Evergreening in Banking

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Abstract

In the dynamic model of banking, a bank’s option to hide its loan losses by rolling over non-performing loans is shown to worsen moral hazard. Contrary to the classic theory, moral hazard may arise even when a bank cannot seek a correlated risk for its loans. The loans seem to be performing and the bank makes a profit although it is de facto insolvent. When the bank’s balance sheet includes hidden non-performing loans, the bank may optimally shrink lending or gamble for resurrection by growing aggressively. To eliminate this type of moral hazard, which is broadly consistent with evidence from emerging economies, a few regulatory implications are suggested.

JEL Classification: G21, G22, G28

Keywords: Banking Crises, Bank Regulation, Bank Failures, Deposit Insurance, Dynamic Moral Hazard, Gambling for Resurrection

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1. Introduction

Banking crises have become extremely frequent and widespread. Since 1980, over 1930 countries, comprising almost three fourths of the International Monetary Fund’s member countries, have undergone severe banking problems (Lingren et al., 1996). Most of all, banking crises have hit emerging economies. In Argentina and Chile, for instance, damages were more than or equal to 25% of GDP (Goldstein & Turner, 1996). This paper explores banking crises when banks can hide their loan losses by rolling over these loans. Although the model is most consistent with banking crises in emerging economies, the results can be generalized to developed countries.

The assumption of banks’ ability to hide their loan losses is clearly supported by evidence. Most of the evidence comes from the emerging economies with relatively weak auditing systems and poor bank transparency. In Russia, for example, the true financial condition of banks is unclear due to fallacious accounting data. The data is impaired by hidden loan losses. “Bad loan problems were substantial although in many cases hidden by repeated rollovers of loans with capitalisation of interest (OECD, 2001, p. 161).” Similar conclusions are drawn by Hansson & Tombak’s (1999, p.217) regarding the Baltic economies.

The ability of banks to roll over problem loans concealed their true solvency and created a false picture of health. Bank profits and thus net worth were overstated. When the hidden problems finally emerged, especially through improved accounting and auditing, the resulting erosion of profits and capital was unexpected.

Rojas-Suarez & Weisbrod (1996) and de Juan (1996, 2003) document abundant evidence from Latin America. “In fact, several banking crises in Latin America have been preceded by capitalization of unpaid interest into new loans to make non-performing loans look healthy (Rojas-Suarez & Weisbrod, 1996, p.18).”

Banks generally hide their loan losses also in developed economies. In Japan, the total volume of non-performing loans in all deposit-taking institutions amounted to ¥25 trillion in 1998 based on old accounting standards. Yet, the introduction of novel, more stringent, standards lifted the burden of non-performing loans to ¥35 trillion (Levy, 1998). Diverse methods were exploited in

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1 Tompson (2000, p. 616) brings forward an example on the poor accounting information of the Russian banks “The results reported by Inkombank for 1998 show a healthy profit of $422 million on RAS (Russian accounting standard) but a loss of $355 million on IAS (International accounting standard).” In reality, Inkombank was clearly insolvent.
hiding. “By the 1990s, Japanese banks were reportedly restructuring nonviable loans by reducing interest rates and extending their maturity. Banks also often recapitalized unpaid interest and opened new credit lines so that borrowers could repay overdue loans (Kanaya & Woo, 2001, p.14).” For novel empirical evidence from Japan and U.S.A, see Peek & Rosengren (2005) and Gunther & Moore (2003).

To explore how the option to hide loan losses affects on the banks’ risk taking and banking crises, this paper develops a dynamic model of financial intermediation. As in Holmström & Tirole (1997), the task of the bank is to monitor its borrowers. Yet, deposit insurance and limited liability tempt the bank to neglect costly monitoring. The main novelty in the analysis is that non-monitoring strategy may be optimal to a bank although it cannot seek a correlated risk for its loans, since the bank can hide its loan losses via loan rollovers. The loans seem to be performing and the bank makes a profit although it is de facto insolvent.

When does *ex ante* moral hazard occur in the model? Moral hazard occurs when the costs of monitoring borrowers are high for banks. Additionally, moral hazard occurs when a bank regulator’s auditing system is weak and bank transparency is poor. In contrast to standard moral hazard, e.g. Holmström & Tirole (1997), the audits are not targeted to evaluate credit risk of performing loans. Instead, the regulator audits banks in order to uncover hidden loan losses. By eliminating the hiding, the regulator can eliminate moral hazard. If the regulator’s auditing system is weak, the bank insolvency is uncovered only by illiquidity: bank’s loan interest income is inadequate for payments on deposits. Illiquidity appears when the share of hidden loan losses on the bank’s balance sheet is sufficiently large.

The model is also utilized to explore *ex post* moral hazard. How does a bank react when it already possesses hidden loan losses? Maybe surprisingly, an optimizing bank may react in two opposite ways.

1. A bank may grow aggressively in order to gamble for resurrection (see Kane, 1989). Rapid growth pushes up the share of fresh, performing loans on the balance sheet, thereby increasing the loan interest income and making it possible to avoid bankruptcy.

2. A bank may shrink the scale of its lending. Given an equity ratio requirement, it can lower the amount of equity by shrinking lending. The “excess equity” can be paid out as a dividend. The bank attempts to pay out as much dividends as possible before as its true financial condition surfaces and it is closed down.
A bank prefers growth to shrinkage if the required equity ratio is small. Intuitively, growth calls for an injection of fresh equity capital. The owners of the bank are unwilling to inject much equity into an insolvent bank. Instead, when the required equity ratio is small, it is optimal to inject fresh equity and gamble for resurrection. The model also discloses that an insolvent bank is ready to pay high interest on deposits in order to fund the growth. The heavier the burden of non-performing loans in the balance sheet, the more the bank favours growth and the more it is ready to pay for deposits.

This paper builds on rich analysis on moral hazard in banking: e.g. Diamond (1984), Kane (1986, 1989), Holmström & Tirole (1997), Hellman & Murdock & Stiglitz (2000), John & Saunders & Senbet (2000), Niinimäki (2001), Cordella & Yeyati (2003). Most of all, the paper relates to the recent articles of Aghion & Bolton & Fries (1999) and Mitchell (2001) in which a bank can hide its true solvency via loan rollovers. The articles shed light on optimal bailout policies. Instead, the purpose of this paper is to construct a simple model that can reproduce some of the stylized facts reported above and thereby investigate the origin of financial crises in emerging economics and the roles of banking sector and moral hazard in this context. Under investigation are both ex ante and ex post moral hazard as well as how the length of lending relationships influence on the magnitude of moral hazard. Some early warning signals on moral hazard behaviour can be observed and a few regulatory recommendations can be given (see Conclusion). For alternative explanations for financial crises in emerging economies see Corsetti & Pesenti & Roubini (1999a, 1999b), Chang & Velasco (2000, 2001), Goldstein (1997) and Goldstein & Turner (1996).

Aghion et al. (1999) find that a tough bank recapitalization policy in which bank managers are always dismissed drives the managers to hide bad loans by rolling them over. This softens the budget constraints of the banks’ borrowers. A mild recapitalization policy creates incentives to overstate bad loans to obtain larger recapitalization. Mitchell (2001) compares three bailout policies: a laissez-faire policy, transfer of bad debt to an asset management company, and cancellation of debt inherited from a previous regime. The government’s choice of policy affects bank behaviour and bank’s borrowers’ behaviour. None of the policies prove to be unconditionally optimal. The analysis is enriched in Mitchell (2000). The optimal bailout policies are also explored by Cordella & Yeati (2003) and Rochet & Vives (2004). See Freixas & Rochet (1997) for an extensive survey on moral hazard in banking.
2. The model

The model has two periods, a fully competitive banking sector and a bank regulator. To shorten the analysis, the operations of banks are greatly simplified. The behaviour of borrowers (=entrepreneurs) is omitted and the banks invest their funds in assets.\(^3\)

2.1 Asset types

An asset requires a unit of investment input. Under bank monitoring, the asset always succeeds. In the absence of monitoring, the asset is risky and succeeds with probability \( p, \ 0 < p < 1 \), in every period and fails with probability \( 1−p \). There are different asset types. The realized asset type is learned by the bank during period-1 subsequent to the investment and the decision whether or not to monitor.

A fast asset lasts for a period. If successful, it produces \( Y \) units output after the period (at the end of a period).

A slow asset lasts for two periods. If successful, it produces \( Y−1 \) units after period-1. If it also succeeds during period-2, it then produces \( Y \) units. When the asset is liquidated after period-1, the liquidation value is 0.

A very slow asset takes for two periods. It produces no interim output after period-1. If successful, it produces \( Y_2 \) after period-2, \( Y_2 > Y^2 \). A liquidation value of a very slow asset is \( Y \). To simplify the analysis, very slow assets, which are really productive, are assumed to occur only under monitoring during period-1.

A failed asset has no value and the failure is definite. Although the bank has monitored during period-1, it can neglect monitoring during period-2. Even if a slow asset or a very slow asset succeeds during period-1, it can fail during period-2, if the bank then neglects monitoring.

2.2 Asset values with and without monitoring

It is assumed that self-interested bankers own and run banks. Initially, bank size is 1 and it is funded with equity capital, \( E, \ 0 \leq E \leq 1 \), and deposits, \( 1−E \). Deposit interest rate is \( r \), which also

\(^3\) The author will thank a referee for this suggestion.
represents the cost of equity to the banker (=he). Banks invest their funds in assets. Given the cost of lending per a loan unit, \( c \), and the cost of bank monitoring, \( m \), zero profit loan interest rate, \( R_i \), \( i \in \{m, nm\} \), is \( R_m = r + m + c \) under monitoring and \( R_{nm} = r + c \) in the absence of monitoring.

When a bank invests in assets at the beginning of period-1, the upcoming asset type is still unknown and the bank lends the funds for a period at the loan interest rate \( 1 + R_i \), but the loan contract includes two options to roll over the loan.

**Rolling over 1** (for slow assets): If the asset pays loan interest, \( R_i \), after period-1, the bank can roll over the loan and delay the repayment of the principal until period-2. The asset then yields \( 1 + R_i \).

**Rolling over 2** (for very slow assets): If an asset proves to be very slow, it does not yield anything after period-1, and the loan is rolled over. The extent of a new loan is \( 1 + R_i \) (unpaid interest is capitalized) and the asset yields \( (1 + R_i)^2 \) after period-2.

More precisely, the bank recognizes the materialized type of the asset during period-1. The type is private information and observable only to the bank (even if the asset yield is assumed to be publicly observable). The bank rolls over the loans that are granted for slow and very slow assets. In this way, the interruption of productive long-term assets can be prevented. The following standard assumption is made.

**Assumption 1.** Under bank monitoring, each asset type has positive NPV (net present value) in every period, but in the absence of monitoring the NPV is negative.

Next, the content of Assumption 1 is detailed. Under monitoring, it is assumed that

\[
Y > 1 + r + m + c. \tag{2.1}
\]

The inequality implies that fast assets and slow assets have positive NPV. Given (2.1) and \( Y_2 > Y_1 \) very slow assets have also positive NPV. In the absence of monitoring, it is assumed that

i. \( pY + b < 1 + r + c \)  
ii. \( p(Y - 1) + b + \delta p(pY + b) < 1 + r + c \)  
iii. \( pY_2 + b < Y(1 + r + c) \). \tag{2.2}
Here $b \geq 0$ represents the private benefit of the asset issuer if his project is not monitored. The first and the second inequality state that fast and slow assets have negative NPV without monitoring. The third inequality gives an equal result for a very slow asset.

Consequently, since assets are productive only under monitoring, the bank grants loans at the interest rate $R_m = r + m + c$. The task of monitoring is delegated to the bank. This does not, however, guarantee that the bank will exert effort in monitoring, which is unobservable to outsiders.

### 2.3 Bank’s balance sheet

To compare how the existence of long-term lending relationships and the chance to hide loan losses affect on the magnitude of moral hazard, two bank types are introduced: Bank A and Bank B. The type can be chosen by a bank at the beginning of period-1 and the choice is unobservable to others. Bank A invests only in fast assets via short-term loans. Bank B invests in all asset types and also has long-term lending relationships with rolled over loans. With monitoring, a stochastic fraction $v$ of Bank B’s assets are very slow. Here $v$ has a support $[\tilde{v}, \bar{v}]$, $0 \leq \tilde{v} < \bar{v} < 1$, continuous density $w$, and distribution $W$. Regarding the rest of the assets, $1-v$, a fixed fraction $s$ of these will be slow and the rest $1-s$ will be fast. Hence, the volumes of financed assets are: $v$ very slow assets, $(1-v)s$ slow assets and $(1-v)(1-s)$ fast assets.

Without monitoring, a stochastic fraction $l$ of the assets fails. Here $l$ has a support $[\underline{L}, \bar{L}]$, $0 < \underline{L} < \bar{L} < 1$, continuous density $f$, and distribution $F$. Regarding the rest of the assets, $1-l$, a fixed fraction $s$ of those will be slow and the rest $1-s$ will be fast. Thus, the volumes of financed assets are: $l$ failed assets, $(1-l)s$ slow assets and $(1-l)(1-s)$ fast assets.

During period-2, the loan rollovers occupy a large share of Bank B’s loan portfolio. Yet, since some assets are fast and mature during period-1, the loan portfolio has room for fresh loans. These funds are invested in fast assets which mature at the end of period-2.

To clarify connections between symbols, it is useful to note that asset’s expected probability to success under shirking meets

\[ p = \int_{\underline{L}}^{\bar{L}} (1-l) f(l) \, dl. \]  (2.3)
Since an average asset is unprofitable in the absence of monitoring, loans are on average also unprofitable

\[
\int \frac{(1-l)(1+R_m)}{l} f(l) \, dl < 1 + r + c. \tag{2.4}
\]

It is now possible to shed light on the roles of different assets. To begin with, recall that very slow assets as well as the assets that fail during period-1 yield no output after period-1. Additionally, the fractions of both asset types are stochastic. Furthermore, the loans, which are granted for these assets, can be rolled over. Since the asset types are unobservable to outsiders, they cannot know whether a bank rolls over a loan in order to delay the repayment of a very slow asset (which is socially valuable) or to hide a default of a loan repayment (which is socially harmful). The role of the slow assets is to show how the illiquidity of these assets helps to eliminate moral hazard. The existence of fast assets makes it possible to investigate the effects of growth and shrinkage. All of this will be detailed below.

The bank regulator (= she), who can act at no cost, insures deposits, sets an equity capital requirement for banks, supervises them and closes down problem banks. The instruments are used by the regulator in such a way that the bank prefers the monitoring strategy to non-monitoring. Therefore, in equilibrium the bank monitors. More precisely, deposits can be fully insured at no cost, since loans are risk-free under monitoring. Additionally, the regulator pre-commits to close down banks that neglect monitoring. A closed bank is liquidated and the liquidation proceeds are first and foremost spent for lending costs and thereafter for deposit payments. The remainder of the proceeds, if any, is paid to the banker.\footnote{Alternatively, it is possible to assume that deposits are not insured. Then, the regulator sets equity capital requirements for banks and supervises them. When she uncovers loan losses, the losses are made public. Unprotected depositors immediately panic in order to withdraw their deposits in time. The bank is liquidated through the panic and the regulator does not have to pre-commit to liquidate problem banks.} As regards to the bank supervision, the regulator can not directly observe whether Bank A monitored or not. Yet, its loans are short-term and thus the bank can be liquidated after each period. If the liquidation uncovers loan losses, the regulator knows that it has neglected monitoring (since monitored loans always succeed) and she closes down the bank. The case of Bank B is more interesting. Again, the regulator can not directly observe whether the bank monitors or not. However, Bank B cannot be liquidated after period-1, since the liquidation process:
would interrupt slow assets and very slow assets. Since liquidation is impossible, Bank B aims to hide its loan losses by rolling over the defaulted loans. The regulator attempts to prevent hiding by auditing banks. She can uncover the hidden loan losses with probability $1 - h$. That is, if a bank has loan losses, it manages to hide them from the regulator with probability $h$. With probability $1 - h$, the regulator uncovers the loan losses, recognizes that the bank has neglected monitoring and closes down the bank. Here $h < 1$ for two reasons; either the regulator does not audit every bank or the quality of the audits is so weak that the regulator cannot uncover every hiding attempt. Even when the regulator cannot uncover a hiding attempt, loan losses surface, if their realized share is so large that the bank is illiquid. The bank is then closed down. The time line is the following.

1.1 The regulator imposes a fixed equity capital requirement, $E$, to banks for both periods.
1.2 Each bank maintains $E$ and attracts the amount $1 - E$ of deposits.
1.3 Banks choose their types (Bank A, Bank B), grant loans and decide whether to monitor or not.
1.4 Without monitoring, some assets fail.
1.5 The end of period-1: fast assets mature and these loans are repaid. Slow assets yield interim loan interest. These loans as well as the loans for very slow assets are rolled over. If bank B has neglected monitoring, it rolls over defaulted loans.
1.6 The regulator audits banks. If she observes loan losses, she liquidates the bank and closes it.
2.1 If a bank is not closed, it can pay the costs of lending and dividends as well as pay back deposits. Simultaneously, the bank attracts fresh deposits for period-2. Thanks to the simultaneity, the interruption of long-term loans is avoided.
2.2 Banks use their liquid funds by granting short-term loans for fresh fast assets. The banks decide whether to monitor or not.
2.3 At the end of period-2, all outstanding loans mature and the banks are closed down. They pay back deposits and the banker receives the remaining returns.

This section explores moral hazard and evaluates an incentive compatible amount of bank equity, when a bank finances fast assets via short-term loans. This means that both the fractions of slow assets and very slow assets are zero.

**Assumption 2.**

i. \((1 - \bar{L})(1 + R_m) - (1 + r) - c > 0.\)

ii. \((1 - \bar{L})(1 + R_m) - c > 0.\)

Assumption 2i generates the problem of moral hazard under short-term lending. Banking may be profitable without monitoring. According to Assumption 2ii, a bank that is fully equity funded can always pay the cost of lending, \(c\). This simplifies the analysis since a bank that is fully equity funded never collapses.

Under full competition, banker’s earnings are zero if he monitors. If the non-monitoring strategy yields positive expected earnings, the banker will neglect monitoring. Without monitoring, the banker’s expected earnings are

\[
\int \left( (1 - l)(1 + R_m) - (1 + r)(1 - E) - c \right) f(l) \, dl - (1 + r)E,
\]

where \((1 - \tilde{l}_{\text{Short}})(1 + R_m) = (1 + r)(1 - E) + c.\)

Since assets are fast, the bank can be liquidated after both periods. Here \(\tilde{l}_{\text{Short}}\) marks the highest realized share of loan losses so that the bank can pay back deposits. The banker’s earnings consist of expected bank returns (the first, positive term) and the costs of injected equity, \((1 + r)E.\) As regards to the bank returns, when a share \(l\) of loans defaults, loan repayments (principal and interest) amount to \((1 - l)(1 + R_m)\), and payments to depositors are \((1 + r)(1 - E).\) If the realized share of loan losses exceeds \(\tilde{l}_{\text{Short}},\) the bank collapses and the banker receives no returns. Appendix A asserts the following Proposition.

**Proposition 1.** When a bank invests only in fast assets, it neglects monitoring without equity capital. There is an incentive compatible amount of equity, \(E_{\text{Short}}^*\), which eliminates moral hazard.
Since banks are identical in both periods, \( E_{Short}^* \) represents the incentive compatible amount of equity in both periods. Intuitively, in the absence of equity the costs of bank formation are zero. If the banker neglects monitoring, the share of loan losses is small with positive probability, the bank makes a profit and the banker’s earnings are positive. If the realized share of loan losses is large, the bank collapses. The banker earns nothing, but he bears no costs either. Thus, the expected earnings from the non-monitoring strategy are positive. To eliminate this, a positive amount of equity capital must be required. The requirement forces bankers to have some of their own capital at risk so that they internalize the inefficiency of non-monitoring.

4. Bank B: long-term lending with monitoring

To study moral hazard under long-term lending, bank B is investigated. With monitoring, a stochastic fraction \( v \) of assets is very slow. Regarding the rest of the assets, \( 1 - v \), a fixed fraction, \( s \), of these is slow, while the rest of the assets are fast.

After period-1, a monitoring bank rolls over \( v \) loans for very slow assets and \( (1-v)s \) loans for slow assets. The latter loans yield loan interest \( (1-v)sR_m \). The \( (1-v)(1-s) \) loans for fast assets mature yielding \( (1-v)(1-s)(1 + R_m) \). The bank pays back deposits, \( 1+r \), and attracts at the same time fresh deposits, \( 1 \), for period-2. In this way, the interruption of long-term loans (and long-term assets) can be avoided. The banker’s earnings during period-1 – that is, the profit of the bank from which the cost of monitoring and the cost of injected bank equity are subtracted – amount to

\[
\int_{v}^{1} (1-v)R_m - (1-E)r - c \ w(v) \, dv - m - E(1+r),
\]

where \( \bar{v}_1 \) denotes the maximum fraction of rolled over loans; \( (1-\bar{v}_1)R_m - (1-E)r - c = 0 \). \(^5\)

\(^5\) It is possible that the realized share of very slow assets, \( \hat{v} \), is so high that the bank collapses if it rolls over all of these loans since \( (1-\hat{v})R_m < (1-E)r + c \). In this case, it is assumed that the bank rolls over only \( \bar{v}_1 \) loans so that \( (1-\bar{v}_1)R_m = (1-E)r + c \). Then, some very slow assets must be interrupted after period-1 (the liquidation value is \( Y \)). The interruption is uneconomical (recall (2.1)).
If \( \hat{v} \) denotes the realized fraction of rolled over slow loans, the earnings amount to \((1 - \hat{v})R_m - r - c - m - E\) or \(-\hat{v}R_m - E\). Here \( \hat{v}R_m \) represents bank’s interest receivables from the rolled over loans for very slow assets. The interest receivables belong to the returns from period-1 even if they are paid after period-2. Because the receivables are not paid out from the bank after period-1, they increase the retained earnings of the bank and thus raise its equity capital. For the same reason, the need of deposits for period-2 is \(1 - E - \hat{v}R_m\).

After period-2, all outstanding loans mature. The loans of very slow assets yield \(v(1 + R_m)^2\) and loans of slow assets yield \((1 - v)s(1 + R_m)\). The rest of the funds were invested in fresh fast assets at the beginning of period-2. These loans yield \([1 - v(1 + R_m) - (1 - v)s](1 + R_m)\).

After payments on deposits, \((1 - E - vR_m)(1 + r)\), the banker’s earnings during period-2 amount to

\[
\int \left[ v(1 + R_m)^2 + (1 - v)s(1 + R_m) + \left[ 1 - v(1 + R_m) - (1 - v)s \right](1 + R_m) - (1 - E - vR_m)(1 + r) - c \right] w(v) \, dv - m.
\]

Given \(v = \hat{v}\), this simplifies to \(\hat{v}R_m(1 + r) + E(1 + r)\). The earnings are positive during period-2, but equal to the present value of the losses from period-1. Thus, the life-time earnings from the monitoring strategy are zero.
5. Long-term lending without monitoring

This section investigates moral hazard under long-term lending when the regulator’s auditing system is imperfect and a bank can sometimes hide its loan losses, \( h > 0 \). Without monitoring, a stochastic fraction \( l \) of the financed assets fails. The rest of the assets are either slow, \((1-l)s\), or fast, \((1-l)(1-s)\). To keep the model simple, a few assumptions have to be made. Most of these can be relaxed, but it makes the model much more complex.

**Assumption 3.**

\[ i. \quad v \leq \bar{v} = \bar{L} < \overline{v} < 1. \]

\[ ii. \quad 1 - (1 + R_m)\bar{L} - (1 - \bar{L})s > 0. \]

\[ iii. \quad (1 - \bar{L})R_m < c. \]

\[ iv. \quad (1-L)(1-s)(1+R_m) < c. \]

\[ v. \quad [1-(1+R_m)\overline{L}](1+R_m)(1-L) + LR_m(1+r) < c + c_h. \]

Assumption 3i ensures that the share of loan rollovers does not reveal that the loans are non-performing. Instead, the regulator may incorrectly interpret these loans as performing and granted for very slow assets. Assumption 3ii guarantees that the bank’s balance sheet has enough room for loan rollovers, since their volume is assumed to be less than 1. Owing to Assumption 3iii, bank’s loan interest income is inadequate to pay interest on deposits, when the share of realized loan losses peaks, \( \bar{L} \). This makes it is possible to explore how illiquidity uncovers an attempt at hiding. According to Assumption 3iv, loans on average are illiquid to the extent that a non-monitoring bank collapses if it is liquidated after period-1. The assumption is supported by evidence, since liquidation of a problem bank rarely yields anything to bank owners. Assumption 3v highlights how loan losses accumulate in the long run, finally making a bank insolvent after period-2.\(^6\) Here \( c_h \) denotes an extra cost of hiding during period-2. The bank needs to manipulate its accounting and information on borrowers, etc.

\(^6\)The following values, for example, meet Assumptions 2 and 3: \( L = 0.05, \bar{L} = 0.3, R_m = 0.3, c = 0.22, m = 0.07, r = 0.01, s = 0.85, c_h = 1 \). If the example is extended by assuming \( Y = 1.31, Y_2 = 1.72 \), and \( p = \frac{1}{7} \), \( 0 \leq b \leq \frac{1}{7} \) the conditions (2.1)-(2.2) are also satisfied.
5.1 Stages of hiding

Suppose that a bank neglects monitoring during period-1 and a stochastic fraction $\hat{l}_1$ of loans default (subscript 1 stresses that the realized loan losses stem from period-1). If these loans are not rolled over, loan losses surface, the regulator closes down the bank and liquidates it. The banker earns the remainder of the liquidation proceeds

$$\text{Max}\left\{0, (1-\hat{l}_1)(1-s)(1+R_m) - (1+r)(1-E) - c\right\} = 0 . \quad (5.1)$$

Given the large fraction of slow assets (s) and the low liquidation value of these loans (0), it is known that $\text{Max}\left\{,0\right\}=0$ (Assumption 3iv). To avoid this, the bank always attempts to hide loan losses via loan rollovers. Next the banker’s expected earnings under the option of hiding are resolved. Four cases arise depending on whether or not the bank manages in the attempt to hide.

With probability $h \, F(\hat{l}_1)$ the bank manages in hiding and makes during period-1 expected returns

$$\pi_i = \int_{\frac{1}{2}}^{\hat{l}_1} (1-l_1)R_m - (1-E)r - c \, f(l_1) \, dl_1 , \quad (5.2)$$

which can be paid out as dividends to the banker at the end of period 1. Here $(1-l_1)R_m$ marks the total loan interest income, which consists of successful fast loans, $(1-l_1)(1-s)R_m$ and successful slow loans, $(1-l_1)sR_m$. A share $l_1$ of loans is defaulted and rolled over. The outsiders do not know whether the loans are rolled over in order to finance very slow assets or to hide loan losses. The second term $(1-E)r$ indicates interest payments on deposits. Only if the loan interest income is adequate to pay interest on deposits, hiding is possible. Thus, the realized share of loan losses, $\hat{l}_1$, has an upper limit, $\tilde{l}_1$, so that the bank can only pay just interest on deposits

$$(1-\tilde{l}_1)R_m - (1-E)r - c = 0 . \quad (5.3)$$

The probability that the realized share of loan losses is low enough, $\hat{l}_1 \leq \tilde{l}_1$, is $F(\tilde{l}_1)$. 


With probability \( h \left[ 1 - F(\tilde{i}_1) \right] \) the regulator’s audit does not uncover loan losses, but their realized share exceeds \( \tilde{i}_1 \). Illiquidity – loan interest income, \((1 - \tilde{i}_1)R_m \), is insufficient for the interest on deposits, \((1 - E)r\) - now uncovers loan losses and the bank is closed down. The banker receives the remainder of the liquidation proceeds, 0, just as in (5.1).

With probability \( 1 - h \), the regulator can uncover loan losses and closes the bank. The banker again receives the remainder of the liquidation proceeds, 0, just as in (5.1).

With probability \( h F(\tilde{i}_1) \) the bank manages to hide during period-1 and achieves period-2. After period-2 all outstanding loans mature. The bank’s expected returns from period-2 are (the ex ante point of view, when the share of period-1 loan losses has not been realized)

\[
\int_{l_1}^{L} \int_{l_2}^{L} \left[ (1 - (1 + R_m)l_1)(1 + R_m)(1 - l_2) - (1 - E - l_1R_m)(1 + r) - c - c_h f(l_1)dl_1 f(l_2)dl_2 = 0 \right].
\]

(5.4)

Again, \( \tilde{i}_t \) denotes the highest share of realized loan losses during period-t, \( t \in \{1,2\} \), so that a bank can pay back deposits. The result, no earnings, follows from the assumption 3v. The inherited non-performing loans from period-1 occupy a large share of the balance sheet, \((1 + R_m)\tilde{i}_1\). The burden of non-performing loans accumulates, since unpaid interest is capitalized; the size of these loans expands from 1 unit to \( 1 + R_m \) units. Given the hidden loan losses, there is relatively little room for fresh loans. Since \( l_2 \geq L > 0 \), a part of the loans again default during period-2. The accumulated non-performing loans erode loan repayments, which is insufficient to cover the costs of banking after period-2 and the bank collapses.

The banker’s total earnings from the non-monitoring strategy consist of expected bank returns (5.2) from which the costs of injected bank equity are subtracted

\[
\Pi = h \pi_1 - (1 + r)E.
\]

(5.5)

Some noteworthy discoveries can be made. To start with, when a realized share of loan losses is low, \( \tilde{i}_1 < \tilde{i}_1 \), and when a hiding attempt is successful, the bank makes a profit, \((1 - \tilde{i}_1)R_m - (1 - E)r - c > 0\). In reality, the bank may be insolvent \((1 - \tilde{i}_1)(1 + R_m) < (1 - E)(1 + r) + c\) and its true equity ratio is negative. Thus, the information value of the official equity ratio, \( E \), is modest
when the bank has been operating for some time and possesses non-performing loans. The insolvency is uncovered only after period-2, when the bank collapses (recall (5.4)). To explain the bank collapse, an outsider may argue that the bank neglected monitoring during period-2 and that a devastating shock – for example, a macroeconomic downturn or financial panic - then hit the banking sector ($\hat{i}_2$ is really high). The explanation is deficient, since the bank has been insolvent for a long time, but it has been able to hide its true financial condition. The shock of period-2 may have a minor effect on the volume of loan losses and any explanation, which stresses the economic factors of period-2 as the actual origin of the bank collapse, is deficient.

If the auditing system is really weak, $h = 1$, insolvency is uncovered after period-1 only if a bank is illiquid, $(1 - \hat{i}_1)R_m < (1 - E)r + c$. Hence, illiquidity provides an important signal of bank insolvency.\footnote{The crucial role of illiquidity as a signal of bank insolvency is documented by De Juan (1996). “When the Spanish banking crisis on the late 1970s and early 1980s began, …, insolvent banks were identified only when they became illiquid (De Juan, 1996, p. 100).” “In the mid 1980s, Argentina suffered a very serious banking crisis that affected mostly new banks and banks run by new bankers. Some two to three hundred banks experienced interventions and/or were liquidated. Practically all were insolvent, but intervention was triggered by illiquidity. Only through illiquidity was the insolvency discovered (De Juan, 1996, p. 93).”}

Owing to the option to hide loan losses, the costs of bank restructuring are likely to skyrocket. To see this, suppose that $l_1 = l_2 = l$. Without hiding, $l(1 + R_m)$ represents the volume of loan losses. With hiding, the volume of hidden loan losses is $l(1 + R_m)$ after period-1. The bank reinvests the rest of the funds, $1 - l(1 + R_m)$, and a fraction $l$ of loans again default during period-2. The loan losses from period-2 account for $[1 - l(1 + R_m)](1 + R_m)l$. The total value of loan losses after period-2 is

$$l(1 + R_m)[1 + (1 - l)(1 + R_m)] > l(1 + R_m). \tag{5.6}$$

For example, if $l = 0.1$ and $R_m = 0.3$, the volume of loan losses accounts for $l(1 + R_m) = 0.1 \times 1.3 = 0.13$ without hiding. Using (5.6) it is easy to see that the volume of loan losses is more than doubled, 0.28, with hiding. Two causes for the doubling up of volume exist. First, the loan losses of period-1 are multiplied by $1 + R_m$, since unpaid loan interest is capitalized in the size of the loan. Second, during period-1, some loans default. The bank keeps on gambling with the rest of
the funds and some of these loans default again during period-2. Hence, the volume of loan losses accumulates period after period and achieves a larger total volume than what can be achieved during a single period. The doubled volume may help to explain why the share of non-performing loans has been extremely large in numerous emerging economies and why the costs of bank restructuring have been so massive.8

When the auditing system is complete, \( h = 0 \), the banker’s expected earnings amount \( \Pi = -(1 + r)E \). The non-monitoring strategy is unprofitable even without bank equity.

**Proposition 2.** When \( h=0 \), the problem of moral hazard disappears under long-term lending. Moral hazard is a lesser problem under long-term lending than under short-term lending.

For the latter part of the Proposition 2, recall Proposition 1. Intuitively, the fact that a significant part of loans are long-term and illiquid together with the fact that the loans mature and are repaid at different times helps to eliminate moral hazard. Since the auditing system is perfect, loan losses are uncovered at the maturity of short-term loans (after period-1), the bank is closed down and liquidated. The loans, which are granted for slow assets, are still ongoing. The fraction of these loans, \( s \), is large and the loans have a minimal liquidation value, 0. Thus, the liquidation value of the whole bank is negative even when the realized share of loan losses is at the lower limit.9 The bank liquidation yields no earnings to the banker and the incentive problem is eliminated.10

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8 In many emerging economies, the share of non-performing loans has proved to be tragic: 80-90% in Kyrgyz Republic, 75% in Bulgaria, 75% in Congo, 60-70% in Cameroon, 40% in Ghana, etc. In developed countries with banking crises, the shares of non-performing loans have been much lower: 13% in Finland, 6% in Norway, etc. (Lindgren et al., 1996).

9 Alternatively, loans may mature at the same time if information on loan losses surfaces in the middle of the loan period. Suppose that at time 0 a bank invests in illiquid assets, which mature at time 2. At time 1 signals become public. A signal indicates whether or not an asset will succeed. If the signals reveal that some assets will fail, the regulator closes down the bank. Given the illiquidity of assets, the liquidation proceeds are so minimal that the banker receives no earnings.

10 Without deposit insurance, a panic of unprotected depositors has the same incentive effect. Suppose that depositors can observe after period-1 that their bank has suffered loan losses and that this information triggers a bank run. Given a low liquidation value of slow assets, the bank collapses. This is enough to eliminate moral hazard. For financial panics, see also Diamond & Dybvig (1983), Niinimäki (2003), Chang & Velasco (2000, 2001) and Rochet & Vives (2004).
5.2 Incentive compatible amount of equity capital for Bank-B

The following result is achieved in Appendix B.

**Proposition 3.** When a bank finances long-term assets and can hide its loan losses, the bank neglects monitoring in the absence of equity capital. There is an incentive compatible amount of equity, \( E_{\text{Long}}^* > 0 \), which makes the non-monitoring strategy unprofitable.

The option to hide loan losses worsens moral hazard. In contrast to Proposition 2, long-term loans without hiding, the incentive compatible amount of equity is now positive. The regulator should recognize the characteristics of the local economy when she sets equity requirements (Appendix C).

**Proposition 4.** The weaker the auditing system (the higher \( h \)), the bigger the incentive compatible amount of equity, \( E_{\text{Long}}^* \), is.

Equity capital and auditing system represent substitutes. In developed economies, where the auditing systems are strong (\( h \) is low), the equity requirement can be small. In contrary, in emerging economies with weak auditing systems more equity capital is needed; it is unlikely that the regulator can uncover hidden loan losses (\( h \) is high) and moral hazard can be eliminated only by requiring more equity.\(^{11}\) For the following result, see Appendix D.

**Proposition 5.** The higher the cost of monitoring for the bank, \( m \), the bigger the incentive compatible amount of equity, \( E_{\text{Long}}^* \), is.

The higher the cost of monitoring, the more profitable the non-monitoring strategy is. Monitoring costs are high in many emerging economies due to the lack of information on the creditworthiness of firms and insufficient creditor rights. Imperfect property titles as well as poor mortgage and

\(^{11}\) There is strong empirical evidence that bank exams have an important role in uncovering financial problems (see Dahl & O’keefe & Hanweek, 1998, and Gunther & Moore, 2003).
commitment registers also raise the costs of monitoring. Consequently, more equity capital ought to be required in these economies.

5.3 Omit earned interest receivables from equity capital

In Section 4, a bank monitored its assets. Since very slow assets yielded no output during period-1, these loans created interest receivables, \( vR_m \), thereby raising the amount of bank equity. Thus, rolled over loans and interest receivables had a positive effect on the bank’s incentives.

Suppose that the required amount of equity is \( LongE^* \). A bank monitors and rolls over \( \hat{v}(1+R) \) loans for very slow assets after period-1. The loans create interest receivables \( \hat{v}R_m \). If the receivables are incorporated in equity, the bank equity amounts to \( LongE^* + \hat{v}R_m \). To extract excessive equity and to restore the requirement \( LongE^* \), the bank can pay out some initially injected equity, \( Min[LongE^*, \hat{v}R_m] \), as dividend.

Unfortunately, this alternative worsens moral hazard, since the regulator is unable to observe the type of the rolled over loans and the real worth of interest receivables. Suppose that the bank neglects monitoring during period-1 and rolls over \( \hat{i}_1(1+R_m) \) non-performing loans having interest receivables \( \hat{i}_1R_m \), which can be incorporated in equity. The equity capital amounts to \( LongE^* + \hat{i}_1R_m \). To restore the required amount of equity, the bank can pay out, \( Min[LongE^*, \hat{i}_1R_m] \), as a dividend. Only after period-2, the truth is uncovered; the rolled over loans proved to be non-performing and interest receivables are worthless.

This increases earnings from period-1 by \( Min[LongE^*, \hat{i}_1R_m] \). During period-2, the bank needs more deposits, \( Min[LongE^*, \hat{i}_1R_m] \), to compensate for the reduction of equity. This is, however, meaningless for the bank, since it will collapse in any case after period-2. Thus, the alternative to incorporate interest receivables into equity capital and pay out initially injected equity increases the returns of the non-monitoring strategy, making it optimal under the equity requirement \( LongE^* \).

\(^{12}\) Demirguc-Kunt & Detragiache (2002) find empirical evidence that countries with weak legislation and enforcement have frequent banking crises.

\(^{13}\) The result is supported by the report of de Juan (1993, p. 24):”... lead bankers to provision less than they should, but will also lead them to capitalize interest, that is, to account for refinanced interest (which in fact will be increased
**Corollary 1.** The regulator should exclude earned interest receivables from bank equity.

### 5.4 Capital requirement

Section 5 resolved the incentive compatible equity ratio, which eliminates moral hazard under long-term lending, $E_{Long}^*$. Section 4 derived the incentive compatible equity ratio, which eliminates moral hazard under short-term lending, $E_{Short}^*$. The regulator imposes equity requirement

$$E^* = \text{Max}\{E_{Short}^*, E_{Long}^*\}. \quad (5.7)$$

This requirement eliminates the moral hazard incentives of both bank types (Bank A, Bank B) during both periods.\(^\text{14}\)

If $E_{Short}^* > E_{Long}^*$, the problem of moral hazard is more severe under short-term lending than under long-term lending. It is easy to see that this is possible. Recall the values of footnote 5 and suppose that a bank operates in the absence of equity capital. Under the support $[L, \bar{L}] = [0.05, 0.3]$, short-term lending yields 0.005 when the realized share of loan losses is at the lower limit, $L = 0.05$. Thus, moral hazard can be eliminated only if $E_{Short}^* > 0$. On the other hand, if the auditing system is so good that banks cannot hide loan losses, $h = 0$, it is known that $E_{Long}^* = 0$ (Proposition 2). Therefore, the case $E_{Short}^* > E_{Long}^*$ is possible.

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\(^{14}\) In this model environment, the moral hazard problem can always be eliminated by using capital requirements. The aim of the paper is to find out the minimal incentive compatible levels of bank capital in different circumstances. How does the bank’s option to hide its loan losses (or the quality of bank supervision, or the costs of bank monitoring, or the ability to pay back initially injected equity capital, etc.) effect on the incentive compatible level of bank capital? Does $E^*$ eliminate gambling for resurrection when it is sufficient to eliminate ex ante moral hazard? The findings may help bank regulators to determine incentive compatible capital requirements in different circumstances and discover alternative forms of moral hazard.
If $E_{\text{Long}}^* > E_{\text{Short}}^*$, the moral hazard problem is more severe when the bank invests in long-term assets than when it invests only in short-term assets. Recall again the values of footnote 5 and suppose that the bank operates in the absence of equity capital. However, the support is now assumed to be $[L, \bar{L}] = [0.1, 0.25]$ and the auditing system is assumed to be bad, $h = 1$. Now short-term lending is unprofitable at the lower limit (-0.06), but long-term lending is profitable (0.04). As a result, the incentive compatible equity ratios satisfy $E_{\text{Short}}^* = 0$, $E_{\text{Long}}^* > 0$. Intuitively, the minimum share of loan losses, 0.1, is so high that the non-monitoring strategy is always unprofitable under short-term lending. Loan losses incur lesser expenses under long-term lending, since banks can hide them by rolling over the defaulted loans.

Consequently, the quality of the bank supervision strongly determines whether the moral hazard problem is more severe under long-term lending or under short-term lending.
6. Gambling for resurrection

So far the size of the bank has assumed to be fixed, 1. The assumption is now dropped. As before, bank size is 1 during period-1, but the bank is allowed to change the scale of its lending during period-2. At the beginning of period-2, the banker faces a difficult choice. He knows that the bank possesses a burden of non-performing loans and that it will subsequently collapse at the end of the period if it maintains the initial scale, 1. This section explores how the banker optimally reacts to non-performing loans during period-2. Thus, the analysis of the previous sections is extended. Alternatively, the section can be interpreted as a separate research asset. When the bank possesses, for whatever reason, non-performing loans in its loan portfolio what is the optimal reaction? Is the equity requirement $E^*$ incentive compatible any more? Subsection 6.1 examines the optimality of shrinkage, whereas gambling for resurrection through growth is explored in subsection 6.2. Interestingly, both alternatives may be profitable.

6.1 Shrink

Suppose that a bank neglected monitoring during period-1 and manages in the attempt to hide. The bank size during period-2 is $S_2$, $S_2 \in [0,1]$. The required equity ratio, $E$, can be smaller than the incentive compatible ratio, $E^*$.\(^{15}\) Given the equity requirement, a shrinking bank can pay out excess equity $(1 - S_2)E$ as dividend in step 2.1 of the timeline. Instead, if the bank grows, the banker needs to inject $(S_2 - 1)E$ more equity in step 2.1.

What is the minimum of $S_2$? The share of non-performing loans, $\hat{l}_i(1 + R_m)$, sets an absolute bottom limit, since these loans must be rolled over. Further, the bank needs to roll over the loans granted for slow assets, since the loans have an extremely low liquidation value, 0. The third

\(^{15}\) Since the equity requirement may be under the incentive compatible level, it is implicitly assumed, that the regulator may underestimate the need of equity. This assumption is chosen for several reasons. First, the incentive compatible amount of equity is dependent on the costs of monitoring, $m$, and on the quality of the auditing system, $h$. If the regulator, for example, undervalues the cost of monitoring in a local economy inadvertently, she will require banks to maintain inadequate equity capital. Given the frequency and costs of recent banking crises, it is obvious that regulators make this kind of mistake. Second, the regulator may be unwilling to raise the equity requirement over the normal level, since this would indicate that the regulator is a bad auditor (Proposition 4). Third and most importantly, it is interesting to investigate how banks operate when the equity requirement is too small. Are the results consistent with evidence? If so, this offers a noteworthy signal that the equity requirement should be raised.
loan type, fast loans, offers a tool to shrink lending. These loans mature after period-1 and the bank could reinvest the funds in fresh short-term loans. Alternatively, the bank can skip the reinvestment and shrink lending. As a result, the bank’s real minimum size during period-2 is

$$S_2 = \hat{l}_1(1 + R_m) + (1 - \hat{l})s < 1. \quad (6.1)$$

Recall from (5.4) that without growth and shrinkage, the bank will collapse after period-2

$$\left[1 - (1 + R_m)\hat{l}_1\right](1 + R_m)(1 - \hat{l}_2) < (1 - E - \hat{l}_1 R_m)(1 + r) + c + c_h, \quad (6.2)$$

with every $\hat{l}_1, \hat{l}_2$, and the banker earns nothing. In contrast, if the bank shrinks lending and pays out excess equity, the banker earns $(1 - S_2)E(1 + r)$. Since the earnings are increasing in $E$ and decreasing in $S_2$, the bank shrinks its scale to $S_2$ and pays out $(1 - S_2)E$ to the banker. Thus, the bank always prefers shrinkage to the initial bank size, 1.

**Proposition 6.** The bank can always increase the banker’s earnings by shrinking lending during period-2 and paying back initially injected equity capital. The earnings from shrinkage are increasing in the required equity ratio and the scale of shrinkage.

Intuitively, under the burden of hidden loan losses, the true financial condition of the bank is bad and it will collapse after the period. Through shrinkage the banker can withdraw equity capital from the bank before than its true financial condition surfaces and the bank is closed down. The shrinked quasi-bank then keeps operating during period-2. After the period the truth surfaces; the loans proved to be mostly rolled over non-performing loans and the bank is almost worthless.

Consider, for example, that the bank size is 1000 million euros and that the required equity ratio is 8%. If the bank shrinks lending by 40%, it can pay out 32 million euros as extra dividend.\(^{16}\)

\(^{16}\) The author was unable to find empirical evidence on shrinkage. Probably, the shrink-asset-size strategy has not been investigated. Some evidence on it might be found from emerging economies. Alternatively, it is possible that bank regulators do not allow a bank to pay back initially injected equity capital, since they anticipate that the bank is following the shrink-asset-size strategy and it is insolvent.
Consequently, shrinkage increases the expected returns of the non-monitoring strategy and equity ratio $E^*$ may be too small to eliminate non-monitoring. Hence, the regulator ought to be alerted when the bank attempts to shrink its lending and pay back initially injected equity.

6.2 Grow

This section analyzes the optimality of growth. The bank can grow during period-2 by granting short-term loans for fast assets, which mature at the end of period-2. The maximum size during period-2 satisfies, $1 \leq S_2 \leq \bar{S}_2$, where $1 < \bar{S}_2 < \infty$. The expected bank returns during period-2 are

$$\pi_2 = S_2 \left[ 1 - \frac{(1 + R_m)\hat{l}_1}{S_2} \right] (1 + R_m)(1 - \hat{l}_2) - (1 - E - \frac{\hat{l}_1 R_m}{S_2})(1 + r) - c - \frac{c_h}{S_2} \int f(l_2) dl_2, \tag{6.3}$$

where the upper limit of period-2 loan losses, $\bar{l}_2(\hat{l}_1, E; S_2)$, satisfies

$$\left[ S_2 - (1 + R)\hat{l}_1 \right] (1 + R_m)(1 - \bar{l}_2) = \left[ S_2(1 - E) - \hat{l}_1 R_m \right](1 + r) + S_2 c + c_h. \tag{6.4}$$

Lemmas 1 and 2 are proved in Appendix F and Lemma 3 in Appendix G.

**Lemma 1.** $\bar{l}_2(\hat{l}_1, E; S_2)$ is decreasing in $\hat{l}_1$, but increasing in $S_2$.

Intuitively, the larger the burden of non-performing loans from period-1, smaller the share of loan losses must be during period-2 so that the bank is still capable of paying back deposits. The larger $S_2$, the more rapidly the bank grows and the more it has fresh performing loans. The relative burden of the non-performing loans from period-1 is then small. Thus, the larger $S_2$, the higher the share of loan losses can be during period-2 so that the bank is still capable of paying back deposits.

**Lemma 2.** $\bar{l}_2(\hat{l}_1, E; S_2)$ would approach $\bar{l}_2(0, E; S_2)$ if $S_2$ could grow without bound. Since $S_2$ is assumed to be finite, it is known that $\bar{l}_2(\hat{l}_1, E; S_2) < \bar{l}_2(0, E; S_2)$ with every $S_2$. 
Intuitively, the upper limit of loan losses during period-2, $\hat{\bar{T}}_2(\hat{l}, E; S_z)$, peaks when the bank has no inherited loan losses from period-1, $\hat{l}_1 = 0$. An identical loan portfolio could be achieved, if the bank could grow without bound. In both cases, the share of the inherited loan losses in the loan portfolio of period-2 is zero. Yet, since growth is assumed to be finite, the share of inherited loan losses is positive and the peak value, $\hat{T}_2(0, E; S_z)$, can not be achieved.

**Lemma 3.** When $E = E_{\text{short}}^*$, growth is unprofitable, but when $E < E_{\text{short}}^*$ growth is profitable if the maximum achievable size, $\bar{S}_2$, is large enough. When $E < E_{\text{short}}^*$, the earnings from growth would approach infinity if $\bar{S}_2$ could grow without bound.

Given Lemma 3, it is enough to detail the optimality of growth when $E < E_{\text{short}}^*$. To start with, (6.2) is rewritten as

\[
(1 + R_n)(1 - L) - (1 - E)(1 + r) - c < \frac{L(1 + R_n)[(1 + R_n)(1 - L) - R_n(1 + r)/(1 + R_n)]}{\bar{S}_2} + c_* ,
\]

when $S_2 = 1$. That is, the bank will be insolvent after period-2 if it retains its initial size, 1, even when the share of loan losses is at the lower limit, $L$, during period-2. This means that $\hat{T}_2(\hat{l}, E; S_z) = L$ in (6.3) and thus bank returns are zero, $\pi_2 = 0$. Suppose now that the bank grows by granting more one-period loans for fast assets during period-2. Since these fresh loans are performing, growth raises the share of performing loans in the loan portfolio and increases the loan interest income. There exist such a minimum bank size, $S_2^{\text{Min}} > 1$, that the bank has a possibility to break even during period-2

\[
(1 + R_n)(1 - L) - (1 - E)(1 + r) - c = \frac{L(1 + R_n)[(1 + R_n)(1 - L) - R_n(1 + r)/(1 + R_n)]}{S_2^{\text{Min}}} + c_* ,
\]

where the L.H.S and the term in brackets are positive (Appendix E). An even more rapid growth pushes up the share of performing loans further so that the L.H.S exceeds the R.H.S. Then,

\[17\] Since the bank gambles for resurrection by granting short-term loans, the incentive compatible level of short-term lending is crucial.
\( \tilde{I}_2(\hat{I}, E; S_2) > L \) in (6.3) and thus \( \pi_2 > 0 \); expected bank returns are positive. Hence, growth increases bank returns with certainty. This does not, however, ensure that growth is profitable for the banker, who must inject fresh equity in the bank in order to maintain the required equity ratio, \( E \). The costs of fresh equity are \( (S_2 - 1)E(1 + r) \).

If \( \bar{S}_2 < S_2^{Min} \), growth is unprofitable, since the maximal achievable size, \( \bar{S}_2 \), is so small that the bank cannot grow out of its problems. Even if the bank expanded the maximal size, \( \bar{S}_2 \), it could not obtain enough performing loans and the bank would collapse. Growth would only incur costs, \( (\bar{S}_2 - 1)E(1 + r) \), to the banker. Thus, shrinkage is more profitable than growth.

When \( \bar{S}_2 > S_2^{Min} \), growth is optimal when \( \bar{S}_2 \) is large enough. If \( 0 < \pi_2(\hat{I}, E; \bar{S}_2) - (\bar{S}_2 - 1)E(1 + r) < (1 - S_2)E(1 + r) \), growth is profitable, but still less profitable than shrinkage. Only if \( \bar{S}_2 \) is so large that \( \pi_2(\hat{I}, E; \bar{S}_2) - (\bar{S}_2 - 1)E(1 + r) > (1 - S_2)E(1 + r) \) growth is more profitable than shrinkage. This is possible, if \( \bar{S}_2 \) is sufficiently large (recall Lemma 3). The foregoing can be summarized as follows (see also Appendix G).

**Proposition 7.** Under the burden of hidden loan losses, the profitability of growth depends negatively on the required equity ratio and on the burden of hidden loan losses, but positively on the growth opportunities \( (\bar{S}_2) \). Growth is unprofitable when the required equity ratio is at the incentive compatible level, \( E^* \). When the required equity ratio is sufficiently small \( (E < E_{Short}^*) \), growth is profitable if growth opportunities are good \( (\bar{S}_2 \) is large). Growth is unprofitable under any equity requirement, if growth opportunities are remote \( (\bar{S}_2 \leq S_2^{Min}) \).

Consequently, the bank can gamble for resurrection by growing rapidly. Nevertheless, more equity capital must be injected into the bank in order to maintain the required equity ratio. Under the burden of hidden loan losses, the bank is de facto insolvent and its expected returns will be relatively modest even with growth. Because of this, the banker is unwilling to inject much equity. The larger the burden of hidden loan losses, the more unpleasant is growth. It is more profitable to trigger the non-monitoring strategy during period-2 with a clean balance sheet than to continue the non-monitoring strategy with an unclean balance sheet. Hence, the equity ratio that eliminates the non-monitoring strategy with a clean balance sheet would most certainly eliminate gambling for
resurrection with an unclean balance sheet. Only if the required equity ratio is small enough, the banker may be willing to inject fresh equity. Growth nevertheless needs to be rapid so that the bank can grow out of its problems. Thus, good growth opportunities are essential.

Recall that the banker’s earnings during period-2 will be zero if the bank retains its original size, 1. Suppose \( E < E_{\text{Short}}^* \); if the maximal achievable size \( \bar{S}_2 \) is large enough, growth is profitable. Alternatively, the banker can increase his earnings by \( (1 - \bar{S}_2)E(1 + r) \) through shrinkage. Hence, both alternatives - growth or shrinkage - are at the same time more profitable than retaining the initial bank size.

Banking literature greatly emphasizes the incentives to gamble for resurrection and grow rapidly (Kane, 1989; De Juan, 1996). In his thorough study of the S&L crisis in 1980s, Kane (1989, p.3) sheds light on the gambling incentives as follows.

Since about 1984, between 600 and 800 thrift institutions have been hopelessly insolvent… the net value of these crippled firms’ assets has sunk so far under water that their managers’ only hope of becoming profitable again has been to expand their firms’ funding base and to invest new funds they rise in the speculative manner. The idea is to “grow out of their problems” by undertaking longshot new lending and funding activities that essentially renew and expand (or “double up”) the lost bets of the past.

Kane lists several examples on rapid growth. According to his findings, the value of the insolvent bank’s stock jumps in the months immediately after an insolvent bank embarks on the high risk strategy to grow out of its insolvency.\(^\text{19}\)

The cited evidence seems to be more optimistic towards rapid growth and gambling for resurrection than the model. Yet, a more careful study discloses that the contradiction is imaginary. Before the S&L crisis, the equity capital requirements were lowered (Kane, 1989, p.54; White, 1991, p.82-83). Furthermore, the amount of bank equity was overstated, because the equity requirements were based on historical book values rather than market values. Most of all, the equity requirements involved five-year averages of net worth and of liabilities and, for de novo

\(^{18}\) The assumption that a bank is insolvent without growth is not critical. Even if the burden of loan losses is so small that the bank may be solvent after period-2 without growth, equity ratio \( E_{\text{Short}}^* \) eliminates growth.

\(^{19}\) Fleming et al. (1996) report an impressive example on loan rollovers and gambling for resurrection. In Lithuania, an insolvent bank, which was refinancing its defaulted borrowers by rolling over their loans, expanded its assets from $16 million in 1993 to the $77 in 1994 and $169 in 1995.
institutions, a twenty-year phase-in period before the thrift had to comply fully with the requirement. “Thus, for a thrift that grew rapidly in short bursts, and especially for a growing de novo thrift, the actual equity capital needed at the margin to support that growth was only a fraction of the nominal net worth requirement (White, 1991, p. 83).”

It appears that the reduction of the equity requirement together with the accounting loopholes, sharply cut the costs of growth, thereby encouraging banks to gamble for resurrection through aggressive growth. This explanation is convenient with Proposition 8.

Consequently, bank’s rapid growth as such is not a problem. Rapid growth is likely to cause problems only if the regulator does not force rapidly growing banks to maintain the normal equity requirement. When it is maintained, gambling for resurrection can be eliminated.

6.3 Growth with climbing interest on deposits

Although the previous subsection offered few interesting results on gambling for resurrection, the analysis had shortcomings. The maximal achievable size was fixed. Further, deposit rate did not react to growth. In addition, some of the results were surprising. Proposition 7 informed that the banks with the largest burdens of hidden loan losses are the most unwilling to grow. When banks operate under fixed growth opportunities, \( \bar{S}_2 \), and possess different volumes of hidden loan losses, it is possible that only the banks with slightest volumes of hidden loan losses will grow. The intuition is, of course, obvious. Growth calls for the injection of fresh equity. The larger the burden of loan losses, the more unwilling the banker is to inject fresh equity. This result, however, contradicts the standard arguments that the most insolvent banks are extremely willing to grow out of their problems, since they have nothing to lose. The purpose of this subsection is to show that the model can be modified in such a way that the most insolvent banks favour growth the most.

Bank size has no fixed upper limit, but the size is implicitly limited by deposit interest rate, which climbs as the bank grows and attracts more and more deposits. The term \( X(S'_2) \) marks the extra costs of deposits a bank must pay over \( r \) when its size exceeds a bank-specific size \( S'_2 \) (here \( X(S'_2) = X'(S'_2) = 0, X''(S'_2) > 0 \) and \( X'''(S'_2) > 0 \) when \( S'_2 > S''_2, X(\infty) = \infty \)). Up to \( S'_2 \), it is thus enough to pay interest \( r \) on deposits. \( S'_2 \) is assumed to be so large and the required equity ratio so small, under \( E_{short} \), that each bank optimally grows. Given the assumptions, the banks’ optimal growth policy is studied.

Under symmetric information, banks monitor. Each bank chooses (at most) its
maximal bank-specific size, $S'_2$, and pays interest $r$ on deposits. If a bank size exceeded $S'_2$, the loan interest, $R_m$, would not cover the cost of lending and the bank would make losses.

Under asymmetric information, each $S'_2$ is unknown to the regulator, who thus cannot impose bank-specific maximum sizes. Suppose that a bank neglected monitoring during period-1, hid its loan losses and keeps on operating during period-2. As regards to its size during period-2, Appendix G provides the following result.

**Proposition 8.** *A non-monitoring bank chooses for period-2 a size which is socially too large (over $S'_2$ ). When the equity requirement is sufficiently low, the chosen size grows with the share of inherited non-performing loans, $\hat{l}_1$.*

Two motives drive a bank to grow. First, a non-monitoring bank avoids the costs of monitoring and thus each successful loan has a positive interest rate spread, $R_m > r + c$. A monitoring bank has no such a spread, $R_m = r + c + m$. Since lending is more profitable for a non-monitoring bank, it will choose a larger size – and pay more for deposits - than a monitoring bank. The size is socially too large even if the bank possesses no hidden loan losses, $\hat{l}_1 = 0$. Second, the heavier the burden of hidden loan losses, the lower the expected bank returns are during period-2. As a result, the heavier the burden of hidden loan losses, the more the bank is ready to pay for deposits in order to grow out of its problems. If the gamble for resurrection succeeds, the bank makes good returns. If unsuccessful, the bank collapses and the regulator suffers the costs of the gamble, most of all the high payments to depositors. Consequently, the most insolvent banks favour growth the most.

Although the regulator cannot use bank size as a tool of control, she can control growth indirectly by supervising interest rates (a monitoring bank pays interest $r$ on deposits)

**Corollary 2.** *High deposit rate reveals a gamble for resurrection to the regulator.*

Unfortunately, the model is too simple to investigate lending at the aggregate level. However, it is possible that hidden loan losses may generate lending booms. In the model above, the supply of loans is generous, because banks must roll over the non-performing loans. Furthermore, the banks gamble for resurrection by growing rapidly and paying high interest on deposits. When the hidden loan losses finally surface, the bubble bursts: most of the financed projects proved to be worthless, banks are insolvent and the value of deposits slumps. The regulator then indemnifies deposits. This
kind of boom-bust cycle may arise even without irrational, over-optimistic investment manias (compare Kindleberger, 2000).

7. Conclusion

According to the standard banking theory, the problem of moral hazard arises between a bank and its depositors - or a deposit insurance agent - if the bank can seek a correlated risk for its loans (e.g. Holmström & Tirole, 1997). This paper has pointed out that moral hazard may arise even when loan risks are quite diversified if the bank can hide its loan losses by rolling over the defaulted loans. The result expands the magnitude of moral hazard and may help to understand recent banking crises.

The paper has also studied how the time frame of the lending relationships affects on the magnitude of moral hazard. The model indicates that when the regulator’s auditing system is weak, moral hazard is more severe under long-term lending, since the bank can then hide its loan losses by extending the maturity of problem loans. The converse occurs, when the auditing system is strong. In that case, moral hazard is remote under long-term lending. First, banks cannot now hide their loan losses. Furthermore, when the regulator uncovers loan losses, she liquidates a bank. Since ongoing long-term loans have a minimal liquidation value, the bank liquidation yields nothing to the banker. This makes the moral hazard behaviour unprofitable.

As to the regulatory recommendations, the model has stressed the importance of the bank supervision/auditing. In a long run, the quality of the bank supervision ought to be strengthened so that banks cannot hide their true financial condition. Simultaneously, the monitoring costs of banks should be reduced so that banks are more motivated to monitor their borrowers. Unfortunately, these improvements take time. In the short run, the regulator optimally raises the equity requirement over the normal ratio, if the quality of the bank supervision is weak, bank transparency is poor or the costs of monitoring borrowers are high for banks. The composition of equity capital must be designed with extreme care so that the amount of equity provides a truthful signal of bank solvency. Interest receivables, for instance, should be omitted form equity capital. A bank’s attempt to shrink its lending and pay out equity capital should alert the regulator; the bank may be insolvent and it may attempt to pay out as much dividends as possible prior to the surfacing of insolvency. Gambling for resurrection through rapid growth can be eliminated simply by forcing rapidly growing banks to maintain the normal equity ratio. If the regulator cannot be sure whether or not the equity ratio is sufficient, aggressive growth, together with high deposit rates,
provides a noteworthy warning that the bank is insolvent and gambling for resurrection.

**Appendix A**

Appendix A proves Proposition 1. *Step 1* shows that the bank neglects monitoring when $E = 0$. Now, $\tilde{I}_{\text{Short}}$ meets $(1 - \tilde{I}_{\text{Short}})(1 + R_m) = 1 + r + c$ or $\tilde{I}_{\text{Short}} = m/(1 + R_m) > 0$. When the realized share of loan losses is lower than $\tilde{I}_{\text{Short}}$, with probability $F(\tilde{I}_{\text{Short}}) > 0$ (recall Assumption 2i), the banker’s earnings are positive. Thus, it is optimal to neglect monitoring. *Step 2* shows that the earnings are negative when the amount of equity is big enough. Let $\overline{E}$ mark the smallest amount of equity that meets $(1 - \overline{L})(1 + R_m) = (1 + r)(1 - \overline{E}) + c$. When $E \geq \overline{E}$, the banker’s earnings

$$\int_{\overline{L}}^{\overline{L}} (1 - l)(1 + R_m) - (1 - E)(1 + r) - c \cdot f(l) \, dl - (1 + r)E,$$

(A.1)

which is negative (recall (2.4)). *Step 3* points out how the earnings are decreasing in equity when $E < \overline{E}$. From (3.1) or

$$\int_{\overline{L}}^{\overline{L}} (1 - l)(1 + R_m) \cdot f(l) \, dl - (1 + r + c),$$

(A.2)

it is easy to see $d\pi/dE = [F(\tilde{I}_{\text{Short}}) - 1](1 + r) < 0$, since $F(\tilde{I}_{\text{Short}}) < 1$. In sum, the earnings are positive when $E = 0$, negative when $E \geq \overline{E}$ and decreasing in $E$ when $0 < E < \overline{E}$. There exist $E_{\text{Short}}^*, 0 < E_{\text{Short}}^* < \overline{E}$, such that the earnings are zero. QED
Appendix B

Appendix B proves Proposition 3 using (5.5) and three steps. *Step 1* indicates that the bank neglects monitoring when $E = 0$. In that case, the banker’s expected earnings are

$$h \int \frac{i}{L} (1 - l_i)R_m - r - c \ f(l_i) \ dl_i,$$

(B.1)

where $(1 - \tilde{i})R_m - r - c = 0$. Since $\tilde{i}_1 > L_1$ (Assumption 2), there exists a positive probability $F(\tilde{i}_1)$ that the bank makes a profit. Hence, (B.1) is positive. Since the bank incurs no costs, the banker’s earnings are positive without monitoring. *Step 2* points out that the earnings are negative without monitoring when $E = 1$. The banker earns $\Pi(l)$ that satisfies

$$h \int \frac{i}{L} (1 - l_i)R_m - c \ f(l_i) \ dl_i -(1 + r) < h \int \frac{\tilde{i}}{L} (1 - l_i)(1 + R_m) \ f(l_i) \ dl_i -(1 + r + c) < -(1 - h)(1 + r + c).$$

(B.2)

*Step 3* indicates that the banker’s earnings are decreasing in equity

$$\frac{d \Pi(E)}{d E} = h \int \frac{i}{L} r \ f(l_i) \ dl_i - (1 + r) < 0.$$  

(B.3)

In sum, the earnings are positive when $E = 0$, negative when $E = 1$ and decreasing in $E$, when $0 < E < 1$. There exists an incentive compatible amount of equity, $0 < E_{Long}^* < 1$, that eliminates the non-monitoring strategy. QED

Appendix C

Appendix C proves Proposition 4: the weaker the auditing system, the bigger the incentive compatible amount of equity. Appendix B showed that $d \Pi/d E_{Long}^* < 0$ (recall (B.3)). Now (5.5) implies $d\Pi(E_{Long}^*)/d h = \pi_i (E_{Long}^*) > 0$. Putting $d \Pi(E_{Long}^*)/d h > 0$ and $d \Pi/d E_{Long}^* < 0$ together provides and a total differential
\[
\frac{d E_{\text{Long}}^*}{d h} = \frac{d \Pi/d h}{-d \Pi/d E_{\text{Long}}^*} > 0.
\]

QED.

**Appendix D**

Appendix D proves proposition 5: the heavier the costs of monitoring, the larger the incentive compatible amount of equity. Recalling \( R_m = r + c + m \) and using (5.5), it is easy to get

\[
\frac{d \Pi(E_{\text{Long}}^*)}{d m} = h \int_l \left(1 - l_i\right) f(l_i) dl_i > 0.
\]

Putting \( d \Pi(E_{\text{Long}}^*)/d m > 0 \) and \( d \Pi(E_{\text{Long}}^*)/d E < 0 \) (recall (B.3)) together gives a total differential

\[
d E_{\text{Long}}^* / d m = \frac{d \Pi / d m}{-d \Pi / d E_{\text{Long}}^*} > 0.
\]

QED

**Appendix E**

The bank returns during period-2 (recall (6.3)) can be rewritten as

\[
\int_l \left[ S_z (1 + R_m)(1 - l_z) - S_z (1 - E)(1 + r) - S_z c - c_h - \right] \\
\hat{t}_i (1 + R_m) \left[ (1 - l_z) (1 + R_m) - \frac{R_m (1 + r)}{1 + R_m} \right] f(l_z) dl_z
\]

The term in the brackets is positive,
\[(1 - l_2)(1 + R_m) - \frac{R_m(1 + r)}{1 + R_m} \geq (1 - \bar{L})(1 + R_m) - \frac{R_m(1 + r)}{1 + R_m} = \]
\[\left\{1 - \bar{L}(1 + R_m) \right\} + R_m \left\{1 - \frac{1 + r}{1 + R_m} \right\} > 0.\]  

(E.2)

The contents of both parenthesis are positive. Since the second term of (E.1) is negative and since the returns from growth are non-negative in total, the first term of (E.1) must be positive even on the upper limit, \(\bar{l}_2\),

\[S_2(1 + R_m)(1 - \bar{l}_2) - S_2(1 - E)(1 + r) - S_2c - c_h > 0. \]  

(E.3)

QED.

Appendix F

Lemma 1 - “\(\bar{l}_2(\hat{l}_1,E;S_2)\) is decreasing in \(\hat{l}_1\), but increasing in \(S_2\)” - is first proved. Now (6.4) provides

\[\frac{d\bar{l}_2}{d\hat{l}_1} = \frac{-(1 + R_m)[(1 + R_m)(1 - \bar{l}_2) - R_m(1 + r)/(1 + R_m)]}{(1 + R_m)[S_2 - (1 + R_m)\hat{l}_1]} < 0,\]  

(F.1)

\[\frac{d\bar{l}_2}{dS_2} = \frac{(1 + R_m)(1 - \bar{l}_2) - (1 - E)(1 + r) - c}{(1 + R_m)[S_2 - (1 + R_m)\hat{l}_1]} > 0,\]

since \((1 + R_m)(1 - \bar{l}_2) - (1 - E)(1 + r) - c > 0\) (Appendix E, (E.3)) and \((1 + R_m)(1 - \bar{l}_2) - R_m(1 + r)/(1 + R_m) > 0\) (Appendix E, (E.2)). Next we prove Lemma 2: “\(\bar{l}_2(\hat{l}_1,E;S_2)\) would approach \(\bar{l}_2(0,E;S_2)\) if \(S_2\) could grow without bound. Since \(S_2\) is assumed to be finite, it is known that \(\bar{l}_2(\hat{l}_1,E;S_2) < \bar{l}_2(0,E;S_2)\) with every \(S_2\)”.

Now (6.4) can be expressed as

\[(1 + R_m)(1 - \bar{l}_2) - (1 - E)(1 + r) - c = \frac{\hat{l}_1(1 + R_m)[(1 + R_m)(1 - \bar{l}_2) - R_m(1 + r)/(1 + R_m)]}{S_2} + c_h.\]  

(F.2)
First, recall (F.1). Then note that in (F.2) both sides are positive and diminishing in $\tilde{l}_2$. The term on the R.H.S is the relative burden of period-1 loan losses. If $S_2$ could grow without bound, the relative burden (the R.H.S, where the numerator is finite) would decline to zero with every $\hat{l}_1$. Thus, $\hat{l}_1$ would be meaningless and $\tilde{l}_2(\hat{l}_1, E; S_2)$ would rise to $\tilde{l}_2(0, E; S_2)$ on the L.H.S. The bank would operate as if it had no inherited loan losses, and (7.6) would simplify to $(1 + R_m)[l - \tilde{l}_2(0, E; S_2)] - (1 - E)(1 + r) - c = 0$ (Notice that $\tilde{l}_2(0, E; S_2)$ is identical to $\tilde{l}_{Short}$ in (3.1)). Yet, since $S_2$ is assumed to be finite, the relative burden of inherited loan losses does not fully vanish and $\tilde{l}_2(\hat{l}_1, E; S_2) < \tilde{l}_2(0, E; S_2)$. QED

Appendix G

This appendix proves Lemma 3: when $E = E_{Short}^{*}$ growth is not optimal, but when $E < E_{Short}^{*}$ growth is optimal if it is rapid enough. The banker’s earnings during period-2, (G.3), can be rewritten as

$$
\Pi_2 = (S_2 - 1) \left\{ \int_\mathcal{L} (1 + R_m)(1 - l_2) - (1 - E)(1 + r) - c \cdot f(l_2) dl_2 - (1 + r)E \right\} 
+ \int_\mathcal{L} \left[ 1 - (1 + R_m)\hat{l}_1 \right] (1 + R_m)(1 - l_2) - (1 - E - \hat{l}_1 R_m)(1 + r) - c - c_h \cdot f(l_2) dl_2.
$$

If the term in parenthesis is denoted by $\pi_2(\hat{l}_1, E; S_2) - (1 + r)E$, (G.1) implies

$$
\Pi_2 > (S_2 - 1) \left[ \pi_2(\hat{l}_1, E; S_2) - (1 + r)E \right] - (1 + r)(1 - E) - c - c_h,
$$

since the lower line of (G.1) exceeds $-(1 + r)(1 - E) - c - c_h$. In (G.2), $S_2 - 1$ is positive and rising in $S_2$. Further, $\Pi_2 > 0$ and approaches to infinity when $S_2$ grows without bound if the term in the brackets is positive. Next, the term is shown to be positive when $E < E_{Short}^{*}$ and $S_2$ is large enough. To begin with, it is useful to denote bank returns as
\[ \pi_2(\hat{l}_1, E; S_2) = \int \pi_2 f(l_2) dl_2, \quad \text{where} \quad \pi_2 = (1 + R_m)(1 - l_2) - (1 - E)(1 + r) - c. \quad (G.3) \]

From Appendix A it is known that without the burden of hidden loan losses, equity requirement \( E_{\text{Short}}^* \) eliminates risk taking: that is, \( \pi_2(0, E_{\text{Short}}^*; S_2) = (1 + r)E_{\text{Short}}^* \) or

\[ \int \pi_2(E_{\text{Short}}^*) f(l_2) dl_2 = (1 + r)E_{\text{Short}}^*. \quad (G.4) \]

Given Appendix A, when \( E < E_{\text{Short}}^* \) there is \( \epsilon > 0 \) so that the banker’s earnings exceed \( \epsilon \)

\[ \int \pi_2(E) f(l_2) dl_2 - (1 + r)E > \epsilon. \quad (G.5) \]

With every \( \epsilon > 0 \), there exist \( \tau > 0 \) so that

\[ \int \pi_2(E) f(l_2) dl_2 > \int \pi_2(E) f(l_2) dl_2 - \epsilon. \quad (G.6) \]

Since \( \bar{I}_2(\hat{l}_1, E; S_2) \) is moving upward in \( S_2 \) (Lemma 1) and approaches \( \bar{I}_2(0, E; S_2) \) when \( S_2 \) grows without bound (Lemma 2), there exist an infinite \( \bar{S}_2 \) so that \( \bar{I}_2(\hat{l}_1, E; \bar{S}_2) > \bar{I}_2(0, E; \bar{S}_2) - \tau \). Then,

\[ \pi_2(\hat{l}_1, E; \bar{S}_2) = \int \pi_2(E) f(l_2) dl_2 > \int \pi_2(E) f(l_2) dl_2 - \epsilon > (1 + r)E, \quad (G.7) \]

which means that \( \pi_2(\hat{l}_1, E; \bar{S}_2) - (1 + r)E > 0 \). Then, the term in brackets of (G.2) is positive. Hence, \( \Pi_2 > 0 \) in (G.2); the banker’s earnings from growth are positive when \( E < E_{\text{Short}}^* \) and growth is rapid enough. Furthermore, the earnings approach to infinity when \( S_2 \) grows without bound.
Regarding $E = E^*_{Short}$, see (G.1). The lower line is non-positive (recall (5.4)). When $E = E^*_{Short}$ the upper line is negative since $\tilde{l}_2(\hat{L}_1, E; S_2) < \tilde{l}_2(0, E; S_2)$. Hence, the banker’s earnings from growth are negative when $E = E^*_{Short}$. From (G.1) it is easy to solve

$$\frac{d \Pi_2}{d \hat{L}_1} = \frac{\hat{L}_2}{2} - (1 + R_m)(1 - l_2)(1 + R_m) - R_m(1 + r)/(1 + R_m) < 0. \quad (G.8)$$

Since the term in brackets is positive (Appendix E), the earnings from growth are decreasing in the volume of hidden loan losses from period-1. QED.

**Appendix H**

**Step 1:** The banker’s earnings from growth are assumed to be positive

$$S_2 \int_{\hat{L}_1}^{\tilde{l}_2} (1 - l_2)(1 + R_m) - (1 - E)(1 + r) - c \cdot f(l_2)dl_2 - (S_2 - 1)(1 + r)E$$

$$- \hat{l}_1(1 + R_m)^2 \int_{\hat{L}_1}^{\tilde{l}_2} 1 - l_2 f(l_2)dl_2 > F(\tilde{l}_2)\left[-\hat{l}_1 R_m (1 + r) + X(S_2) + c_h\right]. \quad (H.1)$$

On the upper limit, $\tilde{l}_2$, bank returns are zero

$$S_2 \left[(1 - \tilde{l}_2)(1 + R_m) - (1 - E)(1 + r) - c\right] - (1 + R_m)^2 \hat{l}_1 (1 - \tilde{l}_2) = - \hat{l}_1 R_m (1 + r) + X(S_2) + c_h. \quad (H.2)$$

Inserting (H.2) into (H.1) indicates
\[
\frac{1}{F(l_2)} \left[ \int_{l_2}^{l_1} (1-l_2)(1+R_m) - (1-E)(1+r) - c \ f(l_2) dl_2 - (1+r)E \right] > \\
\left[ (1-l_2)(1+R_m) - (1-E)(1+r) - c \right] + \frac{1}{2} \frac{(1-R_m)^2}{S_2} \left[ \int_{l_2}^{l_1} f(l_2) dl_2 - (1-l_2) \right] - \frac{(1+r)E}{F(l_2)S_2}. \quad (H.3)
\]

The term in parenthesis is positive. Step 2: From (H.1), it is easy to solve the optimal bank size

\[
\int_{l_2}^{l_1} (1+R_m)(1-l_2) - (1-E)(1+r) - X'(S_2^*) - c \ f(l_2) dl_2 - (1+r)E = 0. \quad (H.4)
\]

Since the L.H.S of (H.4) is positive when \( E < E_{short} \) and \( S_2 = S_2' \) (recall \( X'(S_2') = 0 \)), the size is too large. That is, the size exceeds \( S_2' \) with every \( \hat{l}_1 \) and \( X'(S_2^*) > 0 \). A total differential gives

\[
\frac{dS_2^*}{dl_1} = -\frac{d\hat{l}_2}{dl_1} \Phi \\
\left[ \frac{\frac{d\hat{l}_2}{dS_2^*} \Phi}{\int_{l_2}^{l_1} X''(S_2^*) f(l_2) dl_2} \right], \quad (H.5)
\]

where \( \Phi = (1+R_m)(1-l_2) - (1+r)(1-E) - c - X'(S_2^*) \). The nominator is negative due to the second-order constraint of (H.4) (a more detailed proof is possible, but omitted for brevity). The objective is to show that the case \( dS_2^*/dl_1 > 0 \) is possible. Thus, the denominator must be negative. Since \( d\hat{l}_2/d\hat{l}_1 < 0 \) (Lemma 1), \( \Phi \) has to be negative or \( X'(S_2^*) > (1+R_m)(1-l_2) - (1+r)(1-E) - c \). To study this, (H.4) is first rewritten as

\[
\frac{1}{F(l_2)} \left[ \int_{l_2}^{l_1} (1+R_m)(1-l_2) - (1+r)(1-E) - c \ f(l_2) dl_2 - (1+r)E \right] = X'(S_2^*), \quad (H.6)
\]
and this is then inserted into (H.3),

\[
X'(S_2) - \left[(1-\bar{l}_2)(1+R_m) - (1-E)(1+r) - c\right] > \\
+ \frac{\hat{I}_1(1+R_m)^2}{S_2} \int_{\bar{l}}^{l} 1 - l_2 f(l_2)dl_2 - (1-\bar{l}_2) \right) - \frac{(1+r)E}{F(\bar{l}_2)S_2}.
\]

(H.7)

Hence, \( \Phi \) is negative, when the L.H.S is positive. This is true with certainty when the R.H.S is positive. On the R.H.S the first term is positive and the second term is negative. The R.H.S is positive if the second term is almost zero. This is true when \( E \) is small enough. In sum, when \( E \) is small enough, \( \Phi \) is negative and thus \( ds_2^*/d\hat{l}_1 > 0 \). QED

References


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