To Draft or Not to Draft?
Efficiency, Generational Incidence, and
Political Economy of Military Conscription

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Abstract

We study the efficiency and distributional consequences of establishing and abolishing the draft in a dynamic model with overlapping generations, taking into account endogenous human capital formation as well as government budget constraints. The introduction of the draft initially benefits the older generation while harming the young and all future generations. Its Pareto-improving abolition requires levying age-dependent taxes on the young. These being infeasible, abolition of the draft would harm the old. The intergenerational incidence of the gains and losses from its introduction and abolition helps to explain the political allure of the draft.

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1 Introduction

Governments can recruit their manpower either by hiring workers at market wages or by compulsory labor service. Both methods rely on the government’s power to tax: either as an in-kind tax levied on drafted people in form of forced labor or as pecuniary taxes, raised to be spent on remunerating hired workers. Today’s democracies no longer rely on forced labor – with the notable exception of the military draft and its corollary, civil service.¹ Ten out of the 26 NATO members are still utilizing conscription, among them Germany, Turkey, Greece and the Baltic States.

The draft still heavily intrudes into the lives of young men in several Asian countries with spells of at least two years, in Russia and most other successor states of the defunct Soviet Union as well as throughout Latin America, the Arab World and the Middle East (where draft duration is generally between 24 and 36 months).² Israel, Eritrea and Tunisia deserve mention as, unlike the rest of the world, they also draw women into compulsory military service.

Even though most OECD countries have either abolished the military draft or are debating this, the possibility of re-introducing or maintaining the draft or more general compulsory labor service resurfaces, from time to time. In 2004, the United States issued stop-loss orders that kept thousands of reservists and National Guard members past their agreed terms in Iraq; several critics of President Bush argue that this amounts to a back-door draft. In France, where military conscription has been abolished since 1997, both the Socialist Party and the centre-right UDF advocate a compulsory civil service – an idea that gathered momentum as “un investissement républicain” in the wake of the street riots in fall 2005 (Libération, 2005).

At least since Adam Smith, economists have raised strong reservations about the draft and other forms of involuntary service (for recent surveys, see Sandler and Hartley, 1995, Chapter 6; Warner and Asch, 2001). Most ob-

¹ According to ILO (2001), there are three types of forced labor which involve government coercion (as contrasted to slavery, bonded labor or people trafficking which rely on private coercion): (i) compulsory labor, when people are required by law to work on public construction projects; as is practiced in Cambodia, the Central African Republic, Kenya, Burma (Myanmar), Sierra Leone, Swaziland, Tanzania, and Vietnam; (ii) military work, when civilians are forced by the government authorities to work for military purposes; as is widespread in Burma (Myanmar); (iii) prison labor, i.e., the contracting out of prison labor or the forcing of prisoners to work for profit-making enterprises: as is practised in China, Cote d’Ivoire, Madagascar, and Malaysia, but also in democratic countries such as Australia, Austria, France, Germany, New Zealand, and the United States.

² In North Korea, compulsory military service takes three to ten years, in South Korea 26-30 months, and in China and Taiwan 24 months. For a comprehensive listing of military systems throughout the world, see CIA (2006).
viously, relying on forced labor foregoes the benefits of specialization, as well as it fails to take into account differences in opportunity costs and comparative advantage. Staffing military or hospitals by unmotivated or underpaid draftees easily results in shirking and considerable loss of potential output. In countries like Russia, the army is plagued by a culture of violence against draftees, resulting even according to official reports in hundreds of deaths annually (The Economist, 2005). Less drastically, but still testifying of the significant costs of military draft, former conscripts usually suffer from lower earnings than those exempted from the draft (see Angrist, 1990, and Imbens and van der Klaauw, 1995). To the extent that these earnings reductions are due to a lower stock of human capital of ex-draftees, they constitute a sizable dynamic cost of the draft that will hit society as a whole – not just draftees (Lau et al., 2004).

Even if there were a consensus that the draft is an inefficient system, still a considerable opposition might resist its abolition. Older cohorts who have already delivered their military service, understandably, raise an objection that they would suffer a double burden in case of moving to voluntary forces: first as young draftees and then later in form of higher taxes to finance a professional army. In a democracy in which the majority of citizens has already passed the age of a draft (which typically is in the late teens and early twenties) and where future generations are not represented politically, a majority of voters might well support a draft even if it is inefficient from the perspective of steady-state generations. Correspondingly, middle-aged taxpayers might be tempted to introduce a draft system in order to escape the monetary tax burden of paying for a professional army, thereby neglecting dynamic costs that a draft imposes on future generations.

In this paper, we consider the economic and political dynamics of establishing and abolishing compulsory labor services. These compulsory labor services may consist not only of military draft, but also on work in the social sector or other activities deemed socially valuable. For the sake of simplicity we stick to the military terminology though. In short, we show that setting up a draft system (rather than running an all-volunteer army) distorts the accumulation of human capital, and forces young people to work when they should still be studying. Due to this inefficient timing of work and studies, the draft imposes a larger tax burden than collecting the same resources with distorting wage taxes. Therefore, the draft comes at the cost of a lower steady-state stock of human capital. We derive this result in a one-sector economy where the output can be used for civilian consumption as well as for military purposes. This simplification also implies that a distinction of various types of human capital (say, human capital acquired by military training and human capital accumulated in civilian education) is not needed.
The homogeneity assumption on both goods and human capital deliberately rules out all benefits from specialization but is clearly not realistic. Furthermore, our assumption that draft takes place before investment in education may be viewed as biased against the draft. We therefore extend the model to a two-sector economy with military and civilian production where individuals can acquire sector-specific productivity advantages by choosing their type of education, assuming that education takes place before an eventual draft. Our result that a draft system leads to a lower stock of human capital than a professional army holds also for the more complex economy.

If military draft has been established, it can always be abolished in a Pareto-improving manner by replacing it with age-dependent taxes, collected only from the young. With a positive interest rate, such frontloading of the tax burden results in a lower utility for steady-state generations than if taxes to finance a professional army would be collected from all age cohorts. Thus, it is impossible to fully undo the dynamic costs of once established draft systems again, without additionally hurting at least one generation.

Previous literature on the economic effects of the draft has largely used static and partial equilibrium models. In pioneering studies, Hansen and Weisbrod (1967) and Oi (1967) evaluate the distributive and allocative effects of the draft. They particularly highlight that the draft imposes special in-kind taxes and, due to low payment, implicit income taxes exclusively on the young adult part of the population. Unlike the present paper, these papers do not analyze the generational incidence of starting or ending a draft scheme. Taking as their starting point the U.S. draft scheme during the Vietnam War period, Hansen and Weisbrod (1967) as well as Fisher (1969) assess the cost of replacing compulsory conscription by an all-volunteer force. Both studies estimate substantial increases in budgetary needs in order to finance such a transition, but they do not embed this observation into a general equilibrium model with a government budget constraint, alternative ways of tax finance, and the repercussions of this on the rest of the economy.

Our paper analyzes the effects of starting and ending the draft when the wage tax rate is determined endogenously to balance the government budget constraint. Harford and Marcus (1988) and Lee and McKenzie (1992) study the effects of the draft in a static general equilibrium framework with exogenous productivities. In a dynamic general equilibrium model with human capital accumulation, Lau et al. (2004) derive estimates for the excess burden of the draft that results from its distorting effect on human capital decisions. The analysis is confined to comparing the steady states of economies with and without the draft. Unlike our paper, none of the studies so far has explicitly taken into account the effects of introducing or abolishing the draft on
transition generations.

Modes of recruiting military manpower not only differ in their opportunity costs but also in the direct costs of their administration and implementation. These costs are lower in countries with a well-developed and sophisticated administrative system. Mulligan and Shleifer (2005) offer this as an alternative explanation why some countries adopt a draft system while others do not; they find that countries with an administrative and legal system of French origin are more likely to draft than common law countries. Mulligan and Shleifer (2005) assume that each different conscription system (including the all-volunteer army) is the most efficient solution with some combination of the size of the military force and population. By contrast, we suggest that even if a draft system would be universally less efficient than a voluntary army, it might still be maintained as a political equilibrium due to the intergenerational incidence of tax burden.

Our paper is organized as follows. We introduce an overlapping generations model with a given public sector resource requirement and private investment in education in section 2. We derive steady states with and without a draft in section 3, and study transition in section 4. Section 5 extends the model to a two-sector economy with sector-specific types of education and an eventual draft taking place after private investment in education. It shows that our results also carry over to more complex economies. Section 6 concludes. All proofs are collected in an appendix.

2 Model

Consider an economy with two active overlapping generations. Every generation consists of the same number of ex-ante identical individuals; this number is normalized to one. If necessary, we index time by $t$. We refer to the generation that is in their youth in period $t$ as “generation $t$”.

The economy is a small and open one and there are perfect capital markets where consumption can be shifted over time at an exogenous interest rate of $r \geq 0$ per period.

In each of the two periods of his life, every individual has available a certain time endowment, again normalized to one. During youth a fraction $\alpha$ of the time endowment has to be spent for education. Moreover, the young may be called for service, lasting a fraction $d$ of their time endowment. The rest of the time is used for working and denoted by $\ell$. During working age, individuals work full-time.

Individuals are born with some innate human capital, the stock of which is normalized to unity. During their education period, individuals decide on
how much effort to spend on studying or training. Effort in studying increases the productivity of human capital according to a function $w = w(e)$. This function is strictly increasing, strictly concave, and satisfies $w(0) = 1$. Once acquired, the quality of human capital does not change over the life-cycle.

Individual’s preferences are separable in consumption over the two periods of life and effort for studying. With perfect capital markets, individuals are, as far as consumption is concerned, only interested in the net present value of income over their life-cycle. Effort in education generates a utility cost which in terms of consumption equals $c(e)$; the function $c$ is strictly increasing and convex.

There are two sectors in the economy: a private one and the military. Output $y$ in the private sector is produced by employing labor (measured in efficiency units) in a linear production technology. Thus, for output in period $t$ we obtain:

$$y_t = \ell_t \cdot w(e_t) + 1 \cdot w(e_{t-1})$$

where $\ell_t$ and 1 are the working hours of generation $t$ and $t-1$ in period $t$.

The military is run by the government, and doing this is the only task of the government. We measure output of the military in terms of private consumption and assume that its level is exogenously fixed at $\tilde{m} > 0$. We, thus, do not analyze the politically or economically optimal size of military expenditure but rather suppose that society considers $\tilde{m}$ as a suitable level of national defence and security.

To produce $\tilde{m}$, the government can choose between a professional army (all-volunteer force) and a draft system. In a professional army, the government buys from the labor market the number of labor units that is necessary to produce $\tilde{m}$. It finances military expenditure with a tax on income (private-sector output). Denoting the tax rate in period $t$ by $\tau_t$, government budget balance requires

$$\tau_t y_t = \tilde{m}. \quad (1)$$

With a professional army, the net-present value of an individual’s income is

$$w(e_t) \cdot \left[ (1 - \tau_t)(1 - \alpha) + \frac{(1 - \tau_{t+1})}{1 + r} \right]$$

where $w(e_t)$ is the individual’s wage rate and productivity, $1 - \alpha$ is working time during youth, and $1/(1+r)$ is the present value of an additional unit of

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3In reality, armies that only rely on draftees do not exist. All countries employ some professional soldiers to run their armies (and some professional nurses to staff hospitals and care units). Incorporating this aspect into our model could be done at the cost of some additional complexity. However, none of our results would be qualitatively affected by such a modification.
income during working age (recall that working time is one then). If the tax rate does not change over the life-cycle ($\tau_{t+1} = \tau_t$), we can write life-time income as

$$(1 - \tau_t) \cdot w(e_t) \cdot \Gamma$$

where

$$\Gamma = \frac{2 - \alpha - \alpha r + r}{1 + r} \leq 2 - \alpha$$

for all $r \geq 0$ and $0 \leq \alpha \leq 1$. Lifetime utility with a professional army then amounts to

$$u^p(e_t) = -c(e_t) + (1 - \tau_t) \cdot w(e_t) \cdot \Gamma.$$  

In a draft system, the government recruits a certain number of young individuals before these start their education and employs them in the production of military output for a certain amount of time $d$. Since the productivity of uneducated young is normalized to unity, the length of the draft necessary to produce output $\bar{m}$ is $d = \bar{m}$. As no resources other than labor are needed to operate the military, there is no need to set up a government budget constraint under this regime.

With a draft system, individual’s time spent for work during youth is $1 - \alpha - d$. Provided that (the young expect that) the draft will be maintained also in the next period and, hence, no income taxes are collected, lifetime income equals

$$w(e_t) \cdot \left[ (1 - d - \alpha) + \frac{1}{1 + r} \right] = w(e_t) \cdot (\Gamma - d)$$

and a utility level of

$$u^d(e_t) = -c(e_t) + w(e_t) \cdot (\Gamma - d)$$

will be realized.

3 Steady States

We now compare the steady-state equilibria of economies with a professional army and a draft system. Given that variables are time-invariant, we can omit time subscripts.

In an economy with a professional army, individuals choose $e$ as to maximize (3), thereby taking the tax rate as given. The first-order condition reads as

$$-c'(e) + (1 - \tau) \Gamma w'(e) = 0$$

(5)
and defines optimal education effort as a function of the tax rate. Obviously from (5), the higher taxes the lower educational effort (both $-c(e)$ and $w(e)$ are concave).

Steady-state national income with a professional army is

\[ y(e) = (2 - \alpha) \cdot w(e). \]  

(6)

In an equilibrium the tax rate $\tau$ has to be adjusted as to balance government budget; i.e., from (1) and (6),

\[ \tau = \frac{\bar{m}}{(2 - \alpha)w(e)}. \]  

(7)

The amount of educational investment $e^p$ in an economy with a professional army can therefore be determined from plugging (7) into (5); it is implicitly given by:

\[ -c'(e^p) + w'(e^p) \cdot \frac{(2 - \alpha)w(e^p) - \bar{m}}{(2 - \alpha)w(e^p)} \cdot \Gamma = 0. \]  

(8)

Denote by $V^p$ the steady-state utility arising from individual optimization in an economy with a professional army:

\[ V^p = u^p(e^p) = -c(e^p) + (1 - \tau^p)w(e^p) \]

where $\tau^p = \frac{\bar{m}}{(2 - \alpha)w(e^p)}$.

With a draft system, the optimal effort invested in human capital $e^d$ is determined from maximizing (4); the first-order condition reads as

\[ -c'(e^d) + w'(e^d) \cdot (\Gamma - \bar{m}) = 0 \]  

(9)

where we already substituted $\bar{m}$ for $d$. Denote by $V^d$ the maximum utility obtainable in a draft economy:

\[ V^d = u^d(e^d) = -c(e^d) + w(e^d)(\Gamma - \bar{m}). \]

A comparison of a drafted and a professional army yields the following results:

**Proposition 1** For all levels of military output $\bar{m}$, the effort into human capital and, therefore, national output and private consumption are lower in an economy with a draft system than in an economy with a professional army:

\[ e^d < e^p. \]  

(10)

Moreover, the utility level in the steady-state of an economy with a draft system always falls short of the utility level in the steady-state of an economy with a professional army:

\[ V^d < V^p. \]  

(11)
The first part of Proposition 1 shows that a draft system distorts the accumulation of human capital, relative to a professional army. Empirical evidence for this dynamic cost of the draft has been found by Angrist (1990) and Imbens and van der Klaauw (1995), and computational estimates for its considerable impact on national output are provided in Lau et al. (2004).

To see the intuition for this result, consider by how much military output $\bar{m}$ crowds out marginal incentives for human capital investment under the two regimes of draft and professional army. In the case of the draft, where $d = \bar{m}/w(0)$, we get from (4) that the impact of $\bar{m}$ on the marginal utility from education effort amounts to

$$-\bar{m} \cdot \frac{1}{w(0)} \cdot w'(e).$$

(12)

The disincentives result from the fact that the returns to human capital investment do not accrue over the full length of life but only for the remaining lifetime after draft $d$. In the case of a professional army, military output enters utility in (3) via the tax rate. The disincentives on education effort can be measured by

$$-\tau \cdot \left(1 - \frac{1}{1 + r}\right) \cdot w'(e) = -\bar{m} \cdot \frac{1 - a + \frac{1}{1 + r}}{2 - a} \cdot \frac{1}{w(e)} \cdot w'(e)$$

(13)

where we used the steady-state level of $\tau$ from (7). Comparing (12) and (13), disincentive effects on human capital investment are lower with a professional army than with military conscription for two independent reasons. First, there is a timing effect: The draft hits individuals in the early period of their lives while the burden from tax-financing a professional army is evenly spread over the life-cycle. This front-loading of the draft causes an extra burden which is reflected by the interest rate $r$ on the LHS of (13). The presence of $r$ renders the fraction smaller than one; with $r = 0$ front-loading would not matter and the timing effect disappear. Second, there is a level effect: While in the case of the draft military output is produced using labor of low productivity $w(0)$, it is effectively provided with (average) post-education productivity $w(e) > w(0)$ in the case of a professional army. Again, this leads to higher disincentives for education effort under a draft regime.

The second part of Proposition 1 establishes the superiority of a professional army over a drafted army in terms of steady-state utilities. It adds to a collection of results in the literature that the draft is an economically suboptimal arrangement for recruiting staff to the government sector (for a survey see, e.g., Sandler and Hartley, 1995, Chapter 6). However, these findings are based on static inefficiencies (forgone benefits of specialization,
inefficient job matches etc.). By contrast, Proposition 1 emerges from the intertemporal distortion of human capital investments.

4 Transition Dynamics

4.1 Introducing the draft

Suppose that prior to some date $t$ the economy was running a professional army, but that the government announces plans to introduce a draft system effectively of date $t$, before generation $t$ will decide on their study effort. We assume that this policy change could not be anticipated before period $t - 1$. Clearly, all generations $t'$ with $t' \leq t - 2$ will be unaffected (they are dead already at $t$). From Proposition 1 all generations $t'$ with $t' \geq t$ will suffer from the introduction of the draft, relative to the professional-army scenario. Generation $t - 1$ will, however, welcome the introduction of the draft since it will save the taxes it would, with a professional army, have had to pay during the second period of its working age. Its utility therefore is:

$$V_{t-1} = -c(e^p) + (1 - \tau^p)w(e^p)(1 - \alpha) + w(e^p)/(1 + r) > V^p.$$ 

Hence, introducing the draft is beneficial to those who are old in the period of introduction but harms all future generations.

4.2 Abolishing the draft

Suppose now that prior to some date $t$ the economy was running a conscription system but that the government announces plans to switch to a professional army effectively by date $t$. Again assume that this policy change could not be anticipated prior to period $t$. Can such a move be arranged in a Pareto-improving way?

Introducing a professional army requires (additional) tax revenues. A Pareto-improving transition requires that generation $t - 1$ (who has already delivered its military service under the draft system) must not be harmed by the new taxes. Hence, the taxes due in $t$ can only be levied on the then young generation $t$.

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4If the introduction of military draft were anticipated, then the last generation free from draft would invest more in education, anticipating that its second-period income would not be taxed. This would increase the utility gain of the old transition generation, without affecting the utility loss of the drafted generations.
The necessary tax rate \( \tau_t^a \) to finance abolition of the draft in period \( t \) emerges from the budget constraint:

\[
\tau_t^a \cdot (1 - \alpha)w(e_t) = \bar{m}
\]

where \((1 - \alpha)\) is the working time of generation \( t \) in period \( t \). In period \( t + 1 \), this generation will not pay any taxes. Hence, generation \( t \)'s life-time income amounts to

\[
w(e_t) \cdot \left[ (1 - \alpha)(1 - \tau_t^a) + \frac{1}{1+r} \right].
\]

The same applies to all future generations; we therefore omit time subscripts henceforth. Utility maximization then implies that educational effort satisfies:

\[-c'(e) + w'(e) \cdot \left[ (1 - \alpha)(1 - \tau_t^a) + \frac{1}{1+r} \right] = 0.\]

Plugging in the (time-invariant) tax rate \( \tau_t^a \) implicitly defines the equilibrium value of educational effort \( e^a \):

\[-c'(e^a) + w'(e^a) \cdot \left[ 1 - \alpha + \frac{1}{1+r} - \frac{\bar{m}}{w(e^a)} \right] = 0. \quad (14)\]

Denoting the attending utility level by \( V^a \), we obtain

**Proposition 2** For all levels of military output \( \bar{m} \), provided through a volunteer army, the effort into human capital and, consequently, output and consumption as well as the maximally obtainable utility level for the steady-state generations are never larger if the taxes needed to finance the army are levied exclusively on the young generation rather than spread across all cohorts:

\[ e^a \leq e^p \quad \text{and} \quad V^a \leq V^p. \quad (15)\]

Both inequalities are strict whenever \( r > 0 \).

The first part of Proposition 2 is driven by the same timing effect as its analogue in Proposition 1. Here, we are comparing two scenarios: one where taxes are levied at the same rate over the full life-cycle, and one where taxes are due only in the first period of one’s life. With a zero interest rate, the timing of taxes is irrelevant. With a positive interest rate, front-loading taxes entails a higher burden, even if the amount of taxes that the government raises at any point in time is identical. Lower after-tax return to education reduces private investment in education.\(^5\)

\(^5\)Unlike Proposition 1, the comparison of the two types of financing a professional army in Proposition 2 does not entail a level-effect. In both scenarios, military output is produced using labor of average productivity.
As could be expected, this also involves a loss in utility. Replacing the draft by a professional army that is financed by taxes only on the young leads to a situation that is worse for the steady-state generations than if the draft had never been introduced. Levying the tax on the young is, however, the only way to prevent the generation that is old when the transition from draft to professional army takes place from being harmed. Thus, the welfare of the steady-state generations is lower once the draft is introduced and again abolished than if it had never been there in the first place.

This result does not change if we allow the use of government debt during a transition. To see this, assume that the government has access to credit markets, and faces the same interest rate as individuals. Suppose further that it aims to organize transition from the draft to a professional army so that the young generation at the time of transition would be equally well off as the young generations in a steady-state without draft. This would, however, require public debt to reduce the lifetime tax burden of the initial young generation. As long as the government used public debt to keep the current young generation at the level of the steady-state generations without draft, public debt would accumulate. Once the government started to redeem its debt, the then young generations would be left with a lower utility level than without public debt.

In essence, the impossibility to completely undo the effects of a once-introduced draft system is due to the same mechanism as the impossibility to replace an inefficient pay-as-you-go pension scheme by a funded one: Like the introduction of a pay-as-you-go scheme, introducing a draft amounts to a “present” to the generation that is old at that moment. Such a gift may be revolved, but can never be accomplished such as to make everybody in the future equally well off as without the gift. Military draft differs from a pay-as-you-go social security scheme that could be viewed as an implicit government debt in one important aspect. Given that human capital cannot be transferred between generations and over time, there is no hope for neutrality in the sense of Ricardian equivalence with a draft system. The dynamic costs of the draft in the form of lower investment in human capital will persist even if the government would repay draftees afterwards the value of resources it has confiscated them. For the same reason, the negative impact that replacing wage taxes by draft has on the young and all subsequent generations cannot be undone even in a Barrovian dynasty.

Although abolishing the draft never can re-establish the situation with

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6If the government could borrow at a lower interest rate, then it would be optimal to issue debt to subsidize the first period wage and finance this by taxing the second period wage, independently of whether there is draft or professional army. Assuming that the government and individuals face the same interest rate excludes such free lunches.
a professional army, doing so is nevertheless advisable since it improves the situation relative to a perpetuation of a draft scheme. I.e., relative to a continuation of a draft system, abolishing conscription and entirely front-loading the fiscal burden of professionalization increases the utility levels of those who are young at the point of transition and of the steady-state generations after transition (while leaving those who are old at the time of transition unaffected):

**Proposition 3** Replacing a draft system with a professional army that is financed by taxes that are exclusively levied on the young leads to a Pareto-improvement. It also goes along with an increase in the human capital stock of the economy.

Taking stock of the results, we obtain:

$$V^p > V^a > V^d,$$

where the first inequality is strict if $r > 0$. Running a professional army, thus, is the best option society could choose. The second inequality implies that a draft is an inefficient way of transferring resources to the initially elderly generation. Should society choose to give the initially elderly a windfall gain at the expense of all future generations, it is less costly to do this by starting to levy age-dependent taxes. Should the mistake of introducing a draft system be committed, society will have to pay a price for that, at least if age-dependent taxes are not feasible: either it continues with the draft system, lowering utility levels to $V^d$, or one generation has to face a double burden, first being subject to the draft and then be additionally taxed when the draft is abolished. Taxing the older generation in the transition period for a second time (the first burden imposed on them was their conscription while young) would set the economy on the non-draft time path and would deliver the corresponding high utility for all future generations, in exchange for lower utility for the transition generation. If the transition from the draft system to a volunteer army is organized such as not to impose additional harm on those who have already been hurt by the draft, the tax costs of abolishing the draft are fully loaded on young generations by imposing age-dependent taxes. This would involve steady-state utility that falls in between the levels of perpetual volunteer and conscription systems.

We assumed that the abolition of military draft was not anticipated. If it were, then the last drafted generation would further reduce its investment in education, anticipating the double burden. With age-dependent taxes collected only from the young, anticipating the abolition of draft would not affect older generations drafted themselves.
We analyzed the transition from an all-volunteer force to an army of conscripts or vice versa in a model with only two overlapping generations. Correspondingly, the economy switches from one steady state to the other within just one period. However, this should not be seen as a limitation for the validity of our findings: In a stable $n$-OLG model (where $n > 2$), the transition would take at most $n - 1$ periods – until the first generation affected by the policy change has perished. Transitional dynamics might be more complex then, but the effects and the generational incidence of switching in the recruiting regime would be qualitatively identical.

5 Extension: Military and Civilian Careers

5.1 Modification of the Model

The set-up so far has been based on two simplifying assumptions:

- Although the economy comprises a private and a military sector, education has so far not been sector-specific. This rules out all benefits from specialization and, thus, eliminates one of the classical arguments for running a professional army. However, the assumption also ignores the potential costs arising when individuals switch from one sector to another without having acquired an education that is suited best for the sector into which they enter. This is particularly relevant for professional soldiers most of whom move to jobs in the civilian sector after finishing their military contract.

- We assumed that draftees enter into military service as uneducated individuals with low productivities while a professional army produces its output with educated individuals of average productivity. This assumption (which is responsible for the level effect identified in Section 3) might be regarded as unfairly biased against military conscription. Moreover, it is questionable in so far as in reality (i) draftees are called to service after having finished at least parts of their education and (ii) professional soldiers and draftees do, at least upon entrance into the army, not differ in age and, thus, their human capital.

In this section we dispense with these assumptions. Corresponding with the two-sector economy, individuals can now choose between two types of education: “military” and “civilian”. Choosing an educational type means acquiring a comparative advantage for the respective sector.
In the case of a draft system, people invest in their civilian education before being drafted to the army for a time \( d \) (the rest of the first period is spent with work in the civilian sector). In case of professional army, people invest either in civilian or military education and then decide on the sector to which they want to become employed.

Independently of its type, education is equally costly in terms of effort. Each type of education is applicable in either sector, but possibly with reduced productivity if the types of sector and education do not match. Wages in the civilian sector equal marginal productivities in the civilian sector. However, the market values of military and civilian human capital may differ, and the government may decide to deviate from productivity wages if it recruits soldiers to the army.

With a professional army, people can choose two careers: “military” and “civilian”. “Military career” means to work as a professional soldier in the first period of life and then switch to the civilian sector in the second; “civilian career” means to work in the civilian sector for both periods of life.

As before, individuals decide on the education effort \( e \). Incomes over the life-cycle depend on the choices of career and of education type:

- If education type is “civilian” and career is “civilian”, wage rates in the first and second period are given by \((w(e), w(e))\).
- If education type is “military” and career is “military”, wage rates in the first and second period are given by \((\gamma w(e), (1 - \beta)w(e))\) where \(\beta, \gamma > 0\) and \(\gamma\) is the wage rate that the army pays per unit of effective labor.
- If education type is “civilian” and career is “military”, wage rates in the first and second period are given by \(((1 - \beta)\gamma w(e), w(e))\). We assume that the relative productivity loss \(\beta\) resulting from a mismatch between education and employment is the same here as in the previous item.

Output in the military sector for those with military education is \(w(e)\) and \((1 - \beta)w(e)\) for those with civilian education.

Abusing terminology, we can thus call it a “military [civilian] career” if the individual chooses the military [civilian] option for both education and career, and “mixed career” if someone first acquires civilian education but then enters the military career.

\footnote{Our assumptions below ensure that, with a draft system, investing in military education is always dominated in terms of private returns by investing in civilian education.}
The lifetime utilities in, respectively, a civilian, military, or mixed career are given by

\[
U_c(e) = -c(e) + (1 - \tau) \left[ 1 - \alpha + \frac{1}{1 + r} \right] w(e) \quad (16)
\]

\[
U_m(e) = -c(e) + (1 - \tau) \left[ \gamma(1 - \alpha) + \frac{1 - \beta}{1 + r} \right] w(e) \quad (17)
\]

\[
U_{cm}(e) = -c(e) + (1 - \tau) \left[ (1 - \alpha)(1 - \beta) \gamma + \frac{1}{1 + r} \right] w(e). \quad (18)
\]

In case of a professional army, we assume that the government can decide whether to hire those with military or those with civilian education in the military. We will discuss both options in turn.

### 5.2 Hiring Civilians to the Military

In this section, we assume that the government chooses to hire only those in the military who have completed a civilian education. Clearly, this means that potential gains from utilizing specialized human capital in the army are forgone. Yet, comparing such a scenario with a draft regime (which also employs individuals with civilian education) enables us to show the inferiority of the latter.

In order to compensate soldiers with civilian education for the fact that only the fraction \(1 - \beta\) of their civilian human capital is applicable as military human capital, the government has to set

\[
\gamma = \frac{1}{1 - \beta}. \quad (19)
\]

With this choice, the lifetime utilities of those in civilian and in mixed careers coincide. Civilians and soldiers who acquire civilian education choose the same optimal education effort, \(e^c = e^{cm}\), and an occupational equilibrium with \(U^c(e^c) = U^{cm}(e^{cm})\) is reached. We obtain\(^8\)

**Proposition 4** At all levels of military output \(\tilde{m}\) and positive interest rates \(r > 0\), the government can choose a recruiting strategy for a professional army so that the effort into human capital and, therefore, national output and private consumption are higher in this economy than with a draft system.

Hence, a professional army that employs soldiers with civilian education always dominates a draftee army. The reason is the timing effect, i.e., the

\(^8\)We are grateful to an anonymous referee for indicating this result to us.
specific incidence of the draft tax in the first period of the life-cycle. For \( r = 0 \), a professional army staffed with civilians and a draft system are equivalent; the “level effects” of Section 3 are absent due to the fact that professional soldiers and draftees are equally well educated. Proposition 4 can be regarded as an analogue to the second part of Proposition 2 as military draft in the current scenario can essentially be viewed as an age-specific tax imposed on younger cohorts.

5.3 Hiring Soldiers to the Military

To realize benefits from specialization, individuals with military (rather than with civilian) education should be recruited to a professional army. However, this comes at the drawback that, after finishing their military contract, these individuals will work in the civilian sector for which their human capital is less than ideally suited. In this section, we establish sufficient conditions such that a professional army nevertheless dominates a conscripted army.

Again, an occupational equilibrium requires that all combinations of education and career path that are actually chosen by some individual yield the same life-time utility. If the government pays a wage rate of

\[
\gamma = 1 + \frac{\beta}{(1 - \alpha)(1 + r)}
\]

in the military sector, then the bracketed terms in utility functions (16) and (17) coincide. Hence, civilians and soldiers choose the same optimal amount of education effort, \( e^c = e^m \), and an occupational equilibrium is reached, \( U^c(e^c) = U^m(e^m) \).

Proposition 5 An economy that applies compensation strategy (20) to attract individuals with military training to its professional army will induce a higher effort into human capital than an otherwise identical economy that uses a draft system (i.e., \( e^c \geq e^d \)) if and only if

\[
(1 - \beta) \left[ 1 + \frac{\beta}{(1 - \alpha)(1 + r)} \right] \leq \frac{2 - \alpha - \frac{\bar{w} \beta}{w(e^c)(1 - \alpha)(1 + r)}}{1 - \alpha + \frac{1}{1 + r}}.
\]

As it generally includes \( w(e^c) \), one cannot properly analyze condition (21) without knowing the comparative statics of \( e^c \) in the model parameters.

\( ^9 \)Note that choosing to pay wage \( \gamma \) in the military is only one option for the government when running a professional army. In principle, it could choose also a different \( \gamma \) in which case those choosing a military career would choose a different investment in military human capital.
Holding \( e^c \) parametrically fixed, however, condition (21) is more likely to hold (and, thus, a professional army will more likely induce higher effort in human capital investment than a draft system) if the military budget \( \tilde{m} \) is small. The impact of the other parameters \( \alpha, \beta, r \) is unclear in general.

There are, however, three interesting cases which can be analyzed safely:

**No productivity differences** \((\beta = 0)\): It is easy to verify that, for \( r = 0 \), we obtain \( e^c = e^d \) (no difference between professional army and draft system) while for \( r > 0 \) we have \( e^c > e^d \). The negative impact of the draft system here results from the timing effect identified in Section 3.

**No time cost from studying** \((\alpha = 0)\): In the Appendix we show that inequality (21) strictly holds for all \( r, \beta \geq 0 \). By a continuity argument, a professional army will thus imply higher levels of human capital than military conscription as long as time costs of studying are small, relative to the length of first working period in the life cycle.

**Zero interest rate** \((r = 0)\): Here, (21) simplifies to

\[
e^c \geq e^d \iff \alpha \leq \beta.
\]

Hence, a draft system will lead to higher investments in human capital than a professional army hiring soldiers with military education only if productivity differences between military and civilian employment are small or if professional life (relative to education time \( \alpha \)) is short in the first period. Recall, however, that a professional army hiring soldiers with civilian education would dominate also in these cases. Also, recall that we restricted the analysis to the case in which the government would choose the pay in the military such that civilians and soldiers invest equally in education. In some cases the government may choose a compensation package in which this is not the case. However, choosing such compensation package would only be optimal if it further improved welfare with a professional army, as compared with an army of draftees.

With respect to transition dynamics and political allure, the scenarios presented in this section do not lend to further insights, compared to those in Section 4: Even though the draft system is inferior to the professional army, the older generations will still advocate its introduction and object to its abolition.

We derived all our results assuming individuals to be ex ante identical. Accounting for individual heterogeneity would further strengthen the case for professional army as this would allow to exploit differences in comparative advantage.
6 Conclusion

In this paper, we analyze efficiency and distributional implications of the military draft and other compulsory work services. A specific and, to our knowledge, novel aspect of our paper is its focus on the introduction and abolition of such schemes. We adopt a dynamic framework, taking into account that both the draft and levying wage taxes affect individual incentives to invest in education. The draft forces young people to work for the government, thus postponing their education and entry to the labor market, and shortening their remaining working career. Wage taxes reduce the after-tax return to education, thus also discouraging investment in education.

We show that even if the draft were not plagued by inefficient matches between people and jobs, the lack of specialization, or other static inefficiencies it would still be a worse solution for steady-state generations than levying wage taxes to acquire the same labor input in market wages. The initially older generation, however, gains from the introduction of the draft since it will save the taxes it would, with a professional army, have had to pay. All future generations would lose. Abolishing the draft allows to reduce welfare losses and can always be implemented in a Pareto-improving manner. However, the utility available to steady-state generations after a draft system has been abolished in a Pareto-improving manner still falls short of their utility if a draft system had not been introduced in the first place. The reason for this is that a Pareto-improving elimination of draft requires collecting taxes only from the young, so as not to levy a double burden on the elderly who have already been subject to the draft. Importantly, our results generalize to the case of different types of education and also hold when military draft takes place only after education has been completed. The key to the latter observation is the timing effect. The net present value of the burden that draft imposes exceeds the net present value of wage taxes needed to purchase the same services at market wages.

The draft can be understood and (mis-)used as a device for intergenerational redistribution, as it one-sidedly levies parts of the costs for the provision of government services on the young generations. In ageing European societies that, due to pay-as-you-go financing of pensions and health care, already load the lion’s share of the burden of demographic transitions on younger generations, draft systems acerbate the intergenerational imbalances. It may well be questioned whether – apart from being unnecessarily costly – compulsory military or social services pass any meaningful test for intergenerational fairness.

The specific intergenerational incidence may help to explain the political allure of military draft and its corollaries. In our model with two overlapping
generations, the older cohort benefits from introducing the draft. Moreover, once a draft scheme is installed, its abolition would harm the older generation, at least if one reasonably assumes that levying age-specific taxes is not feasible. The gist of these observations easily extends to models with larger numbers of overlapping generations (which may possibly be of unequal sizes). Since age cohorts beyond the draft age typically outnumber younger cohorts at or below the draft age, both the introduction and the continuance of military draft garner widespread political support – in spite of their inefficiency.

Our approach focused on the economic aspects of military draft, in particular on its dynamic costs. Advocates of military draft tend to refer to its political virtues: embedding democratic controls in the army, reducing the likelihood of war etc. Empirical evidence on these aspects seems to be mixed, to say the least. Analyzing militarized interstate disputes from 1886 to 1992, Choi and James (2003) find that a military manpower system with conscripted soldiers is associated with more military disputes than professional or voluntary armies. Based on cross-sectional data from 1980, Anderson et al. (1996) conclude also that “warlike” states are more likely to employ conscription.

In recent years, quite a number of countries have abolished military draft (while, e.g., similar reforms in pay-as-you-go pension schemes are still unheard of). This seems to contradict our proposition that conscription is, ceteris paribus, unlikely to disappear once installed. However, it should be noticed that the abolition of military draft in many countries paralleled other changes in the social, military, and geopolitical environment. Standing armies for territorial defense have become increasingly obsolete, technological changes have rendered warfare less labor-intensive, and many countries have reduced their military expenditures since the end of the Cold War. All this made the transition from draft to professional army less costly for those opposing it. In our model, these effects could be captured by an increase in the specialization parameter $\beta$ or by a reduction in $\tilde{m}$, the numéraire-equivalent of military output. Moreover, we derived our result in the absence of other intergenerational transfer institutions, most notably pay-as-you-go pensions. These give the older generation a stake in the future productivity of the current young. Therefore, an increase in pensions and health-care costs may well have motivated political support to abolish military draft, as the elderly also share part of the burden through reductions in the tax base for other transfers. A detailed analysis and empirical testing of the link between draft

\footnote{Indeed, the share of military expenditure in GDP for France, Belgium, Italy, The Netherlands, and Spain (all countries that abolished conscription) decreased from 3.5, 2.4, 2.1, 2.5, 1.8 percent in 1989 to, respectively, 2.6, 1.3, 1.9, 1.6, and 1.2 percent in 2002.}
and other intergenerational transfer institutions, however, is left for future research.

Appendix

Proof of Proposition 1

Denote the function on the RHS of (9) by $\phi(e)$; it is strictly decreasing in $e$ and $\phi(e^d) = 0$. Evaluate $\phi$ for $e = e^p$ and replace $-c'(e^p)$ from (8):

\[
\phi(e^p) = -c'(e^p) + w'(e^p) \cdot (\Gamma - \bar{m}) = w'(e^p) \cdot \left[ \Gamma - \bar{m} - \Gamma \cdot \frac{(2 - \alpha)w(e^p) - \bar{m}}{(2 - \alpha)w(e^p)} \right].
\]

The square-bracketed expression is negative if and only if

\[
w(e^p) > \frac{\Gamma}{2 - \alpha}.
\]

This condition always holds due to (2) and $w(e)$ being increasing in $e$ and therefore $w(e^p) > w(0) = 1$. We, thus, have $\phi(e^p) < 0 = \phi(e^d)$ and the claim $e^d < e^p$ follows from the strict monotonicity of $\phi$.

As productivity increases in $e$ and the government always takes away the same absolute amount $\bar{m}$ of resources for the military, the assertions for output and consumption follow immediately.

To establish the utility comparison, denote by $\tau^p$ the equilibrium tax rate in an economy with a professional army. Then the following inequalities hold:

\[
V^p = -c(e^p) + (1 - \tau^p)w(e^p)\Gamma \\
\quad \geq -c(e^d) + (1 - \tau^p)w(e^d)\Gamma \\
\quad = -c(e^d) + w(e^d)\Gamma - \frac{\bar{m}}{(2 - \alpha)w(e^p)}w(e^d)\Gamma \\
\quad > -c(e^d) + w(e^d)(\Gamma - \bar{m}) = V^d.
\]

The second line is by a revealed-preference argument, the third replaces the budget-balancing tax rate $\tau^p$, and the fourth follows from (10).

Proof of Proposition 2

Denote the function on the RHS of (14) by $\psi(e)$. Observe that $\psi(e^a) = 0$ and that $\psi$ is strictly decreasing around $e^a$: $\psi'(e^a) < 0$.\footnote{As $e^a$ is a utility-maximizing choice, this follows from the second-order condition.} Evaluate $\psi$ for...
\( e = e^p \) and replace \(-c'(e^p)\) from (8):

\[
\psi(e^p) = -c'(e^p) + w'(e^p) \cdot \left[ 1 - \alpha + \frac{1}{1 + r} - \frac{\bar{m}}{w(e^p)} \right] \\
= w'(e^p) \cdot \frac{\bar{m}}{w(e^p)} \cdot \left( \frac{\Gamma}{2 - \alpha} - 1 \right) \leq 0 = \psi(e^a)
\]

where the non-positive sign of \( \psi(e^p) \) follows from (2). The sign will be strictly negative whenever \( r > 0 \). We therefore have \( e^p \geq e^a \) due to the local monotonicity of \( \psi \); the inequality being strict in the presence of discounting.

As productivity increases in \( e \) and the government always takes away the same absolute amount \( \bar{m} \) of resources for the military, the assertions for output and consumption follow immediately. For utilities we get the following (in-)equalities:

\[
V^p = -c(e^p) + (1 - \tau^p)w(e^p)\Gamma \\
\geq -c(e^a) + (1 - \tau^p)w(e^a)\Gamma \\
= -c(e^a) + w(e^a) \left( 1 - \alpha + \frac{1}{1 + r} - \frac{\Gamma}{2 - \alpha} \cdot \frac{\bar{m}}{w(e^p)} \right) \\
\geq -c(e^a) + w(e^a) \left( 1 - \alpha + \frac{1}{1 + r} - \frac{\bar{m}}{w(e^p)} \right) \\
\geq -c(e^a) + w(e^a) \left( 1 - \alpha + \frac{1}{1 + r} - \frac{\bar{m}}{w(e^a)} \right) = V^a.
\]

The second line follows by a revealed-preference argument, the third replaces the budget-balancing tax rate \( \tau^p \) and expands terms, the fourth follows from (2), and the fifth from \( e^a \leq e^p \). The inequalities in the fourth and fifth lines will be strict whenever \( r > 0 \).

\textbf{Proof of Proposition 3}

Calculate:

\[
V^a - V^d = -c(e^a) + c(e^d) + w(e^a) \left( \Gamma - \frac{\bar{m}}{w(e^a)} \right) - w(e^d) \left( \Gamma - \bar{m} \right) \\
= \left[ -c(e^a) + w(e^a) \left( \Gamma - \frac{\bar{m}}{w(e^a)} \right) + c(e^d) - w(e^d) \left( \Gamma - \frac{\bar{m}}{w(e^d)} \right) \right] \\
+ \left( w(e^d) - 1 \right) \cdot \bar{m}.
\]

The expression in square brackets in the second line is non-negative as \( e^a \) maximizes \(-c(e) + w(e) \left( \Gamma - \frac{\bar{m}}{w(e)} \right) \). The bracketed expression in the third line is positive since \( w(e^d) > 1 \). Hence, \( V^a > V^d \).
To show that human capital investment will be lower in an economy with a draft than in one with an abolished draft, we evaluate the function \( \phi(e) \) from the proof of Proposition 1 at \( e^a \):

\[
\phi(e^a) = -c'(e^a) + w'(e^a) \cdot (\Gamma - \bar{m}) \\
= w'(e^a) \cdot \left[ -\Gamma + \frac{\bar{m}}{w(e^a)} + \Gamma - \bar{m} \right] \\
= -w'(e^a) \cdot \bar{m} \cdot \left( 1 - \frac{1}{w(e^a)} \right) < 0 = \phi(e^d).
\]

Hence, \( e^a > e^d \) due to the strict monotonicity of \( \phi(e) \).

**Proof of Proposition 4**

With compensation scheme (19), the FOC for \( e^c \) (and thus also \( e^{cm} \)) reads:

\[
-c'(e^c) + (1 - \tau)w'(e^c) \left[ 1 - \alpha + \frac{1}{1 + r} \right] = 0.
\]

Let \( g \in (0, 1) \) be the fraction of individuals choosing to be a soldier. National income at a point in time comprises the output produced by the young plus output produced by the old. In a steady state where, in all periods, everybody chooses identical education effort:

\[
y(e^c) = [g(1 - \alpha)(1 - \beta)\gamma + (1 - g)(1 - \alpha) + g + (1 - g)] w(e) \\
= (2 - \alpha)w(e)
\]

where \( \gamma \) was replaced by \( (1 - \beta)^{-1} \). The exogenous military output \( \bar{m} \) must be produced and financed. Production in the army requires \( \bar{m} = gw(e^c)(1 - \alpha)(1 - \beta) \), leading to an army of size

\[
g = \frac{\bar{m}}{w(e^c)(1 - \alpha)(1 - \beta)}.
\]

Financing the army by a linear income tax requires \( \gamma \bar{m} = \tau y \) or:

\[
\frac{1}{1 - \beta} \bar{m} = \tau(2 - \alpha)w(e^c).
\]

Hence, solving for \( \tau \), we obtain:

\[
\tau = \frac{\bar{m}}{(2 - \alpha)(1 - \beta)w(e^c)} =: \tau^c.
\]
Plugging $\tau$ into the FOC gives:

$$-c'(e^c) + (1 - \tau^c) \left[ 1 - \alpha + \frac{1}{1 + r} \right] w'(e^c) = 0,$$

or, after, replacing $\tau^c$ and re-arranging terms,

$$c'(e^c) = \left[ 1 - \alpha + \frac{1 + r}{1 + r} - \frac{\bar{m}}{(1 - \beta)w(e^c)} + \frac{\bar{m}}{(2 - \alpha)(1 - \beta)w(e^c)} \frac{r}{1 + r} \right] w'(e^c)$$

which implicitly defines the equilibrium value of $e^c$.

Draftees deliver military service of length $d$ after acquiring their full civilian education. Their lifetime utility equals

$$U^c(e) = -c(e) + \left[ 1 - \alpha - d + \frac{1}{1 + r} \right] w(e),$$

and their optimal investment in civilian education is derived from

$$-c'(e^d) + \left[ 1 - \alpha - d + \frac{1}{1 + r} \right] w'(e^d) = 0.$$

Here, the length of the draft spell necessary to produce military output $\bar{m}$ is determined by $\bar{m} = d(1 - \beta)w(e^d)$.

Define

$$\psi(e) := -c'(e) + \left[ 1 - \alpha + \frac{1}{1 + r} - \frac{\bar{m}}{(1 - \beta)w(e)} \right] w'(e). \quad (22)$$

The equilibrium value of $e^d$ is then implicitly defined by $\psi(e^d) = 0$. Observe that

$$\psi'(e) := -c''(e) + \left[ 1 - \alpha + \frac{1}{1 + r} - \frac{\bar{m}}{(1 - \beta)w(e)} \right] w''(e) + \frac{\bar{m}w'(e)^2}{(1 - \beta)w(e)^2}$$

This is negative at least for small values of $\bar{m}/w(e)$. This holds for democratic countries in peacetime, where military output is small, relative to total output. We henceforth assume that $\psi'(e) < 0$. Then,

$$e^e \geq e^d \iff \psi(e^e) \leq 0.$$

Now evaluate $\psi$ at $e^c$:

$$\psi(e^c) = -c'(e^c) + \left[ 1 - \alpha + \frac{1}{1 + r} - \frac{\bar{m}}{(1 - \beta)w(e^c)} \right] w'(e^c)$$

$$= -\frac{\bar{m}}{(2 - \alpha)(1 - \beta)w(e^c)} \frac{r}{1 + r} w'(e^c),$$

which is negative as long as $r > 0$. Hence, $e^c > e^d$. The assertions for output and consumption follow immediately.
Proof of Proposition 5

With \( g \in (0, 1) \) as the fraction of individuals choosing to be a soldier, steady-state national income when everybody chooses the same education effort amounts to:

\[
y(e^c) = [g(1 - \alpha)\gamma + (1 - g)(1 - \alpha) + g(1 - \beta) + (1 - g)] w(e) \\
= \left[ 2 - \alpha - \frac{gr\beta}{1 + r} \right] w(e)
\]

where \( \gamma \) was replaced by (20) in the second line. Production of the exogenous military output \( m \) requires an army of size

\[
g = \frac{\bar{m}}{w(e)(1 - \alpha)}.
\]

Using this and the definition of \( y \), the tax rate necessary to finance the army by a linear income tax (\( \gamma \bar{m} = \tau y \)) turns out to be

\[
\tau^c := \frac{\left[ 1 + \frac{\beta}{(1 - \alpha)(1 + r)} \right] \bar{m} \left[ 2 - \alpha - \frac{\bar{m}r\beta}{w(e^c)(1 - \alpha)(1 + r)} \right] w(e)}{1 - \alpha + \frac{1}{1 + r}}.
\]

The FOC for \( e^c \) (and thus also \( e^{cm} \)) reads:

\[
-c'(e) + (1 - \tau)w'(e) \left[ 1 - \alpha + \frac{1}{1 + r} \right] = 0.
\]

Inserting \( \tau^c \) implicitly defines the equilibrium value of \( e^c \):

\[
-c'(e^c) + (1 - \tau^c) \left[ 1 - \alpha + \frac{1}{1 + r} \right] w'(e^c) = 0.
\]

For a draft system, we can refer to the proof of Proposition 4. Evaluating \( \psi \), as defined in (22), at \( e^c \) yields:

\[
\psi(e^c) = -c'(e^c) + \left[ 1 - \alpha + \frac{1}{1 + r} - \frac{\bar{m}}{(1 - \beta)w(e^c)} \right] w'(e^c)
\]

\[
= \left\{ \tau^c \left[ 1 - \alpha + \frac{1}{1 + r} \right] - \frac{\bar{m}}{(1 - \beta)w(e^c)} \right\} w'(e^c)
\]

\[
= \left\{ \frac{1 + \frac{\beta}{(1 - \alpha)(1 + r)}}{2 - \alpha - \frac{\bar{m}r\beta}{w(e^c)(1 - \alpha)(1 + r)}} \cdot \left[ 1 - \alpha + \frac{1}{1 + r} \right] - \frac{1}{1 - \beta} \right\} \frac{w'(e^c)\bar{m}}{w(e^c)}
\]

where the second line used the implicit definition of \( e^c \). The condition \( \psi(e^c) \leq 0 \) is equivalent to (21).
The case $\alpha = 0$ (Section 5)

For $\alpha = 0$, condition (21) is equivalent to

$$\frac{\bar{m}r\beta}{w(e^c)(1 - \beta)} + (2 + r)\left[1 + \frac{\beta}{1 + r}\right] \leq \frac{2(1 + r)}{1 - \beta}.$$

In order to be able to produce military output by those with civilian education and living in their first period, it has to be that $\bar{m} < (1 - \beta)w(e^c)$. The inequality above is thus strictly satisfied at least if

$$(1 + r)(r\beta(1 - \beta) - 2(1 + r)) + (2 + r)(1 - \beta)(1 + r + \beta) = -r(1 + \beta + r) - \beta^2(2 + 2r + r^2)$$

is non-positive. This holds for all $\beta, r \geq 0$.

References


