Adverse selection in dynamic matching markets

Klaus Kultti
University of Helsinki and HECER

and

Juuso Vanhala
Bank of Finland

and

Timo Vesala
University of Helsinki, RUESG and HECER

Discussion Paper No. 154
March 2007

ISSN 1795-0562
Adverse selection in dynamic matching markets*

Abstract

We study the Akerlofian adverse selection problem in a dynamic matching model where the competitive situation varies across different meetings. The 'lemons principle' is shown to limit the high quality sales within a wider range of quality distributions than in the Walrasian benchmark. High quality goods can nevertheless be traded, albeit less frequently than the low quality goods. For certain quality distributions, there exists a 'partially pooling' steady state where high quality sellers are active whenever at least two buyers compete for the good. Otherwise, the model features cycles in a sense that high quality goods are traded only in non-consecutive periods.

JEL Classification: D82, D83.

Keywords: Adverse selection, search, matching, lemons principle.

Klaus Kultti  
Department of Economics  
University of Helsinki  
P.O. Box 17 (Arkadiankatu 7)  
FI-00014 University of Helsinki  
FINLAND  
e-mail: klaus.kultti@helsinki.fi

Juuso Vanhala  
Monetary Policy and Research Department  
Bank of Finland  
P.O. Box 160  
FI-00101 Helsinki  
FINLAND  
e-mail: juuso.vanhala@bof.fi

Timo Vesala  
Department of Economics  
University of Helsinki  
P.O. Box 17 (Arkadiankatu 7)  
FI-00014 University of Helsinki  
FINLAND  
e-mail: tvesala@valt.helsinki.fi

* We are grateful to Juha Kilponen, Nicolás Porteiro and Juuso Välimäki whose comments and guidance helped to improve the paper. We also thank seminar audiences at the Annual Meeting of the Finnish Society for Economic Research 2006, the Spring Meeting of Young Economists 2006 and ESEM 2006. Financial support from the Yrjö Jahnsson Foundation is gratefully acknowledged. The usual disclaimer applies.
1 Introduction

Akerlof (1970) shows in the pioneering work on adverse selection that when goods for sale come in different qualities and when buyers cannot observe the quality of the good there may be a market failure: only goods of low quality are traded and the potentially gainful trade of high quality goods remains unrealized. This is the ‘lemons principle’ and it may occur when the proportion of high quality goods is too low and their valuation by the sellers (or production cost) sufficiently higher than that of low quality goods. The result is based on an analysis of a static Walrasian market and it depends heavily on restrictions of trading opportunities. It is thus not clear whether the lemons principle could still hold in a dynamic setting. If one thinks of a dynamic model where only low quality goods are traded, and where the quality of new goods or traders is determined randomly, one would expect that the proportion of high quality goods increases over time, making market failure less likely. This intuition is formalized e.g. by Janssen and Roy (2004) who consider a dynamic Walrasian market where new sellers enter the market in each period. The model exhibits cycles, however, because once the average quality in the market - and thereby the competitive price - is high enough for the high quality goods to be traded the proportion of these goods begins to decrease and soon the lemons principle destroys the market for high quality goods.

We study the Akerlofian adverse selection problem in a dynamic matching model. Our purpose is to allow for two important deviations from the Walrasian setting. First, trading in the market is decentralized as transactions are concluded in private meetings between buyers and sellers. Besides being a plausible feature of many empirically relevant durable goods markets, e.g. markets for used cars, the decentralized structure enables an explicit characterization of price formation. Second, as the matching between buyers and sellers is modeled as an ‘urn-ball’ process (first developed by Butters, 1977, and Hall, 1979), the number of competing buyers a seller confronts is determined randomly. Effectively, the local conditions among buyers varies across different trading opportunities. This feature gives rise to the possibility that the feasibility of high quality trades is not only conditional on the relative share of high quality goods in the market but also on the competitive situation at the particular contact. As a result, high quality trades may not be ‘doomed’ to cyclicality but a steady state may arise.
where high quality goods are traded every period in certain competitive environments. The urn-ball model also enables the use of auction rather than bargaining as the price formation method\(^1\). This is convenient as bargaining with incomplete information typically leads to multiplicity of equilibria\(^2\).

We analyze the dynamic trading pattern in all possible distributions over high and low quality goods and study the existence of possible pooling regimes where the goods of different qualities are sold at the same price. We start with the standard case where the trading practice is symmetric in a sense that all seller types trade every period with equal probability. In the analysis, the valuation of future trading opportunities or impatience of agents determines the importance of trading frictions. We show that when trading frictions are significant enough the 'lemons principle' actually inhibits the high quality trades within a wider range of quality distributions than in Akerlof's competitive benchmark. Trading frictions generally improve buyers' ability to earn positive rents from the low quality trades which creates a negative externality on high quality sellers' trading opportunities. As a result, the proportion of the high quality goods has to be relatively high for buyers to be willing to propose a price offer that results in both high and low quality trades. When the discount factor becomes large enough, i.e. the dynamic trading frictions ease off, the quality distributions for which high quality trades are feasible are the same as in the static Walrasian benchmark.

However, if the quality distribution does not support the pooling steady state with the equal frequency of trades, the high quality sellers can still expect to trade, albeit less frequently than the low quality sellers. We consider a partially pooling equilibrium with asymmetric trading where the sellers of low quality goods trade whenever at least one buyer approaches them but the sellers of high quality goods trade only if there is a competitive situation between two or more buyer candidates. We show that there is a range of quality distributions which in the Akerlofian world would make the high quality trades infeasible but in our dynamic setting support the partially pooling equilibrium. Such an asymmetric trading practice, however, ceases to exist as a steady state outcome if the discount factor is high enough. As the agents become more patient, the opportunity cost of sales increases and the low quality sellers can seize an ever increasing fraction of the rent when in contact with a single buyer.

---

\(^1\) As a technical notice, Lu and McAfee (1996) show that in a frictional environment auction but not bargaining is an 'evolutionary stable' mechanism.

\(^2\) See for example Muthoo (1999, ch. 9.8) and Fudenberg and Tirole (1991, ch. 10.4)
candidate. This attracts buyers to deviate and offer a price that will be accepted also by the high quality sellers. As a result, the outflow of high quality goods outruns their inflow and the steady state is no longer sustainable. The fact that an increase in the value of the future trading opportunities limits the sustainability of the partially pooling equilibrium runs counter to the view that a dynamic trading perspective is a necessary precondition for a successful resolution of the lemons problem.

There are also quality distributions that support neither of the proposed steady states. In those cases, the model features pooling trading only in cycles, as in Janssen and Roy (2004). We show that high quality goods are then traded in non-consecutive periods. After the active trading period, the proportion of the high quality sellers immediately drops below the level that would make trading with high quality goods feasible and the lemons principle kicks in.

In the related literature, Blouin (2003) and Inderst and Müller (2002) provide the first formalizations of the lemons problem in search and matching framework. These studies do not, however, consider pooling outcomes but seek to develop price mechanisms that would induce separation in a sense that high and low quality goods could be traded at different prices. Blouin (2003) does not model price formation explicitly but makes the pairwise matched agents play an exogenously given bimatrix price game where the agents can choose between two exogenous price levels, a high and a low price\(^3\). He then proves that there exists a continuum of semi-separating equilibria where low quality sellers quote the low price frequently enough so that high quality goods can be traded at the high price. Inderst and Müller (2002), in turn, consider a competitive search model where different trading environments can coexist. High quality goods are sold at a more congested high-price submarket while low quality goods are sold at less congested low-price submarket, so that the sellers who want to get a high price must accept longer time in circulation. However, as in Blouin (2003), strategic interactions are absent in the model: Since buyers are taken to be the long side of the whole market, prices in different submarkets are determined by the condition that secures the buyers no more than their reservation utilities.

There are also other studies that develop various separating mechanisms to overcome the lemons problem. These papers include Janssen and Roy (2002) who show

\(^3\) Obviously, this practice helps circumvent the indeterminacy problem related to bilateral bargaining models with asymmetric information.
in a dynamic Walrasian model without market entry that the price mechanism can sort the sellers of different qualities into different time periods: Low quality goods are traded earlier at lower prices while high quality sellers have to wait until the average quality of the remaining sellers is sufficiently high for trade to take place. Hendel and Lizzeri (1999) and Hendel et al. (2005), in turn, show that the lemons problem may not arise at all if also the ownership structure is thought endogenous. The idea is that if the quality of the good depreciates over time, the vintage of the good is then a noisy signal of the quality. Market failure can be avoided if high valuation consumers buy new goods and sell relatively ’quickly’ while low valuation consumers self-select to vintage markets.

The rest of the paper is organized as follows: Section 2 characterizes the basic setup of the model. The existence of the pooling steady states is analyzed in Section 3. Section 4 provides a brief discussion on the properties of the pooling cycles. Section 5 concludes.

2 Preliminaries

There are equal numbers of buyers and sellers who come to the market every period. Without loss of generality, we assume that their numbers are normalized to unity. Proportion $q$ of the sellers produce a high quality good with production cost $c_H$. The rest of the sellers produce a low quality good with production cost $c_L$. From the high quality goods the buyers get utility $u_H$, and from the low quality goods they get utility $u_L$. All buyers are identical, and they observe the quality of the good only after they have purchased it. Contracts conditional on the quality of the good are assumed infeasible.

We assume the following ranking of the magnitudes:

$$u_H > c_H > u_L > c_L.$$  

Without loss of generality, we normalize $c_L = 0$.

In a competitive and static setting, this assumption generates market failure whenever $\hat{u} = qu_H + (1-q)u_L < c_H$, or

$$q < \frac{c_H - u_L}{u_H - u_L} \equiv \hat{q}.$$  

4
If this is the case, the high quality sellers are reluctant to trade as the most a buyer is willing to pay is \( \hat{u} \) which is less than the high quality production cost. We make no restrictions on the quality distribution but investigate the functioning of the dynamic search market for all possible values of \( q \); i.e. \( q \in [0,1] \).

Trading in the market is uncoordinated and decentralized. Matching between buyers and sellers is modeled as an 'urn-ball' process à la Butters (1977) and Hall (1979). Buyers ('balls') are assumed to contact the sellers ('urns'). The number of unmatched buyers in the market is \( M \) and the number of unmatched sellers is \( N \). We assume large markets with \( M \to \infty, N \to \infty \), so that the number of buyers a seller expects to meet is given by a Poisson distribution with parameter \( M/N = \theta \). The probability that \( k \) buyers contact a particular seller is \( \frac{e^\theta}{k!} e^{-\theta} \). We assume that the buyers contact sellers in an uncoordinated manner, and to that end we focus on a symmetric strategy. This means that the buyers mix over which sellers to contact. This results in a coordination failure in the market as some 'urns' (sellers) do not receive any 'balls' (buyers) and thereby remain unmatched. Some 'urns', however, receive many 'balls'. But since exactly one 'urn' and one 'ball' establish a match, in a congestion situation some of the 'balls' have to remain unmatched. This coordination failure induces non-trivial frictions in the market because it extends the time span an agent remains unmatched, and the agents discount future returns with the common discount factor \( \delta \).

In the next section we consider the feasibility of pooling steady state equilibrium. We start with a steady state with symmetric trading pattern, which means that all seller types trade with equal probabilities. When such a steady state is infeasible, we show that another pooling steady state may arise where trading is asymmetric in the sense that high quality sellers conduct trades less frequently than the low quality sellers.

3 Steady state equilibria

3.1 Pooling equilibrium

In a pooling steady state equilibrium there is no difference between the probability of trading high and low quality goods; trade takes place whenever at least one buyer
candidate contacts a seller. Obviously, under symmetric trading, the steady state fractions of high and low quality goods must equal the proportions of high quality sellers among the incoming sellers, so that the expected quality of a traded good in a steady state is \( \hat{u} = qu_H + (1 - q) u_L \).

The life time utility of buyers is denoted by \( B \) and the corresponding utilities of high and low quality sellers by \( S_H \) and \( S_L \). Buyers can observe how many other buyers are competing for the same good. With probability \( e^{-\theta} \), only one buyer approaches the seller and the buyer offers \( p^m \). Since we are looking for a steady state equilibrium where all qualities are traded, the price \( p^m \) must cover the cost \( c_H \) plus the loss of the value of remaining unmatched in the market; i.e. \( p^m = c_H + S_H \). The earnings of the high and low quality sellers are \( p^m - c_H = S_H \) and \( p^m - c_L = c_H + S_H \). With probability \( 1 - e^{-\theta} \), there are at least two bidders competing for the same good. The buyers raise their bids upto \( p^c \) where they are driven to their reservation utility levels; i.e., \( p^c \) solves \( \hat{u} - p^c = B \).

Evaluated at the end of a period the life-time utilities are determined by the following equations:

\[
B = \delta \left[ e^{-\theta} (\hat{u} - c_H - S_H) + (1 - e^{-\theta}) B \right],
\]
\[
S_L = \delta \left[ e^{-\theta} S_L + \theta e^{-\theta} (c_H + S_H) + (1 - (1 + \theta) e^{-\theta}) (\hat{u} - B) \right],
\]
\[
S_H = \delta \left[ e^{-\theta} S_H + \theta e^{-\theta} S_H + (1 - (1 + \theta) e^{-\theta}) (\hat{u} - c_H - B) \right].
\]

In a steady state, those who trade are replaced by an equal number of new agents. Since a unit measure of new buyers and sellers enter each period and since exactly one buyer and one seller are required for a complete transaction, there is a steady state with \( M = N \) or \( \theta = 1 \). Given such a steady state configuration, the system can be solved for \( B, S_L \) and \( S_H \) to obtain

\[
B = \frac{\delta}{e - \delta} (\hat{u} - c_H),
\]
\[
S_L = \frac{\delta (e - 2)}{e - \delta} (\hat{u} + \frac{c_H}{e - 2}),
\]
\[
S_H = \frac{\delta (e - 2)}{e - \delta} (\hat{u} - c_H).
\]

The viability of the pooling steady state equilibrium requires that each agent earns non-negative life-time utility and no buyer can profit by offering a different price. Obviously, \( S_L > 0 \) holds and requirements \( S_H, B \geq 0 \) are satisfied whenever
\( \hat{u} \geq c_H \) which corresponds to the condition in Akerlof’s benchmark. The relevant deviation for the buyer is the following: When the buyer is alone with the seller, he bids \( p_{d}^{m} = S_L < c_H + S_H \) so that the transaction is concluded only if the good is of low quality. When there is at least one other buyer candidate, the buyer is anyhow driven to his reservation utility level\(^4\). The life-time utility available from the deviation is given by

\[
B_d = \delta \left\{ e^{-1} (qB_d + (1 - q) (u_L - S_L)) + (1 - e^{-1}) B_d \right\}.
\]

Solving for \( B_d \) gives

\[
B_d = \frac{\delta (1 - q)}{e (1 - \delta) + \delta (1 - q)} (u_L - S_L).
\] \hspace{1cm} (4)

No buyer will deviate from the equilibrium trading practice if \( B \geq B_d \).

**Proposition 1** The pooling equilibrium is feasible when the proportion of high quality sellers satisfies

\[
q \geq \max \{ \hat{q}, q^p \},
\]

where \( q^p \) solves

\[
q^p = \hat{q} + \frac{(1 - q^p)((e - \delta)u_L - \delta(e - 1)c_H)}{e - \delta - \delta q^p(e - 1)}.
\]

The threshold \( q^p \) is greater than the Akerlof benchmark \( \hat{q} \) if

\[
\delta < \frac{e u_L}{(e - 1) c_H + u_L} \equiv \delta_1 < 1.
\]

**Proof.** For proof, see Appendix A.1. \( \blacksquare \)

Proposition 1 implies that the pooling equilibrium is viable within a narrower range of quality distributions than in the Akerlof benchmark when \( \delta \leq \delta_1 \). For \( \delta \in [\delta_1, 1] \), the outcomes coincide. Lower \( \delta \) magnifies the hindrance of the search frictions and increases the attractiveness of the deviating strategy for buyers; i.e. to restrict trades only with low quality sellers. Greater ’impatience’ thus creates a negative externality on the high quality sellers’ trading opportunities and extends the range of parameter values within which the lemons principle is in effect. In fact,

**Remark 1** The threshold \( q^p \) is monotonously decreasing in \( \delta \); i.e.

\[
\frac{\partial q^p}{\partial \delta} < 0.
\]

\(^4\) It can be shown that it does not matter whether the buyer bids \( p^c \) and earns \( B \) or does not place a bid at all and earns \( B_d \). The former option corresponds to the idea of ‘one-time deviation’, which is usually regarded as a sufficient test for the viability of an equilibrium.
Proof. For proof, see Appendix A.2.

Remark 1 thus implies that the 'lemons effect' is the strongest in the static limit of the model; i.e. when traders do not value future trading opportunities at all and have \( \delta = 0 \). In that extreme case, \( q^p = (\hat{q} + u_L)/(1 + u_L) > \hat{q} \). Figure 1 illustrates the viability of the pooling steady state in \( \delta q \)-plane.

3.2 Partially pooling equilibrium

We now address the question what happens if the quality distribution does not satisfy the condition in Proposition 1. We verify the possibility of another steady state where the low quality goods are traded whenever at least one buyer approaches a seller, while the high quality goods are traded only if there are multiple buyer candidates competing for the same good. We call this mode of equilibrium trades as partially pooling.

When there is only one buyer and one seller upon a meeting, the buyer rationally expects that trade will take place only if his trading partner is a low quality seller and the buyer makes a price offer accordingly; i.e. \( p^m = S_L \). When there are two or more buyers at the same meeting, the buyers know that all sellers trade and the winning bid
is the one that drives the buyer to his reservation utility level, which is $B$. Evaluated again at the end of a period the life-time utilities read as follows:

$$S_L = \delta \left\{ e^{-\theta} S_L + \theta e^{-\theta} S_L + (1 - (1 + \theta)e^{-\theta}) (\hat{U} - B) \right\},$$

$$S_H = \delta \left\{ e^{-\theta} S_H + \theta e^{-\theta} S_H + (1 - (1 + \theta)e^{-\theta}) (\hat{U} - c_H - B) \right\},$$

$$B = \delta \left\{ e^{-\theta} (QB + (1 - Q) (u_L - S_L)) + (1 - e^{-\theta}) B \right\}.$$

Note that since the high quality sellers conduct trades less frequently than the low quality sellers, their steady state fraction $Q$ must be greater than $q$, so that $\hat{U} = Q u_H + (1 - Q) u_L > \hat{u}$. However, even though the composition of sellers is now different from the case examined above, the corresponding steady state still features $M = N$, or $\theta = 1$. The steady state share of high quality sellers $Q$ in relation to $q$ then yields:

**Lemma 1** The partially pooling steady state implies

$$Q = \frac{q(e - 1)}{q + e - 2} > q.$$

**Proof.** Denote the measure of high (low) quality sellers in the market by $N_H = Q N$ ($N_L = (1 - Q) N$). Steady state implies that

$$
(1 - 2e^{-1}) N_H = q, \quad (5) \\
(1 - e^{-1}) N_L = 1 - q, \quad (6)
$$

The result follows directly from (5)-(6) and the fact that $Q = \frac{N_H}{N_H + N_L}$. ■

The value equations thus simplify to

$$S_L = \frac{\delta (e - 2)}{e - 2\delta} (\hat{U} - B), \quad (7)$$

$$S_H = \frac{\delta (e - 2)}{e - 2\delta} (\hat{U} - c_H - B), \quad (8)$$

$$B = \frac{\delta (1 - Q)}{(1 - \delta) e + \delta (1 - Q)} (u_L - S_L). \quad (9)$$

The viability of the partially pooling equilibrium requires that the life-time utilities are non-negative. Since $S_L > S_H$, the relevant conditions to be worked through are $S_H \geq 0$ and $B \geq 0$. Moreover, one also has to check that buyers do not have an incentive to deviate from the asymmetric trading pattern. Since $Q$ is such that $\hat{U} \geq c_H$, a solitary
buyer could feasibly propose an offer that would also be accepted by a high quality seller. The buyer’s utility from such a deviation yields

\[ B_d = \frac{\delta}{e} \{(\bar{U} - c_H - S_H) + (e - 1) B_d \}. \]  

(10)

No buyer will ever deviate from the equilibrium trading practice if \( B_d \leq B \).

**Proposition 2** The partially pooling equilibrium is feasible when the proportion of high quality sellers satisfies

\[ \underline{Q}^{pp} \leq Q \leq \overline{Q}^{pp}, \]

where \( \underline{Q}^{pp} \) and \( \overline{Q}^{pp} \) solve

\[
\underline{Q}^{pp} = \hat{q} + \frac{\delta(1 - Q^{pp})((e - 2\delta)u_L - \delta(e - 2)c_H)}{2\delta^2(e - 1 + \overline{Q}^{pp}) + e(\delta - 1 + Q^{pp} + e)}, \quad \text{and} \\
\overline{Q}^{pp} = \hat{q} + \frac{(e - \delta)(1 - Q^{pp})((e - 2\delta)u_L - \delta(e - 2)c_H)}{(e - 2\delta)(e - \delta - \delta Q^{pp}(e - 1))}. 
\]

The interval \([\underline{Q}^{pp}, \overline{Q}^{pp}]\) with the property \( \hat{q} \leq Q^{pp} \leq \overline{Q}^{pp} \) exists if

\[ \delta \leq \frac{eu_L}{(e - 2)c_H + 2u_L} \equiv \tilde{\delta}_2. \]

For the threshold \( \tilde{\delta}_2 \) it holds that \( \tilde{\delta}_1 < \tilde{\delta}_2 < 1 \).

**Proof.** For proof, see Appendix A.3. \( \blacksquare \)

Again, the proportion of high quality sellers has to be large enough, i.e. \( Q \geq Q^{pp} \), in order for the high quality price to be sufficiently high, not only to cover the cost \( c_H \) but also the buyers’ reservation value \( B \). On the other hand, this proportion should not be too large either, i.e. \( Q \leq \overline{Q}^{pp} \), because otherwise the buyer, when being the only purchaser candidate, would become tempted to offer a price that would attract also the high quality sellers to conduct trade. As a result, the partially pooling mode could no longer hold as an equilibrium trading practice.

**Remark 2** For \( \forall \delta \leq \tilde{\delta}_2 \) it holds that

\[ \overline{Q}^{pp} \geq \hat{q}. \]

I.e. there exist proportions of high quality goods such that both the pooling and the partially pooling equilibrium are simultaneously consistent with these steady state quality distributions.
Figure 2: The overlap of pooling and partially pooling outcomes in $Q\delta$-plane.

**Proof.** See Appendix A.4. ■

Hence, regarding the proportion of high quality sellers among all sellers in the market, the two alternative regimes, the pooling equilibrium and the partially pooling equilibrium, can be supported by the same overall quality distributions (see Figure 2). The reason why these alternative outcomes are not mutually exclusive stems from the difference in buyers’ coordination in the trading practice. In the partially pooling equilibrium, buyers choose to conduct high quality trades only in competitive situations and they gain by doing so. Therefore it may not pay to deviate from the partially pooling regime, even though also the completely pooling outcome would be feasible.

Although the feasibility of the partially pooling equilibrium requires that the steady state proportion of high quality sellers is greater than the Akerlofian benchmark $\hat{q}$, we know by Lemma 1 that this does not have to hold for the incoming sellers. In order to get an idea to what extent the partially pooling trading practice may help overcome the lemons problem, we next consider how the proportions of high quality agents among the incoming sellers that support the partially pooling steady state compare to $\hat{q}$. 
Proposition 3  Regarding the proportion of high quality sellers among the new entrants, the partially pooling equilibrium is feasible for \( q \in [\underline{q}^{pp}, \overline{q}^{pp}] \) where \( \underline{q}^{pp} \) and \( \overline{q}^{pp} \) are given by

\[
\underline{q}^{pp} = \frac{(e - 2) Q^{pp}}{e - 1 - Q^{pp}} \quad \text{and} \quad \overline{q}^{pp} = \frac{(e - 2) \overline{Q}^{pp}}{e - 1 - \overline{Q}^{pp}}.
\]

It holds that

(i) if \( \hat{q} \geq (e - 2) u_L \), then \( \underline{q}^{pp} \leq \overline{q}^{pp} \leq \hat{q} \quad \forall \delta \in [0, \hat{\delta}_2] \),

(ii) if \( \hat{q} < (e - 2) u_L \), then \( \exists \tilde{\delta} \) s.t. \( \underline{q}^{pp} \leq \overline{q}^{pp} \leq \hat{q} \quad \forall \delta \in \left[ \hat{\delta}, \tilde{\delta}_2 \right] \). Moreover, \( \exists \delta \in \left[ 0, \hat{\delta} \right] \) s.t. \( \underline{q}^{pp} \leq \hat{q} \leq \overline{q}^{pp} \).

Proof. For proof, see Appendix A.5. 

The main insight in the Proposition 3 is that the partially pooling equilibrium extends to a range of quality distributions which in the Akerlofian benchmark would drive the high quality sellers out of the market. In fact, if \( u_L \) is low enough so that \( \hat{q} \geq (e - 2) u_L \), even the upper bound for the proportion of high quality goods \( \overline{q}^{pp} \) is always lower than the threshold \( \hat{q} \). In this case, the partially pooling equilibrium can be supported only by quality distributions which in the competitive and static model would lead to a market for ‘lemons’ only. Figure 3 illustrates the possible divisions of different pooling regimes in \( \delta q \)-plane. Note that the possible overlap of the pooling and the partially pooling regions in \( \delta q \)-plane is analytically difficult to rule out. However, our numerical simulations through the relevant parameter values confirm that an overlapping region does not exist.\(^5\)

Hence, there is a region in between the two pooling steady states where the proportion of high quality goods among the newly entered sellers is too low to support the symmetric trading pattern but too high for the asymmetric trading practice (i.e. partially pooling) to be feasible. Symmetric trading is not feasible because buyers’ ability to conduct gainful trade with low quality sellers makes them reluctant to offer a price high enough for high quality sellers to be willing to trade. On the other hand, the asymmetric trading pattern is sustainable neither because the buyer, when being the sole purchaser candidate, finds it beneficial to deviate and offer a price that induces also the high quality sellers to trade. As a result, the outflow of high quality goods overruns their inflow and the steady state cannot be sustained. In the region below the asymmetric pooling steady state, in turn, the relative number of good sellers among

\(^5\) The calibration details are available from the authors upon request.
the new entrants is too low to maintain the steady state where high quality sellers exit at a frequency required by the asymmetric trading pattern.

Interestingly, the partially pooling equilibrium ceases to exist when the discount factor is high enough, i.e. $\delta > \delta_2$. As the agents become more patient, the opportunity cost of sales increases and the low quality sellers can seize ever increasing fraction of the rent when in contact with a single buyer candidate. This attracts buyers to deviate and offer a price that will be accepted also by the high quality sellers. The fact that an increase in the value of the future trading opportunities limits the feasibility of the partially pooling equilibrium runs counter to the previously held view that a dynamic trading perspective is a necessary precondition for a successful resolution of the lemons problem.

4 Pooling cycles

The blank areas in Figure 3 represent the combination of the discount factor and the average quality of the incoming sellers that support neither of the pooling steady states. These areas must feature non-steady state equilibria, if any, and it turns out that there exist cyclical equilibria: There are periods when only low quality goods are traded and the proportion of high quality good traders accumulates until it exceeds a threshold beyond which also high quality goods can be gainfully traded. In the
equilibrium that we construct this only happens when a high quality seller meets more than one buyer, and thus there is partial pooling like in Section 3. When high quality goods are traded the share of high quality sellers starts declining, and once it drops below the threshold there is trade in low quality goods only. The length of partial pooling in the cycle is but one period; whenever high quality goods are traded the share of high quality sellers immediately drops below the threshold, and the 'lemons principle' kicks in. The number of periods when only low quality goods are traded varies. In terms of the high quality sellers’ trading opportunities, we observe

**Proposition 4** When neither of the steady states is feasible, high quality goods can be traded only in non-consecutive trading periods.

**Proof.** For proof, see Appendix A.6.

Dynamic markets with adverse selection problems feature trading cycles also in Janssen and Roy (2004). Their focus is, however, on Walrasian prices while in our model prices are determined in many local situations in a non-cooperative way. The results of the two models are very close. Janssen and Roy show that the high quality sellers have to wait longer than low quality sellers until they trade. In our model, the same holds as the low quality sellers trade every time they meet a seller while the high quality sellers may trade only in some periods, and even then they trade only if they meet several buyers.

## 5 Concluding remarks

In this paper, we study the adverse selection problem in an ‘urn-ball’ random matching model with auctions as the price formation method. We first show that the standard pooling equilibrium with equally frequent sales of all qualities is feasible for a narrower range of quality distributions than in Akerlof’s competitive and static benchmark. This is because buyers’ ability to earn positive rents when trading with low quality sellers creates a negative externality on high quality sellers’ trading opportunities. When the pooling steady state with the symmetric trading pattern is not feasible, the high quality sellers can still expect to trade, albeit less frequently than the low quality sellers. There exists another steady state featuring partially pooling in a sense high quality sales occur whenever two or more buyer candidates compete for the good. This
asymmetric trading pattern is supported as a steady state practice even for some quality distribution that in the Akerlofian world would give rise to the 'lemons principle'. Quite interestingly, however, the partially pooling equilibrium ceases to exist as the agents become patient enough, i.e. the discount factor is sufficiently high. This finding is inconsistent with some earlier results suggesting that a dynamic trading perspective is a necessary precondition for a successful resolution of the lemons problem. For the remaining quality distributions which support neither of the proposed steady states, the model exhibits pooling cycles. The cycles are such that high quality goods are sold only in non-consecutive periods. In this respect, dynamic horizon guarantees at least temporary trading opportunities for the high quality sellers.

A Appendix

A.1 Proof of Proposition 1

Conditions $S_H \geq 0$ and $B \geq 0$ are satisfied whenever $\hat{u} \geq c_H$, or $q \geq \frac{c_H - u_H}{u_H - u_L} \equiv \hat{q}$. Using (1) and (4), the non-deviation condition $B \geq B_d$ can be shown to hold if

$$\hat{u} \geq \frac{(1 - q)(e - \delta)u_L + e(1 - \delta)c_H}{e - \delta - \delta q(e - 1)} \equiv \hat{u}^{nd}. \quad (11)$$

We start the proof by showing that $q^p \geq \hat{q}$ for $\delta \leq \hat{\delta}_1$. This is done by examining when $B \geq B_d$ is a stricter condition than $S_H \geq 0$ and $B \geq 0$; i.e. when $\hat{u}_d \geq c_H$. This is the case if $(e - \delta)u_L \geq \delta (e - 1)c_H$, or

$$\delta \leq \frac{eu_L}{(e - 1)c_H + u_L} \equiv \hat{\delta}_1.$$

Remembering that $\hat{u} = qu_H + (1 - q)u_L$ and using the definition of $\hat{q}$, the inequality in (11) can be written in terms of $q$ as

$$q \geq \hat{q} + \frac{(1 - q)((e - \delta)u_L - \delta(e - 1)c_H)}{e - \delta - \delta q(e - 1)},$$

so that the threshold $q^p$ solves this condition with equality.

In order to prove the existence of a unique solution for $q^p$, let us rewrite (11) as the following inequality:

$$-\delta(e - 1)(u_H - u_L)q^2 + [(e - \delta)u_H - \delta(e - 1)u_L]q - e(1 - \delta)c_H \geq 0. \quad (12)$$
It should be obvious that (12) holds whenever \( q \in (q_1, q_2) \), where \( q_1 \) and \( q_2 \) are the roots that make (12) hold with equality. Now, for \( q = 1 \) (12) obtains

\[
u_H \geq c_H,
\]

which holds with strict inequality by assumption. Then it must hold that the roots \( q_1 \) and \( q_2 \) are real and satisfy \( q_1 < 1 < q_2 \). Thus, since \( q \) only takes values between 0 and 1, \( q^p = q_1 \) is unique solution for (12) holding with equality.

The pooling equilibrium thus exists for \( q \in [\max \{q, q^p\}, 1] \).

### A.2 Proof of Remark 1

To establish the claim, write the solution formula for \( q^p \) as

\[
q^p = \frac{(e^{-\delta})u_H - (e - 1)u_L - \sqrt{[\alpha(e^{-\delta})u_H - (e - 1)u_L]^2 - 4(e - 1)(u_H - u_L)e(1 - \frac{1}{\delta})c_H}}{2(e - 1)(u_H - u_L)},
\]

so that \( \delta \) appears only in the numerator. Aggregating the relevant terms obtains

\[
q^p = \frac{a(\delta) - ([a(\delta)]^2 - b(\delta))^{\frac{1}{2}}}{2(e - 1)(u_H - u_L)},
\]

where \( a(\delta) = (e^{-\delta})u_H - (e - 1)u_L \) and \( b(\delta) = 4(e - 1)(u_H - u_L)e(1 - \frac{1}{\delta})c_H \). Then

\[
\text{sign } \frac{\partial q^p}{\partial \delta} = \text{sign} \left( a'(\delta) - \frac{2a(\delta)a'(\delta) - b'(\delta)}{2([a(\delta)]^2 - b(\delta))^{\frac{1}{2}}} \right) = \text{sign} \left( a'(\delta) \left( 1 - \frac{a(\delta)}{([a(\delta)]^2 - b(\delta))^{\frac{1}{2}}} \right) + \frac{b'(\delta)}{([a(\delta)]^2 - b(\delta))^{\frac{1}{2}}} \right) < 0,
\]

because \( a'(\delta) < 0, b'(\delta) < 0 \) and \( a(\delta) / \left( ([a(\delta)]^2 - b(\delta))^{\frac{1}{2}} \right) < 1 \).

### A.3 Proof of Proposition 2

Using (7) in (9), the explicit solution for \( B \) yields

\[
B = \frac{\delta(1 - Q)}{(1 - \delta)e(\delta - (1 + Q))(e - 2\delta)u_L - \delta(e - 2)\hat{U})},
\]

which implies that \( B \geq 0 \) if

\[
\hat{U} \leq \frac{e - 2\delta}{\delta(e - 2)}u_L \equiv \hat{U}_{B \geq 0}.
\]
Then (8) and (13) together imply that \( S_H \geq 0 \) if
\[
\hat{U} \geq \frac{\delta (1 - Q) (e - 2\delta) u_L + e (1 - \delta) (e - \delta (1 + Q)) c_H}{2\delta^2 (e - (1 - Q)) + e (e - \delta (1 + Q + e))} \equiv \hat{U}_{S_H \geq 0}. \tag{15}
\]
Equations (8), (10) and (13), in turn, imply that the buyers’ non-deviation condition, \( B_d \leq B \), holds if
\[
\hat{U} \leq \frac{(1 - Q) (e - 2\delta) (e - \delta) u_L + e (1 - \delta) (e - \delta (1 + Q)) c_H}{(e - 2\delta) (e - \delta - \delta Q (e - 1))} \equiv \hat{U}_{B_d \leq B} \tag{16}
\]
Since \( \hat{U}_{B \geq 0} \geq \hat{U}_{B_d \leq B} \) if
\[
u_L \geq \frac{\delta (e - 2) - c_H}{e - 2\delta}, \text{ or } \delta \leq \frac{eu_L}{(e - 2) c_H + 2u_L} \equiv \bar{\delta}_2,
\]
(16) is stricter than (14) for \( \delta \leq \bar{\delta}_2 \). On the other hand, (15) and (16) imply that it also holds that \( \hat{U}_{B_d \leq B} \geq \hat{U}_{S_H \geq 0} \) if \( \delta \leq \bar{\delta}_2 \). From this it follows that the partially pooling equilibrium can only be feasible for \( \delta \leq \bar{\delta}_2 \). Hence, the relevant conditions restricting the existence of the proposed steady state are \( S_H \geq 0 \) and \( B_d \leq B \). Again using the fact that \( \hat{U} = Q u_H + (1 - Q) u_L \), the expressions for the thresholds \( \underline{Q}^{pp} \) and \( \overline{Q}^{pp} \) are derived by rewriting (15) and (13) in terms of \( Q \) and setting the conditions to hold with equality. It is quite easy to check that \( \hat{q} \leq \underline{Q}^{pp} \leq \overline{Q}^{pp} \) holds whenever \( \delta \leq \bar{\delta}_2 \).

The existence and uniqueness of the thresholds \( \underline{Q}^{pp} \) and \( \overline{Q}^{pp} \) can be proved by following the same steps as in the proof of Proposition 1 in Appendix A.1. First, (15) can be rewritten as
\[
\begin{align*}
-\delta (e - 2\delta) (u_H - u_L) Q^2 &+ e (1 - \delta) [2\delta^2 (e - 1) + e (e - \delta (1 + e))] (u_H - u_L) + e (1 - \delta) \delta c_H \big) Q \\
&+ e (1 - \delta) [(e - \delta) c_H - (e - 2\delta) u_L] \\
&\geq 0.
\end{align*}
\tag{17}
\]
It is again obvious that (17) holds whenever \( Q \in (Q_1, Q_2) \), where \( Q_1 \) and \( Q_2 \) the roots that make (17) hold with equality. For \( Q = 1 \), (17) obtains \( u_H \geq c_H \) which holds with strict inequality by assumption. Then it must hold that the roots \( Q_1 \) and \( Q_2 \) are real and satisfy \( Q_1 < 1 < Q_2 \). Thus, since \( Q \) only takes values between 0 and 1, \( \overline{Q}^{pp} = q_1 \).
is unique solution for (17) holding with equality. Similarly, (16) can be rewritten as

\[-\delta (e - 2\delta) (e - 1) (u_H - u_L) Q^2 +
+ [(e - 2\delta) (e - \delta) (u_H - u_L) + e (1 - \delta) ((e - 2\delta) u_L + \delta c_H)] Q
- e (1 - \delta) (e - \delta) c_H \leq 0. \tag{18}\]

Since (18) obviously holds for \( Q = 0 \) and since for \( Q = 1 \) it obtains \( u_H \leq c_H \) which is inconsistent with the model assumptions, it must hold that there is a unique \( Q^{pp} \) s.t. (18) holds with equality.

**A.4 Proof of Remark 2**

The condition determining \( Q^{pp} \) in Proposition 2 can be rewritten as

\[
\frac{1 - Q^{pp}}{e - \delta - \delta Q^{pp}(e - 1)} A_1 = \hat{q} \quad \text{and} \quad \frac{1 - q^p}{e - \delta - \delta q^p(e - 1)} A_2 = \hat{q},
\]

while the condition for \( q^p \) obtains

\[
q^p - \frac{(1 - q^p)((e - \delta)u_L - \delta(e - 1)c_H)}{e - \delta - \delta q^p(e - 1)} = \hat{q}.
\]

Define

\[
A_1 \equiv (e - \delta)u_L - \frac{\delta(e - 2)(e - \delta)}{e - 2\delta} c_H, \quad \text{and} \quad A_2 \equiv (e - \delta)u_L - \delta(e - 1)c_H,
\]

so that

\[
\frac{1 - Q^{pp}}{e - \delta - \delta Q^{pp}(e - 1)} A_1 = \hat{q} \quad \text{and} \quad \frac{1 - q^p}{e - \delta - \delta q^p(e - 1)} A_2 = \hat{q}.
\]

Obviously, \( A_1 > A_2 \) because \( \frac{(e-2)(e-\delta)}{e-2\delta} < e - 1 \). Consider now the following equation:

\[
Q - \frac{1 - Q}{e - \delta - \delta Q(e - 1)} A = \hat{q}.
\]

Totally differentiating this with respect to \( Q \) and \( A \) gives

\[
\frac{dQ}{dA} = \frac{1 - Q}{e - \delta - \delta Q(e - 1)} \left( \frac{1 - Q}{(e - \delta - \delta Q(e - 1))^2} \right) > 0,
\]

from which it follows that \( Q^{pp} > q^p \) since \( A_1 > A_2 \).
A.5 Proof of Proposition 3

We start the proof by noting that $\partial Q^p / \partial \delta < 0$ (this can be shown by following exactly the same steps as above in the proof of Remark 1 in Appendix A.2), from which it follows that also $\partial \pi^p / \partial \delta < 0$. Then note that for $\delta = 0$ $Q^p = (\hat{q} + u_L)/(1 + u_L)$, which by Lemma 1 implies that for $\delta = 0$

$$\pi^p = \frac{(e - 2) (\hat{q} + u_L)}{e - 1 + (e - 2) u_L - \hat{q}}.$$  

From this it follows that $\pi^p_{\delta = 0} \leq \hat{q}$ if $\hat{q} \geq (e - 2) u_L$. Therefore, since $\partial \pi^p / \partial \delta < 0$, we know that if $\hat{q} \geq (e - 2) u_L$, then $q^p \leq \pi^p \leq \hat{q} \forall \delta \in [0, \tilde{\delta}_2]$, which was our claim in part (i).

To establish the claim in the second part with the case $\hat{q} < (e - 2) u_L$, note first that if $\delta \to \tilde{\delta}_2$ then $Q^p \to \hat{q}$. Thus, it must hold that $q^p \leq \pi^p \leq \hat{q}$ for $\delta$ sufficiently 'close' to $\tilde{\delta}_2$; i.e. $\exists \delta$ s.t. $q^p \leq \hat{q} \leq \pi^p \forall \delta \in \left[\tilde{\delta}, \tilde{\delta}_2\right]$. On the other hand, if $\delta \to 0$ then $Q^p \to \hat{q}$ and it must hold that $\exists \delta \in \left[0, \tilde{\delta}\right]$ s.t. $q^p \leq \hat{q} \leq \pi^p$.

A.6 Proof of Proposition 4

Let $q$ be the minimum proportion of high quality sellers such that a buyer is willing to offer a price that makes the high quality seller willing to trade. Notice that this only happens when a seller meets more than one buyer. We determine the steady state values of good and bad sellers when the proportion of the former is exactly $q$. The steady state stock of bad sellers is $(1 - q) e$, and let the stock of good sellers be $X$. Now, $X$ is determined by

$$\frac{X}{X + \frac{(1-q)e}{e-1}} = q.$$  

Each period measure $q$ of high quality sellers enter the market. The maximum stock of good sellers that can exist in the market is $X + q$. When this happens the good sellers start making trades, and we want to show that the number of entering good sellers who exit, $(X + q) (1 - 2e^{-1})$, is greater than the number of entering good sellers $q$. The inequality

$$(X + q) (1 - 2e^{-1}) > q$$  

becomes, when inserting the formula of $X$,

$$\frac{1 - q}{1 - q e - 1} > \frac{2}{q e - 2}.$$  

19
The inequality is hardest to satisfy when \( q \) is the largest. Inserting the formula of the greatest possible value of \( q \), namely \( q = \frac{(e-2)q}{e-1-q} \), here the inequality becomes \( e > 2 \), which always holds.

This means that whenever the proportion of high quality sellers goes over the threshold \( q \), the good sellers trade only for that period as then the proportion drops below \( q \) right next period. Then the proportion of good sellers increases over time until it goes over the threshold again.

References


