Optimal Taxation with Capital Accumulation and Wage Bargaining

Tapio Palokangas
University of Helsinki and HECER

Discussion Paper No. 26
October 2004

ISSN 1795-0562
Optimal Taxation with Capital Accumulation and Wage Bargaining

Abstract

This paper examines optimal taxation when monopoly unions set wages and some or all households save in capital. The main findings are as follows. Judd's (1985) and Chamley's (1986) classical results of zero taxation on capital income holds also in a unionized economy. Aggregate production efficiency in the steady state can be maintained by subsidies to wages and employment, which are financed by the consumption tax. The optimal employment subsidy is determined by a specific elasticity rule.

JEL Classification: J51, H21

Keywords: wage settlement, optimal taxation, capital accumulation.

Tapio Palokangas

Department of Economics
P.O. Box 17 (Arkadiankatu 7)
University of Helsinki
FI-00014 University of Helsinki
FINLAND

e-mail: tapio.palokangas@helsinki.fi
1 Introduction

This paper considers how taxes should optimally be determined in a unionized economy with capital accumulation. Palokangas (1987 and 2000, Ch. 4) shows that in a static general equilibrium framework, aggregate production efficiency can be maintained even in the presence of labour unions as long as the government can set specific wage and employment taxes. In this study, we examine whether this result also holds true in a dynamic general equilibrium framework, in which private agents save capital and there is a strategic interdependence between investors and labour unions.

In a dynamic model with investment, aggregate production efficiency takes lines up with the Judd-Chamley assertion: capital income should be taxed at a non-zero rate. Because capital functions as an intermediate good, appearing only in the production but not in the utility function, it should not be taxed, if there are enough instruments to separate consumption and production decisions. Chamley (2001) shows that this assertion critically depends on the existence of a perfect bond market, in which private agents take the interest rate as given, households save in bonds, and firms can finance any amount of investment by issuing bonds. Instead a perfect bond market, we assume here that households own shares in firms directly. In such a case, there is a conflict between workers in a firm and households who invest in the firm. We show that zero taxation on capital income holds even when workers are organized in wage-setting monopoly unions.

Lancing (1999) shows that when the capitalists’ utility is logarithmic and the government faces a balanced-budget constraint, the steady-state optimal tax on capital income is generally non-zero. We prove that in a unionized economy this assertion holds only in the very unlikely special case that all of the following three conditions are simultaneously satisfied: (i) the capitalists’ preferences are logarithmic, (ii) there are constant returns to scale in production, and (iii) the capitalists earn no labour income.

Koskela and von Thadden (2002) show that capital income should be taxed at a non-zero rate in a unionized economy, but they base this result on the assumption that a labour union takes capital stock as given. In this paper, we find it inconsistent to assume that while the union takes the

\footnote{Judd (1985), Chamley (1986) and Correia (1996).}
employment effects of its wage policy into account, it simultaneously ignores the investment effects. Hence, we prefer to assume that a union takes the employer’s investment behaviour as a constraint. We show that in such a case the Judd-Chamley result still holds.

In this study, we assume the following. The government is the Stackelberg leader with respect to the private agents, who take the tax rates as given. In each industry, the wage-setting monopoly union is the Stackelberg leader with respect to the firms and investors. There are two groups of households. The capitalists save and earn all profits and a fixed proportion $\alpha$ of all wages. The non-capitalists earn the rest $(1 - \alpha)$ of total wages and consume all of their income. We use parameter $\alpha$ as a measure of income distribution. The model is then an extension of two special cases: for $\alpha = 0$, Judd’s (1985) case in which the capitalists earn only profits and do not work, while the workers earn only wages and do not save; and for $\alpha = 1$, Chamley’s (1986) model of a representative agent who saves and earns both wages and profits.

The remainder of this paper is organized as follows. Section 2 considers firms and workers in an industry and derives profits and labour income as functions of employment, capital stock and taxes. Section 3 examines investment in a single industry, and section 4 the labour union which sets the wage in an industry. Section 5 constructs the optimal program for the government, by which optimal public policy is established in section 6.

2 Firms and workers

We aggregate all products in the economy into a single good which is chosen as the numeraire. There is a fixed number $J$ of similar industries producing this good. In industry $j$, the representative firm (hereafter firm $j$) produces its output $Y_j$ from its capital $K_j$ and labour $L_j$ through technology

$$Y_j = F(K_j, L_j), \quad F_K > 0, \quad F_L > 0, \quad F_{LL} < 0, \quad F_{KK} < 0, \quad F_{KL} > 0,$$  
(1)

where subscript $K$ ($L$) denotes partial derivatives with respect to $K_j$ ($L_j$). Unit labour cost for firm $j$ is given by $v_j = (1 + \tau_W)w_j + \tau_L$, where $w_j$ is the wage in firm $j$, $\tau_W$ the wage tax and $\tau_L$ the employment tax. Firm $j$ takes its unit labour cost $v_j$ and capital stock $K_j$ as given and maximizes its profit
\[ \pi_j = F(K_j, L_j) - v_j L_j - \mu K_j \] by labour input \( L_j \), where and the constant \( \mu \in (0, 1) \) is the rate of capital depreciation. This yields the function

\[ \pi_j = \Pi(K_j, v_j) = \max_{L} [F(K_j, L_j) - v_j L_j - \mu K_j], \]

\[ \Pi_K = \partial \Pi / \partial K = F_K - \mu, \quad \Pi_v = \partial \Pi / \partial v = -L_j, \]

\[ \Pi_{KK}(K_j, v_j) \equiv 0 \iff \Pi(K_j, v_j) = \max_{\ell} [F(1, \ell) - v_j \ell - \mu] K_j = \Pi_K(v_j) K, \]

\[ v_j = F_L(K_j, L_j), \quad w_j = [F_L(K_j, L_j) - \tau L_j] / (1 + \tau W). \]

We assume that for workers the disutility of employment in terms of consumption, \( Z(L_j) \), increases with the level of employment, \( Z' > 0 \). We can then define that labour income in industry \( j \), \( W^j \), is equal to wages \( w_j L_j \) minus the disutility of employment \( Z \). Noting (1) and (2), labour income in industry \( j \) is then obtained as a function of employment, capital and taxes:

\[ W^j = W(L_j, K_j, \tau W, \tau L) = w_j L_j - Z = \frac{F_L(K_j, L_j) - \tau L_j - Z(L_j)}{1 + \tau W}, \]

\[ W_L = \frac{\partial W}{\partial L_j} = \frac{F_{LL} L + F_L - \tau L}{1 + \tau W} - Z', \quad W_K = \frac{\partial W}{\partial K_j} = \frac{F_{KL} L_j}{1 + \tau W}. \] (3)

## 3 Investment

We assume that each capitalist invests only in a single industry, for tractability.\(^2\) The representative capitalist in industry \( j \) (hereafter capitalist \( j \)) earns a fixed proportion \( \alpha_j \) of total labour income \( \sum_k W^k \). The capitalists as a group earn a fixed proportion \( \alpha = \sum_j \alpha_j \) of \( \sum_k W^k \). Because each capitalist is the Stackeberg follower with respect to the wage-setting union in the same industry, he takes unit labour cost \( v_j \) as given.

We assume that capitalist \( j \) takes his wage revenue \( \alpha_j \sum_k W^k \) as given. This can be justified as follows. When a worker invests in capital, the proportion of his portfolio invested in his current employer is so small that he can ignore the effect of his investment on his own wage revenue. Capitalist \( j \)'s budget constraint is given by

\[ \dot{K}_j = dK_j / dt = \alpha_j \sum_k W^k + (1 - \tau_K) \Pi(K_j, v_j) - (1 + \tau_C) C_j, \] (4)

\(^2\)If capitalists invested in all industries, then the levels of investment for all industries would be ‘bang-bang’ controls and the model would be excessively complicated.
where $\dot{K}_j$ is capital accumulation, $\alpha_j \sum_k W^k$ exogenous labour income, $\Pi(K_j, w_j)$ profits, $\tau_K$ tax on capital income, $C_j$ his consumption and $\tau_C$ consumption tax. Capitalist $j$’s instantaneous utility is given by

$$U(C_j) = \begin{cases} \frac{[C_j^{1-\sigma} - 1]}{(1 - \sigma)} & \text{for } \sigma \in (0, 1) \cup (1, \infty), \\ \log C_j & \text{for } \sigma = 1, \end{cases}$$

where the constant $1/\sigma$ is the intertemporal elasticity of substitution.

Capitalist $j$ chooses his consumption to maximize the flow of utility starting at time zero, $\int_0^\infty U(C_j) e^{-\rho t} dt$, where $t$ is time and $\rho > 0$ the rate of time preference, subject to capital accumulation (4) and the functions (3), taking unit labour cost in the industry, $v_j$, and his own labour income $\alpha_j \sum_k W^k$ as given. This leads to the Hamiltonian

$$H^{C_j} = U(C_j) + \theta_j \left[ \alpha_j \sum_k W^k + (1 - \tau_K)\Pi(K_j, v_j) - (1 + \tau_C)C_j \right],$$

where the co-state variable $\theta_j$ evolves according to

$$\dot{\theta}_j = \rho \theta_j - \frac{\partial H^{C_j}}{\partial K_j} = [\rho - (1 - \tau_K)\Pi_K(K_j, v_j)]\theta_j, \lim_{t \to \infty} \theta_j K_j e^{-\rho t} = 0. \quad (6)$$

The first-order condition for the capitalist’s maximization is given by $C_j^{-\sigma} = U''(C_j) = (1 + \tau_C)\theta_j$. Noting this, we can transform the constraint (6) into the capitalist’s Euler equation as follows:

$$\frac{\dot{C}_j}{C_j} = -(1/\sigma)\dot{\theta}_j/\theta_j = [(1 - \tau_K)\Pi_K(K_j, v_j) - \rho] / \sigma. \quad (7)$$

Variables $K_j$ and $C_j$ are governed by (4) and (7). When there are decreasing returns to scale in production, $\Pi_{KK} < 0$, the dynamics is as follows. Because $\partial K_j/\partial K_j = (1 - \tau_K)\Pi_K > 0$, $\partial \dot{K}_j/\partial C_j < 0$, $\partial \dot{C}_j/\partial K_j = (1 - \tau_K)\Pi_{KK} C_j / \sigma < 0$ and $[\partial \dot{C}_j/\partial C_j]_{C_j=0} = 0$, we obtain

$$\frac{\partial \dot{K}_j}{\partial K_j} + \frac{\partial \dot{C}_j}{\partial C_j} \bigg|_{C_j=0} > 0, \quad \frac{\partial K_j}{\partial K_j} \frac{\partial \dot{C}_j}{\partial C_j} \bigg|_{C_j=0} < \frac{\partial K_j}{\partial C_j} \frac{\partial \dot{C}_j}{\partial K_j},$$

and there is a saddle-point solution. Hence, the co-state variable $C_j$ (which represents $\theta_j$) jumps onto the saddle path which leads to the steady state in which $K_j$, $C_j$ and $\theta_j$ are constants, and $\lim_{t \to \infty} \theta_j K_j e^{-\rho t} = 0$ holds.
With constant returns to scale $\Pi_{KK} \equiv 0$, from (2), (4) and (6) we obtain
\[
\left[ \frac{\dot{K}_j}{K_j} + \frac{\dot{\theta}_j}{\theta_j} - \rho \right]_{K_j=0} = \left[ \alpha_j \sum_k W^k - (1 + \tau_C) \frac{C_j}{K_j} \right]_{K_j=0} = (\tau_K - 1)\Pi_K < 0.
\]
This as well implies the transversality condition $\lim_{t \to \infty} K_j \theta_j e^{-\rho t} dt = 0$.

Finally, we examine the special case where $\sigma = 1$, $\Pi(v_j, K_j) = \Pi_K(v_j)K_j$ and $\alpha = 0$. In such a case, the equations (4) and (7) take the form
\[
\dot{K}_j/K_j = (1 - \tau_K)\Pi_K(v_j) - (1 + \tau_C)C_j/K_j, \quad \dot{C}_j/C_j = (1 - \tau_K)\Pi_K(v_j) - \rho.
\]
In this system, the co-state variable $C_j$ jumps to the level $[\rho/(1 + \tau_C)]K_j$, which maintains the steady state $\dot{K}_j/K_j = \dot{C}_j/C_j$. This reconstructs Lancing’s (1999) assertion as follows:

**Proposition 1** If (i) the capitalists’ preferences are logarithmic, $\sigma = 1$, (ii) there are constant returns to scale in production, $\Pi(v_j, K_j) = \Pi_K(v_j)K_j$, and (iii) the capitalists earn no labour income, $\alpha \equiv 0$, then the capitalists’ optimal decisions depend solely on the current rates of return or tax rates, not on future rates of return or tax rates.

To exclude this very special case, we assume that at least one of the conditions (i) – (iii) in proposition 1 is not true.

### 4 Wage settlement

The workers in industry $j$ are organized in monopoly union $j$, which maximizes the value of the flow of labour income $W^j$, discounted by the households’ rate of time preference, $\rho$. Given (3), this target can be written as
\[
\int_0^\infty W^j e^{-\rho t} dt = \int_0^\infty W(L_j, K_j, \tau_W, \tau_L) e^{-\rho t} dt.
\]
(8)

Union $j$ takes wages paid elsewhere in the economy, $\sum_{k \neq j} W^k$, as fixed and sets its wage $w_j$ to maximize its welfare (8), given the capitalist’s Euler equation (7) and capital accumulation (4) and the firm’s responses (3) for the same industry $j$. Because there is a one-to-one correspondence from $w_j$
to \(L_j\) through (2), the wage \(w_j\) can be replaced by employment \(L_j\) as the union’s policy instrument. The union then maximizes by \(L_j\) the Hamiltonian

\[
H_j = W(L_j, K_j, \tau_W, \tau_L) + \xi_j \{ (1 - \tau_K)[F_K(K_j, L_j) - \mu] - \rho \} C_j/\sigma
+ \phi_j \{ \alpha_j W(L_j, K_j, \tau_W, \tau_L) + \alpha_j \sum_{k \neq j} W_k + (1 - \tau_K)\Pi(K_j, F_L(K_j, L_j))
- (1 + \tau_C)C_j \},
\]

(9)

where the co-state variables \(\xi_j\) and \(\phi_j\) evolve according to

\[
\dot{\xi}_j = \rho \xi_j - \partial H_j/\partial C_j = \rho \xi_j + \{ \rho - (1 - \tau_K)[F_K(K_j, L_j) - \mu] \} \frac{\xi_j}{\sigma} + (1 + \tau_C)\phi_j, \\
\lim_{t \to \infty} \xi_j C_j e^{-\rho t} = 0, \quad (10)
\]

\[
\dot{\phi}_j = \rho \phi_j - \partial H_j/\partial K_j = [\rho - (1 - \tau_K)(\Pi_K + \Pi_v F_{KL})] \phi_j - (1 + \alpha_j \phi_j) W_K, \\
\lim_{t \to \infty} \phi_j K_j e^{-\rho t} = 0. \quad (11)
\]

Noting (2) and (3), we obtain the first-order condition for \(L_j\) as follows:

\[
\partial H_j/\partial L_j = (1 + \alpha_j \phi_j) W_L + (1 - \tau_K)[F_{KL} \xi_j C_j/\sigma + \Pi_v F_{LL} \phi_j]
= (1 + \alpha_j \phi_j) \{(1 + \tau_W)^{-1} [F_{Ll}(K_j, L_j) L_j + F_L(K_j, L_j) - \tau_L] - Z'(L_j)\}
+ (1 - \tau_K) [F_{KL}(K_j, L_j) \xi_j C_j/\sigma - L_j F_{LL}(K_j, L_j) \phi_j] = 0. \quad (12)
\]

5 Optimal public policy

The non-capitalists consume their entire income net of taxes, \((1 - \alpha) \sum_j W^j \) / \((1 + \tau_C)\), where \(\tau_C > -1\) is the consumption tax. The government is subject to fixed expenditure \(E\) and finances these by taxing total consumption \(\sum_j C_j + (1 - \alpha) \sum_j W^j / (1 + \tau_C)\), profits \(\sum_j \pi_j\), wages \(\sum_j w_j L_j\) and employment \(\sum_j L_j\). Its budget constraint is therefore given by

\[
E = \tau_C \left[ \sum_j C_j + \frac{1 - \alpha}{1 + \tau_C} \sum_j W^j \right] + \tau_K \sum_j \pi_j + \tau_W \sum_j w_j L_j + \tau_L \sum_j L_j, \quad (13)
\]

where \(\tau_K \leq 1\) is the tax on capital income, \(\tau_W > -1\) the wage tax and \(\tau_L\) the employment tax. We assume a fixed upper limit \(\eta \in [0, \infty)\) for the capital subsidy \(-\tau_K\), so that\(^3\)

\[
-\eta \leq \tau_K \leq 1. \quad (14)
\]

\(^3\)Otherwise, the subsidy \(-\tau_K\) could get an infinite value in the government’s optimal optimal policy.
Total capital accumulation $\sum_j \dot{K}_j$ is equal to production $\sum_j F(K_j, L_j)$ minus the capitalists’ consumption $\sum_j C_j$, the non-capitalists’ consumption $(1 - \alpha) \sum_j W^j/(1 + \tau_C)$, the disutility of employment in terms of consumption, $\sum_j Z(L_j)$, public spending $E$ and capital depreciation $\sum_j \mu K_j$:

$$
\sum_j \dot{K}_j = \sum_j \left[ F(K_j, L_j) - C_j - \frac{1 - \alpha}{1 + \tau_C} W^j - Z(L_j) - \mu K_j \right] - E. \quad (15)
$$

When (15) holds, the goods market is in equilibrium. Then, by Walras’ law, the government budget is balanced and (13) holds as well.

Because there is perfect symmetry throughout industries $j = 1, \ldots, J$, we obtain $K_j = K$, $L_j = L$, $C_j = C$, $W^j = W$, $\xi_j = \xi$ and $\phi_j = \phi$. Noting this and (2), capital accumulation (15), the Euler equation (7) and the constraints (10)-(12) take the form

$$
\begin{align*}
\dot{K} &= F(K, L) - C - (1 - \alpha)W/(1 + \tau_C) - Z(L) - \mu K - E/J, \quad (16) \\
\dot{C}/C &= \left[ (1 - \tau_K)(F_K(K, L) - \mu)/\sigma - \rho/\sigma, \quad (17) \\
\dot{\xi} &= \left\{ \rho + \rho/\sigma - (1 - \tau_K)(F_K(K, L) - \mu)/\sigma \right\} \xi + (1 + \tau_C)\phi, \quad \lim_{t \to \infty} \xi Ce^{-\rho t} = 0, \quad (18) \\
\dot{\phi} &= \left\{ \rho - (1 - \tau_K)(F_K(K, L) - F_{KL}(K, L)L - \mu) \right\} \phi \\
&\quad - (1 + \tau_W)^{-1}(1 + \alpha\phi)F_{KL}(K, L)\phi, \quad \lim_{t \to \infty} \phi Ke^{-\rho t} = 0, \quad (19) \\
(1 + \alpha\phi) \left\{ (1 + \tau_W)^{-1}F_{LL}(K, L)L + F_L(K, L) - \tau L \right\} &+ (1 - \tau_K)(F_{KL}(K, L)\xi C/\sigma - LF_{LL}(K, L)\phi) = 0. \quad (20)
\end{align*}
$$

A representative non-capitalist’s instantaneous utility is given by $V\left( ((1 - \alpha)/(1 - \tau_C))\sum_j W^j \right)$ with $V' > 0$ and $V'' < 0$. We assume that the whole population has the same constant rate of time preference, $\rho > 0$. The social welfare function is then a weighted average of the non-capitalists’ and capitalists’ utilities:

$$
\begin{align*}
\int_0^\infty \left[ V\left( \frac{1 - \alpha}{1 - \tau_C} \sum_j W^j \right) + \vartheta \sum_j U(C_j) \right] e^{-\rho t} dt \\
= \int_0^\infty \left[ V\left( \frac{1 - \alpha}{1 - \tau_C} JW \right) + \vartheta JU(C) \right] e^{-\rho t} dt, \quad (21)
\end{align*}
$$

where constant $\vartheta > 0$ is the social weight of the capitalists.
The government sets taxes $\tau_C$, $\tau_K$, $\tau_L$ and $\tau_W$ for all $j$ to maximize social welfare (21) subject to the dynamics of the economy (16)-(19) and the constraints for the capital tax (14). Because there is a one-to-one correspondence from $(\tau_L, \tau_W)$ to $W$ and $L$ through (3) and (12), employment and wage taxes $(\tau_L, \tau_W)$ can be replaced by labour income $W$ and employment $L$ as control variables. Noting (5), this leads to the Hamiltonian and the Lagrangean

$$
\mathcal{H} = V\left(\frac{1-\alpha}{1-\tau_C} JW\right) + \partial JU(C) + \gamma\{(1-\tau_K)(F_K(K,L) - \mu) - \rho\}C/\sigma
$$

$$
+ \chi\{F(K,L) - C - (1-\alpha)W/(1+\tau_C) - Z(L) - \mu K - E/J\},
$$

$$
\mathcal{L}^G = \mathcal{H} + \nu_1[\tau_K + \eta] + \nu_2[1-\tau_K],
$$

(22)

where the co-state variables $\chi$ and $\gamma$ evolve according to

$$
\dot{\chi} = \rho \chi - \partial \mathcal{H}/\partial C = [\rho + \rho/\sigma - (1-\tau_K)(F_K - \mu)/\sigma] \gamma + \chi - \partial J C^{-\sigma},
$$

$$
\dot{\gamma} = \rho \gamma - \partial \mathcal{H}/\partial K = [\rho + \mu - F_K(K,L)] \chi - (1-\tau_K)F_{KK}C \gamma/\sigma,
$$

$$
limit_{t \to \infty} \chi Ke^{-\rho t} = 0, \quad limit_{t \to \infty} \gamma Ce^{-\rho t} = 0,
$$

(23)

and variables $\nu_1$ and $\nu_2$ satisfy the Kuhn-Tucker conditions

$$
\nu_1[\tau_K + \eta] = 0, \quad \nu_1 \geq 0, \quad \nu_2[1-\tau_K] = 0, \quad \nu_2 \geq 0.
$$

(24)

6 Policy rules

Noting (22), the first-order condition for $\tau_K$ is given by

$$
\partial \mathcal{H}/\partial \tau_K = (\mu - F_K)C \gamma/\sigma + \nu_1 - \nu_2 = 0.
$$

(25)

Assume first $-\eta < \tau_K < 1$, so that $\nu_1 = \nu_2 = 0$. Because $\partial^2 \mathcal{H}/\partial (\tau_K)^2 \equiv 0$, we have to solve for $\tau_K$ through the generalized Clebsch-Legendre conditions:

$$
\frac{\partial}{\partial \tau_K}\left(\frac{d^p}{dt^p} \frac{\partial \mathcal{H}}{\partial \tau_K}\right) = 0 \text{ for any odd integer } p,
$$

$$
(-1)^q \frac{\partial}{\partial \tau_K}\left(\frac{d^{2q}}{dt^{2q}} \frac{\partial \mathcal{H}}{\partial \tau_K}\right) \geq 0 \text{ for any integer } q,
$$

(26)

---

where $t$ is time. Because $C > 0$ and $(F_K - \mu)\dot{C} = 0 > 0$ by (17), equation (25) yields $\gamma = 0$. Differentiating (25) with respect to time $t$ and noting (2), (7), (17), (23) and $\gamma = 0$, we see that the Clebsch-Legendre conditions (26) hold:

$$\frac{d}{dt} \left( \frac{\partial H}{\partial \tau K} \right) = (\mu - F_K) \frac{C}{\sigma} \bigg|_{\gamma = 0} = \Pi_K C \frac{\partial J C^{-\sigma}}{\partial \tau K} - \chi = 0,$$  
$$\frac{\partial }{\partial \tau K} \frac{d}{dt} \left( \frac{\partial H}{\partial \tau K} \right) = 0,$$  
$$\frac{\partial }{\partial \tau K} \frac{d^2}{dt^2} \left( \frac{\partial H}{\partial \tau K} \right) = -\Pi_K C \frac{\partial J C^{-\sigma} \dot{C}}{\partial \tau K} = 0,$$  
$$\frac{\partial }{\partial \tau K} \frac{d^2}{dt^2} \left( \frac{\partial H}{\partial \tau K} \right) = -\Pi_K C \frac{\partial J C^{-\sigma}}{\partial \tau K} \left[ \frac{\partial J C^\sigma}{\partial \tau K} + \frac{\partial J C^\sigma}{\partial \tau K} \right] = (\Pi_K)^2 \frac{\partial J C^\sigma}{\partial \tau K} > 0.$$

Given (27), we obtain

$$\chi = \partial J C^{-\sigma}. \quad (30)$$

From $\gamma = 0$, (2), (17), (23), (28) and (30) it follows that

$$0 = \dot{\chi} + \sigma \partial J C^{-\sigma - 1} \dot{C} = (\rho + \mu - F_K)\chi + \partial J C^{-\sigma}[(1 - \tau K)\Pi_K - \rho]$$
$$= (\rho - \Pi_K)\partial J C^{-\sigma} + \partial J C^{-\sigma}[(1 - \tau K)\Pi_K - \rho] = -\partial J C^{-\sigma} \tau K\Pi_K,$$

which is equivalent to $\tau K = 0$. This yields the following result:

**Proposition 2** The steady-state capital tax $\tau K$ should be zero.

Noting $\gamma = 0$ and (22), the first-order conditions for $W$ and $L$ are

$$\partial H/\partial W = (1 - \alpha)(J V' - \chi) = 0, \quad \partial H/\partial L = \chi (F_L - Z') = 0. \quad (31)$$

Given (30) and (31), we obtain $F_L = Z'$, $\partial C^{-\sigma} = \chi/J = V'$ and the results:

**Proposition 3** (i) In the steady state, labour income $W$ should be kept by the wage tax $\tau W$ such that a capitalist’s marginal utility of income, $C^{-\sigma}$, times the capitalists’ social weight $\vartheta$ is equal to a non-capitalist’s marginal utility of income, $V'$.

(ii) In the steady state, employment $L$ should be kept by the employment tax $\tau L$ such that the marginal product of labour is equal to the marginal disutility of employment, $v = F_L = Z'$. 

9
Result (i) means that the wage tax should be used to maintain socially optimal income distribution, and result (ii) that the employment tax should be used to maintain efficient labour allocation.

Because the marginal disutility of employment is a non-observable variable, we have to find another expression for proposition 2(ii). The system (15) and (7) produces a steady state in which $K$ and $C$ are kept constant. Given $\dot{C} = 0$, (2), (7) and proposition 2, we obtain

$$\rho = F_K(K, L) - \mu = \Pi_K. \tag{32}$$

Hence, the elasticity of the marginal product of capital, $F_K - \mu$, with respect to employment $L$ in industry $j$, when capital $K$ is kept constant, is

$$\epsilon = \frac{L}{F_K - \mu} \frac{\partial (F_K - \mu)}{\partial K} = \frac{1}{\rho} LF_{KL} > 0. \tag{33}$$

Noting (2), (3), (32), propositions 2 and 3 and the symmetry, conditions (11)-(12) become

$$\dot{\xi} = \rho \xi + (1 + \tau_C)\phi = 0, \quad \lim_{t \to \infty} \phi Ke^{-\rho t} = 0, \tag{34}$$

$$\dot{\phi} = LF_{KL}\phi - (1 + \tau_W)^{-1}(1 + \alpha\phi)] = 0, \quad \lim_{t \to \infty} \phi Ke^{-\rho t} = 0, \tag{35}$$

$$0 = (1 + \alpha\phi)\{(1 + \tau_W)^{-1}[F_{LL}L + F_L - \tau_L] - Z'\} + F_{KL}\xi C/\sigma - LF_{LL}\phi. \tag{36}$$

Choosing $1 + \alpha\phi = (1 + \tau_W)\phi$ we obtain $\dot{\phi} \equiv 0$ by (35). Noting this and choosing $\xi = -(1 + \tau_C)\phi/\rho$, we obtain $\dot{\xi} \equiv 0$ by (34). Inserting these results and (33) into the equation (36) and noting proposition 3(ii), we obtain $0 = F_{KL}C\xi/\sigma - (\tau_L + \tau_W v)\phi$. From this and the definition (33) it follows that

$$-\tau_L - \tau_W v = -F_{KL}C/\sigma \frac{\xi}{\phi} = (1 + \tau_C)F_{KL}C/\rho\sigma = (1 + \tau_C)\frac{C\epsilon}{L\sigma} > 0.$$

This outcome can be rephrased as follows:

**Proposition 4** In the steady state, total subsidies to wages and employment, $-\tau_L - \tau_W v$, are positive and they must be financed by consumption taxation. The employment subsidy $-\tau_L$ should then be equal to $(1 + \tau_C)\frac{C}{L} \frac{\epsilon}{\sigma} + \tau_W v$, where $(1 + \tau_C)\frac{C}{L}$ is the capitalist’s consumption expenditure per employment, $\epsilon$ the elasticity of the marginal product of capital with respect to employment, $\tau_W v$ the wage tax per employment and $1/\sigma$ the intertemporal elasticity of substitution for a capitalist.
This tax rule changes the slope of the labour demand function so that the unions set the marginal products of labour, $F_L$, equal to its members’ marginal disutility of employment, $V$.

From (32) and proposition 3(ii), it follows that the value $(K^*, L^*)$ for $(K, L)$ in the steady state with $\gamma = 0$ is determined by two equations $F_K(K^*, L^*) = \rho + \mu$ and $F_L(K^*, L^*) = Z'(L^*)$. Noting (24) and (25), we obtain the following. If $\gamma > 0$ ($\gamma < 0$), then the capital subsidy (tax) should be raised to the maximum, $-\tau_W = \eta$ ($\tau_W = 1$), so that the capitalist accumulates (exhausts) capital, $\dot{K} > 0$ ($\dot{K} < 0$). This can be rephrased as:

**Proposition 5** The equilibrium values $K^*$ and $L^*$ for capital $K$ and employment $L$ are given by $F_K(K^*, L^*) = \rho + \mu$ and $F_L(K^*, L^*) = Z'(L^*)$. The government should encourage (discourage) investment in industry $j$ as long as capital $K$ is above (below) its equilibrium level $K^*$, $\dot{K} > 0$ ($\dot{K} < 0$).

Since the system ends up with a steady state in which $K$, $C$, $\chi$ and $\gamma$ are constants, conditions $\lim_{t \to \infty} K\chi e^{-\rho t} = 0$ and $\lim_{t \to \infty} C\gamma e^{-\rho t} = 0$ hold.

### 7 Conclusions

This paper examines optimal taxation in a unionized economy. Workers form a union, which raises their wage above the marginal disutility of employment. Some (or all) households specified as capitalists save and earn a fixed proportion of all wages, while the others specified as non-capitalists spend all of their income. A labour union takes the firms’ and capitalists’ employment and investment behaviour as constraints in wage settlement. Wages are determined at the level of a single firm, at the level of the whole economy, or at some intermediate level. The government can tax consumption, employment, wages and capital income. The main findings of this paper are the following.

Zero taxation of capital income applies to unionized economies as well. Aggregate production efficiency can be maintained by the taxes on consumption, wages and employment, which are commonly used in modern industrial economies. The sum of wage and employment subsidies per worker should be positive and the consumption tax is needed to finance these. The wage

---

5The wage and employment taxes are equivalent to progressive labour taxation.
tax should be set so as to keep the marginal utility of a non-capitalist equal to the marginal utility of a capitalist times the capitalists’ weight in the social welfare function. There is a specific elasticity rule for the determination of the employment subsidy. This changes the slope of the labour demand curve, so that the union sets its wage equal to the marginal disutility of employment. These tax rules hold for any proportion of wages earned by the capital-saving households.

References:


