Entrepreneurship, Financiership, and Selection

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Abstract

We develop an equilibrium model of the market for entrepreneurial finance where all agents are endowed with some wealth and a project whose quality is their private information. All agents are capital constrained and need to choose whether to invest as entrepreneurs or financiers, or not to invest. We compare this economy to one where finance comes from the outside. Removing outside investors tends to improve efficiency by raising the cost of capital and creating advantageous selection where the agents with productive projects become entrepreneurs and those with unproductive ones become their financiers. If funding is easier to come by, entrepreneurship becomes attractive also for unproductive agents. Financial liberalization may therefore have harmful efficiency effects due to adverse selection. In our model insufficient wealth generally holds back business creation, but the markets for entrepreneurial finance can nonetheless exhibit too much activity.

JEL Classification: D58, G14, G20, G28, G32

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I. INTRODUCTION

A key choice a would-be entrepreneur encounters is how to use her personal wealth. Should she really become an entrepreneur, invest her wealth in the potential project of her own and raise the rest from outside? Or is it more profitable to invest the assets in the financial market to finance the projects of others? Or is it better yet to put aside the investment opportunities all together and just to consume the assets? These considerations are inherent in any financial market from microfinance of developing countries with sparse investment and savings choices to the sophisticated markets of rich countries where access to interest-bearing savings account and mutual funds is widely available and where successful entrepreneurs become private equity investors and vice versa. Yet, the majority of the economic and finance literature has overlooked this choice and its implications on financial market efficiency. This choice could matter especially in the much studied context where the quality of potential entrepreneurs' ventures is private information, since low quality project holders might find it more profitable to give up their entrepreneurial aspirations and invest in the projects of others. The defining feature of financial markets might be advantageous rather than adverse selection.

The aim of this paper is to explore entrepreneurship and the functioning of financial markets under asymmetric information when the roles of agents are determined within the model. In a departure from most of the existing literature, all agents in our model are capital constrained and have an investment project whose quality is their private information. There is an occupational choice in the sense that agents choose whether to participate in financial markets and, if they participate, whether to become entrepreneurs or financiers. This creates a natural framework to
study whether entrepreneurship, and by implication, a market for financial claims 
emerge in equilibrium and whether the eventual markets are efficient.

We find that the financial market without outside financiers works remarkably 
well despite asymmetric information and the absence of financial institutions 
mitigating the asymmetric information problem: Pareto or interim efficient outcomes 
prevail for a large set of parameter values for which partial equilibrium models would 
predict an inefficient pooling equilibrium or even a collapse of the market. The 
finding has an economic implication: importing finance from outside can decrease the 
efficiency of a financial market.

Our model includes both economies where the total wealth is sufficient to 
implement all projects with positive net present value, and wealth constrained 
economies. In turns out that an aggregate shortage of liquidity matters. Pareto 
efficient and inefficient equilibria exist both in wealth constrained and unconstrained 
economies, as does autarky. Contrary to what one might expect, the economy-level 
wealth constraint does not necessarily dilute the performance of the financial market.
When wealth is scarce relative to the economy's productive potential, agents with low 
quality projects prefer financing agents with higher entrepreneurial potential to 
investing in the projects of their own. Relaxing the wealth constraint can, however, 
lower interest rates to the extent that it induces adverse selection since agents with 
low quality projects begin to seek funding. This means that increasing wealth may 
cause a shift from a Pareto efficient equilibrium into a Pareto inefficient one.

We build on the large and well-established literature on financial markets with 
asymmetric information emerging from Stiglitz and Weiss (1981) and de Meza and 
Webb (1987). In Section II we explore the role of entrepreneurs' wealth in a 
conventional set-up with outside investors and replicate some key results of the
literature. If entrepreneurs are very poor so that those with low quality projects have higher pledgeable incomes than high-quality project holders, a Stiglitz-Weiss type financial market emerges where marginal entrepreneurs have productive projects. If there is sufficient wealth to render the pledgeable incomes of high-quality entrepreneurs higher than that of low-quality entrepreneurs, the market is of de Meza-Webb type where marginal entrepreneurs have unproductive projects. There is too much entrepreneurial activity and increases in wealth generate efficient exit from entrepreneurship. When entrepreneurial wealth is very high, a Pareto efficient separating equilibrium arises. Wealth thus acts akin to entrepreneurs' risk-aversion in de Meza and Webb (1990).

In Section III we close the economy so that provision of entrepreneurial finance becomes endogenous. All agents now encounter a choice between entrepreneurship, financing others’ ventures, and remaining outside financial markets. To the best of our knowledge, Boyd and Prescott (1986) and Shleifer and Wolfenzon (2002) are the only studies besides ours where there is a genuine choice between investing as an entrepreneur and a financier. Whereas the focus in Shleifer and Wolfenzon (2002) has little to do with our analysis, Boyd and Prescott's model (1986) is close to ours. There are agents have investment projects whose quality is their private information and who can choose whether to invest their endowment (of time) in their project or evaluate the quality of a project. They show that if the agents form intermediary-coalitions that evaluate projects, they can do better than in decentralized markets. Our model is simpler than theirs in that we do not allow for financial institutions that provide information. However, Boyd and Prescott (1986) focus on an economy where the aggregate liquidity constraint is not binding. Indeed, in Section III.A we provide an example which shows how the market outcome of Boyd and
Prescott (1986) is a special case of our model. In many other cases – in particular when aggregate liquidity constraint binds or agents are rich enough - there is no need for information provision by financial institutions since the markets are efficient. We also highlight the comparative statics over agents' initial wealth.\footnote{In Boyd and Prescott (1986), agents' (time) endowment is normalized to unity whereas their projects have variable scale. As also shown by Boyd and Prescott, however, variable scale is irrelevant in the absence of financial intermediaries, since all executed projects have maximum scale regardless of their type. There are some other differences, e.g., in Boyd and Prescott all projects are profitable gross of project evaluation costs whereas in our case low quality projects have negative net present values.}

In Section IV we, motivated by Holmström and Tirole (1998), consider impacts of imperfect storage technology. Although the efficiency of storage has trivially major effects on financial market participation, it has only minor effects on efficiency, except that the scope of the Pareto efficient equilibrium in non-wealth constrained economies is increasing in the efficiency of storage. At the limit where there is no storage, the Pareto efficient equilibrium disappears.

Policy implications are collected into Section V. Our model is very stylized and certainly does not correspond exactly to any real-world financial market environment. Nonetheless, we feel that that our finding concerning the detrimental effects of outside financiers and entrepreneurial wealth have important bearings on two current policy debates. First, if financial market liberalization amounts to a large inflow of funds from outside investors without a change in the composition of potential entrepreneurs, it may have adverse consequences. That financial liberalization can have a dark side is well known but most of the literature stresses moral hazard as a major problem. In contrast, our explanation stems from adverse selection and capital inflows. A similar point is elaborated by Giannetti (2005) and its regulatory implications by Morrison and White (2004).
The second policy implication we want to raise concerns the promotion of entrepreneurship. In our model wealth and entrepreneurial activity are generally positively correlated, which is in line with existing evidence (e.g., Evans and Jovanovic, 1989, and Black, de Meza and Jeffrey, 1996). Nonetheless, echoing the argument forcefully advanced in the work of de Meza and Webb (1987, 1990, 1999), we find that too low cost of capital attracts too many entrepreneurs and, as a result, neither asymmetric information nor insufficient wealth necessarily provides a reason to subsidize entrepreneurs or their finance.\(^2\) But here the conclusion emerges as part of equilibrium when the agents' have intermediate wealth and the economy-level wealth constraint does not bind. When it binds, increases in wealth cause entry of productive entrepreneurs and a case for subsidizing business creation may arise.

Besides the aforementioned articles, our study is also inspired by Holmström and Tirole (1997), Caballero and Krishnamurthy (2001) and Aghion, Bacchetta, and Banerjee (2004) who emphasize that both microeconomic and economy-wide financial constraints influence the performance of financial markets. From this more macroeconomic perspective our study has also a link to the emerging literature on the impact of adverse selection over the business cycle (e.g., Eisfeldt, 2004, and House, 2006).

Finally, Section VI summarizes our findings.

II. THE MODEL WITH OUTSIDE INVESTORS

There is a unit mass of risk-neutral potential entrepreneurs who have access to a project of size \(I\), and unlimited entry by risk-neutral outside investors without a project of their own. A proportion \(h\) (\(0<h<1\)) of potential entrepreneurs are high (H)
types who are endowed with a positive NPV project, the rest are low (L) types with a negative NPV project. The projects have two-point return distribution: we assume that $p_H R_H > I > p_L R_L$ and $R_L > R_H$, where $p_i$ is the success probability and $R_i$ the return (conditional on success) of an entrepreneur of type $i$, $i \in \{H, L\}$. Failed projects yield zero regardless of their type. Project success and wealth are verifiable, but project type is private information following, e.g., Bolton and Scharfstein (1990).\(^3\)

Potential entrepreneurs have some liquid funds $A$, $0 < A < I$, which they entirely invest either in their own project or in the storage technology.\(^4\) For the moment we assume that storage is perfect, converting initial wealth to consumption at a zero rate of return. We present most of our analysis using a graph in the $(A, h)$-space where natural parameter boundaries are given by $h \leq 1$, and $A < I$.

The financial market works as follows. First, potential entrepreneurs decide whether to invest their initial funds in their project or in storage. If they initiate the project, the rest of required funds ($I-A$) needs to be raised from outside investors. Contract terms stipulate the conditional payment from the entrepreneur to outside investors in case of success.\(^5\) Once financing needs have been settled, entrepreneurs execute their projects. Successful entrepreneurs compensate outside investors according to contract terms, and consumption takes place.

The potential entrepreneurs' individual rationality condition is given by

$$\pi_i^* \equiv p_i (R_i - R_b) \geq A \quad \forall i, \, i \in \{H, L\},$$

\(^3\) An alternative assumption used, e.g., in de Meza and Webb (1999) is that only payments from entrepreneurs to financiers are verifiable and that entrepreneurs cannot hide income in case they default.

\(^4\) It is cheaper for H-type entrepreneurs to use their own rather than outside funds. As a result, L-type entrepreneurs have no other option but to follow and invest all their wealth in their own projects. See de Meza and Webb (1987).

\(^5\) As there is no outside collateral in our model, introducing collateral into the contract does not change the final wealth of an entrepreneur (of either type) in case of default: This is zero for any level of collateral. Collateral requirements cannot thus be used as a screening device in our model. As is well
where superscript \( e \) denotes entrepreneurship so that \( \pi_i^e \) is the expected profits of a type \( i \) entrepreneur and \( R_B \) is the (fixed) payment that a successful entrepreneur pays to her investors. Equation (1) shows how the potential entrepreneurs’ wealth determines the opportunity cost of entrepreneurship. The richer she is, the larger should her expected payoff be to make entrepreneurship lucrative.

Because of competitive supply of finance, the cost of financing is determined by outside investors’ zero-profit condition, i.e.,

\[
R_B = \frac{I - A}{\bar{p}},
\]

where \( \bar{p} \) is the average success probability of those who become entrepreneurs. From (1) and (2) we observe four potential equilibrium outcomes. First, a Pareto efficient separating equilibrium where only H-types become entrepreneurs (\( \bar{p} = p_H \)), occurs if \( \pi_H^e \geq A > \pi_i^e \). Second, a Pareto inefficient pooling equilibrium where all potential entrepreneurs also actually become entrepreneurs (\( \bar{p} = h p_H + (1-h) p_L \)), occurs if both \( \pi_H^e \) and \( \pi_L^e \) are larger than \( A \). Third, there may be a semi-separating equilibrium where all H-types are entrepreneurs, but L-types split with some becoming entrepreneurs and others opting out of the financial market.\(^6\) In this case \( \bar{p} = [h p_H + \mu_L (1-h) p_L] / [h + \mu_L (1-h)] \) where \( \mu_L \) denotes the proportion of L-types that become entrepreneurs.\(^7\) Now L-types’ indifference condition (1) and outside investors’ zero profit condition (2) give \( \mu_L = h[p_L (p_H R_L - I) - (p_H - p_L) A] / p_L (1-h) \) and \( R_B = (p_L R_L - A) / p_L \).

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\(^6\) It is easy to show that a semi-separating equilibrium where H-types are indifferent between entrepreneurship and using storage does not exist when outside finance is available.

\(^7\) Since there is a continuum of potential entrepreneurs, we model mixed strategies by a distributional approach where the proportion \( \mu_L \) of L-types use the pure strategy of becoming an entrepreneur and the proportion \( 1-\mu_L \) use the pure strategy of investing in storage.
Finally, if both $\pi_H^e$ and $\pi_L^e$ are smaller than $A$, the financial market does not open and all agents invest in storage. We term this outcome autarky.

In Figure 1 we have depicted the $(A, h)$-values for which each of the four different equilibria exist. The downward sloping line is H-types' IR constraint in the pooling equilibrium and the upward sloping line is the corresponding constraint of L-types. The L-types' IR constraint also divides the pooling and the semi-separating equilibria so that $\mu_L=1$ holds above the line and $\mu_L<1$ below it. In the semi-separating equilibrium, H-types' IR constraint is given by the vertical 
\[ \hat{A} \equiv p_L p_H (R_L - R_H) / (p_H - p_L) \]-line, and the vertical 
\[ \bar{A} \equiv \hat{A} + p_L (p_H R_H - I) / (p_H - p_L) = p_L (p_H R_L - I) / (p_H - p_L) \]-line is L-types' IR-constraint. As a result, Figure 1 shows that the pooling equilibrium exists in the upper part to the left of $\bar{A}$ and above the IR constraints. Below the pooling case, autarky prevails to the left of $\hat{A}$ and the semi-separating equilibrium between $\hat{A}$ and $\bar{A}$. The Pareto efficient equilibrium exists to the right of $\bar{A}$.

**FIGURE 1 HERE**

The two vertical lines and the three regions they define will also be crucial in the subsequent sections. On the one hand, the Pareto efficient equilibrium emerges for $A>\bar{A}$, since the opportunity cost of entrepreneurship rises with wealth. The opportunity cost matters more for L-types, because the expected return on their project is lower. The opportunity cost trivially exceeds L-type entrepreneurs' expected profit when $A \geq p_LR_L$ but, given the equilibrium $R_B>0$, the L-type entrepreneurs' expected profit is less than $A$ in the whole region to the right of $\bar{A} < p_LR_L$. When $A \leq \bar{A}$, the level of wealth becomes low enough to tempt L-types to bet it in their projects. Higher interest rates cannot be charged to prevent L-types from becoming
entrepreneurs in competitive financial market. This suggests that if the supply of finance was restricted, e.g., by a monopoly supplier, the economy could reach Pareto efficiency also to the left of \( A \). The suggestion will be confirmed in the next section. On the other hand, financial markets have difficulties in opening up to the left of \( \hat{A} \). This observation has attracted much attention in the literature: As articulated by Stiglitz and Weiss (1981), the markets can collapse because of adverse selection.

The observation that financial markets and entrepreneurship generally emerge when \( A \geq \hat{A} \) but not when \( A < \hat{A} \) can be explained by adopting the concept of pledgeable income (Holmström and Tirole 1997, 1998), defined as the maximum amount of an entrepreneur can promise to pay back to a financier. From (1) we see that the pledgeable income of type \( i \) agent is given by \( R_i - A/p_i \). When potential entrepreneurs are poor \( (A < \hat{A}) \), limited liability makes L-type entrepreneurs' pledgeable income higher than that of H-types. Because L-types can always match the maximum repayment that H-types are able to offer, adverse selection may lead to the collapse of the financial market. But since increases in wealth raise L-types' liability more, the pledgeable incomes of H- and L-type entrepreneurs are equal when \( A = \hat{A} \) and, when \( A > \hat{A} \), H-types' pledgeable income exceeds that of L-types. There bad projects cannot drive out good ones and financial markets can operate. In contrast, the inefficiency in the middle area \( (A \in [\overline{A}, \hat{A}]) \) is that there is too much entrepreneurship, as in de Meza and Webb (1987).

To further relate our findings to Stiglitz and Weiss (1981) and de Meza and Webb (1987), note that Stiglitz and Weiss (1981) consider a mean-preserving spread between projects. Here it would mean that \( p_H R_H = p_L R_L \) and, consequently, L-type entrepreneurs’ pledgeable income would always be higher than H-type entrepreneurs’
there could be underinvestment since insufficient wealth holds back H-type entrepreneurs more easily. In this case the Pareto efficient equilibrium cannot exist. De Meza and Webb (1987) in turn assume that the project returns conditional on success are the same. Here it would mean that \( R_H = R_L \) and, consequently, \( \hat{A} = 0 \). The pledgeable income of H-type entrepreneurs would exceed that of L-type entrepreneurs, as in the area to the right of \( \hat{A} \) in Figure 1. That area is characterized by overinvestment. Increases in wealth cause efficient exit from entrepreneurship, since marginal entrepreneurs (in the semi-separating equilibrium) are those of low quality.

We summarize the results of the model with outside investors in the following proposition:

**PROPOSITION 1:** With outside investors,

a. When \( A \geq \bar{A} \), the Pareto efficient separating equilibrium exists.

b. When \( A < \bar{A} \), the Pareto inefficient pooling equilibrium exists above the L-type entrepreneurs’ IR constraint.

c. When \( \hat{A} \leq A < \bar{A} \), the Pareto inefficient semi-separating equilibrium exists below the L-type entrepreneurs’ IR constraint.

d. When \( A < \hat{A} \), the financial markets collapses to autarky below the H-type entrepreneurs’ IR constraint.

e. When \( A \leq \hat{A} \), an increase in entrepreneurial wealth (eventually) helps the market out of autarky but when \( A > \hat{A} \), it (eventually) leads to exit.

### III. THE MODEL WITHOUT OUTSIDE INVESTORS

The absence of outside investors limits the amount of funds available for investment. A natural consequence is that all agents face a choice between becoming entrepreneurs or financiers. We regard an economy as wealth constrained if the total initial wealth of all agents is insufficient to finance all H-types’ projects.
Correspondingly, an economy is not wealth constrained if the total wealth exceeds the financing needs of all H-type projects.

The financial market works as in the previous section. Since we allow no financial institutions that gather and process information, the financial market in our model could be interpreted as a frictionless (stock) market, a mutual fund, or a microfinance institution. After the projects have been implemented, the total payments from all entrepreneurs are divided evenly among all financiers. Thus it is as if a financier buys a stake in the average implemented project, instead of implementing her own project. Loosely speaking, it makes no difference whether one envisions a financial market where some potential financiers come together to finance one project (or a few projects), or a market where all financiers buy a similar stake in every implemented project. Both result in the same expected payment to financiers.

The market collapses to autarky when all agents resort to the storage technology and there are neither entrepreneurs nor financiers. In a Pareto efficient allocation all or as many H-type projects as possible are financed whereas no L-type projects receive finance. Correspondingly, in a Pareto inefficient equilibrium at least some L-type projects are carried out.

Let us denote the proportion of type i agents that become entrepreneurs by \( \mu_i \). With \( \mu_i \in [0,1] \), we have a 3x3 matrix of potential equilibria as shown in Table 1.\(^8\) It is immediately clear that three out of the nine cannot exist. If no H-type agent becomes an entrepreneur, the potential financiers’ individual rationality constraint is violated. Similarly, due to our assumption that \( A < I \), it is impossible that all agents become entrepreneurs. The remaining six configurations cannot be excluded a priori. They consist of autarky and five cases where financial markets emerge as an

\[^8\] These nine categories can be split further according to whether all type i agents participate or not.
equilibrium outcome. We name the five potential equilibria with financial markets according to what occupations \((e = \text{entrepreneur}, f = \text{financier}, s = \text{user of storage technology})\) agents of type \(i\) choose. For example, \(H^eL^s\) (column one, row one in Table 1) is the equilibrium where all H-type agents become entrepreneurs, and L-types split between becoming financiers and using storage.

TABLE 1 HERE

Both Pareto efficient equilibria are in the first column of Table 1. Of these, the one in the last row is strictly better than the one in the middle row. Similarly, in the middle column, the equilibrium in the last row is more desirable than the one in the middle row. The equilibrium in the last column is the worst of the five equilibria with economic activity.

An equilibrium is now constrained by four conditions. The first arises from the individual rationality (IR) constraints we saw in the previous section, with a slight but crucial modification. Now all agents compare the expected profits from becoming active, either as an entrepreneur or as a financier, investing in storage. The second set of conditions comes from the incentive compatibility (IC) constraints of both agent types, which guide the choice between entrepreneurship and being a financier. The third relationship equalizes the supply of funds from financiers with the demand of funds by entrepreneurs. Finally, contract terms are determined by equating the expected payments by successful entrepreneurs to the expected compensation for financiers.

Denoting expected profits of a type \(i\) agent from activity \(j\) by \(\pi_i^j\), the IR constraints are

\[
(2) \quad \pi_i^j \geq A \quad \forall i, j, A \in \{H, L\}, \quad j \in \{e, f\}. 
\]

The IC constraints can be written as
Depending on the equilibrium (see Table 1) and agent type, the IC or IR constraint or both may bind, and the IC constraint may hold strictly one way (e.g., all H-type agents become entrepreneurs) or the other (e.g., all L-type agents become financiers).

The equality of demand and supply of funds is given by

\[ (I - A)[\mu_H h + \mu_L (1 - h)] = A[(1 - \mu_H - \chi_H)h + (1 - \mu_L - \chi_L)(1 - h)], \]

where \( \mu_i \) and \( \chi_i \in [0,1] \) denote the proportion of type \( i \) agents who become entrepreneurs and who employ the storage technology. The left hand side of (5) captures the demand. Each entrepreneur lacks \( I - A \) of funds to be able to carry out her project and the term in the square brackets is the equilibrium mass of entrepreneurs. In the right hand side of (5) we have the supply of funds from financiers, whose equilibrium mass can be seen from the term in the square brackets.

Finally, the expected payments by entrepreneurs must equal the expected payments received by financiers

\[ R_h[\mu_H h p_H + \mu_L (1 - h) p_L] = R_F[(1 - \mu_H - \chi_H)h + (1 - \mu_L - \chi_L)(1 - h)]. \]

The term in the square brackets on the right-hand side of (6) is the equilibrium mass of financiers as in (5), whereas the term on the left now equals the expected equilibrium mass of successful entrepreneurs. In (6) \( R_h \) is, as before, the fixed payment an entrepreneur has promised to pay back in case of success, and \( R_F \) is the expected payment received by a financier.

Solving the range of parameters where conditions (3)-(6) hold for all five equilibria with economic activity (see Table 1) is a straightforward but tedious exercise. We consider the equilibrium \( H^e L^f \) as an example and then graphically describe the remaining equilibria, relegating calculations to the Appendix.
A. Example: $H^e L^{ef}$

In $H^e L^{ef}$, $\mu_H = 1$ and $0 < \mu_L < 1$, i.e., all H-type agents are entrepreneurs and L-type agents become either entrepreneurs or financiers (i.e., nobody chooses the storage technology, $\chi_i = 0$, $i \in \{L,H\}$). This equilibrium corresponds to the decentralized market equilibrium in Boyd and Prescott (1986) and is comparable to the semi-separating equilibrium with outside financiers (Section II). As it turns out, this equilibrium also displays plausible empirical implications and the de Meza-Webb type results and policy recommendations. The example also illustrates one of our main results of how a decrease in initial wealth can improve the efficiency of a financial market.

Since financial market participation is complete in this equilibrium, we require that both types' IR constraints are satisfied, i.e.,

\[(7) \quad R_F \geq A.\]

L-type agents’ IC constraint must hold with equality, which means that

\[(8) \quad p_L (R_L - R_B) = R_F.\]

The left hand side gives the expected return for an L-type agent from becoming an entrepreneur and the right hand side gives the expected return from becoming a financier. As L-type agents split between the two occupations, they must be indifferent between them.

Because all H-type agents prefer entrepreneurship to being financiers, their expected return from entrepreneurship must be at least as large as that of becoming a financier, i.e.,

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9 The equilibrium $H^e L^{ef}$ (see Section A in the Appendix) perhaps corresponds even more accurately to the semi-separating equilibrium of Section II. However, it is immaterial whether $H^e L^{ef}$ or $H^e L^{ef}$ is chosen as a benchmark because, as we will show, they together span a smaller proportion of the parameter space than the semi-separating equilibrium in the model with outside financiers.
The aggregate demand and supply for finance is balanced when
\[ (I - A)[h + \mu_L (1 - h)] = A(1 - \mu_L)(1 - h) \]
holds.

Finally, the (expected) repayments from successful entrepreneurs must equal the payments received by their financiers:
\[ R_h [hp_h + \mu_L (1 - h)p_L] = R_F (1 - \mu_L)(1 - h). \]

Conditions (8), (10) and (11) determine the endogenous variables \( \mu_L, R_h, \) and \( R_F \). Solving first for \( \mu_L \) from (10) gives
\[ \mu_L^* = \frac{A - hI}{(1 - h)I}. \]
The proportion of L-types who become entrepreneurs has to be less than unity. This is guaranteed by our assumption \( I > A \). As \( \mu_L^* \) also has to be nonnegative in \( H^e L^e \), (12) immediately reveals that \( H^e L^e \) can only exist if \( A/I \geq h \). In other words, \( H^e L^e \) cannot exist in a wealth constrained economy where the total wealth is insufficient to finance all H-types’ projects.

Using (11), (10) and (7) we can solve for the equilibrium payments \( R_h^* \) and \( R_F^* \). They are given by
\[ R_h^* = \frac{(I - A)}{L[hp_h + (1 - h)p_L]} p_L R_L \]
and
\[ R_F^* = \left[1 - \frac{p_L(I - A)}{L[hp_h + (1 - h)p_L]} \right] p_L R_L. \]

Equations (13) and (14) suggest that for H-types, the payments \( R_h^* \) and \( R_F^* \) are independent of project outcome, whereas for L-types the payments are functions of
project outcome. The payment from a successful L-type entrepreneur to her financier is fixed, although in equilibrium, it is a function of her project return.

After solving for the endogenous variables, we still need to find the parameter values satisfying the agents’ IR and IC constraints (7)-(9). The L-type IC constraint (8) binds as they split in their occupational choices. Since H-types prefer entrepreneurship to becoming financiers, and being a financier is at least as rewarding as investing in storage, their IR constraint (7) does not bind. This means that the relevant constraints are the IR constraint (7) for L-types and the H-type IC constraint (9). Substituting (14) into (7) shows that the L-type IR constraint is satisfied (guaranteeing that no L-type agent stores her wealth) if

\[ A \leq \frac{p_L R_L I h (p_H - p_L)}{I h (p_H - p_L) + p_L (I - p_L R_L)}. \]

The H-type IC constraint (guaranteeing that no H-type agent prefers becoming a financier to entrepreneurship) is satisfied if

\[ A \geq I - \frac{(p_H R_H - p_L R_L) [h p_H + (1-h) p_L]}{(p_H - p_L) p_L R_L}. \]

In Figure 2 we use the \((A, h)\)-space to represent the set of parameter values for which \(H^L L^f\) exists.

The \(h=A/I\) diagonal divides economies into wealth constrained (above), and non-wealth constrained ones (below). \(H^L L^f\) only exists in non-wealth constrained economies as suggested by (12). There L-types’ IC constraint binds, whereas H-types’ IR constraint does not. H-types’ IC constraint (16) is a decreasing line in the \((A, h)\)-space so that poor economies with a small proportion of good projects fail to satisfy this constraint. L-types’ IR constraint (15) is a monotonically increasing curve that
starts at the origin and cuts the diagonal once. Below the curve, some L-types prefer not to participate.

Let us consider how a decrease in wealth affects the equilibrium in Figure 2. Decreasing $A$ reduces the funds available to entrepreneurs. Because of the tightened financial market, $R^*_B$ increases, meaning that entrepreneurs are worse off. Since in this equilibrium H-types prefer entrepreneurship to becoming a financier and L-types are indifferent, any marginal change in $A$ has a first-order impact on L-types’ occupational choice, but H-types continue to be entrepreneurs. Therefore, exit from entrepreneurship occurs ($\mu^*_L$ declines). From a financier's point of view, the improvement in the quality of entrepreneurial pool and the increase in the lending rate increase her payoff. However, a decline in $\mu^*_L$ also means that the ratio of entrepreneurs to financiers diminishes, driving financiers' returns downwards to the extent that the net result is a lower payoff per financier ($R^*_F$). Hence, decreasing $A$ dilutes the payoff from both entrepreneurship and finance. By definition, L-types remain indifferent, but H-types’ returns to entrepreneurship wane relatively faster.

If $A$ continues to decrease, it will inevitably also affect H-types’ actions. Thus the outcome depends on the proportion of H-types in the economy. If the proportion is sufficiently high, the financial market will eventually run out of funds. When this happens, all L-types are financiers. In terms of Figure 2, we hit the $\mu^*_L \geq 0$ constraint, and some H-types must also become financiers. The new equilibrium is Pareto efficient. If the proportion of H-types in the market is sufficiently low, however, there is less risk of running out of funds. Therefore H-types' IC constraint (the downward sloping line in Figure 2) is breached before the $\mu^*_L \geq 0$ constraint. This will cause an
abrupt drop in the average quality of entrepreneurs, leading to a collapse of the market.

The example suggests a role of financial intermediaries that would mitigate the asymmetric information problem. Based on Boyd and Prescott (1986), we conjecture that such intermediaries would emerge if agents had access to a project evaluation technology. Our focus is however on the effects of wealth. The example illustrates how a decrease in wealth tightens the financial market, which is beneficial from the efficiency point of view. The underlying reason is the same as in de Meza and Webb (1987): marginal entrepreneurs are of low quality and can be driven out by higher interest rates. A major difference to de Meza and Webb (1987) is that here the problem of overinvestment emerges as part of equilibrium: the economy’s total initial wealth relative to the proportion of high quality projects is too large. Moreover, there is a positive relationship between wealth and entrepreneurship. This is at odds with the prediction of de Meza and Webb (1987) but has empirical appeal. In the next Section we describe the all equilibria of the economy and show that interest rates can sometimes be endogenously high enough to discourage L-types from becoming entrepreneurs.

**B. Existence and Efficiency of Equilibria**

Following a similar procedure as for the case of $H' L^e$ in Section III.A., we derive in the Appendix the values of the endogenous variables and determine the conditions for the existence of the six candidate equilibria. Here we present graphically the equilibria

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10 De Meza and Webb (1999) add moral hazard to their basic framework to generate a positive relationship between wealth and entrepreneurship. Our example shows that one does not need moral hazard considerations to obtain the de Meza-Webb type results and policy recommendations while maintaining the empirically plausible relationship between wealth and business creation.
and describe their efficiency properties. At the end of the Section, we summarize our main results and explain them.

In Figure 3, we employ the labeling of Table 1 to indicate the areas in which each equilibrium exists in the \((A, h)-space\). There are two key lines: the \(h=A/I\) diagonal and the vertical \(\hat{A} \equiv p_L p_H (R_L - R_H)/(p_H - p_L)\)-line, familiar from Section II. The diagonal not only divides the economies into wealth and non-wealth constrained ones, but is also a border of various equilibria in many cases. To the right of the vertical \(\hat{A}\)-line the equilibria are unique.

**FIGURE 3 HERE**

Let us first examine non-wealth constrained economies, i.e., those below the diagonal in Figure 3. Because there is no aggregate shortage of liquidity, this region has similarities to the case of outside finance of Section II. Corresponding to the Pareto efficient separating equilibrium with outside finance (see Figure 1), we find a Pareto efficient equilibrium, \(H^eL^f\), in the right part of Figure 3. All H-type projects are financed, and L-types are indifferent between financing the H-types and using the storage technology. On the right hand side of the vertical \(p_L R_L\)-line, L-types prefer investing in storage to entrepreneurship by assumption. Because of costly financing \((R_L^e>0)\), L-types continue to find storage superior some distance to the left of the \(p_L R_L\)-line.

Going further to the left, once we hit the vertical \(\bar{A}\)-line, a Pareto inefficient semi-separating equilibrium, \(H^*L^{f*}\), emerges much as in the case of outside finance. All H-type agents are entrepreneurs, but so are some L-type agents. Below the L-type IR-curve (equation (15)), the L-type participation constraint is satisfied with equality through some L-types opting for storage. Above the L-type IR-curve, we have the

\[\text{In the Appendix, we present a separate figure for each equilibrium.}\]
Pareto inefficient equilibrium $H^e L^f$ that was characterized in Section III.A. All L-type agents participate and thus nobody uses storage even though available assets exceed the financing needs of H-type entrepreneurs. Demand and supply of funds is equated through some L-type agents becoming entrepreneurs. H-type entrepreneurs’ IC (and IR) constraint is given by (16). Autarky prevails to the left of it.

We then turn to wealth-constrained economies, i.e., the area above the diagonal in Figure 3. Here the contrast with the case of outside finance is stark. In the middle and right part we find relatively rich economies with a high proportion of H-types. There, an equilibrium exists where all L-types become financiers, and H-types mix in their occupations between entrepreneurship and finance ($H^d L^f$). All funds are directed into H-type projects, and therefore the equilibrium is Pareto efficient.

Moving to the left, entrepreneurship becomes an option to L-types when we reach the vertical $\hat{\lambda}$-line. Between it and another vertical line, $\hat{\lambda}(llp_H R_H)$, we have one to three equilibria in the upper part of Figure 3. One is the same $H^d L^f$ as on the right hand side of that line. Another is $H^d L^f$ where both L- and H-types can be found among entrepreneurs and financiers. The third equilibrium is $H^d L^e$ where all L-types are entrepreneurs and H-types mix their occupations between entrepreneurship and being a financier. Since there are L-type agents among entrepreneurs in $H^d L^f$ and $H^d L^e$, they are Pareto inferior to $H^d L^f$.

Once we cross $\hat{\lambda}(llp_H R_H)$ financial markets cease to operate except for a small area close to the $h=1$ border. There we find $H^d L^e$ where all L-types are entrepreneurs. Although the equilibrium is inefficient, it survives thanks to a very high proportion of H-type entrepreneurs. The lower is the agents’ wealth, the higher the needed proportion of H-type agents to avoid the collapse of the financial market.
To conclude the discussion on the efficiency of equilibria, we investigate whether the Pareto inefficient equilibria are interim (incentive) efficient in the sense of Holmström and Myerson (1983).\textsuperscript{12} In the non-wealth constrained economies the equilibria $H^e L^d$ and $H^e L^f$ are not interim efficient. The social planner could achieve efficiency, albeit not a Pareto improvement, by imposing a high enough $R_H$ ($R_H = (p_L R_L - A)/p_L$) or otherwise taxing entrepreneurial profits or subsidizing inactivity. This would work because in this region the pledgeable income of H-type agents is higher than that of L-type agents. In wealth-constrained economies the equilibria $H^d L^d$ and $H^d L'$ cannot be interim efficient to the right of $\hat{A}(I/p_H R_H)$ because the Pareto efficient $H^d L'$ exists there. To the left of $\hat{A}(I/p_H R_H)$, a social planner cannot simultaneously discourage L-type entrepreneurship and encourage H-type entrepreneurship as the pledgeable income of L-types is much larger than that of H-type agents.\textsuperscript{13}

We summarize the above discussion in the following proposition:

**PROPOSITION 2:** Without outside investors,

a. **equilibria are typically unique:** multiple equilibria can only exist in an area shaped by $\hat{A}(I/p_H R_H)$, $\hat{A}$, (16) and (D.20) (see the Appendix for (D.20),

b. **the unique equilibrium is autarky if the level of initial wealth is sufficiently low and** Pareto efficient if the level of initial wealth is sufficiently high.

\textsuperscript{12} Loosely, in an interim incentive efficient equilibrium a benevolent social planner encountering the same informational imperfections as the individual agents cannot improve upon the market outcome without violating the agents’ individual rationality and incentive compatibility constraints.

\textsuperscript{13} If the social planner were allowed to dictate the agents’ occupations, efficiency could also be improved in some other cases. In the region to the left of $\hat{A}(I/p_H R_H)$, the planner could raise efficiency by randomly allocating agents into entrepreneurship. This would be feasible when $h \geq (I - p_L R_L)/p_H R_H$ for L-type agents to use storage. Improvement on autarky would be possible between $\hat{A}(I/p_H R_H)$ and $\hat{A}$ and below (15), if the social planner could force some agents to use storage. With positive probability, a high-enough proportion of L-type agents would use storage, pushing the proportion of H-type agents in the active population above the threshold ($\hat{A}/I$) needed to obtain economic activity.
c. in the intermediate range of initial wealth, both Pareto efficient and inefficient equilibria exist,

d. the Pareto inefficient equilibria with active financial markets are not interim efficient in the intermediate range of initial wealth, and

e. the threshold levels of wealth that prevent the market from collapsing to autarky and yield a Pareto efficient equilibrium are higher in a non-wealth constrained economy than in a wealth constrained one.

Although parts a-d) of Proposition 2 apply both to wealth and non-wealth constrained economies, we emphasize that the aggregate wealth constraint matters. Most clearly this can be seen from part e) of Proposition 2: in wealth constrained economies $A \geq \hat{A}$ is a sufficient condition for a Pareto efficient equilibrium whereas in non-wealth constrained economies it is only a sufficient condition to avoid a collapse of the market to autarky.

That the equilibria are typically more efficient when the aggregate wealth constraint binds suggests that opening up the financial market to outside investors might have adverse efficiency effects. Indeed, a comparison of Figures 1 and 3 reveals striking findings: Only to the right of $\overline{A}$ outside finance dominates over endogenous finance, because in wealth constrained economies, outside finance allows the execution of all positive NPV projects. In non-wealth constrained economies, the equilibria coincide. To the left of $\overline{A}$, outside finance reduces the efficiency apart from the upper left hand corner where the equilibrium $H^d L^e$ prevails. We obtain the following result:

**PROPOSITION 3:**

a. When $A < \hat{A}(I/pR_H)$ and $A > \overline{A}$ the outcome with outside investors weakly Pareto dominates the outcome without outside investors.
b. When \( A \in [\bar{A}(l/p_HR_H), \bar{A}] \), the outcomes without outside investors Pareto dominate the outcomes with outside investors if equilibrium \( H^fL' \) prevails.

c. When \( A \in [\bar{A}, \bar{A}] \), the outcomes without outside investors Pareto dominate the outcomes with outside investors.

In other words, only if the economy is very poor (\( A < \bar{A}(l/p_HR_H) \)) or very rich (\( A > \bar{A} \)), opening up the financial markets to outside investors may yield efficiency gains. In the intermediate range of wealth, allowing outside finance can reduce the efficiency of the financial markets: With outside finance, the equilibrium is an inefficient pooling or semi-separating equilibrium while without, the equilibrium may even be Pareto efficient.

The difference in the two cases lies in the opportunity cost of entrepreneurship. Without outside financiers, all agents cannot be entrepreneurs. The relative scarcity of funds raises the interest rates and the opportunity cost of entrepreneurship. This makes entrepreneurship less attractive, particularly in wealth constrained economies. When \( A > \bar{A} \), the H-type entrepreneurs’ pledgeable income exceeds the one of the L-type entrepreneurs. In such an environment the higher opportunity costs discourages foremost L-type entrepreneurs, improving the quality of the entrepreneurial pool. The same logic does not apply when \( A \leq \bar{A} \), because there the pledgeable income of L-type entrepreneurs is higher. The higher opportunity cost first affects H-types’ choice, causing an adverse effect on the average quality of entrepreneurs. In wealth constrained economies the efficient \( H^fL' \) equilibrium can nonetheless be supported for some parameter values, since the high quality of the entrepreneurial pool raises the returns on finance sufficiently to keep L-types as financiers.
C. Wealth and Entrepreneurship

Proposition 3 shows some efficiency effects of wealth, but removing outside financiers also affects the way wealth and entrepreneurship is linked. As expected, the link in non-wealth constrained economies has similarities with the case of outside finance: when \( A < \bar{A} \), insufficient wealth suppresses entrepreneurial activity because it leads to autarky, and when \( A \geq \bar{A} \), increases in wealth cause efficient exit of L-type entrepreneurs in equilibrium \( H^e L^{ef} \). However, as the example of Section III.A shows, increases in wealth stimulates inefficient entry of L-type entrepreneurs in equilibrium \( H^e L^f \).

In wealth constrained economies the relationship between wealth and entrepreneurship is quite different: in all equilibria except \( H^f L^f \), wealth is positively associated with efficient entry of H-type entrepreneurs. In \( H^f L^f \) H-type entrepreneurs are replaced by L-types as wealth rises. The aggregate amount of entrepreneurship in \( H^f L^f \) is nonetheless increasing in wealth.

The relationship between wealth and entrepreneurship can succinctly be written as follows:

**PROPOSITION 4:** Wealth and entrepreneurship are (weakly) negatively correlated only if storage is used. Otherwise, they are (weakly) positively correlated.

The negative relationship between wealth and entrepreneurship arises only if some agents invest in storage also in the case of outside finance. But there storage is used for a much wider range of parameter values. Moreover, with outside financiers, increases in wealth cannot lead to entry of H-type entrepreneurs once the financial markets open up.\(^{14}\)

\(^{14}\)The wealth constraint also affects the distribution of economic rents: In a wealth constrained economy, L-types earn rents, whereas H-types earn rents if the aggregate wealth constraint does not
IV. IMPERFECT STORAGE TECHNOLOGY

So far we have assumed an exogenous storage technology that fully converts the initial liquid assets to consumption goods. This assumption, while standard, is not realistic. For example, poor people in developing countries have no safe storing place and they need invest whatever extra liquid funds they have in livestock that may die, in jewellery that may be stolen, etc. In the richer world, available storage technologies such as cash are generally better but their efficiency hinges on the stability of monetary policy. Moreover, the assumption is not necessarily harmless. For instance, removing the storage technology eliminates bank runs in Diamond and Dybvig’s (1983) model and its variations.

To verify whether our findings are sensitive to the efficiency of the storage technology, we now assume that storage is imperfect so that \( A \) depreciates at rate \( 1 - \delta \), \( \delta \in [0,1] \). The only difference to the previous model is that the agents' IR constraints (3) should be rewritten as

\[
\pi_i^j \geq \delta A \ \forall i, j, \ i \in \{H, L\}, \ j \in \{e, f\}.
\]

When \( \delta \) is close to unity, our previous analysis is robust to the introduction of imperfect storage by continuity. As one might expect, however, the equilibria will change if \( \delta \) becomes small, because all agents are willing to invest either as financiers or as entrepreneurs even if their returns are small. To get an idea of the changes, let us reconsider the example of Section III.A (\( H^e L^f \)). To guarantee that all agents participate, we require that bind. H-types only earn rents as entrepreneurs, but L-types may earn them also as financiers. The rents are studied in more detail in the discussion paper version (HECER DP, 2006).
(18) \( R_p \geq \delta A \).

All other equations remain unchanged except that the L-type IR constraint (15) now takes the form

\[ A \leq \frac{p_L R_L h(p_H - p_L)}{(p_H - p_L) + p_L (\delta l - p_L R_L)}. \]

When \( \delta \) is close to unity, (19) remains a monotonically increasing curve in the \((A, h)\)-space. Decreasing \( \delta \) shifts the curve to the right, increasing the range of parameters where \( H^f L^{cf} \) exists. It can be shown that when \( \delta \) approaches zero, \( H^f L^{cf} \) exists for all parameter values in the non-wealth constrained region in so far as H-types' IC constraint (16) holds. This is quite natural, since without storage, the L-types' IR constraint is trivially satisfied.

For the rest of the section we focus on the case when \( \delta=0 \). Besides shortening the discussion, letting \( \delta=0 \) generalizes our model. When the agents no longer have an access to an exogenous storage technology, investing either as an entrepreneur or as a financier becomes the only way to transfer initial wealth to a consumption good.

Although our model lacks a second investment period, the exercise is similar in spirit to Holmström and Tirole (1998) who evaluate whether financial markets alone are able to supply enough liquidity and transform wealth over time.

FIGURE 4 HERE

The results of this exercise are summarized in Figure 4 (the calculations are available upon request). The equilibria \( H^f L^{fs} \) and \( H^f L^{cf} \), where storage is a viable option in the basic model with \( \delta=1 \), cease to exist. Instead, \( H^f L^{cf}, H^f L^{cf} \) and \( H^f L^c \) exist for larger parameter value ranges. \( H^f L^f \) remains unchanged. The largest change in efficiency occurs in non-wealth constrained economies for \( A > \bar{A} \), where the inefficient \( H^f L^{cf} \) exists instead of the efficient \( H^f L^{fs} \). As a result, only Pareto
inefficient equilibria exist in non-wealth constrained economies. Without storage, L-types are certain to invest either as entrepreneurs or as financiers. Once the needs of all H-type entrepreneurs are satisfied, it is impossible to prevent the remaining L-types from splitting between entrepreneurship and financiership. For $A \leq \bar{A}$, removing storage causes only modest changes to financial market performance, suggesting that financial markets alone can take care of transformation of wealth.

V. POLICY IMPLICATIONS

Though there are several limitations to our simple model, we boldly offer some policy recommendations. The first deals with financial market liberalization. If financial market liberalization means the introduction of outside investors without projects of their own, the predictions are rather clear. Liberalization can help a very poor country from autarky and generate a Pareto efficient separating equilibrium in a rich country. But liberalization is likely to result in a deterioration of entrepreneurial quality and the performance of financial markets in countries with medium initial wealth (compare the middle sections of Figures 1 and 3).

Our findings also have implications on widely adopted policies that seek to promote entrepreneurship (see, e.g., European Commission, 2001). These policies are often motivated by the observation that personal wealth facilitates entrepreneurship. Although designing an optimal budget-balancing tax-subsidy policy is beyond the scope of our study, our findings can be read to support the findings of de Meza and Webb (1987 and 1999) who argue that neither asymmetric information nor

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15 For instance, future work should consider more than two types of agents, heterogeneity in agents’ wealth, non-Walrasian market clearing, and more dynamic environment. In particular, we think that paying closer attention to coalition formation and the effect of creditor concentration on financial market efficiency in an equilibrium model of entrepreneurship and financial markets is a promising avenue for further research. Advances in this direction are made by Bris and Welch (2005).

16 This is reminiscent of Aghion et al. (2004) where financial market liberalization destabilizes an economy at an intermediate level of financial development which, in their model, is directly related to the initial wealth of the economy.
insufficient wealth is necessarily a reason to subsidize entrepreneurs or their finance.\textsuperscript{17} In our model this conclusion follows in particular if the aggregate wealth constraint of an economy is not binding. In that case, there is too much lending and entrepreneurship for a wide range of parameter values. This applies even if business creation is increasing in the level of wealth. The less efficient is the storage technology, the larger is the parameter space where entrepreneurial activity is excessive in spite of the positive correlation between wealth and entrepreneurship. However, a caveat should be borne in mind: in wealth constrained economies insufficient wealth can hold back productive entrepreneurs and a case for subsidies may arise. Encouraging entrepreneurship by making finance cheaper can also work if the economy is very poor and the market for entrepreneurial finance does not otherwise open up.

Another straightforward “policy experiment” concerning promotion of entrepreneurship is to move the vertical $\bar{A} \equiv p_L (p_H R_L - I) / (p_H - p_L)$-line to left. This increases the set of parameter values for which a Pareto efficient equilibrium exists in non-wealth constrained economies. Our experiment suggests that if entrepreneurship policies such as education and advice unconditionally raise the success probabilities or returns on successful projects without upgrading the L-type projects to positive NPV projects, they may be misguided.

As the discussion on wealth and entrepreneurship in Section III.A shows, shocks to model parameters change the values of the endogenous variables even if the equilibrium type remains the same as before the shock. When the parameters initially are close to a border, even small shocks may change the type of equilibrium. A

\textsuperscript{17} The optimal tax-subsidy policies in the market for entrepreneurial finance under asymmetric information are elaborated by Boadway and Keen (2005). Their results are consistent with our claim here.
decrease in initial wealth may shift the economy from a Pareto efficient equilibrium to an inefficient one (e.g., from $H^*L^f$ to $H^*L'^f$), or even to autarky (e.g., from $H^fL^f$ to autarky). But in our model also an increase in wealth may reduce efficiency: Increasing wealth may move an economy from a Pareto efficient $H^fL^f$ equilibrium to an inefficient $H^*L'^f$ (from point 1 to point 2 in Figure 3). However, increasing wealth is an effective tool in raising an economy out of autarky.

Finally, let us consider the role for financial intermediaries that collect and analyze information. There is no need for such financial institutions in the Pareto efficient equilibria. However, Pareto improving financial intermediary-coalitions could arise in equilibrium $H^*L'^f$, as indicated by Boyd and Prescott (1986). Extending the insights from Boyd and Prescott (1986) beyond $H^*L'^f$, financial intermediaries could improve upon the market equilibrium when we have low initial wealth (autarky), moderate initial wealth if $H^fL'^f$ prevails, or moderate to high initial wealth in a non-wealth constrained economy ($H^*L'^f$ and $H^*L'^{fs}$). The welfare-enhancing prospects of financial intermediaries further increase if the storage technology is inefficient (i.e., when $\delta$ is small).

Perhaps the most surprising rationale for financial intermediaries with market power comes from the observation that competitive financial markets can drive interest rates too low from an efficiency point of view. As in de Meza and Webb (1987), we show in Section II how competition between outside financiers results in the oversupply of funds for a wide range of parameter values. With endogenous finance, such overinvestment occurs in non-wealth constrained economies, generating the inefficient equilibria $HFL'^f$ and $HFL'^{fs}$.

These results concerning entrepreneurship policy, the role of initial wealth and the need of financial intermediaries under an inefficient storage technology all support the
notion that the creation of microfinance institutions might be a less wasteful antipoverty tactic than development aid, debt forgiveness or artificially making credit to poor cheaper, e.g., via loan rate regulation.\textsuperscript{18}

VI. CONCLUSIONS

In this paper we study whether, despite asymmetric information and capital constraints, the markets for entrepreneurial finance can endogenously emerge in equilibrium, and the efficiency of the eventual markets. In our model all agents encounter capital constraints but can choose whether they invest in their own project or finance others' ventures. In the usual partial equilibrium setting the only equilibria are autarky without financial markets and entrepreneurship, and a Pareto inefficient pooling equilibrium. We first show that a Pareto efficient separating equilibrium arises, if potential entrepreneurs are sufficiently rich, whereas a semi-separating Pareto inefficient equilibrium exists under intermediate entrepreneurial wealth.

We then exclude outside investors and find that the market for entrepreneurial finance continues to work. If anything, the market works more efficiently than with outside finance: a Pareto efficient equilibrium emerges in a wealth constrained economy for a wide range of parameter values where inefficient equilibria characterize the market with outside finance. We also find that in many cases business creation rises with wealth but this does not necessarily rationalize subsidies to entrepreneurs or their financiers. While these results are similar in spirit to de Meza and Webb (1987, 1990, 1999) who argue that a major concern in the markets for entrepreneurial finance is overinvestment, we also identify circumstances where business creation should be subsidized.

\textsuperscript{18} See Eeckhout and Munshi (2005) for the effects of interest rate regulation on microfinance.
Our findings differ from the conventional wisdom derived from partial equilibrium models. The findings suggest that, in the face of asymmetric information, the simplest type of financial markets may perform their role in resource allocation and asset transformation well, and that while increasing the proportion of high-quality entrepreneurs is a remedy for removing inefficiency, injecting capital into the market may not be.

REFERENCES


Table 1

<table>
<thead>
<tr>
<th>TYPES OF EQUILIBRIA</th>
<th>$\mu_L = 0$</th>
<th>$0 &lt; \mu_L &lt; 1$</th>
<th>$\mu_L = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_H = 0$</td>
<td>AUTARKY</td>
<td>Not possible</td>
<td>Not possible</td>
</tr>
<tr>
<td>$0 &lt; \mu_H &lt; 1$</td>
<td>$H^\ell L^f$</td>
<td>$H^\ell L^f$</td>
<td>$H^\ell L^f$</td>
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<tr>
<td>$\mu_H = 1$</td>
<td>$H^\ell L^\mu$</td>
<td>$H^\ell L^\mu$, $H^\ell L^\mu$</td>
<td>Not possible</td>
</tr>
</tbody>
</table>

Notes:

$\mu_i$ = the proportion of $i$ type agents that become entrepreneurs in equilibrium.
Figure 1

- Pareto inefficient pooling equilibrium
- L-type IR $\mu \leq 1$
- Pareto efficient separating equilibrium
- Pareto inefficient semi-separating equilibrium
- Autarky
Figure 2 \( (H^e L^e) \)

\[ \mu_{L^*} > 0 \text{ (11)} \]

- L-type IR (14)
- H-type IC (15)
Superscript $e = \text{entrepreneurship}$

$f = \text{financiers}$

$s = \text{storage technology}$
Figure 4

\[ \frac{I^H}{P_R^H} \frac{I^L}{P_R^L} \]

Pareto efficient equilibria

AUTARKY

Pareto inefficient equilibria
In this Appendix, we go through all possible equilibria besides autarky. For each equilibrium, we present
- the constraints,
- the equilibrium values of endogenous variables, and
- the equilibrium existence conditions.

We shorten the exposition by using the following notation: \( \Delta p = p_H - p_L \), \( \Delta R = R_L - R_H \), \( \gamma = p_H R_H - I \), \( \lambda = I - p_L R_L \), and \( \Delta W = \gamma \lambda = p_H R_H - p_L R_L \). The definitions have obvious interpretations. Since our approach to solve the model is rather mechanical, we explain the solution for the first equilibrium in more detail than for the subsequent equilibria. We also omit intermediate steps as these are straightforward (albeit sometimes tedious).

A. \( H^e L^{ef} \) and \( H^e L^{ed} \)

\( H^e L^{ef} \) is described in the example in Section III.A, so we first define \( H^e L^{ed} \) and then merely characterize its relation to \( H^e L^{ef} \). In \( H^e L^{ed} \), all H-types are entrepreneurs and L-types are indifferent among entrepreneurship, financing, and using the storage technology, i.e., \( \mu_H = 1 \), \( \mu_L \in (0,1) \), \( \chi_L = 0 \), and \( \chi_L \in (0,1) \). The situation here is otherwise similar to \( H^e L^{ef} \) described in Section III.A except that \( \chi_L \) is strictly positive. This means that (7) must hold as an equality, i.e.,

\[ R_F = A. "L- and H-type IR" \] (A.1)

The agents’ IC constraints are as before in (7) and (9), i.e.,

\[ p_L (R_L - R_B) = R_F. "L-type IC" \] (A.2)
\[ p_H (R_H - R_B) \geq R_F. "H-type IC" \] (A.3)

The economy level “budget constraint” (10) becomes

\[(1 - \mu_L - \chi_L)(1 - h)A = [h + \mu_L (1 - h)](I - A) \] (A.4)

"Equality of supply and demand for funds"

and, similarly, the financial market equilibrium condition (11) is

\[ h \mu_H + \mu_L (1 - h) p_L R_B = R_F (1 - \mu_L - \chi_L)(1 - h). \] (A.5)

"Financial market transactions"

Conditions (A.1)-(A.5) constrains \( H^e L^{ed} \). Equation (A.3) restricts the range of parameters and an equation system consisting of (A.1), (A.2), (A.4) and (A.5) determines the values of the endogenous variables \( R_B, R_F, \chi_L \), and \( \mu_L \). The equilibrium value of the expected payment received by financier, \( R_F^* \), equals \( A \) by (A.1). Then, solving (A.1) and (A.2) for \( R_B \) gives

\[ R_B^* = \frac{p_L R_L - A}{p_L}. \] (A.6)

Upon substituting (A.1) and (A.6) into (A.5) we have two equations, (A.4) and (A.5), that determine the remaining two endogenous variables, \( \chi_L \) and \( \mu_L \). After somewhat involved algebra they can be written as
\[ \mu_L^* = \frac{h}{1-h} \left[ \frac{(p_L R_L - A)\Delta p}{p_L \lambda} - 1 \right] = \frac{h}{1-h} \left( \frac{R^*_L \Delta p}{\lambda} - 1 \right) \]  
and
\[ \chi_L^* = \frac{1}{1-h} \left[ 1 - \frac{(p_L R_L - A)\Delta p h}{p_L \lambda A} \right] = \frac{1}{1-h} \left[ 1 - \frac{R^*_L \Delta p h}{\lambda A} \right]. \]

where the last equalities come from (A.6).

The equilibrium exists if \( \chi_L^* \) and \( \mu_L^* \) given by (A.7) and (A.8) satisfy our initial assumptions \( \mu_L \in (0,1) \) and \( \chi_L \in (0,1) \), and if the agents' IC and IR constraints are satisfied with \( R^*_L \) given by (A.6). The first four existence conditions are

\[ \mu_L^* < 1 \iff A > p_L \left( R_L - \frac{\lambda}{h \Delta p} \right), \]  
\[ \mu_L^* > 0 \iff A < p_L \left( R_L - \frac{\lambda}{\Delta p} \right) = \hat{A} + \frac{p_L Y}{\Delta p} = \overline{A}, \]  
\[ \chi_L^* < 1 \iff A < 1 + \frac{p^*_LR_L I \Delta p}{I \Delta p + p_L \lambda} = \frac{p^*_LR_L I \Delta p}{p^*_L I - p^*_L R_L}, \]  
\[ \chi_L^* > 0 \iff A > \frac{p^*_LR_L I \Delta p}{I \Delta p + p_L \lambda} = \frac{p^*_LR_L I \Delta p}{I(p_L + h \Delta p) - p^*_L R_L}. \]

Since L-type IC and IR bind by (A.1) and (A.2), the fifth existence condition comes from H-type IC (A.3). If it is satisfied, H-type IR (A.1) also trivially holds. Inserting (A.1) and (A.6) into (A.3) shows that H-type IC holds if

\[ A \geq \hat{A}. \]  

Equations (A.9)-(A.13) define the range of parameters for which \( H^* \) \( L^* \) exists. Since the critical values of \( A \) in (A.11) and (A.12) are strictly larger than the respective critical values in (A.10) and (A.9), the binding critical values are given by (A.10) and (A.12). They in turn cross each other at the diagonal \( h = \frac{A}{I} \). This means that \( H^* \) \( L^* \) only exists in non-wealth constrained economies. In terms of the (A, h)-space, \( H^* \) \( L^* \) exists in the area between the vertical lines (A.13) and (A.10), and below the curve (A.12), as depicted in Figure A.1.

When (A.12) (which is identical to equation (15)) is violated, the H-type IC changes from (A.13) to (16), i.e., to \( A \geq I - I \Delta W \left( p_L + h \Delta p \right) / \Delta p p_L R_L \). Thus, \( H^* \) \( L^* \) exists in the range of parameters described in Section III.A., i.e., in the area shaped by curve (A.12), the downward sloping line (16) and \( h = \frac{A}{I} \) diagonal. Note also that curve (A.12), the vertical \( \hat{A} \) line and the downward sloping line cross at the same point where \( h = h_i \equiv \frac{\hat{A} \lambda}{I \Delta W} \).
B. $H^{eS}L^f$ and $H^{eS}L^{fs}$

We first prove that $H^{eS}L^{fs}$ cannot exist. In this equilibrium $\mu_H \in (0,1)$, $\mu_L = 0$ and both $\chi_H$ and $\chi_L \in (0,1)$. The equilibrium is constrained by the following five conditions:

\[ R_F \geq A, \text{ "L- and H-type IR"} \] (B.1)

\[ p_L (R_L - R_H) \leq R_F, \text{ "L-type IC"} \] (B.2)

\[ p_H (R_H - R_B) = R_F, \text{ "H-type IC"} \] (B.3)

\[ [(1 - \mu_H - \chi_H)h + (1 - \chi_L)(1 - h)]A = \mu_H h(I - A), \] (B.4)

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and

\[ h\mu_H p_H R_R = R_F \left[ (1 - \mu_H - \chi_H)h + (1 - \chi_L)(1 - h) \right], \] (B.5)

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In $H^{eS}L^{fs}$ (B.1) holds with equality. Solving (B.4) for $\mu_H$ yields

\[ \mu_H = \frac{A}{h(I - A)}, \] (B.6)

Using (B.3) and (B.6) in (B.5) yields $R_B$ as

\[ R^*_B = \frac{R_H (I - A)}{I}. \] (B.7)

Inserting (B.7) back into (B.3) gives

\[ R^*_F = p_H R_H \frac{A}{I}. \] (B.8)

Since $R^*_F$ in (B.8) is strictly larger than $A$, the initial assumption (B.1) that the agents’ IR constraints bind is invalid. This means that the equilibrium cannot exist for positive $\chi_H$ and $\chi_L$.

$H^{eS}L^f$ can be characterized by setting $\chi_H = \chi_L = 0$ in (B.6). This means that

\[ \mu_H^* = \frac{A}{h(I - h)}. \] (B.9)

Equation (B.9) gives two equilibrium existence conditions:

\[ \mu_H^* < 1 \iff A < hI, \] (B.10)

and

\[ \mu_H^* > 0 \iff A > 0. \] (B.11)

By means of (B.7) and (B.8) the third existence condition, the L-type IC constraint (B.2), can be written as

\[ A \geq \frac{I}{p_H R_H}. \] (B.12)

Equations (B.10)-(B.12) define the range of parameters for which $H^{eS}L^f$ exists. As shown in Figure A.2, the equilibrium exists in wealth constrained economies for $A \in [\hat{A}l/p_H R_H, I]$. 

FIGURE A.2 HERE
C. \( H^e L^f \)

In this equilibrium, \( \mu_H = 1 \), \( \mu_L = 0 \), \( \chi_H = 0 \) and \( \chi_L \in (0,1) \). In words, all H-types are entrepreneurs and L-types are either financiers or use the storage technology. The five basic conditions constraining the equilibrium are

\[
R_F = A, \text{ "L-type IR"} \tag{C.1}
\]
\[
p_L (R_L - R_H) \leq R_F, \text{ "L-type IC"} \tag{C.2}
\]
\[
p_H (R_H - R_B) \geq R_F, \text{ "H-type IC and IR"} \tag{C.3}
\]

\[
(1 - \chi_L)(1 - h)A = h(I - A), \tag{C.4}
\]

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and

\[
h p_H R_B = R_F (1 - \chi_L)(1 - h). \tag{C.5}
\]

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The equilibrium value of \( R_F \) trivially equals \( A \) by (C.1). By substituting (C.1) into (C.5), the other endogenous variables, \( \chi_L \) and \( R_B \), can be solved from (C.4) and (C.5). They are given by

\[
\chi_L^* = \frac{A - hI}{A(1 - h)} \tag{C.6}
\]

and

\[
R_a^* = \frac{I - A}{p_H}. \tag{C.7}
\]

From (C.6) we see that \( \chi_L^* < 1 \) by assumption \( A < I \). Similarly, inserting (C.1) and (C.7) into (C.3) shows that H-types' IC and IR constraints are equivalent to \( p_H R_B > I \) which holds by assumption. Thus, \( H^e L^f \) is defined by two existence conditions. First,

\[
\chi_L^* \geq 0 \iff A \geq hI \tag{C.9}
\]

must hold. Second, the L-type IC constraint (C.2) must hold. Employing (C.1) and (C.7), it can be rewritten as

\[
A \geq \hat{A} + \frac{p_L \gamma}{\Delta p} = \overline{A}, \tag{C.10}
\]

where the right hand side equals (A.10). Equations (C.9) and (C.10) show that \( H^e L^f \) only exists in non-wealth constrained economies for \( A \in [\hat{A} + p_L \gamma/\Delta p, I] \) (see Figure A.3).

FIGURE A.3 HERE

D. \( H^{ef} L^e \) and \( H^{ef} L^f \)

We first prove that \( H^{ef} L^e \) cannot exist. The set-up of \( H^{ef} L^e \) practically mirrors \( H^e L^{ef} \) of Section A of the Appendix, because here \( \mu_H \in (0,1), \mu_L = 1, \chi_H \in (0,1) \) and \( \chi_L = 0 \). In words, all L-types are entrepreneurs, and H-types are indifferent between entrepreneurship, financing, and using the storage technology. The five basic constraints in \( H^{ef} L^e \) are
\[ R_F = A, \text{"L- and H-type IR"} \] (D.1)
\[ p_L(R_L - R_H) \geq R_F, \text{"L-type IC"} \] (D.2)
\[ p_H(R_H - R_B) = R_F, \text{"H-type IC"} \] (D.3)
\[ (1 - \mu_H - \chi_H)hA = [1 - h + \mu_H \delta](I - A). \] (D.4)
"Equality of supply and demand for funds",

and
\[ \left[ h \mu_H p_H + (1 - h)p_L \right] R_B = R_F (1 - \mu_H - \chi_H)h. \] (D.5)
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An equation system consisting of (D.1) and (D.3)-(D.5) determines the values of the endogenous variables, \( R_B, \chi_H, \) and \( \mu_H. \) Solving (D.1) and (D.3) for \( R_B \) gives
\[ R_B^* = \frac{p_H R_H - A}{p_H}. \] (D.6)

Upon substituting (D.1) and (D.6) into (D.5) we have two equations (D.4) and (D.5) that determine the remaining two endogenous variables, \( \chi_H, \) and \( \mu_H. \) After somewhat involved algebra they can be written as
\[ \mu_H^* = 1 - \frac{1 - h}{\delta} \left[ 1 - \frac{\Delta p (p_H R_H - A)}{p_H \gamma} \right] = 1 - \frac{h}{\delta} \left( 1 - \frac{R_H^* \Delta p}{\gamma} \right) \] (D.8)
and
\[ \chi_H^* = 1 - \frac{1 - h}{\delta} \left[ 1 - \frac{\Delta p (p_H R_H - A)I(1 - h)}{p_H \gamma A} \right] = 1 - \frac{h}{\delta} \left[ 1 - \frac{R_H^* \Delta p I (1 - h)}{\gamma A} \right] \] (D.9)

Equations (D.8) and (D.9) provide four equilibrium existence conditions:
\[ \mu_H^* \leq 1 \iff A \leq \frac{p_H \gamma (1 - h) \delta}{\Delta p (1 - h)}, \] (D.10)
\[ \mu_H^* \geq 0 \iff A \geq \frac{p_H (I - p_L R_H)}{\Delta p}, \] (D.11)
\[ \chi_H^* \leq 1 \iff A \leq \frac{p_H R_H I \Delta p}{I \Delta p + p_H \gamma} = \frac{p_H R_H I \Delta p}{p_H^2 R_H - p_L I}, \] (D.12)
and
\[ \chi_H^* \geq 0 \iff A \geq \frac{p_H R_H I (1 - h) \Delta p}{I (1 - h) \Delta p + p_H \gamma} = \frac{p_H R_H I (1 - h) \Delta p}{p_H^2 R_H - I (p_L + h \Delta p)}. \] (D.13)

Since H-types' IC and IR bind, and L-types' IR is satisfied through their IC, the L-type IC is the fifth equilibrium existence condition. It is satisfied if
\[ A \leq \hat{A}. \] (D.14)

The equilibrium may exist between the vertical lines (D.11) and (D.12), which is a nonempty set. However, the vertical line (D.14) is smaller in value than the vertical line (D.11). This means that the equilibrium cannot exist for positive \( \chi_H. \)
In contrast, $H^e L^e$ does exist. To see this, note first that in $H^e L^e$, (D.1) holds with a weak inequality. Then, let $\chi_L = 0$ in (D.4) to get

$$\mu^*_H = \frac{A - (1-h)I}{hI}. \tag{D.15}$$

Substituting (D.15) and (D.3) for (D.5) and letting $\chi_L = 0$ yields

$$R^*_n = \frac{p_\mu R_H (I - A)}{I \left(p_L + h\Delta p \right)}. \tag{D.16}$$

Inserting (D.16) back into (D.3) gives

$$R^*_F = \frac{p_\mu R_H \left(p_\mu A - \Delta p(1-h) \right)}{I \left(p_L + h\Delta p \right)}. \tag{D.17}$$

From (D.15) we see that $\mu^*_H < 1$ holds by our assumption that $A < I$. An equilibrium existence condition is thus

$$\mu^*_H \geq 0 \iff A \geq (1-h)I. \tag{D.18}$$

The H-type IR is now $R^*_F \geq A$, which - using (D.17) - can be expressed as

$$A \geq \frac{p_\mu R_H I \Delta p(1-h)}{I \Delta p(1-h) + p_\mu \gamma} = \frac{p_\mu R_H I \Delta p(1-h)}{p^2_\mu R_H - I \left[p_L + h\Delta p \right]}. \tag{D.19}$$

Similarly, by means of (D.16) and (D.17) the L-type IC (D.2) is given by

$$A \leq I - \frac{\Delta W \left(p_L + h\Delta p \right)}{\Delta p \mu R_H R_H} = \frac{I}{p_\mu R_H \left[1 + \Delta W(1-h) \right]}. \tag{D.20}$$

Conditions (D.18)-(D.20) define the range of parameters for which $H^e L^e$ exists. This is shown in the $(A,h)$-space in Figure A.4.

Conditions (D.18)-(D.20) practically mirror those of $H^e L^e$ described in Section III.A. Equation (D.18) defines the downward sloping $h = 1 - A/II$ diagonal that starts from the $(A=0, h=0)$ corner and ends in the $(A=I, h=0)$ corner. The L-type IC constraint (D.20) is a downward sloping line that cuts the $h = 1 - A/II$ diagonal at the same point as the vertical $\hat{A} l l p_L R_L$ line. H-types’ IR constraint (D.19) is a monotonically downward sloping curve that starts from the $(A=0, h=1)$ corner and cuts the $h = 1 - A/II$ diagonal once. H-types’ IR and L-types’ IC constraints and the vertical $\hat{A}$ line cross at the same point at

$$h = h_2 = 1 - \frac{\gamma}{\Delta Wl}. \text{ In sum, } H^e L^e \text{ exists above the H-type IR curve (D.19) and below the L-type IC line (D.20). This area exists in the upper-left corner of the (A, h)-space where } A \in [0, \hat{A}] \text{ and } h \in [h_2, 1].$$

$E. \; H^f L^f$ and $H^{dfs} L^{dfs}$

We first prove that $H^f L^{dfs}$ cannot exist for a non-trivial set of parameters. In this equilibrium $

\mu_H \in (0, 1), \; \mu_L \in (0, 1), \; \chi_H \in (0, 1) \; \chi_L \in (0, 1). \text{ In words, all agents are indifferent between
entrepreneurship, financing, and using storage. The agents' IR and IC constraints bind, i.e., it must hold that

\[ R_F = A, \text{ "L- and H-type IR"} \]  
\[ p_L (R_L - R_H) = R_F, \text{ "L-type IC"} \]  

and

\[ p_H (R_H - R_H) = R_F, \text{ "H-type IC"} \]  

Solving (E.2)-(E.3) for \( R_H \) gives

\[ R_H^* = \frac{\Delta W}{\Delta p}. \]  

Thus, there is a unique value of

\[ A = p_H (R_H - \frac{\Delta W}{\Delta p}) = p_L (R_L - \frac{\Delta W}{\Delta p}) = \hat{A}. \]  

for which this equilibrium can exist. This means that only \( H^L \) (where \( \mu_H \in (0, 1), \mu_L \in (0, 1) \), and \( \chi_H = \chi_L = 0 \)) may exist for a non-trivial range of parameters.

\( H^L \) is constrained by the following five basic conditions:

\[ R_F \geq A, \text{ "L- and H-type IR"} \]  
\[ p_L (R_L - R_H) = R_F, \text{ "L-type IC"} \]  
\[ p_H (R_H - R_H) = R_F, \text{ "H-type IC"} \]  

\[ [(1 - \mu_H)h + (1 - \mu_L)(1 - h)]A = [\mu_L (1 - h) + \mu_H h](I - A), \]  

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and

\[ [(1 - \mu_H)h + (1 - \mu_L)(1 - h)]R_F = [p_L \mu_L (1 - h) + p_H \mu_H h]R_H. \]  

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Equation system (E.9)-(E.12) determines the values of the endogenous variables, \( R_F, R_H, \mu_L, \) and \( \mu_H \). Solving (E.9) and (E.10) for \( R_H \) and \( R_F \) gives

\[ R_H^* = \frac{\Delta W}{\Delta p}. \]  

and

\[ R_F^* = p_H (R_H - \frac{\Delta W}{\Delta p}) = p_L (R_L - \frac{\Delta W}{\Delta p}) = \hat{A}. \]  

Substituting (E.13) and (E.14) into (E.12) and solving (E.11) and (E.12) for \( \mu_L \) and \( \mu_H \) yields

\[ \mu_H = \frac{1}{h\Delta W} \left( A - \frac{\Delta p_L R_L}{I} \right) \]  

and
\[
\mu^*_L = \frac{1}{(1-h)\Delta W} \left( \frac{A p_H R_H}{I} - \hat{A} \right). \tag{E.16}
\]

Equations (E.15) and (E.16) yield four equilibrium existence conditions:

\[
\mu^*_H < 1 \iff A > \frac{I}{p_H R_H} \left( \hat{A} - h \Delta W \right), \tag{E.17}
\]

\[
\mu^*_H > 0 \iff A < \frac{I}{p_H R_H} \hat{A}, \tag{E.18}
\]

\[
\mu^*_L < 1 \iff A < \frac{I}{p_H R_H} \left[ \hat{A} + (1-h) \Delta W \right], \tag{E.19}
\]

and

\[
\mu^*_L > 0 \iff A > \frac{I}{p_H R_H} \hat{A}. \tag{E.20}
\]

From equations (E.8)-(E.10) we see that agents’ IC constraints bind and IR constraints are satisfied if

\[
A \leq \hat{A}. \tag{E.21}
\]

This is the fifth equilibrium existence condition. However, we see that if condition (E.21) holds, (E.18) also holds. The equilibrium is thus defined by equations (E.17), and (E.19)-(E.21). Since (E.19) is identical to (D.20) we know that it cuts the vertical \( \hat{A} \)-line at \( h = h_2 \) where \( h_2 \equiv 1 - \frac{\hat{A} \gamma}{\Delta W} \) as defined in Section D of the appendix. This means that when \( h \) is large, i.e., \( h \in [h_2,1] \), the downward sloping line (E.19) and the vertical line (E.20) are the binding constraints. For \( h \in [h_3, h_2] \) where \( h_3 \equiv \frac{\hat{A}}{p_H R_H} \), the binding constraints are the vertical lines (E.20) and (E.21). For \( h \in [h_1, h_3] \), where \( h_1 \equiv \frac{\hat{A} \lambda}{I \Delta W} \) as defined in Section A of the appendix, the binding constraints are (E.17) (which is identical to (15)) and (E.21). For \( h < h_1 \), the equilibrium does not exists, since (E.17) is violated.

FIGURE A.5 HERE

In Figure A.5 we illustrate how in terms of the \((A, h)\)-space, \( H' \cap L' \) exists in a parallelogram between the vertical lines (E.20) and (E.21) and the downward sloping lines (E.17) and (E.19). This parallelogram exists for \( A \in (\hat{A} / p_H R_H, \hat{A}) \) and \( h \in [h_1,1] \).
Figure A.1 \((H^e L^{efs})\)

\[
\chi_L^{*} > 0 \quad (A.12)
\]

\[
\mu_L^{*} > 0 \quad (A.10)
\]

\[
\mu_L^{*} > 0 \quad (A.10)
\]

\[
\chi_L^{*} > 0 \quad (A.12)
\]
Figure A.2 \((H_{\text{ref}}L^{f})\)
Figure A.3 \( (H^e L^{fs}) \)

\[ \chi L > 0 \quad \text{(C.6)} \]

\[ \chi t^* > 0 \quad \text{(C.6)} \]

L-type IC
\( \text{(C.10)} \)
Figure A.4 \((H^e L^e)\)

\[ \hat{A} \]

- L-type IC (D.20)
- H-type IR (D.19)
- \(\mu_{ii}^* > 0\) (D.18)

I  A
Figure A.5 ($H_f^eL_f^e$)

\[ \mu_l^* > 0 \quad (E.20) \]

\[ \mu_H^* > 0 \quad (E.18) \]

\[ \mu_l^* < 1 \quad (E.19) \]

H- and L-type IR and IC

\[ \hat{I} \hat{A} \]

\[ \frac{\hat{I}}{P_{hl}R_h} \]

\[ \hat{A} \]

h=1

h=1