buyer and seller

As a natural coordination device, it is assumed that when a buyer and a seller meet in order to trade, they first draw a new shared signal $s \in [0,1]$ from a quality-specific distribution, F_h or F_l, corresponding the seller's quality. Then, the seller makes the buyer a price offer, $p \in [c_l, u_h]$, which the buyer either accepts, $a = 1$, or rejects,

$a = 0$. If the buyer accepts the offer, they trade for this price and exit the market but, if the buyer rejects the offer, they return back to the market.

Note that, as the shared signal is observable by both the buyer and the seller, it may condition all of the later actions of their meeting, i.e. the seller's price offer and the buyer's acceptance probability of a price offer. Nevertheless, because the signals and the actions in a meeting not observable by outsiders, they have no effect on any of the following meetings in which the buyer and seller might engage. Signals from earlier meetings are therefore not part of relevant trading history.[13]

The distribution functions $F_\theta : [0, 1] \to [0, 1]$ are continuous and supported on the unit interval $[0, 1] = \mathrm{cl}\{s | f_\theta(s) > 0\}$[14], where f_θ denotes the density function related to F_θ. It is assumed that a higher signal is indicative of a higher quality and that extreme signals are perfectly revealing:

Assumption 1

$$\frac{\partial}{\partial s} \frac{f_h(s)}{f_l(s)} \geq 0 \textit{ for all } s,$$

and

$$\lim_{s \to 0} \frac{f_h(s)}{f_l(s)} = 0,$$

and

$$\lim_{s \to 1} \frac{f_h(s)}{f_l(s)} = \infty.$$

The first part of this assumption just says that the signals s satisfy the standard monotone likelihood ratio property (MLRP). Observe also that, by the second part of this assumption and by continuity, any positive likelihood ratio is attainable under an appropriately chosen signal $s \in (0, 1)$.[15]

Notice that before the signal, information is perfectly asymmetric as standard. However, after the value of the signal is revealed, also the buyer has a bit of information about the seller's quality. As the signals are inexact, the buyer's belief $E(u_\theta | s)$ is nevertheless likely to be biased. Moreover, as the signal is shared by both parties, the seller knows exactly what the bias $E(u_\theta | s) - u_\theta$ is. This can have an

[13] This lack of information linkage between different meetings in the market distinguishes our work from earlier settings where inverse dynamics arise, e.g., Taylor (1999), Kaya and Kim (2015) and Zhu (2012).

[14] The closure of a set A contains all the points a whose every neighborhood $B(a)$ intersects with the set A: $cl(A) = \{a | \forall B(a) : A \cap B(a) \neq \emptyset\}$ where $B(a)$ is an arbitrary open set such that $a \in B(a)$.

[15] This implies that it is possible for both high and low quality sellers to emit also a highly misleading signal, $E_\gamma(u|s) \approx u_l = \lambda$ for $\theta = h$ and $\gamma_h \in (0, 1)$ or $E_\gamma(u|s) \approx u_h = 1$ for $\theta = l$ and $\gamma_h \in (0, 1)$.

effect on the following signaling game. Because the signals are noisy, the bias and the incentives to trade could thus vary from one meeting to another.

Namely, recall that in a static Walrasian market, there is no trade between a buyer and a seller if the buyer's beliefs are too low compared to the seller's beliefs: if the buyer estimates her valuation is $E(u)$ but the seller knows the value is higher, $1+g$. Here the situation is basically the same but more subtle. First, if the buyer's beliefs are too low after the shared signal, the seller has still a chance to affect them by the price offer. Second, both have also a chance to wait until they get a more favorable, higher signal. They need not trade in their first match.

Timing, strategies, equilibria, and organization of paper

The timing within a period can now be captured compactly by:

t.1 Entry of new buyers and sellers,

t.2 Buyers and sellers are matched,

t.3 Meetings of these matched pairs,

- *m.1* a shared signal is drawn and shown to this pair,
- *m.2* the seller offers buyer a price,
- *m.3* the buyer accepts the price or rejects the price,

t.4 Exit (if trade) or stay (no trade).

We concentrate on stationary Markovian equilibria in pure or randomized behavioral strategies $\boldsymbol{\sigma} = (\sigma_h, \sigma_l, \sigma_b)$: (i) The sellers' (mixed) strategies $\sigma_\theta(p,s) = Pr(p_\theta < p|s) : [0,1] \to \Delta\,[0, 1+g]$ map a price distribution to every signal s for high quality sellers, $\theta = h$, and low quality sellers, $\theta = l$, respectively. (i) The buyers' (mixed) strategies $\sigma_b(p,s) = Pr(a = 1|p,s) : [0, 1+g] \times [0,1] \to \Delta\,\{0,1\}$ map an acceptance probability to each pair (p,s) of a price and a shared signal.

The solution concept that we apply is perfect Bayesian equilibrium (PBE). A PBE is a pair $(\boldsymbol{\sigma}, \boldsymbol{\pi})$ that consists of a strategy profile $\boldsymbol{\sigma}$ and a belief system $\boldsymbol{\pi}$ such that (i) the strategy profile $\boldsymbol{\sigma}$ is consistent with sequential rationality given the belief system $\boldsymbol{\pi}$ and (ii) the belief system $\boldsymbol{\pi}$ is derived from the strategy profile $\boldsymbol{\sigma}$ whenever possible. To capture the full set of stationary Markovian PBE in this signaling game, we consider maximal punishments off the equilibrium path.[16]

[16] All PBE we consider satisfy the Intuitive Criterion (Cho and Kreps, 1987).

The rest of this paper is structured followingly: We first prove in Section 3 that there exists no efficient equilibrium in the lemons market nor in a better market when there is a possibility to delay trade and additional coordinating information given by the shared signal. We then move on to characterize the full set of equilibria for exogenous average quality in the market and outside options in Section 4. In furtherance of generality, this is done using mixed strategies.

We thereby study what kinds of dynamics can be sustained when average market quality and outside options arise endogenously in a stationary Markovian equilibrium. With pure strategies, we prove in Section 5 that in the lemons market only standard dynamics result from this but in a better market also inverse dynamics are possible.[17] In the next two Sections 2.1 and 2.2 we show how the average market quality and outside options are determined in an equilibrium.

2.1 Average quality in market is endogenous

In this section we show how the average quality in the market adjusts from its entry level value if there are possible differences in the relative liquidity of low quality goods and high quality goods. This adjustment will constrain equilibrium behavior in the long run in important ways because the quality in the market pins down also the buyer's beliefs about an average seller's quality. Later we prove that it is the key factor preventing us from obtaining inverse dynamics in the lemons market in Section 5 and the key factor working against us in this attempt also in a better market.

Note that, if high quality goods are less liquid than low quality goods (standard dynamics), their share goes up as they keep accumulating in the market and the buyers therefore become more optimistic about the quality of the sellers they meet; the opposite takes place if the low quality is traded faster (inverse dynamics). We next show what these buyer's beliefs are (i) before the signal (and before the price) and (i) after the signal (but before the price). Thereafter, depending on whether the sellers use separating pricing strategies, also the price offer can affect the buyer's beliefs.

To keep the market stock of each quality m_θ constant as required in a stationary equilibrium, the (exogenous) per period entry flow of each quality, 1/2, should equal the (endogenous) per period exit flow of that quality, $\tau_\theta m_\theta$:

[17] Inverse dynamics do not arise in a lemons market with mixed strategies either.

$$m_\theta = \frac{1}{2\tau_\theta}. \tag{1}$$

The exit rate is now simply given by trade probability

$$\tau_\theta = \int_0^{1+g} \int_0^1 \sigma_b^\star(p,s) d\sigma_\theta^\star(p,s) dF_\theta(s), \tag{2}$$

where $\sigma_b^\star$ and $\sigma_\theta^\star$ denote the optimal mixed strategies.

Hence, before the signal is observed but without any further revelation by the price offer, the expected buyer value of a random product equals

$$E_\gamma(u) = \underbrace{\frac{m_h}{m_h + m_l}}_{=:\gamma}(1+g) + \underbrace{\frac{m_l}{m_h + m_l}}_{=1-\gamma}\lambda. \tag{3}$$

After the signal is viewed, on the other hand, the expected buyer value of purchasing from that particular seller is given by Bayesian updating

$$E_\gamma(u|s) := \underbrace{\frac{m_h f_h(s)}{m_h f_h(s) + m_l f_l(s)}}_{=:\gamma(s)}(1+g) + \underbrace{\frac{m_l f_l(s)}{m_h f_h(s) + m_l f_l(s)}}_{=1-\gamma(s)}\lambda. \tag{4}$$

Note that, the probabilities of that the seller has a high quality good can also be expressed as

$$\gamma = \frac{1}{1 + \frac{\tau_h}{\tau_l}} \text{ and } \gamma(s) = \frac{1}{1 + \frac{\tau_h}{\tau_l}\frac{f_l(s)}{f_h(s)}}, \tag{5}$$

before the signal and after the signal, respectively, and the average entrant quality is given by

$$E_{\frac{1}{2}}(u) = \frac{1}{2}(1+g) + \frac{1}{2}\lambda.$$

2.2 Values of outside options are endogenous

In this section we present the buyer's problem and the seller's problem. Once matched with a pair, the buyer and the seller each solve their respective optimal stopping problem: both of them could either trade with their current partner and pocket the related immediate payoff or turn back to the market and keep on search-

ing in order to trade with some later trading partner. In particular, the seller in our model could offer such a high price that the buyer would always reject it; for simplicity, when a seller does so we just say that the seller has *quit* the meeting without trading.

As is standard in dynamic optimization problems, any solution to both optimal stopping problems will induce some continuation values for the buyer and the seller. They were absent in a static market and can now reduce the trading incentives. The values of these endogenous outside options are denoted by δV_b for buyers and by δV_θ for sellers. A larger value of outside options makes it obviously less attractive to trade. In Section 3 we show that even in a better market the outside options will be inflated in such a way that it will be impossible to sustain efficient trading for the lowest signals.

A buyer, who has been made a price offer p, decides whether to accept it or reject it by solving the following Bellman equation

$$V_b(p,s) := \max_{\sigma_b \in [0,1]} \sigma_b \left(E_\gamma(u|p,s) - p\right) + (1-\sigma_b)\,\delta V_b. \tag{6}$$

The optimal solution to this problem is

$$\sigma_b^\star(p,s) = \begin{cases} 1 & \text{for } E_\gamma(u|p,s) - p > \delta V_b, \\ [0,1] & \text{for } E_\gamma(u|p,s) - p = \delta V_b, \\ 0 & \text{for } E_\gamma(u|p,s) - p < \delta V_b. \end{cases} \tag{7}$$

A seller, who is endowed with a product of quality $\theta = h, l$, chooses the price offer by solving the following Bellman equation

$$V_\theta(s) := \max_{p\in[0,1]} \sigma_b^\star(p,s)\,(p - c_\theta) + (1-\sigma_b^\star(p,s))\,\delta V_\theta, \tag{8}$$

or, equivalently, the simpler problem

$$\max_{p\in[0,1]} \sigma_b^\star(p,s)\,(p - c_\theta - \delta V_\theta)\,. \tag{9}$$

To refer to them all at once, we can collect them into $\mathbf{V} = (V_b, V_h, V_l)$.

The buyer's problem is to sample one seller at a time sequentially, drawing new payoffs, $E_\gamma(u - p|s)$, for each new seller they encounter. These payoffs are distributed

independently across the bilateral meetings according to a given distribution with no recall option.[18] It is well known that the solution to such an optimal stopping problem is characterized by a cutoff policy where, if the expected payoff is below the cutoff, the buyer accepts the offer but, if the expected payoff is above the cutoff, the buyer rejects the offer. At an optimum, this cutoff will be equal to the value of outside options, δV_b.

The seller's problem is instead like that of a monopolist whose costs are $c_\theta + \delta V_\theta$, i.e., the seller reservation value plus the seller continuation value, and the demand is given by $\sigma_b^\star(p, s)$, i.e., the probability of trade for a given price. Observe, however, that in contrast to the standard monopoly problem, the seller's problem is not very well behaved in this case where the price can also act as a signal of quality: due to the usual flexibility with off path beliefs, even a slightest deviation from the anticipated price offer can make the buyer extremely suspicious of the seller's quality and thus reject the price offer.[19]

Indeed, note that equilibrium analysis pins down $\sigma_b^\star(p, s)$ only for equilibrium price-signal pairs (p, s). For those pairs that lie off the equilibrium path, we let the buyers' beliefs collapse to as negative as possible: we let the buyer in those cases conjecture that the seller's quality is low for sure. Another way to put this is to say that we analyze equilibria with maximal punishments off the equilibrium path. In this way we can look for the possibility of sustaining inverse dynamics in the largest possible equilibrium set, all of which are consistent with the Intuitive Criterion (Cho and Kreps, 1987).[20]

Based on Section 2.1 and Section 2.2, a stationary Markovian equilibrium can be defined as follows:

Definition 3 *A stationary Markovian PBE is a tuple* $(\boldsymbol{\sigma}, \boldsymbol{V}, \boldsymbol{m}, \boldsymbol{\tau}, E_\gamma(u), E_\gamma(u|s), E_\gamma(u|p, s))$ *such that:*

[18]To see where these payoffs come from, note that each date the buyer draws a new signal s from a signal distribution that is a mixture of high and low signal distributions, F_l and F_h. Furthermore, the seller's pricing strategies, $\sigma_l^\star$ and $\sigma_h^\star$, are stationary for all possible signal values by construction.

[19]For instance, under the usual flexibility with beliefs off the equilibrium path in games of signaling, the price elasticity of the demand, $\frac{\partial \sigma_b^\star(p,s)}{\partial p} \frac{p}{\sigma_b^\star(p,s)}$, can get infinite for some (p, s). This illustrates that it is generally not possible to resort to, say, basic tools of calculus to tackle the seller's problem.

[20]For completeness, we present the Intuitive Criterion in Appendix. Observe that this standard refinement is not violated by the kind of extreme beliefs we just described because typically, if low quality sellers gain from a deviation when taken for high quality sellers, then also high quality sellers would do so.

1. *Buyer strategy:* $\sigma_b^\star(p,s)$ *maximizes (6) given* $E_\gamma(u|p,s)$ *and* V_b *for all* (p,s).

2. *Seller strategies:* $\sigma_\theta^\star(p,s)$ *maximizes (8) given* $\sigma_b^\star(p,s)$ *and* V_θ *for all* (p,s).

3. *Buyer value:* V_b *satisfies (6).*

4. *Seller values:* V_θ *satisfy (8).*

5. *Stocks:* m_θ *satisfy (1).*

6. *Flows:* τ_θ *satisfy (2).*

7. *Average quality in the market and buyers' beliefs: (i)* $E_\gamma(u)$ *satisfies (3) where* γ *is given by (5), (ii)* $E_\gamma(u|s)$ *satisfies (4) where* $\gamma(s)$ *is given by (5) for all* s *and (iii)* $E_\gamma(u|p,s)$ *are consistent with* $\sigma_\theta^\star(p,s)$ *for all* (p,s) *on the equilibrium path and with the Intuitive Criterion (see App. for Def. 7) for all* (p,s) *off the equilibrium path.*

3 Welfare

As a necessary ingredient for the our later analysis of trading dynamics, we first make some general remarks about the effects of the shared signal on welfare. Since the traders are paired each period and there are strictly positive gains from trade in both qualities, efficient trading requires that every match results in trade. If there is instead a possible delay in trade, total surplus depends on the length of time it takes to trade in different qualities; inefficiency is manifested in decreased liquidity. While also the transition path to the steady state might matter, for stationary equilibria the standard welfare measure is the weighted sum of the values to a generation of buyers and sellers (e.g., Moreno and Wooders (2010, p. 388)):

$$S = V_b + \frac{1}{2}V_h + \frac{1}{2}V_l = \frac{\tau_h}{2}\frac{1-\lambda}{1-\delta} + \frac{\tau_h}{2}\frac{\lambda}{1-\delta}.$$

One of our most striking findings in this paper is now the non-existence of efficient equilibria both in the lemons market (in which the static Walrasian market fails) and in a better market (in which the static Walrasian market need not fail) when the shared signal is added to the game form. This strong result implies that both the lemons market and a better market are, in effect, negatively affected by asymmetric information. Interestingly, this general finding does not even depend on whether we have a signaling game or screening game, or some other form of bargaining in the

bilateral meetings; the proof makes this clear. Inefficiency arises merely from the interplay of dynamics and the noisy information revelation:

Proposition 1 *Any equilibrium is inefficient. (i) In a market where* $g > 0$ *the following statement holds:* $\exists s' \in (0,1) : \forall s < s' : E_\gamma(u|s) < \lambda + g$*; buyers and high quality sellers cannot hence trade with probability one for so low signals* $s < s'$*. (ii) In a market where* $g = 0$ *the following statements hold:*

1. *If* $V_h = 0$*, then* $V_b > 0$*. If* $V_b = 0$*, then* $V_h > 0$*.*

2. *Suppose that* $V_b > 0$*. Then,* $\exists s' \in (0,1) : \forall s < s' : E_\gamma(u|s) - p < \delta V_b$ *even for* $p = \lambda + g$ *(the minimal price for which a high quality seller may sell); a buyer would thus never accept so high a price.*

3. *Suppose that* $V_h > 0$*. Then,* $\exists s' \in (0,1) : \forall s < s' : p - \lambda - g < \delta V_h$ *even for* $p = E_\gamma(u|s)$ *(the maximal price for which a buyer might buy); a high quality seller would thus never offer so low a price.*

Proof. Observe first that after the signal $s \in [0,1]$ is viewed but without any costly revelation by the price offer, the maximal price that a buyer would be willing to accept is $E_\gamma(u|s) - \delta V_b$ (to compensate the buyer for the loss of the search option) and the minimal price that a high quality seller would be willing to offer is $\lambda + g + \delta V_h$ (to compensate for the cost and the loss of the search option). For the lowest possible signal $s = 0$ these prices would be $\lambda - \delta V_b$ for the buyer and $\lambda + g + \delta V_h$ for the seller. In the lemons market and in a better market that have a positive gap $g > 0$, it is thus immediate that there is no price that both could agree on in some open the neighborhood of the lowest possible signal $s = 1$.

For the other case where the gap is zero $g = 0$, note that all goods can be traded in the first match, as efficient trading requires, only if there is a price $p(s) \in [\lambda + \delta V_h, E_\gamma(u|s) - \delta V_b]$, that is acceptable to both the buyer and the seller for almost all s, especially, for the cases in which $E_\gamma(u|s)$ is close to λ. This is possible only if the following conditions hold: (i) $\delta V_b = 0$ and, thus, $p(s) = E_\gamma(u|s)$, for almost all s, since the use of any price below it would raise the buyer value V_b over zero *and* (ii) $\delta V_h = 0$ and, thus, $p(s) = \lambda$, for almost all s, since the use of any price above it would raise the seller value V_h over zero. But (i) and (ii) are clearly incompatible because $\lambda < E_\gamma(u|s)$ for all $s \in (0,1)$.[21]

[21] While we do not cover the case with a negative gap $g < 0$, it is clear from this proof that the inefficiency result may not extend to that case. Indeed in the setup where only the cost varies with type by Lauermann and Wolinsky (2016), the market is always efficient. Their paper studies information aggregation by prices.

To complete the proof, note that signaling or screening by the price offer does not help in this case because raising beliefs up from $E_\gamma(u|s)$ to some $E_\gamma(u|p,s)$, that would need to be higher, is not possible without inducing welfare costs in terms of possible delay in trade: higher prices cannot be accepted for certain because, otherwise, low quality sellers would have an incentive to mimic high quality sellers, which would violate the underlying incentive constraints. Hence, there exists no equilibrium where everything is traded in the first match. ■

When the gap is positive $g = c_h - u_l > 0$, it is intuitive that this wedge between low utility $u_l = \lambda$ and high cost $c_h = \lambda + g$ makes it impossible to trade under the lowest most signal realizations $s \approx 0$ because the beliefs are then about as low as they can be, $E_\gamma(u|s) \approx u_l$; mark that under the assumed information structure there is a non negligible probability of observing so low signals $s \in [1, \epsilon]$, for some tiny $\epsilon > 0$, ever for high quality sellers, $supp(F_h) = supp(F_l) = [0, 1]$. Without any such gap $g = c_h - u_l = 0$, however, the insight behind this result is as follows: Since additional information revelation makes it impossible to trade all goods in the first match without giving a fraction of the information rents either to buyers or to high quality sellers, it makes it also impossible to trade all goods for the lowest signals, which would provide them with almost no rents. With positive outside options, it is thus indeed better either for the buyers or for the high quality sellers to wait for higher signals than to trade under an unfavorably low one. This implies that the trade will break down for the lowest signal values.

In an economics job market application for example (which may not map one for one with the model but serves to illustrate the insights), if a candidate who considers himself or herself of high quality utterly fails in the job talk, the person can still try to signal his or her quality in negotiating the contractual terms with the department or put the hope in the other flyouts, the market scramble or in the next year. Since a high quality candidate however has rather high outside options of employment elsewhere, c_h, and in the economics job market, δV_h, there is no point in accepting a lower salary offer or inferior contract terms due to the failure – a post doc with the same teaching load as for the aspired assistant professor position. Thereby, since it is more difficult for say the micro fraction of the hiring committee to persuade the macro fraction to back a micro candidate who did poorly in the job talk, this can lead to an inefficiently long job search process; also high quality candidates sometimes do badly. Likewise, low quality candidates might also have their moment of luck earning them a good placement.

We think that these effects of correlated information on dynamic trading are important and unexpected. They show that the information based coordination in bilateral meetings can be a two-edged-sword. Most of the time signals take the parties beliefs' about the seller's quality closer together. This is fine because it makes the sorting problem easier for the buyers.

Nevertheless, because the signals are noisy, sometimes they also take the parties' beliefs quite far apart. While this is rare, from time to time a high quality seller can make a very bad impression on a buyer. If this disagreement among a buyer and a high quality seller is strong, it will be best for both the buyer and the seller to split up since they cannot agree on the terms of trade.[22]

In contrast, with either fully asymmetric or fully symmetric information, all gains from trade could still be realized in a better market:

Remark 2 *Consider a better market where the shared signal is uninformative. Then, there exist a continuum of efficient equilibria where the price offer that is made by both sellers is always $p \in [\lambda + g, E_\gamma(u)]$. All goods are traded in the first match. The buyer value is $V_b = E_\gamma(u) - p \geq 0$ and the seller values are $V_h = p - \lambda - g \geq 0$ and $V_l = p > V_h \geq 0$.*

Remark 3 *Consider any market where the shared signal is perfectly revealing. Then, there exist a unique efficient equilibrium where the price offer made by high quality sellers is $p_h = 1 + g$ and that made by low quality sellers is $p_l = \lambda$. All goods are traded in the first match. The buyer value is $V_b = 0$ and the seller values are $V_h = 1 - \lambda$ and $V_l = \lambda$.*

Corollary 1 *The shared signal can be welfare-reducing.*

We note that the underlying idea is akin to the so called Hirshleifer effect, which refers to a decrease in overall welfare with additional information as opportunities for risk sharing are removed (Hirshleifer, 1971). Later we demonstrate more specifically that in a better market the problem with shared signals is that pooling no longer constitutes an equilibrium for the lowest signals. In a better market, shared signals thus eliminate the above efficient pooling equilibria.

To show next that the shared signal can also have welfare improving effects, we consider the lemons market where the gains from trade are the same for both qualities, i.e., $\lambda = 1 - \lambda = \frac{1}{2}$. To make sure the average entrant quality is low enough for a lemons market, we assume that the gap is twice as large as this, i.e., $g = 1$. We

[22] On the flip side, a low quality seller can also make a very good impression on a buyer. This generates the inverse trading dynamics we explore in Section 5.

then compare trading in pure strategies with no shared signal, the standard case in other words, and with a shared signal distributed as

$$f_h(s) = 2s \text{ for all } s \in [0,1] \text{ and } f_l(s) = 2 - 2s \text{ for all } s \in [0,1]\,,$$

such that the increasing likelihood ratio is given by

$$\frac{f_h(s)}{f_l(s)} = \frac{s}{1-s} \text{ such that } \lim_{s\to 0} \frac{f_h(s)}{f_l(s)} \to 0 \text{ and } \lim_{s\to 1} \frac{f_h(s)}{f_l(s)} \to \infty,$$

which satisfies all our previously made assumptions.

It is next rather easy to become convinced that without a shared signal and with only pure strategies the following cyclic trading pattern is an equilibrium and indeed the surplus maximizing one: in odd periods $t = 2k - 1$, only low quality is traded for price $p = \lambda$ and, in even periods $t = 2k$, both low and high quality is trades for price $p = \lambda + g$. This empties the market so that the same cycle could then continue anew. Average utility over this cycle would hence be

$$2S = \lambda + \delta(1 - \lambda) + 1.$$

Note that, in odd periods, average quality in the market is

$$\frac{1}{2}(1+g) + \frac{1}{2}\lambda = 1 + \frac{1}{4} < \frac{3}{2} = \lambda + g$$

whereas, in even periods, average market quality is higher

$$\frac{2}{3}(1+g) + \frac{1}{3}\lambda = \frac{8}{6} + \frac{1}{6} = \frac{3}{2} = \lambda + g$$

because the high quality from the earlier period remains still there. Moreover, observe that in this bordering case, where the average market quality equals high cost in even time periods, both high quality sellers and buyers just break even: $V_b^t = V_h^t = 0$ for all t. The full surplus goes to the low quality sellers. To make sure that it is indeed an equilibrium for them to accept the lower price $p = \lambda$ in odd periods and not just wait for the the higher price $p = \lambda + g$ in even periods, the common discount factor has to be sufficiently low so as to guarantee that

$$\lambda \geq \delta(\lambda + g), \text{ which gives } \delta \leq \frac{1}{3}.$$

This is of course a rather low number for δ but it suffices for our purposes – to show that the availability of the shared signal can also increase welfare. If the

common discount factor is higher, a resembling equilibrium would still exits but then in between the periods when prices are $p = \lambda$ (only low quality trades) and $p = \lambda + g$ (also high quality trades), there would be an intermediate market freeze period in which no seller trades before the final clearing period. This equilibrium is essentially as in the example by Moreno and Wooders (2010, pp. 393-394).

Also to give the reader a flavor of how our model works, we are going to contrast this next with the equilibrium payoff attainable in exactly the same lemons market with pure strategies but now with a shared signal available in the bilateral meetings. We postulate that the following trading strategies then constitute an equilibrium: If the signal is below $s' = \frac{2}{3}$, low quality sellers make a separating price offer, $p_l = \lambda - \delta V_b$, and high quality sellers make a separating price offer, say, $p_h = 1 + g - \delta V_b$, where we are later going to derive an upper bound for the buyer value V_b.

In anticipation of our subsequent results in the following section, both of these prices are chosen in the way that the buyer just breaks even. To prevent low quality sellers from mimicking high quality sellers with simple pure strategies, p_l must be accepted and p_h must be rejected. However, if the signal is above $s' = \frac{2}{3}$, both sellers make a pooling price offer $p = \lambda + g$. Buyers accept it as the expected utility for that high signals is at least

$$E_{\frac{1}{2}}(u|s = s')\frac{1}{1+\frac{1}{2}}(1+g) + \frac{\frac{1}{2}}{1+\frac{1}{2}}\lambda = \frac{1}{1+\frac{1}{2}}2 + \frac{\frac{1}{2}}{1+\frac{1}{2}}\frac{1}{2} = \frac{3}{2} = \lambda + g,$$

where we have used in the denominator $\frac{f_l(s')}{f_h(s')} = \frac{1-s'}{s'} = \frac{1-\frac{2}{3}}{\frac{2}{3}} = \frac{1}{2}$. Note that we did not even take into account the fact that average market quality is going to be higher than average quality at entry in this kind of an equilibrium; above we have just set $\frac{\tau_h}{\tau_l} = 1$. Specifically, if we assumed instead that the described trading pattern is a stationary equilibrium where $\frac{\tau_h}{\tau_l} = \frac{1-F_h(\frac{2}{3})}{1} = \frac{5}{9}$, that would then obviously give an even higher expected utility to buyers

$$E_\gamma(u|s = s') = \frac{1}{1+\frac{5}{9}\frac{1}{2}}(1+g) + \frac{\frac{5}{9}\frac{1}{2}}{1+\frac{5}{9}\frac{1}{2}}\lambda = \frac{1}{1+\frac{5}{9}\frac{1}{2}}2 + \frac{\frac{5}{9}\frac{1}{2}}{1+\frac{5}{9}\frac{1}{2}}\frac{1}{2} > \frac{3}{2} = \lambda + g.$$

We can now approximate buyer value from upward by

$$
\begin{aligned}
V_b &= \frac{1}{1+\frac{5}{9}\frac{1}{2}}\left(\left(1-F_h\left(\frac{2}{3}\right)\right)(1+g-(\lambda+g))+F_h\left(\frac{2}{3}\right)\delta V_b\right)\\
&+\frac{\frac{5}{9}\frac{1}{2}}{1+\frac{5}{9}\frac{1}{2}}\left(\left(1-F_l\left(\frac{2}{3}\right)\right)(\lambda-(\lambda+g))+F_l\left(\frac{2}{3}\right)\delta V_b\right)\\
&= \frac{1}{1+\frac{5}{9}\frac{1}{2}}\left(\frac{5}{9}(1-\lambda)+\frac{4}{9}\delta V_b\right)+\frac{\frac{5}{9}\frac{1}{2}}{1+\frac{1}{2}}\left(\frac{1}{9}(-g)+\frac{8}{9}\delta V_b\right)\\
&\implies V_b = \frac{5(1-\frac{1}{9})}{23-8(1+\frac{5}{9})\delta},
\end{aligned}
$$

and calculate from this a lower bound on seller value

$$
V_l = F_l(s')\left(\lambda-\delta V_b\right)+\left(1-F_l(s')\right)(\lambda+g) = \frac{1}{9}(\lambda-\delta V_b)+\frac{8}{9}(\lambda+g).
$$

It should be clear from this construction that high quality sellers and buyers have always an incentive to stick with their postulated equilibrium strategies; in particular, any off equilibrium path offers will be rejected. However, low quality sellers who get a low signal $s < s'$ face a potentially non trivial tradeoff between offering a revealing price $\lambda - \delta V_b$ or waiting in hope to draw a higher signal in the future and, therefore, obtaining their continuation value δV_l; this value also accounts for the possibility of pooling with high quality sellers for price $\lambda + g$. Low quality sellers are thus willing to make the separating price offer p_l if

$$
\lambda - \delta V_b \geq \delta\frac{1}{9}(\lambda-\delta V_b)+\delta\frac{8}{9}(\lambda+g).
$$

Which gives us an upper bound for the common discount factor $\delta' \approx 0.310$, that thus guarantees the existence of this second kind of an equilibrium.

We can thus conclude this exercise by juxtaposing the average surplus over the cycle in the first equilibrium,

$$
S_1 = \frac{1}{2}\left(\frac{1}{2}+\frac{1}{2}\right)+\frac{1}{2}\left(\frac{1}{2}+\frac{\delta}{2}\right) = \frac{1}{2}+\frac{1}{4}(1+\delta),
$$

and lower bound for the surplus in the second equilibrium,

$$
S_2 = \frac{1}{2}+\frac{1}{2}\frac{5}{9}.
$$

It is clear that the latter surplus is greater than the former surplus if the common

discount factor is small $\delta < \frac{1}{9}$.

Corollary 2 *The shared signal can be welfare-enhancing.*

It is worth mentioning anew that, if low quality sellers are patient, they might have no incentive to make their separating price offer but instead wait until they get a shared signal that is high enough to allow them pool with high quality sellers. Namely, if that were feasible in the long run, it would shift us from standard trading dynamics to the case of inverse dynamics because high quality would then trade more often than low quality, $\tau_h = 1 - F_h(s')$ versus $\tau_l = 1 - F_l(s')$. We explore when this is actually possible in more detail in Section 5. Further, note that in the above examples we confined our analysis to pure strategies only. In the next Section 4 we consider trading also in randomized strategies.

4 Characterization of equilibrium strategies with exogenous average quality and outside options

This is now the paper's most technically challenging section. In this section we characterize the full set of equilibria which could arise in the bilateral meetings under any particular, favorable circumstances and suggest some natural refinements for the equilibrium set. We also give conditions for existence.[23] Observe that this is still basically a static problem because the average quality in the market γ is determined by the subsequent trading history and the outside options $\mathbf{V}$ are determined by what is done in the following continuation equilibrium; both of them are hence fixed when the matched buyer and seller choose their actions. Especially, neither γ nor $\mathbf{V}$ depends on what is done in the specific bilateral meeting, whose size is zero relative to the larger market. Thus, we can take the average quality γ, the outside options $\mathbf{V}$, and the shared signal s, that realizes in this meeting, as our starting point and study what kinds of equilibria are sustainable with this data.[24]

Definition 4 *A meeting-specific PBE with exogenous data* $(\gamma, \boldsymbol{V}, s)$ *is a tuple* $(\boldsymbol{\sigma}, E_\gamma(u), E_\gamma(u|s), E_\gamma(u|p,s))$ *such that:*

[23]As for notation, here we use the 4-parameter notation, u_θ and c_θ, and not the simpler 2-parameter notation, λ and g, which comes in handy in later fixed point analysis in Section 5.

[24]Note that, while some of the following pricing patterns may not be sustainable in a stationary equilibrium, they could arise more generally in an appropriate non-stationary equilibrium.

1. *Buyer strategy:* $\sigma_b^\star(p,s)$ *maximizes (6) given* $E_\gamma(u|p,s)$ *for all* p*.*

2. *Seller strategies:* $\sigma_\theta^\star(p,s)$ *maximizes (8) given* $\sigma_b^\star(p,s)$ *for all* p*.*

3. *Average quality in the market and buyers' beliefs: (i)* $E_\gamma(u)$ *satisfies (3) and* $E_\gamma(u|s)$ *satisfies (4) where* γ *and* $\gamma(s)$ *are both consistent with (5),*[25] *and (ii)* $E_\gamma(u|p,s)$ *are consistent with* $\sigma_\theta^\star(p,s)$ *for all* p *on the equilibrium path and with the Intuitive Criterion (see App. for Def. 7) for all* p *off the equilibrium path.*

These meeting-specific equilibria come here in three different kinds: pooling, separating, and semi-pooling, in which a seller is mixing between pooling and separting. Withing each type, there might exist a continuum of equilibria that are consistent with the Intuitive Criterion. Most deviations can be attributed to low quality sellers because, if the sellers of high quality would benefit from a deviation, then also the sellers of low quality would benefit from that deviation: it is thus perfectly reasonable for the buyer to expect that the deviant seller is a low quality seller.[26] As said in some cases we also simply say that the seller "quits" – whether the sellers' pricing strategies happen to be pooling, separating or semi-pooling. The idea is that the seller can also make such a high price offer, $p = 1 + g$ for example, that the buyer almost never accepts it. The inevitable end result is then that the seller goes back to the market and continues searching for better outside options.

Definition 5 *Consider a meeting-specific PBE with exogenous* $(\gamma, \boldsymbol{V}, s)$*.*

1. *A profile of pricing strategies* $\sigma_h^\star(p,s), \sigma_l^\star(p,s) \in \Delta[c_l, u_h]$ *is* pooling *if both sellers only make pooling offers, i.e., if* $supp(\sigma_h^\star(p,s)) = supp(\sigma_l^\star(p,s))$*.*

2. *A profile of pricing strategies* $\sigma_h^\star(p,s), \sigma_l^\star(p,s) \in \Delta[c_l, u_h]$ *is* separating *if both sellers only make separating offers, i.e., if* $supp(\sigma_h^\star(p,s)) \cap supp(\sigma_l^\star(p,s)) = \emptyset$*.*

Otherwise, a profile of pricing strategies is described as semi-pooling.

Observe that our data and, in particular, the outside options and the shared signal now determines which kinds of meeting-specific equilibria are supportable:

Proposition 2 *Consider a meeting-specific PBE with exogenous* $(\gamma, \boldsymbol{V}, s)$*.*

[25] Note that we could think, with no loss of generality, that γ defines through (5) some underlying $\boldsymbol{\tau}$ consistent with it. This $\boldsymbol{\tau}$ then, together with the shared signal s, defines $\gamma(s)$ through (5).

[26] We later on suggest a way to refine the equilibrium set based on how the sellers would prefer to coordinate their strategies. The idea is close to that presented by Nöldeke and Samuelson (1997) as we apparently end focusing on mixtures of what they call Riley equilibria and Hellwig equilibria.

1. *There exists a pooling equilibrium iff* $E_\gamma(u|s) - \delta V_b \geq c_h + \delta V_h$ $(IR-b, IR-h)$ *and* $E_\gamma(u|s) - \delta V_b \geq c_l + \delta V_l$ $(IR-b, IR-l)$. *In a pooling equilibrium, the price offer* p *is between max* $\{c_l + \delta V_l, c_h + \delta V_h\}$ *and* $E_\gamma(u|s) - \delta V_b$. *If* $p < E_\gamma(u|s) - \delta V_b$ *and the acceptance probability is given by*

$$\sigma_b^\star(p,s) = 1 \qquad \text{for } p < E_\gamma(u|s) - \delta V_b \text{ and}$$
$$\sigma_b^\star(p,s) \in [0,1] \qquad \text{for } p(s) := E_\gamma(u|s) - \delta V_b.$$

2. *There exists a separating equilibrium iff* $u_h - \delta V_b \geq c_h + \delta V_h$ $(IR-b, IR-h)$ *and* $u_l - \delta V_b \geq c_l + \delta V_l$ $(IR-b, IR-l)$. *In a separating equilibrium, the price offers are* $p_h = u_h - \delta V_b$, *for the high quality seller, and* $p_l = u_l - \delta V_b$, *for the low quality seller, and the acceptance probabilities are given by*

$$\sigma_b^\star(p_h, s) \in \left[0, \frac{p_l - (c_l + \delta V_l)}{p_h - (c_l + \delta V_l)}\right] \text{ and } \sigma_b^\star(p_l, s) = 1. \quad (IC - l)$$

3. *There exists a semi-pooling equilibrium iff*

$$u_l \geq c_l + \delta V_l + \delta V_b \qquad \text{and} \qquad u_h > c_h + \delta V_h + \delta V_b, \text{ or}$$
$$E_\gamma(u|s) > c_h + \delta V_h + \delta V_b \qquad \text{and} \qquad E_\gamma(u|s) > c_l + \delta V_l + \delta V_b.$$

In a semi-pooling equilibrium, except for a knife-edge case $c_l + \delta V_l = c_h + \delta V_h$, *there could be at maximum one pooling price* p *in use and at maximum one separating price* p_l *or* p_h *in use: If the high quality seller is mixing between* p *and* $p_h > p$ *the low quality seller only using* p *whereas if the low quality seller is mixing between* p *and* $p_l < p$, *the high quality seller is only using* p. *(See App. for the full characterization.)*

Above IR refer to individual rationality constraints and IC to incentive compatibility constraints.

In a pooling equilibrium, both low quality sellers and high quality sellers use the same price p. If the price leaves the buyer positive surplus, it must be accepted for probability one; otherwise, the buyer can also mix between accepting and rejecting the price. For the sellers the best such equilibrium is of course the one where the price keeps the buyers at their outside options and the buyers accept it for probability one nonetheless; it combines the best of both worlds.

Note also that, low quality sellers can separate from high quality sellers when-

ever they want by offering any price below the high cost; high quality sellers would never use that low prices. High quality sellers cannot do so themselves.[27,28] Notwithstanding, low quality sellers are willing to separate only if their continuation value is below the payoff they get from trading for their low revealing price. For higher values of outside options, there does hence not exist a separating equilibrium. The existence of a pooling equilibrium depends instead on the level of the signal. If the signal is very high, buyers rest assured that the seller's quality is high and are thus willing to accept a higher price.[29]

In a separating equilibrium, both sellers offer the buyer a revealing price, a lower price p_l for the low quality sellers and a higher price p_h for the high quality sellers. The former is accepted for certain but the latter one has to be accepted for a probability less than one to stop the low quality sellers from mimicking the high quality sellers.[30] Both prices must additionally keep the buyers at their outside options to honor buyers and sellers' optimality conditions.

In a semi-pooling equilibrium, either the low quality sellers mix between a low and a high price while the high quality sellers only use the high price or the high quality sellers mix between a low and a high price while the low quality sellers only use the low price. To prevent the low quality sellers from mimicking, the high price must be accepted less often than the low price. To keep the buyers mixing in accepting and rejecting it, they must be kept at their outside options. That is, several fixed point conditions, i.e., the revenue equivalence condition for the mixing buyer and the mixing seller, plus, the incentive condition for the low quality seller, should be holding all at once.

To sum up, the key message of Proposition 2 is that, for any $\gamma \in (0,1)$ and assuming quite naturally that $u_h - \delta V_b > c_h + \delta V_h$ and $c_h + \delta V_h \geq c_l + \delta V_l$, by

[27] Due to the harsh off the equilibrium path beliefs that would arise as low quality sellers might also have an incentive to offer those higher prices.

[28] To be even more precise, if was so that $c_l + \delta V_l > c_h + \delta V_h$, high quality seller could separate by offering $p = c_l + \delta V_l - \epsilon$ for some tiny $\epsilon > 0$. However, then the cutoff signal for pooling would be given by $E(u|s') = c_l + \delta V_l > c_h + \delta V_h$ so that the high quality seller would only be willing to do so for low signals $s < s'$. This would imply that low quality trades only for $s \geq s'$ but high quality trades for all signals. In the long run, this would not generate assumed stationary equilibrium payoffs $c_l + \delta V_l > c_h + \delta V_h$. It is a good exercise to show that the payoff that low quality seller gets from pooling with the high is at most c_h plus the payoff that high quality seller gets from pooling with the low in a stationary equilibrium. Furthermore, this kind of behavior is not an equilibrium without additional tweaking with tie breaking assumptions because high quality sellers have an incentive to elevate their price offer until $\epsilon \to 0$.

[29] In some cases there may not exist a pooling equilibrium nor a separating equilibrium; then the only remaining option for sellers is to quit.

[30] Observe that in many applications there might exist natural ways to interpret or purify the randomized strategies, for example, by perturbing the players' payoffs à la Harsanyi (1973).

Proposition 1 there always exists a cutoff signal $s' \in (0,1)$ such that a pooling equilibrium exists if and only if $s \geq s'$, where $E_\gamma(u|s') - c_h = \delta V_h + \delta V_b$ (the expected gains from trade must be sufficient to compensate both the high quality seller and the buyer for the loss of their outside options). A separating equilibrium exists instead if and only if $u_l - c_l \geq \delta V_l + \delta V_b$ (the gains from trade must be sufficient to compensate both the low quality seller and the buyer for the loss of their outside options). These constraints for existence are presented in Table 1.

$s' : E_\gamma(u|s') = c_h + \delta V_h + \delta V_b$	$s \geq s'$	$s < s'$
$u_l \geq c_l + \delta V_l + \delta V_b$	$\exists$ pooling eq.	$\nexists$ pooling eq.
$u_l \geq c_l + \delta V_l + \delta V_b$	$\exists$ separating eq.	$\exists$ separating eq.
$u_l < c_l + V_l + \delta V_b$	$\exists$ pooling eq.	$\nexists$ pooling eq.
$u_l < c_l + V_l + \delta V_b$	$\nexists$ separating eq.	$\nexists$ separating eq.

Table 1: Existence of meeting-specific pooling equilibria and separating equilibria.

Now one way to refine the equilibrium set is to concentrate on equilibria that both types of sellers would like to play if they were able to coordinate to an equilibrium they both prefer; the idea is like spirit with the concept of undefeated equilibrium by Mailath et al. (1993) but now we only consider seller payoffs:[31]

Definition 6 *Consider a meeting-specific equilibrium with data $d = (\gamma, \boldsymbol{V}, s)$. The equilibrium is* seller maximal *if there exist no other equilibrium that both sellers weakly prefer and either high quality sellers or low quality sellers strictly prefer. Otherwise, the latter equilibrium* defeats *the former equilibrium.*

Remark 4 *Consider a meeting-specific PBE with exogenous $(\gamma, \boldsymbol{V}, s)$.*

1. *Any pooling equilibrium is defeated by the best pooling equilibrium where the price offer is $p = E_\gamma(u|s) - \delta V_b$ and the acceptance probability is $\sigma_b^\star(p,s) = 1$.*

2. *Any separating equilibrium is defeated by the best separating equilibrium where the acceptance probabilities are*

$$\sigma_b^\star(p_h, s) = \frac{p_l - (c_l + \delta V_l)}{p_h - (c_l + \delta V_l)} \text{ and } \sigma_b^\star(p_l, s) = 1.$$

[31]More generally one might want to do a related exercise for stationary equilibria where γ and $\mathbf{V}$ are endogenous. Our initial analysis suggests that, to find the stationary equilibrium with maximal welfare, the level of the pooling price offer might need to be adjusted so that both the buyer participation constraint $(IR - b)$ or a seller participation constraint $(IR - h)$ are binding at the cutoff signal s' (above it, sellers should pool, below it, they should separate).

3. *Any semi-pooling equilibrium is defeated by the best pooling equilibrium or by the best separating equilibrium.*

4. *The best separating equilibrium is defeated by the best pooling equilibrium if*

$$E_\gamma(u|s) - c_h \geq \sigma_b^\star(p_h, s)\,(u_h - c_h) + (1 - \sigma_b^\star(p_h, s))\,\delta V_h.$$

5. *The best pooling equilibrium is not defeated by the best separating equilibrium.*

Crucially, we find that, in search for seller maximal meeting-specific equilibria, it is possible to ignore as defeated all semi-pooling equilibria and zoom in on the best pooling equilibrium and the best separating equilibrium. This result arises because the seller who is mixing in prices has to be indifferent between playing a higher price and a lower price whereas the other seller would be better off if the ratio in which the first seller mixes was degenerate; a seller maximal meeting-specific equilibrium should consequently either be fully pooling or fully separating.

Moreover, if the signal is high (if there exists a pooling equilibrium), all sellers are best off relying on this costless high signal but, if the signal is low (if there exists only a separating equilibrium), they need to opt for possibly costly signaling. Otherwise (if there exists no pooling equilibrium nor a separating equilibrium), they just quit. Observe also that, if a stationary equilibrium (Def. 3) is constructed of seller maximal meeting-specific equilibria (Def. 4), a Diamond (1971)-like holdup result arises and buyers get no rents; buyer value V_b is thus zero.

5 Standard dynamics and inverse dynamics with endogenous average quality and outside options

In this last section we show as the key result of this paper that inverse dynamics are possible in a better market but not in the lemons market. Average quality and outside options are endogenized and we only consider trading with pure strategies. Note first that obtaining inverse dynamics requires we are in the lowest two rows of Table 1 where the low quality sellers are no willing to make a separating price offer. Otherwise, the low quality sellers would have an incentive to guarantee that they trade in the first match either (i) by pooling with the high quality sellers, for

some of the highest signals when this is possible, or (ii) by making a revealingly low price offer. Since Proposition 1 demonstrates that all high quality goods cannot be traded in the first match, this would imply that low quality is traded faster than high quality. A necessary condition for inverse dynamics is thus

$$\lambda < \delta V_b + \delta V_l. \tag{10}$$

5.1 Lemons market

As separating equilibria cannot exist in this case and semi-pooling equilibria are infeasible with pure strategies, the only way of trading here is to sell both types of goods for the same pooling price $p(s)$ for some strict subset of signals $S \subset [0,1]$. By Proposition 1 we know that the largest set of signals is $[s',1]$, where the value of the cutoff $s' \in (0,1)$ is given by

$$s' : E_\gamma(u|s') = \max\{\delta V_l + \delta V_b, \lambda + g + \delta V_h + \delta V_b\}. \tag{11}$$

In consequence, if the sellers pool whenever they can, average quality in the market adjusts until

$$\gamma = \frac{1}{1+\frac{\tau_h}{\tau_l}} = \frac{1}{1+\frac{1-F_h(s')}{1-F_l(s')}}.$$

However, there could also in principle be gaps in the set of signals for which the sellers are trading. The set of feasible trading signals $[s',1]$ could be partitioned into a set of actual trading signals $S_T \subset S$ and another set of signals $S_{NT} \subset$ for which the sellers do not trade due to harsh off path beliefs.

$$S_T = [s_0, s_1] \cup [s_2, s_3] \cup \ldots \cup [s_k, s_{k+1}] \cup \ldots \text{ where } s_0 \geq s'.$$

Nevertheless, due to the monotone likelihood ration property, $\frac{f_h(s_k)}{f_l(s_k)} < \frac{f_h(s_{k+1})}{f_l(s_{k+1})}$ for $s_k < s_{k+1}$, introducing these kinds of gaps of length $l_k = s_{k+1} - s_k$ can increase the average market quality only up to a certain point because

$$\frac{\tau_h}{\tau_l} = \frac{\sum_{k=1}^{\infty}(F_h(s_{2k-1}) - F_h(s_{2(k-1)}))}{\sum_{k=1}^{\infty}(F_l(s_{2k-1}) - F_l(s_{2(k-1)}))} \geq \frac{\sum_{k=1}^{\infty} f_h(s_{2(k-1)})l_{2(k-1)}}{\sum_{k=1}^{\infty} f_l(s_{2(k-1)})l_{2(k-1)}} \geq \frac{f_h(s')}{f_l(s')}.$$

In the lemons market where $g > 1-\lambda$, there could thus only exist stationary

equilibria where low quality is more liquid than high quality. This is because at the lowest signal s' for which the sellers are trading if we assume inverse dynamics, buyer expectations are necessarily below those at entry:

$$\gamma(s) = \frac{1}{1 + \frac{1-F_h(s')}{1-F_l(s')}\frac{f_l(s')}{f_h(s')}} < \frac{1}{2},$$

because, by the monotone likelihood ratio property,

$$\frac{1-F_h(s')}{f_h(s')} > \frac{1-F_l(s')}{f_l(s')} \text{ for all } s',$$

or because, with the kind of gaps we just described,

$$\gamma(s) \leq \frac{1}{1 + \frac{f_h(s')}{f_l(s')}\frac{f_l(s')}{f_h(s')}} = \frac{1}{2}.$$

This entails that in the lemons market where the average quality at entry is already too low to sustain immediate trading the high quality seller cannot be willing to trade for the highest pooling price offer that the buyer would accept at s'. This contradicts the possibility of obtaining inverse dynamics.[32]

Corollary 3 *Inverse dynamics cannot arise in the lemons market.*

5.2 Better market

For a better market where $g \leq 1-\lambda$ we find, as a novelty, that also inverse dynamics are possible:

Proposition 3 *As long as the following limit condition holds in a better market*

$$E_\gamma(u|s') - (\lambda+g) = \frac{1}{1 + \frac{1-F_h(s')}{1-F_l(s')}\frac{f_l(s')}{f_h(s')}}(1-\lambda) - \frac{\frac{1-F_h(s')}{1-F_l(s')}\frac{f_l(s')}{f_h(s')}}{1 + \frac{1-F_h(s')}{1-F_l(s')}\frac{f_l(s')}{f_h(s')}} g \to a > 0 \text{ as } s' \to 1,$$

there exists a stationary Markovian equilibrium where high quality is traded faster than low quality. The equilibrium is characterized by a cutoff signal s': below the cutoff, for $s < s'$, sellers return to the market and, above the cutoff, for $s \geq s'$, they offer a seller maximal price $p(s) = E_\gamma(u|s)$, that buyers accept with probability one.

[32] Note that existence is not an issue in this case. There exist for instance a multiplicity of stationary Markovian equilibria with standard dynamics, where both sellers make a pooling price offer for higher signals and offer separating prices for lower signals.

High quality goods are traded with probability $1 - F_h(s')$ *and low quality goods are traded with probability* $1 - F_l(s')$*, which is lower.*

The proof is based on a fixed point argument and on showing that the outside options to low quality sellers can be bounded away from zero in equilibria like this. For low enough gains from trade in low quality, λ, the necessary condition (10) for this kind of an equilibrium can therefore be satisfied.

We can also illustrate this by showing a couple of examples. To make our task easier, though, we let the pooling price be at a buyer maximal level $p(s) = \lambda + g$ instead of the seller maximal level $p(s) = E_\gamma(u|s)$, that we have in the formal proof. This allows us to obtain $\mathbf{V}$ without solving an integral.

In this case, the buyer value is derived as

$$
\begin{aligned}
V_b &= \gamma \left(F_h(s')\delta V_b + (1 - F_h(s'))(1 - \lambda)\right) \\
&\quad + (1 - \gamma) \left(F_l(s')\delta V_b + (1 - F_l(s'))(-g)\right) \\
V_b &= \frac{1}{1-\delta} \frac{1 - F_l(s')}{2 - F_h(s') - F_l(s')}(1 - F_h(s'))(1 - \lambda) \\
&\quad + \frac{1}{1-\delta} \frac{1 - F_h(s')}{2 - F_h(s') - F_l(s')}(1 - F_l(s'))(-g) \\
&\Longrightarrow V_b = \frac{(1 - F_h(s'))(1 - F_l(s'))}{2 - F_h(s') - F_l(s')} \frac{1 - \lambda - g}{1 - \delta}.
\end{aligned}
$$

The cutoff signal can be obtained by solving

$$
\begin{aligned}
&\left(\frac{(1 - F_l(s'))f_h(s')}{(1 - F_h(s'))f_l(s') + (1 - F_l(s'))f_h(s')} - \frac{\delta}{1-\delta} \frac{(1 - F_l(s'))(1 - F_h(s'))}{2 - F_h(s') - F_l(s')}\right)(1 - \lambda) = \\
&\left(\frac{(1 - F_h(s'))f_l(s')}{(1 - F_h(s'))f_l(s') + (1 - F_l(s'))f_h(s')} - \frac{\delta}{1-\delta} \frac{(1 - F_l(s'))(1 - F_h(s'))}{2 - F_h(s') - F_l(s')}\right)(-g)
\end{aligned}
\tag{12}
$$

and the existence again requires that the necessary condition (10) holds. We can now use the same distributions for shared signal as previously

$$
f_h(s) = 2s \text{ for all } s \in [0, 1] \text{ and } f_l(s) = 2 - 2s \text{ for all } s \in [0, 1],
$$

such that

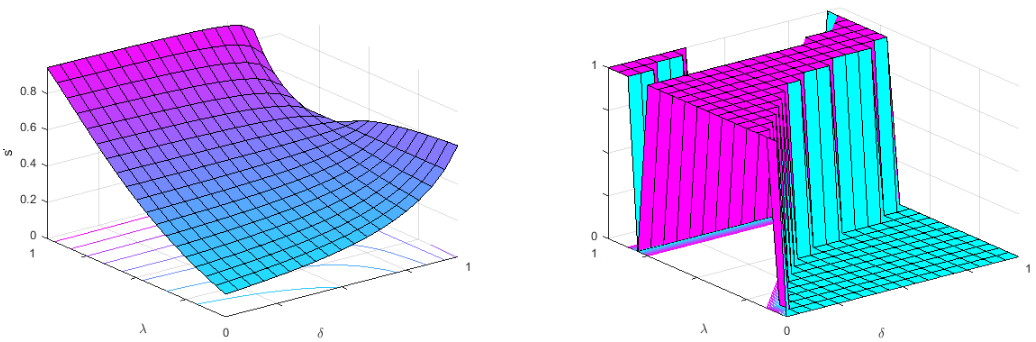
$$F_h(s) = s^2 \text{ for all } s \in [0,1] \text{ and } F_l(s) = s - s^2 \text{ for all } s \in [0,1]\,,$$

to find numerically that an equilibrium like this exists for a range of parameter values depicted in Figure 2b where 2a gives the related cutoff.

For instance the following parameter values would be consistent with inverse dynamics:

$$g = 0.1, \delta = 0.3, \lambda = 0.2 \text{ and } g = 0.1, \delta = 0.65, \lambda = 0.75.$$

It is also easy for anybody to check from above that, if we set the gap to zero $g = 0$ to facilitate the calculation, then $s' = 0.5$ solves (12) for $\delta \approx 0.55$. Cond. (10) will then be satisfied for low enough values of $\lambda \leq 0.15$.



(a) (b)

Figure 2: The value of the cutoff signal that satisfies Eq. (12) for $(\lambda, \delta) \in (0,1)^2$ and $g = 0.1$ (a) and an indicator of whether Cond. (10) is holding for $(\lambda, \delta) \in (0,1)^2$ and $g = 0.1$ (b).

Corollary 4 *Inverse dynamics could arise in a better market.*

6 Closing remarks

Though a useful abstraction in economics, information on common transaction values remains rarely purely private all the way up to the moment when the terms of trade are negotiated by a buyer and a seller. In particular, when trading partners meet face-to-face, some payoff related information is often transmitted under both traders' eyes.

We provide an example of a market in which this opportunity to get additional information in the bilateral meetings, through which trading is conducted, causes a reduction in liquidity in a setup where the static Walrasian market may not fail. Indeed, all goods could be traded efficiently in the first match if the shared signals were removed.

This dismal outcome arises here because the possibility to wait for higher signals, which give rise to better prices, makes either the buyers or the high quality sellers too picky to trade for the lowest signals. The effect works through an increase in the outside options; it does not depend, for example, whether we have a signaling game or a screening game. Hence, we obtain a classic trade-off between rent extraction and efficiency.

Based on these findings, we think it might be appropriate to reconsider what is meant by a lemons market. While in a static market, only the case where average quality is low is bound to fail, in a dynamic setting with some information percolation in bilateral meetings or otherwise, the market is inefficient independent of the average quality. In absence of reliable evidence, buyers and sellers may for instance have an incentive to pay excessive attention to rumors or to coordinate their actions based on only mildly relevant signals. According to our welfare results, this can then reduce market liquidity.

We also study what kinds of dynamics can be sustained when average quality in the market and outside options arise endogenously in a stationary Markovian equilibrium. We are particularly keen to find out whether inverse trading dynamics that have been largely ignored by the literature are possible with our rich communication opportunities. We show that high quality can be traded faster than low quality in a better market but not in the lemons market. We argue that both cases are relevant because neither market is efficient due to the additional coordinating information and the option to delay trade.

Our rudimentary, extended analysis reveals further that from an efficiency perspective it might be better to stick with standard dynamics. Notwithstanding, we hypothesize that subsidizing low quality sellers or otherwise increasing their outside options, say through some of the contemplated market interventions to steer clear from market illiquidity, can engender the prevalence of less efficient inverse dynamics – which seems to be a common trading pattern in markets for skilled labor. We think

that this relation between dynamics and welfare merits a closer investigation.[33]

Appendix

INTUITIVE CRITERION (Cho and Kreps, 1987)

Definition 7 *Consider an equilibrium belief system π. Denote by $\Theta(p') \subset \{h, l\}$ the subset of sellers who prefer the disequilibrium price offer p' to the equilibrium price offer p when considered high quality sellers. The equilibrium fails the Intuitive Criterion if there is a seller type $\theta' \in \Theta(p')$ who prefers p' to p for whatever the buyers' belief, as long as the buyer takes him for a seller type $\theta \in \Theta(p')$.*

In our case the belief system fails the Intuitive Criterion if (a) $\pi(p') = E_\gamma(u|p') > u_l$ but the high quality seller would always lose by the deviation to p' whereas the low quality seller would gain from it even if his type was known or (b) $\pi(p') = E_\gamma(u|p') < u_h$ but the low quality seller would always lose by the deviation to p' whereas the high quality seller would gain from it even if his type was known.

PROOFS OF REMARKS 2 AND 3

Note first that, due to the discounting, it is better to trade at once than wait for the repetition of this same stationary equilibrium. In Lemma 2, the seller is willing to offer p provided that any higher price offer is rejected because of the low off path beliefs that would arise. In Lemma 3, such adverse beliefs present no threat as the matched buyer and seller both know the quality and sellers can therefore use $p_h = u_h$ and $p_l = u_l$ to extract full surplus. Since buyer value is here zero, buyers have no reason to reject these offers. ■

PROOF OF PROPOSITION 2

Pooling meeting-specific PBE

As both sellers make the same price offer p for the associated signal realization, s, its use prompts no updating of buyers' beliefs: $E_\gamma(u|s,p) = E_\gamma(u|s)$; these are given by Eq. (4). Consequently, to make the buyers accept a pooling price offer $p(s)$, it is necessary that $p \leq E_\gamma(u|s) - \delta V_b$ and, to make both types of sellers offer that price, it is necessary that $p \geq \max_{\theta=l,h}\{c_\theta + \delta V_\theta\}$. In other words, for a pooling meeting-specific PBE exist, the expected utility to buyer $E_\gamma(u|s)$ must be high enough to compensate the traders from the loss of their continuation values and costs $c_\theta + \delta V_\theta + \delta V_b$ for $\theta = l, h$. Otherwise, any price offer would do as profitable deviations (i) to higher prices or (ii) to lower prices (in hope to increase the acceptance probability $\sigma_b^\star(p,s)$ if it is below unity), can be prevented by attributing such deviations to low quality sellers (see Footnote 28 for some additional discussion about this topic). By Cond. (7), if $p < E_\gamma(u|s) - \delta V_b$, the buyers should accept

[33] That analysis should account for both the surplus accrued during the convergence path and the steady state welfare: transition to a steady state with inverse dynamics may generate more surplus than a transition to a steady state with standard dynamics, depending on relative gains from trade λ and the gap g.

the pooling price with probability one but, if $p = E_\gamma(u|s) - \delta V_b$, they could also randomize between accepting and rejecting the pooling price. ■

Separating meeting-specific PBE

As different sellers use different prices in a separating equilibrium, p_l for the low and p_h for the high, buyers' beliefs become degenerate after the price offer has been made: $E_\gamma(u|s, p_l) = u_l < E_\gamma(u|s)$ and $E_\gamma(u|s, p_h) = u_h > E_\gamma(u|s)$. Thus, to make both parties willing to trade for these prices, according to Cond. (6) and Cond. (9), it must hold that $p_l \in [c_l + \delta V_l, u_l - \delta V_b]$ and $p_h \in [c_h + \delta V_h, u_h - \delta V_b]$. To keep the low quality sellers from mimicking the high quality sellers, it should also hold that

$$\sigma_b^\star(p_l, s)(p_l - c_l) + (1 - \sigma_b^\star(p_l, s)))\,\delta V_l \geq \sigma_b^\star(p_h, s)(p_h - c_l) + (1 - \sigma_b^\star(p_h, s))\,\delta V_l.$$

Note that, since $p_l < p_h$, the low price must be accepted more frequently than the high price, $\sigma_b^\star(p_l, s) > \sigma_b^\star(p_h, s)$, to satisfy the incentive condition for the low quality sellers. (Obviously, the high quality sellers have no incentive to mimic the low quality sellers as the low price p_l is below the high cost c_h.) Therefore, to make the buyers randomize between accepting and rejecting the high price p_h, it is necessary that this price keeps the buyers at their continuation values, $p_h = E_\gamma(u|p_h, s) - \delta V_b$.

Moreover, also the low price p_l must keep the buyers at their continuation values, $p_l = E_\gamma(u|p_l, s) - \delta V_b$, and it must additionally be accepted with probability one, $\sigma_b^\star(p_l, s) = 1$. To see why this is so observe that, if the former requirement $p_l = E_\gamma(u|p_l, s) - \delta V_b$ would not hold, there would be a profitable deviation for the seller from p_l to $p_l + \eta$ to keep the acceptance rate the same but to increase the price offer and, if the latter requirement $\sigma_b^\star(p_l, s) = 1$ would not hold, there would be a profitable deviation for the seller from p_l to $p_l - \eta$ to make the buyer accept the price for certain, for some tiny $\eta > 0$. As buyers' beliefs are already the harshest possible for p_l it is impossible to discipline such deviations by out of equilibrium path beliefs, which could not be worse still.

It is clear from above that a separating equilibrium cannot exist unless $c_l + \delta V_l \leq u_l - \delta V_b$ and $c_h + \delta V_h \leq u_h - \delta V_b$. Given the usual freedom with off path beliefs, these necessary conditions for existence are also sufficient. ■

Semi-pooling meeting-specific PBE

We first show that both sellers cannot randomize their prices simultaneously:

To the contrary, suppose the sellers mix between two pooling prices, p^1 and p^2 such that $a^1 := \sigma_b^\star(p^1, s)$ and $a^2 := \sigma_b^\star(p^2, s)$. With no loss of generality, we now can assume that $p^1 < p^2$ such that $a^1 > a^2$. By individual rationality, the prices should be such that $p^i - c_\theta \geq \delta V_\theta$ if price p^i is to be used by sellers of quality $\theta = h, l$. Now, to keep the high quality sellers mixing between p^1 and p^2, it should hold that

$$a^1\left(p^1 - c_h\right) + \left(1 - a^1\right)\delta V_h = a^2\left(p^2 - c_h\right) + \left(1 - a^2\right)\delta V_h$$

$$a^1 p^1 = \left(a^1 - a^2\right)\left(c_h + \delta V_h\right) + a^2 p^2$$

and, to keep the low quality sellers mixing between p^1 and p^2, it must hold that

$$a^1\left(p^1-c_l\right)+\left(1-a^1\right)\delta V_l=a^2\left(p^2-c_l\right)+\left(1-a^2\right)\delta V_l$$

$$a^1p^1=\left(a^1-a^2\right)\left(c_l+\delta V_l\right)+a^2p^2.$$

Clearly, both of them cannot be satisfied at the same time unless $c_h+\delta V_h=c_l+\delta V_l$. Instead, for the most natural case of $\delta V_h+\lambda>\delta V_l$, if the high quality mixes between p^1 and p^2, then the low quality prefers to use p^1 only and, if the low quality mixes between p^1 and p^2, then the high quality prefers to use p^2 only. In the former (latter) case, p^2 is a separating (pooling) price and p^1 is a pooling (separating) price.

We then derive conditions for existence and give ideas about characterization:

Note that separating prices are perfectly revealing such that $E_\gamma(u|p_l,s)=u_l$ and $E_\gamma(u|p_h,s)=u_h$ for any s. This implies that $p_l\in[c_l,u_l]$ and $p_h\in[c_h,u_h]$ for the sellers to offer them and for the buyers to accept them. As mentioned two cases could arise: (i) If the higher price p^2 is the pooling price so that low quality sellers partially separate from the high, then $E_\gamma(u|p^2,s)=E_\gamma(u|s)+\nu$ for some positive ν. (ii) If the lower price p^1 is the pooling price so that high quality sellers partially separate from the low, then $E_\gamma(u|p^1,s)=E_\gamma(u|s)-\nu$ for some positive ν. Similarly as in Eq. (4), the buyers' beliefs are given by Bayes' rule

$$\begin{aligned}
E_\gamma(u|p^2,s)&:=\frac{m_hf_h(s)}{m_hf_h(s)+r^2m_lf_l(s)}u_h+\frac{r^2m_lf_l(s)}{m_hf_h(s)+r^2m_lf_l(s)}u_l,\\
E_\gamma(u|p^1,s)&:=\frac{r^1m_hf_h(s)}{r^1m_hf_h(s)+m_lf_l(s)}u_h+\frac{m_lf_l(s)}{r^1m_hf_h(s)+m_lf_l(s)}u_l,
\end{aligned}$$

where r^2 is the probability of pooling for low quality sellers in case (i) and r^1 is the probability of pooling for high quality sellers in case (ii). A pooling price p^i has to lie within $\left[c_\theta+\delta V_\theta,E_\gamma(u|p^i,s)-\delta V_b\right]$ for $\theta=l,h$ and for $i=1,2$. Note that, the first type (i) of semi-pooling equilibrium exists if

$$\begin{aligned}
u_l&\geq c_l+\delta V_l+\delta V_b\\
u_h&>c_h+\delta V_h+\delta V_b.
\end{aligned}$$

The first condition is needed to induce low quality sellers to separate and the second one to make pooling possible for some mixing rate $r^1>0$. The second type (ii) of semi-pooling equilibrium exists if

$$\begin{aligned}
E_\gamma(u|s)&>c_h+\delta V_h+\delta V_b\\
E_\gamma(u|s)&>c_l+\delta V_l+\delta V_b.
\end{aligned}$$

The first condition is necessary to make sure that high quality sellers are willing to pool (and thus also separate) and the second one guarantees that also for low quality sellers for some mixing rate $r^2>0$.

We can show that the separating prices are unique for both low quality sellers and high quality sellers by the same logic as for the separating equilibrium: Case (i): $p^2 = E(u|p^2, s) - \delta V_b$ and $\sigma_b^\star(p^2, s) < 1$ (to keep the buyers mixing and to prevent the low quality sellers from mimicking the high quality sellers) and $p_l = u_l - \delta V_b$ (otherwise, there is a profitable deviation to $p_l + \epsilon$ for a higher price) and $\sigma_b^\star(p_l, s) = 1$ (otherwise, there is a profitable deviation to $p_l - \epsilon$ for a higher acceptance rate). Case (ii): $p_h = u_l - \delta V_b$ and $\sigma_b^\star(p_h, s) < 1$ with $(1 - \sigma_b^\star(p^1, s))(E(u|p^1, s) - p^1 - \delta V_b) = 0$ (if p^1 is not accepted for sure is must not leave the buyer any surplus; if it however does so, if should be accepted for certain). The constraints on acceptance probabilities $a^1 := \sigma_b^\star(p^1, s)$ and $a^2 := \sigma_b^\star(p^2, s)$ we already derived above. ■

PROOF OF REMARK 4

Cases 1 and 2: These cases are obvious because neither seller is harmed by a higher price or a higher acceptance probability and, at least, one of the sellers benefits from them strictly.

Case 3: As we are looking for a seller maximal equilibrium, to simplify the notation, it is without loss to assume that $V_b = 0$. Otherwise, we just have to subtract V_b from each price p.

Semi-pooling equilibrium of type (i): Suppose that both sellers use a pooling price p and the low quality sellers mix between the pooling price p and a separating price p_l. The maximal pooling price $E_\gamma(u|s, p) \in [E_\gamma(u|s), u_h]$ clearly depends on the ratio r^2 in which the low quality sellers mix. To keep the low quality sellers indifferent between offering the pooling price and the separating price, as $p_l = u_l$ and $\sigma_b^\star(p_l) = 1$, in the best case the acceptance probability of the higher price offer p must be equal to $\sigma_b^\star(p, s) = \frac{u_l - c_l - \delta V_l}{p - c_l - \delta V_l}$.

Observe that, for any suitable γ, $\mathbf{V}$, and s there can exist many such equilibria with different r^2 and p. As low quality sellers are mixing between p_l and p, what they get is constant over all such equilibria. However, as high quality sellers are pooling to p, what they expect to obtain equals

$$V_h(s) = \frac{u_l - \delta V_l}{p - \delta V_l} (p - c_h - \delta V_h) + \delta V_h.$$

This is either maximized by the highest feasible price $p = u_h$ ($r^2 = 0$) if $\frac{p - c_h - \delta V_h}{p - c_l - \delta V_l} < 1$, or by the lowest feasible price $p = E_\gamma(u|s)$ ($r^2 = 1$) if $\frac{p - c_h - \delta V_h}{p - c_l - \delta V_l} > 1$. That is, any semi pooling equilibrium of this type would be defeated either by the best separating equilibrium or by the best pooling equilibrium.

Semi-pooling equilibrium of type (ii): Suppose that both sellers use a pooling price p and the high quality sellers mix between the pooling price p and a separating price p_h. The maximal pooling price $E_\gamma(u|s, p) \in [c_h, E_\gamma(u|s)]$ clearly depends on the ratio r^1 in which the high quality sellers mix. To keep the high quality sellers indifferent between offering the pooling price and the separating price, the acceptance probability of the higher price offer p_h must be equal to $\sigma_b^\star(p_h, s)\,(u_h - c_h - \delta V_h) = \sigma_b^\star(p, s)\,(p - c_h - \delta V_h)$. Observe also that, if $\sigma_b^\star(p_h, s)\,(u_h - c_h - \delta V_h) = \sigma_b^\star(p, s)\,(p - c_h - \delta V_h)$ is satisfied, then

$\sigma_b^\star(p_h, s)\,(u_h - c_l - \delta V_l) \leq \sigma_b^\star(p, s)\,(p - c_l - \delta V_l)$ is satisfied, such that the low quality seller have no incentive to mimic the high quality sellers as long as $c_h + \delta V_h > c_l + \delta V_l$ (we leave the other case for the reader to think about).

Note again that, for any suitable γ, $\mathbf{V}$, and s, there could exist many such equilibria with different r^1 and p. As high quality sellers are mixing between p_l and p and low quality sellers are pooling to p, what they expect to obtain equals

$$V_l(s) = \sigma_b^\star(p, s)\,(p - c_l) + (1 - \sigma_b^\star(p, s))\,\delta V_l,$$

$$V_h(s) = \sigma_b^\star(p, s)\,(p - c_h) + (1 - \sigma_b^\star(p, s))\,\delta V_h.$$

Both of them are maximized by the highest feasible price $p = E_\gamma(u|s)$ ($r^1 = 1$) and the highest feasible acceptance probability $\sigma_b^\star(p, s) = 1$. That is, any semi-pooling equilibrium of this type would be defeated by the best pooling equilibrium.

As semi-pooling equilibria are either as of type (i) or of type (ii), any semi-pooling equilibrium is defeated by either the best pooling equilibrium or by the best separating equlibrium, or the sellers' individual rationality constraints bind.

Cases 4 and 5: As $p(s) := E_\gamma(u|s) > p_l$, the low quality sellers are always better off if they they play the best pooling equilibrium than if they play the best separating equilibrium. This is not always the case for the high quality sellers, however. To see this, denote by $s^h \in (0, 1)$ the (higher) signal that solves

$$p(s) - c_h = \sigma_b^\star(p_h, s)\,(p_h - c_h) + (1 - \sigma_b^\star(p_h, s))\,\delta V_h \text{ where } \sigma_b^\star(p_h, s) = \frac{u_l - c_l - \delta V_l}{u_h - c_l - \delta V_l}$$

and by $s^l \in (0, 1)$ the (lower) signal that solves $p(s) - c_h = \delta V_h$. Since $p(s) := E_\gamma(u|s)$ is larger for larger signals s, for any $s \in \left[s^h, 1\right]$, the high quality sellers are better off if they they play the best pooling equilibrium than if they play the best separating equilibrium, whereas for signals $s \in \left(s^l, s^h\right)$, it is the opposite.

Notice also that by Proposition 1, individual rationality constraints will always bind for sufficiently low signal values. If $p(s) - c_h < \delta V_h$, pooling is not feasible and, $p_l < c_l + \delta V_l$, separation is impossible. In the former case the high quality sellers would rather quit than pool, in the latter case the low quality sellers would rather quit than separate. ■

PROOF OF PROPOSITION 3

For simplicity of exposition, we consider a better market where $g = 0$. It is immediate to extend the analysis to better markets where $1 - \lambda > g > 0$.

We study cases where, for signals above a cutoff s', both sellers make as high a pooling price offer as is feasible

$$p(s, s') := \frac{1}{1 + \frac{1-F_h(s')}{1-F_l(s')}\frac{f_l(s')}{f_h(s')}} u_h - \frac{\frac{1-F_h(s')}{1-F_l(s')}\frac{f_l(s')}{f_h(s')}}{1 + \frac{1-F_h(s')}{1-F_l(s')}\frac{f_l(s')}{f_h(s')}} u_l \text{ for } s \geq s' \in (0, 1),$$

and obtain $p(s, s') - c_\theta$ whereas, for signals below the cutoff s', they quit and hence receive their outside options δV_θ.

Note that while we do not initially pay attention to the fact that the sellers may not have no incentive to do so but, ultimately, we are interested to find a pair $(\lambda, s'(\lambda))$ of exogenous λ and endogenous s' for which this would constitute a seller maximal equilibrium: (i) IR-h should be satisfied as an equality for a pooling price offer $p(s)$ at s'

$$p(s',s') - c_h = \delta V_h = \frac{\delta \int_{s'}^1 (p(s',s) - c_h)\, dF_h(s)}{1 - \delta F_h(s')}$$
$$\iff$$
$$p(s',s') - \lambda = \delta V_h = \frac{\delta \int_{s'}^1 (p(s',s) - \lambda)\, dF_h(s)}{1 - \delta F_h(s')} \tag{13}$$

and (ii) IR-l should not be satisfied for the low separating price offer p_l at s'

$$u_l - c_l < \delta V_l = \frac{\delta \int_{s'}^1 (p(s',s) - c_l)\, dF_l(s)}{1 - \delta F_l(s')} \iff \lambda < \frac{\delta V_l = \delta \int_{s'}^1 (p(s',s))\, dF_l(s)}{1 - \delta F_h(s')}. \tag{14}$$

For later use, note that (13) can also be rewritten as

$$p(s',s') - \underbrace{\frac{1-\delta}{1 - \delta F_h(s')}}_{:=M(s')>1} \lambda = \underbrace{\frac{\delta \int_{s'}^1 p(s',s) dF_h(s)}{1 - \delta F_h(s')}}_{=:\delta W_l(s')}.$$

This is applied to show the monotonicity of $\overline{s}$ and $\underline{s}$.

We also need to define a new mapping

$$g^5(s') := p(s',s') - \lambda - \delta V_h(s').$$

for auxiliary purposes. Note that, g^5 is continuous in s' for the intermediate range of signal values $s' \in (0,1)$. In addition, by what we have assumed for Proposition 3, its limit is positive for $s' \to 1$

$$g^5(s') \to a - \delta \underbrace{V_h(1)}_{=0}$$

and negative as $s' \to 0$

$$g^5(s') \to -\lambda - \delta \underbrace{V_h(0)}_{>0}.$$

This entails that for any $\lambda \in \left[0, \overline{\lambda}\right]$ there always exists a fixed point signal, denoted by $s'(\lambda) \in (0,1)$, such that (13) holds.

Observe next that although we present no proof to show that the fixed point is unique, it appears safe to assume that g^5 does not have more than a finite number of roots which satisfy (13). A maximal root and a minimal root therefore exist. They are denoted by $\overline{s}(\lambda)$ and $\underline{s}(\lambda)$, respectively. We next show that both $\overline{s}(\lambda) \in (0,1)$ and $\underline{s}(\lambda) \in (0,1)$ are increasing in λ. This can be seen from the following

$$g^5(\overline{s}(\lambda),\lambda) = p(\overline{s}(\lambda)),\overline{s}(\lambda)) - M(\overline{s}(\lambda))\lambda - \delta W_h(\overline{s}(\lambda)) = 0$$
$$\Longrightarrow g^5(\overline{s}(\lambda),\lambda+\varepsilon) = p(\overline{s}(\lambda),\overline{s}(\lambda)) - M(\overline{s}(\lambda))(\lambda+\varepsilon) - \delta W_h(\overline{s}(\lambda)) < 0, \tag{15}$$

and

$$g^5(\underline{s}(\lambda),\lambda) = p(\underline{s}(\lambda),\underline{s}(\lambda)) - M(\underline{s}(\lambda))\lambda - \delta W_h(\underline{s}(\lambda)) = 0$$
$$\Longrightarrow g^5(\underline{s}(\lambda),\lambda-\varepsilon) = p(\underline{s}(\lambda),\underline{s}(\lambda)) - M(\underline{s}(\lambda))(\lambda-\varepsilon) - \delta W_h(\underline{s}(\lambda)) > 0. \tag{16}$$

It is also apparent from (15) that, as $g^5(1,\lambda+\varepsilon) > 0$, the largest root for $\lambda+\varepsilon$ must lie between $\overline{s}(\lambda)$ and 1,

$$\overline{s}(\lambda) < \overline{s}(\lambda+\varepsilon),$$

Likewise, (16) makes it clear that, as $g^5(0,\lambda-\varepsilon) < 0$, the smallest root for $\lambda-\varepsilon$ must lie between 0 and $\underline{s}(\lambda)$,

$$\underline{s}(\lambda) > \underline{s}(\lambda-\varepsilon).$$

Altogether this implies that all those pairs (λ, s') that we take an interest in lie in $\left[0,\overline{\lambda}\right] \times \left[\underline{s}(0),\overline{s}(\overline{\lambda})\right]$.

As the next step, we define two sets for each arbitrary $\overline{\lambda} \in (0,1)$ where the first set is included in the second one

$$S(\overline{\lambda}) = \left\{(\lambda,s')|\lambda \in \left[0,\overline{\lambda}\right], s' = s'(\lambda)\right\} \subset \overline{S}(\overline{\lambda}) = \left\{(\lambda,s')|(\lambda,s') \in \left[0,\overline{\lambda}\right] \times \left[\underline{s}(0),\overline{s}(\overline{\lambda})\right]\right\}.$$

For the two related minimization problems, the value of the latter is thereby bounded by the value of the former as

$$\min_{(\lambda,s')\in S(\overline{\lambda})} V_l(\lambda,s') \geq \min_{(\lambda,s')\in \overline{S}(\overline{\lambda})} V_l(\lambda,s') =: \underline{V}_l.$$

As the last step, we then note that the value $\underline{V}_l$ is well-defined as the function V_l is continuous in (λ, s') and the set $\overline{S}(\overline{\lambda})$ is compact. The value $\underline{V}_l$ is also positive as $V_l(\lambda, s')$ is positive for any pair $(\lambda, s') \in \overline{S}(\overline{\lambda})$. As a result, we have a positive minimum for low quality seller values in this equilibrium type; in particular, the low quality seller values do not get smaller and smaller as λ does. This permits us to conclude that both (13) and (14) are satisfied for $\lambda < \delta\underline{V}_l$. ■

References

George A Akerlof. The market for "lemons": Quality uncertainty and the market mechanism. *The quarterly journal of economics*, pages 488–500, 1970.

Paul Asabere and Forrest Huffman. Price concessions, time on the market, and the

actual sale price of homes. *The Journal of Real Estate Finance and Economics*, 6(2):167–174, 1993.

Max R Blouin. Equilibrium in a decentralized market with adverse selection. *Economic Theory*, 22:245–262, 2003.

Braz Camargo and Benjamin Lester. Trading dynamics in decentralized markets with adverse selection. *Journal of Economic Theory*, 153:534–568, 2014.

In-Koo Cho and David M Kreps. Signaling games and stable equilibria. *The Quarterly Journal of Economics*, 102(2):179–221, 1987.

In-Koo Cho and Akihiko Matsui. Search, adverse selection and market clearing. *Manuscript*, 2015.

Brendan Daley and Brett Green. Waiting for news in the market for lemons. *Econometrica*, 80:1433–1504, 2011.

Peter Diamond. A model of price adjustment. *Journal of economic theory*, 3(2): 156–168, 1971.

Veronica Guerrieri, Robert Shimer, and Randall Wright. Adverse selection in competitive search equilibrium. *Econometrica*, 78(6):1823–1862, 2010.

Daniel Hamermesh, Xin Meng, and Junsen Zhang. Dress for success - does primping pay? *Labour Economics*, 9(3):361–373, 2002.

John C Harsanyi. Games with randomly disturbed payoffs: A new rationale for mixed-strategy equilibrium points. *International Journal of Game Theory*, 2(1): 1–23, 1973.

James Heckman and George Borjas. Does unemployment cause future unemployment? definitions, questions and answers from a continuous time model of heterogeneity and state dependence. *Economica*, 47(187):247–283, 1980.

James Heckman and Burton Singer. A method for minimizing the impact of distributional assumptions in econometric models for duration data. *Econometrica*, pages 271–320, 1984.

Jack Hirshleifer. The private and social value of information and the reward to inventive activity. *The American Economic Review*, pages 561–574, 1971.

Roman Inderst. Contractual signaling in a market environment. *Games and Economic Behavior*, 40(1):77–98, July 2002.

Roman Inderst and Holger M Muller. Competitive search markets for durable goods. *Economic Theory*, 19:599–622, 2002.

Maarten Janssen and Santanu Roy. Dynamic trading in a durable good market with asymmetric information. *International Economic Review*, 43(1):257–282, 2002.

Maarten Janssen and Santanu Roy. On durable goods markets with entry and adverse selection. *The Canadian Journal of Economics / Revue canadienne d'Economique*, 37(3):552–589, 2004.

Ayca Kaya and Kyungmin Kim. Trading dynamics with private buyer signals in the market for lemons. *Manuscript*, 2015.

Kyungmin Kim. Endogenous market segmentation for lemons. *The RAND Journal of Economics*, 43(3):562–576, 2012.

Klaus Kultti, Eeva Mauring, Juuso Vanhala, and Timo Vesala. Adverse selection in dynamic matching markets. *Bulletin of Economic Research*, 2012.

Stephan Lauermann and Asher Wolinsky. Search with adverse selection. *Econometrica*, 84(1):243–315, 2016.

George J Mailath, Masahiro Okuno-Fujiwara, and Andrew Postlewaite. Belief-based refinements in signalling games. *Journal of Economic Theory*, 60(2):241–276, 1993.

Markus Mobius and Tanya Rosenblat. Why beauty matters. *The American Economic Review*, pages 222–235, 2006.

Diego Moreno and John Wooders. Decentralized trade mitigates the lemons problem. *International Economic Review*, 51(2):383–399, 2010.

Diego Moreno and John Wooders. Dynamic markets for lemons: performance, liquidity, and policy intervention. *Forthcoming: Theoretical Economics*, 2016.

Georg Nöldeke and Larry Samuelson. A dynamic model of equilibrium selection in signaling markets. *Journal of Economic Theory*, 73(1):118–156, 1997.

Curtis R Taylor. Time-on-the-market as a sign of quality. *Review of Economic Studies*, 66(3):555–578, 1999.

Haoxiang Zhu. Finding a good price in opaque over-the-counter markets. *Review of Financial Studies*, 25(4):1255–1285, 2012.