PHYSICAL FEATURES OF THE BALTIC SEA
Pentti Mälkki and Rein Tamsalu

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ERRATA

p.56, eq. 3.1 reads $\frac{\partial}{\partial z} \bar{t}$ should be $-\frac{\partial}{\partial z} \bar{t}$

p.66, eq.3.39 reads $+ \frac{(z)}{0} \frac{q_h}{H - h}$ should be $- \frac{(z)}{0} \frac{q_h}{H - h}$

p.93, line 20 reads LEONOV et al. 1977 should be LEONOV et al. 1979
PHYSICAL FEATURES OF THE BALTIC SEA
Pentti Mäikki and Rein Tamsalu
PREFACE

This monograph is the result of the physical studies carried out by a team of scientists within the framework of the Working Group on the Gulf of Finland under the Finnish-Soviet Commission for Scientific and Technological Cooperation. When dealing with the problems of the Gulf, the team considered it necessary to include many problems common to the entire Baltic Sea. This is due to the fact that the connection between the Gulf of Finland and the Baltic Sea is open and wide, the Gulf forming the head of the entire estuarine sea. Moreover, a substantial part of the fresh water flowing into the Baltic Sea first enters the Gulf of Finland. Thus, the processes in the two basins are strongly interconnected and cannot be treated independently.

Due to the background of this monograph, the treatment of various processes is somewhat biased. Emphasis has been given to those processes which have a direct connection with environmental questions. Many others have been discussed only superficially and some have been omitted. The authors found it necessary to try to cover a wide spectrum, even if the treatment is far from ideal. The Baltic Sea has been dealt with in many books, but all their treatments of the physical processes are fairly short and specialised. It is, therefore, hoped that the present text will give an improved overview of various aspects of the study of the Baltic Sea. Numerical modelling will certainly improve the understanding of the general circulation of the Baltic in the near future. Similarly, several ongoing studies on specialized processes, such as the impact of the topography, air-sea interaction and the inflow through the Danish Straits, and studies on internal waves will broaden the view of the basic mechanisms which regulate many environmental chemical and biological processes.

The authors would like to express their gratitude to Profs. Harald Velner and Aarno Voipio, the chairmen of the Working Group, for their continuous support since the initial stages of the production process. We would also like to thank numerous colleagues for their valuable comments. Special thanks are due to Mrs. Anna Damström for revising the English of the manuscript, and to Mrs. Hilkka Raunisto for drawing the figures.

Helsinki and Tallinn, May 1985

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R. Tamsalu
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INTRODUCTION

The hydrography of the Baltic Sea is determined mainly by four factors: the interaction between the atmosphere and the sea, the water exchange through the Danish Sounds, the river discharge into the sea, and the variable topography of the basin.

The water exchange and river runoff are of special importance for the internal processes in the sea, as they determine the stratification of the water masses into a relatively homogeneous upper layer and stably stratified lower layer. Due to the stratification, the atmosphere exerts its influence mainly on the homogeneous upper layer. During the summer, this layer is heated and a seasonal thermocline is established. During late autumn and winter, the thermal stratification vanishes from the upper layer, resulting in the characteristic two-layer structure mentioned above. The bottom morphology separates the subhalocline water masses into separate basins, delimited by high sills. The environmental conditions may vary considerably from basin to basin, depending on the degree of isolation.

The purpose of this monograph is to examine a set of characteristic features of the Baltic Sea hydrography using simple analytic similarity profiles to describe the stratification. In the presence of stratification, strong high-frequency movements occur. However, a considerable part of the short-term irregularities are filtered out by integration over an inertial period (some 14.4 h in the Central Baltic). Universal profiles have been used previously for describing density structure, e.g. by O. M. PHILLIPS (1966) to study the buoyancy and circulation of the Red Sea, and by KITAIGORODSKY and MIROPOLSKY (1970) in studies of thermal stratification of the seas. In the present text, we use these profiles separately for thermal stratification in the upper layer and for the stratification due to salinity in the lower layer.

In the first chapter, general features of the morphology of the Baltic Sea are described, the climatology of the sea region is discussed on the basis of published atlases, and the hydrological conditions, especially the annual and long-term cycle of the river discharge are reviewed. The air-sea interaction is also briefly discussed, and although we do not intend to discuss in detail the influence of the ice conditions, a climatic description of the variability of the ice cover is presented.

The second chapter deals with the salinity structure of the Baltic Sea and its modelling. We discuss briefly the variability of the salinity on the basis of observations made during this century. The similarity profile for the salinity is then described, on the basis of empirical data. Finally, modelling of the salt influx through the Danish Sounds is discussed, using both the classical Knudsen approach and a stationary and non-stationary diffusion approach. Numerical data on the behaviour of the salinity structure in the Baltic Proper are presented.

In the third chapter, the thermal structure of the Baltic Sea is considered. The layers below the halocline behave independently of the processes in the surface layer. Their thermal structure is mainly determined by the properties of the inflowing water and influxes through the Sounds. In the surface layer, long-term variations are connected with the course of the atmospheric temperature. The seasonal cycle is described both by a mixed-layer model developed by NIILER and KRAUS (1977), and by a model based on
the universal similarity profile of the temperature. The latter model includes variation below the mixed layer.

The fourth chapter considers the circulation of the sea caused by wind and baroclinicity. The model of Kuzin and Tamsalu (1974) is used to present the reaction of the basin to a variable pressure field (geostrophic wind) in simplified conditions. Assuming quasigeostrophy, the variability of the dynamic fields is discussed in terms of the bottom topography and baroclinicity.

The fifth chapter examines a full range of waves in the Baltic Sea. The discussion begins with a short review of the recent theoretical and empirical results for wind waves, including the fetch dependence and spectral form of the waves. A short introduction to barotropic eigenoscillations and tides in the Baltic Sea, is followed by a discussion of the topographic waves both in a sloping coastal region and in the open sea. The emphasis is laid on internal waves. The non-linear waves have an especially great influence on several processes occurring in the sea. It has been shown that soliton waves may cause instability in the pycnocline (Ri less than 0.25), thus deforming the density structure and influencing both the baroclinic circulation and the exchange processes between the upper and lower layers. Non-stationary waves of this kind can also form a microstructure in the density field.

In dealing with the material presented in this volume, lengthy description has often been avoided by brief reference to more general books of physical oceanography. Studies found particularly useful by the authors in compiling this book were »Geophysical Fluid Dynamics« (1979) by Pedlosky, »Dynamics of Internal Gravity Waves in the Ocean« (1981) by Miropolsky, »The Physics of the Ocean« (1978), edited by Monin and Kamenkovich, and »The Baltic Sea« (1981), edited by Voipio.
1 EXTERNAL FACTORS

1.1 Morphology of the Baltic Sea

The topography of the sea bottom and the coastal regions has an important influence on the processes occurring in the Baltic Sea. Among the characteristic features are the narrow and shallow connection with the North Sea through the Danish Sounds, the division of the sea into different basins and bays, and, in the long-term, the constant land uplift in the northern parts of the sea. A comprehensive survey of the geology, evolution and geomorphology of the Baltic Sea has been presented by Winterhalter et al. (1981).

Along the major part of the Swedish coast and the entire Finnish coast, the basement of the Baltic Sea consists of Precambrian rock, covered by only thin layers of sediment. In these regions the bottom topography is very variable. In the south-western and southern parts of the sea the bottom consists of sedimentary rocks, ranging in age from Cambrian to Tertiary. Although the bottom topography is very variable here as well, the variation is on a larger scale, which gives the bottom a more regular shape. The bottom material ranges from bedrock to soft sediment-covered bottoms, the latter occurring in regions with calm dynamic conditions in the deepest parts of the sea.

The Baltic Sea is connected with the North Sea by the Danish Sounds, where the water depth above the sill is about 18 metres. On the North Sea side of the Sounds, the Kattegatt region forms a relatively shallow (about 26 m deep) mixing region, through which the North Sea water enters the Baltic. On the Baltic Sea side the Arkona Basin has a depth of about 40 metres, from there a deep furrow leads to the Bornholm basin, which has a maximum depth of about 100 metres. Another depression with a maximum depth of over 100 m, is the Gdansk basin. In the central Baltic a ridge around Gotland lies between the Gotland Deep (depth less than 240 m) east of Gotland, and the Landsort Deep (maximum depth 459 m), which is located on a major fault line NW of Gotland. North of Gotland, the Fårö Deep extends down to more than 200 m. The Gulf of Finland is a direct continuation of the Baltic Proper without any notable sill. The Gulf of Riga, isolated from the Baltic Proper by a shallow sill, has a maximum depth of some 50 metres.

In the northern part of the Baltic Proper, shoals with a depth of approximately 30–40 metres isolate the Aland Sea, Archipelago Sea and Gulf of Bothnia from the southern basins. Some narrow depressions lead through these shoals to the Aland Sea, which is a relatively deep and steep-sided basin, with a maximum depth of some 300 metres. East of Aland is the Archipelago Sea, a mosaic of islands and depressions. Narrow channels lead from these basins to the Bothnian Sea. This has a large depression extending from the central part to the northwestern corner, where the maximum depth is 280 metres. The topography of the Finnish side is fairly regular, with an average slope of $2 \times 10^{-3}$. On the Swedish side the slope is considerably steeper and the bottom is more rugged. A shallow sill (some 25 m) in the northern Quark forms the southern limit of the Bothnian Bay, which has an irregular bottom topography and a maximum depth of some 140 m.

The mean depth of the different parts are: Baltic Proper, 67 m, Gulf of Riga 28 m, Gulf of Finland 38 m, Aland Sea 77 m, Bothnian Sea 68 m and Bothnian Bay 43 m. A bathymetric map of the Baltic Sea is presented in Fig. 1.1.
Fig. 1.1. Bathymetric map of the Baltic Sea.
1.2 Climatic conditions over the Baltic Sea

The Baltic Sea is situated in latitudes where the inter-annual variations in solar radiation, pressure, wind and temperature are rather high. The distribution of the oceans and continents, and also the orographic effects of the Norwegian mountains determine the prevailing conditions and modify the routes and life history of weather disturbances.

In the free atmosphere, westerly winds generally prevail, and although their direction and magnitude at different heights have a clear seasonal cycle, the pattern is fairly regular. A dominant feature of the climate of the Baltic Sea is the location of the polar front. In summer it lies far north of the Baltic region; in October it reaches the northernmost parts of the Baltic and then lies close to the central regions of the sea during the winter months, retreating northwards during the spring. Thus, in the southernmost parts of the sea, the climatic conditions are closer to those over the North Sea, whereas towards the north and east the climatic conditions have a more continental character, with a considerable amplitude in the seasonal cycle of the air temperature. The heat balance is characterized by local heat losses, which are compensated by meridional convergence and transport from lower latitudes.

In the surface layer, a significant feature influencing the Baltic Sea climate is the relatively low areas bordering the entire southern Baltic. Variable, but predominantly westerly winds bring moist, relatively warm maritime air into the southern Baltic. Towards the north the cold season becomes more marked; in the northernmost parts of the Gulf of Bothnia, the climate has an almost completely continental character. The surface pressure field has on the average the strongest gradients during the months October to February, with isobar directions from SW to W towards NE to E. The weakest gradients and also on the average the highest surface pressures are found in March to mid-June in the southern Baltic, and in April to July in the northern Baltic.

During the months when the average pressure field has the strongest gradient, the variability of the synoptic weather conditions is also greatest. The polar front forms a band touching the central and northern Baltic Sea, and its spatial meandering (Rossby waves) with interconnected polar jet streams guides weather disturbances into the Baltic Sea. In the northern Baltic western weather disturbances usually bring relatively little moisture, due to the influence of the Norwegian mountains. The vectoral mean is about 30 % of the scalar mean velocity.

During the spring and early summer, the pressure field over the Baltic Sea is fairly uniform. This is partly connected with strong stability over the sea surface. Synoptic maps often reveal long periods with a blocking high over the Baltic Sea and anticyclonic circulation. During such periods the weather statistics show weak winds for up to 70 % of the time, i.e. less than 3 on the Beaufort scale. The wind direction in the surface layer does not have a clear preference: the vectoral mean velocity is about 40 % of the scalar mean.

The surface layer temperature over the Baltic Sea has a considerable amplitude. In the southern regions, hardly ever reached by the polar front, westerly winds bring warmer Atlantic air in the wintertime and the winter temperature tend to be close to zero centigrade. Towards the north and east the degree of continentality gradually increases, this is seen especially clearly in the mean temperatures of the coldest months. The horizontal variation in the air temperature above the sea in the summer months lies within 2°C over the entire Baltic Sea, while during the winter months the mean temperatures in the different parts deviate by more than 10°C. The range of variability in the southern Baltic is about 17°C, in the northern Gulf of Bothnia about 27°C.

In Fig. 1.2 to 1.5 (DEFANT, 1972) the surface temperatures $T_a$ are presented for the months January, April, July and September. Over the open sea, the air temperature is
Fig. 1.2. Atmospheric surface temperature over the Baltic Sea in January (Defant, 1972).

Fig. 1.3. Atmospheric surface temperature over the Baltic Sea in April (Defant, 1972).
Fig. 1.4. Atmospheric surface temperature over the Baltic Sea in July (Defant, 1972).

Fig. 1.5. Atmospheric surface temperature over the Baltic Sea in September (Defant, 1972).
fairly uniform in July, during the other months a distinct north-south gradient is found. A very typical feature, which influences the entire climate over the Baltic Sea, is that during both the summer and winter months there are large differences between the surface temperatures over the sea and over land areas. In winter, the surface temperatures of the sea are higher than those of the air, and the atmospheric temperature is moderated by a heat flux from the sea. The lower stability over the sea is clearly seen in the wind statistics, as well. During the summer months, between May and August, the sea surface temperature is lower than that of the atmosphere and the surface layers of the atmosphere are cooled by a downward heat flux, resulting in considerable temperature gradients in the coastal regions.

As stated above, the seasonal and day-to-day variability in weather condition clearly exceeds the mean values. In particular, wind velocities and directions vary with the passage of cyclones, although a westerly component is dominant. As an example of wind statistics, Fig. 1.6 shows four seasonal wind roses, classified into two groups: weak winds (less than 5 m/s, direction unspecified), and moderate to strong winds (The Baltic Sea Waves and Wind Atlas, unpublished manuscript). The predominance of westerly to south-westerly winds during the autumn and winter, and of weak winds in the summer season is apparent.

A practical tool for some forecast purposes is to present typical weather situations in maps of surface pressure fields. In the Baltic Sea Waves and Wind Atlas, 44 typical wind situations are classified. Of these, the four most recurrent wind fields, when weak winds are disregarded, are presented as pressure gradient fields in Fig. 1.7. The corresponding winds are of the order of 5 to 9 m/s.

Fig. 1.6. Wind roses over the central part of the Baltic Sea: A) December-February, B) March-May, C) June-August, D) September-November.
The total precipitation falling on the Baltic Sea is difficult to measure. Long-term observation stations are only found on coasts and islands, and much is thus left for interpolation. Recent studies with weather radars (e.g. HEIKINHEIMO and PUHAKKA, 1980) indicate that during the early summer stability the coastal stations will give overestimates of the precipitation and during the autumn and winter underestimates. The annual precipitation varies between 400 and 800 mm (DEFANT, 1972). Overall estimates for annual precipitation vary between 400 and 550 mm/y (EHLIN, 1981). Precipitation is higher in the southern parts and diminishes towards the Bothnian Bay. A seasonal cycle is evident: the maximum is reached in August, the minimum (between 30 and 40 mm/month) in February–March. The same seasonal cycle is evident in the humidity, which varies between 3.5 and 12 g m⁻³.

1.3 Air-sea interaction

The annual cycle of sea surface temperature and the depth of the mixed layer shows a complicated balance between incoming heat through short-wave radiation and outgoing losses through long-wave radiation, evaporation and turbulent convection of heat. Of these components of the heat balance, only short-wave radiation and long-wave back
radiation can be measured directly. Estimates of evaporation and turbulent heat conduction must be made with semi-empirical equations. These equations are obtained by theoretical considerations and complex field experiments over a more or less rough water surface. As observed by Charnock (1981), in spite of intensive research during the past 20 years, many uncertainties still exist, leading to scatter in estimates of free parameters. Moreover, a major part of the studies deal with neutral atmospheric stratification, which is often a rough approximation of the surface layer. For better estimates, special studies of the heat balance are required.

In the Baltic Sea, heat balance measurements have been carried out mainly in the coastal regions (e.g. Launiainen, 1979) and during the stable winter stratification (Joffre, 1981). Launiainen was able to show that in a semi-enclosed basin close to a power plant the heat content of the bay could be determined fairly accurately on the basis of atmospheric parameters. This indicates that a well-equipped network of meteorological stations could enable accurate estimation of the annual heat balance. However, due to the sparsity of the present networks, not all the necessary parameters can be determined. There is, therefore, a need for simplified formulae for determining various terms in the heat balance. Such formulae have been proposed during past decades by several authors. The formulae for turbulent fluxes are basically all of the form:

\[ Lq_e = -C_E \rho a g (q_{eh} - q_{ex}) |u_a| \]
\[ q_T = -C_H c_p \rho a g (T_a - T_s) |u_a| \]

with variable empirical coefficients \(C_E\) and \(C_H\). If it is borne in mind that an equation for average evaporation or heat flux depends on turbulence, the actual stability condi-
tions and their occurrence frequencies, then the empirical coefficients can be interpreted as a kind of local statistical summaries of those environmental conditions. Despite the differences in numerical values, all the studies carried out reveal similar variation in the annual heat balance.

Using observational data from Finngrundet, HANKIMO (1964) estimated the heat balance for the period March 1961 – February 1962. His results are found in Fig. 1.8. According to these, during the spring the heat losses were about 15 W m\(^{-2}\), during the summer months they were close to zero, after which they increased and reached a maximum of about 100 W m\(^{-2}\) in December. A heat gain due to solar radiation dominates until the end of August, after which heat losses dominate. Long-wave radiation from the water surface remains practically constant, reflected radiation varies between 1 and 5 % of the incoming radiation.

POMERANETS (1964) estimated heat fluxes through the surface in the Baltic Proper and the Gulf of Finland, his averaged results are presented in Fig. 1.9. As can be seen, the net flux is approximately sinusoidal in form. Due to the limited dimensions of the basin, horizontal differences in amplitudes are not large. Fig. 1.10 shows the geographical distribution of the amplitude of net heat flux. The maximum amplitudes are found in the central parts of the sea, the variations along the coastline are not large.

In treating the structure of the surface layer, an additional element to be considered is the buoyancy structure, defined as follows:

\[
\overline{w^T_{z_0}} = g \left[ \alpha_T \overline{w^T_{z_0} + \beta_S \overline{w^S_{z_0}}} \right] = B_T - B_S = B_0 ,
\]

where \( b = \frac{-g (\rho - \rho_0)}{\rho_0} \)

\( \rho_0 \) is reference density.
Fig. 1.11. Annual course of evaporation rate $q_E$, precipitation rate $P_s$, as well as those of buoyancy fluctuations due to salinity, $B_s$, and due to temperature, $B_T$.

\[ \rho \quad \text{actual density} \]
\[ g \quad \text{acceleration due to gravity} \]
\[ \alpha_T \quad \text{coefficient of thermal expansion} \]
\[ \beta_S \quad \text{coefficient of density change due to salinity} \]

and $w^T$, $w^S$ describe autocorrelation functions of turbulent fluxes. The first term at the right-hand side can be evaluated as was described above, i.e.

\[ \overline{w^T} = \frac{1}{c_p \rho_0} \left( R_0 - I_0 + L q_e + q_T \right) \]  \(1.4\)

where $R_0$ is flux due to long-wave radiation
$I_0$ penetrative component of solar radiation
$L$ latent heat of vaporisation
$q_T$ convective heat transport
$q_e$ evaporation rate

The buoyancy flux due to salinity fluctuation is mainly regulated by the salt balance in the surface resulting from precipitation and evaporation, as follows

\[ g \beta_S \overline{w^S} = \beta_S (q_e - P_s) S_s = B_s \]  \(1.5\)

According to Brogmus (1952), the annual evaporation of water is about 180 km$^3$ and annual precipitation about 200 km$^3$. Fig. 1.11 presents a mean seasonal cycle of evaporation, obtained by averaging the results of Palmén and Söderman (1966), Simojoki (1949) and Brogmus (1952). In the same graph, the seasonal variation of
precipitation according to WYRTKI (1954) is also presented. The graphs for $B_T$ and $B_s$ show the familiar shape of the buoyancy fluctuation. As the scales indicate, the salt contribution to buoyancy is only of the order of 1%. Thus, the contribution of the salinity fluctuation to the buoyancy fluctuation seems to be negligible. However, this estimate does not take into account the influence of the winter ice in the northern parts of the Baltic Sea. With ice thicknesses of the order of 50 cm, the contribution of brine salinity may have some importance.

A key parameter in the air-sea interaction studies is the wind stress, usually parameterized with a quadratic form:

$$\tau = C_t \rho_a u |u|$$  

In most calculations the stress estimate has to be based on geostrophic wind. As mentioned previously, the wind conditions at the surface are strongly influenced by surface layer stability. This can be taken into account by determining the drag coefficient $C_t$ and also the veering angle $\alpha$ as functions of e.g. the Richardson number $R_i$ and Rossby number $R_o$ (see ZILITINKEVICH, MONIN and CHALIKOV, 1978).

1.4 River runoff

The great latitudinal extent of the Baltic Sea, from 54° N to almost 66° N, results in great variability in hydrological conditions. In the northernmost parts of the drainage basin the seasons are markedly different from in the southern parts. In the north, the snow cover storage of precipitation lasts until late May. Therefore, the maximum runoff into the Gulf of Bothnia is found at the end of May and beginning of June. The drainage basins of the rivers discharging into the Gulf of Finland have a high percentage of lakes, which delays the spring flood until early summer. Discharge into the Gulf of Riga has its maximum in April, at which time it is three times the annual average. South of the watershed of the Gulf of Riga, the snow cover is less important and runoff is maximal during the first quarter of the year and lowest after midsummer. The total inflow into the Baltic has a maximum in May-June, after which it slowly decreases till the end of year. The annual mean runoff into the Baltic has been estimated by several authors at some 440 km$^3$ yr$^{-1}$. Of the rivers discharging into the Baltic, the River Neva contributes approximately 20%, with a mean runoff of some 2600 m$^3$s$^{-1}$. According to MIKULSKI (1970, 1972), during the period 1951–1970 the Gulf of Bothnia had an inflow of 185 km$^3$ yr$^{-1}$; the corresponding figures for the Gulfs of Finland and Riga and for the Baltic Proper are 114, 29 and 110, respectively. A comprehensive discussion of the hydrology of the Baltic Sea can be found in the paper of EHLIN (1981). The average hydrological balance for the reference period 1931–1960 is shown in Fig. 1.12 (JACOBSEN, 1980).

During the past century, the river runoff has fluctuated considerably. KALEIS (1976) has determined seven low and high runoff periods for the rivers Neva, Daugava and Namunas. ASTOK and TAMSA (1975) made a spectral analysis of a long time series of runoff values for the River Neva, presented here in Fig. 1.13. Notable in this spectrum are the significant peaks with periods of 28.6, 10.9, 6.2, 3.3 and 1.0 years. The time series consists of monthly runoff records, so that the shorter periods apparent in the spectrum are due to asymmetry in the annual cycle. Significant long-term fluctuations can also be seen in the smoothed curves of long time series presented for the Götaälv, Vistula, and Vuoksi by HUPPER et al. (1979), and HVÄRINEN and VEHVIÄINEN (1981), and given in Fig. 1.14.
Even in the 10-year running means, variations of the order of 40% of the mean discharge occur. Fluctuation of this magnitude gives rise to long-term variability in the salinity structure of the Baltic Sea. In Fig. 1.15 the long-term fluctuations in the discharge of the rivers Neva, Daugava and Nemunas have been correlated with changes in the salinity in the Bornholm Deep reported by Kalejs (1976). Despite the approximate character of the salinity fluctuations, a negative correlation seems quite obvious. Although the long-term fluctuations in discharge seem to be clearly reflected in the salinity structure, the same is not true of the annual variation. As stated earlier, the maximum runoff occurs at different times of the year in different regions. Locally, the influence of
Fig. 1.14. Long term fluctuations of the runoff of some rivers, 10 years and 30 years running means, reproduced from Launiainen (1982).

Fig. 1.15. Long term discharges of rivers Daugava, Namunas and Neva (continuous line) and mean salinities in the Bornholm Deep (histogram, 0 to 90 m mean) according to Kaleis (1976).
the spring maximum can be detected, but wind-induced mixing and currents homogenize
the upper layer and the seasonal signal is very weak, often detectable only by careful
statistical analysis. According to Fig. 1.16, the inflow maximum is of the order of 150
% of the mean inflow, being larger in the Gulfs of Bothnia and Riga. Regular observa-
tions along the coast reveal seasonal signal smaller than 6% of the mean salinity.

1.5 Water exchange through the Danish Sounds

The exchange of water and substances between the North Sea and the Baltic is one
of the most important mechanisms regulating the hydrographical environment of the
Baltic Sea. The overall salinity distribution, both horizontal and vertical, depends on
the balance of river input and salt influx through the sounds. The thermal structure of
the layers below the primary halocline is independent of the influence of the atmosphere,
being determined by the properties of the inflowing water and mixing conditions below
the halocline. The chief mechanism of aeration of those deeper layers is the influx of
oxygen-rich water during inflow periods. If strong influxes occur very seldom, the oxygen
consumption exceeds the input and an oxygen deficit develops.

The water and salt exchange has been investigated by many oceanographers during
the past 80 years. A hydrographical theorem for the Danish Sound was presented by
Knudsen (1899), which described the advective transfer of water and salts of a two-
layer liquid. The investigations of Jacobson (1925), Hei (1944), Wyrtki (1953),
Soskin (1963) and many others have suggested the following mechanism for water ex-
change through the Danish Sounds:

- A two-layer system of currents prevails at moderate and weak winds. In the upper
  layer the current is directed towards the North Sea, in the lower layer towards the
  Baltic Sea.
- The current structure changes during periods of strong winds, when inflow or out-
  flow occurs in the whole water column.

Figure 1.17 (according to Soskin, 1963) shows the longitudinal distribution of
salinity during a calm period (1.17a), during a period of strong easterly winds (1.17b),
and during a period of strong westerly winds (1.17c). Since weak to moderate winds
are most common, two-layer exchange should predominate in the Sounds. According
to the current measurements carried out by **Kruse, Jacobsen and Nielsen** (1980) during the period 1974–1977, the predominant type is non-stationary one-layer flow. Salinity measurements show predominance of a two-layer structure.

In view of the complex topography of the entrance region, and of the strong density gradients present, it is possible that several types of flow may prevail with rapid changes to another type. The analyses applied by **Svansson** (1980) and **Astor and Otson** (1977) indicate that fluctuation in the level of the North Sea is the chief driving force for water exchange variation. According to these studies, the barotropic or weakly baroclinic one-layer type flow has a short life span (of the order of 10–11 days) with a large amplitude, while the two-layer type of flow is quasi-stationary and not so intense.

### 1.6 Ice conditions

The Baltic Sea is annually partly covered with ice. The northernmost Bothnian Bay, eastern parts of the Gulf of Finland and all the northern coastal regions freeze every winter. The 50 per cent probability line for ice coverage lies approximately at the latitude 59°N. In the southern Baltic, ice seldom extends far seawards from the coastal region. Fig. 1.18 (reproduced from SMHI & Merentutkimuslaitos, 1982) shows the freezing probability as a percentages for different regions of the Baltic Sea. The probability of total ice coverage is very low. On the basis of all available material, **Jurva** (1952) produced a diagram for the extent of the ice cover. It is presented (as completed by **Palosuo, 1966**,
Fig. 1.18. Probability of ice coverage occurrence in the Baltic Sea (100 % equivalent to ice cover every year).

Fig. 1.19. Maximum ice extent of the Baltic Sea during winters 1720–1979, reproduced from Alenius and Makkonen, (1981).
and others) in Fig. 1.19. The graphs show that during the past 260 years the sea has been entirely covered by ice about three times in a hundred years. The last winter with total coverage occurred in the 1940’s. The time series does not seem to have any systematic features, it merely shows large variability of the winter climate (see e.g. ALENIUS and MAKKONEN, 1981).

For the purpose of ice services, Jurva outlined a cartographic method for estimating the seasonal run of the ice coverage in the Baltic Sea. According to this method, the ice cover develops in consecutive phases, though the phases may occur at different times during different winters. For example, the variability in the time of freezing is about two months, the variability in the recession about one month. In the autumn, when the sea surface temperature is higher than the air temperature, heat is lost by conduction and evaporation. As a consequence, the sea surface cools on the average by 0.5 to 1.0 degrees in ten days. In the Bothnian Bay, ice formation at the coast begins on the average at the end of November. After the coastal regions have frozen, the ice boundary moves towards the open sea, in the Gulf of Finland from east to west. The thickness of the ice increases, and in mid-January, when the ice edge generally lies south of Vaasa in the Gulf of Bothnia, and at the longitude of Kotka in the Gulf of Finland, freezing begins in the central parts. During an average winter, the Gulf of Bothnia, Gulf of Finland and Gulf of Riga are entirely covered with ice. In the southern parts of the Baltic, ice forms in the Gdansk Bay and in the Belt region. In the central parts of the Baltic Proper ice occurs to a notable extent only during severe winters, approximately once in a decade. During these severe winters, the area of the ice cover is greatest in February. The ice thickness is greatest in February—March, when in the Bothnian Bay it is between 50 and 80 cm, in the Bothnian Sea 25 to 40 cm, in the Gulf of Finland 20 to 50 cm and in the Gulf of Riga 20 to 30 cm. During severe winters the ice thickness in the northern Baltic north of Gotland reaches about 15 to 20 cm.

The coastal zones of the Baltic Sea are usually covered by fast ice, which extends a few tens of kilometres seawards from the coast, usually to the borderline of the archipelago or the limit of the depth range where pack ice can extend down to the bottom. On the open sea the ice drifts freely, forming ridges or leads, depending on the weather conditions. Leads are mainly found at the border of the fast ice; they may be tens of kilometres wide. Ridging occurs when floats meet obstacles. At the margin of the fast ice this ridging occasionally extends down to the bottom. The deepest ridge recorded was about 28 metres deep. In such cases, a limited coastal region may be isolated from water exchange with the open sea.

The decay of the ice cover begins in March. Due to increasing solar radiation, the retreat of the ice boundary is fairly regular, as can be seen in Fig. 1.20. The differences in the duration of the separate phases are also smaller than during the growth phase. By the beginning of April, the ice has usually disappeared from the Baltic Proper, one month later the Bothnian Sea and the Gulf of Finland are free of ice and finally, during May, the Bothnian Bay becomes free. The final phase in ice melting is that of ice hummocks at the coasts. Remnants of hummocks may occasionally occur in the Bothnian Bay at midsummer.

The average number of ice-covered days varies considerably, as is shown in Fig. 1.21. In the Bothnian Bay the ice winter lasts 4 to 6 months, in the Bothnian Sea and the Gulf of Finland 2 to 4 months, and in the Baltic Proper less than one month. On the average Finland is completely surrounded by ice 3 months of the year.

The ice cover has an important role in the energy exchange between the sea and the atmosphere. During the cold winter months it isolates the sea from the colder air, hence preventing heat flux from the sea to the atmosphere. In the autumn, the climate over
the sea is milder than over the land, these differences disappear when the ice cover is established. In southern and south-western Finland the winter climate remains mild due to the short duration of the ice cover in the northern Baltic. The same is even more true of the coastal regions of the Baltic Proper. The wind stress is transmitted to the water through the ice cover in winter. With low ice concentrations, the transmitted energy is approximately the same as during the ice-free season, but according to LISITZIN (1957) a dense pack-ice field over the Bothnian Sea reduces the amplitude of the variation in the sea level. In the spring, the salinity of the Baltic Sea ice is fairly low, and its melting creates thin layer of less saline water, which has an impact on the early stages of the primary production at the ice edge.

The dynamics of sea ice is usually described with the ordinary equations of fluid dynamics. Since the ice field consists of separate floes, ranging in diameter from metres to kilometres, the scales of the motion must be adjusted accordingly. If the concentration of ice is low (open pack ice), the ice is found to move with a velocity of some 2.5 % of
the wind velocity, in a direction some $20^\circ$ to the right of the wind direction. At higher concentrations, internal friction between the floes diminishes the ice velocity. Since there is no permanent current field in the Baltic, the movement of ice depends greatly on the winds. A thorough discussion of the dynamics of the Baltic Sea ice can be found in a series of publications by Leppäranta (1981a, 1981b).
2 THE SALINITY REGIME OF THE BALTIC SEA

2.1 General features

During its comparatively short existence (some 12,000 years) the Baltic Sea has passed through many stages. It has been both a freshwater lake (the Baltic Ice Lake, the Ancylus Lake) and a saline water body. At present it is a brackish-water sea.

As was discussed in the previous chapter, the salinity of the Baltic is regulated by the inflows of saline and river water. The inflow of saline water into the Baltic Sea depends on the hydraulic properties of the interconnecting straits. In the course of history, the cross-section area and the depth of the Danish sounds has varied. Studies of the marine sediments have revealed variation presented schematically in Fig. 2.1. The Baltic Ice Lake and the Ancylus Lake were freshwater bodies without any connection with the Atlantic. The Yoldia Sea was connected with the North Sea by a wide sound located at the edge of the glacier north of the present sounds. During the Littorina Sea phase, the connection with the North Sea was in its present place, but the channel was wider and deeper, enabling a stronger inflow and thus higher salinities.

Along the passage towards the Baltic Sea, the salinity decreases gradually. In the Kattegatt the surface layer salinity is of the order of 20 %o, while that of the bottom layer may be close to 34 %. Beyond the Belts, the water passing the Darss sill still has a salinity of the order of 17 %. During its passage towards the central parts of the Baltic Sea, this water sinks down and forms the deep water of the basin. Mixing and diffusion during the passage reduce the salinity of the inflowing water (HELA and KRAUS, 1959, see also KALLE, 1942). According to KALEIS (1970), the time required for water to pass from the Darss sill to the Gotland Deep is about 4 to 8 months. The salinities recorded in the Gotland Deep are 11 to 14 %o.

![Fig. 2.1. Presumed historical evolution of the Baltic Sea salinity (according to Kessel and Punning, 1972).](image)
The deepest layer of the Baltic Sea may be said to be under the influence of the Kattegatt waters. Correspondingly, the surface layer waters are strongly influenced by river discharges. The surface salinity varies from 2% in the Neva Bay to some 9% in the region of the Arkona basin. In the entire Baltic Proper, between the southern boundary of the Finnish archipelago and Bornholm, the surface layer salinity is about 6.5 to 7.5%. A summary of the salinity distribution in the Baltic Sea has been presented by Bock (1971).

The upper layer is fairly homogeneous during the time when thermal stratification is absent. It terminates with a strong pycnocline, where salinity increases more than one per mille over a vertical distance of a few metres. At the entrances to the Gulf of Bothnia and Gulf of Riga the pycnocline usually lies below the top of the sill, so that the circulation brings less saline water to these bays. In these bays there is weak salinity stratification, which is at least partly destroyed by convective mixing in the autumn.

2.2 Long-term variations in the salinity

Since it is a function of the water exchange and river runoff, the salinity of the Baltic water masses may be expected to vary. All the observations made during this century show that the salinity has somewhat increased (HELA, 1966a). Increases in salinity occur during major inflows, when saline water penetrates the deepest basins. These inflows occur at irregular intervals. During a strong saline inflow, the old bottom-layer water partly mixes with the new inflow and is partly pushed aside, causing a slight salinity increase in other parts of the Baltic Proper as well. The importance of these saline influxes for the entire water mass has been emphasized by many authors, among others Matthäus (1977, 1979) and Fonselius (1969). An increase in the bottom layer salinity might imply increasing static stability. Fonselius made a careful analysis of the stability conditions in the central parts of the Baltic Sea and was able to show that there is no indication of such an increase in static stability. This is because there is a slight increase in the surface layer salinity as well. Thus no change is apparent in the mixing conditions between the upper and lower layers of the Baltic Sea.

The data series obtained from the central parts of the Baltic are in general more irregular than those observed in the coastal regions. Research vessels do not visit the open-sea stations more than a few times a year and the data may therefore be disturbed by seasonal signals. The regular coastal station data contain much more information on salinity variations. The observation network along the Finnish coast was established at the beginning of the century. Of particular interest are the data from the Utö station, located at the SW edge of the Finnish archipelago. Observations at this station began in the year 1919 and continued without interruption until the mid 1970's. After a short break the observation activity has begun again. The deepest observations were made at this station, at a depth of 90 m. It is visited by the pilot house personnel every ten days for measurements of water temperature and salinity sampling. Lauriainen (1982) has analysed both the seasonal and long-term variation of the Utö observations, and the seasonal variation is shown in Fig. 2.2. In the bottom layer, at depths of 80 and 90 metres, there is a clear seasonal cycle: the salinity is highest (about 8%) in July and August and somewhat below 7% in December—January. The fact that the surface and bottom salinities are very close to each other in winter may be partly explained by the proximity of the coast and weak static stability during the winter months. In summer, on the other hand, there is a well-defined
thermocline, which prevents mixing down to the halocline. The surface layer salinity minimum occurs some 2.5 months after the maximal runoff, which implies a mean flow of a couple of centimetres per second. The same average velocity has been observed by other means as well. Fig. 2.3 shows three-year running means of the Utö salinities at a depth of 80 metres. As can be seen, the salinity remained fairly constant till the middle of the 30's, after which an increase occurred, this change agreeing with observations in
Fig. 2.4. Long term variability of the surface layer salinity in the Gulf of Finland and river runoff into Gulf of Finland (Launiainen, 1982).

Fig. 2.5. Long term variability of the surface layer salinity in the Gulf of Bothnia (Utö, Säppi 61°29'N, 21°21'E, Valassaaret 63°25'N, 24°04'E, Ulkokalla 64°20'N, 23°27'E; according to Launiainen, 1982).

The increase of salinity continued until the mid 50's and the salinity has since remained at a constant level, or even decreased slightly. In the same figure, a graph according to Mikulski (1980) shows similar variation in the river discharge to the Gulf of Finland. The graphs show that during periods of increasing river runoff salinity decreases and vice versa. It should be noted that the salinity data set is taken
below or within the halocline. The data set showing this correlation with runoff is strongly smoothed and obtained at a coastal station, so that it is not possible to be sure that a similar dependence will be found below the halocline in the open sea as well.

As was stated in Chapter I, the greatest river discharge is that of the River Neva at the head of the Gulf of Finland. From the Neva estuary, there is a buoyancy driven flow towards the Baltic Sea. Due to the rotation of the earth this flux is concentrated on the Finnish side of the Gulf of Finland, but it is obscured by more energetic and very variable wind-driven flow. However, the influence of river discharge on the salinity is clearly evident on the north side of the gulf, as can be seen in Fig. 2.4, which correlates the data from three fixed stations with the respective integral discharge (note the reverse scale of the runoff values). Synchronous variations occur, although with different amplitudes. Towards the eastern station Tammio (60°25'N, 27°25'E) the dependence on integrated runoff increases and the slight trend of increasing salinity still evident in the Tvärminne (59°51'N, 23°15'E) data disappears.

In the same study, Launiainen also made a comparison of the salinity variations at the stations Säppi in the Bothnian Sea, Valassaaret in the northern Quark and Ulkokalla in the Bothnian Bay. The results are presented in Fig. 2.5. Due to the shallow coast of the Gulf of Bothnia, no deep stations are available. The mean salinities decrease towards the north, but the same features as were found at Utö can be detected as far north as Valassaaret, where they are seen mainly in the bottom layer. It may be concluded that the gradual increase of salinity extends all over the Baltic, except for the heads of the bays, where the influence of rivers and estuarine features are most clear.

There are few other regular observations from deep water besides the Utö records. In the Åland Sea, the personnel of Märket lighthouse carried out observations until the automation of the lighthouse. The data from this station indicate the same general trend as the Utö data, with the exception that there is an approximately constant difference between the surface and bottom layer salinities. As mentioned above, the water in the Åland Sea originates from the surface layer of the Baltic Proper, and the surface layer salinity is influenced by the cyclonal circulation of the Bothnian Sea, which gives it somewhat lower values.

### 2.3 Vertical large-scale structure of the salinity

According to the above discussion, the vertical structure is of two-layer character, with an occasional third layer formed by exceptional salt intrusions. The upper layer down to the halocline at depth $h_s$ may be considered practically homogeneous. The lower layer, between $h_s$ and $H$, has a considerable salinity gradient. For modelling purposes we take the depth $H$ to be either the bottom depth or, in the case of the deepest basins, the depth of the secondary halocline, approximately 125–130 metres. When the secondary halocline is absent, the layer below this depth is fairly homogeneous with constant salinity.

The formation of the upper homogeneous layer is physically similar to the formation of the seasonal thermocline: the stable stratification in the surface due to fresh water from rivers and saline water from the Danish Sounds is destroyed by the action of the wind. Deepening of the halocline is restricted to seasons without a seasonal thermocline, and its ultimate limit is a strong buoyancy gradient which cannot be eroded by the prevailing winds. During the summer months, both the lower portion of the isosaline layer and the halocline are decoupled from the atmospheric influence due to the formation of the seasonal thermocline.
Fig. 2.6. Vertical profiles of salinity in various regions of the Baltic Sea (1 is Gulf of Finland, 2 is Gotland Deep, 3 is Bornholm Deep, 4 is Fehmarn Belt), according to Fonselius (1969) and Soskin (1963).

Fig. 2.7. Self-similarity profile of the salinity.

The lower layer with thickness $H - h_s$ is stably stratified and has, if any, only patchy turbulence of very limited extension. The momentum and kinetic energy induced from the upper layer appear largely in the form of internal waves. These waves, due to several mechanisms of instability, participate in the redistribution of salinity.

Fig. 2.6 shows typical salinity profiles from several regions of the Baltic Sea. Despite the variable character of the profiles, they can be modified to a uniform type by taking for the interval $(h_s, H)$ the following dimensionless variables:

$$
\theta_s = \frac{S(z) - S_s}{S_H - S_s} ; \quad \xi = \frac{z - h_s}{H - h_s}
$$

where $S_s$ is salinity in the homogeneous surface layer

$S_H$ salinity at the bottom

For $S(z)$, we may as well use the dimensionless coordinate $\xi$. With this notation, the vertical structure of the salinity can be written in the following form:

$$
S = S_s \quad \text{for} \quad 0 \leq z \leq h_s
$$

$$
S = S_s + \theta_s(S_H - S_s) \quad \text{for} \quad h_s \leq z \leq H
$$

A similar model was proposed by Kitaigorodsky and Mirovolsky (1970) for describing the distribution of temperature in the surface layer. Later, an analogous model was used in the studies of Reschetov and Chalikov (1977), Barrenblatt (1978), Linden (1975), Kalatsky (1978) and some others. The similarity of the salinity distribution patterns in the Baltic Sea has been pointed out by Tamsalu (1979).

If the boundary conditions may be presented as:

$$
\theta_s = 0 \quad \text{when} \quad \xi_s = 0
$$

$$
\theta_s = 1 \quad \text{when} \quad \xi_s = 1
$$
Fig. 2.8. Time series of bottom salinity $S_H$, mean salinity $\bar{S}$, surface salinity $S_s$ and the non-dimensional ratio $\gamma = (\bar{S} - S_s)/(S_H - S_s)$ in the Bornholm Deep.

and the integral conditions as

$$
\int_0^1 \theta_s \, d\xi_s = \kappa_s ; \quad \int_0^1 \int_0^1 \theta_s \, d\xi_s \, d\xi_s = \overline{\kappa_s}.
$$

A fourth order polynomial is found for $\theta(\xi)$. The values for $\kappa_s$ and $\overline{\kappa_s}$ have been determined experimentally to be 0.6 and 0.2, respectively. The polynomial fulfilling the conditions presented above is the following:
\[ \theta_s(\xi_s) = 6\xi_s^2 - 8\xi_s^3 + 3\xi_s^4 \quad (2.4) \]

Fig. 2.7 shows the dependence of \( \theta \) on the dimensionless coordinate. The continuous line marks curve (2.4) and points show the empirical observations of salinity.

If we integrate equation (2.2) between the surface and depth \( H \), using for the similarity function equation (2.4), we obtain the depth of the halocline \( h_s \) as a function of depth, mean salinity, and surface and bottom salinities:

\[ h_s = H \left[ 1 - \frac{1}{\kappa_s} \frac{\bar{S} - S_s}{S_H - S_s} \right] \quad (2.5) \]

Fig. 2.8 shows the long-term variations of the salinity variables in equation 2.5 for Bornholm Deep. Although there is considerable variation with time especially in the bottom salinity \( S_H \), the non-dimensional ratio \((\bar{S} - S_s)/(S_H - S_s)\) has a relatively steady value, as can be seen from graph 2.8b. The ratio varies within the limits 0.32 and 0.39. Correspondingly, the dependence of the differences of the mean and bottom salinities from the surface salinity is presented for long-term means from several regions in Fig. 2.9. As a first approximation we may take the dependence to be linear, that is their ratio is constant. Within the limits of this approximation, the halocline depth is a linear function of the total depth (if the depth exceeds the secondary halocline depth, we take the secondary halocline as the bottom). Fig. 2.10 shows the dependence of the halocline depth on total depth according to Kaleis (1983). As is seen, the linear dependence is valid in these data for both August and October for depths of more than 60 metres. In October the thermal stratification has diminished and the convective mixing deepens the halocline. The linear dependence is probably not valid in shallow regions.

Using the same long-term averages, a further proportionality can be obtained for the surface and bottom salinities vs. mean salinity. This dependence is shown in Fig. 2.11, and can be presented as follows:

\[ S_s = \frac{2}{3} S_0 + \frac{1}{3} \bar{S}(x,y,t) \]
\[ S_H = -\frac{4}{3} S_0 + \frac{7}{3} \bar{S}(x,y,t) \quad (2.6) \]

where \( S_0 \) is the salinity below which a permanent halocline disappears. For salinities less than \( S_0 \) the above dependence is no longer valid. Substitution of equations (2.6) for the surface and bottom salinities in eq. (2.5) gives

\[ h_s = H \left[ 1 - \frac{1}{3\kappa_s} \right] = 0.44H , \quad (2.7) \]

provided \( \kappa_s = 0.6 \). This linear dependence is shown as the continuous line in Fig. 2.10.

The value presented for \( h_s \) in equation (2.7) may be considered a minimum value for the halocline. In the autumn, when the water mass is thermally homogeneous down to the halocline, wind mixing tends to penetrate to greater depths. In coastal regions where \( S_H - S_s \) is not high, mixing may extend deeper down, since smaller amounts of mechanical energy are needed for erosion of the pycnocline. Correspondingly, in regions where \( S_H - S_s \) is high, deepening may be negligible.

On the basis of the above, the salinity structure in the Baltic Proper may be described as a first approximation by equations (2.2; 2.4; 2.6; 2.7) alone. The only independent
Fig. 2.9. Dependence of \((S_H - S_s)\) on \((\bar{S} - S_s)\) at various regions of the Baltic Sea.

Fig. 2.10. Relationship between halocline depth and total depth in August (A) and October (B) according to Kaleis.
parameter left is the mean salinity of the profile, \( \bar{S} \). As seen in Fig. 2.11, the equations are not valid for the low salinities occurring in estuarine conditions, as in the eastern Gulf of Finland, Gulf of Riga, and the Bothnian Bay. By definition, the structure proposed here describes a long-term mean, not one single synoptic situation. In the presence of short-term variability, such as internal waves and synoptic eddies, the similarity profile may still be valid, but the linear relationships (2.6) and (2.7) are not accurate. When slow processes are considered, such as the influx of water through separate basins, the similarity profile presented above allows a physically relevant approximation of the development of the salinity structure. Such model approximation is presented in the next section.

2.4 Modelling the water and salt exchange through the Danish Sound

The first model for estimating the saline water flux through the Danish Sounds was presented by Knudsen (1899). If the water is assumed to be homogeneous on each side of the sound, the balance of mass and salt in a two-layer type of exchange can be written as follows:

\[
Q_1 - Q_2 = Q_R \\
Q_1 S_1 - Q_2 S_2 = 0 ,
\]

where \( Q_1 \) and \( Q_2 \) represent transport of water out of and into the sounds, \( S_1 \) and \( S_2 \) the salinities of the respective water masses and \( Q_R \) is the river runoff.

Assuming that the salinity of inflowing water \( S_1 = 17.4 \% \) and that of the outflowing water \( 8.7 \% \), we have

\[
Q_1 = 2Q_R \quad \text{and} \quad Q_2 = Q_R .
\]
For prediction of the Baltic Sea salinity, an additional relation of the type

\[ Q_1 = f(S_2 - S_1, Q_R) \]  

must be added to close the problem, provided that the inflow is caused only by buoyancy-driven flow. A system of this type was studied theoretically and experimentally by Stommel and Farmer (1953). In their paper the authors used the following equation of critical internal flow:

\[ \frac{Q_1^2}{\eta^2} + \frac{Q_2^2}{(1 - \eta)^2} = \frac{\Delta p}{\rho^2} g H^3 \]  

where \( \eta = h_s/H \).

Using equations (2.8) and (2.9) and assuming some diffusive transport in the interface between the upper and lower layers, Welander (1974) presented a non-stationary nonlinear equation system for the prediction of outflow, inflow, Baltic Sea salinity and diffusive transport.

The extensive experimental studies in the Danish Sounds (Kruse, Jacobsen and Nielsen, 1980) and theoretical considerations (Svansson, 1980; Astok and Otsman, 1977) both show that non-stationary one-layer type flow predominates in the water exchange. In that case, the relation (2.8) may be modified to read

\[ Q_1 - Q_2 + Q_T = Q_R \]  

where \( Q_T \) is non-stationary one-layer type water exchange.

In order to estimate the stationary inflow, we take the equation of the turbulent transfer of salt:

\[ \nabla \cdot (uS) - \mu \nabla^2_h S - \nu \frac{\partial^2 S}{\partial z^2} = 0 \]  

where \( S \) is salinity, \( u \) velocity vector, \( \nabla^2_h \) horizontal Laplacian, \( \mu, \nu \) are coefficients of horizontal and vertical turbulent diffusion.

Integrating the equation (2.13) over the cross-section area of the Sounds and taking into account the self similarity profile of salinity (2.2), we obtain

\[ \frac{d}{dx} \left[ Q_s S_s + Q_h (S_s + \kappa_s (S_H - S_s)) - \mu A_T \frac{dS}{dx} \right] = 0 \]  

where, in addition to the symbols in earlier notations, \( A_T \) is the cross-section area of the channel. Equation (2.14) states that the inflow, outflow and diffusive flux of salt are the same at any cross-section. By integrating (2.14) and keeping in mind the Knudsen relations, it is possible to calculate inflow and outflow at any section. The last term in eq. (2.14), the diffusive salt flux, varies locally and vanishes at the ends of the channel. As discussed by Svansson, the value of horizontal eddy diffusivity is very difficult to determine. Assuming that the eddy diffusivity is \( 5 \times 10^3 \) m\(^2\)s\(^{-1}\), the cross-section area 0.225 km\(^2\) and salinity gradient 0.04 \( \% \) km\(^{-1}\), we obtain for diffusive flux 1350 km\(^3\) \( \% \) year\(^{-1}\), which should be compared with the customary value for \( Q_s S_s \) (of the order of 7600 km\(^3\) \( \% \) year\(^{-1}\)).
Astok and Otsman (1977) and Svansson (1980) studied the non-stationary inflow and proposed the following equation:

$$\frac{d^2 Q_T}{dt^2} + R \frac{dQ_T}{dt} + \omega_0^2 Q_T = \omega_0^2 \left( \frac{d\xi}{dt} + A_B \frac{dP}{dt} \right),$$

2.15

where $\xi$ is fluctuation of North Sea level

$P$ air pressure difference between North Sea and Baltic

$\left( \frac{A_T g}{A_B L} \right)^{1/2}$ frequency of eigenfluctuation of the system in the absence of friction

$R$ friction coefficient $A_T$ cross-section area of the sounds $A_B$ surface area of the Baltic Sea $g$ acceleration due to gravity $L$ length of the sounds

The analysis of equation (2.15) and the numerical results indicate that the fluctuation in the North Sea level is one of the main driving forces at all frequencies considered.

The period range considered was from 2 days up to one year, the direct influence of wind in eq. (2.15) is neglected, the indirect influence is included with the sea level fluctuations.

Many numerical experiments carried out with the above model of Svansson, Astok and Otsman have revealed good correlations between calculated and observed inflows.

The total value of water exchange consists of stationary and non-stationary components. Thus, if

$$Q_T + Q_1 > 0$$

2.16

the inflow prevails and, correspondingly, if

$$Q_T + Q_2 < 0$$

2.17

the outflow is dominant. This is the case irrespective of the form of the current and salinity profiles. Most of the time, either (2.16) or (2.17) is valid, i.e. a one-layer type of current prevails in the Sounds.

The salt exchange can be examined in a similar way by dividing the salinity into stationary and non-stationary components:

$$S = \tilde{S} + S'$$

2.18

The stationary component is determined by the stationary near-bottom inflow current, and mixing and river discharge in the upper layer. The stationary distribution corresponds to that in Fig. 1.17a. The non-stationary component depends mainly on non-stationary water exchange. Since observations have shown that this part of the water exchange is vertically homogeneous, the non-stationary component of salinity is vertically homogeneous as well.

If the diffusion is disregarded, the salinity fluctuations in the Sounds can be described by the following equation

$$\frac{\partial S'}{\partial t} + u \frac{\partial S}{\partial x} = 0$$

2.19
In (2.19), $S$ corresponds to the vertically averaged salinity and the velocity $u' = Q_T/A_T$. The fluctuating flow can be determined from the water level fluctuations of the Baltic Sea, assuming that there is no drastic variability in the river runoff:

$$A_B \frac{d\bar{x}_B}{dt} = -Q_T,$$

where $\bar{x}_B$ is the mean sea level of the Baltic. Inserting (2.20) into (2.19) and integrating, we obtain for local salinity fluctuations

$$S' = S'_0 + \omega_1(\bar{x}_B - \bar{x}_0B),$$

where the subscript $0$ indicates the corresponding value at time $t = 0$. Thus, the variations of salinity in the Danish Sounds are directly proportional to the fluctuations of the mean sea level of the Baltic Sea, as has been experimentally discovered by many authors, among them HELA (1944) and SVANSSON (1972).

The division proposed here: stationary regime due to near-bottom inflow and river discharge, and non-stationary regime due to fluctuations in sea level is a way of treating the complex structure in the Danish Sounds by isolating the processes. As was noted, the non-stationary part depends on the Baltic Sea water level, which in turn depends on the fluctuations of the North Sea, as discussed by ASTÓK and OTSMAN and by SVANSSON. Dependence on the weather is implied by inclusion of the pressure field in the equation (2.15).

### 2.5 Horizontal large-scale structure of the salinity

Due to the two opposing processes responsible for the salinity structure of the Baltic Sea, the most saline waters are found near the Danish Straits and the least saline waters at the heads of isolated Gulfs. Figure 2.12 shows the salinity profile along the main axis of the Baltic Sea from the Skagerrak up to Neva Bay at the head of the Gulf of Finland. Along this axis the overall change of salinity is some 30%. The inflow of ocean water and river discharges are not the only mechanisms responsible for the distribution of salinity. Fig. 2.13 shows the distribution of surface salinity in the Baltic Sea in May 1970 (MATTHÄUS, 1979). Dots indicate long-term average values. The map clearly shows a tendency for higher salinities to concentrate in the southern and south-eastern parts,
Fig. 2.13. Horizontal distribution of surface salinity according to Matthäus (1979).

Fig. 2.14. Salinity distribution close to bottom.

while lower values occur in the western and north-western parts. This demonstrates clearly the influence of the Coriolis effect on the slow mean currents and, consequently, on the distribution of surface salinity. The lower value of salinity in the region of the Gdansk Bay is probably due to inflow from the River Wisla.

The vertical stratification of salinity is well demonstrated in Fig. 2.14, which shows the horizontal distribution of salinity in the vicinity of the bottom $H(x,y)$. When compared with the map of the bottom topography (Fig. 1.1), the map also reveals the direction of movement of the saline bottom water. From the Arkona Basin we may follow the course of the highest bottom salinities to the Bornholm Basin, and from there via Stolpe Channel to the Gdansk Basin and Gotland Deep.

The factors influencing the salinity structure: inflow of ocean water, river water inflow, bottom topography, currents and macroturbulence may be combined in a set of equations which, together with the boundary conditions, describe the behaviour of the sea. We may also treat the Baltic Sea as a large estuary and apply the semi-empirical turbulent diffusion equation:

$$\frac{\partial S}{\partial t} + \nabla \cdot (uS + I_s) = 0,$$

where $u$ is velocity vector

$I_s$ salt flux vector.

It is of interest to study the salinity as the function of the longitudinal coordinate only, i.e. averaging the conditions over the transverse cross-section. The problems involved in deriving the one-dimensional equation are discussed by Okubo (1964). If we assume that evaporation and precipitation balance each other, the one-dimensional equation for salinity will have the following form:
\[
\frac{\partial}{\partial t} [A \tilde{S}] + \frac{\partial}{\partial x} \left[ A \tilde{u} \tilde{S} - A\mu \frac{\partial \tilde{S}}{\partial x} \right] = 0,
\]

where tildes (\(\tilde{\cdot}\)) indicate averaging over the cross-section, \(A(x)\) is the cross-section area and \(\mu\) is the coefficient of horizontal diffusion.

In a stationary situation eq. (2.23) and the incompressibility condition yield

\[
\frac{d}{dx} \left[ A\mu \frac{d\tilde{S}}{dx} - A\tilde{u} \tilde{S} \right] = 0
\]

\[
\frac{d}{dx} \left[ A\tilde{u} \right] = -Q_R,
\]

where, as earlier, \(Q_R\) is river discharge.

We take the entrance to the Kattegat as the origin, and assume that all the river discharges are concentrated in the estuary of the River Neva. (Since the Gulf of Finland and the Gulf of Bothnia are responsible for some 68% of total river runoff, this may be acceptable for the Baltic Proper). The mean velocity of the profile is defined as follows:

\[
\tilde{u} = -\frac{\tilde{Q}_R}{A(x)}
\]

At the entrance to the Kattegat \((x = 0)\) the assumed salinity of the North Sea is

\[
\tilde{S} = \tilde{S} = 32 \%
\]

and in the estuary of the River Neva \((x = L)\) the boundary condition is

\[
A\mu \frac{d\tilde{S}}{dx} + \tilde{Q}_R \tilde{S} = 0
\]

With these boundary conditions, equation (2.24) may be presented in the form

\[
A(x) \frac{d\tilde{S}}{dx} + B \tilde{S} = 0 ; \quad B = \frac{\tilde{Q}_R}{\mu},
\]

which has the solution

\[
\tilde{S} = \tilde{S}_0 \exp \left\{ -B \int_{x_0}^{x} \frac{dx'}{A(x')} \right\}
\]

In the calculations of the longitudinal profile of the mean salinity, the Baltic Sea is divided into four parts: the Kattegat (interval \(0, L_1\)), the Belt Sea \((L_1, L_2)\), the Baltic Proper \((L_2, L_3)\) and the Gulf of Finland \((L_3, L)\). The coefficient of horizontal diffusion is taken as \(5 \times 10^3 \text{ m}^2\text{s}^{-1}\).

After the structure of the mean salinity has been determined, the vertical structure of the salinity will be calculated using the similarity profile described in Section 2.3. This procedure has been applied for determining the influence of variations of river discharge on the salinity structure of the Baltic Sea. The results of this calculation are shown in Figs. 2.15 to 2.17.
Fig. 2.15. Vertical distribution of salinity according to steady state similarity model.

Fig. 2.16. Same as Fig. 2.15 but river runoff reduced by 20%.

Fig. 2.17. Same as Fig. 2.15 but river runoff increased by 20%.
Fig. 2.15 shows the longitudinal section of salinity calculated as described above with river runoff $Q_R = 430 \text{ km}^3\text{y}^{-1}$. According to Mikulski (1970, 1972), this runoff corresponds to the long-term mean of the basin. Comparison with the observed profile in Fig. 2.12 reveals that the characteristic features are well reproduced in the calculated profile. Fig. 2.16 shows the salinity distribution calculated with the assumption that river runoff is reduced by 20% of the present mean value. As can be seen, the increase of salinity in the deep basins is drastic, while the conditions in the surface layer are altered only slightly. A corresponding calculation of salinity with increased river runoff is shown in Fig. 2.17. According to this figure, the mean salinity north of Gotland falls below 7%, resulting in a relatively weak or completely missing halocline.

The calculations presented above are based on two-dimensional considerations: salt diffusion equation with integration over transverse sections, and vertical variability according to the similarity profile. Apart from river runoff, horizontal movements are not included. An interesting time scale for the processes involved may be obtained by a simple dimensional argumentation. The diffusion coefficient has been taken as $5 \times 10^3 \text{ m}^2\text{s}^{-1}$. If the length scale is taken as the length of the basin, some 1300 km, the relevant time scale obtained is some 10 years. This is the time needed for stabilization towards the equilibrium.

Finally, we would like to consider the three-dimensional structure of salinity. Assuming that the vertical salinity stratification can be described with reasonable accuracy by the relationships (2.2) and (2.6), we need to define the vertical average salinity $S(x,y,t)$. To obtain this, the equation (2.22) is integrated from surface to bottom ($z$ increasing downwards) with the following boundary conditions:

\[
\begin{align*}
\text{at the surface } z = 0 & & l_z = 0 ; & w = 0 & \quad 2.31 \\
\text{at the bottom } z = H & & l_z = 0 ; & w = u \cdot \nabla H & \quad 2.32
\end{align*}
\]

If the velocity components $(u,v)$ and salinity are expressed as a sum of the vertical average and deviation from this average:

\[
\begin{align*}
u = \bar{u} + u^* ; & \quad v = \bar{v} + v^* ; & \quad S = \bar{S} + S^* \\
\bar{u} = \frac{1}{H_0} \int_0^H udz ; & \quad \bar{v} = \frac{1}{H_0} \int_0^H vdz ; & \quad \bar{S} = \frac{1}{H_0} \int_0^H sdz ,
\end{align*}
\]

the integral of (2.22) can be expressed as:

\[
\begin{align*}
\frac{\partial \bar{S}}{\partial t} + \bar{u} \frac{\partial \bar{S}}{\partial x} + \bar{v} \frac{\partial \bar{S}}{\partial y} + \frac{1}{H} \left[ \frac{\partial}{\partial x} \int_0^H u^* S^* dz + \frac{\partial}{\partial y} \int_0^H v^* S^* dz \right] \\
+ \frac{1}{H_0} \int_0^H \nabla \cdot I dz = 0 .
\end{align*}
\]

In deriving (2.34), the continuity equation

\[
\frac{\partial \bar{H} u}{\partial x} + \frac{\partial \bar{H} v}{\partial y} = 0
\]

has been used. Using eqs. (2.6) and (2.7), we obtain for the integrated fluxes:
\[
\int_0^H u^*S^*dz = \frac{2}{3\kappa_s} H(\bar{S} - S_0) \int_0^H u^* \theta^* dz',
\]
\[
\int_0^H v^*S^*dz = \frac{2}{3\kappa_s} H(\bar{S} - S_0) \int_0^H v^* \theta^* dz'.
\]

The velocities \((u, v)\) may be determined from the geostrophic relationship and the equation of state:

\[-fv = g \frac{\partial \xi}{\partial x} - \frac{\partial}{\partial x} \int_0^z \rho dz\]
\[lu = g \frac{\partial \xi}{\partial y} - \frac{\partial}{\partial y} \int_0^z \rho dz\]
\[b = g\beta(\bar{S} - S_0)\]

In these equations, \(f\) is Coriolis parameter, \(g\) acceleration due to gravity, \(b\) buoyancy, \(\beta = 7.8 \times 10^{-2} \text{ (\%)}^{-1}\), and \(\xi\) is deviation of the sea surface from the mean level. Integration of (2.37) and (2.38) from surface to bottom, taking into account the similarity conditions (2.6) and (2.7), and also (2.33) and (2.39), gives the deviations from the mean velocity as:

\[u^* = -\frac{g\beta}{f} \left[ \varphi_1 \frac{\partial(\bar{S} - S_0)}{\partial y} + \varphi_2(\bar{S} - S_0) \frac{\partial H}{\partial y} \right]\]
\[v^* = \frac{g\beta}{f} \left[ \varphi_1 \frac{\partial(\bar{S} - S_0)}{\partial x} + \varphi_2(\bar{S} - S_0) \frac{\partial H}{\partial x} \right]\]

Using these values, equations (2.36) give the integral of the baroclinic fluxes as:

\[H^{-1} \left[ \int_0^H \frac{\partial}{\partial x} \int_0^H u^*S^*dz + \int_0^H \frac{\partial}{\partial y} \int_0^H v^*S^*dz \right] = C \frac{g\beta}{f} J(M, H)\]

The constant \(C = 0.6, M = \bar{S} - S_0\) and \(J\) is Jacobian \(J(a, b) = \frac{\partial a}{\partial \bar{a}} \frac{\partial b}{\partial \bar{b}} \frac{\partial \bar{a}}{\partial V^2} - \frac{\partial a}{\partial \bar{a}} \frac{\partial b}{\partial \bar{b}} \frac{\partial \bar{a}}{\partial V^2}\). The second integral, divergence of salt fluxes, is parametrized in a traditional way:

\[\frac{1}{H} \int_0^H \nabla \cdot \mathbf{F} dz = \mu \nabla^2 \bar{S} = \mu \left( \frac{\partial^2 \bar{S}}{\partial x^2} + \frac{\partial^2 \bar{S}}{\partial y^2} \right)\]

If we now non-dimensionalize the equation (2.34) in the integrated and parametrized form by notating:

\[x = Lx'; \quad y = Ly'; \quad t = \frac{L}{U} t';
\]
\[\bar{u} = Uu'; \quad \bar{v} = Uv'; \quad H = DH';\]
Fig. 2.18. Horizontal distribution of salinity for a square basin with variable topography, numerical solution of eq. 2.45 (see text).
The velocities \((u,v)\) may be determined from the geostrophic relationship and the equation of state:

\[
-fv = g \frac{\partial \xi}{\partial x} - \frac{\partial}{\partial x} \int_0^z b \, dz
\]

\[
u u = g \frac{\partial \xi}{\partial y} - \frac{\partial}{\partial y} \int_0^z b \, dz
\]

\[
b = g\beta (S - S_0).
\]

In these equations, \(f\) is Coriolis parameter, \(g\) acceleration due to gravity, \(b\) buoyancy, \(\beta = 7.8 \times 10^{-4} (\%)^{-1}\), and \(\xi\) is deviation of the sea surface from the mean level. Integration of (2.37) and (2.38) from surface to bottom, taking into account the similarity conditions (2.6) and (2.7), and also (2.33) and (2.39), gives the deviations from the mean velocity as:

\[
u^* = -\frac{g\beta}{f} \left[ \varphi_1 \frac{\partial (S - S_0)}{\partial y} + \varphi_2 (S - S_0) \frac{\partial H}{\partial y} \right]
\]

\[
u^* = \frac{g\beta}{f} \left[ \varphi_1 \frac{\partial (S - S_0)}{\partial x} + \varphi_2 (S - S_0) \frac{\partial H}{\partial x} \right]
\]

\[\varphi_1 = \varphi_1(\xi); \quad \varphi_2 = \varphi_2(\xi).\]

Using these values, equations (2.36) give the integral of the baroclinic fluxes as:

\[
H^{-1} \left[ \frac{\partial}{\partial x} \int_0^H u^* S^* \, dz + \frac{\partial}{\partial y} \int_0^H v^* S^* \, dz \right] = C \frac{g\beta}{f} J(M,H).
\]

The constant \(C = 0.6\), \(M = S - S_0\) and \(J\) is Jacobian \(J(a,b) = \frac{\partial a \partial b}{\partial x \partial y} - \frac{\partial a \partial b}{\partial y \partial x}\). The second integral, divergence of salt fluxes, is parametrized in a traditional way:

\[
\frac{1}{H} \int_0^H \nabla \cdot I \, dz = \mu \nabla^2 S = \mu \left( \frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} \right).
\]

If we now non-dimensionalize the equation (2.34) in the integrated and parametrized form by notating:

\[x = Lx'; \quad y = Ly'; \quad t = \frac{L}{U} t'; \]

\[\bar{u} = Uu'; \quad \bar{v} = Uv'; \quad H = DH'.\]
Fig. 2.18. Horizontal distribution of salinity for a square basin with variable topography, numerical solution of eq. 2.45 (see text).
Fig. 2.19. Time evolution of vertical salinity structure along the section given by the broken line in Fig. 2.18f (see text).
\[ \bar{S} = S_0(1 + s) \; ; \quad \beta S_0 = \frac{\Delta \rho}{\rho} \; ; \quad \text{Ro} = \frac{U}{fL} \]
\[ \text{Re} = \frac{UL}{\mu} \; ; \quad L_D = \sqrt{\frac{\Delta \rho}{\rho} gD/f} \; ; \quad \text{Bu} = L_D L^{-1} \]

where the non-dimensional numbers have the meaning:

- \( L_D \) is Rossby (baroclinic) deformation radius
- \( \text{Ro} \) Rossby number
- \( \text{Re} \) Reynolds number, and
- \( \text{Bu} \) Burger number,

we obtain the following equation:

\[ \frac{\partial s}{\partial t} + u' \frac{\partial s}{\partial x} + C \frac{\text{Bu}}{\text{Ro}} J(s^2, H') - \frac{1}{\text{Re}} \nabla'^2 s = 0 \]  \hspace{1cm} (2.44)

In the Baltic Sea we may choose the scales as follows: Scale depth = 60 m, scale length = 100 km, velocity scale \( U = 0.1 \text{ ms}^{-1} \), internal Rossby radius = 13 km. These scales give the ratio \( \text{Bu} \text{Ro}^{-1} = 1 \). With an eddy diffusivity scale of \( 10^4 \text{ m}^2\text{s}^{-1} \), the turbulent Reynolds number equals 1. Taking the vertical mean velocity as zero, we obtain from (2.44)

\[ \frac{\partial s}{\partial t} + J(s^2, H') - \nabla'^2 s = 0 \]  \hspace{1cm} (2.45)

which is easily solved numerically. The evolution of the mean salinity structure in a square basin which has two open corners and variable bottom topography is shown in

Fig. 2.20. Model simulation of mean salinity of the Baltic Sea.
Fig. 2.18. The last diagram of Fig. 2.18 shows both the bottom topography and a broken line defining the cross-section presented in Fig. 2.19. The latter figure shows the evolution of the vertical salinity structure as a function of non-dimensional time. With the scales chosen, one time step corresponds to about 11.5 days. Finally, Fig. 2.20 shows the horizontal distribution of vertically integrated salinity $\bar{S}$ in a basin approximating to the Baltic Sea.
3 TEMPERATURE REGIME OF THE BALTIC SEA

3.1 General features

During past decades a large number of studies have been devoted to the spatial and temporal structure of the Baltic Sea temperature. The first studies were presumably those of M. Knudsen (1899) and R. Witting (1905), and subsequent studies deserving special mention are those of I. Jacobsen (1908) and G. Granqvist (1938, 1952). Recently, the temperature regime in the eastern coastal region has been investigated by I. Leder (1972).

From the beginning of the century, a number of lightships have made temperature and salinity observations in different parts of the Baltic Sea. The bulk of the information available on the seasonal cycle and vertical distribution of temperature in different seasons has been obtained from these observations. With the automation of navigation aids, these manned stations have disappeared one by one, and now very few fixed stations remain around the Baltic Sea.

Many of the studies mentioned above deal with observations at fixed coastal stations. Simojoki (1946, 1949) studied the heat balance on the basis of observations made at Bogskär, Helä (1951) analysed the data of Finngrundet light vessel, Hankimo (1964) estimated the annual energy balance using the data from Finngrundet as well, and Laevastu (1960) made use of a large amount of observations around the Baltic in his study of the heat balance. Antonov, Kubareva and Sergeev (1972) determined the statistical characteristics of temperature at the Gedser Rev Lightship.

Lentz (1971) has presented all the available observations in monthly mean maps of the horizontal thermal structure in different layers. W. Matthäus (1977) has gathered most of the temperature data from research vessels cruising on the Baltic Proper during the past 80 years and used them to describe the thermal structure of the basin.

The variations in the upper layer temperatures of the Baltic Sea are due to solar heat or atmospheric forcing, those in the lower layers are mainly caused by the inflow of warm water from the North Sea. Due to the strong stratification, the layer below the primary halocline is practically decoupled from the surface layer, as far as heat exchange is concerned. This decoupling becomes even more apparent during the summer season when the seasonal thermocline is being formed, and a cold interfacial layer lies between the surface and deeper layers. Except for periods of bottom water renewal, and occasional patchy turbulence in the stably stratified layer, the main mechanism for heat transfer in a vertical direction is conduction, which according to the observations is a very slow process.

During the summer season, the Baltic Sea can be said to consist of four layers: a mixed layer, extending from the surface down to the depth of the seasonal thermocline (0–20 m), followed by a cold interfacial layer, which due to heating from above is stably stratified. This layer terminates at the depth of the permanent halocline (40–60 m). In this layer the temperature decreases with depth from the surface temperature (15–18 °C) to the temperature of the mixed layer at the end of the previous winter (usually 2–3 °C). We may call this water mass »old winter water«. From the primary halocline downwards the temperature increases. This water mass is renewed by influx from the ocean (with
considerable mixing), so that there is a clear positive correlation between the temperature and salinity. At the lowest limit, the so-called secondary halocline, the water temperature is about 5–6 °C. The secondary halocline is present in the deepest basins after the intrusion of very saline water. This water mass flows in at the very bottom with moderate to weak mixing during its course; it is usually almost isothermal.

In shallow gulfs and bays, and in the regions where the salinity is fairly low, the two last-mentioned layers are absent. During the winter season, heat losses and mechanical mixing by wind induced erosion extend the isothermal layer down to the primary halocline. In shallower regions this results in a homogeneous layer from surface to bottom. In regions with low salinity but considerable depth, such as some parts of the Gulf of Bothnia, there is an inverse thermocline during the winter season.

3.2 Annual and long-term variations in the Baltic Sea heat balance

The annual variability of the heat balance of the Baltic Sea is determined by radiation, air-sea exchange and the formation of sea ice. During the winter months, surface cooling decreases the temperature, in the northern regions close to freezing point. At the salinities of the Baltic Sea, the density maximum occurs between 2 °C and 3 °C, so that winter deepening of the mixed layer through convective heat transfer occurs only until the maximum density temperatures are reached. In the Baltic Proper, the homogeneous winter layer usually extends down to the depth of the primary halocline. In the Gulf of Bothnia, the halocline is much weaker and the winter mixing extends far lower. Homogeneous layers extending from the surface down to more than 80 metres have been observed. In the Baltic Proper, homogeneous layers deeper than the average depth of the halocline (50 to 70 m) are only occasionally observed in strongly baroclinic cases. Due to increased solar radiation, the surface temperature begins to rise in April and reaches its maximum value by the end of July, after which the temperature of the surface layer decreases. While most of the mixed layer modelling theories assume that the vertical heat flux downwards through the thermocline is zero, all observations from the Baltic Sea show that the deeper layer temperatures increase during the summer. This is clearly demonstrated in the papers of Granqvist (1938), and also by Launiainen (1982). Fig. 3.1 shows the annual course of temperature at different depths at Utö during the past 40 years. The time for the maximum surface temperature is about the end of July, that for 20 metres the end of August, and for 40 metres the end of October. By October the vertical stability is determined mainly by salinity.

The increase of temperature below the seasonal thermocline may be partly due to conduction, partly due to large-scale convection. It may also be due to coastal mixing according to the following process: When the seasonal thermocline is formed, a substantial part of the wind energy is transferred to internal waves. In the upper layer, the zone with sufficiently large Väisälä-Brunt frequency is fairly limited. Hence the internal waves move practically horizontally, until they reach the coastal region and breaking of the waves occurs. This breaking creates horizontal temperature (and density) gradients, which tend to become balanced by diffusive fluxes.

Fig. 3.2 shows the seasonal variations of the surface layer temperatures at four fixed stations along the coast of the Gulf of Bothnia. Despite the large horizontal distances between the different stations, the range of variation between the stations is fairly small, both at the surface and at a depth of 20 metres. Throughout the seasons the average temperature difference between the northern Baltic Proper (Utö) and the northernmost Bothnian Bay (Ulkokalla) is 2 °C. This reveals the same uniformity in climatic conditions as was noted in the maps of the air temperature over the Baltic Sea (Figs. 1.2 to
Fig. 3.1. Seasonal cycle of the mean temperature at different depths near Utö (Launiainen, 1982).

Fig. 3.2. Seasonal cycle of the surface temperatures (upper curves) and 20 m temperatures (lower curves) at various stations in the Gulf of Bothnia, according to Launiainen (1982).

1.5). The year-to-year variability, expressed as the standard deviation from the mean seasonal curves in Figs. 3.1 and 3.2, is of the order of 2°C in the surface layer and 1°C in the deeper layers. In some coastal regions, which favour strongly baroclinic currents, the variability is as much as twice the above figures. On the other hand, in some places such as the station Seili in the central Archipelago Sea, the standard deviations are half of the above figures.
Fig. 3.3. Air temperatures at Uppsala ($U_a$) and water temperatures at Finngrendet ($F_w$) during summer months (Launiainen, 1982).

Fig. 3.4. The first three Fourier coefficients of temperature variations in Arkona (A), Bornholm (B) and Gotland (C) deeps, according to Matthäus (1977, 1979).

Fig. 3.5. Annual course of surface temperature (continuous), halocline temperature (dash-dotted) and bottom temperature (dashed) for Arkona (A), Bornholm (B) and Gotland (C) deeps.
The variations in salinity could be shown to depend on water exchange. Similarly, the variations in the long-term means of the sea surface temperature can be correlated with climatic variations in the air temperature. Launainen (1982) compared temperatures at different observation stations with some climatic series of air temperature. Fig. 3.3 shows the long-term air temperatures at Uppsala during the summertime and the simultaneous surface water temperatures at Finngrundet. As can be noted, there is considerable variability during and between different decades, but the temperature difference between the air and water is generally less than 0.5°C. The long-term variability may be traced back to climatic fluctuations. An exceptionally warm period occurred at the end of the 1930's, when the mean of the summer season (June to November) was some 12°C. At the same time, the sea surface temperature was more than 1 °C higher than during subsequent and previous years.

The large-scale temperature variation in the Baltic Proper has been studied by Matthäus (1977, 1979). His Fourier analysis shows that the one-year harmonics make the main contribution to the temperature variability (see Fig. 3.4). The amplitudes of different modes are largest at the surface, have an intermediate minimum in the »winter water» range and decrease towards the bottom. In the southernmost basins there is a secondary maximum at approximately the depth of the halocline.

For modelling purposes it is necessary to know the range of variation of the surface temperature $T_s$, the temperature of the primary halocline $T_r$ (which is the lowest temperature observed during the summer season) and the bottom temperature $T_H$. Fig. 3.5 shows the annual course of these temperatures for different basins. As can be seen, during the winter the two uppermost temperatures coincide. They begin to deviate in April when the maximum density temperature is reached. During the summer the primary halocline temperature changes very little; for modelling purposes it can be taken as constant. In October and November, when convective mixing reaches the halocline depth, the halocline temperature first increases, then decreases as the cooling proceeds. The bottom temperature $T_H$ is, as stated earlier, dependent on the water inflow from the North Sea. Due to this, the temperature variations in the western basins are greater than those in the Gotland Deep.

The spatial variation of the surface temperature (the Bothnian Sea and the Bothnian Bay excluded) is trivial, as can be seen in Fig. 3.6, which shows both the long-term average surface temperature (map A) and the amplitude of the annual variation (map B). Deeper in the sea the horizontal temperature structure is not as uniform as in the well-mixed layer. Figs. 3.7 and 3.8 show the long-term means (A) and the yearly amplitudes (B) for the cold interfacial layer and the bottom layer, respectively. In both layers the means decrease towards the north. The amplitudes are highest around Bornholm, reaching a minimum in the central parts. The cold interface temperature amplitudes increase somewhat towards the coastal regions, possibly due to ambient baroclinicity.

According to the thermal structure, the Baltic Sea can be divided into the following regions:

a) the Baltic Proper (coastal areas excluded)
b) the gulfs and coastal areas.

The regime of the Baltic Proper depends on the active region of the Danish Sounds. Here the warm bottom layer is formed and convective mixing is stopped by the halocline. In the gulfs and coastal areas the heat regime is formed mainly by the interaction between the atmosphere and the sea, and during the winter season the whole water column is mixed, unless the total depth is too great. The broken line in Figs. 3.6 to 3.8 shows the putative boundary between the two different types of regions.

Due to their horizontal uniformity, the dependence of the temperatures of the
Fig. 3.6. Long term mean of surface temperature (A) and its yearly amplitude (B) on the basis of Matthäus (1977) data.

Fig. 3.7. Long term mean of halocline temperature (A) and its yearly amplitude (B) on the basis of Matthäus (1977) data.

Fig. 3.8. Long term mean of bottom temperature (A) and its yearly amplitude (B) on the basis of Matthäus (1977) data.
uppermost layer may well be described by relatively simple models, which are the subject of the next chapter.

3.3 The development of the sea surface temperature and the seasonal thermocline

During the winter months the Baltic Sea is well mixed down to the permanent halocline. When the net incoming radiation exceeds the sum of heat losses due to long-wave radiation, heat conduction and evaporation, the formation of the seasonal thermocline begins. At this time of the year the winds over the Baltic are usually weak, so that the mixed layer is very thin or completely lacking and the temperature gradient in the surface is strong. This gradient is partly destroyed by mechanical mixing by the wind. In the mixing process, the sea surface temperature decreases because colder water from below is carried into the surface layer. During the heating season secondary thermoclines are formed. Storms can cause enough turbulence to destroy these secondary buoyancy gradients and, if the wind force is strong enough, the thermocline is eroded downward.

The heat balance of the sea surface becomes negative in early August. After this, the mechanical mixing due to wind is enhanced by convection, which decreases the temperature difference between the surface layer and the water below. In October, the sea surface temperature (SST) has fallen close to 7 °C and in late November or December it approaches the value of maximum density of the surface layer, i.e. about 2.5 °C. By this time, the depth of the thermocline has reached the depth of the halocline. In the Gulf of Bothnia the halocline is usually very weak, so that the depth of the mixed layer can increase considerably during the winter season. In the Baltic Proper, once the halocline depth is reached, the density gradient is steep enough to prevent further deepening.

A very large number of papers have been devoted to the modelling of the seasonal thermocline during past decades. A comprehensive discussion of the most used one-dimensional model is presented by, for example, NIILER and KRAUS (1977), and another valuable approach is used by ZILITINKEVICH, CHALIKOV and RESNYANSKY (1979). The references in the latter paper alone comprise more than 100 titles. In order to clarify the model results given below, the derivation of the main equations and their parametrization will be briefly presented here.

Thermocline models usually neglect horizontal advection, which limits their applicability to regions where no permanent current system is found. In the Baltic Sea, such regions occur mainly in the central parts, but the Gulf of Finland, for example, belongs almost entirely to regions where 1D models are less suitable. The same is valid for coastal regions several internal Rossby radii from the coastline.

The governing equations for thermocline models are derived from equations of momentum, heat and turbulent energy. In general, linearized equations of momentum are used in the following form:

\[
\frac{\partial u}{\partial t} + 2\Omega \times u = \frac{\partial}{\partial z} \tau ,
\]

where \( u \) is horizontal velocity with components \((u, v)\)

\( \Omega \) angular velocity of earth rotation

\( \tau \) turbulent stress in the interior \((\tau^u, \tau^v, \tau^w)\)

The corresponding equation for temperature change can be presented as follows:
\[
\frac{\partial T}{\partial t} + \frac{\partial}{\partial z} \left( \overline{w T'} \right) = -\frac{1}{\rho_0 c_p} \frac{\partial}{\partial z} \frac{1}{\alpha_T g^2 w'T'} \quad \text{(3.2)}
\]

where \( T \) is temperature
\( \overline{I_wT} \) penetrative component of solar radiation
\( \overline{w'T'} \) heat flux due to turbulence
\( c_p \) specific heat of water
\( \rho_0 \) reference density (constant)

In equation (3.2), molecular heat conduction has been omitted, and the possible influence of mean vertical motions as well. In the time scales dealt with here these are negligible. The equation of turbulent kinetic energy reads:

\[
\frac{\partial e}{\partial t} + \frac{\partial}{\partial z} \left( \frac{\overline{w'u'u'} + \overline{w' e'}}{\rho_0} \right) + \tau \frac{\partial u}{\partial z} = \alpha_T g \overline{w'T'} - \epsilon \quad \text{(3.3)}
\]

where \( e \) is turbulent kinetic energy \( \left( \frac{1}{2} \overline{u'u'} \right) \)
\( \overline{w'p'} \) turbulent pressure flux
\( \alpha_T \) coefficient of heat expansion
\( \epsilon \) dissipation of energy

Many simplifications and assumptions are necessary when the above equations are applied to thermocline evolution modelling. Roughly speaking, the available models can be divided into two groups depending on the degree of vertical resolution. The first group consists of slablike models, in which the whole surface layer is considered as a single entity. The parametrization required for these models is far simpler than for the second group of models, which take into account the vertical variability of all the terms included. In order to have vertical resolution in the models, the second-order turbulent fluxes such as \( w'T' \) must be parametrized using higher order closure schemes, which are difficult to verify. Therefore, the discussion here is restricted to the first category of models.

From equations (3.1) to (3.3), simpler integral forms are obtained by integrating the equations through the mixed layer down to the seasonal thermocline at depth \( h \). The procedure is essentially the same as that of Kraus and Turner (1967), Denman and Miyake (1973), Nilier (1975) and Nilier and Kraus (1977). In addition to these integrated equations, the necessary equation for potential energy is obtained by integrating equation (3.2) twice: first from the surface to depth \( z \), after that from the surface to the boundary of the mixed layer, \( h \). As a result of these integrations, the following equations emerge:

\[
\frac{\partial (hu)}{\partial t} + 2\Omega x (hu) = \tau_0 \quad \text{(3.4)}
\]

\[
h \frac{\partial T_s}{\partial t} = \overline{w'T'}_0 - \overline{w'T'}_h + \frac{1}{c_p \rho_0} \left( \frac{1}{1 - e^{-\gamma h}} \right) \quad \text{(3.5)}
\]

\[
h \frac{\partial e'}{\partial t} = \left( \frac{\overline{w'p'}}{\rho_0} + \overline{w' e'} \right)_0 - \left( \frac{\overline{w'p'}}{\rho_0} + \overline{w' e'} \right)_h + \int_0^h \left( \frac{\partial u}{\partial z} + \alpha_T g \overline{w'T'} + \epsilon \right) dz \quad \text{(3.6)}
\]

\[
h^2 \frac{\partial T_s}{\partial t} = h \overline{w'T'}_0 - \int_0^h \overline{w'T'} \, dz + \frac{hI_o}{c_p \rho_0} \left[ 1 + \frac{1}{\gamma h} - \frac{1}{\gamma h} e^{-\gamma h} \right] \quad \text{(3.7)}
\]

In deriving these equations, we have assumed that penetrative solar radiation has an exponential profile, that both velocity and eddy kinetic energy are independent of depth,
and that the bottom stress $\tau_h = u \frac{\partial h}{\partial t}$. Due to the mixing, the surface layer temperature is taken as homogeneous.

The assumption of a zero velocity gradient may be acceptable in the major part of the mixed layer. At the boundary layer close to the thermocline, however, strong velocity shears occur. Denman and Miyake (1973), Niler (1975) and Niler and Kraus (1977) estimated the contribution of this shear layer at the limit $\delta h \to 0$. As a result, the terms including the shear production of turbulence (in equation 3.6 the first term of the integral on the right-hand side) appear as a boundary condition. Equations (3.4)
to (3.7) contain several unknown functions and their parametrization with known properties of the surface layer determines the quality of the model. A review of different parametrization schemes is presented in NILER and KRAUS (1977) and in tabular form in the article by ZUJTINKEVICH, CHALIKOV and RESNIANSKY (1979). Chiefly depending on the parametrization of the terms of r.h.s. integral in eq. 3.6, and the turbulent fluxes of kinetic energy at the surface and thermocline, models with slightly differing properties can be found. Short-term modelling of the thermocline erosion can be done if heat exchange with the atmosphere is disregarded, i.e. only the mechanical influence of wind stress and internal processes in the mixed layer are considered. For seasonal models the annual cycle of surface fluxes of heat must be retained. In these models, the influence of convection during the autumn must be considered as well.

As an additional closure approximation, POLLARD, RHINES and THOMPSON (1973) suggested that during active thermocline erosion the flow is marginally stable, i.e. Richardson number $Ri = N^2/u^2 = 1$. As pointed out by NILER and KRAUS (1977), this represents a very special case, which is not valid for long-time scales. The influence of surface layer currents on thermocline erosion in the Baltic Sea has been clearly demonstrated by KRAUSS (1981). In his experiment the wind forcing due to passage of a cyclone generated a strong inertial current in the surface layer, which mixed this layer mechanically and eroded the thermocline layer by more than 10 metres. During this mixing period, the thermocline layer became diffuse but turned sharp again after a few inertial cycles. Measurements with moored thermistor chains have revealed similar events in other regions of the Baltic sea as well, e.g. in the Bothnian Sea in 1976. Fig. 3.9 shows the case analysed by KRAUSS and Fig. 3.10 the evolution of the thermocline in the Bothnian Sea during a strong wind. In the latter case, the neighbourhood of the coast must be taken into account, so that it cannot be directly compared with the former. Several modellers have introduced inertial oscillations into the initial stages of deepening. These movement are ambient and contain some 20 to 30 % of the total kinetic energy in the surface layer (e.g. ALENIUS and MÅLKKI, 1978). They have group velocity close to zero and consequently do not transfer energy from the site of generation. They therefore fit well into 1D models also.

The parametrization of the unknowns in eqs. (3.5) to (3.7) presupposes knowledge of the behaviour of the variables at the upper and lower boundaries. A detailed discussion of the various ways used is found in NILER and KRAUS (1977). Only some of the main results will be summarized here. The equations contain turbulent heat fluxes $w'T'$ at the upper and lower boundaries and changes of potential energy due to these fluxes. At the upper boundary, the flux is due to radiation balance, convective heat transport and evaporation. Thus the surface flux can be presented as the following sum:

$$w'T_0 = (R_0 + q_T + Lq_e - l_0) \frac{1}{\rho_0 c_p} = \frac{q_0 - l_0}{\rho_0 c_p} .$$

3.8

Correspondingly, during the entrainment due to cold water intrusion into the mixed layer the flux at the lower boundary is:

$$w'T_h = \Delta T \frac{\partial h}{\partial t} ,$$

3.9

where $\Delta T$ is temperature change through the thermocline. The models assume that the heat flux below the thermocline is due only to the penetrative component of the short-wave radiation $I$. The Baltic Sea is optically a better absorber of energy than ocean waters, so that practically all the solar radiation is absorbed in the surface layer alone.
As can be seen, e.g. in the paper by Töyräinen (1977), the bottom layer temperature increases slightly during the summer season. The possible mechanisms of this temperature increase were discussed in Section 3.2.

The integral of $w'T'$ over the mixed layer, when multiplied by $\alpha_T \rho_g$, describes the potential energy changes due to turbulent fluxes from the boundaries to the interior of the mixed layer in eqs. (3.6) and (3.7). The surface fluxes involving turbulent energy and pressure fluctuations are induced by the wind forcing and are taken as proportional to the wind energy:

$$\left( \frac{w'p'}{\rho_0} + w'e' \right) = -m_1 u^3,$$

where $u_*$ is friction velocity of the wind ($= \sqrt{\tau_0/\rho_0}$)

$m_1$ empirical constant.

Correspondingly, in the bottom mixed layer the energy flux is seen as deepening of the thermocline. In order to be able to parametrize this, one must assume that there is no turbulence below the thermocline and, furthermore, that there is no radiation of energy into the internal waves. When the stratification is stable, the first assumption may be valid, but, in such conditions, the internal waves are strongest in the thermocline layer and part of the energy is propagated downwards. In the Baltic Sea, the isosaline and usually thermally homogeneous layer below the thermocline tends to keep the thermocline sharp. Thus the internal waves can move only within a fairly shallow channel. It may be assumed that they transfer energy only in a horizontal direction, with negligible loss of energy to the interior. With this assumption the bottom boundary flux for energy can be parametrized as:

$$\left( \frac{w'p'}{\rho_0} + w'e' \right) = e \frac{\partial h}{\partial t}.$$

The turbulence production due to Reynolds stresses within the mixed layer has been assumed to be zero, since we assumed the velocity field to be uniform. At the bottom of this layer there is a strong shear of mean velocity, which erodes the thermocline correspondingly. If velocity is assumed to have a linear profile throughout the layer, the limiting contribution from this layer becomes

$$\left( \frac{w'p'}{\rho_0} + w'e' \right) = \frac{1}{3} \frac{\partial u}{\partial z} + \frac{1}{3} C_D u^3,$$

where the last term is due to drag at the boundary.

Finally, the overall dissipation of turbulent energy in the mixed layer can be considered to be the sum of three separate terms.

A part of the wind-induced energy is dissipated within the surface layer; this is taken as proportional to the forcing. During the erosion of the thermocline, some energy is dissipated, which can be taken as proportional to the erosion velocity. A part of the dissipation is related to heat convection. Altogether, the overall dissipation is thus:

$$\int_0^h \epsilon dz = (m_1 - m) u^3 + (1 - s) \frac{1}{2} u^2 \frac{\partial h}{\partial t} + (1 - n) \frac{1}{4} (q_0 + |q_0|).$$

The equation contains three new constants ($m, n, s$); the choice of these constants is discussed in Nilger and Kraus (1977).
Using the parametrization schemes presented in eqs. (3.8) to (3.13), the original equations (3.5) to (3.7) can be written as follows:

\[
h \frac{\partial T}{\partial t} = \frac{q_0 - I_0 e^{-\gamma h}}{(c_p \rho_0)} - \Delta T \frac{\partial h}{\partial t}
\]

\[
\left( e + \frac{u^2}{2} \right) \frac{\partial h}{\partial t} + \frac{C_p}{3} u^3 + m u^3 = \alpha_T g \int_0^h w T' dz + \frac{h}{4} (q_o + q_0)| 3.15
\]

\[
\frac{h^2}{2} \frac{\partial T}{} = \frac{h}{c_p \rho_0} \left( q_0 - \frac{I_0 e^{-\gamma h}}{\gamma h} \right) - \int_0^h w T' \, dz .
\]

Together with the equations of motion (3.4), we have five equations for six unknowns: velocity (u, v), temperature T, thermocline depth h, potential energy change \( \int w' T' \), and the turbulent energy flux at the bottom, edh/dt. The other variables in these equations can be measured with suitable accuracy. According to GORDEYEYEV and KAGAN (1976), the last-mentioned unknown edh/dt = 0. Thus the system is closed.
In different cases different treatment of these equations is required. The entrainment process is irreversible, so that in cases in which the calculations show \( \frac{dh}{dt} < 0 \), the entrainment velocity must be set at equal to zero. The thermocline depth can then be obtained more directly by eliminating the integral of the heat fluxes from the equations.

The numerical solution of the above equations in the form presented by Kraus and Turner (1967) was applied to the northern Baltic by Tyrväinen (1978). Her results, together with another model, described in the next section, are presented in Fig. 3.11. The numerical experiment with the Kraus-Turner model shows that the differences between experimental and calculated results are largest in the autumn, when the variation of the bottom layer temperature \( T_b \) is great.

3.4 A seasonal thermocline model with determination of bottom layer temperature

The vertical structure of temperature in the Baltic Sea can be described by the following equations, into which a dimensionless function and dimensionless depth have been introduced:

\[
\Theta_T = \frac{T_s - T(z)}{T_s - T_H}; \quad \eta = \frac{z - h}{H - h},
\]

where \( T_s \) is surface temperature
\( T_H \) bottom temperature
\( h \) thickness of thermally homogeneous layer
\( H \) depth of the sea.

Assuming that \( \Theta_T \) is a function of \( \eta \) only (Kitaigorodskii and Mironovskii, 1971), we may determine \( \Theta_T \) as a fourth order polynomial with the following boundary conditions:

\[
\eta = 0 \quad \Theta_T = 0
\]
\[
\eta = 1 \quad \Theta_T = 1 \quad \Theta_T' = 0
\]

\[
\int_0^1 \Theta_T d\eta = \kappa_T \quad \int_0^1 \eta \Theta_T d\eta d\eta' = \bar{\kappa}_T.
\]

In these equations, the functions \( \kappa_T \) and \( \bar{\kappa}_T \) are to be determined from the empirical data as follows:

\[
\kappa_T = (T_s - \int_0^1 T d\eta)/(T_s - T_H); \quad \bar{\kappa}_T = (T_s - \int_0^1 \eta T d\eta d\eta')/(T_s - T_H).
\]

In considering the low frequency variations, the small-scale variability of these functions has to be filtered out by averaging with respect to time.

Fig. 3.12 shows the variability of \( \kappa_T \) and \( t^{-1} \int_0^T \kappa_T dt' \). The data were obtained by bathythermograph measurements at 15-minute intervals. The function \( \kappa_T \) varies between 0.5 and 0.9, but after one inertial period (some 14 hours) \( t^{-1} \int_0^T \kappa_T dt \) and \( t^{-1} \int_0^{t'} \kappa_T dt \) remain fairly stable with the values \( \kappa_T = 0.75 \) and \( \bar{\kappa}_T = 0.3 \), respectively. The data obtained during another period when the mixed layer decreased (\( \frac{dh}{dt} < 0 \)) showed that the time average stabilizes close to a value of \( \kappa_T = 0.6 \). Thus, with these different values of \( \kappa_T = 0.75 \), \( \kappa_T = 0.6 \) the similarity function \( \Theta_T \) has the following forms:
The dependence of $\Theta_T$ on $\eta$ according to the experimental data of Matthäus (1977) is presented in Fig. 3.13. The figure shows that the observations form a cluster between the two curves, as presented by equations (3.19).

On the basis of the above, the large-scale temperature structure can be expressed by the following equations:
\[ T = T_s(t) \quad 0 \leq z \leq h \]

\[ T = T_s(t) + \Theta_T(\eta) [T_H(t) - T_s(t)] \quad h \leq z \leq H . \]

The change of temperature in time is, according to eq. (3.2)

\[ \frac{dT}{dt} + \frac{dq}{dz} = 0 , \]

where \( q = \frac{w^0}{T^0} + \frac{1}{c_p \rho_0} \).

In the homogeneous layer \((z \leq h)\) the equation is simply

\[ \frac{dT_s}{dt} + \frac{dq}{dz} = 0 , \]

where \( T_s \) is surface temperature,

while in the layer \((h, H)\) the eq. (3.21) has the form

\[ (1 - \Theta_T) \frac{dT_s}{dt} + \Theta_T \frac{dT_H}{dt} - (\eta - 1) \frac{d\Theta_T}{d\eta} \frac{\partial(T_s - T_H)}{\partial t} \frac{\partial h}{\partial t} + \frac{1}{H - h} \frac{\partial q}{\partial \eta} = 0 . \] 3.23

Integrating (3.22) over the mixed layer, we obtain

\[ h \frac{dT_s}{dt} = q_s - J_0 - q_h , \]

where \( q_s = \frac{q_0}{c_p \rho_0} ; \quad J_0 = l_0 e^{-\gamma h} / (c_p \rho_0) ; \quad q_h = \frac{w^0}{T^0} \).

Integrating the eq. (3.22) twice, first from 0 to \( z \) and then from 0 to \( h \), one obtains

\[ \frac{h^2}{2} \frac{dT_s}{dt} = h q_s - J_0 / \gamma - \int_0^h \frac{w^0}{T^0} dz . \] 3.25

Similarly, integrating eq. (3.23) with respect to \( \eta \) between the limits 0 and 1 and using the boundary condition \( q_H = 0 \), we obtain

\[ (1 - \kappa_T) \frac{dT_s}{dt} + \kappa_T \frac{dT_H}{dt} + \kappa_T (\frac{T_s - T_H}{H - h}) \frac{\partial h}{\partial t} \frac{q_h}{H - h} = 0 . \] 3.26

Double integration of (3.23), first from 0 to \( \eta \), then from 0 to 1, yields

\[ \left[ \left( \frac{1}{2} - \kappa_T \right) \frac{dT_s}{dt} + \kappa_T \frac{dT_H}{dt} + 2 \kappa_T \left( \frac{T_s - T_H}{H - h} \right) \frac{\partial h}{\partial t} \frac{q_h}{H - h} \right] \frac{q_h}{H - h} = 0 . \] 3.27

where \( \nu = (q_h - \int_0^1 q d\eta) / q_h . \)
The system of equations (3.24) to (3.27) may be reorganized for determining the unknowns $T_s$, $T_H$, $h$ and $q_h$, as follows

\[
\frac{\partial T_s}{\partial t} = \frac{q_h - q_h - J_0}{h} \quad \text{3.28}
\]

\[
\frac{\partial T_H}{\partial t} = C_1 \frac{q_h}{H-h} - C_2 \frac{q_h - q_h - J_0}{h} \quad \text{3.29}
\]

\[
\frac{\partial h}{\partial t} = C_3 \frac{q_h}{T_s - T_H} - C_4 \left( \frac{H-h}{T_s - T_H} \right) \frac{q_h - q_h - J_0}{h} \quad \text{3.30}
\]

\[
q_h = \frac{2}{h} \int_{0}^{h} w' T'_w \, dz - q_h + J_0 \left( \frac{2}{\gamma h} - 1 \right). \quad \text{3.31}
\]

where $C_1 = (2\kappa_T - \kappa_T \nu) / (\kappa_T \kappa_T)$

$C_2 = (2\kappa_T - \kappa_T \kappa_T - 1/2 \kappa_T) / (\kappa_T \kappa_T)$

$C_3 = (\kappa_T \nu - \kappa_T) / (\kappa_T \kappa_T)$

$C_4 = (1/2 \kappa_T - \kappa_T) / (\kappa_T \kappa_T)$

The system of equations (3.28) to (3.31) is not closed, since the integrals $\int_{0}^{h} w' T'_w \, dz$ and $\nu = (q_h - \int_{0}^{h} q_d \eta) / q_h$ are still unknown. The first integral has been discussed in section 3.4. Using the equation (3.15) with conditions $n = 1$, $C_D = 0$ and $\varepsilon = 0$, the integral $\int_{0}^{h} w' T'_w \, dz$ can be determined as follows:

\[
\alpha_T \int_{0}^{h} w' T'_w \, dz = \mu u^3 + \frac{s}{2} \frac{\partial h}{\partial t}. \quad \text{3.32}
\]

According to Kato and Phillips (1969), $m = 1.25$. Later on, Garnich and Kitaigorodski (1978) and Kantha (1977) showed that $m$ is not constant. According to Resnyansky (1975), $m$ can be expressed as follows:

\[
m = a_1 - \frac{h}{\mu \lambda} (a_2 + a_3), \quad \text{3.33}
\]

where $a_1, a_2, a_3$ are empirical constants

$\lambda = u^2 / 2 \Omega$ is Ekman length

$\mu = \lambda / L$ Monin-Kazansky stratification parameter

$L = u^3 / (\alpha g q_s)$ Monin-Obuchov length.

In order to determine the second integral $\nu$, equations (3.26) and (3.27) can be written in the following form:

\[
\bar{\beta} \left[ 1 - \kappa_T (1 - \bar{n} - \bar{p}) \right] = 1 \quad \text{3.34}
\]

\[
\bar{\beta} \left[ \frac{1}{2} - \kappa_T (1 - n - 2p) \right] = \nu, \quad \text{3.35}
\]
where \( \bar{\beta} = \frac{H - h}{q_h} \frac{\partial T_s}{\partial t} \)
\( \bar{n} = \left( \frac{\partial T_H}{\partial t} \right) / \left( \frac{\partial T_s}{\partial t} \right) \)
\( \bar{\rho} = \frac{T_s - T_H}{H - h} \frac{\partial h}{\partial t} / \frac{\partial T_s}{\partial t} \)

From equations (3.34) and (3.35) we obtain

\[
\nu = \left[ \frac{1}{2} - \kappa_{T1} (1 - \bar{n} - 2\bar{\rho}) \right] / \left[ 1 - \kappa_{T1} (1 - \bar{n} - \bar{\rho}) \right]. 
\]

The treatment of empirical data, as presented in Fig. 3.14 shows that the variation of \( \nu \) is considerable, ranging between the values \( 0.2 < \nu < 1.1 \). After approximately one inertial cycle however, the time average of \( \nu \) seems to stabilize close to the value \( \nu = 0.6 \). Determination of this variable is not necessary, when the thermocline is not deepening, since in this case \( q_h = 0 \).

Consequently, the system of equations (3.28) to (3.31) is closed, and taking for parameters \( \kappa \) and \( \nu \) the values

\[
\kappa = \{ 0.75, 0.6 \}; \quad \nu = \{ 0.6 \},
\]

can be expressed in the form

\[
\frac{\partial T_s}{\partial t} = \frac{q_s - q_h}{h} \]

\[
\frac{\partial T_h}{\partial t} = \left\{ \begin{array}{c}
0 \\
1/6 \end{array} \right\} \frac{q_s - q_h}{h} + \left\{ 2/3 \right\} \frac{q_h}{H - h} \]

\[
\frac{\partial h}{\partial t} = \frac{H - h}{T_h - T_s} \left[ \left\{ 1/3 \right\} \frac{q_s - q_h}{h} + \left\{ 2/3 \right\} \frac{q_h}{H - h} \right].
\]
Here \( J_0 = 0 \)
\[
q_h = -\frac{a_0}{h} u_w^3 - b_0 q_s \geq 0 .
\]

If \( s = 0 \), then \( a_0 = 2m/\alpha_T g \) and \( b_0 = 1 \), otherwise
\[
a_0 = 6m h^2 \left( \frac{T_s - T_h}{b_1 - b_2} \right)
\]
\[
b_0 = \frac{b_1 + b_3}{b_1 - b_2}
\]
\[
b_1 = 3\alpha_T g h^2 (T_s - T_h)
\]
\[
b_2 = su \cdot u H
\]
\[
b_3 = su \cdot u (H - h) .
\]

In the last case, the equation of motion (3.4) will be added to equations (3.37) to (3.39).

The above equations were applied to the entrance to the Gulf of Finland using as atmospheric input the same data as Tyrväinen (1978). The results of the model run for one season are presented in Fig. 3.11 together with the KT-model results of Tynär (1978). Comparison of the two different models shows that during the spring and summer seasons, when the variability of the bottom temperature \( T_H \) is insignificant, both models result in approximately the same time evolution of both the surface layer temperature and thermocline depth. During the autumn season, however, when the bottom layer temperature increases, the similarity model based on equations (3.37) to (3.39) gives values more closely related to the observed temperature and thermocline values than those calculated by the Kraus-Turner model.

In the Baltic Proper, depth \( H \) coincides with the depth of the primary halocline \( h_s \), since thermocline erosion seldom exceeds this depth due to a considerable density jump in the halocline. Below the primary halocline and above the secondary halocline (which lies approximately at a depth of some 125 to 130 m), the self-similarity function for temperature is different. It coincides with the corresponding salinity function. Four regions can thus be distinguished in the overall stratification of temperature: a homogeneous upper layer (present during the summer and autumn), a thermocline with a considerable density change represented by the similarity function \( \Theta_T \), a layer between the primary and secondary haloclines and a homogeneous bottom layer below the secondary halocline. The self-similarity functions for these layers for the Gotland Deep are presented in Fig. 3.15.

Instead of using temperature, one can treat the erosion of the thermocline using equations for turbulent buoyancy fluxes as follows:

\[
\frac{\partial b}{\partial t} + u \cdot \nabla b - \mu \nabla^2 b = -\frac{\partial B}{\partial z}, \tag{3.40}
\]

where
\[
b = -g \left( \frac{\rho - \rho_0}{\rho_0} \right) = g[\alpha_T (T - T_0) - \beta_s (S - S_0)]
\]
\[
B = w' b' + \alpha_T g l / (c_p \rho_0)
\]
\[
B_{z=0} = B_s = \frac{g}{\rho_0} \left[ \frac{\alpha_T}{c_p} (q_0 - 1_0) - \beta_s (q_e - q_s) \right]
\]
Fig. 3.15. Schematic presentation of self-similarity functions for Gotland Deep.

\[ P_0 \] is precipitation rate

\[ S_s \] surface salinity

\[ u \] velocity with components \((u, v, w)\) along the axes \(x, y\) and \(z\), respectively

\[ \nabla_h^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \]

\( \mu \) is coefficient of horizontal turbulent diffusion.

Similarly to the treatment of equations (3.26) and (3.27), integration and double integration of equation (3.40) yields a system of equations for sea stratified in four layers (for the case of thermocline erosion):

\[
\frac{\partial b_s}{\partial t} + u_1 \cdot \nabla b_s - \mu \nabla_h^2 b_s = \frac{B_s - B_h}{h_T} \tag{3.41}
\]

\[
\frac{\partial b_h}{\partial t} + u_2 \cdot \nabla b_h - \mu \nabla_h^2 b_h = \frac{2}{3} \frac{B_h - B_H}{\delta_t} \tag{3.42}
\]
\[
\frac{\partial b_H}{\partial t} + u_4 \cdot \nabla b_H - \mu \nabla^2 b_H = \frac{B_D}{\delta_3}
\]
3.43

\[
\frac{\partial \delta_1}{\partial t} + \nabla \cdot (u_2 \delta_1) - \mu \nabla^2 \delta_1 = F_1
\]
3.44

\[
\frac{\partial \delta_2}{\partial t} + \nabla \cdot (u_3 \delta_2) - \mu \nabla^2 \delta_2 = F_2
\]
3.45

\[
B_h = \frac{2m}{h_T} u_3^3 - B_s
\]
3.46

\[
B_H = -c_1 \frac{\delta_2}{\delta_1} b_h + c_2 B_D + c_3 \delta_2 (u_3 - u_4) \nabla b_H + c_4 \delta_2 (u_2 - u_1) \nabla b_H
\]
3.47

\[
B_D = \frac{2m_1}{\delta_3} v_*^3
\]
3.48

In these equations,

- \(b_s\) is surface buoyancy
- \(b_h\) is buoyancy at the cold interface
- \(b_H\) is buoyancy in the bottom layer below the secondary halocline
- \(u_i\) is velocity vector in layer \(i\) (horizontal)
- \(\delta_i = h_s - h_T\)
- \(\delta_2 = H - h_s\)
- \(\delta_3 = D - H\)
- \(D\) is depth of the sea

\[
F_1 = \frac{\delta_1}{b_s - b_h} \left[ \frac{2}{3} \frac{B_h - B_H}{\delta_1} - \frac{1}{3} \left( \frac{B_s - B_h}{h_T} + (u_2 - u_1) \cdot \nabla b_s \right) \right]
\]

\[
F_2 = \frac{\delta_2}{b_h - b_H} \left[ \frac{5}{3} \frac{B_H - B_D}{\delta_2} - \frac{4}{9} \frac{B_h - B_H}{\delta_1} - \frac{B_D}{\delta_3} + (u_4 - u_3) \cdot \nabla b_H + \frac{2}{3} (u_2 - u_3) \cdot \nabla b_h \right]
\]

\(u_*\) is friction velocity of the wind
\(v_*\) is friction velocity near the bottom

\[
c_1 = \frac{0.2}{c_0^2} ; \quad c_2 = \frac{2.6 - 3\nu_2}{c_0} ; \quad c_3 = \frac{0.6}{c_0}
\]

\[
c_4 = \frac{0.1}{c_0} ; \quad c_0 = 2 - 3\nu_2 + \frac{0.2 \delta_2}{3\delta_1}
\]

\[
\nu_2 = \left( B_H - \int_0^1 B_d \eta_3 \right)/(B_H - B_D) \approx 0.6
\]

During the winter, when the thermocline and halocline depths coincide, i.e. \(h_s = h_T\), the buoyancy structure may be calculated without equations (3.43) and (3.45).
Inserting into equations (3.41) to (3.48) the following buoyancy structure:

\[
\begin{align*}
    & b = b_s & 0 \leq z \leq h_T \\
    & b = b_s + \theta_1(\eta)(b_h - b_s) & h_T \leq z \leq h_s \\
    & b = b_h + \theta_2(\eta)(b_H - b_h) & h_s \leq z \leq H \\
    & b = b_H & H \leq z \leq D,
\end{align*}
\]

we find, instead of equations (3.40), the following equation for buoyancy:

\[
\frac{\partial b}{\partial t} + u \cdot \nabla b - \mu \nabla^2 b = F_B,
\]

where

\[
F_B = \begin{cases}
    \frac{B_s - B_h}{h_T} & 0 \leq z \leq h_T \\
    (1 - \theta_1) \left[ \frac{B_s - B_h}{h_T} + (u_2 - u_1) \cdot \nabla b_s \right] + \frac{2}{3} \theta_1 \frac{B_h - B_H}{\delta_1} \\
    + (1 - \eta) \frac{d\theta_1}{d\eta} F_1 & h_T \leq z \leq h_s \\
    (1 - \theta_2) \left[ \frac{B_h - B_H}{h_T} + (u_3 - u_2) \cdot \nabla b_h \right] + \theta_2 \left[ \frac{B_D}{\delta_3} - (u_4 - u_3) \cdot \nabla b_H \right] \\
    + (1 - \eta) \frac{d\theta_2}{d\eta} F_2 & h_s \leq z \leq H \\
    \frac{B_D}{\delta_3} & H \leq z \leq D.
\end{cases}
\]

Correspondingly, the temperature structure can be expressed as follows:

\[
\frac{\partial T}{\partial t} + u \cdot \nabla T - \mu \nabla^2 T = F_T,
\]

where in the upper layer between 0 and \( h_T \)

\[
F_T = \frac{q_s - q_H}{h_T}.
\]

In the lower layer the forcing function \( F_T \) has to be determined using the entrainment condition

\[
B = w_e \Delta b \\
q = w_e \Delta T,
\]

where \( \Delta b \) and \( \Delta T \) are the buoyancy and temperature jumps, respectively, and \( w_e \) is the entrainment velocity. They are interrelated: the use of equations (3.52) yields
Assuming horizontal homogeneity, the numerical solution for the above equations can be found. The numerical result for the Gotland Deep is presented in Fig. 3.16 together with experimental data according to Matthäus (1977). The numerical results presented in Fig. 3.16a seem to agree qualitatively with the observed time evolution.

In the case of a nonhomogeneous sea, the solution of equation (3.51) requires a set of equations which describe the horizontal circulation.
4 CIRCULATION

4.1 General circumstances and basic equations

A vast number of systematic and detailed hydrographical observations have been carried out in the Baltic Sea during past decades. However, due to lack of simultaneous current velocity measurements, it is not possible to obtain a correct picture of the circulation. To organize comprehensive current observations is a very complicated and expensive operation, so that at present, and presumably also in the future, the main instrument for studies of the dynamics of the Baltic Sea will be numerical modelling.

Currents are generated here mainly by forcing in the boundary layers, due to wind stress in the sea surface, and other atmospheric forcing, due to fresh water inflow at the river mouths and saline water influxes in the Danish Sounds. The internal structure of buoyancy together with topographic effects determines the variability of the circulation in space and time. In the oceans the buoyancy structure is mainly determined by variations in temperature, but in the Baltic Sea a substantial role in the currents is played by the salinity structure. The vertical distribution of temperature has an important effect during the summer season; in particular, the depth of the thermocline determines the depth of drift currents.

Storm surges have received considerable attention in the Baltic Sea due to the economic losses involved. They are of short duration and can be well described with simple barotropic models (Uusitalo, 1960, Henning, 1962, Laska, 1966). Due to the strong stability conditions, the buoyancy structure may, however, play the main role in the circulation processes on longer time scales (Kowalik, 1972, Sarkisyan, 1977).

In order to describe the circulation, an initial system of equations for large-scale currents is used. A detailed analysis of these equations may be found, for example, in the monographs of Pedlosky (1979) and Kamenkovich (1977). Starting from the basic equations for momentum transfer, conservation of mass, transfer of heat and diffusion of salts, averaged equations are derived in order to describe average hydrographical characteristics. Many assumptions are made, some of them common, some specific for the Baltic Sea.

Briefly, the assumptions adopted as follows:
1. The equation of transfer of entropy is replaced by the equation of heat conduction in fluid;
2. Molecular processes are completely disregarded;
3. Since the extent of the Baltic Sea is limited, the effect of the curvature of the earth is disregarded (f-plane);
4. The horizontal components of the earth rotation vector are disregarded;
5. The vertical components of the momentum equation is replaced by the equation of hydrostatics;
6. Except in the equation of hydrostatics and terms related to buoyancy, the density is replaced by mean density (Boussinesq approximation).

With these approximations, the equations describing the thermohaline processes in the Baltic Sea are as follows:
\[
\frac{\partial u}{\partial t} + \nabla \cdot (uu) - fu = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} - \nabla \cdot (u'u) \quad 4.1
\]

\[
\frac{\partial v}{\partial t} + \nabla \cdot (uv) + fu = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} - \nabla \cdot (u'v') \quad 4.2
\]

\[
O = -\frac{1}{\rho} \frac{\partial p}{\partial z} + g \quad 4.3
\]

\[
\nabla \cdot u = 0 \quad 4.4
\]

\[
\frac{\partial T}{\partial t} + \nabla \cdot (uT) = -\frac{1}{\rho_0 c_p} \frac{\partial I}{\partial z} - \nabla \cdot (u'T') \quad 4.5
\]

\[
\frac{\partial S}{\partial t} + \nabla \cdot (uS) = -\nabla \cdot (u'S') \quad 4.6
\]

\[
\rho = \frac{(p_0 + p_1)a_0}{p_2 + a_0(p_0 + p_1)} \quad 4.7
\]

In these equations, according to MANABE and BRYAN (1972):

- \( T \) is temperature
- \( S \) salinity
- \( u \) velocity, components \((u, v, w)\) along axes \((x, y, z)\)
- \( I \) flux of heat radiation
- \( c_p \) specific heat capacity
- \( \rho \) density
- \( \rho_0 \) mean density
- \( a_0 = 0.698 \)
- \( a_1 = 1.000027 \)
- \( p_0 = 5980 + 38T - 0.375T^2 + 3S \)
- \( p_1 = T + \rho_0 g z/1.013 \cdot 10^6 \)
- \( p_2 = 1799.5 + 11.25T - 0.0745T^2 - (3.8 + 0.1T)S \)
- \( f \) Coriolis parameter
- \( g \) acceleration due to gravity
- \( T' \) temperature fluctuation
- \( S' \) salinity fluctuation
- \( u' \) fluctuation of velocity, with components \((u', v', w')\).

The vertical axis is directed downwards; the \( y \)-axis to the left of the \( x \)-axis.

The system of equations (4.1) to (4.7) can only be solved numerically. Within the limits of the diagnostic problem with given density fields, the circulation problem for the Baltic Sea was solved by SARKISYAN, STASKEVICH and KOWALIK (1975) and by KALEIS et al. (1974). The prognostic problem, i.e. all main fields should be found quantitatively, was solved by TAMASALU and KULLAS (1972 to 1979) for the Baltic Sea, and by SIMONS (1976, 1978) for the south-western part of the Baltic Sea. In this connection, we will not discuss all the numerical solutions, but only examine some specific problems.
4.2 Baroclinic circulation in the Baltic Sea

The system of equations (4.1) to (4.7) is very complicated. With the assumption that the equation of state depends linearly on temperature and salinity, and that horizontal mixing is negligible, the system of equations (4.1) to (4.7) can be written as follows:

\[
\frac{\partial u}{\partial t} + u \cdot \nabla u + f \times u = -\nabla \left( \frac{p}{\rho_0} \right) - \frac{\partial}{\partial z} (w u')
\]

\[
\frac{\partial p}{\partial z} = b
\]

\[
\nabla_h \cdot u + \frac{\partial w}{\partial z} = 0
\]

\[
\frac{\partial b}{\partial t} + u \cdot \nabla b + w \frac{\partial b}{\partial z} = - \frac{\partial}{\partial z} (b w')
\]

where \( u \) is horizontal velocity vector with components \((u, v)\)

\[
b = \frac{(\rho - \rho_0)}{\rho_0} g \text{ buoyancy}
\]

\[
f \text{ Coriolis vector (0, 0, f)}
\]

One of the complicated problems in modelling the circulation of the Baltic Sea and other water bodies is the parametrization of the turbulent transfer of heat (or buoyancy) and momentum, i.e. terms of type \(-w'T', w'u', w'v'\). The traditional approach is to use an analogy with the corresponding molecular transfer, i.e. parametrize these terms by

\[
-w'T' = K_T \frac{\partial T}{\partial z} ; \quad -w'u' = K_T \frac{\partial u}{\partial z}
\]

where the coefficients \( K \) and \( K_T \) are turbulent coefficients for vertical momentum and heat exchange. This analogy with molecular processes is artificial, it may be justified if it is known that the energy of mean motion is converted into the energy of turbulent motion. This does not always occur, as was shown, for example, by Malkki (1975), who analysed records of current observations in the northern Baltic. Even with the assumption of reasonable energy exchange, the coefficients are functions of the state of motion, often very difficult to define properly.

The stratification of water pays an essential role in the formation of small scale (vertical) turbulence. Within the surface layer, in the interval 0 to \( h_s \), the water is quasi-homogeneous, vertical stratification is very weak and the turbulence is intensive. Below this quasi-homogeneous layer the water is stably stratified and turbulence may occur only intermittently. In the surface layer, turbulence is caused by breaking of wind waves and during the cold seasons by convective mixing. In the stably stratified layer, turbulence is mainly due to instability of the internal wave fields. In this layer, therefore, the derivative of turbulent transfer of momentum \( w'u' \) is so small as to be negligible and the geostrophic relationship \((f \times u = -\nabla (p/\rho_0))\) has a cardinal role in the equation (4.8).

On the other hand, the turbulent transfer of heat \( w'T' \) plays an essential role in the formation of the structure of temperature in the stably stratified layer.

Using the self-similarity temperature profile for the Baltic Sea as described in Chapter 3, the turbulent transfer of heat may be determined as follows:
\[ \overline{w'T'} = q_h \left( \frac{2}{3} \tilde{\xi} + \frac{1}{3} \tilde{\xi}^2 \right) \] (\( \geq 0 \)),

where \( \tilde{\xi} = 1 - \eta = (H - z)/(H - h_T) \).

With the definition

\[ K_T = -\frac{\overline{w'T'}}{dT/dz} \]

the heat transfer coefficient takes the following form:

\[ K_T = \frac{q_h}{9} \frac{H - h_T}{T_s - T_h} \left( 1 + 2\tilde{\xi}^{-3}\tilde{\xi}^2 \right) \] 4.13

In this equation, \( q_h \) is a function of friction velocity, and of surface flux, as in equations (3.37) to (3.39). In cases when \( K_T \) equals zero, one may equate \( K_T = K_w \), the term \( K_w \) arising from interaction of internal waves.

According to Garrett and Munk (1972), the shear instability of internal waves in the ocean may support a coefficient of vertical turbulent mixing of the order of 1 cm\(^2\)s\(^{-1}\) in the region of the thermocline. Taking into account the fact that the Baltic Sea is more strongly stratified than the ocean (the maximum value of the Väisälä-Brunt frequency \( N = \left[ g/\rho_0 (\partial \rho/\partial z) \right]^{1/2} \) exceeds that in the ocean by about one order of magnitude), it may be assumed that the vertical turbulent transfer coefficient in the pycnocline is somewhat smaller in the Baltic Sea than in the ocean, say \( K_w \sim 0.1 \text{ cm}^2\text{s}^{-1} \).

Inserting characteristic values for the Baltic Sea (\( h_T = 20 \text{ m}, H = 60 \text{ m}, v_* = 1 \text{ cm s}^{-1} \)) in equation (4.13) and using the boundary conditions of no heat flux in the surface and \( \tilde{\xi} = 1 \), we obtain the estimate \( K_T = 1/3 \).

Simojoki (1946), Hella (1966b), Krems and Matthäus (1973) and others have determined the coefficient of vertical turbulent mixing by inserting observation data in the semiempirical equations with the turbulent diffusion. The results obtained by those authors are in accordance with this estimate for the coefficient of vertical turbulent mixing.

To determine the vertical transfer coefficient due to wind, Kullenberg (1974) has suggested the following empirical relationship:

\[ K = A U_a^2 N^2 \left| \frac{dU}{dz} \right| \]

where \( U_a \) is wind velocity
\( A \) an empirical constant, of the order of 2 to 8 x 10\(^{-8}\).

The mixing conditions in the Baltic Sea are investigated and presented in detail by Kullenberg (1981).

If the pressure and buoyancy are presented as the sum of their mean and fluctuating parts

\[ p = \overline{p}(z) + p'(x, y, z, t) \]
\[ b = \overline{b}(z) + b'(x, y, z, t) \],
the equations (4.8) to (4.12) can be written, omitting horizontal advection terms, in the following form:

\[ \frac{\partial u}{\partial t} + f \times u = -\nabla \left( \frac{p'}{\rho_0} \right) + \frac{\partial}{\partial z} \left( K \frac{\partial u}{\partial z} \right) \]  

\[ \frac{\partial p'}{\partial z} = b' \]  

\[ \nabla_h \cdot u + \frac{\partial w}{\partial z} = 0 \]  

\[ \frac{\partial b'}{\partial t} + w \frac{\partial b'}{\partial z} = \frac{\partial}{\partial z} \left( K b \frac{\partial b'}{\partial z} \right) \]  

Marchuk (1969) has formulated a method of approaching the problem of the dynamics of a baroclinic ocean, based on analysis of the spectral structure of oceanographic characteristics.

Kuzin and Tamsalu (1974) have used his method to solve the problem (4.14) to (4.17) for a baroclinic sea of constant depth.

The boundary conditions applied by Kuzin and Tamsalu (1974) are as follows:
- on the free surface of the sea \( a_0 \) (\( z = 0 \)):

\[ -\rho_0 \overline{w' u'} = \tau_0 ; \quad \overline{b' w'} = B_0 ; \quad w = -\frac{1}{g \rho} \frac{\partial p}{\partial t} \]  

- in the layer \( z = h_s \) (at halocline depth)

\[ -\rho_0 \overline{w' u'} = 0 ; \quad \overline{b' w'} = 0 ; \quad w = 0 \]  

In these boundary conditions, \( \tau_0 \) denotes the wind stress vector and \( \overline{b' w'} \) the buoyancy flux through the free boundary surface.

At the horizontal boundaries, which are assumed to be vertical (cylinder-like) walls,

\[ u_n = \begin{cases} f_1 & \text{at the boundary} \ a_1 \ 
0 & \text{at the boundary} \ a_2 \end{cases} \]  

where \( u_n \) denotes the velocity component normal to the wall, \( f_1 \) is a function and \( a_2 \) denotes the closed boundary and \( a_1 \) the part of the boundary which is open and connects the basin with adjacent basins. The following integral constraints must be fulfilled:

\[ \int_{a_0 \cdot a_1 \cdot a_2} \overline{b' w'} ds = 0 ; \quad \int_{a_1 \cdot a_2} f_1 ds = 0 \]  

This method has been used in calculations carried out to determine the baroclinic circulation in the Baltic Sea.

The model was first applied to a basin with constant depth and a stationary wind field as described in Fig. 4.1A, the wind velocity being \( 10 \text{ m s}^{-1} \). No buoyancy flux through the sea surface was allowed. After the initiation of the wind stress field, a baroclinic circulation pattern was established. The maximum barotropic velocity is \( 2.5 \text{ m s}^{-1} \), while the maximum baroclinic velocity equals \( 17 \text{ cm s}^{-1} \). The current patterns in the
surface and at 50 m depth are shown in Fig. 4.1B and C. The wind pattern applied calls forth two circulation patterns in the lower layers, as seen in Fig. 4.1C and a corresponding pressure field, Fig. 4.1D. Higher pressure prevails in the eastern part and low pressure in the western part of the basin. A powerful north-eastward flux (5 to 6 cm s$^{-1}$) dominates in the central parts of the basin. The occurrence of such a flux corresponds with the idea of prevailing currents in the Baltic Sea.

Secondly, the model was applied for a non-stationary forcing field with the wind (pressure) varying with time. The results of this calculation are presented in the sequence of Figs. 4.2. The atmospheric pressure field at consecutive intervals is presented in the upper squares, with the corresponding pressure fields at a depth of 50 m. Arrows denote the direction of the circulation. The variability of the atmospheric forcing, although somewhat artificial in the example, causes considerable variability in the basin pressure gradients, leading to considerable variability in the circulation patterns.

The weakness of the above experiment lies in the fact that neither formation of buoyancy fields due to lateral sources (the Danish Sounds and rivers), nor the influence of
bottom relief has been considered. Diagnostic calculations, such as those made by Kaleis et al. (1974) and Sarkisyan et al. (1975), have shown that major factors causing circulation below the upper homogeneous layer are baroclinicity and variation of the bottom topography. This can be illustrated by the following simple example.

Let us assume that the buoyancy gradient in the Baltic Sea is mainly determined by the salinity gradient. We may further assume that the velocity field is approximately in geostrophic balance \( f \times u = -\nabla (p/\rho_0) \). Using the salinity profile presented in equation (2.2), and writing the equation for the deviation from the mean velocity, we obtain:

\[
f \times u^* = \varphi_1 H \nabla \bar{b} + \varphi_2 \bar{b} \nabla H,
\]

where

\[
\begin{align*}
  u^* &= u(x, y, z) - v(x, y) \\
  \bar{b} &= H^{-1} \int_0^H b \, dz \\
  v &= H^{-1} \int_0^H u \, dz.
\end{align*}
\]

The functions \( \varphi_1 \) and \( \varphi_2 \) are obtained from the density structure and are, for the upper homogeneous and lower stratified layer, respectively, as follows:

- in the layer \( 0 \leq z \leq h_s \)
  \[
  \begin{align*}
  \varphi_1 &= -0.29 + 0.15 \xi_1 \\
  \varphi_2 &= 0.42
  \end{align*}
  \]
Fig. 4.3. Functions $\varphi_1$ and $\varphi_2$ in Eq. 4.21.

- in the layer $h_s \leq z \leq H$

$$\varphi_1 = -0.14 + \frac{2}{3\kappa_s} \left[ \frac{1}{6} \xi_1^2 + \frac{1}{6} \xi_2 \xi_1 \frac{\partial \theta_s}{\partial z} \right]$$

$$\varphi_2 = 0.42 - \frac{2}{3\kappa_s} \left[ \frac{4}{5} + \xi_2 \right] \theta_s - \frac{1}{6} \theta_s \frac{\partial \xi_2}{\partial z}$$

In these equations, the coordinates $\xi_1$ and $\xi_2$ are

$$\xi_1 = \frac{z}{h_s} ; \quad \xi_2 = \frac{z - h_s}{H - h_s}$$

The vertical distribution of the functions $\varphi_1$ and $\varphi_2$ is shown in Fig. 4.3.

Making use of equation (4.21) and the given variations in the mean salinity (buoyancy*) for the section Öland—Liepaja, it is possible to calculate the circulation structure for this section. Results of this calculation, together with the transverse profile of mean salinity, are presented in Fig. 4.4. The figure shows that in the slope regions rather strong circulation arises. In the lower layers, the current in the slope region is directed northwards on the eastern side and southwards on the western side. In the upper layer the currents are weaker and flow in the opposite directions. Such a circulation pattern may occur only in the case that the buoyancy isopleths at the slopes of the deep regions follow the bottom relief. According to Kaleis (personal communication) such a situation is almost always possible during the intrusion of saline water. During stagnation periods the buoyancy isopleths are perpendicular to the slopes of the deeps.

* The buoyancy gradient in the Baltic Sea is formed mainly by the salinity gradient. This can be seen in Fig. 4.5, where the geostrophic circulation is calculated both from the real buoyancy structure (dashed line) and from the buoyancy structure formed only by the salinity (continuous line).
Fig. 4.4. Current velocities (cm/s) in the transversal section between Öland and Liepaja (Eq. 4.21, A) and vertically averaged salinities (B).

Fig. 4.5. Vertical distributions of geostrophic currents, as calculated from real buoyancy structure (dashed) and from salinity structure (continuous line).
The diagnostic calculations of Sarkisyan, Staskevich and Kowalik (1975) have shown that one of the main factors in the formation of a circulation pattern in the Baltic Sea is the interaction between the bottom topography and buoyancy. Fig. 4.6 presents the integrated baroclinic circulation function (Sarkisyan et al., 1975).

The integral circulation is dominated by extensive cyclonic circulation, the centre of which is situated in the region of the Gotland Deep. In the southern part, two anticyclonic circulation patterns are found, one south of Bornholm, the other in the Bay of Gdansk. North of Bornholm there is a pronounced cyclonic circulation pattern. In the northern Baltic Proper, north-east of the Landsort Deep anticyclonic exchange of water prevails; the Gulf of Finland is characterized by closed cyclonic circulation of relatively weak intensity.
5 WAVE MOTION

The range of wave-type phenomena to be discussed in this chapter extends from sub-inertial topographic waves to wind-induced waves in the sea surface. Obviously, this wide range limits the discussion to some of the most recent results of studies carried out in the Baltic Sea.

Wind waves have received considerable attention in various countries around the Baltic Sea due to their importance to shipping and the planning of shipping routes and harbours. In addition to collection of basic wave climatology data, the generation and propagation of wind waves have been studied. Standing oscillations, or seiches, have been investigated intensely, especially before the era of modern computer facilities. In recent years, one-dimensional seiche models have been replaced by two-dimensional models, which have enabled inclusion of the effect of the earth’s rotation, hence changing the picture considerably. Topographic waves have been studied in the Baltic Sea only since the early 70’s, when the possibilities for comparison of theoretical models with observational data improved considerably. Research on internal waves in the Baltic Sea dates back more than 20 years. However, there are still relatively few studies which include non-linear effects and their role in the internal movements of the sea area.

5.1 Wind waves

The geomorphology of the basin together with the local climatology determine the behaviour of wind waves observed in the Baltic Sea. Because of the proximity of the coastline, wind waves cannot be expected to reach the dimensions observed in oceanic areas; nor can we expect long-period swell travelling from outside the sea area.

In this region, waves are predominantly fetch-limited, i.e. the growth rate depends on the distance from the windward shore. KAHMA (1981a, 1981b) has determined the relationship between wave amplitude, wind force and fetch, and a similar dependence for the frequency of dominant waves. According to Kahma, the relationships may be expressed in non-dimensional variables as follows:

\[
\tilde{\omega}_m = 20 \tilde{x}^{-1/3} \\
\tilde{\sigma} = 6 \times 10^{-4} \tilde{x}^{1/2}.
\]

In these equations,

- \(\tilde{\omega}_m = \omega u_{10}^{5/8} \) is non-dimensional frequency of the peak of the spectrum,
- \(\tilde{\sigma} = g u_{10}^2 \) non-dimensionalized square root of the variance of surface displacement, and
- \(\tilde{x} = g x u_{10}^{-2} \) non-dimensional fetch.

Figure 5.1 shows these relationships together with the observational data used. Equation (5.1) is valid for the measuring range, which limits the frequencies to less than some
0.8 Hz. As can be seen, the energy levels observed by Kahma are considerably higher than those observed in the JONSWAP expedition in the North Sea (Hasselman et al., 1973; dashed lines in the figures). The reasons for this considerable deviation are so far obscure. As possible candidates Kahma mentions scaling parameters which have not been included in the equations.

The wave spectra observed by Kahma are not represented by the well-known Phillips’ law, but rather by the equation

\[ S(\omega) = a u_{10}^2 \omega^{-4} \]  

where \( a = 4.5 \times 10^{-3} \)
\( u_{10} \) is wind velocity at 10 m level.

The equation for non-dimensional frequency, (5.1a), is easily obtained as a combination of equations (5.1b) and (5.2). The calculations of Kahma (1981b) also show that the flux of momentum during wave growth can be estimated well using linear theory. His estimate for the flux to waves amounts to some 50 to 100% of the total momentum flux. These estimates are valid for stationary wind conditions, with no wave propagation from various source regions. They can be used for wave growth estimates in similar conditions; for forecasting purposes more complicated models are necessary.

During the past decade, a considerable amount of wave climatological observations have been carried out in the Baltic Sea, using various types of wave recorders (MIAS Catalogue of Wave Data, 1982). The observations carried out by the Finnish Institute of Marine Research, for example, show the significant wave height (\( H_{1/3} = 3.13a \)) to be in most cases less than 1 m, especially during the summer months. During the autumn, the number of cases with higher waves increase. In the Gulf of Bothnia, the highest waves observed have been some 5.5 m, in the Baltic Proper even higher significant waves have been observed. On the basis of the known relationships it can be estimated that in severe circumstances waves higher than 10 m may occur.

As an example of wave statistics, Fig. 5.2 shows a summary of records made by FIMR in the Bothnian Sea in the year 1973–1975 (Kahma, 1976). The data cover late summer (July to September) and autumn (October to December). The directional statistics (Fig. 5.2A) show clearly the dependence of fetch for the measuring site off Rauma and Uusikaupunki and, therefore, the data can be considered representative of the neighbourhood of the measuring site only. The statistics on wave height (Fig. 5.2B) reveal the increase
of wave heights during the late autumn period; notably the mode of the observations has increased from 0.5 m to some 1 m. The observational data sets are of different size, and during the summer months cases with weak winds are numerous. Statistics for only a few years can certainly not be expected to give more than an approximate picture of conditions in a particular region. As observational work is both expensive and time-consuming, a comprehensive picture of the wave conditions in various regions in the Baltic Sea is unlikely to be obtained in the near future.

5.2 Seiches and tides

Due to the semi-enclosed character of the Baltic, water level observations show predominantly fluctuations due to tilt in the water surface; variations with longer periods have lower amplitudes. In the central parts of the Gulf of Finland, the water level may rise more than 100 cm above MSL during periods of south-westerly winds. Simultaneously, a corresponding deviation in the opposite direction may be observed in the southern part of the Baltic Sea. When the wind direction changes or the weather disturbance disappears, return flow towards the equilibrium level begins. As a result, a damped oscillation, called a seiche, can often be observed in water level recordings. On the basis of the original theory of Chrystal (1906), Neumann (1941) calculated the theoretical periods of one-dimensional seiches in the Baltic. Since then, several authors have presented similar calculations, in particular Krauss and Magaard (1962) and most recently Wübben and Krauss (1979). The last-mentioned study is of particular interest, because it describes the oscillations using a two-dimensional model, which includes the rotation of the earth. Earlier one-dimensional models gave the basic mode of the system Baltic Sea — Gulf of Finland as a period of 27.7 hours, a value seldom observed in water level records. Calculations with a two-dimensional model give a corresponding period for the second mode, if rotation is omitted. With rotation, the period of the second mode is reduced to 26.4 hours, which is close to the observed values. The water movement is no longer unidirectional along the basin, except in narrow basins like the Gulf of Finland. The variations have a cyclonal phase progression with amphidromic points in the interior according to the mode number. As a peculiarity, it may be mentioned that the Gulf of Riga has a considerable influence on the second mode, due to its co-oscillation with the Baltic Proper.

The analysis of tide gauge records for periods of notable seiches has shown that their attenuation can be expressed with the linear damping coefficient $5.5 \times 10^{-8} \text{s}^{-1}$. The same value has also been obtained in studies of damped inertial oscillations by Kullen-
BERG and HELA (1942) and by MÄLKKI (1975). With this linear damping, seiches die out during a few periods.

Compared with tides in the Baltic, seiches are a far more dominant periodic phenomenon. The amplitudes of tidal elevations seldom exceed a value of 10 cm, except in the proximity of the Danish Sounds (LISITZIN, 1943, 1944). The prevailing type of tides is mixed and to amplitudes are usually less than 3 cm. Despite this low value, some coastal regions may have notable tidal currents. A summary of spring tide ranges for both diurnal and semidiurnal tidal components, and a classification of tides in the Baltic Sea can be found, for example, in MAGAARD (1974).

5.3 Topographic waves

The presence of topographic eddies has been demonstrated both in numerical experiments (KIELMANN et al., 1976; KIELMANN, 1978) and in synoptic surveys (AITSAM and ELKEN, 1981; TALPSEPP, 1983). Both types of analyses show that eddy-like phenomena occur and proceed in a direction determined by the topography, the shallower regions lying to the right of their path of progress. Because of the salinity and temperature structure of the Baltic Sea, AITSAM and ELKEN conclude that anticyclonic eddies have a cold centre in the thermocline and a salty centre in the halocline. Correspondingly, cyclonic eddies have warm and fresh centres in thermocline and halocline. The type of waves observed by these authors were bottom-trapped, and the migration speed of the eddies agreed approximately with the values predicted by RHINES' (1970) theory. With length scales of the order of 30 to 40 kilometres and using observed data on stratification, the migration speed was calculated to be some 1.9 cm s\(^{-1}\). Consecutive hydrographic surveys revealed a migration speed of some 1.5 cm s\(^{-1}\). TALPSEPP (1983), who analysed the velocity measurements of the same experiments, found that the strongest periodic variability corresponded to 6-day to 8-day oscillations in regions where the bottom slope is strongest. The energy levels in the period ranges diminished clearly in regions where the bottom slopes were moderate or weak. Trapping close to the bottom was strongest near the steepest bottom slopes.

The numerical simulation of KIELMANN (1978) for the Bornholm basin reveals similar eddies, which migrate with a speed of some 20 kilometres per day. Both cyclonic and anticyclonic eddies were observed, KIELMANN ran the simulation with a 4-layer model which, to save computer time, was taken as vertically homogeneous. The wind was allowed to vary sinusoidally with a period of four days. After an excitation period of 3 days the forcing was allowed to vanish, and the simulation was run for an additional four days. The most notable vortices migrate along the bottom contours with shallower water on the right. Kielman also discusses the generation of the vortices, on the basis of the results of the numerical simulations. Starting from the conservation of potential vorticity:

\[
(H + \eta) \frac{D}{Dt} \left( \frac{f + \xi}{H + \eta} \right) = \nabla \times \left( \frac{\tau_0 - \tau_b}{\rho H} \right) + A_H \nabla^2 \xi,
\]

which in the case of a barotropic sea can be expressed as

\[
\frac{\partial \xi}{\partial t} = \frac{f}{H} u \cdot \nabla H + \nabla \times \frac{\tau_0 - \tau_b}{\rho H} + A_H \nabla^2 \xi,
\]

as a linear approximation, he concludes that the dominant generation of vorticity is due
to the term $\frac{f}{H} \mathbf{u} \cdot \nabla \mathbf{H}$. Model runs with $f = 0$ showed vorticity generation in the regions where the slope was steepest. Vortices in shallower water, where the currents are mainly due to Ekman drift, stop more rapidly after the forcing ceases. The major region of eddy generation, according to these simulations, is the entrance through Stolpe Channel to the Bornholm basin.

On the basis of these studies, it can be concluded that in the open Baltic Sea the wind forcing over a variable bottom topography may cause various kind of topographic vortices, especially in shallower regions. In the case studies by AITSAM and ELKEN, the generation mechanism may be different, since the bottom waters were uncoupled from direct influence of the wind by a strong thermocline. The authors attribute the eddy generation to baroclinic instability. Satellite pictures (e.g. HORSTMANN, 1983) often show eddy-like phenomena in the surface layers of the Baltic Sea. These may be slowly varying thermocline eddies. So far, there is little information about the character and behaviour of these eddies.

In addition to the above-mentioned types of topographic waves, others can be found in the coastal regions in the Baltic Sea. The coastline on the eastern side of the Bothnian Sea seems to be especially suitable for coastally trapped waves. On a sloping coast, barotropic Kelvin waves cannot occur, but observations reveal velocity fluctuations which may be interpreted as coastally trapped modes. The most recent review of this kind of phenomena is presented by HENDERSHOTT (1981). When the bottom slope $\alpha$ is even, as is the case along the coast of the Bothnian Sea, the wave equation for water levels varying sinusoidally both horizontally and with time, yields the following equation for the amplitude function:

$$x \frac{\partial^2 h}{\partial x^2} + \frac{\partial h}{\partial x} + \left[ \left( \frac{\omega^2 - \frac{f_0^2}{\omega}}{\alpha g} - \frac{f_0 k}{\omega} \right) - xk^2 \right] h = 0 , \tag{5.4}$$

where $h$ is amplitude function
$k$ alongshore wave number
$\alpha$ bottom slope
$x$ axis normal to the coast.

The equation (5.4) has a solution

$$h(x) = h_0 e^{-kx} L_n(2kx) , \tag{5.5}$$

where $L_n(x)$ is Laguerre polynomial of the order of $n$
$h_0$ amplitude.

When solution (5.5) is inserted into equation (5.4), the dispersion relation becomes

$$s^3 - s[1 + (2n + 1)K] - K = 0 , \tag{5.6}$$

where the non-dimensional variables $s$ and $K$ are

$$s = \omega / f_0$$

$$K = kag / f_0^2$$

In the case of an infinite slope, the waves remain in the slope region, in the case of a level bottom at some distance from the slope, the boundary conditions are fairly com-
Fig. 5.3. Spectra of current meter moorings off Pori in August-September 1976 at three mooring stations (Alenius and Mäkki, 1978).
plicated and can be found most easily by numerical methods. Solutions of this type have been discussed by Csanady (1976), with an application to Lake Ontario.

On the Finnish side of the Bothnian Sea, the slightly curved coastline extends more than 200 kilometres towards the north, with an average bottom slope of $2 \times 10^{-3}$. The width of the bottom slope is approximately 60 km, after which the bottom is even for some 20 kilometres.

Fig. 5.3 shows current velocity spectra measured off the coastline close to Pori at different sites. The dominant feature in these spectra is the inertial oscillation, but other periodic oscillations can be observed as well. Of particular interest in this connection are the maxima with periods in the range of 57 to 65 hours, corresponding to about $(0.21-0.23) \times$ inertial frequency. Following the reasoning of Csanady (1976), these could be interpreted as barotropic topographic waves with a wave length of about 390 km. The total length of the evenly sloping coastal strip is some 200 kilometres, thus this wave length is not an unreasonable estimate for the gravest mode. The observed spectra at station VO show considerable barotropy at this frequency range, as do also those at station V01, some 9 nautical miles NE of V0. This maximum seems to be missing at station V11, which is situated NW of station V0. For a barotropic wave, the amplitude should decrease with increasing distance from the coastline, as is the case in the observations. The phase difference between the alongshore components at stations V0 and V01 is very low, of the order of $5^\circ$, with a delay towards the north. The coherences between these stations are of the order of 0.85. Both the phase and coherence, and the decay of amplitude in the offshore direction support the idea of a long coastal topographic wave, although the calculation is rather crude. The spectra presented in Fig. 5.3 are so scaled that the proportion of kinetic energy in a certain frequency range is directly proportional to the area below the curve in the respective interval. As can be seen, the frequency range of coastally trapped topographic waves has a considerable energy level. Together with the inertial frequency range, it is the major contributor to current variations. It may also be the major mechanism for the dispersion of pollutants along the coast, since the velocities transverse to the coast seem to be of minor importance in this frequency range.

5.4 Inertial oscillations

A pronounced phenomenon in the Baltic Sea current variations is the appearance of inertial oscillations in almost all current records. This can be seen in Fig. 5.3, and in other published data (e.g. Aalenius and Målck, 1978; Målck, 1975; Kielland, Krauss and Keuncke, 1973). In the open sea observations these oscillations form some 30% of kinetic energy in summer conditions, in coastal regions their proportion depends on stratification and the shape of the coastal boundary layer. Thus, in the coastal regions of the Baltic Proper, where the halocline together with thermal stratification determines the stability, the greater part of kinetic energy is found in low frequency (almost barotropic) oscillations. In the Gulf of Bothnia stratification is weaker and in coastal regions inertial oscillations have a more dominant role. Inertial oscillations can also be observed in time series of thermistor chains practically everywhere in the Baltic. Examples are given in Figs. 3.9 and 3.10. During the summer season, the oscillations are roughly in the opposite phase above and below the thermocline, and the periods are very close to the theoretical inertial period, say, $\omega = f \pm 2\%$ (Målck, 1975; Aalenius and Målck, 1978). The spectrum band is very narrow; frequencies more than 5% above the inertial frequency are seldom observed.

Pure inertial oscillations are obtained from the equations of motion (4.14) as a special case when the pressure gradients and frictional terms are neglected. Rotation takes place circularly polarized in a clockwise direction, the radius of the rotational circle being
For typical velocities in the surface layer of the Baltic Sea and for the Coriolis parameter $f = 1.25 \times 10^{-4}$, this yields a radius of the order of $R = 800$ m. The phase velocity of the wave is given by the equation

$$c = \frac{\omega}{k^2} k = \pm \frac{\Omega \cdot k}{k^3} k$$

For pure inertial oscillation the group velocity equals zero; even for frequencies slightly above the inertial frequency the group velocity is very small. Thus, the energy is maintained within the region of origin of the oscillations. Observations from the Baltic Sea indicate that the wave number of inertial oscillations deviates from zero (MÄLKKI, 1975). The presence of this wave number can be explained, for example, by the presence of lateral friction. Consider the case (e.g. KRAUSS, 1973, MÄLKKI, 1975) in which linearized equations of motion contain a term for horizontal eddy friction:

$$\frac{\partial u}{\partial t} + \Omega \times u = - \frac{1}{\rho} \nabla p + \nu \nabla^2 u.$$ 

A solution of periodic variation with exponential decay exists for both the pressure and velocity fields:

$$u = u_1 e^{-\lambda t} \cos(k \cdot r - \omega t) + u_2 e^{-\lambda t} \sin(k \cdot r - \omega t)$$

$$p = p_1 e^{-\lambda t} \cos(k \cdot r - \omega t) + p_2 e^{-\lambda t} \sin(k \cdot r - \omega t).$$

Equation (5.10) is valid if the coefficients of the trigonometric terms vanish when (5.10) is inserted into equation (5.9), i.e. if

$$-\lambda u_1 - \omega u_2 + \Omega \times u_1 = -\frac{p_2}{\rho} k - \nu k^2 u_1$$

$$\omega u_1 - \lambda u_2 + \Omega \times u_2 = -\frac{p_1}{\rho} k - \nu k^2 u_2.$$

As a result, the amplitude functions $u_1^2$ and $u_2^2$ are equal to each other. They are perpendicular to both each other and the wave number vector $k$. The damping coefficient is found to be

$$\lambda = \nu k^2$$

and the frequency is

$$\omega = \pm \frac{\Omega \cdot k}{k}$$

which, in the case of purely horizontal motion, equals the Coriolis frequency $f$.

Observations of inertial oscillations in the Baltic Sea (e.g. GUSTAFSSON and KULLENBERG, 1936; KULLENBERG and HELA, 1942; MÄLKKI, 1975) suggest a damping coefficient of the order of $10^{-5} s^{-1}$. With a constant eddy viscosity $\nu = 10^3 \text{ m}^2 \text{s}^{-1}$, this corresponds to a scalar wave number of the order of $10^{-4} \text{ m}^{-1}$. 

MALIKI (1975) analysed inertial movements in a coastal region of the Baltic Proper and found wave numbers somewhat higher than the above figure. He assumed this to be due to the proximity of the coastline. A constant eddy viscosity is a rough approximation; moreover, its numerical value can vary, especially in the coastal regions. Therefore, damping can be expected to vary to some extent at different sites.

The generation of inertial movements has been studied by several authors (see e.g. POLLARD, 1970; POLLARD and MILLARD, 1970; WELLER, 1982). They are related to variations in the local wind stress, with fairly slow propagation from the origin. WELLER found that despite homogeneous wind stress, the variations in inertial response were considerable even at short distances. He considered the influence of ambient geostrophic currents and found that divergent flow damps the motion, whereas vorticity causes frequency shifts in either direction from the inertial frequency. In the Baltic Sea, the presence of lateral boundaries may have considerable influence, since the stratified flow may have patterns of internal Poincaré waves. These modes are concentrated close to the inertial frequency, and depending on the forcing conditions, may have considerable amplitudes.

5.5 Internal waves

Internal waves are always found in water bodies which have stable stratification, if the state of equilibrium of the fluid particles is destroyed by external forces. The disturbing forces may be fluctuations of atmospheric pressure, wind-induced Ekman suction, fluctuations of the sea surface and also fluid flow over uneven bottom topography, etc.

In the water masses of the Baltic Sea, the stratification is predominantly stable and, due to the above-mentioned generation mechanisms, internal waves are a common feature in all observational data sets.

The frequency spectrum of internal gravity waves lies within the limits of the Coriolis frequency $f$ and the maximum of the Väisälä-Brunt-frequency $N_0$: $f < \omega < N_0$, where $\omega = 2\pi/T$. The period $T$ of the waves thus lies within the interval from a few minutes up to half a day. Observations of both current velocity fluctuations and displacements of isotherms have proved the occurrence of internal waves in the Baltic Sea in the whole frequency range described above.

In the investigations of the Kiel Bight, KRAUSS and MAGAARD (1961) found a concentration of periods in the range 1.5 to 30 minutes. A special study of HOLLAN (1966) has shown that fluctuations in the Kiel Bight are mainly concentrated in periods within the range 1 to 10 minutes.

Fluctuations with periods in the interval 5 to 6 hours are, according to KRAUSS and MAGAARD (1961), a general phenomenon in the Baltic Sea. They are observed in the Gulf of Finland, in the Arkona Basin and in the region of Darss Sill.

Experimental investigations of internal waves in the Baltic Sea have mainly concerned fluctuations of temperature, salinity and oxygen concentration. For example, in August 1964 in the vicinity of Darss Sill, hourly fluctuations of temperature with a range of 8.2 °C and fluctuations of salinity of the order of 3.5 ‰ were established.

The largest variations of hydrographical characteristics are observed in the depths with maximum Brunt-Väisälä frequency, i.e. in the pycnocline. With increasing distance from the pycnocline, either towards the surface or towards the bottom, the amplitudes decrease. In regions where there are two separate maxima of the Väisälä-Brunt frequency (due to the thermocline and halocline) there are also two maxima of amplitudes.

Experimental investigations carried out in several regions of the World Ocean have shown (MIROPOLSKY, 1981) that outside the boundary layer there is weak turbulence,
which is intermittent in character. The most plausible source of this intermittent turbulence and also of the microstructure of the hydrophysical fields may be the internal waves. They take part in the redistribution of fluxes of heat, salt, momentum and mechanical energy. We may assume that internal waves in mutual interaction can generate the universal profiles of temperature and salinity which were discussed in previous chapters. This may be the major role of the internal waves in the structure and dynamics of the Baltic Sea water masses. 

The literature on linear internal waves is extensive, but since the linear internal waves presumably do not take part in the redistribution of fluxes of heat, salt and momentum, efforts should be directed towards the investigation of non-linear internal waves. We will therefore restrict our attention to these waves in the subsequent subsections.

5.5.1 Non-linear stationary waves

The uneven distribution of forcing together with variable topography provides many possibilities for the formation of non-linear internal waves. For example, the flow over sills (LONG, 1953) in stratified conditions may cause considerable non-linearities in the downstream region (MAGAARD, 1965). The instabilities of the internal waves near the bottom layer may, on the other hand, cause bottom turbulence (LOZOVATSKY et al., 1977). During the past decade, considerable progress has been made in the study of non-linear waves (e.g. LONG, 1972, LEONOV and MIROPOLSKY, 1975a and 1975b, LEONOV et al., 1977 etc.). A comprehensive monograph on internal waves has been published by MIROPOLSKY (1981).

MIROPOLSKY (1981) has shown that in a shallow sea (a water body of finite depth) the dispersion relation of the nth mode has the following form, if the Coriolis effect is disregarded:

\[ \omega_n = c_{r,n} k - d_n k^3 + O(k^5) , \]  

5.11

where

\( c_{r,n} \) is phase velocity

\( d_n \) constant depending on the kind of stratification

\( k \) wave number.

In the dispersion relation, the operators \(-\partial / \partial t + i \omega\) and \(\partial / \partial x \rightarrow ik\) have been taken into account. Correspondingly, the differential equation for the amplitude function reads as follows:

\[ \frac{\partial u_n}{\partial t} + c_{r,n} \frac{\partial u_n}{\partial x} + d_n \frac{\partial^3 u_n}{\partial x^3} = 0 . \]  

5.12

When non-linear waves are examined, the characteristic for the hydrodynamics of these waves, the non-linear term \( \delta_n u_n \frac{\partial u_n}{\partial x} \), should be added. In this term, \( \delta_n \) stands for the constant depending on stratification. As a result we have the well-known KORTEWEG-DE VRIES (1895) equation

\[ \frac{\partial u_n}{\partial t} + (c_{r,n} + \delta_n u_n) \frac{\partial u_n}{\partial x} + d_n \frac{\partial^3 u_n}{\partial x^3} = 0 . \]  

5.13

For waves which travel with constant phase velocity, we may take as independent variable the wave phase \( \theta_n = x - c_{r,n} t \). As a result, instead of eq. (5.13) we obtain the following form:
\[
\frac{d}{d\theta_n} \left[ d_n \frac{d^2 u_n}{d\theta_n^2} + c_{r,n} u_n + \frac{1}{2} \delta_n u_n^2 \right] = 0. \tag{5.14}
\]

The integration of equation (5.14) with respect to \( \theta \) yields:

\[
\frac{d^2 u_n}{d\theta_n^2} + \alpha_n u_n + \gamma_n u_n^2 = z, \tag{5.15}
\]

where \( z \) is constant of integration.

As a boundary condition, it is necessary to have a) for a solitary wave

\[
u_n \bigg|_{\theta = \pm \infty} = 0 \tag{5.16}
\]

and b) for periodic non-linear waves

\[u_n(\theta_n + \lambda) = u_n(\theta_n), \tag{5.17}\]

where \( \lambda \) is wave length.

The stream function \( \psi \), defined with equations

\[
u = \frac{\partial \psi}{\partial z}; \quad w = -\frac{\partial \psi}{\partial x},
\]

where \( \nu \) is longitudinal velocity, and \( w \) cross-section velocity,

can be determined for plane waves from the relationship

\[\psi_n = u_n(\theta_n)w(z). \tag{5.18}\]

In equation (5.18), the function \( w_n \) is determined from the solution of the boundary problem

\[w'' + \beta_n \Omega w = 0 \tag{5.19}\]

\[w(0) = w(1) = 0.\]

Here

\[\beta_n = \frac{H^2 N^2}{c_{r,n}}; \quad \Omega = \frac{N^2(z)}{N_0^2},\]

\[N^2(z) \text{ is the Väisälä-Brunt frequency } \left( = \frac{g}{\rho_0(z)} \frac{\partial \rho_0}{\partial z} \right).\]

\[\rho_0(z) \text{ density distribution, undisturbed by the wave}\]

\[H \text{ depth of the sea}\]

\[g \text{ acceleration due to gravity.}\]

The unknown coefficients \( \alpha_n \) and \( \gamma_n \) depend on stratification \( \rho_0(z) \) and on the eigenfunction \( w_n \).

In Boussinesq approximation (which is only possible with the condition \( w = 0 \)), the coefficients are as follows:
The coefficients $\alpha_n$ and $\gamma_n$ with the condition $\omega \neq 0$ have been discussed by Leonov, Miropolsky and Tamsalu (1979).

For solitary waves, defined by equations (5.15) and (5.16), we find

$$u_n = \frac{3}{2\gamma_n n^2(\xi_n/2)^2}; \quad \xi_n = \frac{\theta_n}{\lambda_n}.$$  \hspace{1cm} 5.20

For periodic, weakly non-linear waves, the corresponding equation is as follows:

$$u_n = \frac{u_0}{n} \left[ \frac{d\eta}{\sqrt{\gamma_n \frac{u_0^2}{\lambda_n}}(\xi_n, s)} \right] - \frac{E(s)}{K(s)} \frac{1}{\lambda_n} \int_0^\lambda u_n d\xi_n = 0.$$  \hspace{1cm} 5.21

In these equations, $dn(x, s)$ are the elliptic Jacobi functions with $x$ and the modulus $s$ ($0 < s < 1$); $K(s)$ and $E(s)$ are the elliptic integrals. When $s = 1$, the periodic wave degenerates into a solitary wave, and when $s = 0$ into an infinitesimal one.

The above theory was applied to the Baltic Sea, using as experimental data the density structure observed by Kielmann et al. (1973) for the Arkona Basin. The depth in this area, $H = 47$ m, and the maximum of the Väisälä-Brunt frequency, $N_0 = 5.6 \times 10^{-2}$ s$^{-1}$. The distribution of the Väisälä-Brunt frequency, normalized into dimensionless form $\Omega = N^2(z)/N_0^2$, with slight smoothing, is presented in Fig. 5.4 (broken line).

In the same figure, the first three eigenfunctions $w_n$ are presented in non-dimensional form, normalized to satisfy the condition $\int_0^1 w_n^2 d\eta = 1$. The spectral problem was solved by numerical methods. Along with the eigenfunction, the parameters $\alpha_n$ and $\gamma_n$ were defined. The phase velocity, $c_{f,n} = H N_0 \theta_n^{1/2}$, and the addition to the phase velocity, $\tilde{c}_{f,n} = H N_0 u_0^2/(2\alpha_n \beta_{on}^3)$, were calculated. For simplicity, the non-dimensional wave number was taken as constant, with the value $K_0 = 0.05$. The wave length $\lambda$ for the infinitesimal wave was equal to $1.23$ km. The other characteristics of the weakly non-linear internal wave are given in Table 1.

**Table 1. The parameters of the three first modes of the internal wave.**

<table>
<thead>
<tr>
<th>Mode</th>
<th>$10^2\alpha_n$</th>
<th>$\beta_{on}$</th>
<th>$\gamma_n$</th>
<th>$c_{f,n}/\text{ms}^{-1}$</th>
<th>$T_n/\text{h}$</th>
<th>$E_n/\text{m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.6</td>
<td>19.0</td>
<td>0.26</td>
<td>0.61</td>
<td>0.6</td>
<td>4.05</td>
</tr>
<tr>
<td>2</td>
<td>1.6</td>
<td>13.4</td>
<td>3.00</td>
<td>0.23</td>
<td>1.6</td>
<td>0.35</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>30.6</td>
<td>-55.00</td>
<td>0.15</td>
<td>2.4</td>
<td>0.22</td>
</tr>
</tbody>
</table>

The parameter $s$ for cnoidal waves was chosen as equalling 0.99. In this case the wave length of the cnoidal wave was equal to $2.14 \times \lambda$, where $\lambda$ is the wave length of the infinitesimal wave. In Fig. 5.5 the functions $u_n(\theta_n)$ for different mode numbers are...
shown. The amplitude functions $u_n(\theta_n)$ decrease as the modulus increases. The solitary wave differs from the cnoidal wave only in that its minimum transforms into zero.

The streamlines of the horizontal structure of non-linear periodic internal waves are presented in Leonov, Miropolsky and Tamsalu (1979). The flow of fluid is disintegrated into cells, the number of cells along the vertical coordinate corresponds to the mode number. They have a layered cellular structure. As an example, the spatial distribution of the transverse velocity field for the first mode is presented in Fig. 5.6c.

For solitary waves and as an approximation of cnoidal waves which are not far from soliton, we have $\rho = \rho_0(\xi - \psi/c_0)$. Using the definition of relative density $\sigma = (\dot{\rho}(\eta) - 1) \times 10^3$, we obtain for this density

$$\sigma = [\rho(\eta) - 1] \times 10^3 + 10^3K_0^2\rho_0(\eta)\Omega(\eta)w(\eta)u(\xi) \label{5.22},$$

where $\dot{\rho}(\eta)$ is the vertical distribution of fluid density undisturbed by waves. Fig. 5.6A shows the distribution of $\sigma$ values for the first mode of the cnoidal wave. The specific feature of these curves is the obvious concentration of isocrests in the region of maximum amplitudes of the function $u(\xi)$. Secondary non-linear effects which cause violent
mixing of stratified fluid are possible in the regions with large horizontal and vertical velocity gradients. It is often supposed (e.g. MILES, 1963) that local hydrodynamic instability may occur in the regions where the local Richardson number $R_i < 0.25$. In Fig. 5.6B isopleths of the Richardson number are shown for the first mode of the cnoidal wave. As can be clearly seen, the minimum $R_i$ occurs in the region of large density gradients (shaded parts in Fig. 5.6B). In these regions, the Richardson number is less than 0.25, so that zones with possible intermittent turbulence are formed.
5.5.2 Non-linear non-stationary waves

The non-stationary internal waves can be studied by introducing a stream function, defined by the equations

\[ u = \frac{\partial \psi}{\partial z}, \quad w = -\frac{\partial \psi}{\partial x}, \]

into the equations of motion. With the use of Boussinesq approximation, the equations for stream function and continuity are as follows:

\[ \rho_0(z) \left[ \frac{\partial}{\partial t} \nabla^2 \psi + J(\nabla^2 \psi, \psi) \right] = g \frac{\partial \rho}{\partial x} \]

\[ \frac{\partial \rho}{\partial t} + J(\rho, \psi) = 0, \]

where \( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \)

\( J \) is Jacobian \( \left( J(a, b) = \frac{\partial a}{\partial x} \frac{\partial b}{\partial y} - \frac{\partial a}{\partial y} \frac{\partial b}{\partial x} \right) \).

The use of rigid lid approximation can be expressed by the following boundary conditions:

\[ \frac{\partial \psi}{\partial x} \bigg|_{z=0} = \frac{\partial \psi}{\partial x} \bigg|_{z=-H} = 0. \]

We may assume that the stream function \( \psi \) and the density field \( \rho \) may be expressed by the following power series:

\[ \psi(x, z, t) = \sum_n \epsilon^n \psi_n(\xi, \tau, z, \theta) \]

\[ \rho(x, z, t) = \rho_0(z) + \sum_n \epsilon^n \rho_n(\xi, \tau, z, \theta). \]

In these equations,

- \( \epsilon \) is a small parameter
- \( \theta \) wave phase (»fast»)
- \( \xi = \epsilon^2 x \) »slow» horizontal coordinate
- \( \tau = \epsilon^2 t \) »slow» time coordinate.

The theory of propagation of a weakly non-linear internal wave packet in stratified fluid has been presented among others by BORISENKO et al. (1976). It is shown that the mean currents and density field are formed by non-linear interaction. The vertical structure of these mean currents and density has the peculiarity of fine structure. This circumstance can explain the generation of complicated fine structure, almost always found in CTD soundings in the sea.

The currents and density microstructure formed by the propagation of weakly non-linear wave packages in stratified fluid can be shown to be

\[ \psi(x, z, t) = a(\xi, \tau) \phi(z) \cos \theta + a^2(\xi, \tau) \left[ \phi(z) + x_1(z) \cos 2\theta \right] + O(a^3) \]
\[
\rho(x, z, t) = \rho_0(z) + a(\xi, \tau) \frac{d\rho_0}{dz} \frac{k}{\omega} \phi(z) \cos \theta + a^2(\xi, \tau)[R(z) + \chi_2(z) \cos 2\theta] + O(a^3),
\]

where \( a(\xi, \tau) \) is amplitude of the internal wave,
\( k \) horizontal wave number,
\( \omega \) frequency of the internal wave,
\( \phi \) eigenfunction of the linear vertical operator.

The eigenfunction \( \phi \) satisfies the following equation:

\[
\phi'' + k^2 \frac{N_0^2(z) - \omega^2}{\omega^2} \phi = 0,
\]

with the boundary conditions

\[
\phi|_{z=0} = \phi|_{z=-H} = 0,
\]

where \( H \) is depth of the sea and \( N_0^2 \) is the square of the Väisälä-Brunt frequency. The functions \( \chi_1 \) and \( \chi_2 \) can be expressed as

\[
\chi_1(z) = -\frac{k \phi(z)}{3\omega^3} \int_{-H}^{0} \frac{dN^2}{dz} \phi^3 dz
\]

\[
\chi_2(z) = -\frac{k}{\omega} \frac{d\rho_0}{dz} \chi_1(z) + \frac{1}{2} \frac{k^2}{\omega^2} \frac{d^2\rho_0}{dz^2} \chi_1^2(z).
\]

The mean current \( \phi(z) \) (analogous to Stokes current in the theory of surface waves), and the density function \( R(z) \) satisfy the following equations:

\[
\phi'' + N_0^2(z) c_g^{-2} \phi = F_1(z)
\]

\[
\phi|_{z=0} = \phi|_{z=-H} = 0
\]

\[
R(z) = F_2(z),
\]

where

\[
F_1(z) = c_g^3 \left[ \frac{2 - \frac{c \phi}{c_g} + \left( \frac{c \phi}{c_g} \right)^2}{ \frac{d\phi}{dz} \frac{d\phi}{dz} + 2 \phi^2 \frac{dN_0}{dz} } \right] N_0^2(z) \frac{d\phi}{dz} \frac{d\phi}{dz} + 2 \phi^2 \frac{dN_0}{dz}
\]

\[
F_2(z) = \rho_0(z) \left[ N_0^2(z) \phi(z) - c_g^2 \left[ 1 - \frac{c \phi}{c_g} \right] N_0^2 \frac{d^2\phi}{dz^2} + \phi^2 \frac{dN_0}{dz} \right]
\]

\[
c_g = \omega/k \quad \text{is phase velocity}
\]

\[
c_g = \omega/dk \quad \text{group velocity.}
\]

For sufficiently small \( c_g \), i.e. when the frequency \( \omega \to N_0 \), the equation (5.30) describes the fine structure of the current. Correspondingly, small oscillations of \( \phi(z) \) together with the equation (5.31) describe the fine structure of the density.

The above theory was applied to the Baltic Sea by Leonov, Miropolsky and Tamsalu (1977). In order to do this, the amplitude \( a \), the frequency \( \omega \) and the Väisälä-Brunt frequency \( N_0(z) \) must be taken as known. The vertical profile of the stability frequency was obtained from Kielmann et al. (1973) for the Arkona Basin. It is presented
in non-dimensional form, averaged over a time of 10 hours in Fig. 5.7 (line 1), and in smoothed form (line 2).

The spectral problem (equation 5.29) was solved numerically for different values of $\omega$. The first eigenfunction for $\omega = 4.15 \times 10^{-2} \text{ s}^{-1}$ is given in line 3 in Fig. 5.7. The
Table 2. Parameters of the weakly non-linear first wave mode.

<table>
<thead>
<tr>
<th>$10^2 \omega /s^{-1}$</th>
<th>$T = 2\pi /\omega /s$</th>
<th>$\lambda = 2\pi /k /m$</th>
<th>$c_\phi /cm/s^{-1}$</th>
<th>$c_g /cm/s^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.84</td>
<td>132</td>
<td>11</td>
<td>8.7</td>
<td>1.2</td>
</tr>
<tr>
<td>4.15</td>
<td>150</td>
<td>24</td>
<td>15.9</td>
<td>3.8</td>
</tr>
<tr>
<td>3.28</td>
<td>192</td>
<td>60</td>
<td>24.3</td>
<td>15.0</td>
</tr>
<tr>
<td>2.15</td>
<td>294</td>
<td>125</td>
<td>43.0</td>
<td>26.0</td>
</tr>
</tbody>
</table>

choice of this frequency gives wave solutions only in a narrow depth interval of some 10 m (dashed vertical interval in Fig. 5.7). The wave characteristics for different frequencies are given in Table 2.

Equations (5.30) and (5.31) were solved numerically using the frequency $\omega = 4.15 \times 10^{-2} s^{-1}$, the calculated parameter values $c_\phi$ and $c_g$, and $\phi$. The amplitude of the wave was set as equal to 40 centimetres. The calculation was started with the initially smoothed Väisälä-Brunt frequency, as shown in line 2, Fig. 5.7. Fig. 5.8 shows the fine structure of the hydrophysical characteristics formed by the propagation of weakly non-linear wave packages.

In Fig. 5.9 (dashed line) the resulting Väisälä-Brunt frequency, determined from the equation

$$\Phi(z) = N_{0 \max}^{-2} \left[ N_0^2(z) + \frac{g}{R(z)} \frac{dR}{dz} \right],$$

is shown, together with the original time-averaged distribution. As can be seen, the computed microstructure, especially in the region of the narrow wave guide, closely follows the observed (continuous) curve. As was noted by LEONOV et al. (1977), even better agreement could have been achieved by varying the amplitude and frequency, and taking into consideration other vertical modes of $\phi$, but even the first approximation shows qualitative agreement, suggesting the generation of microstructure by non-linear waves.

In conclusion, it may be stated that the non-linear wave motion is a very important phenomenon in the sea. The soliton type internal waves, for example, many produce shear instability in the halocline, thus regulating the exchange processes between the upper and lower layers. The propagation of non-linear internal wave packages, as shown above, produces microstructure in the hydrophysical fields. It may be presumed that the self-similarity (universal) structure of both the salinity and temperature fields, and possibly also those of other substances, are due to the instabilities and microstructures produced by these non-linear waves.
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