The Effect of Labour Tax Progression under Nash Wage Bargaining and Flexible Outsourcing

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Abstract

This paper studies in the presence of flexible outsourcing the effects of outsourcing costs, productivity of outsourcing, wage tax and tax exemption in an imperfectly competitive labour markets when labour unions and firms negotiate wages and the impacts of labour tax progression on domestic wage setting and employment. The wage elasticity of domestic labour demand is higher than in the case of strategic outsourcing and a decreasing function of the outsourcing cost, an increasing function both of the productivity of outsourcing and of the wage rate. With sufficiently strong (weak) labour market imperfections a lower outsourcing cost has a wage-moderating (wage-increasing) effect. Finally, increasing the degree to tax progression, to keep the relative tax burden per worker constant, has a wage-moderating effect and a positive effect on domestic employment and a negative effect on outsourcing.

JEL Classification: H22, J41, J51.

Keywords: Outsourcing, wage negotiation, labour tax progression, employment.

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I. Introduction

High wage differences across countries constitute an important explanation for the currently significant business practice of international outsourcing (see e.g. Sinn (2007) for an overview and Stefanova (2006) concerning the East-West dichotomy of outsourcing). How governments can control when their countries are exposed to increasing international integration of their economies? This paper provides some answers to this question for the case of labour market tax reform in the presence of outsourcing when domestic labour markets are imperfectly competitive. It is assumed in this paper that firms are flexible enough to decide upon the amount of outsourcing activity simultaneously concerning domestic labour demand after the wage rate have been negotiated by labour unions and firms.

This paper designs a model to answer the following questions: What are the effects of outsourcing costs, productivity of outsourcing and domestic wage rate on the wage elasticity of labour demand in the case of flexible outsourcing? What are the effects of outsourcing costs and substitutability between outsourcing and domestic labour and wage tax and tax exemption on wage formation in an imperfectly competitive labour markets when labour unions and firms negotiate wages? Finally, what are the effects of labour tax reform on domestic wage setting and employment under flexible outsourcing? In this analysis the fully-balanced public sector budget aspect is not considered, because only some sector may engage outsourcing, but not the whole economy.

It is shown that in the presence of flexible outsourcing the wage elasticity of domestic labour demand is higher than in the case of strategic outsourcing. It is a decreasing function of the outsourcing cost, and an increasing function both of the degree of substitutability between domestic labour and outsourcing and of the wage rate of domestic labour. With sufficiently strong (weak) labour market imperfections

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1 Amiti and Wei (2004) as well as Rishi and Saxena (2004) emphasize to the big difference on labour costs as the main explanation for the strong increase in outsourcing of both manufacturing and services to countries with low labour costs.

2 This issue have been analyzed in the absence of outsourcing e.g. Koskela and Vilmunen (1996), and Koskela and Schöb (1999), (2002a), (2002b)
a lower outsourcing cost has a wage-moderating (wage-increasing) effect. With a
monopoly labour union, a lower outsourcing cost moderates wages and in the absence
of labour market imperfections there is no relationship between outsourcing cost and
wage formation. In the presence of flexible outsourcing the wage tax has a positive
effect and the tax exemption a negative effect on wage negotiation. In the presence of
flexible outsourcing increasing the degree of tax progression under Nash wage
bargaining, to keep the relative tax burden per worker constant, has a wage-
moderating effect and a positive effect of domestic employment and a negative effect
on outsourcing. In this case qualitative results on wage formation and domestic
employment are similar in the absence of outsourcing.

This paper proceeds as follows. Section II presents the basic structure of
theoretical framework as well as time sequence of decisions in terms of wage
formation, outsourcing and domestic labour demand. Domestic labour demand and
outsourcing are studied in section III, whereas the focus on wage determination
through Nash bargaining in the presence of linearly progressive wage tax is in section
IV. The effects of labour tax progression on domestic wage setting and employment
are analyzed in section V. Finally, conclusions are presented in section VI.

II. Basic Framework

In this paper the focus is to study the effects of wage tax policy on wage negotiation
and domestic labour demand when outsourcing is flexible to decide simultaneously
both domestic labour demand and outsourcing after the wage negotiation is set by the
labour union and the firm. The time sequence is described in Figure 1.

Figure 1: Time sequence of decisions
At stage 1 the government behaves as a Stackelberg leader and fixes two labour tax parameters. To raise revenues the government can employ a wage tax \( t \), which is levied on the wage \( w \), minus a tax exemption \( a \). The tax base per worker for the wage tax \( t \) equals \((w-a)\). In the presence of a positive tax exemption \( a \), the marginal tax rate \( t \) exceeds the average tax rate \( t(1-a/w) \) so that the tax system is linearly progressive.\(^3\) The net-of-tax wage, the worker receives, is given by \( w^* = w(1-t)w + ta \).

At stage 2 the labour union and the firm negotiate wage formation using the Nash bargaining approach by taking tax parameters as given and anticipating the consequences that wage setting will have for the domestic labour demand and outsourcing.

At stage 3 both the domestic labour demand and outsourcing is decided simultaneously by the firm by taking tax parameters and wage setting as given. Skaksen (2004) has analyzed this timing structure in the absence of labour taxation by assuming the firms could decide outsourcing after the determination of domestic wage. Also Brown and Scheffel (2007) have developed in the absence of labour taxation a simple two-stage game between a monopoly union and a firm by assuming that the union sets wages before the firm decides on the degree of outsourcing and the level of production.

To derive an explicit solution a decreasing returns to scale production function is presented as

\[
R(L, M) = \frac{\delta}{\delta - 1} \left( L + \gamma M \right)^{\frac{\delta - 1}{\delta}}, \quad \delta > 1
\]

where \( L \) is the amount of labour employed in-house and \( M \) denotes the firm’s labour input acquired from external suppliers through outsourcing. The parameter \( \gamma > 0 \) captures the productivity of outsourcing relative to the domestic labour input. In the case of perfect substitutability we have \( \gamma = 1 \).

\(^3\) For a seminal paper about tax progression, see Musgrave and Thin (1948), and for another elaboration, see e.g. Lambert (2001, chapters 7-8).
The analysis starts with an investigation of domestic labour demand and outsourcing and in the subsequent sections it is characterized wage bargaining and tax policies by applying backward induction.

III. Domestic Labour Demand and Outsourcing

In the case of flexible outsourcing the firm decides simultaneously on domestic in-house employment $L$ and outsourcing $M$, where the costs of outsourcing are convex, so as to maximize the profit function when the price of the output good is normalized to unity

$$\max_{L,M} \pi = \frac{\delta}{\delta-1} (L + \gamma M)^{\frac{\delta-1}{\delta}} - wL - \frac{1}{2}cM^2, \quad (2)$$

by taking both the negotiated wage and the cost of outsourcing as given. The first-order conditions are

$$\pi_L = \left( L + \gamma M \right)^{\frac{1}{\delta}} - w = 0, \quad (3a)$$
$$\pi_M = \left( L + \gamma M \right)^{\frac{1}{\delta}} \gamma - cM = 0. \quad (3b)$$

These give the following domestic labour demand and outsourcing

$$L = w^{-\delta} - \gamma M = w^{-\delta} - \gamma^2 \frac{w}{c}, \quad (4a)$$
$$M = \gamma \frac{w}{c}. \quad (4b)$$

Domestic labour demand is a negative function of the wage rate and the productivity of outsourcing, and a positive function of the cost of outsourcing, while outsourcing is a positive function of wage rate and productivity of outsourcing and a negative
function of the cost of outsourcing. This means that higher outsourcing will decrease domestic labour demand.\footnote{See e.g. Görg and Hanley (2005) and Hijzen et al. (2005) for evidence based on various data sets.}

The wage elasticity of labour demand under flexible outsourcing, which turns out to be important later on, can be expressed as

\[
\eta^f (w, c, \gamma) = -\frac{L_0 w}{L} = \frac{\delta w^s + \gamma^2 w}{L} = \delta (1 + \gamma \frac{M^*}{L^*}) + \gamma \frac{M^*}{L^*} = \delta + (1 + \delta) \gamma \frac{M^*}{L^*}. \quad (5)
\]

The wage elasticity of labour demand is higher than in the case of strategic outsourcing when outsourcing is determined by firms before wage negotiation, i.e. \( \eta^f > \eta^r = \delta (1 + \gamma \frac{M}{L}) \) (= the wage elasticity of labour demand under strategic outsourcing\footnote{This case has been analyzed in Koskela and Stenbacka (2008) by studying the impact of strategic outsourcing on equilibrium unemployment.}), and it depends on parameters \( \gamma \) and \( w \) as well as the cost of outsourcing \( c \) via the share of outsourced production \( M/L \). The outsourcing elasticities are constant and equal to one, i.e. \( \frac{M_0 w}{M} = \frac{M \gamma}{M} = -\frac{M c}{M} = 1. \)

The relationship between the wage rate and the wage elasticity of domestic labour demand is positive

\[
\eta_w = (1 + \delta) \gamma \left( \frac{LM_c - ML_c}{L^2} \right) = (1 + \delta) \gamma \frac{M_c}{L} \left( \frac{M_0 w}{M} - \frac{L_0 w}{L} \right) = (1 + \eta^f)(1 + \delta) \gamma \frac{M^*}{wL^*} > 0 , \quad (6)
\]

and the relationship between the outsourcing cost and the wage elasticity of domestic labour demand is negative

\[
\eta_c = (1 + \delta) \gamma \left( \frac{LM_c - ML_c}{L^2} \right) = (1 + \delta) \gamma \frac{M_c}{cL} \left( \frac{M_0 c}{M} - \frac{L_0 c}{L} \right) = -(1 + \delta) \gamma \frac{M^*}{cL} \left( 1 + \gamma \frac{M^*}{L^*} \right) < 0 . \quad (7)
\]
According to this higher outsourcing due to lower outsourcing cost will increase the wage elasticity of domestic labour demand, which lies in conformity with empirical evidence from various data sets (see e.g. Slaughter (2001), Senses (2006) and Hasan et al. (2007)). Also the wage elasticity depends positively on the productivity of outsourcing, i.e.

\[
\eta_\gamma = (1 + \delta) \left[ \frac{M}{L} + \gamma \frac{LM\gamma - ML\gamma}{L^2} \right] = (1 + \delta) \left[ \frac{M}{L} + \frac{M}{L} \left( \frac{M\gamma - L\gamma}{M} \right) \right]
\]

\[
= 2(1 + \delta) \frac{M^*}{L} \left(1 + \gamma \frac{M^*}{L} \right) > 0.
\]

These results can be presented in

**Proposition 1:** In the presence of flexible outsourcing
(a) the wage elasticity of domestic labour demand is higher than in the case of strategic outsourcing and
(b) it is a decreasing function of the outsourcing cost, an increasing function both of the productivity of outsourcing and the wage rate of domestic labour.

**IV. Wage Determination via Nash Bargaining under Linearly Progressive Wage Tax**

We now proceed to investigate wage determination by applying the Nash bargaining solution following the right-to-manage (RTM) approach so that wage negotiation takes place in anticipation of optimal labour and outsourcing decisions by the firm (see e.g. Cahuc and Zylberberg (2004), chapter 7). In the presence of a positive tax exemption the marginal tax rate exceeds the average tax rate \( t(1-a/w) \) and the net-of-tax wage is \( w(1-t)+ta \). The labour union’s objective function in the presence of linearly progressive wage taxation is assumed to be

\[
\hat{U} = (w(1-t)+ta)L'L + b(N-L'), \tag{8}
\]

where the tax base for the wage tax \( t \) equals
\((w-a)L^*\), and \(L^*\) denotes the total domestic employment and there is a positive tax exemption \(a\). \(b\) is the (exogenous) outside option available to union members and \(N\) is the number of union members \(\(N \geq L^*\)\) and the threat point is \(U^o = Nb\) so that the relevant target function of the labour union is \(U = \hat{U} - Nb = L^*(w(1-t) + ta - b)\).

The indirect profit function by substituting the optimal domestic labour demand (4a) and the optimal outsourcing (4b) into the profit function (2) can be written after the calculations as follows:

\[
\pi^* = \frac{w^{1-\delta}}{\delta - 1} + \frac{\gamma^2 w^2}{2c} = \frac{w^{1-\delta}}{\delta - 1} + \frac{\gamma \omega M^*}{2c}.
\]

Following the Nash bargaining approach the firm and the labour union negotiate with respect to wage rate so as to solve the following optimization problem

\[
\max_{\omega} \Omega = \left[ L^*(w(1-t) + ta - b) \right]^\beta \left[ R(L^*, M^*) - wL^* - \frac{1}{2} M^* \right]^{1-\beta}
\]

s.t. \(L^* = w^{\delta} - \gamma^2 \frac{w}{c}\),

where the relative bargaining power of the labour union is \(\beta\) and that of the firm is \(1 - \beta\). The first-order condition for the negotiated wage rate can be written as

\[
\Omega_w = 0 \iff \beta \frac{U_w}{U} + (1 - \beta) \frac{\pi^*_w}{\pi^*} = 0,
\]

where

\[
\frac{U_w}{U} = \frac{1}{w} \left[ \frac{w(1 - \eta^f(w, \gamma, c)(1-t) + \eta^f(w, \gamma, c)(b-ta))}{w(1-t) - (b-ta)} \right] > 0,
\]

and

\[
\frac{\pi^*_w}{\pi^*} = -\frac{1}{w} \omega L = -\frac{1}{w} \frac{R_L L}{R - R_L M - \frac{1}{2} \omega M} = -\frac{1}{w} \frac{2(\delta - 1)}{2 - \delta + \eta^f}.
\]

(see Appendix A concerning (11b)).
Substituting (11a) and (11b) into the first-order condition (10) gives the following Nash bargaining solution for the negotiated wage (see Appendix A)

\[ w^N = \frac{\beta \eta' (2-\delta + \eta') + (1-\beta)2(\delta-1)}{\beta (\eta'-1)(2-\delta + \eta') + (1-\beta)2(\delta-1)} \hat{b} = A' \hat{b} \tag{12} \]

where \( \hat{b} = \frac{b-t\hat{a}}{1-t} \) and using the notation \( Z = 2 + (1+\delta)\frac{M^*}{L} \) and

\[ \eta' - \delta = (1+\delta)\gamma \frac{M^*}{L} \]

the mark-up \( A' \) can be written as follows

\[ A' = \frac{\beta \eta' (2+(1+\delta)\gamma \frac{M^*}{L}) + (1-\beta)2(\delta-1)}{\beta (\eta'-1)(2+(1+\delta)\gamma \frac{M^*}{L}) + (1-\beta)2(\delta-1)} = \frac{\beta \eta' Z + (1-\beta)2(\delta-1)}{\beta (\eta'-1)Z + (1-\beta)2(\delta-1)} > 1 \] \( \geq \beta > 0 \).

It is important to mention that equation (12) is not an explicit form for the wage rate under outsourcing because the mark-up both in terms of the numerator and the denominator also depends in a non-linear way on the wage ratio via the ratio between outsourcing and domestic labour demand (see equation (4a)). According to (12) the negotiated wage rate depends positively on \( \hat{b} \) and the relative bargaining power of the labour union, and negatively on the wage elasticity of domestic labour demand. In the case of the monopoly labour union under outsourcing we have the following implicit form

\[ w^N \bigg|_{\beta=1} = \frac{ \eta(2-\delta + \eta)}{(\eta-1)(2-\delta + \eta)} \hat{b} = \frac{\eta}{(\eta-1)} \hat{b} \tag{13} \]

In the absence of outsourcing the Nash bargaining solution (12) for the wage rate is explicit, i.e. \( w^N \bigg|_{\beta=0} = \frac{\beta + \delta - 1}{(\delta - 1)} \hat{b} \) as well as in the case of monopoly labour union, i.e.

\[ w^N \bigg|_{\beta=1, M=0} = \frac{\delta}{(\delta - 1)} \hat{b} . \]
By differentiating the negotiated wage (12) with respect to the outsourcing cost \( c \) gives (see Appendix B)

\[
\frac{dw^N}{dc} = \frac{A^f w}{1 - \frac{A^f w}{A^f}}
\]

(14)

where

\[
1 - \frac{A^f w}{A^f} > 0
\]

(15a)

and

\[
\begin{align*}
\frac{A^f w}{A^f} & > 0 \quad \text{as} \quad \beta > 0, \\
& \frac{2\delta(\delta - 1)}{(1 + \delta)(2 + (1 + \delta)\gamma \frac{M^*}{L})^2 + 2\delta(\delta - 1)}.
\end{align*}
\]

(15b)

The mark-up will decrease in the lower outsourcing cost if the relative bargaining powers of labour union is higher than the low threshold determined in (15b). This threshold is inversely related to the wage elasticity. Lower outsourcing cost increases the wage elasticity of domestic labour demand by decreasing the mark-up. This is the dominant effect as long as the labour union has a sufficiently strong bargaining power. Also wage is affected by the negative effect on profit according to (10) and when the labour union has a sufficiently low bargaining power, higher outsourcing due to lower outsourcing cost moderates profit reducing effect of a higher wage. In this case more outsourcing induces an increase in the wage when the bargaining power lies with the firm to a sufficient degree.\(^6\)

Under monopoly union \( A^f \bigg|_{\beta=1} = \frac{\eta^f}{\eta^f - 1} \) so that in this case the lower outsourcing cost will decrease the mark-up, i.e. \( A^f \bigg|_{\beta=1} = \frac{-\eta^f}{(\eta^f - 1)} > 0 \). In the absence of labour market imperfections, this effect is zero.

\(^6\) This has been analyzed in Koskela and Stenbacka (2008) in the presence of strategic outsourcing.
This result can be summarized in.

**Proposition 2:** With sufficiently strong (weak) labour market imperfections a lower outsourcing cost has a wage-moderating (wage-increasing) effect. With a monopoly labour union, a lower outsourcing cost moderates wages and in the absence of labour market imperfections there is no relationship between outsourcing cost and wage formation.

In terms of the wage tax and the tax exemption differentiating (12) gives

\[
\frac{dw}{dt} = \frac{A^t}{\left(1 - \frac{A^t w}{A^t}\right)} \frac{b-a}{(1-t)^2} > 0 \quad \text{as } b-a > 0
\]  

(16a)

\[
\frac{dw}{da} = -\frac{A^t}{\left(1 - \frac{A^t w}{A^t}\right)} \frac{t}{(1-t)} < 0
\]  

(16b)

so that in the presence of flexible outsourcing wage tax has a positive effect and tax exemption a negative effect of wage negotiation.

V. Effects of Labour Tax Progression on Wage Negotiation and Employment

Now the analysis concentrates on the effects of tax progression for wage negotiation and employment by looking as tax reform that increases tax progression while keeping the average tax burden per worker constant, i.e. that

\[
t - \frac{ta}{w} = t^n
\]  

(17)
is constant. The average tax rate progression (ARP) is given by the difference between the marginal tax rate \( t \) and the average tax rate \( t'' \), \( \text{ARP} \equiv t - t'' \). The tax system is progressive if \( \text{ARP} \) is positive and tax progression is increased if the difference increases. Government can raise the degree of tax progression when it increases \( t \) and adjusts \( a \) upwards such that \( t'' \) remains constant. In this analysis the fully-balanced public sector budget aspect is not considered, because only some sector may engage outsourcing, but not the whole economy.

First the analysis focuses the wage effect of this tax reform under Nash domestic wage bargaining between the labour union and the firm.

Differentiating (17) with respect to \( t \), \( a \) and \( w \) to keep it constant gives

\[
da = \frac{(w-a)}{t} dt + \frac{a}{w} dw \quad \text{and the total wage effect is} \quad dw = w_i dt + w_a da \ .
\]

Substituting the RHS of \( da = \frac{(w-a)}{t} dt + \frac{a}{w} dw \) for \( da \) in \( dw = w_i dt + w_a da \) gives

\[
\left. \frac{dw^N}{dt} \right|_{da=0} = \frac{\left( w_i + \frac{(w-a)}{t} w_a \right)}{\left( 1 - \frac{w_a a}{w} \right)} \tag{18}
\]

where \( 1 - \frac{w_a a}{w} > 0 \) according to (16b) and

\[
\left( w_i + \frac{(w-a)}{t} w_a \right) = \frac{A_f}{A'} \left[ b - a - (w-a)(1-t) \right] = - \frac{A_f}{A'} \left[ b - a \right] \frac{w(1-t) + ta - b}{(1-t)^2} < 0 \tag{19}
\]

so that \( \left. \frac{dw^N}{dt} \right|_{da=0} < 0 \). A higher degree of tax progression under Nash wage bargaining, keeping the relative tax burden per worker constant, will decrease the wage rate in the presence of flexible outsourcing. This also happens in the absence of outsourcing. The employment and outsourcing effects of this tax reform is by using equations (4a), (4b) and (18) are
\[
\frac{dL}{dt} \bigg|_{\Delta r=0} = L^* \frac{dw^N}{dt} \bigg|_{\Delta r=0} > 0 \tag{20a}
\]
\[
\frac{dM}{dt} \bigg|_{\Delta r=0} = M^* \frac{dw^N}{dt} \bigg|_{\Delta r=0} < 0 \tag{20b}
\]

The wage moderating effect of tax progression, to keep the relative tax burden per worker constant, increases domestic labour demand and decreases outsourcing in the presence of flexible outsourcing. This also happens in the absence of outsourcing, which has been analyzed in the earlier literature, which has been mentioned in introduction.

These results can be summarized in.

**Proposition 3:** In the presence of flexible outsourcing increasing the degree of tax progression under Nash wage bargaining, to keep the relative tax burden per worker constant, has

(a) a wage-moderating effect, a positive domestic employment and a negative effect on outsourcing, and

(b) qualitative results on wage formation and domestic employment are similar in the absence of outsourcing.

**VI. Conclusions**

This paper has presented: What are the effects of outsourcing costs, productivity of outsourcing and domestic labour and domestic wage rate on the wage elasticity of labour demand in the case of flexible outsourcing? What are the effects of outsourcing costs and substitutability between outsourcing and domestic labour and wage tax and tax exemption on wage formation in an imperfectly competitive labour markets when labour unions and firms negotiate wages? What are the effects of labour tax reform on domestic wage setting and domestic employment as well as on outsourcing under flexible outsourcing.
It has been shown that in the presence of flexible outsourcing the wage elasticity of domestic labour demand is higher than in the case of strategic outsourcing and a decreasing function of the outsourcing cost, and an increasing function both of the productivity of outsourcing and the wage rate of domestic labour. With sufficiently strong (weak) labour market imperfections a lower outsourcing cost has a wage-moderating (wage-increasing) effect. In the presence of flexible outsourcing wage tax has a positive effect and tax exemption a negative effect of wage negotiation. In the presence of flexible outsourcing increasing the degree of tax progression under Nash wage bargaining, to keep the relative tax burden per worker constant, has a wage-moderating effect and a positive effect of domestic employment and a negative effect on outsourcing. In this case qualitative results on wage formation and domestic employment are similar in the absence of outsourcing.

References:


**Appendix A: Nash bargaining solution**

Taking labour demand (4a) and outsourcing (4b) into account we find that
\[
\frac{\pi^*_w}{\pi^*} = -\frac{1}{w} \frac{wL}{\pi} = -\frac{1}{w} \frac{R_L L}{R_R L - \frac{1}{2} R_M M} \\
= -\frac{1}{w} \frac{(L + \gamma M)^{1/\delta} L}{\left[\frac{\delta}{\delta - 1} (L + \gamma M)^{1/\delta} - (L + \gamma M)^{1/\delta} L - \frac{1}{2} \gamma M (L + \gamma M)^{1/\delta}\right]} = -\frac{1}{w} \frac{\delta - 1}{1 + \frac{(1 + \delta)}{2} \frac{\gamma M}{L}},
\]

which gives (11b). Substituting (11a) and (11b) into the first-order condition (10) gives
\[
(\beta w(1-\eta^f)(1-t) + \beta(b-ta)\eta^f)(2-\delta + \eta^f) = (1-\beta)(w(1-t)-(b-ta))2(\delta - 1)
\]
and its solution implies the Nash bargaining solution (12). QED.

**Appendix B:**

By implicit differentiation of (12) with respect to the wage rate and outsourcing cost gives
\[
\frac{dw}{dc} = \frac{A^f \hat{b}}{1 - A^f \hat{b}}
\]
and substituting \( \hat{b} = \frac{w}{A^f} \) from (12) for \( \hat{b} \) gives
\[
\frac{dw}{dc} = \frac{A^f w}{A^f} \frac{A^f}{1 - A^f w}.
\]
Using the notation \( X = \beta(\eta^f - 1)Z + (1 - \beta)2(\delta - 1) \), where \( Z = 2 + (1 + \delta)\gamma M^* / L \),

differentiating the mark-up \( A^f = \frac{X + \beta Z}{X} \) from (12) with respect to outsourcing cost \( c \) gives
\[
A^f = X \left( \beta Z \eta^f_c + \beta \eta^f (Z_c) - (X + \beta Z) (\beta Z \eta^f_c + \beta(\eta^f - 1)Z_c) \right)
X^2
\]
\[
= -\beta^2 Z^2 \eta^f_c + (\beta X - \beta^2 Z(\eta^f - 1))Z_c X^2
\]
\[
= \frac{1}{X^2} \beta^2 Z^2 \eta^f_c + \beta(1 - \beta)2(\delta - 1)Z_c X^2
\]

(B1)
By using \( Z_c = \delta \gamma \frac{M_L}{L}(M_c - \frac{L_c}{L}) = -\delta \gamma \frac{M^*}{L}(1 + \gamma \frac{M^*}{L}) = \frac{\delta}{1 + \delta} \eta^f < 0 \) (B2) can be written as

\[
A^f_c = \frac{-\beta \eta^f}{1 + \delta} \left[ \beta \left( Z^2(1 + \delta) + 2\delta(\delta - 1) \right) - 2\delta(\delta - 1) \right] \frac{1}{X^2} \tag{B2'}
\]

The effect of outsourcing cost on the mark-up under Nash wage bargaining depends on the relative bargaining power of the labour union as

\[
A^f_c > 0 \quad \text{as} \quad \beta > \frac{2\delta(\delta - 1)}{(1 + \delta)(2 + (1 + \delta)\gamma \frac{M^*}{L})^2 + 2\delta(\delta - 1)} \tag{B3}
\]

so that

\[
\frac{A^f_w}{A^f_c} > 0 \quad \text{as} \quad \beta > \frac{2\delta(\delta - 1)}{(1 + \delta)(2 + (1 + \delta)\gamma \frac{M^*}{L})^2 + 2\delta(\delta - 1)} \tag{B4}
\]

Differentiating the mark-up with respect to the wage gives by using \( Z_w = \frac{\delta}{1 + \delta} \eta^f w < 0 \)

\[
A^f_w = \frac{X(\beta Z \eta^f Z_w + \beta \eta^f Z_w + (\beta Z \eta^f + \beta (\eta^f - 1) Z_w))}{X^2} \tag{B5}
\]

\[
= \frac{-\beta \eta^f}{1 + \delta} \left[ \beta \left( Z^2(1 + \delta) + 2\delta(\delta - 1) \right) - 2\delta(\delta - 1) \right] \frac{1}{X^2}
\]

so that the effect of the wage rate on the mark-up is

\[
A^f_w < 0 \quad \text{as} \quad \beta > \frac{2\delta(\delta - 1)}{(1 + \delta)(2 + (1 + \delta)\gamma \frac{M^*}{L})^2 + 2\delta(\delta - 1)} \tag{B6}
\]

By using (B3) and (B5) the equation (B1) can be expressed as follows
\[
\frac{d\omega}{dc} = \left( 1 - \frac{A'_w}{A} \right) = \frac{\beta \eta \omega \left( (1+\delta)Z^2 + 2\delta(\delta-1) \right)}{(1+\delta)X(X+\beta Z) + \beta \eta \omega \left( (1+\delta)Z^2 + 2\delta(\delta-1) \right)}
\]

where the denominator is positive so that we have the conclusion

\[
\frac{d\omega}{dc} \begin{cases} > 0 & \text{as } \beta \begin{cases} > \frac{2\delta(\delta-1)}{(1+\delta)(2+(1+\delta)\gamma \frac{M^*}{L})^2 + 2\delta(\delta-1)} & \text{. } \text{QED.} \end{cases} \end{cases}
\]