ON THE STRUCTURE AND MECHANICS OF PACK ICE IN THE BOTHNIAN BAY
MATTI LEPPÄRANTA

ON TWO-PEAKED WAVE SPECTRA
KIMMO K. KAHMA

STUDIES ON NITROGEN FIXATION IN THE GULF OF BOTHNIA
ILKKA RINNE, TERTTU MELVASALO, ÅKE NIEMI AND LAURI NIEMISTÖ

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ABSTRACT
Theoretical and empirical studies are made for the March—April period when thermal changes of ice mass are small. The ice pack in the research basin is fractured into separate floes the diameters of which are, considering the relative areal coverage, fairly evenly distributed from tens of meters to 4—5 km. The thickness of level ice is \(\sim 1 \text{ m} \) and the amount of deformed ice is comparable. Rates of deformation of pack ice are \(\sim 10^{-6} \text{ s}^{-1} \) and their temporal scale is several hours. Ice drift follows the wind rather well with a response time shorter than one hour. Ice velocity spectra show no clear peak at the Coriolis period. The governing forces in ice drift are the surface shear stresses of wind and water on ice and the internal friction within the ice. Estimates of the bulk and shear viscosities of pack ice vary in the range \(10^7 \text{ to } 10^{10} \text{ kg s}^{-1} \), while the compactness is 0.89—0.95. Largest viscosities result when the compactness is at its highest and at the same time either deformation rates are small or ice is ridging. Dissipation of kinetic energy in internal deformation processes is significant, and the main energy sinks are the friction in shearing between ice floes and the production of potential energy in ice ridges; the former is several times larger than the latter.
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1. INTRODUCTION

From the geophysical viewpoint a sea ice cover is a thin layer of complicated material between the atmosphere and the ocean, and is an important link in the exchange of energy between the two. The term "pack ice", as defined in the nomenclature of the World Meteorological Organization (WMO 1970), is used in a wide sense to include any area of sea ice other than fast ice, which forms and remains fast along the coast. Pack ice drifts and its material structure changes due to dynamic and thermodynamic forcing from the atmosphere and the ocean.

The present work consists of studies of the structure and mechanics of pack ice in the Bothnian Bay during the spring period, when thermal growth and decay of ice are small. This work is part of a long-term program which began in the early 1970's and aims at developing methods for sea ice forecasting in the Baltic Sea.

1.1. REVIEW OF EARLIER RESEARCH IN THE BALTIC SEA

For many years information on ice conditions in the open sea has been obtained from icebreakers but little of this material was saved during the war. Therefore, the best earlier data is available from reconnaissance flights in the 1930's. More extensive observations were realized under the auspices of the Baltic Ice Week in 12—18 February 1938 (Granqvist 1938). Unfortunately, as this winter was mild, there was very little ice to observe. But it was made clear that the nature of the ice pack in the Baltic Sea is dynamic; thus, even though it may remain stationary during calm frosty periods, it is movable throughout the whole winter.

A detailed description of the dynamical features of the pack ice in the Central Baltic was given by Palosuo (1953). His data was mainly based on reconnaissance flights during the severe ice season 1941/42. In the 50's and 60's sea ice research was concerned with climatology and small scale properties of ice (e.g., Rodhe 1952, Palosuo 1963 and 1965a), and our knowledge of pack ice mechanics remained of qualitative nature. A mention must be made of the paper of Lisitzin (1957), which showed that water level variations are considerably damped in winter in the Bothnian Bay due to the presence of the ice cover.

Attention was first concentrated on the mechanical state of pack ice, when research on the structure of ice ridges began in the late 60's (Palosuo 1975). New insight into the ice conditions over the entire Baltic Sea was provided by satellite images (Brosin & Neumeister 1972). In the 70's, sea ice studies increased vigorously due to the needs arising from the expanding winter navigation in Finnish and Swedish waters. Pack ice mechanics thus became a main field of research.

Development of models for short-term ice forecasting started (Udin & Ullerstig 1973, Valli & Leppäranta 1975) and the first field study on ice motion was performed (Omstedt et al. 1974). An extensive remote sensing experiment was done in the Bothnian Bay in March 1975 (Blomquist et al. 1976). Soviet scientists studied ice motion from aerial photographs.
(Shirokov 1977) and a field study on mechanical properties of ice was undertaken by the Japanese (Tabata 1975). To improve our understanding of pack ice mechanics more field experiments were performed in the late 70's and theoretical studies commenced (Udin & Omstedt 1976; Joffre 1978; Leppäranta 1979, 1980b, 1981). Operative short-term ice forecasting began in Finland in the ice season 1976/77 (Leppäranta 1977) and the results so far have been satisfactory (Leppäranta 1980a).

1.2. PRESENT STAGE OF PACK ICE MECHANICS

Most of the research work is, and has been, done by American and Soviet scientists in the Arctic seas. Recent progress has been reviewed by Doronin & Kheisin (1975) and Hibler (1980). A considerable amount of work has been done in the lower latitudes, where seas are ice-free during the summer, e.g., in the Baltic Sea and in the Okhotsk Sea (Tabata 1972). As was noted already by Zubov (1945), such easily accessible seas could and should be studied in order to increase our understanding of pack ice in general.

The basic mechanical feature of a pack ice cover is that the ice is fractured into separate floes, the interactions of which give rise to the mechanical properties of pack ice on the geophysical scale. There is still much, in the first place the break-up phenomena of ice floes, which is not well understood. Problems in connection with the different scales of phenomena were discussed by Rothrock (1975b, 1979). The mechanical state of pack ice is presently described by a thickness distribution function (Thorndike et al. 1975). Although the dynamic continuity condition of this function is not yet completely solved, it is generally believed that the stress field within the ice pack can be treated as a function of the strain-rates and the thickness distribution. The kinetic energy budget is becoming an important point in the research (Coon & Pritchard 1979), but no previous analysis of field data exists.

One of the main questions in pack ice mechanics is the constitutive law on the geophysical scale. Commonly, this is assumed to be a viscous law based on the general form of Glen (1970). An exception is the elastic-plastic law of Coon et al. (1974), which has its origins in the small scale behaviour of pack ice. Hibler (1977) showed that the viscous and plastic approaches are not contradictory, but that a viscous law results from an averaging of stochastic variations in deformation rates even though the nonaveraged law is plastic. Some tests of the constitutive laws have been made on the basis of momentum balance (e.g. Rothrock et al. 1980), but the main support for the different laws comes from model calculations of ice drift. However, the model tests have been too qualitative and verification methods need to be improved (Rothrock 1979).

Quite recently the marginal ice zone and the seasonal and annual variations of the world ice cover have become main fields of research. Both these are of importance to studies of our climate.

A NOTE ON NOTATION

We shall consider here the horizontal motion of sea ice and consequently vectors and tensors are two-dimensional. The vertically upward unit vector \( \mathbf{k} \) appears in some formulas in cross-products with vectors to express the rotation through the right-angle in the horizontal plane. The directions of vectors are expressed in mathematical form: zero direction points eastward and counterclockwise rotation counts positive. The wavy line above a quantity, e.g. \( \overline{H} \), stands for spatial averaging.
season 1941/42). The coastal area gets its snow cover in late November — early December. The average snow thickness is 25—45 cm in January and 35—60 cm in March (Kolkki 1969). This generally gives rise to a 5—20 cm snow ice layer on the ice cover. The currents in the basin are caused by air pressure fields over the Baltic Sea and they are very variable with no significant permanent component (Palmén 1930). The tidal amplitude is only about 1 cm.

The course of the ice season in the Bothnian Bay

Jurva (1937) has made a detailed classification of the phases of the ice season in the Baltic Sea on the basis of the extension and thickness of fast ice. From the viewpoint of pack ice mechanics, we can distinguish the following transition times for a given basin:

- **T1** freezing of coastal waters begins,
- **T2** first freezing of the basin,
- **T3** maximum ice extent in the whole Baltic Sea,
- **T4** the coastal waters start to melt,
- **T5** disappearance of ice.

The periods defined by these transitions differ in the typical thermal changes of ice mass: [T1, T2] — rapid ice formation and growth all over the basin; [T2, T3] — rapid freezing of open areas; [T3, T4] — thermal changes small and slow; [T4, T5] — rapid melting of ice. The stages T1—T5 have been determined for the Bothnian Bay for the ice seasons 1963/64—1979/80 from published ice charts (Inst. Mar. Res. 1963—1980) (Table 1). In the present work we shall study pack ice mechanics in the period [T3, T4], which we shall call "early spring".

### TABLE 1. The times T1—T5 for the Bothnian Bay on the basis of the ice seasons 1963/64—1979/80.

<table>
<thead>
<tr>
<th></th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
<th>T4</th>
<th>T5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>9 Nov</td>
<td>18 Jan</td>
<td>2 Mar</td>
<td>28 Apr</td>
<td>27 May</td>
</tr>
<tr>
<td>St dev (days)</td>
<td>17</td>
<td>38</td>
<td>17</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>Earliest</td>
<td>13 Oct</td>
<td>10 Dec</td>
<td>31 Jan</td>
<td>5 Apr</td>
<td>16 May</td>
</tr>
<tr>
<td>Latest</td>
<td>7 Dec</td>
<td>26 Feb</td>
<td>24 Mar</td>
<td>14 May</td>
<td>3 Jun</td>
</tr>
<tr>
<td>1978/79</td>
<td>2 Nov</td>
<td>21 Dec</td>
<td>22 Feb</td>
<td>8 May</td>
<td>1 Jun</td>
</tr>
</tbody>
</table>

Early spring in the Bothnian Bay

It must first be noted that, although thermal mass changes occur during early spring, they are slow compared to the dynamical time scale of pack ice. In addition, ice temperature slowly increases, thus causing brine migration and softening of floes. On average the period covers most of March and April (Table 1). The mean air temperature in Oulu is —6.6 °C in March and 0.3 °C in April (Kolkki 1969).

The dominant driving force on the pack ice is the wind stress. The prevailing wind direction during March — May is southwest, although northerly winds are also common (Fig. 2). Hence the long-term ice movement has a back and forth character with small overall displacements, as has been verified from the drift buoy observations of Sahlberg.
The time is the official Finnish time (GMT +2 hours). A few geographical names for the Bothnian Bay region used in the text are shown in Fig. 4. The terminology for sea ice follows the standard terminology of WMO (1970). Some new oceanographic terms have been used following the recent recommendations of IAPSO (1979) (International Association of Physical Sciences of the Ocean), e.g. "Coriolis period" in favour of "inertial period".

2. RESEARCH BASIN AND DATA MATERIAL

2.1. BOTHNIAN BAY

The Bothnian Bay, the northernmost basin of the Baltic Sea, is a shallow brackish water basin (Fig. 1). The length of the basin between the sill in the Quark and Tornio is 315 km; it has maximum width 180 km, surface area 36 500 km² and average depth 43 m (Tulkki 1977). The salinity of the water is $3 - 4 \times 10^{-3}$ and the cooling process consequently similar to that in fresh water basins, i.e. the maximum density of water occurs at $3.1 - 3.3 \, ^\circ C$, which is well above the freezing point of about $-0.2 \, ^\circ C$ (these values resulted from the formulas in Neumann & Pierson 1966). In freezing salt rejection occurs and the salinity of new ice is on average about $\frac{1}{3}$ of that of the surface water, as indicated by the measurements of Palosuo (1963). The properties of ice in the Bothnian Bay are hence significantly different from those of fresh water ice.

In the course of the winter, level ice can attain a thickness of 50—100 due to thermal growth. The maximum measured fast ice thickness is 115 cm (close to Tornio, in the ice

Figure 1. Bathymetric map of the Bothnian Bay (Tulkki 1977).
(1978). Winds over the Baltic Sea cause water transport. In March—April the daily variation of water level is on average 15.4 cm with a maximum of 65 cm in Kemi (Lisitzin 1952). The currents can be as large as 50 cm/s in extreme cases. In recent sea ice experiments a maximum current speed of $\sim$ 20 cm/s at 20 m depth has been measured. The water is homogeneous to a depth of 20—40 m with the temperature close to freezing point (Fig. 3). Further down, the temperature and salinity increase continuously.

Winds and currents drive the ice pack, which has high mobility, since freezing of openings is either slow or absent. The maximum measured ice drift speed is 60 cm/s and typical deformation rates are $10^{-3} - 10^{-2}$ h$^{-1}$. The compactness of ice on the basin scale is usually greater than $\sim$0.8, which means that internal stresses within the ice pack are significant.

2.2. FIELD EXPERIMENT SEA ICE 1979

On the basis of the knowledge gained from the earlier expedition SEA ICE 1977 (Leppäranta 1980b), a new experiment SEA ICE 1979 was planned (these are referred to here as SI77 and SI79, respectively). The experiment was carried out during 6—15 April 1979 with a total of twenty people participating in the field work. The research basin was, as before, the Bothnian Bay (Fig. 4). Two manned bases were used: the research vessel Aranda, moored to a vast ice floe (diameter about 4 km), and the Ulkokalla islet. The distance between these bases was 14—25 km. The floe broke in the experiment period.
Figure 4. Charts on ice situation at the beginning and at the end of the experiment S179 (left side), and LANDSAT image over the Bothnian Bay on 17 April 1979 (right side).
The observation program

The structure of pack ice: Aerial photographs were taken over the research area on 10, 13 and 21 April on scales from 1:7 500 to 1:30 000. Ice compactness, ridge density and the structure of ice floes were determined from them. Ice and snow thicknesses were measured in about 170 points. Visual observations were made continuously.

Motion of one ice floe: The translational motion of the floe to which Aranda was moored was measured by recording the Decca-coordinates of the ship at half-hour intervals. The accuracy of displacements was 10—20 m. Estimation of ice drift was done as in Leppäranta (1980b). The rotational motion of the floe was determined with the ship's gyrocompass for the same time intervals with an accuracy of 0.05 degrees.

Pack ice deformation: Five sets of reflector prisms were installed on the ice at distances of about 3 km from Aranda. Their positions were measured with a laser geodimeter and a theodolite at half-hour intervals from the ship on which a special observation platform had been constructed. The geodimeter was mounted on the theodolite, and the five positions could be determined within 1—2 minutes. The measurement error was of the order of centimeters, which is negligible compared to the measured deformations. Some temporal discontinuities in the time series were caused by heavy snowfall. No problems with icing of the reflectors occurred.

The Decca trisponder system was used for deformation measurements on a ~10 km scale. The system measures distances with radio signals, with an accuracy of about 3 m. The distances from two unmanned drifting stations to the observation stations of Aranda and Ulkokalla were continuously shown on digital displays wherefrom the values were manually taken at half-hour intervals. The drifting stations were located in the area between Aranda and Ulkokalla.

Surface wind: An automatic observation mast was installed on the floe at Aranda. Integrated wind speed and momentary wind direction at an altitude of 10 m were recorded at one-minute intervals.

Currents: Speed integrating and momentary direction recording current meters were submerged to a depth of 20 m at Aranda and at three of the reflector masts. The threshold speed is 3—4 cm/s and the recording interval was ten minutes. Unfortunately the instrument at one mast was lost in ridging of ice and that at another mast did not function properly.

Hydrographical observations were made twice a day with reversing thermometers and Nansen bottles. The temperature of the ice was measured continuously and weather observations were made every six hours.

Additional data

The Finnish Institute of Marine Research (FIMR), the Finnish Meteorological Institute (FMI) and the Swedish Meteorological and Hydrological Institute (SMHI) provided observations over the whole Bothnian Bay. Daily ice charts were obtained from FIMR, hourly water level elevations at 7 coastal stations were taken from FIMR and SMHI, weather maps at three-hour intervals were taken from FMI, and atmospheric surface pressure at 11 stations at six-hour intervals were obtained from FMI and SMHI.
Figure 5. The path of Aranda in SI79. Notation 13:2 means 13th April 2.00 hrs. The time interval between successive dots is one hour and between successive arrows six hours.
Weather and ice conditions during the research period

The fast ice boundary in the Bothnian Bay was stationary (Fig. 4). In the beginning the sea outside the fast ice area was covered with very close pack ice except for the lead on the eastern side. During the first days a high pressure stretched from northern Russia to Finland, with winds variable and low or moderate. Consequently, the ice movement at Aranda remained small (Fig. 5). The air temperature was in the range —3 to 0.5°C.

On 12 April a weak low pressure entered in the central Scandinavia. Northeasterly winds of nearly 10 m/s were observed and Aranda drifted southwest. During 14—15 April an intense cyclone passed the Bothnian Bay from west to east. The strong northerly wind opened a wide lead at the northern fast ice boundary and pressed the ice heavily against the Finnish side south from about 64°30′N. The wind brought cold air and a minimum air temperature of —10°C was observed. When the experiment ended in the morning of 15 April, Aranda had drifted an integrated path of 61.4 km, the net displacement being 23.3 km in the direction of 266.7 degrees (Fig. 5); the total drift time had been 8 days and 9 hours.
3. STRUCTURAL PROPERTIES OF PACK ICE

3.1. PACK ICE AREA

Islands and grounded ice ridges hold the coastal ice motionless and cause the formation of the consolidated fast ice area, the boundary of which is stationary during early spring. Depending on the severity of the winter, the fast ice area extends to a little above or below the 10 m depth contour (Fig. 6). The relative area with depth less than 10 m is 0.173 (Tulkki 1977). In the Arctic, where ice is thicker and ridges larger, fast ice extends to greater depths, e.g. to 25 m on the Siberian coast (Zubov 1945).

Although the mechanism of break-up of a consolidated ice sheet in a region bounded by islands and/or grounded ridges is not known, an analysis of the observations of Palosuo (1971) reveals a clear connection between ice thickness, wind speed, size of the region and break-up (Fig. 7). It is noteworthy that there seems to be an inflection point for the region diameter between 3 and 7 km; in wider regions ice sheets become easily unstable. Considering that the wind speed seldom exceeds 15 m/s, with a mean fast ice thickness of 75 cm, linear extrapolation on Fig. 7 leads to an estimate of 13 km for maximum width of

![Figure 6. The fast ice boundary in the Bothnian Bay in a mild winter (1975) and severe winter (1979).](image)
stable regions. Since the wind speed here has, more or less, a parametric role characterizing also the conditions in the sea, we cannot generalize the result quantitatively to an arbitrary basin.

Next, we want to determine how large ridges can occur with a spacing not more a given $l$. The solution is obtained directly from the equations

$$ h_k = y h_s, \quad \mu = \mu_0 \exp\left\{-\lambda_1 (h_s - h_{so})\right\}, \quad l = \mu^{-1}. \tag{3.1} $$

The first eq. describes a simple structural relationship between the keel depth $h_k$ and sail height $h_s$ of ridges. The ridges with deeper keels than 5 m in Palosuo (1975) gave, through linear regression, an estimate of $y = 6.9$; the number of ridges was 12 and the correlation 0.92. In the second eq. $\mu$ and $\mu_0$ are the densities of ridges with sail height greater than $h_1$ and $h_{so}$, respectively, and the exponential term is the integral of the probability density function of Wadhams (1980) (Eq. 3.3.b) from $h_1$ to infinity, i.e. the probability that the sail height is greater than $h_1$. From Leppäranta (1981) we have $\mu_0 \approx 10 \text{ km}^{-1}$ for $h_{so} = 30 \text{ cm}$ and $\lambda_1$ in the range $1/8$ to $1/37 \text{ cm}^{-1}$. Then, with $l = 13 \text{ km}$, Eqs. (3.1) give $h_k = 4.8$ or $14.5 \text{ m}$ for $\lambda_1 = 1/8$ or $1/37 \text{ cm}^{-1}$, respectively, and with $y$, $\mu_0$ and $h_{so}$ fixed as above; the solution is most sensitive to $\lambda_1/y$. The fast ice boundary in Fig. 6 lies within the depth range obtained for $h_k$.

The pack ice domain in the Bothnian Bay is simply connected and surrounded by the fast ice region except for the channel of 25—30 km width between the Bothnian Bay and Bothnian Sea. The length of the pack ice domain is 275 km, maximum width 150 km and the surface area about the same as that of the region deeper than 10 m, i.e. 30 190 km$^2$. The composition of pack ice on the large scale is rather uniform except for a small northeast mass gradient. It is notable that no highly deformed shear zone exists close to the fast ice boundary such as, e.g., the region of about 50 km width which occurred in the Beaufort Sea (Hibler et al. 1974a, Tucker et al. 1979).
3.2. INTERNAL STRUCTURE OF PACK ICE PARTICLES

The structure of level ice sheet

Snow-covered level sea ice sheet has three distinct layers of frozen water. The top layer is snow, the middle layer snow ice (frozen slush) formed from melted snow or from a mixture of sea water (or rain water) and snow, while the bottom layer is black ice formed from sea water only (e.g. Weeks & Lee 1958). The term ”black ice” has been adopted for the bottom layer, because it is used in the description of lake ice (e.g. Adams & Jones 1971).

<table>
<thead>
<tr>
<th>Snow</th>
<th>Snow ice</th>
<th>Black ice</th>
<th>Place</th>
<th>Time</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>860—890</td>
<td>905—915</td>
<td>Lake Sääksjärvi</td>
<td>Apr 1963</td>
<td>Palosuo (1965b)</td>
</tr>
<tr>
<td>300—400</td>
<td>—</td>
<td>915</td>
<td>Bothnian Bay</td>
<td>Apr 1977</td>
<td>Keinonen (1977)</td>
</tr>
</tbody>
</table>

In pack ice sheets of $\frac{1}{2}$ m thickness the snow ice layer can account for as much as $\frac{1}{3}$ of the total. Its density is somewhat less than that of black ice (Table 2; see also Weeks & Lee 1958); there are no measurements of the density of snow ice in the Baltic Sea and hence the results of Palosuo (1965b) from a lake in South Finland have been cited. The mechanical strength of the ice sheet is mainly determined by the black ice layer.

The salinity of pack ice in the Bothnian Bay during early spring is close to $0.5 \times 10^{-3}$ (Keinonen 1977, Omstedt & Sahlberg 1978); Palosuo (1963) reports values less than $0.2 \times 10^{-3}$ from the fast ice area. During SI79 the temperature of ice at 35 cm from the upper surface (11 cm from the bottom) varied between $-0.2$ and $-0.4$ °C. Thus the relative volume of brine is $\sim 0.09$ (Schwerdtfeger 1963). Observations of the air content of Baltic Sea ice do not exist; however, Palosuo (1965b) measured 0.01 in Lake Sääksjärvi and the density measurements of Keinonen (1977) in Table 2 suggest that the air content should be about the same in the Bothnian Bay. Consequently, in early spring the properties of ice depend mainly on the brine content.

Spatial variation of ice sheet thickness

It is well known that the thickness of a pack ice sheet is extremely variable, due to

(i) uneveness in the snow cover,
(ii) overriding of ice sheets (rafting),
(iii) opening of leads and polynyas.

The thermal factor (i) gives the continuous component, while the mechanical factors (ii & iii) give the discrete component to the thickness distribution function. The high degree of variability is clearly seen in the SI79 data (Figs. 8 and 16).

The effect of the horizontal variation of snow thickness is two-fold: firstly, the insulating effect of the snow cover varies and secondly, the amount of snow available for snow ice
formation and growth varies. The influence is reinforced by the thermal conductivity being much less for horizontal heat flow than for vertical flow due to the geometry of the brine pockets (Schwerdtfeger 1963). Hence horizontally non-uniform heating or cooling tends to remain a local phenomenon. Uusitalo (1957) observed that over a horizontal distance of 3 m the snow thickness increased from zero to 10 cm while ice thickness decreased from 25 to 15 cm; the age of the ice sheet was about two weeks.

The mechanical processes are of great importance. Rafting causes discrete increments in ice thickness, and the opening and freezing of leads and polynyas create ice areas of different thermal history. The overall implication of the mechanical processes is the multimodal structure of the thickness distribution, as will be seen later.

**Deformed ice**

Ice sheets thicker than 3—11 cm break under compression or shear into small blocks which are then forced upwards or downwards. The process leads to irregular fields of uneven broken ice with blocks having only a low elevation above the level ice surface, or it may result in ridges which have approximately triangular cross-sections in their upper and lower portions and appear horizontally as narrow curvilinear formations (see Fig. 37). The former will be called light deformed ice and the latter ridged ice. The distinction between them is made on the basis of their height above the level ice surface: deformed ice with higher elevation than some \( h_{so} \) (the cutoff height) is called ridged ice.
The question is then whether such a height \( h_{so} \) exists that the definitions above become meaningful? The results of Leppäranta (1981) seem promising. The height \( h_{so} = 30 \text{ cm} \) was chosen mainly on grounds of observational techniques (a laser profilometer used from an icebreaker deck). However, the resulting ridge densities corresponded quite well with those determined earlier visually and from accurate aerial photographs (scale 1:9 000). Another point is that the inclination angle of sails \( \varphi \), decreases with decreasing sail height and the triangular shape of the cross-sections vanishes into the randomness of ice block orientations; at \( h_1 = 30 \text{ cm} \), \( \varphi \approx 10^\circ \) (Leppäranta 1981). It should be noted that \( h_{so} \) depends on the scale of the level ice thickness and, consequently, should be larger in the Arctic seas than in the Baltic Sea.

Below \( h_{so} \) will always be 30 cm. The totally frozen layer of deformed ice is not more than 1 m (Palosuo 1975). The porosity of ridges is \( \approx 0.4 \) (Keinonen 1977) and the same value will be used for all deformed ice.

**Ridged ice**

The cross-sectional area of a ridge sail, \( A_s \), including voids is (Leppäranta 1981)

\[
A_s = \frac{h_s^2}{a + bh_s},
\]

(3.2.a)

where \( a \) and \( b \) are empirical constants. The denominator is equal to \( \tan \varphi \), and describes the relationship between \( \varphi \) and \( h_s \). From observations it was estimated that \( a \approx 0.106 \) and \( b \approx 0.383 \text{ m}^{-1} \); \( a \) is not significantly different from zero and with the condition \( a = 0 \) we have \( b = b_s \approx 0.442 \text{ m}^{-1} \). The cross-sectional area of ice blocks in a ridge is obtained from the isostatic principle and structural properties of ridges:

\[
A_r = x A_s,
\]

(3.2.b)

where the factor \( x \) is \( \approx 13 \) (Leppäranta 1981).

Based on the assumptions of a) geometric similarity of ridge cross-sections and b) equal probability of ridge heights yielding the same net deformation, Hibler et al. (1972) showed that

\[
\rho(h_s) \propto \exp \left\{-\lambda A_s(h_s)\right\} S(h_s - h_{so}),
\]

(3.3.a)

where \( \rho(\cdot) \) is the probability density function, \( \lambda \) a distribution shape parameter and \( S \) the Heaviside function, \( S(x) = 1 \) or 0 if \( x \geq 0 \) or \( x < 0 \), respectively; it was further assumed that \( A_s \propto h_s^2 \). Wadhams (1980) proposed an exponent term linéar in \( h_s \), which gave a better fit to his observations:

\[
\rho(h_s) = \lambda_1 \exp \left\{-\lambda_1(h_s - h_{so})\right\} S(h_s - h_{so}).
\]

(3.3.b)
This was supported by Tucker et al. (1979) for the Beaufort Sea and by Leppäranta (1981) for the Gulf of Bothnia. It seems that the general form (3.3.a) is valid, but due to the positive correlation of $h_s$ and $\varphi_s$, the linear exponent term is a better approximation than the quadratic one.

From the assumption of spatially random occurrence Hibler et al. (1972) showed that the distribution of ridge spacings follows the exponential distribution. Reasonably good agreement with observations was found by Mock et al. (1972), who then derived, assuming directional isotropy in ridging, the relation

$$\frac{L_r}{A_i} = \frac{\pi}{2\mu_s},$$

(3.4)

where $L_r$ is the total length of ridges in the ice-covered area $A_i$. Although the assumption was shown to be rather unrealistic, the overall validity of Eq. (3.4) was good.

The exponential distribution fits the ridge spacings in the Gulf of Bothnia (Leppäranta 1981). In SI79 four aerial photographs on a scale 1:30 000 were taken near Ulkokalla. The scale was small so that the effective cutoff height became necessarily greater than 30 cm; ridge densities determined from the pictures were 2—4 km$^{-1}$. Six lines, three perpendicular to the other three, were drawn on each picture and the number of ridges crossing each line was counted. Using the analysis of variance, the hypothesis of independence of the ridge density on direction was accepted at the 5% level of significance for all the pictures. Then the total length of ridges was measured with a curvimeter for each picture; the mean of $(L_r/A_i)\mu^{-1}$ was 1.65 (Table 3), which supports the prediction of Eq. (3.4) that $(L_r/A_i)\mu^{-1} = \pi/2 \approx 1.57$.

| TABLE 3. Linear ($\mu$) and areal ($L_r/A_i$) ridge densities in the 1:30 000 photos I—IV. Unit km$^{-1}$. |
|----------------|----------------|----------------|----------------|----------------|
|                | I              | II             | III            | IV             |
| \(\mu\)        | 4.0            | 2.7            | 2.8            | 2.5            | 3.00          |
| \(L_r/A_i\)    | 5.1            | 5.2            | 5.0            | 4.5            | 4.95          |
| \((L_r/A_i)\mu^{-1}\) | 1.27          | 1.93           | 1.76           | 1.78           | 1.65          |

On 2nd April 1978 six aerial photo tracks (scale 1:9 000) of 20—25 km length were taken near Ulkokalla. The mean (linear) ridge density was 8.0 km$^{-1}$ and the hypothesis of its uniform distribution w.r.t. direction was accepted at the 5% level of significance.

The equivalent thickness of ridged ice $h_r$ satisfies, by definition, the equation

$$A_i h_r = L_r A_i.$$

(3.5)

Integration of Eq. (3.2.a) gives a rather cumbersome result involving the exponential integral. However, its nonlinearity is not strong and thus $A_i h_r/(a + b h_r)$. Then, from Eqs. (3.2.b), (3.4) and (3.5),
Figure 9. Equivalent thickness of ridged ice per one ridge in a kilometer \((h_r \mu^{-1})\) versus ridge density \(\mu\).

\[
\tilde{h}_r = \frac{\pi}{2} \times \mu \frac{\tilde{h}_s^2}{a + b\tilde{h}_s}.
\]

(3.6)

Hibler et al. (1974b) used the eq. \(\tilde{h}_r = 10 \pi \mu \tilde{h}_s^2\), which agrees with our formula at \(\tilde{h}_s = 142\) cm. For smaller \(\tilde{h}_s\), as in the Bothnian Bay, our formula gives larger values.

Sail height measurements were not performed in SI79 and further simplifications must be made. The observations in Leppäranta (1981) show that, in the Bothnian Bay, one ridge per kilometer accounts for on average 2.21 cm equivalent level ice thickness (Fig. 9). From an aerial photo track (scale 1:12 500) of 14 km length over Aranda the ridge density of 9.7 km\(^{-1}\) was determined; consequently, \(h_r \approx 21.4\) cm.

**Light deformed ice**

In the derivation of the theoretical distribution of ridge sail height (Hibler et al. 1972) one of the main points was the assumption of the existence of a one-to-one mapping \(h_s \rightarrow A_r\). Let us suppose now that light deformed ice can be treated as "quasi ridges" so that a one-to-one mapping \(h_s \rightarrow A_r\), exists for all \(h_s \geq 0\). The functional form does not need to be the same for \(h_s < h_{s_0}\) as for \(h_s \geq h_{s_0}\). In fact, a rectangular cross-section would be more realistic for light deformed ice; such a form is also supported by the observations of Fukutomi & Kusunoki (1951). Then we can proceed similarly to Hibler et al. (1972) and obtain

\[
p(A_r) \propto \exp (-\lambda' A_r) S(A_r).
\]
The ratio of the volume of light deformed ice to that of ridged ice is obtained then through integration:

\[
D = \frac{\int_{0}^{\infty} A_T p dA_T}{\int_{0}^{\infty} A_T p dA_T} = \frac{\exp (\lambda' A_{r0})}{1 + \lambda' A_{r0}} - 1,
\]

where \( A_{r0} = A_s(h_{so}) \). Using the linear approximation \( A_s \propto h_s \) for \( h_s \geq h_{so} \), we have \( \lambda' A_{r0} = \lambda_i h_{so} \) (see Eq. 3.3.b). With the typical value \( \lambda_i h_{so} = 1.5 \) (corresponding to a mean sail height of 50 cm, when \( h_{so} = 30 \) cm), \( D \approx 0.8 \). That is, the volume of light deformed ice should be of the same order of magnitude as that of ridged ice.

In the Swedish SEA ICE-75 experiment ice thickness measurements were made at intervals of 10 m along a line of 1 km length (Udin 1976); the resulting distribution shows a wide spread (Fig. 10). The fast ice thickness was 60 cm at the same latitude. If we can consider the values higher than 80 cm as originating from light deformed ice, then we can conclude the following: that the relative coverage of light deformed ice was 0.16, its mean thickness 100 cm and, consequently, its equivalent thickness 16 cm. The mean thickness of level ice was 33 cm and the ridge density in the study area about 10 km\(^{-1}\) (Udin, personal communication) giving, through Fig. 9, an equivalent thickness of 22 cm for ridged ice. Hence, the value of the ratio \( D \) was \( \approx 0.73 \), which supports well our theoretical prediction above.

![Figure 10. Ice thickness distribution in a line of 1 km length. Produced from the measurements in Udin (1976).](image-url)
Ice floes and compactness

Mechanically ice floes are elastic plates lying on the water foundation. They tend to be convex with size varying over a wide range of values, from several meters to several kilometers in diameter. Ice floes consist of level ice patches and deformed ice. The thickness can vary considerably, as a floe can be made up of portions of different origin.

The horizontal size and shape of ice floes are described by the maximum and minimum floe diameters $l_{max}$ and $l_{min}$, respectively, and the surface area $A_f$. These diameters have been taken as the sides of the smallest rectangle which covers the floe. The characteristic floe diameter, elongation and shape factor are defined as

$$l_f = \sqrt{l_{max} \cdot l_{min}},$$
$$e_f = \frac{l_{max}}{l_{min}},$$
$$x_f = \frac{A_f l_f^{-2}}{1}.$$

The four aerial photographs on the 1:30 000 scale taken in S179 have been analyzed. Their total coverage was 200 km$^2$. The surface area (with a planimeter), $l_{min}$ and $l_{max}$, were determined for those floes with $l_{max} \geq 0.3$ km, smaller floes being considered as a single integrated entity. The total number of floes analyzed was 197 and the maximum surface area was 11 km$^2$. Thus the Aranda floe was exceptionally large. In addition, to get information on the size distribution of smaller floes, the diameters $l_{min}$ were determined from the aerial photo tracks (scale 1:9 000) taken on 2nd April 1978 near Ulkokalla; the total coverage of these pictures was 250 km$^2$ and floes for which $10 \, m < 1.5 \, km$ could be studied.

Naturally, small floes are abundant and the probability density of $l_f$ decreases rapidly with increasing $l_f$ (Fig. 11). It is noted that the logarithmic slope is not constant but decreasing and consequently the exponential distribution would underestimate the frequency of large floes. From the principle of random occurrence Hibler et al. (1972) derived the exponential distribution for ridge spacings, the validity of which has been verified through observation. The occurrence of cracks should thus be random, but it is related in a complicated way with break-up processes of floes and we can only see that the result is a decreasing logarithmic slope in the floe diameter distribution. The size of the largest floes lies somewhere near the inflection point in the curve of stability of bounded fast ice regions (Fig. 7).

The floe size distribution can be fruitfully studied in terms of the relative areal coverage of floes of different size (Fig. 12). The data from S179 show a fairly uniform distribution except for the large number of floes with $l_f < 0.3$ km. The slight maximum at $l_f$ between 0.5 and 1 km was also present in the data from 2nd April 1978. The representative floe size should be based on the areal coverage; the distribution in Fig. 12 gives us the mean 1.46 km and the standard deviation 1.15 km for $l_f$. The elongation of ice floes is typically between 1 and 2 and the shape factor between 0.6 and 0.9 (Fig. 13); for circular floes $e_f = 1$ and $x_f = \pi/4$. No correlation with $l_f$ is seen either for $e_f$ or for $x_f$, only the dispersion decreases with increasing $l_f$. 
Figure 11. Floe size distributions near Ulkokalla. The whole spectrum could not be studied in both cases and consequently the vertical scale is non-overlapping (i.e. only the slopes can be compared).

Figure 12. Relative areal coverage of ice floes (SI79). Class interval 0.3 km.

Figure 13. Shape of ice floes (SI79).
The quantity ice compactness \( \chi \) is, by definition, the ratio of the ice-covered area to the total area of a given particle,

\[
\chi = \frac{A_i}{A}.
\]

(3.7)

Since thin ice is deformed by much smaller forces than thick ice, it should be in some cases better to define compactness as the relative areal coverage of ice with thickness above a given value; e.g. Hibler (1979) used in his model the value of 50 cm, which is about one-fifth of the overall average thickness in the Arctic Ocean.

According to Doronin & Kheisin (1975) the friction in contacts between ice floes becomes noticeable at \( \chi \approx 0.7 \). The piling of ice blocks begins, when the most floes are in contact; Thorndike et al. (1975) proposed a value of \( \chi = 0.85 \) for this. The observations of Shirokov (1977) show that the dimensionless ice speed first decreases linearly due to the increasing number of contacts and at \( \chi \approx 0.9 \) starts to drop abruptly, which evidently is the start of the piling mode (Fig. 14). In SI79 ridging began after the compactness had increased to \( \approx 0.95 \). In the case of equal-sized circular floes the loosest and densest packings with no freedom of relative motion are \( \pi/4 \approx 0.79 \) and \( \pi/2\sqrt{3} \approx 0.91 \), respectively (e.g. Thorndike et al. 1975). Even without freezing \( \chi \) can increase from these values because, as shown for arbitrary material in Harr (1977), the size of ice floes is distributed over a wide range. Due to frictional effects the piling mode may start when there still are some free paths between floes.

![Figure 14. Speed of pack ice, scaled with the speed of single floes not in contact with other floes, versus compactness. Reproduced from Shirokov (1977).](image)

3.3. DESCRIPTION OF MASS OF PACK ICE

In a pack ice particle \( \mathcal{E} \), with surface area \( A \), the mass of ice is \( \int_{\mathcal{E}} \varrho_i h dA \), where \( \varrho_i \) is the vertically integrated mean ice density, which is considered here to be constant. In case of
open water \( h \) is defined to be zero. The areal mass density of ice is then
\[
m = \phi_i \frac{1}{A} \int_{\xi} h \, dA.
\]

Doronin (1970) used the two-component description
\[
m = \phi_i \bar{h} \chi,
\]
where \( h \) is the mean total ice thickness in \( \xi \cap \{ h > 0 \} \). Thorndike & Maykut (1973) (see also Thorndike et al. 1975) proposed the thickness distribution function
\[
\Gamma(\xi) = \frac{1}{A} \int_{\xi} S[\xi - h(x,y)] \, dA,
\]
where \( S \) is the Heaviside function, i.e. \( \Gamma(\xi) \) equals the relative area with thickness less than \( \xi \). This approach ignores the spatial moments of the ice mass in \( \xi \). Through the use of (3.9) the dynamic-thermodynamic coupling of ice mass continuity can be very elegantly treated.

Leppäranta (1979) derived a generalization of Eq. (3.8) through decomposing \( \bar{h} \) into level ice thickness \( \bar{h}_1 \) and ridges:
\[
m = \phi_i (\bar{h}_1 + a \mu \bar{h}_s^2) \chi,
\]
where the dimensionless number \( a \) (\( \approx 50 \)) describes the structure, size distribution and spatial distribution of ridges. The term \( a \mu \bar{h}_s^2 \) is the equivalent thickness of ridged ice and should be replaced by the more realistic expression (3.6), and the concept of "level ice" should include in this connection also light deformed ice. Eq. (3.10) has the advantage that the amount of ridged ice can be handled in physically clear and easily measurable terms. In addition, the hydrodynamic roughness of pack ice can be parametrized in terms of \( \mu \) and \( h_s \) (Arya 1973).

Our observations show that the spatial variation of ice thickness is concentrated in wavelengths shorter than \( \sim 1 \) km, and in the case of compactness and ridged ice the variations occur mainly in wavelengths shorter than a few kilometers. Thus, when the length scale is \( \sim 10 \) km or more, the assumption of spatial homogeneity is realistic and integration of ice mass through (3.9) does not destroy any essential information. The difficulty with the Thorndike-Maykut distribution is how to integrate over deformed ice. Thorndike & Maykut (1973) introduced the condition \( \Gamma = 1 \) at some \( \xi = \bar{h}_{\text{max}} \) to avoid arbitrary large thicknesses. Instead, a discontinuity at \( \xi = \bar{h}_{\text{max}} \) could be used to describe ridged ice by
\[
\bar{h}_r \chi = \bar{h}_{\text{max}} \left[ 1 - \Gamma(\bar{h}_{\text{max}}) \right].
\]

Another equation is needed to relate \( h_r \) with \( \mu \) and \( h_s \),
\[
(\mu, \bar{h}_s) \rightarrow \bar{h}_r.
\]
This is generally not one-to-one and hence $\mu$ and $h_s$ must be always known in addition to $\Gamma$. The slope of ridge keels is about 45 degrees (Palosuo 1975) and thus the ridge width $d_r$ is $2h_k$. Using the first of Eqs. (3.1), Eq. (3.4) and Eq. (3.7) the relative areal coverage of ridged ice becomes

$$\chi_r = \frac{L_r d_r}{A} = \eta \mu h_s \chi_r.$$  \hspace{1cm} (3.11)

Now, we can express $h_{\text{max}}$ and the jump in $\Gamma$ as

$$1 - \Gamma(h_{\text{max}}) = \chi_r,$$

$$h_{\text{max}} = \tilde{h}_{\text{r}} \chi_r / \chi_r.$$ \hspace{1cm} (3.12)

Note that, from Eqs. (3.6) and (3.11), $h_{\text{max}}$ is independent of $\mu$. Furthermore, $h_{\text{max}}$ equals the mean vertical size of ridges integrated over length and width.

Figure 15. Distributions of ice thickness in 1 km x 1 km area. The graph in the middle has been produced from Udin (1976) and that in the bottom from Omstedt et al. (1974).
The thickness distribution of pack ice is quite dissimilar from that of fast ice (Fig. 15). The former has a large dispersion and a multi-peaked structure. Observations in SI79 gave a complete thickness distribution for a region with a diameter of 12 km (Fig. 16). Ridged ice was included through Eqs. (3.11) and (3.12): taking the values $\gamma = 6.9$, $\mu = 9.7$ km$^{-1}$ and $\Tilde{h}_r = 21.4$ cm given earlier and choosing $\chi = 0.92$ and $h_s = 50$ cm we find that the values $\chi_s = 0.10$ and $h_{\text{max}} = 197$ cm result. The compactness was between 0.89 and 0.95 during SI79 and is fixed to the mean 0.92 in Fig. 16. Some integrated mass characteristics for the area around Aranda are given in Table 4.

![Image of ice thickness distribution with classes labeled R.](image)

Table 4. Average mass characteristics around Aranda (within an area of 12 km diameter).

<table>
<thead>
<tr>
<th>Value</th>
<th>Equivalent ice thickness (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ice thickness</td>
<td>50.4 cm</td>
</tr>
<tr>
<td>Ridge density</td>
<td>9.7 km$^{-1}$</td>
</tr>
<tr>
<td>Snow thickness</td>
<td>3.3 cm</td>
</tr>
<tr>
<td>Total</td>
<td></td>
</tr>
<tr>
<td>Compactness</td>
<td>0.92</td>
</tr>
<tr>
<td>Areal mass density</td>
<td>613 kg m$^{-2}$</td>
</tr>
</tbody>
</table>
4. DEFORMATION AND CONSTITUTIVE PROPERTIES OF PACK ICE

The length scales of pack ice deformation are:

(i) small scale, \( L \approx 10^{-2} - 10^0 \) km,
(ii) mesoscale, \( L \approx 10^1 - 10^2 \) km,
(iii) large scale, \( L > 10^2 \) km.

Small scale deformation is concerned with clearly distinguishable physical processes such as break-up of ice floes and ridging. Mesoscale elements consist of many ice floes. The mechanical behaviour of pack ice in the mesoscale has been approximated with continuum models with rather good results for the ice velocity field. The intermediate region between small scale and mesoscale is, so far, rather unknown. The largest scales of motion are \( \sim 10^3 \) km, observed in the Arctic Ocean (Thorndike & Colony 1980).

4.1. SMALL SCALE DEFORMATION

Inhomogeneities in the external forces and/or properties of floes cause relative motion which may result in floe collisions. High compactness collisions necessarily lead to shear with floes in contact or overthrusting. Thin sheets of ice override each other over large distances resulting in a local doubling of ice thickness. Quite often the sheets fracture along lines in the direction of the relative motion and form a finger rafting pattern (Weeks & Anderson 1958b, Dunbar 1960). As the ice thickness increases, the bending moments in overriding become so large that small pieces break off from the sheets and these then start to pile up and pile down. If the ice sheets are of equal thickness, the one that is forced down is the one that breaks (Coon 1974). Parmerter (1975) derived theoretically the maximum ice thickness for rafting:

\[
hrf = 14.2 \frac{(1 - \nu^2)}{\sigma_c Y} \cdot \frac{2}{g},
\]

where \( g \) is the acceleration due to gravity and \( \nu, \sigma_c \) and \( Y \) are Poisson's ratio, tensile strength and Young's modulus, respectively, of ice. Poisson's ratio is approximately constant for sea ice, \( \nu = 1/3 \) (Weeks & Assur 1967) and the tensile strength is expected to be close to the flexural strength (Weeks & Assur 1967, Doronin & Kheisin 1975). Then, using the mean values for \( \sigma_c \) and \( Y \) from Table 5, we arrive at \( h_{rf} = 5 \) cm. However, \( h_{rf} \) is sensitive to \( \sigma_c \) and 3—11 cm is a realistic range; Weeks & Anderson (1958a) report that sometimes sea ice can take a large "superload" without breaking. In the Bothnian Bay thicknesses of 15—20 cm are frequently observed for ice blocks in ridge sails and rafting of ice with greater thickness than the upper limit of our range is unusual.
TABLE 5. Measurements of mechanical properties of ice in the Baltic Sea. (BB = Bothnian Bay)

<table>
<thead>
<tr>
<th>Flexural strength (MN m⁻²)</th>
<th>Young’s modulus (GN m⁻²)</th>
<th>Place</th>
<th>Time</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.36 ± 0.06 (n = 17)</td>
<td>4.2 ± 0.9 (n = 17)</td>
<td>Gulf of Finland</td>
<td>winter 1971</td>
<td>Enkvist (1972)</td>
</tr>
<tr>
<td>0.59 ± 0.10 (n = 7)</td>
<td>5.7 ± 1.0 (n = 6)</td>
<td>northern BB</td>
<td>Jan 1973</td>
<td>Määttänen (1973)</td>
</tr>
<tr>
<td>0.32 ± 0.11 (n = 28)</td>
<td>3.1 ± 1.3 (n = 26)</td>
<td>southern BB</td>
<td>Mar 1975</td>
<td>Tabata et al. (1975)</td>
</tr>
</tbody>
</table>

When \( h > h_{fr} \), small pieces break off with a length

\[
l_{br} \sim \frac{\pi}{4} \left[ \frac{Y h^3}{3\rho g (1 - v^2)} \right]^{1/4}
\]

(Coon 1974); e.g., for \( h = 20 \) or 60 cm \( l_{br} \) is 4.7 or 10.8 m, respectively. Under continuous compressive stress the broken pieces accumulate in pressure ridges. The process was successfully modeled by Parmeter & Coon (1972) with a one-dimensional model; they could show that there is a maximum vertical size, depending on ice strength and thickness, to which a ridge can grow. Leppäranta (1977) studied the Parmeter-Coon model using ice properties representative for the Baltic Sea. Taking into account that the model does not produce voids, the maximum heights were comparable to the data of Palosuo (1975) giving the size of the largest ridges as \( h_s + h_k \sim 25 \cdot h \).

Ridging under shear results in long straight ridge lines and the sail and keel tend to become steeper than in compression. But the general image is that from above ridge links look like a random zigzag pattern, as was discussed in section 3.2. Sometimes clear finger ridging formations are observed (e.g. Palosuo 1975). The width of the fingers is of the order of tens of meters.

Several mechanisms causing break-up of ice floes into large pieces have been studied: e.g., thermal cracking (Evans & Untersteiner 1971), loads due to isostatic imbalance in ice sheets (Schwaegler 1974) and ocean waves penetrating into ice (Wadhams 1978). It was emphasized by Rothrock (1975b) that whatever the dominant mechanism is, it must work continuously since pack ice tends to conserve its structure as an ensemble of separate floes. The observations in section 3.2 showed that in the Bothnian Bay there is no definitely favoured band in the floe size spectra.

4.2. DEFORMATION OF PACK ICE PARTICLES

In SI79 measurements were made with the geodimeter and theodolite on the \( \sim 6 \) km scale (GT-array) and with the Decca trisponder system on the \( \sim 10 \) km scale (TR-array). The total displacements of the GT-reflectors with respect to Aranda were 1/2—1 km, of which most was due to about 10° clockwise rotation of the whole array including the Aranda floe (Fig. 17). In the beginning the TR-array was nearly an equilateral triangle, but one of the two unmanned stations behaved quite differently from the other and Aranda, and after some days the two stations lay in opposite directions w.r.t. Aranda (Fig. 18).

Let \( \mathbf{X} \) be the position of a particle in a given reference configuration and \( \mathbf{x} \) the position of the particle in the configuration at time \( t \). As was noted by Pritchard (1974), there is no
preferred reference configuration for an ice pack and we can choose the initial configuration to be such, i.e. $x = X$ at $t = 0$. In view of our observation techniques it is natural to take the Lagrangian frame and we describe pack ice deformation through

$$x = x(X, t).$$

Physical deformation (i.e. motion excluding rigid displacement and rotation) occurs when the distance $l = |x^{(0)} - x^{(0)}|$ between two particles changes. Gorbunov & Timokhov (1968) described deformation using the diffusion coefficient $D = \Delta \rho / \Delta t$. Their observations, when $\chi \lesssim 0.7$, gave $D \approx 1 \text{ m}^2\text{s}^{-1}$ on the length scale $L \sim 1 \text{ km}$ which fits quite well into the $D$ versus $L$ diagram of Okubo & Ozhimov (1970) for tracer spots in the ocean. However, it must be borne in mind that except for the very open pack ice the compactness of ice is too high so that we could speak about diffusion. Consequently, for pack ice the coefficient $D$ is only a nominal diffusion coefficient and, in fact, $\Delta l$ may have either sign. The SI79 data gave $D \sim 10^{-1}$ and $10 \text{ m}^2\text{s}^{-1}$ for the GT- and TR-array, respectively (Table 6); during the experiment the compactness was greater than 0.89. Detailed measurements of shifts between ice floes were given by Legen'kov et al. (1974), showing the clearly random nature of the relative movement of individual floes.
TABLE 6. Statistics for the deformation measurements in SI79. The positions are defined w.r.t. Aranda. (dp = displacement, abs = absolute value).

<table>
<thead>
<tr>
<th></th>
<th>GT-array</th>
<th></th>
<th>TR-array</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of obs</td>
<td>188</td>
<td>217</td>
<td>227</td>
</tr>
<tr>
<td>Mean distance (m)</td>
<td>2346</td>
<td>3129</td>
<td>3038</td>
</tr>
<tr>
<td>Mean direction (deg)</td>
<td>109</td>
<td>50</td>
<td>339</td>
</tr>
<tr>
<td>Abs of mean dp (m)</td>
<td>3.8</td>
<td>3.3</td>
<td>3.3</td>
</tr>
<tr>
<td>Mean of abs dp</td>
<td>9.4</td>
<td>6.5</td>
<td>7.2</td>
</tr>
<tr>
<td>&quot;Diffusion&quot; coefficient (m²s⁻¹)</td>
<td>0.16</td>
<td>0.08</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Continuum deformation: theory

For a more advanced treatment of deformation restrictions must be placed on the function \( x \): we assume that \( x \) is differentiable to any required order. Then the movement must be
smoothed both in time and in the space domain (Nye 1973a). The spatial smoothing must be made in a length scale large enough that the material can be considered as a continuum. According to the experience with dynamical modeling of sea ice (Leppäranta 1980a), $L_c \approx 10$ km can be considered as a continuum length scale in the Baltic Sea and thus our GT-array lies in the lower continuum limit. In the Arctic Ocean $L_c$ is taken from $\geq 10$ km (Hibler et al. 1974c) up to 100 km (Rothrock 1975b). Evidently the lower limit of $L_c$ depends on the floe size, which in turn depends on ice thickness. In the following we shall treat the ice pack using the framework of general continuum mechanics (e.g. Hunter 1976) applied to two dimensions.

The method was first used by Hibler et al. (1973) in the Arctic Ocean and by Leppäranta (1980b) in the Baltic Sea. The Lagrangian deformation gradient is

$$ F = (\nabla x)^T, $$

where $T$ stands for matrix transpose. In component form, $F_{ij} = \partial x_i / \partial x_j$. Note that $\det F$ is the Jacobian of the transformation $x \rightarrow \mathbf{x}$, i.e. the ratio of the areal mass densities. The displacement gradient is then

$$ \mathbf{U} = F - I, $$

where $I$ is the unit tensor.

The finite strain tensor is $\frac{1}{2} (\mathbf{U} + \mathbf{U}^T + \mathbf{U}^T \cdot \mathbf{U})$; in small deformation theory the product $\mathbf{U}^T \cdot \mathbf{U}$ is ignored, and the symmetric and antisymmetric parts of $\mathbf{U}$ become the (infinitesimal) strain and rotation tensors,

$$ \varepsilon = \frac{1}{2} (\mathbf{U} + \mathbf{U}^T), $$

$$ \omega = \frac{1}{2} (\mathbf{U} - \mathbf{U}^T), $$

respectively. In the two-dimensional theory $\omega$ has only one independent component: $\omega_{11} = \omega_{22} = 0$, $\omega_{12} = - \omega_{21} = \frac{1}{2} (U_{12} - U_{21})$. The component $\omega_{12}$ will also be denoted by $\omega$, when there is no danger of misunderstanding. In the infinitesimal approximation the Lagrangian and Eulerian strain and rotation are equal.

The principal axes values $\varepsilon_1$ and $\varepsilon_2$ of the strain tensor are obtained as the roots of the characteristic polynomial

$$ \det (\varepsilon - \lambda I) = (\lambda^2 - (\text{tr} \varepsilon)\lambda + \det \varepsilon). \quad (4.1) $$

These are real-valued and given by

$$ \varepsilon_{1,2} = \frac{1}{2} \text{tr} \varepsilon \pm \left[ (\frac{1}{2} \text{tr} \varepsilon)^2 - \det \varepsilon \right]^{1/2}, $$

and we choose the "+" sign for $\varepsilon_1$ so that $\varepsilon_1 \geq \varepsilon_2$, by definition. The principal axes values represent the maximum and minimum normal strains, positive values representing axial extension and negative values axial contraction. The directions of the principal axes are
equal to the directions of the eigenvectors $\Lambda^{(i)}$ and $\Lambda^{(j)}$. These are obtained from the homogeneous equations
\[(\varepsilon - \varepsilon_k \mathbf{1}) \cdot \Lambda^{(k)} = 0 \quad (k = 1, 2),\]
which determine the directions uniquely, unless $\varepsilon$ is spherical ($\varepsilon = \varepsilon_{11} \mathbf{1}$). In non-spherical cases the directions are
\[
\alpha_1 = \begin{cases} 
\arctan \left( \frac{\varepsilon_{11} - \varepsilon_{12}}{\varepsilon_{12}} \right), & \text{if } \varepsilon_{12} \neq 0, \\
0, & \text{if } \varepsilon_{12} = 0 \text{ and } \varepsilon_1 = \varepsilon_{11}, \\
\pi/2, & \text{if } \varepsilon_{12} = 0 \text{ and } \varepsilon_1 = \varepsilon_{22},
\end{cases}
\]
\[
\alpha_2 = \alpha_1 + \pi/2.
\]

The coefficients $\text{tr}\varepsilon$ and $\text{det}\varepsilon$ of the characteristic polynomial (4.1) are invariants of the strain tensor. In pack ice studies $\text{det}\varepsilon$ is generally replaced by $[(\text{tr}\varepsilon)^2 - 4 \text{ det}\varepsilon]^{1/2}$ (e.g. Rothrock 1975b); i.e. we use the invariants
\[
\varepsilon_1 = \text{tr}\varepsilon = \varepsilon_1 + \varepsilon_2,
\]
\[
\varepsilon_{11} = [(\text{tr}\varepsilon)^2 - 4 \text{ det}\varepsilon]^{1/2} = \varepsilon_1 - \varepsilon_2.
\]
In the infinitesimal approximation (i.e. neglecting the second and higher powers of $U$), $\varepsilon_1 \sim \text{det } F - 1$, the relative change of the areal mass density, and $\varepsilon_{11}$ is equal to twice the maximum shear strain.

We must now face the question whether (and if yes, then how) an ice pack "remembers" the initial configuration and the strain history. The ice compactness, which is a certain time integral of deformation, is an important quantity, since it tells us much about how the ice may deform during the next time-step. Convergence or divergence naturally hardens or softens, respectively, the material. In the absence of collisions of floes the structural changes are reversible, but break-up of floes and especially overriding and ridging change the structure of particles irreversibly. The elastic-plastic model of Coon et al. (1974) gives strain-hardening behaviour in ridging through producing larger ridges the more heavily the ice is deformed. This is supported by observations in the Arctic Ocean (e.g. Hibler et al. 1974b). In the Bothnian Bay, however, the density and size of ridges seem to be uncorrelated (Fig. 9), suggesting a behaviour independent of the ridging component of strain.

From the viewpoint of constitutive models for pack ice we need to know, in general, the thickness distribution and the rate at which deformation occurs. Hence for the time $t + \Delta t$ the configuration at the time $t$ can be taken as the reference configuration. The displacement during the time interval $\Delta t$ is given by
\[
u = x (X, t + \Delta t) - x (X, t) = v (x, t) \Delta t + o (\Delta t^2),\]
where $v$ is ice velocity and $o(\Delta t^2)/\Delta t \to 0$ as $\Delta t \to 0$. When $\Delta t$ and consequently the displacement is small, the Lagrangian and Eulerian displacement gradients become equal: $U = \nabla_x u \approx \nabla_v u$, where the subscript of $V$ tells us in which frame the derivation is carried out. Then, we have

$$\frac{1}{\Delta t} U = \frac{1}{\Delta t} \nabla_x u \to \nabla_v v, \text{ as } \Delta t \to 0.$$ 

The symmetric and antisymmetric parts of $\nabla_x v$ are the strain-rate and vorticity tensors

$$\dot{e} = \frac{1}{2} [\nabla_x v + (\nabla_x v)^T],$$

$$\dot{\omega} = \frac{1}{2} [\nabla_x v - (\nabla_x v)^T],$$

respectively. The principal axes and invariants of the strain-rate tensor are found similarly as in the case of the strain tensor above.

The analysis of observations of pack ice deformation starts from fitting certain kinds of functions to the measured displacements, whereafter the characteristics of the deformation can be calculated using the functional relationships above (Thorndike 1970, Hibler et al. 1974c). For the present study, the deformation was estimated as follows: The Taylor polynomial of $x(X, t)$ at Aranda (subscript 'o'), for the observation at a fixed moment of time $t$, is written as

$$x(X, t) - x(X_o, t) = \sum_{k=0}^{k_o} \frac{1}{k!} [(X_i - X_{oi}) \frac{\partial}{\partial X_i}]^k x \bigg|_{X = X_o, t} \text{ (4.2)}$$

$$+ \text{ residual.}$$

The measurements provided vectors $x - x_o$ and $X - X_o$, and the partial derivatives in the above equation were estimated through linear regression. A necessary condition to obtain the derivatives to the order of $k_o$ is that at least $k_o(k_o + 3)/2$ vector pairs $(x - x_o, X - X_o)$ are known. Hence, for the GT-array $k_o \leq 2$ and for the TR-array $k_o \leq 1$.

The first-order derivatives in Eq. (4.2) give us the deformation gradient $F$, from which the strain and rotation can be calculated. In order to study the rate of deformation the time-series of the positions were first smoothed through averaging over three-hour intervals. Then consecutive positions ($\Delta t = 30 \text{ minutes}$) were used for the initial and final configurations and the velocity gradient for each time interval was estimated by $\nabla_v v \approx U/\Delta t$.

The GT-array results

The field study of the deformation of the GT-array lasted 6 days and 7 hours. At 15.00 hrs on 14th April the measurements had to be terminated due to a heavy snowfall during which ridging was observed around Aranda. Deformations of several per cent were measured...
The principal axes values had opposite signs and extension occurred mainly in the direction of the long axis of the Bothnian Bay. The rotation of the whole array was quite similar to the rotation of the Aranda floe in the center. It is notable that the infinitesimal approximation for the total deformation failed during the latter half of the study period: $\varepsilon_1$ should be close to $\det F - 1$, but after 12th April this was far from the case due to the large rotation of the array. When the calculations were made in the coordinate frame rotating with the array the validity of the infinitesimal approximation became better (Table 8). Rotation is often but not necessarily always most critical for small deformation theory and in general the infinitesimal approximation cannot be used in time scales more than a few days.

### Table 7. Total Lagrangian deformation in the GT-array. The exact time is 12.00 hrs for each day. Initial time 8th April 5.30 hrs.

<table>
<thead>
<tr>
<th>Day</th>
<th>$\det F$</th>
<th>$\varepsilon_1 \times 10^3$</th>
<th>$\varepsilon_2 \times 10^3$</th>
<th>$\sigma_1$ deg</th>
<th>$\sigma_2$ deg</th>
<th>$\varepsilon_1 \times 10^3$</th>
<th>$\varepsilon_1$ deg</th>
<th>$\varepsilon_1$ Aranda floe deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1.007</td>
<td>8</td>
<td>-1</td>
<td>-42</td>
<td>48</td>
<td>7</td>
<td>9</td>
<td>-0.6</td>
</tr>
<tr>
<td>9</td>
<td>1.021</td>
<td>23</td>
<td>-2</td>
<td>29</td>
<td>-61</td>
<td>21</td>
<td>25</td>
<td>-0.3</td>
</tr>
<tr>
<td>10</td>
<td>1.028</td>
<td>31</td>
<td>-4</td>
<td>72</td>
<td>-18</td>
<td>27</td>
<td>35</td>
<td>-1.4</td>
</tr>
<tr>
<td>11</td>
<td>1.030</td>
<td>49</td>
<td>-20</td>
<td>70</td>
<td>-20</td>
<td>29</td>
<td>69</td>
<td>-2.7</td>
</tr>
<tr>
<td>12</td>
<td>1.020</td>
<td>53</td>
<td>-33</td>
<td>51</td>
<td>-39</td>
<td>20</td>
<td>86</td>
<td>-2.2</td>
</tr>
<tr>
<td>13</td>
<td>1.010</td>
<td>32</td>
<td>-37</td>
<td>73</td>
<td>-17</td>
<td>-5</td>
<td>69</td>
<td>-7.1</td>
</tr>
<tr>
<td>14</td>
<td>0.989</td>
<td>3</td>
<td>-49</td>
<td>64</td>
<td>-26</td>
<td>-46</td>
<td>52</td>
<td>-10.7</td>
</tr>
</tbody>
</table>

### Table 8. Total Lagrangian deformation in the GT-array in the coordinate frame rotating with Aranda. The exact time is 12.00 hrs for each day. Initial time 8th April 5.30 hrs.

<table>
<thead>
<tr>
<th>Day</th>
<th>$\varepsilon_1 \times 10^3$</th>
<th>$\varepsilon_2 \times 10^3$</th>
<th>$\varepsilon_1 \times 10^3$</th>
<th>$\varepsilon_1$ deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>8</td>
<td>-1</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>9</td>
<td>23</td>
<td>-2</td>
<td>21</td>
<td>25</td>
</tr>
<tr>
<td>10</td>
<td>31</td>
<td>-4</td>
<td>27</td>
<td>35</td>
</tr>
<tr>
<td>11</td>
<td>50</td>
<td>-19</td>
<td>31</td>
<td>69</td>
</tr>
<tr>
<td>12</td>
<td>53</td>
<td>-32</td>
<td>21</td>
<td>85</td>
</tr>
<tr>
<td>13</td>
<td>40</td>
<td>-29</td>
<td>11</td>
<td>69</td>
</tr>
<tr>
<td>14</td>
<td>21</td>
<td>-31</td>
<td>-10</td>
<td>52</td>
</tr>
</tbody>
</table>

The Jacobian $\det F$ is equal to $h(0)\dot{x}(0)/h(t)\dot{x}(t)$, where the argument refers to time. Because the mean thickness can be considered constant during the measurement period, $\det F$ gives us the relative changes in ice compactness. On the other hand, the analysis of aerial photographs gave the absolute compactness of $X = 0.915$ on 10th April at noon. Consequently, the time-series of compactness can be calculated (Fig. 19).
Clearly the small deformation theory is valid in our study of the rate of deformation. The results show characteristic features similar to those observed earlier in the Bothnian Bay (Leppäranta 1980b). They are summarized in the following (Table 9, Figs. 20—22). 1° Much of the power of the time-series lies in periods shorter than ~ half a day. Longer periods are present in vorticity. 2° The principal axes values usually have opposite signs. This implies that local openings and closings are due more to mesoscale shear than to mesoscale spherical strain. 3° Large deformation rates are of the order of $10^{-6}$ s$^{-1}$; thus, during periods of ~ 10 hours deformations of 0.03—0.04 are produced.

![Figure 19. The compactness of ice during S179 estimated from the Lagrangian deformation gradient and aerial photographs on 10th April.](image)

| TABLE 9. Statistics for the strain-rate $\epsilon$ and vorticity $\omega$ in the GT-array. Unit $10^{-6}$ s$^{-1}$. |

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Maximum</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_1$</td>
<td>0.44</td>
<td>0.66</td>
<td>3.0</td>
<td>-0.5</td>
</tr>
<tr>
<td>$\epsilon_2$</td>
<td>-0.50</td>
<td>0.55</td>
<td>0.1</td>
<td>-2.4</td>
</tr>
<tr>
<td>$\epsilon_3$</td>
<td>-0.06</td>
<td>0.70</td>
<td>2.8</td>
<td>-2.6</td>
</tr>
<tr>
<td>$\epsilon_{11}$</td>
<td>0.94</td>
<td>1.00</td>
<td>4.4</td>
<td>0.0</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.79</td>
<td>0.77</td>
<td>3.4</td>
<td>0.0</td>
</tr>
</tbody>
</table>

A similar form for the strain-rate ellipse has been found in the Arctic Ocean (Hibler et al. 1973, Hibler et al. 1974c). Due to the differences in ice thickness, the variation of deformation is of lower frequency in the Arctic Ocean than in the Baltic Sea.
Figure 20. The rate of deformation in the GT-array. Unit $10^{-6}$ s$^{-1}$. 
Figure 21. The principal axes of the strain-rate in the GT-array. Notation 10:7 means 10th April 7.00 hrs.
The TR-array results

The deformation of the TR-array was followed for 7 days and 20 hours (Fig. 18). Due to some problems with the measurement system a continuous time-series for the motion of both the unmanned stations could not be obtained. In the beginning the array was nearly an equilateral triangle. Its deformation was of the same order of magnitude as the deformation of the GT-array (Fig. 23), but the results from the two arrays are not well correlated. There is a long gap, from 8th April at 17.00 to 12th April at 13.00, in the data from the northern unmanned TR-station and during that period the TR-array experienced a total (relative) contraction of 0.01 while the GT-array extended by 0.02. On 13th April the northern station moved quite differently from Aranda and the southern station (Fig. 18): there was at its largest about 40° difference in the drift direction while the speeds were ≥ 20 cm/s. It is known that this behaviour did not follow from the wind field. On 13th April the coastal currents in the region flowed southwest and it is possible that the Ulkokalla islet and grounded ridges around it forced a strong turning in the water flow shifting the northern TR-station towards the open sea (cf. Figs. 5 and 18).

As the triangle had finally deformed to nearly a straight line, the two-dimensional tensors could no more be reliably estimated. We can still look at the drift data of Aranda and the unmanned stations together (Fig. 24). After 14th April 18.00 hrs the northern station again moved in a different direction. During that time the whole ice pack was pressing on the eastern fast ice boundary and there seemed to be a slip line between Aranda and the northern station.
Discussion of pack ice deformation

The horizontal scales of deformation start from the size of ice floes, $10^{-2} - 10^0$ km, and go up to the size of the basin, $1 - 3 \times 10^1$ km. In the region between them we have the mesoscale phenomena. As the scale increases, the nature of the deformation becomes smoother, since gradients are taken over more and more numerous floes. The time-scale of deformation in the mesoscale region is of the order of several hours.

Inhomogeneities in the mass characteristics can favour certain kinds of deformations. However, our conclusion in section 3.3 was that variations of pack ice mass are concentrated in wavelengths shorter than a few kilometers and thus mesoscale mass differences are small. We can separate the inhomogeneities into those in a) ice thickness, b) free paths between floes (i.e. compactness) and c) the surface roughness of ice. The first of these was discussed in Leppäranta (1980b). His results show that at a wind speed of $\sim 5$ m/s a thickness gradient of $\sim 5$ cm/km could produce deformation rates of $\sim 10^{-6}$ s$^{-1}$ in the GT-array. The estimated gradient was 3.3 cm/km southwest for level ice and light deformed ice and its effect could not be distinguished. Evidently, due to the high compactness during the experiment, the effect was smoothed out by the internal friction within the ice together with the homogeneity of the mean thickness in scales larger than the GT-array.

Differences in free paths between ice floes are very important. Doronin & Kheisin (1975) stated that near the coast openings may tend to orient in the direction of the fast ice boundary and thus the ice pack can easily shear along the boundary. Our aerial photographs indicate a random distribution of openings on scales of a few kilometers. The
same result, as was concluded in section 3.3, held also for the distribution of ridges; thus the surface roughness, from the viewpoint of ridging, should be uniform.

The fast ice boundary causes heavy deformations to occur, when the ice pack is in direct contact with the boundary and is not moving away from it. Hibler et al. (1974a) described the shear zone on the Alaskan coast of the Beaufort Sea. Kawamura (1977) showed that the convergence and vorticity of pack ice correlate well with the onshore and alongshore, respectively, drift components off the coast of Hokkaido in the Okhotsk Sea. During the study period of the GT-array a small lead occurred at the fast ice boundary and thus the GT-array could not feel the fast ice. On 14th April the lead closed up, an event that was well reflected in the behaviour of the TR-array (Figs. 18 and 24): from Aranda toward the boundary, ice was compressing, while between Aranda and the northern TR-station the ice was shearing in the direction of the boundary. The zone compressing on the fast ice boundary was therefore not wider than about 20 km. This is in agreement with the earlier studies of Leppärinta (1980b) in the Bothnian Bay giving no indication of the influence of the fast ice boundary on compression of a 10 km scale array which was located 20—30 km away from the boundary.
The external forces — winds and currents — tend to deform pack ice, on the one hand due to inhomogeneities in the surface roughness of ice and, on the other hand due to the deformation fields in the atmosphere and sea themselves. It was stated above that, considering the ridges, the surface roughness was uniform in our data. McPhee (1980) reported that the observed ice velocity gradient in free drift (i.e. in the absence of internal friction) could be well explained on the basis of the surface wind gradient, and thus variations in the surface roughness of the ice were insignificant. Hibler (1974) derived from the linear drift theory analytical connections between the atmospheric surface pressure and pack ice deformation and gave empirical data which to some degree supported his theory. In the Bothnian Bay, some correlation between the vorticities of ice drift and geostrophic wind was reported in Leppäranta (1980b); no significant correlation was found from the SI79 data. Especially at the length scales where our data have been obtained, the geostrophic wind field does not tell us much about the gradient of the surface stress. In the paper of McPhee cited above it was further concluded that the correlation between the ice velocity gradient and the geostrophic wind gradient was poor.

Data on the current velocity in SI79 at Aranda and at one GT-array reflector (the one north-northwest from Aranda, see Fig. 17) at the 20 m depth are available. The current

![Figure 25. The velocity difference between Aranda and one reflector mast: ice (solid line) and current at 20 m depth (dashed line). The upper figure is for the absolute value and the lower for direction. In the lower figure the scale on the right refers to the frame in which y-axis is directed from Aranda to the reflector.](image-url)
meters are not very reliable at low speeds and hence only one day of the data, 13th April, is worth looking at in detail (Fig. 25). The ice speed was then 10—25 cm/s and the current speed 5—15 cm/s. The difference in velocity between Aranda and the reflector was less than 0.4 cm/s for ice and one order of magnitude larger for the current. There are some connections, although not very strong, between the velocity differences.

There seems to be no characteristic length scale in the mesoscale region for pack ice deformation in the Bothnian Bay. The mesoscale processes follow, in a very complicated way, from the basin-wide atmospheric and oceanic forcing and from the boundary geometry of the pack ice domain. Consequently, there is no favoured length scale $L_c$ for continuum models. From the viewpoint of the Cauchy stress formalism $L_c$ must be large enough so that each particle will consist of sufficiently many floes and, on the other hand, $L_c$ must be small enough so that the surface forces on each particle will be sufficiently uniform. The scale of $\sim 10$ km lies close to the lower limit and results from computer models (e.g. Leppäranta 1980a) tell us that we can go up to about 30 km with $L_c$.

The cause of the large variability of the pack ice deformation in time-scales of less than some hours is uncertain. One explanation could be compressional waves, which were discussed in Kheisin & Ivchenko (1976). They give speeds of $\sim 10$ m/s for such waves in conditions of high compactness and thus the waves could travel across the Bothnian Bay in 2—3 hours.

4.3. CONSTITUTIVE PROPERTIES OF PACK ICE

It was observed already by Nansen (1902) that the relation between ice drift and wind velocity varies much and he explained this on the basis of the internal friction within the ice; he stated that internal friction increases with increasing compactness. This reasoning was confirmed by Sverdrup (1928), who tried to formulate the internal friction empirically through a frictional force proportional to ice speed and directed against ice drift.

What we need is a constitutive law which gives the stress tensor $\Sigma$ (two-dimensional) within the ice pack as a function of the state of the ice, deformation etc. The equation of motion tells us that the frictional force within the ice is equal to the divergence of $\Sigma$. It must be remembered that we are here dealing with mesoscale stresses within the ice pack. Locally the stress can be much larger. E.g., the ice stress felt by a ship can exceed the mesoscale stress by two orders of magnitude (Kheisin 1978).

The first physical constitutive law for pack ice was proposed by Laikhtman (1958). He considered the motion of an ensemble of ice floes analogous to that of a turbulent fluid. With the classical approach to the Reynolds stresses, Laikhtman obtained

$$\Sigma = 2 \eta_e \dot{e}', \quad (4.3)$$

where $\dot{e}' = \dot{\epsilon} - \frac{1}{2} (\text{tr} \dot{\epsilon}) I$ is the strain-rate deviator and $\eta_e$ the dynamic eddy viscosity coefficient. If the gradient of $\eta_e$ is neglected, the divergence of $\Sigma$ becomes equal to $\eta_e \nabla \cdot \mathbf{v}$. Campbell (1965) could simulate the general features of ice drift in the Arctic Ocean with his numerical model using Laikhtman's frictional force. Doronin (1970) emphasized the importance of compactness and stated that $\eta_e = \eta_e (\lambda)$. 
A general viscous law was proposed by Glen (1970):

\[ \Sigma = \xi (\text{tr} \dot{\varepsilon}) I + 2 \eta \dot{\varepsilon}, \]

(4.4)

where \( \xi \) and \( \eta \) are the bulk and shear viscosity coefficients, respectively, and they can depend on the mass characteristics of ice and the strain-rate invariants. It should be noted that Eq. (4.4) is the classical viscous law not involved with the Reynolds stresses, but their approximation through (4.3) gives a similar form to the turbulent and viscous shear stresses. The data from SI79 allowed calculation of the Reynolds stresses directly: the components of the covariance tensor of ice velocity were at their largest \( 10^{-3} - 10^{-2} \text{m}^2\text{s}^{-2} \) and hence the Reynolds stresses were \( 1 - 10 \text{Nm}^{-1} \), which is definitely too small to give a significant internal friction. The same conclusion has been reached for the Arctic Ocean by Rothrock (1975b). Consequently, the viscosity of ice must be analogous to the classical viscosity and not the eddy viscosity.

Glen's law has been widely applied with constant viscosities to give the internal friction as

\[ \nabla \cdot \Sigma = \xi \nabla (\nabla \cdot \mathbf{v}) + \eta \nabla^2 \mathbf{v}. \]

A slightly more general form was used by Campbell & Rasmussen (1972): \( \eta = \text{constant} \) and \( \xi = 0 \) or \( \eta \) for \( \nabla \cdot \mathbf{v} \geq 0 \) or \( \nabla \cdot \mathbf{v} < 0 \), respectively. This treatment of the bulk viscosity is physically supported by the ice pack giving high resistance to compression and small resistance to tension. Estimates given for the linear viscosity coefficients vary much (Table 10). Hibler (1977) developed a non-linear viscous law which gives high resistance to small deformations, thus approximating the plastic behaviour of pack ice.

**TABLE 10. Estimates for linear viscosities of pack ice. Unit kg s\(^{-1}\).**

<table>
<thead>
<tr>
<th>Bulk viscosity</th>
<th>Shear Viscosity</th>
<th>Area</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-)</td>
<td>(~10^{12})</td>
<td>Central Arctic</td>
<td>Campbell (1965)</td>
</tr>
<tr>
<td>(-)</td>
<td>(~3 \times 10^8)</td>
<td>Kara Sea</td>
<td>Doronin (1970)</td>
</tr>
<tr>
<td>(~10^{10} - 10^{12})</td>
<td>(~10^{10} - 10^{12})</td>
<td>Central Arctic</td>
<td>Hibler &amp; Tucker (1977)</td>
</tr>
<tr>
<td>(~10^8)</td>
<td>(~10^8)</td>
<td>Baltic Sea</td>
<td>Leppäranta (1980a)</td>
</tr>
<tr>
<td>(~10^7 - 10^{10})</td>
<td>(~10^7 - 10^{10})</td>
<td>Bothnian Bay</td>
<td>This work</td>
</tr>
</tbody>
</table>

The theory of particulate media (e.g. Harr 1977) suggests that the viscosities for a material such as pack ice should be sensitive to compactness. Doronin (1970) assumed only a direct proportionality between \( \eta \) and \( X \), but in the range \( X \gtrsim 0.8 \) \( \eta \) can probably vary through one or two orders of magnitude, and consequently the viscosity gradient would become significant. Hibler (1979) had in his model the proportionality of \( \eta \propto \exp[-20(1-X)] \), which seems to give the correct magnitude of variability.

There are two additional terms which have sometimes been used in connection with Eq. (4.4). Nye (1973b) showed that the two-dimensional pack ice stress is actually the three-dimensional Cauchy stress integrated through the thickness of ice plus a horizontal hydrostatic load of water on the inclined (due to nonzero thickness gradient) lower surface of ice. However, the latter term can be present only in compact ice (\( X = 1 \)); in the spring
time, when there is always some open water, the horizontal hydrostatic load integrates to zero in length scales larger than the floe size.

The second term is a hydrostatic pressure within the ice pack (Rothrock 1970). The pressure should result from the equation of state for pack ice, but formulation of such has not yet been successful. Hibler (1979) assumed the pressure to be a function of the compactness and mean thickness only — this can give rise to stresses in an ice cover at rest. Kheisin & Ivchenko (1976) assumed that the pressure is proportional to the displacement divergence — this necessarily brings a reference configuration into the picture and hence does not seem very realistic. To conclude, even though the idea of the hydrostatic pressure itself is acceptable, its formulation is at present far from satisfactory. The equation of state should include not only mass characteristics but also the spatially fluctuating part of the kinetic energy. The close packing of floes and the importance of their collisions evidently makes the equation of state quite complicated.

Completely different from the viscous approach is the treatment of pack ice as an elastic-plastic medium (Coon et al. 1974, Pritchard 1975). The reasoning here is based on small scale mechanical behaviour and features predicted by the elastic-plastic law agree with observations. However, the viscous and plastic approaches are not contradictory, since a viscous law results from averaging stochastic variations in deformation rates even though the nonaveraged law is plastic (Hibler 1977).
5. MOVEMENT OF PACK ICE

5.1. STATEMENT OF THE PROBLEM

When we formulate the equation of motion for pack ice, we must take into account the Coriolis acceleration and integrate through the thickness of the ice. This equation becomes

$$\rho l h \left( \frac{d\mathbf{v}}{dt} + f \mathbf{k} \times \mathbf{v} \right) = \nabla \cdot \mathbf{\Sigma} + \mathbf{r} + \mathbf{b}.$$ \hspace{1cm} (5.1)

Here, $f$ is the Coriolis parameter defined as twice Earth’s angular velocity times the sinus of the latitude, $\mathbf{k}$ is the unit vector vertically upward, and $\mathbf{r}$ and $\mathbf{b}$ are the surface and body force vectors, respectively. The vector $\mathbf{r}$ equals the sum of the surface stresses of air and water, $\tau_a$ and $\tau_w$, respectively, on ice, while $\mathbf{b}$ includes only the component of the gravitational force $\mathbf{G}$ parallel to the sea surface. Thus Eq. (5.1) can be written as

$$\rho l h \left( \frac{d\mathbf{v}}{dt} + f \mathbf{k} \times \mathbf{v} \right) = \nabla \cdot \mathbf{\Sigma} + \tau_a + \tau_w + \mathbf{G}.$$ \hspace{1cm} (5.2)

The steady-state form of Eq. (5.2) was used by Campbell (1965) in his numerical for the Arctic Ocean.

When the stress within the ice pack is insignificant, the situation is called "free drift" (McPhee 1980). The well-known rule

$$v \sim 0.02 w, \quad \psi_a \sim \psi_w \sim 30^\circ,$$ \hspace{1cm} (5.3)

where $(v, \psi_a)$ and $(w, \psi_w)$ are the velocities of ice and surface wind, respectively, in polar coordinates, was given by Nansen (1902) on the basis of the drift of his polar exploration vessel Fram. Ekman (1902) showed that Eqs. (5.3) result from the steady-state free drift equation without sea surface tilt,

$$\rho l h f \mathbf{k} \times \mathbf{v} = \tau_a + \tau_w.$$ \hspace{1cm} (5.4)

Observations gave a large scatter to (5.3) and attempts were made to explain this by friction within the ice (Nansen 1902, Sverdrup 1928), by variations in the properties of the atmospheric and oceanic boundary layers (Sverdrup 1928, Rossby & Montgomery 1935) and by the influence of the ice mass itself (Shuleikin 1938). The inertial term was added to Eq. (5.4) by Shuleikin (1938) and Shvets (1946), whose calculations agreed with the observed small inertia of ice.

During the 1970's considerable progress was made in understanding the stresses within the ice, as was discussed in section 4.3, and in parametrizing the surface stresses. However, the Nansen-Ekman law (5.3) is still a good approximation in the Arctic summer, when ice compactness is low (McPhee 1980). In the Baltic Sea the proportionality factor and the deviation angle must be slightly adjusted due to the small ice thickness (Leppäranta 1980b).
The stress divergence in Eq. (5.2) in general means that the ice drift problem can be solved only through numerical models. As pointed out by Rothrock (1979), modeling has now advanced to the stage where more attention should be paid to verification in the field.

The spectra of the ice velocities in S177 and S179 are shown in Fig. 26. The velocities were Lagrangian in all cases and the calculations were made with the equations given in Målkki (1975). We cannot see any clear peak at the Coriolis period (which is 13.3 hours), in contrast to what has been observed in the Arctic Ocean (McPhee 1978, Thorndike & Colony 1980) and in the Okhotsk Sea (Ono 1978). The explanation is that in the Baltic Sea, firstly, the ice is so thin that it responds to wind on a much shorter time-scale than the Coriolis period and, secondly, wind is the governing external force in ice drift. In the current spectra in the Baltic

Figure 26. Spectra of ice velocity.
Sea, on the other hand the Coriolis peak is very clear (Mälkki 1975, Alenius & Mälkki 1978) but it is not strong enough from the viewpoint of ice motion; the data of Ono were taken from a coastal region, where the current effects on ice drift as much as the effect of winds.

**The inertia of ice and the Coriolis acceleration**

Observers are often struck by the rapid response of drifting ice to wind. In S179 impressive evidence of this fact was obtained (Fig. 27): in less than one hour the wind direction turned by 120 degrees, and the wind speed was greater than 8 m/s; the ice drift followed the wind with a time-lag clearly shorter than our temporal resolution of 30 minutes. This is in agreement with the theoretical free drift calculations of Leppäranta (1980b) showing that for $h \sim 1/2$ m and $w \sim 8$ m/s ice speed accelerates from zero to 0.9 times the steady-state speed in $\sim 10$ minutes. The time-series in S179 illustrate well the close temporal connection between ice drift and wind (Fig. 28).

![Figure 27. The path of Aranda and wind vectors at the end of S179. The time interval between successive dots and arrows is 30 minutes.](image)
The observations gave total accelerations $\frac{dv}{dt}$ typically in the range $10^{-6} - 10^{-4}$ ms$^{-2}$. In the Eulerian frame $\frac{dv}{dt}$ equals the sum of the local acceleration $\frac{\partial v}{\partial t}$ and the advective acceleration $v \cdot \nabla v$. From the results in section 4.2 we see that the components of the velocity gradient are of the order of $10^{-6}$ s$^{-1}$ at their largest; since ice speed is $\sim 10^{-1}$ ms$^{-1}$, we conclude that the local acceleration is the dominant part of the total acceleration.

The Coriolis acceleration has a noticeable effect on the ice drift in the Bothnian Bay, but the main cause of the difference between the directions of ice drift and wind lies below the ice. The wind stress is transmitted through the ice to the oceanic boundary layer, where the Coriolis effect works on the whole layer, resulting in the turning of the tangential stress between ice and water. In the equation of motion for pack ice we can have a constant Coriolis parameter over the whole Bothnian Bay.

**The surface stresses**

Generally the parametrization starts from the form

$$\tau \propto q \times (\text{relative velocity})^2$$

resulting from dimensional analysis in hydrodynamics. The problem then lies in the dimensionless proportionality factor, the so-called drag coefficient $C$. Linear stress laws (Ekman 1902) are sometimes used, because they reduce the free drift equation to a first-order linear differential equation. The turning of wind and current in the planetary boundary layers introduces an additional stress law parameter, the angle of turning $\theta$. To
simplify the notation in the subsequent formulas, rotations will be expressed using a rotation operator $\Theta$

$$\Theta = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

which gives the rotation of $\theta$ to a given vector. In other words, we have the relationship

$$\Theta \cdot \mathbf{v} = \cos \theta \mathbf{v} + \sin \theta \mathbf{k} \times \mathbf{v}.$$ 

Then, the quadratic and linear stress laws become,

$$\tau = \rho C v_R \Theta \cdot \mathbf{v}_R,$$

$$\tau = \rho \sqrt{Kf} \Theta \cdot \mathbf{v}_R,$$

respectively. Here, $v_R$ is the relative velocity and $K$ the kinematic eddy viscosity coefficient of the fluid in question. The parameters $C$, $\theta$ and $K$ depend, first of all, on the reference altitude for the relative velocity. The linear law was used by Rothrock (1975c) in his ice drift model for the Arctic Ocean. We shall first discuss the parameters in the quadratic law.

The effect of the hydrostatic stability on the stress parameters was presented by Sverdrup (1928) and Rossby & Montgomery (1935). In stable conditions $C$ can be reduced by a factor of 1/2 from near-neutral conditions. Baroclinity can also be important, a discussion of which is given in Joffre (1978). Banke & Smith (1971) showed that the drag coefficient for the surface wind can vary by a factor of 2 in ice covers of different small scale surface roughness. This referred to characteristic features of level ice, with or without snow-cover, and did not include the form drag due to ridges, which are a mesoscale roughness feature. Arya (1973) showed theoretically that the form drag for the air stress is proportional to the product of ridge density and sail height and predicted that it can be even larger than the skin friction drag, especially in stable conditions. Observations by Banke & Smith (1975) supported Arya’s theory. Observational evidence also indicates the importance of the form drag for the water stress (J.D. Smith 1973). In summary, if a fixed drag coefficient is used for arbitrary ice and external conditions, errors of a factor of 1/2—2 are expected, in extreme cases even as much as a factor of 1/3—3.

Since $w \gg v$, the relative velocity for the atmosphere is $w$. In the present work the surface wind (altitude 10 m) is used and the angle of turning becomes zero. Thus

$$\tau_a = \rho_a C_a w w.$$  \hfill (5.5) 

For relatively flat snow-covered ice $C_a \approx 1.5 \times 10^{-3}$ in near-neutral conditions (e.g., S.D. Smith 1972, Banke et al. 1976, Joffre 1978). In April in the Bothnian Bay the stratification is generally near-neutral during day-time and slightly stable during night-time. According to the measurements of Banke & Smith (1975) the form drag coefficient $C_{a,s}$ is $\approx 0.1 \times \mu h_s$; for S179 $\mu = 9.7$ km$^{-1}$ and $h_s \approx 50$ cm, and consequently $C_{a,s} \approx 0.5 \times 10^{-3}$. Thus, we can conclude that the drag coefficient for S179 is $1.5 \times 10^{-3} - 2.0 \times 10^{-3}$ and the use of the
constant value $C_a = 1.75 \times 10^{-3}$ should be accurate within ±14% in most cases. The density of air was taken as 1.3 kg m$^{-3}$, corresponding to the surface layer during the experiment.

In the water it is convenient to choose the reference depth as the bottom of the layer of the frictional influence of ice, 20—30 m, which approximately coincides with the bottom of the upper homogeneous layer (Fig. 3). The current at this depth is denoted by $u_b$. During S179 the sea depth was close to 40 m (Fig. 5). The water stress must be formulated using the relative velocity $u_b - v$ and thus

$$
\tau_w = q_w C_w \left. u_b - v \right| \Omega_w \cdot (u_b - v).
$$

(5.6)

Field measurements involving the water stress are not nearly so numerous as in the case of the wind stress. Generally the upper homogeneous layer in the Arctic Ocean has a depth in the above range for S179 (e.g., Hunkins 1966, Hunkins et al. 1980).

Hunkins (1975b) integrated the momentum equation from the ice-water interface to the depth $H$, where he assumed zero stress and the current velocity $u$ equal to the geostrophic current velocity $u_g$. He obtained

$$
\left( \frac{\partial}{\partial t} + f \hat{k} \times \right) \int_{-H}^{0} q_w (u - u_g) \, dz = \tau_w.
$$

![Figure 29](image.png)

Figure 29. The geostrophic drag coefficient estimated from the S177 current profiles.
From measured current profiles Hunkins (1975a) estimated \( \tau_w \), and then obtained \( C_w \approx 2 \times 10^{-3} \) and \( \theta_w \approx 30^\circ \) for \( |u_s - v| > 20 \text{ cm/s} \); a wide scatter was found in the parameters for \( |u_s - v| < 20 \text{ cm/s} \). The method was used for the current measurements in SI77, when the sea depth was \( \approx 80 \text{ m} \); the observation levels forced \( H \) to 20 m. The results repeat the general features observed by Hunkins (Fig. 29). The value of \( C_w \) lay between \( 3.0 \times 10^{-3} \) and \( 3.5 \times 10^{-3} \) for large \( |u_s - v| \) and the boundary layer, angle varied in the range \( 0 \) to \( 40^\circ \). The geostrophic drag parameters should be applicable in Eq. (5.6).

The wind factor \( Na = v/w \) and the deviation angle \( \Delta \psi = \psi_a - \psi_t \) of steady free drift provide information on \( C_w \) and \( \theta_w \), since the wind stress parameters are usually better known (McPhee 1980). This is especially true in the Baltic Sea where

\[
Na \approx \left( \frac{\theta_a C_a}{\theta_w C_w} \right)^{1/2}, \quad \Delta \psi \approx \theta_w,
\]

for \( w \geq 7 \text{ m/s} \). The drag parameters obtained with this method are representative for the bottom of the layer of frictional influence of the ice. The use of the relations (5.7) for the Bothnian Bay has given consistent results (Table 11). The difference in the estimates for the drag coefficient \( C_w \) is due to assuming different drag coefficients for wind, and the accuracy in fixing the northern direction for the observation mast is not better than 4 degrees. In the Arctic Ocean the ratio \( C_w/C_a \) seems to be larger than in the Bothnian

Bay (Table 11), which suggests that with increasing size of ridges the form drag increases faster in the water than in the air.

It is concluded that the turning angle in Eq. (5.6) can be taken as 20 degrees for the SI79 data. To choose the drag coefficient, we can argue that \( C_w \) must be consistent with \( C_a \) so that there would be no kinetic energy losses within the ice pack during steady free drift. That is, we take \( C_w = 3.0 \times 10^{-3} \) from Table 11. This value is also supported by the profile results above. We can roughly say that a realistic range for \( C_w \) in the Bothnian Bay should be \( 2.5 \times 10^{-3} \) to \( 3.5 \times 10^{-3} \) and hence our estimate is accurate within \( \approx 17 \% \) except for very low speeds of which we know very little. It must be emphasized that \( C_a \) and \( C_w \) were not chosen independently but they were forced to satisfy a basic argument about the kinetic energy of ice. The density of water is 1003 kg m\(^{-3}\) in the upper homogeneous layer (Fig. 3).

In some simplified calculations in the subsequent sections we shall need the kinematic eddy viscosity coefficient \( K_w \) for water. This is taken as a constant and chosen so that the linear and quadratic water stresses become equal at \( v = 15 \text{ cm/s} \). Thus, we have \( K_w \approx 16 \text{ cm}^2\text{s}^{-1} \) used by Rothrock (1975c).

<table>
<thead>
<tr>
<th>( C_a/C_w ) (observed)</th>
<th>( C_a \times 10^3 ) (assumed)</th>
<th>( C_w \times 10^3 ) (estimated)</th>
<th>( \theta_w ) (estimated)</th>
<th>Area</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>2.75</td>
<td>5.5</td>
<td>[21°, 26°]</td>
<td>Central Arctic</td>
<td>McPhee (1980)</td>
</tr>
<tr>
<td>1.7</td>
<td>1.5</td>
<td>2.5</td>
<td>16°</td>
<td>Bothnian Bay</td>
<td>Leppärinta (1980b)</td>
</tr>
<tr>
<td>1.7</td>
<td>1.75</td>
<td>3.0</td>
<td>20°</td>
<td>Bothnian Bay</td>
<td>This work</td>
</tr>
</tbody>
</table>
The body force

The body force $G$, which we can simply call the tilt force, is expressed as

$$G = -q_i h g \beta ,$$

(5.8)

where $\beta$ is the water-level gradient. Sometimes $\beta$ is approximated from the geostrophic flow in which case $G = q_i h f \mathbf{k} \times \mathbf{u}_g$.

The parameters in the momentum equation, except for the ice stress, are shown in Table 12. These will be used below, unless otherwise stated. Except in section 5.2, we shall work in the Eulerian frame.

### TABLE 12. The standard parameters for the equation of motion of pack ice in the Bothnian Bay.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density of ice</td>
<td>$\rho_i$</td>
<td>$910 \text{ kg m}^{-3}$</td>
<td></td>
</tr>
<tr>
<td>Coriolis parameter</td>
<td>$f$</td>
<td>$1.3 \times 10^{-4} \text{ s}^{-1}$</td>
<td>latitude $64^\circ 30'\text{N}$</td>
</tr>
<tr>
<td>Gravity acceleration</td>
<td>$g$</td>
<td>$9.8 \text{ m s}^{-2}$</td>
<td></td>
</tr>
<tr>
<td>For air:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>$\rho_a$</td>
<td>$1.3 \text{ kg m}^{-3}$</td>
<td></td>
</tr>
<tr>
<td>Drag coefficient</td>
<td>$C_a$</td>
<td>$1.75 \times 10^{-3}$</td>
<td>surface wind</td>
</tr>
<tr>
<td>For water:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>$\rho_w$</td>
<td>$1003 \text{ kg m}^{-3}$</td>
<td></td>
</tr>
<tr>
<td>Drag coefficient</td>
<td>$C_w$</td>
<td>$3.0 \times 10^{-3}$</td>
<td>current beneath the boundary layer</td>
</tr>
<tr>
<td>Turning angle</td>
<td>$\theta_w$</td>
<td>$20 \text{ deg}$</td>
<td></td>
</tr>
<tr>
<td>Eddy viscosity</td>
<td>$K_w$</td>
<td>$16 \text{ cm}^2\text{s}^{-1}$</td>
<td></td>
</tr>
</tbody>
</table>

The dimensionless form of the momentum equation

Using the expressions (5.5), (5.6) and (5.8) for the wind stress, water stress and tilt force and after decomposing the total acceleration, the momentum equation (5.2) becomes

$$q_i h \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} + f \mathbf{k} \times \mathbf{v} \right) = \nabla \cdot \mathbf{\Sigma} + \rho_a C_a \mathbf{w} \mathbf{w}$$

\[+ \rho_w C_w \left| \mathbf{u}_b - \mathbf{v} \right| \nabla \cdot \left( \mathbf{u}_b - \mathbf{v} \right) - q_i h g \beta . \]

(5.9)

Then we introduce the scales $V$ — velocity, $T$ — time and $L$ — horizontal length, taken as constants. The above eq. is first scaled with $q_i V^2$:

$$\left( \frac{h}{V} \right) \frac{\partial \left( \mathbf{v}/V \right)}{\partial t} + h \left( \frac{\mathbf{v}}{V} \right) \cdot \nabla \left( \frac{\mathbf{v}}{V} \right) + \left( \frac{hf}{V} \right) \mathbf{k} \times \left( \frac{\mathbf{v}}{V} \right)$$
\[
\frac{1}{q_i V^2} \mathbf{v} \cdot \Sigma + C_a \left( \frac{q_a w}{e_i V} \right) \left( \frac{w}{V} \right) 
\]

\[
+ C_w \left( \frac{q_w u_b - v}{q_i V} \right) \mathbf{\Theta}_w \cdot \left( \frac{u_b - v}{V} \right) - \left( \frac{h g}{V^2} \right) \beta. 
\]

Now, scaling velocities, time, horizontal length, densities and stress with \( V, T, L, q_i \) and \( q_i V^2 L \), respectively, and denoting the dimensionless quantities with a subscript asterisk, we have

\[
\left( \frac{h}{VT} \right) \frac{\partial v_*}{\partial t_*} + \left( \frac{h}{L} \right) v_* \cdot \mathbf{v}_* + \left( \frac{h f}{V} \right) \mathbf{k} \times \mathbf{v}_* 
\]

\[
= v_* \cdot \Sigma_* + C_a q_{a_*} w_* w_* 
\]

\[
+ C_w q_{w_*} \left| u_{b_*} - v_* \right| \mathbf{\Theta}_w \cdot (u_{b_*} - v_*) - \left( \frac{h g}{V^2} \right) \beta. 
\]

Some of the dimensionless terms in Eq. (5.10) have a form similar to terms familiar from general and geophysical fluid dynamics (e.g. Landau & Lifschitz 1959, Pedlosky 1979). Hence we denote

- \( Sr = VT/h \) (the Strouhal number),
- \( \delta = h/L \) (the aspect ratio),
- \( Ro = V/hf \) (the Rossby number),
- \( Fr = V^2/hg \) (the Froude number).

Furthermore, it is suggested that \( \Sigma \) depends on \( q_i, h, V, T, L \) and \( \chi \). Using the \( n \)-theorem from the dimensional analysis (e.g. Li & Lam 1964) it is seen that we can form four independent dimensionless products out of them; for example, we can choose \( \Sigma_* \), \( \delta \), \( Sr \) and \( \chi \) and, consequently, \( \Sigma_* = \Sigma_* (\delta, Sr, \chi) \). Then Eq. (5.10) can be written

\[
Sr^{-1} \frac{\partial v_*}{\partial t_*} + \delta v_* \cdot \nabla v_* + Ro^{-1} \mathbf{k} \times v_* = v_* \cdot \Sigma_* (\delta, Sr, \chi) 
\]

\[
+ C_a q_{a_*} w_* w_* + C_w q_{w_*} \left| u_{b_*} - v_* \right| \mathbf{\Theta}_w \cdot (u_{b_*} - v_*) - Fr^{-1} \beta. 
\]

The dimensionless surface stresses are of the form \( C \) times a quantity of the order of unity, i.e. \( \sim 10^3 \). The inertial time-scale is thus obtained from \( Sr \sim 10^3 : T \sim 10^3 h V^{-1} \). Expect for
very low ice speeds this is much smaller than $f^{-1}$ in the Baltic Sea; in fact, no clear peak was seen in the velocity spectra at the Coriolis frequency (Fig. 26). The length scale of deformation $L$ is $V$ divided by the strain-rate scale: the results from section 4.2 give $L \sim 10^5$ m. Thus, $\delta \sim 10^{-5}$, and consequently the advective acceleration is not important.

The Coriolis acceleration becomes important for $Ro \lesssim 10^3$ and the sea surface tilt at $Fr \lesssim 10^{-3}$ ($\beta \lesssim 10^{-6}$). In the Baltic Sea such is the case only when $V$ is small, $\lesssim 5$ cm/s. Evidently, $\Sigma_\alpha$ is very sensitive to $X$ and becomes meaningful when $X$ is greater than about 0.8 (e.g., see Fig. 14).

5.2. MOTION OF SINGLE ICE FLOE

A single ice floe, a flat plate floating on the sea surface, has three degrees of freedom in its movement: the translational velocity $V$ and the angular velocity $\omega$ with respect to the vertical axis. The velocity distribution is written

$$v = V + \omega \hat{k} \times r,$$

where $r$ is the radius vector w.r.t. a given point; here we measure $r$ w.r.t. the center of mass (see Fig. 30). It is assumed that the floe is not in contact with other floes and that the floe is large enough that the stresses of wind and water on the vertical sides of the floe can be neglected.

![Figure 30. Surface stress resultant $\Delta r$ acting on an ice floe.](image)

The motion of a floe is influenced by certain mass characteristics, which are integrals, over the floe surface $\Omega$, of the form

$$\frac{1}{A} \int_{\Omega} \Pi (r, h) \, dA,$$

where $A_f$ is the surface area of the floe. The geometric center lies at the tip of the inhomogeneity vector $r_{ih}$ (Fig. 30) which is obtained by using $\Pi = r$ in (5.13). The integrated
surface stresses act through the geometric center (provided the surface roughness is uniform), while the integrated body force, Coriolis effect and the inertial force act through the mass center. This may give rise to eigenrotation. The radius of gyration \( r_t \) is defined through

\[
q_i \tilde{h} A f r_t^2 = I_z,
\]

where \( I_z \) is the moment of inertia about the vertical axis. In our case,

\[
I_z = \int_{\Omega} q_i h r^2 dA
\]

(e.g. Landau & Lifschitz 1976), and thus \( r_t \) is obtained by using \( \Pi = (h/\tilde{h}) r^3 \) in (5.13).

Only the mean ice thickness needs to be known when the translational motion is studied, but for the rotational motion we must, in addition, know the location of the mass center, the inhomogeneity vector and the radius of gyration. The most difficult point is to locate the mass center and since that is generally not known, the rotation of individual floes tends to appear chaotic (e.g., Gorbunov & Timokhov 1968, Legen’kov et al. 1974). It must be noted that the influence of the inhomogeneity of the surface roughness could be included by changing \( r_t \) to the vector pair \( (r_{ih, a}, r_{ih, w}) \) defining the points of action of the integrated wind and water stresses separately.

The thickness measurements of the Aranda floe showed a clear inhomogeneity with a northeast gradient (Fig. 8). The mass of the floe was analyzed through interpolating the thickness into a number of elements and evaluating the integrals over the floe surface numerically (Table 13). At least the upper surface of the floe was rather homogeneous with no clear ridges.

<table>
<thead>
<tr>
<th>TABLE 13. The mass characteristics of the Aranda floe.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface area</td>
</tr>
<tr>
<td>Mean thickness</td>
</tr>
<tr>
<td>Mean areal mass density</td>
</tr>
<tr>
<td>Inhomogeneity vector:</td>
</tr>
<tr>
<td>absolute value</td>
</tr>
<tr>
<td>direction</td>
</tr>
<tr>
<td>Radius of gyration</td>
</tr>
</tbody>
</table>

The motion of the floe must satisfy the conservation laws of linear and angular momentum. Integrating over the surface of the floe leads to Euler’s equations of motion for a rigid body (e.g. Landau & Lifschitz 1976) which for our special case become

\[
q_i \tilde{h} \left( \frac{dV}{dt} + f \hat{k} \times V \right) = \frac{1}{A_f} \int_{\Omega} F dA, \tag{5.14.a}
\]

\[
q_i \tilde{h} r_j \frac{d\omega}{dt} = \hat{k} \cdot \frac{1}{A_f} \int_{\Omega} r \times F dA, \tag{5.14.b}
\]
where \( \mathbf{F} \) is the external force density, \( \mathbf{F} = \tau_a + \tau_w + \mathbf{G} \). Assume now that the wind stress and sea surface slope are constant over \( \Omega \). Then we obtain through direct calculation that

\[
\frac{1}{A_f} \oint r \times \tau_a \, dA = \mathbf{r}_{ih} \times \tau_a \quad \text{and} \quad \frac{1}{A_f} \oint r \times \mathbf{G} \, dA = \mathbf{0}.
\]

Integration of the water stress is more complicated, since it depends on \( \mathbf{v} \) and consequently on \( \mathbf{r} \). Let us, for simplicity, take for \( \tau_w \) the linear law which can be written through using Eq. (5.12) as

\[
\tau_w = e_w \sqrt{K_{w'}} \Theta_w \cdot (\mathbf{u}_b - \mathbf{v}) - e_w \sqrt{K_{w'}} \omega \Theta_w \cdot (\mathbf{k} \times \mathbf{r}).
\]

The first term on the right-hand side is constant over \( \Omega \) (assuming \( \mathbf{u}_b \) is) and will be denoted by \( \tau_w' \). We have then, after integration, from Eqs. (5.14)

\[
q_i \hat{h} \left( \frac{d\mathbf{v}}{dt} + f \mathbf{k} \times \mathbf{v} \right) = \tau_a + \tau_w' - e_i h \beta \quad \text{(5.15.a)}
\]

\[
- e_w \sqrt{K_{w'}} \omega \Theta_w \cdot (\mathbf{k} \times \mathbf{r}_{ih}),
\]

\[
q_i \hat{h} \frac{2}{I} \frac{d\omega}{dt} = \mathbf{k} \cdot \mathbf{r}_{ih} \times (\tau_a + \tau_w') - e_w \sqrt{K_{w'}} \cos \theta_w \omega \tilde{r}^2,
\]

where \( \tilde{r}^2 \) is the mean square radius of \( \Omega \). For homogeneous floes (i.e. \( \theta = \text{constant} \)) \( r_{ih} = 0 \). The equation for linear momentum now takes on its familiar form, and only the damping term is present on the right-hand side of the equation for angular momentum. For homogeneous floes \( \tilde{r}^2 = r_I^2 \), and the damping eq. has the solution \( \dot{\omega} = -\omega_0 \exp (-t/t_d) \), where

\[
t_d = \frac{q_i h}{e_w \sqrt{K_{w'}} \cos \theta_w}.
\]

In the Bothnian Bay \( t_d \approx 20-30 \) minutes.

Let us consider inhomogeneous floes. For the Aranda floe \( r_{ih} = 236 \) m and \( r_I = 1320 \) m (Table 13). In Eq. (5.15.a) the "stress speed" due to rotation is given by \( \dot{\omega} r_{ih} \). It will be shown below that, for such floes as the Aranda floe, the angular velocity is necessarily small enough that the last term on the right-hand side of Eq. (5.15.a) can be ignored, and the resulting equation is the same as for the free drift of pack ice particles to be discussed in the next section.

On the right-hand side of Eq. (5.15.b) we have the external moment due to the imbalance of the surface stresses and the damping term. Consider the case when \( \tau_a + \tau_w' = (\Delta \tau, 0) = \text{constant} \) (Fig. 30). Then Eq. (5.15.b) can be written as

\[
q_i \hat{h} \frac{2}{I} \frac{d^2\omega}{dt^2} + e_w \sqrt{K_{w'}} \cos \theta_w \tilde{r}^2 \frac{d\omega}{dt} + r_{ih} \Delta \tau \sin \omega = 0. \quad \text{(5.16)}
\]
The discriminant of the characteristic equation of the linearized form of Eq. (5.16) \((\sin \omega \Re \omega)\) is non-negative, if

\[
\frac{r_{1h}^2}{\hbar r_i^2 \Delta r} \geq \frac{4 q_i \Delta r}{(e_w \sqrt{K_w f} \cos \theta_w)^2}.
\] (5.17)

The right-hand side is \(\sim 10\) and thus the inequality is satisfied unless the floe is strongly inhomogeneous.

E.g., for the Aranda floe the left-hand side of (5.17) is \(\sim 5 \times 10^3\). The non-negativeness of the discriminant means that the damping term is very efficient and the linearized differential equation has a non-oscillatory solution. Thus the floe rotates back to the equilibrium orientation \((\omega = 0)\) with an angular speed of the order of

\[
\omega \sim \frac{r_{1h} \Delta r \sin \omega}{e_w \sqrt{K_w f} \cos \theta_w r_i^2}.
\]

For such floes as the Aranda floe the above result gives an angular speed of less than \(\sim 10^{-5}\) s\(^{-1}\).

In practice the surface stress resultant varies in time scales of from less than one hour up to one day and in such periods floe rotations are not more than a few degrees. The variation causes aperiodic oscillations in the rotation of floes. The solutions of Eqs. (5.14) with the quadratic water stress have been numerically studied by the present author and the results supported the above conclusions based on linear water stress.

The kinetic energy per unit area for ice floes can be divided into the translational and rotational kinetic energies through

\[
q = q_{tr} + q_{rot}
\]

\[
= \frac{1}{2} e_i \hbar \nu^2 + \frac{1}{2} e_i \hbar r_i^2 \omega^2
\]

and consequently

\[
\frac{q_{rot}}{q_{tr}} = \left( \frac{r_i \omega}{\nu} \right)^2.
\]

Our observations give \(|r_i \omega| \leq 0.5\) cm/s and hence the kinetic energy of floes is nearly purely translational.

The observed angular velocity of the Aranda floe and the vorticity of the GT-array fit very well together (Fig. 31). Hence in the case of high compactness the contact stresses of surrounding floes determine the rotation of individual floes. This is in agreement with earlier results (e.g., Hibler et al. 1974c, Leppäranta 1980b). The spectrum of the angular
velocity of the Aranda floe was calculated using the maximum entropy method described, e.g., in Alenius (1980). A slight maximum is seen at the Coriolis frequency and the noise level is rather high (Fig. 32).

![Figure 31. Angular velocity of the Aranda floe and vorticity of the GT-array (three-hour floating averages).](image1)

![Figure 32. The spectrum of the angular velocity of the Aranda floe.](image2)
5.3. MOTION OF PACK ICE PARTICLES

The equation of motion of pack ice has been given in general form by Eq. (5.2) and with explicit expression for the external forces by Eq. (5.9). Two assumptions which simplify the ice drift problem considerably are: (i) free drift, i.e. \( \Sigma \equiv 0 \) and consequently \( \mathbf{v} \cdot \Sigma \equiv 0 \); (ii) stagnant ocean (beneath the boundary layer), i.e. \( \mathbf{u}_b \equiv 0 \) and \( \beta \equiv 0 \).

Free drift

Let us first consider steady free drift over a stagnant ocean. The wind velocity is assumed here constant. Then, from Eq. (5.9), the equation of motion is written in explicit form as

\[
e_i h / \mathbf{k} \times \mathbf{v} = e_a C_a w w - e_w C_w v \theta_w \cdot \mathbf{v}.
\]

Scaling this with \( e_i V^2 \) and choosing

\[
V = \sqrt{\frac{e_a C_a w}{e_w C_w}}
\]

as the velocity scale, we have, after rearrangement of the terms,

\[
\frac{e_w C_w}{e_i} \left( \frac{\mathbf{v}}{V} \right) \theta_w \cdot \left( \frac{\mathbf{v}}{V} \right) + Ro^{-1} \mathbf{k} \times \left( \frac{\mathbf{v}}{V} \right) = \frac{e_w C_w}{e_i} \left( \frac{\mathbf{w}}{w} \right).
\]

Taking the square of the modulus gives us

\[
\left( \frac{\mathbf{v}}{V} \right)^4 + 2 \sin \theta_w Ro^{-1} \left( \frac{\mathbf{v}}{V} \right)^3 + Ro^{-2} \left( \frac{\mathbf{v}}{V} \right)^2 = 1,
\]

where

\[
Ro_{iw} = \frac{e_w C_w}{e_i} Ro.
\]

The Rossby number \( Ro = V/hf \) depends only on the state of the ice, whereas the number \( Ro_{iw} \) above arises as a convenient dimensionless quantity in free drift. The product \( \nu/V \cdot Ro_{iw} \) gives then the ratio of the Coriolis effect to the water stress. Below, in the case of non-steady free drift, the Strouhal number will be treated similarly. The dimensionless speed \( \nu/V \) can be solved iteratively from Eq. (5.21) and the kinetic energy budget provides an other equation for the deviation angle \( \Delta \psi \) between wind and ice drift (see section 6.1). That is,

\[
\frac{\nu}{V} = \frac{\nu}{V} \left( Ro_{iw}, \theta_w \right),
\]

\[
\Delta \psi = \Delta \psi \left( Ro_{iw}, \theta_w \right). \tag{5.22}
\]
The solutions are given in Fig. 33. Note that, as \( h \to 0 \), \( \nu/V \) approaches unity and \( \Delta \psi \) approaches \( \theta_n \); in addition \( \nu/V \) equals the proportion of the real wind factor relative to the theoretical wind factor for \( h = 0 \).

Figure 33. The solution of the steady free drift over a stagnant ocean. The relative wind factor equals the ratio of the real wind factor to the hypothetical wind factor for zero ice thickness.

Removal of the assumption of stagnancy of the ocean requires corrections to the solutions (5.22). The tilt force adds the term \(-Fr \beta\) to the right-hand side of Eq. (5.20) and the current \( u_b \) changes the solutions (5.22) by approximately \( u_b/V \). Especially if we can approximate \( u_b \) by the geostrophic current \( u_g \) and \( \beta \) by the tilt resulting from the geostrophic balance, we need only replace \( \nu \) by \( \nu - u_g \) in Eq. (5.18) and then the solutions (5.22) become exact for the relative velocity \( \nu - u_g \).

The proportionality factor in Eq. (5.19) is \( \approx 0.027 \) (Table 12). That is, as \( h \to 0 \) the wind factor approaches 0.027. For the conditions in SI79, the number \( Ro_a^{-1} \) is 0.58 or 0.10 for \( V = 5 \) or 30 cm/s, respectively. For over a little more than half of the observation period the ice drift seemed to follow the free drift estimation quite well (Fig. 28). Significant deviations occurred from a) noon of 10th April to the early hours of 12th April, b) 13th April 18.00 hrs to 14th April 08.00 hrs and c) from 14th April 20.00 hrs onwards.

Let us look at the inertial term. It was shown in section 5.1 that the advective acceleration is much smaller than the local acceleration and we therefore assume that \( \nu \) depends on time only. In the one-dimensional case the equation of motion on a stagnant ocean is written as

\[
\rho \frac{d \nu}{dt} = \rho_a C_a w^2 - \rho_w C_w v^2.
\] (5.23)
Let $V$ be as above (Eq. 5.19), $T$ a time scale and $v_o$ the initial ice speed at time zero. Then $V$ is equal to the steady-state speed and the solution of Eq. (5.23) is

$$\frac{v}{V} = \mathcal{F} \left[ S_{iw} \left( \frac{t}{T} \right) + \mathcal{F}^{-1} \left( \frac{v}{v_o} \right) \right], \text{ for } w > 0,$$

$$\frac{v}{v_o} = \left[ 1 + S_{iwo} \frac{t}{T} \right]^{-1}, \text{ for } w = 0.$$ 

Here $\mathcal{F}(x)$ is $\tanh(x)$ or $\coth(x)$ for $V < v_o$ or $V > v_o$, respectively, $S_{iw} = \frac{g_w C_w}{\ell_i}$, and $S_r$ and $S_{ro}$ are defined through using the velocity scales $V$ and $v_o$, respectively. In the trivial case $v_o = V$, $\mathcal{F}(x) = 1$ and $\mathcal{F}^{-1}$ is meaningless. For $h = 75$ cm and $V$ or $v_o$ $\approx 10$ cm/s the natural inertial time scale becomes $\sim \frac{1}{2}$ hours. Numerical solutions of the two-dimensional non-steady free drift equation also support the rapid response of the Baltic sea ice to the wind (Leppäranta 1980b). These theoretical calculations agree with our observed time-series (Fig. 28). We can conclude that the changes of ice velocity in the Baltic Sea are more controlled by the inertia of the atmosphere than by the inertia of the ice itself.

The balance of forces on pack ice

Deviations from the general free drift force balance are caused by the internal friction within the ice pack. From Eq. (5.2) we have

$$\mathbf{R} \begin{array}{c} \text{Def} \\ \mathbf{v} \cdot \Sigma = \rho_l h \left( \frac{dv}{dt} + \int \mathbf{k} \times \mathbf{v} \right) - \tau_a - \tau_w - \mathbf{G}. \end{array}$$

That is, the residual of the free drift forces gives us the divergence of ice stress. The idea was used by Hunkins (1975b) who found that the residual is of the same order of magnitude as the governing external forces. Similar results were obtained in the Bothnian Bay by Udin & Omstedt (1976) and Leppäranta (1979).

The results from SI79 agree with the earlier work. Some selected cases are shown in Fig. 34; they are also denoted in the velocity time series (Fig. 28). The forces are based on three-hour averages using the formulas (5.5), (5.6) and (5.8) for the external forces and the parameters given in Table 12. The thickness of ice was taken as 73 cm (Table 4). The current measurements at the depth of 20 m were used for $u_b$ and the sea surface slope was determined through regression analysis from the routine water-level measurements at the Finnish and Swedish coastal stations. When some vector is missing in the diagrams, it means that the estimate is close to zero. The cases are representative for the given instant of time $\pm 1\frac{1}{2}$ hours. They are characterized by:

a&B) Steady ice motion at the speed of about 10 cm/s. The wind stress is balanced by the water stress, Coriolis effect and residual force ("residual").
Figure 34. Selected force balances from S179; $\mathbf{M}$ — inertial force, $\mathbf{M}_c$ — Coriolis effect, $\mathbf{R}$ — residual, $\tau_w$ — wind stress, $\tau_a$ — water stress, $\mathbf{G}$ — tilt force.

c&d) Rather steady ice motion with low wind factor. In case d) the wind stress and residual are one order of magnitude larger than the other terms.

e) The ice speed is close to the free drift speed, but there is a current of about 5 cm/s flowing in the same direction as the ice drift. Consequently, the water stress becomes smaller than the wind stress, which gives rise to a large residual.
f) Ice motion is dropping fast (Fig. 28) and the wind stress and residual are dominant. Evidently the drop was caused by compacting of the ice pack after a large southwest displacement (Fig. 18).

g) Acceleration of ice drift in nearly free drift state. The residual is acting as a driving force which may be due to transmission of ice stress from the west, whence the storm was coming.

h) The rapid turning of ice drift in the presence of a high wind (see Fig. 27).

i) The ice pack between Aranda and fast ice boundary has become compact and ice movement has radically diminished. The wind stress and residual are dominant.

Analysis of all the force diagrams from SI79 confirm that the wind stress, water stress and residual are the governing forces and that the other forces are on average one order of magnitude smaller (Table 14). Although we must be critical of the residual method due to large uncertainties in estimating the forces (especially the surface stresses), it does tell us that there is a frictional force usually present in pack ice with compactness higher than $\sim 0.9$, which force is comparable with the governing external forces.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inertia</td>
<td>3</td>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>Coriolis effect</td>
<td>7</td>
<td>7</td>
<td>32</td>
</tr>
<tr>
<td>Wind stress</td>
<td>62</td>
<td>87</td>
<td>420</td>
</tr>
<tr>
<td>Water stress</td>
<td>28</td>
<td>40</td>
<td>210</td>
</tr>
<tr>
<td>Tilt force</td>
<td>4</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>Residual</td>
<td>41</td>
<td>55</td>
<td>260</td>
</tr>
</tbody>
</table>

The linear viscous internal friction is written as (see section 4.3)\[
\nabla \cdot \Sigma = \xi \nabla (\nabla \cdot \mathbf{v}) + \eta \nabla^2 \mathbf{v}.
\]

The second-order derivatives on the right-hand side were estimated from the measurements of the deformation of the GT-array. The method is analogous to the method for estimating the velocity gradient in section 4.2. Using consecutive positions for the reference and final configurations, we have

\[
\frac{\partial^2 v_i}{\partial x_j \partial x_k} \approx \frac{1}{\Delta t} \frac{\partial}{\partial x_k} \left( \frac{\partial x_i}{\partial x_j} \right),
\]

and the right-hand side can be estimated through using $k_\alpha = 2$ in Eq. (4.2). Thus, we have the right-hand side of Eq. (5.24) except for the viscosity coefficients. These should then be obtained through scaling the gradient of divergence and Laplacian of the ice velocity into the force diagrams (Fig. 35).
The force diagrams for which GT-array data were available were studied together. In 61% of the cases the direction of at least one viscous force estimate deviated by less than 45 degrees from the direction of the residual. For such cases the vectors $\nabla (\nabla \cdot v)$ and $\nabla^2 v$ were scaled so that the length became equal to the length of the residual and the scaling coefficients were taken as the viscosity estimates. There were five distinct periods during each of which the viscosity estimates remained within one order of magnitude (Table 15).

During the first three periods there was an overall decrease in compactness from 0.94 to 0.89 (Fig. 19) and the viscosities were of the order of $10^7$ kg s$^{-1}$. Then, during the last two periods the compactness increased back to 0.95 and the viscosities were large. In the last period there was an indication that the bulk viscosity was $10^{11}$ kg s$^{-1}$ for small divergence and $10^{10}$ kg s$^{-1}$ for small convergence. The high viscosities for small deformation rates are in agreement with the nonlinear viscous law of Hibler (1977).

**TABLE 15.** Estimated linear viscosities from force diagrams and deformation measurements in S179. $fit$ is the relative number of cases when at least one viscous term could be consistently estimated. $\nabla$ and $|\dot{\epsilon}|$ are the velocity and strain-rate scales, respectively.

| Day: hours | Shear v. kg/s | Bulk v. kg/s | Fit % | Comp. | Change of comp. | $V$ cm/s | $|\dot{\epsilon}|$ x $10^6$ s |
|------------|---------------|--------------|-------|-------|-----------------|----------|-----------------|
| 8:11-17    | $2 \times 10^7$ | $2 \times 10^7$ | 67    | 0.94  | 0               | 10       | 0.5             |
| 9:11-23    | $1 \times 10^7$ | $2 \times 10^7$ | 80    | 0.92  | 0               | 5        | 1.5             |
| 10:08-23   | $1 \times 10^7$ | $1 \times 10^7$ | 33    | 0.91  | -0.1            | 5        | 2.0             |
| 13:5-14:2  | $2 \times 10^8$ | $-$           | 100   | 0.93  | 0.3             | 15       | 1.0             |
| 14:05-14   | $3 \times 10^9$ | $5 \times 10^9$ | 75    | 0.95  | 0               | 25       | 0.1             |
However, the vectors $\nabla (\nabla \cdot \mathbf{v})$ and $\nabla^2 \mathbf{v}$ do not agree well with the residual of the force diagrams. This may be due to uncertainties in calculating the residual and due to the fact that the number of reflector masts (five) was the lowest possible needed to estimate the second order derivatives. Consequently, random movements of any floe where a mast was located have a large effect on the estimates; if the number of masts had been larger, the random movements would have been taken care of by the residual in the regression analysis.

From 14th April 16.00 hrs onwards we have no deformation data for the GT-array, and then the ice pack was pressing heavily against the Finnish coast, ridging occurred around Aranda, and the minimum ice speed was about 0.005 times the wind speed (Fig. 28). The boundary layer analysis below will show that during the ridging period the viscosities were of the order of $10^9$ kg s$^{-1}$.

Our results thus indicate that for the conditions in SI79 the viscosities vary in the range $10^7$ to $10^{10}$ kg s$^{-1}$. Hibler & Tucker (1977) found two orders of magnitude seasonal variation in the Central Arctic and Rothrock (1975b) already pointed out that several orders of magnitude variation is found in the values given by different authors (cf. Table 10).

**Boundary layer flow of pack ice**

Let us direct the $x$-axis along the fast ice boundary, with the $y$-axis perpendicular to it (Fig. 36), and assume that the ice velocity depends only on the coordinate $y$. Furthermore, let the ocean be stagnant. The equation of motion (5.2) becomes then

$$\rho_t h (\frac{\partial}{\partial y} \mathbf{v} + f \mathbf{k} \times \mathbf{v}) = \nabla \cdot \Sigma + \tau_a + \tau_w.$$

Our conclusion in section (5.1) was that the advective term is negligible. Then we take the linear viscous internal friction and the linear water stress:

$$\rho_t h f \mathbf{k} \times \mathbf{v} = \zeta \mathbf{v} \left( \frac{\partial^2 \mathbf{v}}{\partial y^2} \right) + \eta \frac{d^2 \mathbf{v}}{dy^2} + \tau_a - \rho_w \sqrt{K_w} \sigma_w \cdot \mathbf{v}.$$
Laikhtman (1958) solved the above equation with $\zeta = 0$. Instead, we shall neglect the Coriolis acceleration and the turning angle in the water. Then the boundary layer equation becomes, in component form,

$$\eta \frac{d^2 v_x}{dy^2} + \tau_{ax} - \rho_w \sqrt{K_w} v_x = 0,$$

$$\frac{d^2 v_y}{dy^2} + \tau_{ay} - \rho_w \sqrt{K_w} v_y = 0. \quad (5.25)$$

With the boundary conditions

$$v_x = v_y = 0, \quad \text{for } y = 0,$$

$$v_x = v_{x, \infty} = \tau_{ax}/\rho_w \sqrt{K_w}, \quad \text{for } y \to \infty,$$

$$v_y = v_{y, \infty} = \tau_{ay}/\rho_w \sqrt{K_w}, \quad \text{for } y \to \infty,$$

the solution is

$$v_x = v_{x, \infty} [1 - \exp (-y/L_x)], \quad L_x = [\eta/\rho_w \sqrt{K_w}]^{1/2},$$

$$v_y = v_{y, \infty} [1 - \exp (-y/L_y)], \quad L_y = [(\zeta + \eta)/\rho_w \sqrt{K_w}]^{1/2}.$$

Our data from the ridging situation in the early hours of 15th April give $v/v_{\infty} \approx 0.4$ at $y \approx 20$ km. It is very difficult to say how much exactly would be the ratios for the shear and compression components separately, but it can be concluded that $\zeta$ and $\eta$ are of the same order of magnitude and $\zeta, \eta \sim 10^6$ kg s$^{-1}$.

The pack ice boundary layer was considered in the framework of common turbulent boundary layer theory by Takizawa (1976 and 1979) in the Okhotsk Sea. The assumptions are then, however, not realistic, and the resulting logarithmic velocity profile led to viscosities of the order of $10^4 - 10^6$ kg s$^{-1}$, which is definitely too low. Analyzing the data of Takizawa with Eqs. (5.25), the viscosities are found to be somewhere in the range $5 \times 10^7$ to $5 \times 10^8$ kg s$^{-1}$. The mass characteristics of the pack ice in the Okhotsk Sea are quite similar to those in Baltic Sea and the last mentioned viscosities are rather close to our results.

In reality, the pack ice boundary layer flow is very complicated and the above analysis gives us only average viscosities for a time-scale of several hours. There seems to be large amplitude oscillations in ice velocity, when ice is pressing against the fast ice boundary (Leppäranta 1980b). It is very unlikely that the oscillations could be simulated with a linear viscous constitutive model.
6. MECHANICAL ENERGY BUDGET

6.1. EQUATION OF KINETIC ENERGY

Discussion of the equation of kinetic energy of pack ice has only recently begun. For example, it is found in the paper of Kheisin (1977); Coon & Pritchard (1979) regarded it as a powerful tool in future sea ice research. Analyses of field data have not been given earlier.

Let us first multiply the equation of motion (5.9) scalarly with \( \mathbf{v} \). The result is

\[
\frac{1}{2} \frac{\partial}{\partial t} q_{ih} \left( \frac{\partial v^2}{\partial t} + \mathbf{v} \cdot \nabla v^2 \right) = \mathbf{v} \cdot (\nabla \cdot \mathbf{v}) + \dot{\varepsilon}_a C_a \mathbf{w} \cdot \mathbf{w} + \varepsilon_a C_a w \cdot \mathbf{w} \cdot \mathbf{v}
\]

(6.1)

\[
+ \varepsilon_w C_w \left[ \mathbf{u}_b - \mathbf{v} \right] [((\Theta_w \cdot \mathbf{u}_b) \cdot \mathbf{v} - \cos \theta_w v^2] - \dot{q}_i h g \beta \cdot \mathbf{v},
\]

where use has been made of \( \mathbf{k} \times \mathbf{v} \cdot \mathbf{v} = 0 \). On the left-hand side we have the total rate of change of the kinetic energy per unit area \( q = \frac{1}{2} q_{ih} v^2 \). The terms on the right-hand side give the rate of work done on unit area of ice by the internal and external forces: The first term is the rate of work done by the internal friction and through direct calculation we can see that (Coon & Pritchard 1979)

\[
\mathbf{v} \cdot (\nabla \cdot \mathbf{v}) = \nabla \cdot (\mathbf{v} \cdot \mathbf{v}) - \text{tr} (\dot{\varepsilon} \cdot \Sigma),
\]

(6.2)

where \( \nabla \cdot (\mathbf{v} \cdot \mathbf{v}) \) is the rate of work by the surrounding ice and \( -\text{tr} (\dot{\varepsilon} \cdot \Sigma) \) is the dissipation of kinetic energy in internal deformation processes. The second and fourth terms give the rate of work done by wind and gravity, respectively. The third term can be decomposed into the rate of work done by current and the negative of the rate of work done by ice on the oceanic boundary layer. Then, Eq. (6.1) can be written as

\[
\frac{\partial q}{\partial t} + \mathbf{v} \cdot \nabla q = \nabla \cdot (\mathbf{v} \cdot \mathbf{v}) - \text{tr} (\dot{\varepsilon} \cdot \Sigma) + \varepsilon_a C_a \mathbf{w} \cdot \mathbf{w} \cdot \mathbf{v}
\]

\[
+ \varepsilon_w C_w \left[ \mathbf{u}_b - \mathbf{v} \right] [((\Theta_w \cdot \mathbf{u}_b) \cdot \mathbf{v} - \cos \theta_w v^2] - \dot{q}_i h g \beta \cdot \mathbf{v},
\]

(6.3)

\[
\frac{\partial q}{\partial t} + \mathbf{v} \cdot \nabla q = \nabla \cdot (\mathbf{v} \cdot \mathbf{v}) - \text{tr} (\dot{\varepsilon} \cdot \Sigma) + \varepsilon_a C_a \mathbf{w} \cdot \mathbf{w} \cdot \mathbf{v}
\]

\[
+ \varepsilon_w C_w \left[ \mathbf{u}_b - \mathbf{v} \right] [((\Theta_w \cdot \mathbf{u}_b) \cdot \mathbf{v} - \cos \theta_w v^2] - \dot{q}_i h g \beta \cdot \mathbf{v},
\]

Let us consider the steady free drift over a stagnant ocean. Then the kinetic energy input from the wind is transmitted wholly to the oceanic boundary layer:

\[
\varepsilon_a C_a \mathbf{w} \cdot \mathbf{w} - \varepsilon_w C_w \cos \theta_w v^3 = 0.
\]

Since \( \mathbf{w} \cdot \mathbf{v} = w \nu \cos \omega \), we have

\[
\varepsilon_w C_w \cos \theta_w v^3 \left( \frac{\varepsilon_a C_a}{\varepsilon_w C_w} \cdot \frac{\cos \omega}{\cos \theta_w} \cdot \left( \frac{w}{\nu} \right)^2 - 1 \right) = 0,
\]

(6.4)
and we have thus a simple condition between the wind factor and deviation angle:

\[
\frac{\cos \Delta \psi}{\cos \theta_w} = \frac{\varrho_w C_w}{\varrho_a C_a} \left(\frac{v}{w}\right)^2
\]

We also note that in general the left-hand side of Eq. (6.4) gives the amount of kinetic energy in excess of the free drift balance.

6.2. ENERGY DISSIPATION WITHIN PACK ICE

According to Rothrock (1975a), kinetic energy is dissipated within pack ice through

(i) generation of gravitational potential energy in ridging,
(ii) frictional losses in the rubble piles of ridges during ridging,
(iii) fracture of ice sheets,
(iv) frictional losses in shearing between floes.

We can still add to these

(v) frictional losses in overriding of ice sheets.

For the Arctic seas, Rothrock concluded that the sinks (i) and (ii) due to ridging are nearly equal and argued that (iii) is small compared to (i) and (ii), while (iv) is not greater than (i) and (ii). The point (v) is probably meaningful only in rafting of very thin ice.

The expression for the potential energy per unit area \( P \) of the ice-water system relative to the sea surface was derived by Rothrock (1975a). It consists of the potential energy of the ice and the energy required to displace the water

\[
P = \frac{1}{2} \varrho_i g h'^2 + \frac{1}{2} (\varrho_w - \varrho_i) g h''^2,
\]

where \( h' \) and \( h'' \) are the freeboard and draft of the ice, respectively. This eq. can be written as

\[
P = \frac{1}{2} \frac{\varrho_i}{\varrho_w} (\varrho_w - \varrho_i) g h^2 + \frac{1}{2} \varrho_w g (h'' - \frac{\varrho_i}{\varrho_w} h)^2,
\]

where the latter term gives the energy due to the departure from isostasy and will be neglected below. It can be meaningful (Rothrock 1975a), but its estimation requires information not currently available to us.

The potential energy of a ridge is defined here as the sum of the energies of the rubble piles above and below the level ice sheet. The quantities in the following formulas are explained in Figs. 37 and 38. The potential energy per unit width of the ridge sail is

\[
P_s' = (1 - \nu) \frac{d_s}{2} \frac{\varrho_s g}{\tan \varphi_s} \int y \tan \varphi_s (z + h') dx dz
\]

\[
= (1 - \nu) \frac{\varrho_s g}{\tan \varphi_s} (\frac{1}{2} h_s^3 + h' h_s^2).
\]
The quantities for determining the potential energy of an ice ridge are depicted in Figure 37.

\[ h' = (1 - \frac{\epsilon_i}{\epsilon_w}) h, \quad h'' = \frac{\epsilon_i}{\epsilon_w} h, \]  

\[ \gamma \text{ Def } \frac{h_k}{h_s} = \left( \frac{\epsilon_s}{\epsilon_w - \epsilon_k} \right) \frac{\tan \varphi_k}{\tan \varphi_s} \]  

The ridge keel is treated similarly. Using the isostatic conditions, the potential energy of the ridge per unit width becomes

\[ P_r' = P_s' + P_k' = (1 - \nu) \cdot \frac{\epsilon_s g}{\tan \varphi_s} \left( \frac{1 + \gamma}{3} h_s^3 + h h_s^2 \right). \]  

The term \( hh_s^2 \) arises from our definition in which the rubble piles above and below the level ice sheet are considered. It would vanish if the potential energy of the ridge had been...
defined relative to the sea surface; consequently, the sail height would then be the elevation above the sea surface. Eq. (6.7) includes four ridge characteristics, which are chosen as follows: \( v = 0.4 \) and \( \rho_s = 870 \text{ kg m}^{-3} \) (Keinonen 1977); \( \gamma = 6.9 \) (from Palosuo 1975, see section 3.1);

\[
\varphi_s = \arctan (b_0 h_s), \ b_0 = 0.442 \text{ m}^{-1}
\] (6.8)

(see section 3.1).

The potential energy of ridges per unit ice-covered area in a pack ice particle is

\[
P_r = L_r \rho_r A_i^{-1},
\]

where \( L_r \) is the total length of ridges and \( A_i \) the area of the particle. Using the formulas (3.3.b), (3.4), (6.7) and (6.8), and neglecting the spatial variation of ice thickness in integration, we have

\[
P_r = \frac{n}{2} \mu (1-v) \frac{\rho_s g}{b_0} \left\{ \frac{1+\gamma}{3} \left[ \tilde{h}_s^2 + (\tilde{h}_s - \tilde{h}_{so})^2 \right] + h \tilde{h}_s \right\}.
\]

Insertion of the values for the parameters gives

\[
P_r = 18.2 \text{ kJ m}^{-3} \times \left\{ 2.63 \left[ \tilde{h}_s^2 + (\tilde{h}_s - \tilde{h}_{so})^2 \right] + h \tilde{h}_s \right\} \mu.
\] (6.9)

Rothrock (1975a) studied the frictional losses in the rubble pile during ridge formation on the basis of a simple Coulomb friction model. He obtained the following expression for the frictional force per unit width of a ridge:

\[
F_{fr} = k_{fr} (\rho_w - \rho_k) g \frac{h_{k}^2}{2 \tan \varphi_k}.
\]

Here \( k_{fr} \) is the dynamic coefficient of friction; Rothrock used \( k_{fr} \approx 0.1-0.4 \). Ryvlin (1973) concluded that \( k_{fr} \) for ice on ice and steel on ice are about equal and recommended the value \( k_{fr} = 0.1 \), which we shall take here. Using Eq. (6.6. b) the force \( F_{fr} \) can be expressed in terms of the sail characteristics through

\[
F_{fr} = k_{fr} \rho_i g \frac{h_{s}^2}{2 \tan \varphi_s}.
\]
The work done per unit width against friction in building up a ridge is

\[ W_{fr} = \int_0^l F_{fr}(h_s(y)) \, dy, \] (6.10)

where \( l \) is the length of the ice sheet needed for the rubble pile and \( h_s\) is the instantaneous sail height when the ice sheet has advanced the length \( y \) in the ridge, i.e. \( h_s(0) = 0 \) and \( h_s(h_s) = h_s \) (the final sail height). It is assumed that the geometric structure of the ridge is similar during the whole ridging process, i.e. \( \varphi_s = \arctan \left( \frac{h_s}{b_o} \right) \). The conservation of mass implies that

\[ h_y = \frac{A_r}{h}, \]

where \( A_r \) is the instantaneous cross-sectional area of the ridge. Hence \( h_s = h_y b_o / \kappa \) and \( l = \kappa h_y / b_0 h \), and we can integrate Eq. (6.10) directly:

\[ W_{fr} = \frac{k_{fr} \rho_s g}{2 \tan \varphi_s} \frac{h_s^3}{3 b_o h}. \] (6.11)

The ratio of the work done against friction to the potential energy is, from Eqs. (6.7) and (6.11)

\[ \frac{k_{fr}}{6(1-\nu)} \frac{\kappa}{b_0 h} \left( \frac{1+\nu}{3} + \frac{h_s}{h} \right) \rightarrow \frac{1}{2}. \]

E.g., with \( h_s \approx \frac{h_s}{b_0 h} \) m the ratio is 0.5 and increases to 0.7 as \( h_s \rightarrow \infty \). Exactly by the same way as in the case of the potential energy we can integrate Eq. (6.11) to have the work done against friction per unit area in a pack ice particle. The result is written

\[ W_{fr} = \frac{\pi}{2} \mu \frac{k_{fr} \rho_s g}{2 \tan \varphi_s} \frac{h_s^2}{3 b_o h^2} \left( \tilde{h}_s^2 - (\tilde{h}_s - \tilde{h}_{so})^2 \right). \]

The rest of the energy dissipation is a big question mark. Parmerter & Coon (1972) showed with their pressure ridging model that the energy loss due to fracture of ice is small compared to the potential energy. Frictional losses in shearing between floes are considered meaningful (e.g. Rothrock 1975a) but there is no estimate, empirical or theoretical, of their magnitude.

It was shown in the previous section that the rate of dissipation of kinetic energy per unit area in internal deformation processes is given by \( \dot{\rho} \). Assuming the viscous constitutive law (Eq. 4.4), the dissipation becomes
which is a quadratic expression of the strain-rate invariants. In contrast, the plastic constitutive law of Coon et al. (1974) gives a linear form.

6.3. EMPIRICAL STUDIES OF MECHANICAL ENERGY BUDGET

We start from the paper of Lisitzin (1957) showing that the water level variations in the Bothnian Bay are considerably damped in winter compared to ice-free situations (Fig. 39). Let us fix the origin in the southwest corner of the Bothnian Bay, direct the x-axis in the direction of the long axis of the basin and define the zero water level as that in the origin. The potential energy of the piled-up water in the Bothnian Bay is then

$$\mathcal{F}_w = \int_0^L \int_{y_1}^{y_2} \int_0^x \rho_w g z \, dx \, dy \, dz ,$$

where $L$ is the length of the basin, $y_2(x) - y_1(x)$ is the width and $\xi(x,y)$ is the water level elevation. Assume further that the sea surface tilt is constant and directed northeast. The water level elevation at the northeast corner, $\xi(L,0)$, is denoted by $H$. Then, we have

$$\mathcal{F}_w = \frac{1}{2} \rho_w g \left( \frac{H}{L} \right)^2 \int_0^L (y_2 - y_1) x^2 \, dx .$$

(6.12)
Since the boundary geometry of the basin varies in a complicated way, the integral in Eq. (6.12) must be numerically evaluated. The result was \( \int_0^L (y - y_i)x^2 dx \approx 1.13 \times 10^{21} \text{ m}^4 \). Inserting this value and \( L = 315 \text{ km} \), and dividing Eq. (6.12) with the area of the Bothnian Bay, \( A = 36500 \text{ km}^2 \), we obtain the potential energy per unit area

\[
P_w = \mathcal{F}_w/A = 0.156 \times c_w g H^2.
\]

From Lisitzin (1957) we can deduce that

\[
H = B w^2,
\]

where \( w \) is the wind speed and \( B = B_f \approx 2.36 \times 10^{-3} \text{ m}^{-1} \text{s}^2 \) or \( B = B_i \approx 0.788 \times 10^{-3} \text{ m}^{-1} \text{s}^2 \) for ice-free or ice-covered cases, respectively. The difference in the potential energy corresponding to energy loss within the ice pack is thus

\[
\Delta P_w = 0.156 \times c_w g w^4 (B_f^2 - B_i^2).
\] (6.13)

It should be noted that there is no significant difference in the energy input from the wind between ice-free and ice-covered situations in the Baltic Sea (Joffre 1978) and hence Eq. (6.13) should truly describe the energy dissipation within the ice pack.

For \( w = 10 \text{ m/s} \) Eq. (6.13) gives \( \Delta P_w = 76 \text{ J m}^{-2} \). The time needed for this energy loss is of the order of one day. During such storm conditions the ridge density was estimated by Leppäranta (1980b) to have increased by \( \approx 1.5 \text{ km}^{-1} \) in about 12 hours. Then, with \( h, h_i \approx 50 \text{ cm} \) and \( h_{so} = 30 \text{ cm} \), Eq. (6.9) gives \( \approx 17 \text{ J m}^{-2} \) for the production of potential energy in ridging. From the data in Leppäranta (1981) we can see that the total potential energy in ridges in winter is \( \approx 200 \text{ J m}^{-2} \). The meteorological conditions tell us, through Eq. (6.13), that several times more energy is lost within the ice cover.

Grounded ridges close to the fast ice boundary extend to high elevations and locally store large amount of potential energy. Since the sail heights of large grounded ridges nearly equal the keel depths, the main portion of the potential energy of such ridges is in the sails. Choosing \( h, \sim 7 \text{ m} \) and \( \varphi, \sim 45 \text{ degrees} \), Eq. (6.5) gives an order of magnitude estimate of \( 6 \times 10^5 \text{ J per unit length} \) for the potential energy in the sail. The length of the large grounded ridges in the Bothnian Bay is not more than \( \sim 1000 \text{ km} \). Consequently, the potential energy per unit area of the whole basin becomes \( \sim 10 \text{ J m}^{-2} \) which is one order of magnitude smaller than the potential energy of the floating ridges.

Although Eq. (6.13) is quite sensitive to the wind speed, it is concluded that the potential energy production is not the major part of the kinetic energy dissipation. In the Artic Ocean, on the other hand, the friction in shearing between floes is considered not to be more important than the production of potential energy (Rothrock 1975a). Our results do not necessarily contradict this, since the potential energy increases rapidly with the sail height of ridges which approximately scales with ice thickness, whereas the friction in shear increases only linearly with ice thickness.
Kinetic energy budget in SI79

The terms of the kinetic energy equation (6.3), except for those concerned with the ice stress, were estimated from the SI79 data (Fig. 40, Table 16). They were averaged over three-hour intervals. The drift measurements give us the total rate of change of the kinetic energy of ice, but it was seen from the GT-array data that the advection part is much smaller than the local rate of change. A large part of the kinetic energy input from the wind is almost directly transmitted to the oceanic boundary layer. Wind is clearly the dominant source of kinetic energy and the currents can be either a source or a sink.

Figure 40. The kinetic energy budget in SI79. The bars on the left describe the uncertainty (due to the drag coefficients) of the first three terms in the list. The letters (a)-(i) refer to the force balance cases in section 5.3.

Figure 41. Consumption of kinetic energy within the ice pack in SI79. The letters (a)-(i) refer to the force balance cases in section 5.3.
The ice stress gives rise to the transmission of energy from the surrounding ice and the internal dissipation of energy (Eq. 6.2). Subtracting from the rate of change of kinetic energy the other estimated terms gives us the sum of transmission and dissipation (Fig. 41, Table 16). The error in the residual is mainly due to the errors in our estimates of the wind and water stresses. It was concluded in section 5.1 that $\tau_a$ and $\tau_w$ are accurate within about 15% most of the time (except for the low speed situations). Thus, the sum $\tau_a + \tau_w$ is accurate within $0.15 \times (\tau_a^2 + \tau_w^2)^{1/2}$.

It is seen (Fig. 41) that the residual is nearly always negative. Expecting that the average transmission over a long time should be small, it follows that the dissipation is generally more important than the transmission. The residual was at its largest on 13th April and during the last 12 hours of the observation period. In the former case southwest ice drift was slowing down due to compacting of ice and it is probable that then a significant portion of the residual was due to transmission of ice stress to southwest from Aranda. In the latter case ice was pressing towards the fast ice boundary and ridging occurred.

The dissipation of kinetic energy is evidently at largest in ridging. But then the compactness of ice is high and hence the frictional dissipation in shearing between floes is also at largest. At least in the continuum time and length scales the frictional dissipation seems to be the major energy sink in the Bothnian Bay.

| TABLE 16. Kinetic energy budget in SI79. Unit $10^{-3}$ J m$^{-2}$ s$^{-1}$.                   |
|---------------------------------|-------|--------|--------|--------|
|                                  | Mean  | Standard deviation | Maximum | Minimum |
| Rate of change                  | 0.31  | 1.55   | 10.11  | -4.72  |
| Input from the wind             | 10.33 | 20.68  | 134.50 | -0.14  |
| Input from the current          | 2.31  | 5.72   | 32.88  | -1.69  |
| Input from the tilt             | 0.09  | 0.48   | 1.25   | -1.97  |
| Loss to the oceanic boundary layer | -8.44 | 29.34  | 0.00   | -89.68 |
| Dissipation + transmission within ice | -5.98 | 13.80  | 9.66   | -93.93 |
7. CONCLUSIONS

The pack ice area in the Bothnian Bay (Fig. 4) coincides approximately with the area of depth greater than ten meters which covers 83 per cent of the basin. The structure and mechanical behaviour of the pack ice during early spring (Table 1), when thermal changes of ice mass are small, were studied both from the theoretical viewpoint and on the basis of a field experiment performed in April 1979. The field data are representative for conditions of high ice compactness in early spring. Comparisons with recent results from the Arctic seas have been made.

The ice pack in the study basin is fractured into separate floes the sizes (diameters) of which are, considering the relative areal coverage, rather evenly distributed from tens of meters up to 4—5 kilometers (Fig. 12). The ice mass is composed of level ice of \(~1/2\) m thickness and deformed ice. The latter component was divided into light deformed ice with low elevations above the level ice surface and ridges, and it is shown that the masses of the two types of deformed ice are nearly equal and typically somewhat smaller than the mass of level ice. The distribution of ice thickness in the 12 km length scale was processed from observations (Fig. 16).

Measurements of pack ice deformation were made in two arrays in the length scales of 6 and 10 km (Figs. 17 and 18). The rates of deformation in the arrays did not correlate well, although the arrays were partly overlapping. The rates of deformation were at largest \(~10^{-6}\) s\(^{-1}\) and the principal strain-rates tended to have opposite signs (Figs. 20—22, Table 9). In one fixed direction the rates of deformation in the ice and sea were to some degree correlated, but in the latter medium were one order of magnitude larger (Fig. 25).

The observed ice drift followed the wind rather closely with response times shorter than one hour (Fig. 28). The spectra of ice velocity showed no clear peak at the Coriolis period, which can be explained on the basis of the small inertia of ice and the governing role of the wind in ice drift.

It has been shown through theoretical analysis that observed inhomogeneities in the thickness of a single ice floe can give rise to eigenrotation. In the case of high ice compactness, as during the field experiment, the rotation of an individual floe followed well the mesoscale vorticity of pack ice (Fig. 31) due to the dominant role of the contact stresses between floes.

The forces affecting ice drift were estimated from observations and the most important forces were found to be the surface stresses of wind and water on ice and the internal friction (Fig. 34, Table 14). The parameters of the quadratic water stress law were estimated from current profiles and the drift data: for the current beneath the layer of frictional influence of the ice, the drag coefficient was \(3 \times 10^{-3}\) and the angle of turning 20 degrees.

The parameters of the linear viscous constitutive law for pack ice were estimated from the force diagrams and deformation data. The range \(10^7\) to \(10^{10}\) kg s\(^{-1}\) was obtained for both the shear and the bulk viscosity coefficients (Table 15). During a decrease of compactness from 0.94 to 0.89 the order of magnitude was \(10^7\) kg s\(^{-1}\), but when the compactness increased back to 0.94, the order of magnitude was \(10^8\) kg s\(^{-1}\). The highest values, \(10^9\) to \(10^{10}\) kg s\(^{-1}\), resulted a) for small deformation rates when compactness was 0.95 and b) during ridging.

The kinetic energy budget of pack ice was estimated from observations (Figs. 40—41, Table 16). The main source of kinetic energy was the wind, while the current acted either as a sink or as a source. A large part of the kinetic energy was used in the work of the ice on
the oceanic boundary layer. The dissipation of kinetic energy seemed to be more significant than the transmission to or from the surrounding ice. Different types of energy sinks in dissipation were studied. The major sink was concluded to be the friction in shearing between ice floes. Then came the production of potential energy in ice ridges, which was several times smaller than the major sink.

Several problems treated here will need further study. The structure of small ice ridges and light deformed ice is still very poorly understood. The dynamic coupling of sea ice and the ocean should be examined to find a more realistic model for the time-dependent ice drift problem. Furthermore, the stress at the ice-water interface is rather roughly approximated with the present standard formulas, when the relative motion is small. The constitutive law for pack ice should be studied closely using numerical models and from the point of view of mechanical energy.

In the future it is needed to extend the present work to earlier and later stages of the ice season, which will introduce the difficult problem of coupling of dynamics and thermodynamics. Then we can tackle on seasonal studies of the ice cover in the Baltic Sea.

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## LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$A$</td>
<td>horizontal area or cross-sectional area of a portion of a ridge</td>
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<tr>
<td>$C$</td>
<td>drag coefficient</td>
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<tr>
<td>$\mathcal{E}$</td>
<td>pack ice particle</td>
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<td>$f$</td>
<td>Coriolis parameter</td>
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<td>$\mathcal{F}$</td>
<td>deformation gradient or force</td>
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<td>Froude number</td>
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<td>$g$</td>
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<td>shear viscosity of pack ice</td>
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<tr>
<td>$\theta, \Theta$</td>
<td>angle of turning and rotation operator</td>
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\( \kappa \)  ratio of the cross-sectional area of a ridge to that of the ridge sail
\( \lambda \)  distribution shape parameter or argument of characteristic polynomials of tensors
\( \mu \)  (linear) ridge density (dimension: 1/length)
\( \nu \)  porosity of ridges or Poisson's ratio for ice
\( \rho \)  density
\( \Sigma \)  ice stress tensor (dimension: force/length)
\( \tau \)  surface stress
\( \varphi \)  inclination angle of ridge sail and keel
\( \chi \)  compactness of ice
\( \Delta \mu \)  angle between the directions of wind and ice drift (deviation angle)
\( \omega, \dot{\omega} \)  rotation and angular velocity or vorticity
\( \Omega \)  ice floe surface

**Subscripts:**
- \( a \)  air
- \( f \)  ice floe
- \( i \)  ice
- \( k \)  ridge keel
- \( r \)  ridge
- \( s \)  ridge sail
- \( w \)  water
- \( * \)  dimensionless quantity
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ON TWO-PEAKED WAVE SPECTRA

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ABSTRACT

An experimental investigation was made of the hypothesis that in suitable conditions the Phillips resonance mechanism could, together with the Miles mechanism, generate waves having a two-peaked spectrum. A profile of consecutive buoys in the Bothnian Sea was used to measure the wave growth in steady-state fetch-limited conditions during 7 months in 1976. Data were obtained in conditions comparable to those during which two-peak wave growth has previously been reported. Most of those our spectra which were measured in these conditions had very little energy at the frequencies at which the resonance peak should appear, and our data contained several spectra in which the absence of a considerable resonance peak could not be understood by any of the reasons given in the two-peak hypothesis. Although some two-peak spectra were observed during a certain period, the evolution of the low-frequency peak with fetch, time and wind speed was inconsistent with Phillips' theory. Although weather maps did not indicate it, swell seemed to be the most probable explanation to the observed two-peak structure.

Our results therefore support calculations based on Phillips' theory (using recent data about the turbulent pressure fluctuations) which indicate that the direct atmospheric forcing is negligible in the principal stage of wave development.

1. INTRODUCTION

In 1976 a wave experiment was carried out in the Southern part of the Bothnian Sea. Four consecutive wave buoys were used to measure wave growth in fetch limited conditions.

During a long period of offshore wind in October 1976 the spectra were found to change in shape with time. At the beginning of the period the spectra had only one significant peak, at the end a two-peak structure appeared. The subject of this paper is the two-peak structure. Other results of this experiment are discussed eg. in Kahma (1981).

A two-peak structure caused by swell is a common feature in spectra measured in the open sea. For this reason an arbitrarily chosen spectrum is always likely to contain two peaks rather than just one. In this case, however, our first analysis of the weather maps indicated a very small probability of swell. Later we analysed the data from all Finnish weather stations along the Gulf of Bothnia and found that during the times when the two-peaked spectra were observed there were some wind observations which supported the possibility of swell in the measuring area.
On the basis of the wind data only it was, however, by no means obvious that swell was the explanation. It has been pointed out in several papers that swell is difficult to identify from the synoptical weather maps and therefore spectra containing a low-frequency peak which could be interpreted as swell have often been excluded from the data (e.g. Moskowitz 1964).

In this paper we therefore study the alternative hypothesis of Krylov, Strekalov and Cypluhin (1976). This hypothesis suggests that in suitable conditions the Phillips resonance mechanism (Phillips 1957) alone could be effective enough to produce a significant low-frequency peak in addition to the high-frequency peak generated by the Miles mechanism (Miles 1957). In addition to its theoretical interest, this hypothesis, if true, would have important consequences for practical wave prediction models in the relatively small sheets of water which are typical in the archipelago along the Finnish coast. At high wind speeds the long waves of a low-frequency peak would cause much larger movements of ships than the short waves of the high-frequency peak. Two-peaked spectra which agree with the growth relations proposed in the hypothesis have also been observed on the Finnish coast. It is therefore important that the possibility of a two-peaked wave growth (even in rare conditions) can be excluded.

Results which did not support the two-peak hypothesis were obtained in Kahma (1979). In the present study new data from an experiment in 1979 is taken into account. This data gives more evidence against the two-peak hypothesis and clarifies the problem of the correct wind speed, which could not be solved in the previous study.

2. THE TWO-PEAK HYPOTHESIS AND BACKGROUND

The Phillips-Miles theory gives the following equation for the growth of the wavenumber spectrum (e.g. Phillips 1977)

\[ F(k, t) = \frac{\pi \Pi(k, \omega)}{\varrho \omega c'(k)} \left( \frac{\sinh \mu \omega t}{\mu \omega} \right) \]

where \( c(k) \) is the phase speed of the waves with wave number \( k \), \( \varrho \) is the density of water, \( \omega \) is the angular frequency, \( \Pi(k, \omega) \) is the spectrum of pure atmospheric turbulent pressure fluctuations, and \( \mu \) is the dimensionless coupling coefficient associated with the Miles mechanism. When \( \mu \omega t << 1 \) the equation is reduced to the Phillips equation

\[ F(k, t) = \frac{\pi \Pi(k, \omega)}{\varrho \omega c'(k)} \]

which describes the stage of development of a wave component when only the resonance mechanism is effective. Phillips and Katz (1961) already suggested that waves having phase speed equal to the wind speed could grow to their saturation limit by the resonance mechanism alone.

A direct answer to the question requires knowledge of the three-dimensional spectrum \( \Pi(k, \omega) \) above the waves, which is difficult to measure. Reasonable measurements of the frequency spectrum \( \Pi(\omega) \) above the waves were not available before Elliott (1972a, b).
Calculations based on the Phillips-Miles theory using recent measurements of $\Pi (k, \omega)$ (Snyder, Dobson, Elliott and Long, 1981) do not support the possibility that the Phillips mechanism would be effective enough. We must, however, keep in mind that for waves having phase speed equal to the wind speed also the growth rates predicted by the Miles mechanism (Miles, 1957) are about an order of magnitude too small compared with the observed growth rates.

The problem can also be approached by empirical wave generation studies in which the evolution of the wave spectrum can be determined. In one such experiment Snyder and Cox (1966) found some indication that the resonance mechanism could be effective at least for waves with phase speed equal to the wind speed.

Clearer data about two-peaked growth was obtained in an experiment by Barnett and Wilkerson (1967) in the Atlantic Ocean. Profile measurements by radar altimeter from aircraft in steady-state fetch-limited conditions supported the possibility that for waves with phase speed equal to the wind speed the resonance mechanism was effective enough so that the waves could reach the saturation range even without the exponential growth by the Miles mechanism. At short fetches the spectra had two peaks of almost equal energy and the growth of the low-frequency peak was linear, which was in accordance with the Phillips mechanism.

Comparing the observed growth rate with that predicted by the Phillips theory Barnett and Wilkerson found, however, that the three-dimensional spectrum $\Pi(k, \omega)$ of atmospheric turbulence pressure fluctuations should be about 50 times greater than the spectrum extrapolated from Priestley's (1965) data. As Priestley's data were obtained for low wind speeds over close-mown grass, Barnett and Wilkerson suggested that the extrapolation to high wind speed in strong atmospheric instability was the main reason for the discrepancy. When this scaling factor of about 50 was used, the dependence of the spectrum on the frequency was also satisfactory.

The hypothesis of two-peak spectra generated by a steady wind was more explicitly discussed by Krylov, Kuznetsov and Strekalov (1973, hereafter referred to as KKS). In this paper they used two-peak data together with the one-peak laboratory data by Hidy and Plate (1966) and Mitsuyasu (1968) to determine the properties of the two-peaked spectra. The conclusion in KKS was that, if the wind is steady and the water deep enough for waves to reach the wind speed, the spectrum should contain two peaks of almost equal energy: one at a low, constant frequency and another at a higher frequency which moves towards lower frequencies with increasing fetch. At long fetches the high-frequency peak becomes saturated and merges with the low-frequency peak. From Fig. 2. in KKS the following fetch relations in deep water can be deduced:

$$\tilde{\omega}_1 = 5.0 \cdot 10^{-4} \tilde{X}^{0.44}$$

$$c_{\gamma}/U_{10} = 1.0 \quad (\tilde{\omega}_1 = 1 \text{ in deep water})$$

$$\tilde{\omega}_2 = \begin{cases} 7.75 \cdot 10^{-4} \tilde{X}^{0.44} \\ 0.028 \end{cases} \quad \tilde{X} < 10^1$$

$$\tilde{\omega}_2 = \begin{cases} 16.5 \tilde{X}^{-0.325} \\ 1.26 \end{cases} \quad \tilde{X} > 10^1$$

$$\tilde{X} < 10^1$$

$$\tilde{X} > 10^1$$
where
\[ \tilde{\sigma}_i = \frac{g \sigma_r}{U'_{10}} \quad \text{dimensionless standard deviation of surface displacement} \]
\[ \tilde{\omega}_i = \frac{\omega U_{10}}{g} \quad \text{dimensionless peak frequency} \]
\[ \tilde{\sigma}_i^2 = \text{variance of the peak } i \]
\[ \omega_i = \text{frequency of the maximum of the peak } i \]

and index \( i = 1 \) is associated with the peak generated by the Phillips resonance mechanism and \( i = 2 \) with the peak generated by the Miles mechanism.

The scattering of this data was fairly small compared with other data from fetch-limited conditions. The only exceptions were \( \tilde{\omega}_2 \) and \( \tilde{\sigma}_2 \) at short fetches; this was explained by the influence of the kinematic viscosity, which might be important in the initial stage of the development, and which is not included in the dimensionless formulas.

A more complete description of this hypothesis was given in a monograph by Krylov, Strekalov and Cypluhin (1976), hereafter referred to as KSC.

The fetch relations, which are mainly based on new data, are given in the following form:

\[
\tilde{\sigma}_1 = \begin{cases} 1.0 \cdot 10^{-2} \hat{X}^{0.56} & 10^4 \leq \hat{X} < 10^7 \\ 56 & \end{cases}
\]
\[ c_i/U_{10} = 1 \]
\[
\tilde{\sigma}_2 = \begin{cases} 1.2 \cdot 10^{-2} \hat{X}^{0.5} & 10 < \hat{X} < 10^6 \\ 28 & \end{cases}
\]
\[
\tilde{f}_2 = \begin{cases} 1.0 \hat{X}^{0.5} & 10 < \hat{X} < 10^6 \\ 1/170 & \end{cases}
\]

where
\[ \tilde{\sigma}_i = \frac{g \sigma_r}{u_*^2} \quad \hat{X} = \frac{gX}{u_*^2} \]
\[ \tilde{f}_i = \frac{u_* f_i}{g} \quad f = \frac{\omega}{2\pi} \]

As the relations are scaled by the friction velocity \( u_* \), it is not possible to compare these relations directly with those in KKS, which are scaled by \( U_{10} \). However, from table 2.6 in KSC it can be seen that, with the exception of Kononkova and Kuznetsov (1973), in all cases in which the resonance peak exists the friction velocity \( u_* \) is calculated using a constant drag coefficient \( C_{f0} = 10^{-3} \), and therefore the relations can be written:
\[ \tilde{\alpha}_1 = \begin{cases} 3.16 \times 10^{-4} \tilde{X}^{\frac{1}{5}}, & 10 < \tilde{X} < 10^4 \ \\ 0.056, & 5 \times 10^4 < \tilde{X} \end{cases} \] (2)

\[ c_1/U_{10} = 1.0 \quad (\tilde{\alpha}_1 = 1 \text{ in deep water}) \] (3)

\[ \tilde{\alpha}_2 = \begin{cases} 4.05 \times 10^{-4} \tilde{X}^{\frac{1}{5}}, & 10^{-2} < \tilde{X} < 10^4 \ \\ 0.028, & 10^4 < \tilde{X} \end{cases} \] (4)

\[ \tilde{\omega}_2 = \begin{cases} 19.9 \tilde{X}^{\frac{1}{5}}, & 10^{-2} < \tilde{X} < 10^4 \ \\ 1.17, & 10^4 < \tilde{X} \end{cases} \] (5)

The relations for the high-frequency peak are close to those given by Hasselman et al. (1973). The differences (about 10 ... 20 \%) between these and the relations in KKS (1973) are not significant, but larger than could be expected from the scattering of the data used in KKS.

KSC emphasize that there are many factors, some of them as yet little studied, which can limit or wholly prevent the development of the resonance peak. The factors they discuss are variability of the wind speed and shallow water; of the possible unknown influences they mention stratification of the atmospheric boundary layer.

The solution of the Phillips resonance equations in the case of a constant finite depth h is derived in KSC, and can be written in the form

\[ F_h(k, t) = F(k, t) \tanh^2 kh \] (6)

where \( F(k, t) \) is the original solution (1) of Phillips (1957) in deep water. In addition they point out that in shallow water, when the phase speed of waves cannot reach the wind speed, no resonance peak can develop.

The quantitative effect of variable wind speed on the growth of the resonance peak is not given in KSC but two experiments (Barnett and Wilkerson 1967; Kononkova and Kuznetsov 1973) are mentioned in which the variation in the wind speed was no more than \( \pm \) 10 \%.

3. EFFECTS OF THE EXPERIMENTAL CONDITIONS ON THE RESONANCE PEAK

To explain observations of one-peaked spectra KSC suggested factors which should prevent or limit the growth of the resonance peak. In addition there are some conditions in which the spectrum predicted by the two-peak hypothesis is identical to the spectrum predicted by alternative unimodal models. Here we discuss the requirements for the data that can be used to study the two-peak hypothesis described in KSC.

The first requirement is the absence of swell. If there is no directional information of waves, two-peaked spectra measured in places where swell can travel in from an ocean are rather suspect even when the synoptic maps do not show swell. Wave measurements on a profile can help to distinguish between swell and the resonance peak, but there exist data
about swell decay over a sloping bottom (e.g. Hasselmann et al. 1973) which show that in certain conditions swell may have properties very similar to those predicted for the resonance peak.

From equations (4) and (5) it can be seen that the high-frequency peak becomes saturated when the dimensionless fetch \( \tilde{X} \) reaches 10. Although the saturation limit of the high-frequency peak \( \tilde{\omega}_2 = 1.17 \) is not equal to that of the resonance peak \( \tilde{\omega}_1 = 1.0 \) (in deep water), the peaks will eventually merge, as can be seen from Fig. 4.19 in Phillips (1977). According to this data the longest dimensionless fetch \( \tilde{X} \) at which the peaks are still separate is in the range 3000 . . . 5000.

When the wind is not stationary, equation (1) is no more valid and must be replaced by the equation

\[
F(k, t) = \frac{1}{Q_0^2 c^2} \int_0^t \int_0^t \Omega(k, \tau', \tau'') \sin \omega(t - \tau') \sin \omega(t - \tau'') d\tau' d\tau''
\]

(e.g. Phillips 1977) where

\[
\Omega(k, \tau', \tau'') = \frac{1}{2\pi} \int \hat{P}(x, \tau') \hat{P}(x + r, \tau'') e^{-ikr} dr
\]
is the wave-number spectrum of the turbulent pressure fluctuations \( \hat{P}(x, t) \). When the wind and the fluctuations are stationary, the three-dimensional spectrum \( \Pi \) is defined as (e.g. Phillips, 1977)

\[
\Pi(k, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Omega(k, \tau) e^{i\omega \tau} d\tau, \quad \tau = \tau' - \tau''.
\]

As no data is available to us about \( \Omega(k, \tau', \tau'') \) in nonstationary conditions it is not possible to derive any quantitative information on \( F(k, t) \) in variable wind. If the change in the mean wind speed is slow enough so that \( \Pi \) can be locally defined, two qualitative features can be suggested. First, there will be a shift in the angular frequency and wave number at which the direct atmospheric forcing is most effective, as can be seen from the equations of \( \omega \) and \( k \) for waves with phase speed equal to the wind speed:

\[
kU_{10} = \omega = \frac{g}{U_{10}} \tanh \frac{\omega_U h}{U_{10}}
\]

In deep water \( |\Delta \omega_\omega/\omega_\omega| \simeq |\Delta U/U_{10}| \), otherwise \( |\Delta \omega_\omega/\omega_\omega| > |\Delta U/U_{10}| \). Secondly, \( \Pi \) itself will vary proportionally to \( u_s^2 \) (Snyder and Cox 1966, Elliot 1972a). Both these effects will probably broaden and blur the resonance peak, but they cannot explain why the variance of the peak

\[
\sigma_1^2 = \int S(\omega) d\omega
\]

should be zero (\( \omega_{\min} \) is the frequency at which the high-frequency peak begins to rise).
Fig 1. The dimensionless frequency $\omega_j/\omega_{\text{deep}}$ as a function of dimensionless depth $\tilde{h} = gh/U^2$. Here $\omega_j$ is the frequency of waves with phase speed equal to the wind speed at depth $h$, Eq. (7), and $\omega_{\text{deep}}$ is the corresponding frequency in deep water.

Therefore, if we want to accept the suggestion of KSC that too large variations in the wind speed can explain the absence of the resonance peak, then, to be on the safe side, we must use the about $\pm 10\%$ criterium in the wind speed. In these conditions, according to KSC, the normal development of the resonance peak has been observed.

In stable atmospheric stratification $\Pi(k, \omega)$ is smaller than in neutral stratification. Near-neutral or unstable stratification would therefore be suitable for experiments, as in the measurements by Kononkova and Kuznetsov (1973) or Barnett and Wilkerson (1967).

In ideal conditions the water would be deep enough so that the waves with phase speed equal to the wind speed would not be affected by the bottom in any part of the fetch. However, in fetch-limited studies there is usually a region where the depth is not sufficient.

The growth of the spectrum $F(k, \omega)$ in shallow water of constant depth is given by Eq. (6). When this is compared with the growth in deep water it should be noted that in addition to the factor $\tanh^2(kh)$ there is a significant effect in $\Pi(k, \omega)$ due to the change in $\omega$ and $k$. The combined effect is greatly dependent on the functional form of $\Pi(k, \omega)$, and it is even possible that the maximum growth in shallow water is not in the waves with $c = U_{10}$ but in waves of a slightly higher frequency.

In Fig. 1, the ratio $\omega_j/\omega_{\text{deep}}$ calculated from the equation (7) is given as a function of the dimensionless depth $\tilde{h} = gh/U_{10}^2$. This ratio rises from zero to one very rapidly with increasing depth; the region from shallow water, where the depth is insufficient for waves to reach the wind speed, to practically deep water is fairly short.

To get an idea of the effect of the finite depth on the resonance peak it seems reasonable to choose a subfetch $X_S$ from the deep end of the fetch and to use the following parameters

$$\tilde{h} = gh/U_{10}^2 = c_{\text{max}}^2(h)/U_{10}^2$$

$$q = \frac{1}{\omega_j(h)} \left( \frac{\Delta \omega}{2} \right)^2 + \left( \frac{\Delta \omega_{\text{s}}}{2} \right)^2 \right)^{\frac{1}{2}}$$
where $\bar{h}$ is the average depth in $X_s$, $\omega_j(h)$ is the angular frequency at depth $h$ from eq. (7), $\Delta\omega_h$ is the variation of $\omega_j$ in $X_s$ at depth $h$ due to the variation in the wind speed, and $\Delta\omega_h$ is the variation of $\omega_j$ in $X_s$ at wind speed $U_{10}$ due to the variation in the depth.

The information necessary to determine these parameters is not given for all of the two-peak data used in KSC, but at the measuring point typically $gh/U_{10} = \bar{h} < 2$. The depth $h = \lambda/4$ at which the wave component begins to be considerably influenced by the bottom corresponds to $\bar{h} = 1.6$. The $\pm 10\%$ variation in the wind speed corresponds to $\eta = 0.1$ in deep water.

According to KSC the growth of the resonance peak is linear with fetch. If $\bar{h}$ (the average dimensionless depth in the subfetch $X_s$) and $\eta$ are acceptable in $X_s$, it seems reasonable to assume that the maximum reduction by the shallow water in the beginning of the fetch can be approximated by the ratio $X_s/X$.

The appearance of a resonance peak has certainly been reported under conditions much less optimal than those outlined above. In the data from Rybinsk Reservoir (Kononkova and Kuznetsov, 1973) a low-frequency peak was consistently found in all the spectra at 3.6 m/s wind speed. At the measuring point nearest to the shore the depth was 0.8 m, which gives a maximum phase speed $c_{max}(h) = (gh)^{1/2} = 2.8$ m/s. This contradicts the statement in KSC, that the resonance peak does not develop if $c_{max} < U_{10}$. If, however, the interpretation of this peak as a resonance peak (Kononkova and Kuznetsov, 1973) was correct, even $\bar{h} = 0.6$ would be acceptable, when some indication only of the resonance peak is looked for. The low-frequency peak of the Rybinsk data shows, however, another feature which makes its interpretation as a resonance peak questionable. The frequency $\omega_j$ does not show any indication of shifting towards lower frequencies in shallow water. This could perhaps be understood as a combined effect of $\Gamma(k, \omega)$ and the factor $c^{-\tanh kh}$, if $\Gamma(k, \omega)$ has a suitable form, but this seems unlikely.

4. OTHER EXPERIMENTAL DATA

Since several single-peaked spectra have already been published, the data used to reach conclusive results on the two-peak hypothesis should fulfill the requirements discussed in the preceding section. The requirements are so stringent that there are not many experiments which could be used for this purpose.

In KSC three experiments were referred to in which no resonance peak was observed. As they gave no detailed discussion of why the peak was missing, we shall discuss below each of these three cases.

The first is the data from O.W.S. Weather Explorer in the Northern Atlantic (Pierson 1959). KSC explained the absence of the resonance peak in these spectra by the rapid growth of the wind speed from 9 to 32 m/s. The data show that both the predicted resonance frequency and the main peak frequency shift to lower frequencies at a rate of about 6% in three hours with a time delay of 9 hours. There is no two-peak structure, but the forward faces of the spectra are not very steep. Therefore the amount of energy predicted by the relation (2) using 9 hours duration time is not ruled out by the observed spectra.
The second reference is the paper by Burling (1959). His fetch-limited steady-state spectra were measured in a small reservoir in well-defined conditions, some of which were excellent for two-peak study: \( \pm 5\% \) variability of wind speed, resonance peak in deep water conditions and stratification only slightly unstable. His data therefore does not support the possibility that the factors suggested by KSC would explain the absence of the resonance peak. There is, however, one slightly questionable point in this data. The frequency range in the published spectra is 0.4 . . . 1.8 Hz, and the description of the method of Fourier analysis gives the impression that the lowest frequency analysed was about 0.35 Hz. As the highest resonance frequency in the Burling experiment would be 0.31 Hz, it is possible in principle that a small low-frequency peak could have existed. This possibility cannot be completely excluded, as the square of the significant wave height determined by manual analysis was 1.1 to 1.5 times higher than the square of the significant wave height calculated from the spectra. Burling himself explained this difference by assuming that the manual analysis was biased and \( H_{\bar{y}} \) (the average height of the highest one-fourth of the waves) was measured instead of \( H_{\bar{y}} \) (the average height of the highest one-third of the waves, significant wave height), because the small waves were ignored.

The third reference is to the JONSWAP experiment (Hasselmann et al. 1973). This wave profile experiment was made off Sylt in the North Sea, where the bottom slope is gentle and the equipment could be used for fetch-limited wave generation studies in off-shore winds, and refraction and dissipation studies in the case of swell. The high-quality data contain several steady-state fetch-limited spectra, but none of them contain a low-frequency peak which could not be explained by swell (Ewing and Worthington, private communication 1978).

The suitability of the profile for studying swell unfortunately also limits the wind speeds at which the resonance peak can develop. At typical wind speeds the depth is just in the range where the resonance peak begins to be considerably affected by the bottom. On the other hand, for about half of the measuring points the fetch is too long (\( \bar{X} \geq 5000 \)); thus the high-frequency peak is indistinguishable from the resonance peak. Still, when the wind

<table>
<thead>
<tr>
<th>Point No</th>
<th>( X/\text{km} )</th>
<th>( X'/\text{km} )</th>
<th>( \bar{h}/\text{m} )</th>
<th>( g X/\bar{U}^2 )</th>
<th>( g \bar{h}/\bar{U}^2 )</th>
<th>( q )</th>
<th>( X'/X )</th>
<th>Comments</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>4</td>
<td>200</td>
<td>0.8</td>
<td>-</td>
<td>0</td>
<td>waves cannot reach the wind speed</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>7</td>
<td>400</td>
<td>1.4</td>
<td>-</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
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<td>4</td>
<td>2</td>
<td>10</td>
<td>800</td>
<td>2.0</td>
<td>0.22</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>6.5</td>
<td>3</td>
<td>11</td>
<td>1300</td>
<td>2.2</td>
<td>0.15</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>9.5</td>
<td>5</td>
<td>11.5</td>
<td>1900</td>
<td>2.3</td>
<td>0.12</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>14</td>
<td>10</td>
<td>12</td>
<td>2800</td>
<td>2.4</td>
<td>0.11</td>
<td>0.7</td>
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</tr>
<tr>
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<td>20</td>
<td>16</td>
<td>12</td>
<td>4000</td>
<td>2.4</td>
<td>0.11</td>
<td>0.8</td>
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</tr>
<tr>
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<td>27</td>
<td>22</td>
<td>13</td>
<td>5400</td>
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<td>0.11</td>
<td>0.8</td>
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<td>37</td>
<td>32</td>
<td>14</td>
<td>7400</td>
<td>2.8</td>
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<td>0.9</td>
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<tr>
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<td>47</td>
<td>15</td>
<td>10400</td>
<td>3.0</td>
<td>0.10</td>
<td>0.9</td>
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<td>80</td>
<td>75</td>
<td>18</td>
<td>16000</td>
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</tr>
<tr>
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<td>0.09</td>
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<td></td>
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<td>25</td>
<td>32000</td>
<td>5.0</td>
<td>0.09</td>
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<td></td>
</tr>
</tbody>
</table>
speed in the JONSWAP experiment is less than 10 m/s, the waves can reach the wind speed in a considerable portion of the fetch for most of the points for which $X < 5000$, and therefore some indication of a resonance peak should appear.

In Table 1 some parameters of the resonance peak are given at 7 m/s wind speed, which we found to be the optimum for the observation of the resonance peak under the conditions of the JONSWAP experiment. The determination of the subfetch is, of course, not unique, and the ±8 % variation in the wind speed which is used in the calculation of $q$ is arbitrary and is not based on the wind data from the JONSWAP experiment.

On the basis of the wind speed history and the distribution of dimensionless fetches in the spectra which are accepted for the fetch relations, there seem to be at least some spectra in which the parameters $h$, $q$ and $X_s/X$ are suitable if variation in the wind speed is less than ±8 %. According to the published information this may be slightly too small a value for the typical variation in the JONSWAP experiment. Another limiting factor might be that the spectra were in most cases measured under stable stratification. This decreases the turbulent pressure spectrum $\Pi (k, \omega)$ and hence reduces the growth of the resonance peak.

In any event, the JONSWAP data show that, if the two-peak hypothesis is true, the development of the resonance peak should be limited to either near-neutral or unstable atmospheric stratification, or to very small variations in the mean wind speed.
5. THE 1976 AND 1979 EXPERIMENTS

5.1. MEASUREMENTS

The main experiment was carried out in the Southern part of the Bothnian Sea in 1976 (Fig. 2). A profile of four consecutive wave buoys was used to measure the wave growth in offshore wind conditions. An additional experiment was made in 1979, in which only one buoy was used but at different positions.

The wave measuring equipment consisted of Datawell Waverider buoys. The recording and digitizing system resembles closely that described by Wilson (1975). The measurements began at points C and D in May and the whole profile was operating between September and December 1976. Data were collected at intervals of three hours for 15 minutes from each buoy. Unfortunately buoy A was so far from the receiver at Kuuskajaskari Weather Station that a proper registration could be obtained only under the best radio conditions.

Between Aug 24th and Oct 23rd 1976 the wind was measured at two small flat islets (points K and L, Fig. 2) by automatic weather masts (Aanderaa). These masts measured the wind speed and direction at 10 m altitude and the air temperature at 2 m altitude every 15 minutes. During the 1979 experiment the wind was measured on board R/V Aranda and at
Fig. 3. The bottom topography along the profile. The solid line is the depth $\lambda/4$ at which the influence of the bottom begins to be considerable; the dashed line the depth $0.6\lambda$ at which the bottom has almost no influence. The wavelength $\lambda$ was calculated from (5) using a wind speed 25 m/s. The depths $\lambda/4$ for waves with phase equal to the wind speed are given on the right for different wind speeds.

Point L. The wind was measured also at Kuuskajaskari Weather Station and at Mäntyluoto pilot station. In addition to these measurements there were available sea wind estimates given out by the Finnish Meteorological Institute at intervals of 12 hours (Lange 1973).

The wind measuring points at Kuuskajaskari and Mäntyluoto are so close to the shore that in offshore winds they give a poor representation of the conditions in the open sea. Earlier studies in this area (Kahma, 1977) show that the wind-wave relations are in better agreement with previously published relations, if the sea wind estimates are used instead of the wind measurements at Kuuskajaskari. Therefore the wind measurements at Kuuskajaskari and Mäntyluoto were used only as controls.

From point K no acceptable wind speed data was obtained, since the datalogger malfunctioned. The speed was usually too high by a factor that varied between 1.5 and 2, and the time history did not agree with the other measurements. No similar errors were found in the data from point L, for which the time history and correlation with both the Kuuskajaskari data and the sea wind estimates was acceptable (Fig. 4). During the 1979 experiment comparison wind measurements were made which showed that the wind measurements at point L are representative for the wave measuring area (Kahma and Leppäranta, 1981).

Data from two current meters (Fig. 2) were available (Grönvall, 1977) during the best offshore situation in Oct 1976. Other, more extensive current studies have been carried out in the area (Alenius and Mälkki, 1978), partly simultaneously with the wave measurements, but unfortunately not during the best offshore winds.

Because the shoreline is irregular, the average shoreline was approximated by a least-squares straight line. The uncertainty caused by sheltering islands was estimated to be less that ±1 km. If the sheltering by islands is neglected altogether (which clearly must be unrealistic), the average shoreline ought to be shifted only 2 km inland from the position in Fig. 2.
The bottom depth is shown in Fig. 3. At a depth $h = (ng)/(2\omega^2)$ corresponding to $h = \lambda/4$, a wave component of frequency $\omega$ begins to be considerably influenced by the bottom. For $\omega_m$ this depth is given by the upper continuous line; the frequency $\omega_m$ was calculated from equation (5) at 25 m/s offshore wind speed. As the greatest offshore wind speed during the measurements was 13 m/s (17 m/s according to the sea wind estimate) the depth could be regarded as effectively infinite for the single-peaked spectra.

Due to the limited data-processing resources available for this study, only a small fraction of the measured spectra were digitized. In choosing the situations for analysis, the first criterium was that the offshore wind had been blowing for at least 24 hours. The second criterium was a very small probability of swell in the profile area, as estimated from the history of wind direction on the basis of weather maps.

The best conditions occurred during the period from Oct 11th to Oct 19th 1976, when a high pressure area slowly moved from the northern part of Finland to SE of Lake Ladoga and caused a wind turning slowly from 40° to 140°. The variations in the 15 min averages of wind speed were sometimes very small (less than ±5 % of the long-time averages).
The history of the wind at point L and the sea wind estimates are shown in Fig. 4; the weather maps at the beginning and end of the period are given in Fig. 5. The air-sea temperature difference was negative all the time (Fig. 6) and the stratification unstable (bulk Richardson number $-0.04...-0.07$).

Another suitable period would have been from June 15th to June 16th 1976. Unfortunately, only the two innermost buoys C and D were operating at the time and the weather mast at point L had not yet been installed. The data from this period was therefore not used.

The conditions during the 1979 experiment were somewhat similar to the best period in 1976. A high pressure caused a cold offshore wind to the measuring area. The stratification was about the same (bulk Richardson number $-0.05...-0.09$), but the wind speed was lower ($5...8$ m/s), and there were only a few sufficiently long periods of steady wind. This time the wave measurements were made with one buoy which was moved by R/V Aranda to different positions along the profile.

Two additional cases of offshore wind were analysed, representing situations which were rejected because the absence of swell was not ensured by the weather maps.

For the analysis the different generation cases were divided into periods during which the 15 min averages of the wind did not vary more than $\pm 18$ % in speed and $\pm 20^\circ$ in direction. The average over such a period was used as a wind parameter. A spectrum was accepted, if the wind fulfilled these conditions for at least the stabilizing time, which was about 1.5 h at point D, 3 h at point C and 6 h at point B. The variation of the 15 min averages of wind speed during the stabilizing time (point B) can be seen from the table 2.
5.2 RESULTS

The accepted spectra were divided into the following groups according to their measuring time:

<table>
<thead>
<tr>
<th>group</th>
<th>measuring time</th>
<th>number of spectra</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Oct 13 01.00 ... Oct 15 19.45</td>
<td>55</td>
</tr>
<tr>
<td>2</td>
<td>Oct 18 07.30 ... Oct 18 13.45</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>Oct 18 16.00 ... Oct 18 19.45</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>Oct 19 01.15 ... Oct 19 13.45</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>Oct 29 14.50 ... Oct 30 11.15</td>
<td>91</td>
</tr>
</tbody>
</table>

(The division into groups is slightly different from that in Kahma 1979 and 1980).

The spectra in groups 1, 2 and 5 have (with one exception) a one-peak structure; an example is shown in Fig. 7. The two-peak structure is most clearly seen in group 4. In group 2 the forward faces of the spectra were disturbed by a very small low-frequency peak, and in group 3 a large low-frequency peak appeared in the spectra.

Of the accepted 91 spectra 72 were measured for conditions in which the average depth in the subfetch chosen for each point (Fig. 3) was sufficient so that waves could reach the wind speed. The stratification was unstable and the dimensionless fetch was in most of the
TABLE 2. Summary of the data. For convenience $X$, $\omega_\phi$, $\sigma$ and $\bar{h}$ are tabulated in dimensionless form $\tilde{X} = gX/U_{10}$, $\tilde{\omega}_\phi = \omega_\phi U_{10}/g$, $\tilde{\sigma} = \sigma U_{10}/gU_{10}$ and $\tilde{h} = h U_{10}$, together with the scaling wind speed $U_{10}$.
Parameters $c(\omega_\phi)$, $\omega_\phi$, $\sigma_1$, $\sigma_2$ are defined in section 2., while $h$, $g$, $X$, $\delta$ are defined in section 3.

<table>
<thead>
<tr>
<th>$U_{10}$</th>
<th>$\tilde{X}$</th>
<th>$\tilde{\omega}_\phi$</th>
<th>$\tilde{\sigma}$</th>
<th>$\tilde{h}$</th>
<th>$\Delta U/2U$</th>
<th>$\delta$</th>
<th>$X_e/X_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>1.00 B</td>
<td>10.8 3923 1.369 .00349</td>
<td>4.2 .09 .09 .7</td>
<td>13</td>
<td>1.15 C</td>
<td>10.8 1423 1.790 .0232</td>
<td>1.9 .09 .15 .4</td>
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### group 2 Oct 18

<table>
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<tr>
<th>$U_{10}$</th>
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<th>$\tilde{\omega}_m$</th>
<th>$\tilde{\sigma}$</th>
<th>$\tilde{\eta}$</th>
<th>$\Delta U/2U$</th>
<th>$q$</th>
<th>$\chi_s/X$</th>
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<tbody>
<tr>
<td>18.730 C</td>
<td>9.5</td>
<td>1904</td>
<td>1.763</td>
<td>0.250</td>
<td>2.4</td>
<td>0.09</td>
<td>10</td>
</tr>
<tr>
<td>18.745 D</td>
<td>9.5</td>
<td>800</td>
<td>2.189</td>
<td>0.085</td>
<td>1.1</td>
<td>0.14</td>
<td>9</td>
</tr>
<tr>
<td>18.10.00 B</td>
<td>9.5</td>
<td>5081</td>
<td>1.264</td>
<td>0.037</td>
<td>5.4</td>
<td>0.14</td>
<td>14</td>
</tr>
<tr>
<td>18.10.15 C</td>
<td>9.5</td>
<td>1904</td>
<td>1.811</td>
<td>0.026</td>
<td>2.4</td>
<td>0.14</td>
<td>17</td>
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<td>18.10.30 D</td>
<td>9.5</td>
<td>800</td>
<td>2.176</td>
<td>0.019</td>
<td>1.1</td>
<td>0.12</td>
<td>3</td>
</tr>
<tr>
<td>18.13.00 B</td>
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<td>5081</td>
<td>1.204</td>
<td>0.035</td>
<td>5.4</td>
<td>0.12</td>
<td>12</td>
</tr>
<tr>
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<td>1904</td>
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<td>0.018</td>
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### group 3 Oct 18

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<th>$\tilde{\omega}_m$</th>
<th>$\tilde{\sigma}_2$</th>
<th>$\tilde{\eta}$</th>
<th>$\Delta U/2U$</th>
<th>$q$</th>
<th>$\chi_s/X$</th>
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<td>5081</td>
<td>1.694</td>
<td>0.024</td>
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<td>10</td>
</tr>
<tr>
<td>18.16.15 C</td>
<td>9.5</td>
<td>1904</td>
<td>1.609</td>
<td>0.022</td>
<td>2.4</td>
<td>0.10</td>
<td>12</td>
</tr>
<tr>
<td>18.16.30 D</td>
<td>9.5</td>
<td>800</td>
<td>2.151</td>
<td>0.012</td>
<td>1.1</td>
<td>0.10</td>
<td>3</td>
</tr>
<tr>
<td>18.19.00 B</td>
<td>9.5</td>
<td>5081</td>
<td>1.125</td>
<td>0.041</td>
<td>5.4</td>
<td>0.13</td>
<td>13</td>
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<tr>
<td>18.19.15 C</td>
<td>9.5</td>
<td>1904</td>
<td>1.325</td>
<td>0.036</td>
<td>2.4</td>
<td>0.13</td>
<td>16</td>
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<tr>
<td>18.19.30 D</td>
<td>9.5</td>
<td>800</td>
<td>1.602</td>
<td>0.029</td>
<td>1.1</td>
<td>0.13</td>
<td>3</td>
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### group 4 Oct 19

<table>
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<th>$U_{10}$</th>
<th>$\tilde{\chi}$</th>
<th>$\tilde{\omega}_m$</th>
<th>$\tilde{\sigma}_2$</th>
<th>$\tilde{\eta}$</th>
<th>$\Delta U/2U$</th>
<th>$q$</th>
<th>$\chi_s/X$</th>
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<tbody>
<tr>
<td>19.1.15 C</td>
<td>7.6</td>
<td>2839</td>
<td>1.600</td>
<td>0.031</td>
<td>3.7</td>
<td>0.09</td>
<td>0.9</td>
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<tr>
<td>19.1.30 D</td>
<td>7.6</td>
<td>1256</td>
<td>1.843</td>
<td>0.013</td>
<td>1.7</td>
<td>0.09</td>
<td>0.20</td>
</tr>
<tr>
<td>19.4.00 B</td>
<td>7.6</td>
<td>7753</td>
<td>1.532</td>
<td>0.031</td>
<td>8.5</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>19.4.15 C</td>
<td>7.6</td>
<td>2839</td>
<td>1.541</td>
<td>0.027</td>
<td>3.7</td>
<td>0.09</td>
<td>0.09</td>
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<tr>
<td>19.4.30 D</td>
<td>7.6</td>
<td>1256</td>
<td>1.807</td>
<td>0.017</td>
<td>1.7</td>
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<td>0.20</td>
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<td>19.7.00 B</td>
<td>7.6</td>
<td>7753</td>
<td>1.361</td>
<td>0.037</td>
<td>8.5</td>
<td>0.09</td>
<td>0.09</td>
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<td>19.7.20 C</td>
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<td>2839</td>
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<td>0.026</td>
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<td>0.09</td>
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<td>7.6</td>
<td>1256</td>
<td>2.164</td>
<td>0.017</td>
<td>1.7</td>
<td>0.09</td>
<td>0.20</td>
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<td>8280</td>
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<td>8.5</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>19.10.15 C</td>
<td>7.6</td>
<td>3089</td>
<td>1.541</td>
<td>0.026</td>
<td>3.7</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>19.10.30 D</td>
<td>7.6</td>
<td>1343</td>
<td>1.911</td>
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<tr>
<td>19.13.00 B</td>
<td>7.6</td>
<td>8280</td>
<td>1.323</td>
<td>0.039</td>
<td>8.5</td>
<td>0.11</td>
<td>0.11</td>
</tr>
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<td>3089</td>
<td>1.678</td>
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<td>3.7</td>
<td>0.11</td>
<td>0.11</td>
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<tr>
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<td>7.6</td>
<td>1343</td>
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<td>0.010</td>
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<td>0.11</td>
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### group 5 Oct 29 ... Oct 30

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<tr>
<th>$U_{10}$</th>
<th>$\tilde{\chi}$</th>
<th>$\tilde{\omega}_m$</th>
<th>$\tilde{\sigma}$</th>
<th>$\tilde{\eta}$</th>
<th>$\Delta U/2U$</th>
<th>$q$</th>
<th>$\chi_s/X$</th>
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</thead>
<tbody>
<tr>
<td>29.14.50 E15</td>
<td>5.6</td>
<td>3197</td>
<td>1.418</td>
<td>0.034</td>
<td>4.1</td>
<td>0.10</td>
<td>10</td>
</tr>
<tr>
<td>29.15.05 E15</td>
<td>5.6</td>
<td>3197</td>
<td>1.408</td>
<td>0.027</td>
<td>4.1</td>
<td>0.10</td>
<td>10</td>
</tr>
<tr>
<td>29.16.00 E17</td>
<td>5.6</td>
<td>5507</td>
<td>1.244</td>
<td>0.036</td>
<td>7.9</td>
<td>0.10</td>
<td>10</td>
</tr>
<tr>
<td>29.16.15 E17</td>
<td>5.6</td>
<td>5507</td>
<td>1.322</td>
<td>0.044</td>
<td>7.9</td>
<td>0.10</td>
<td>10</td>
</tr>
<tr>
<td>30.10.15 E15</td>
<td>7.9</td>
<td>1825</td>
<td>1.638</td>
<td>0.025</td>
<td>2.0</td>
<td>0.12</td>
<td>22</td>
</tr>
<tr>
<td>30.10.30 E15</td>
<td>7.9</td>
<td>1825</td>
<td>1.758</td>
<td>0.024</td>
<td>2.0</td>
<td>0.12</td>
<td>22</td>
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<tr>
<td>30.10.45 E15</td>
<td>7.9</td>
<td>1825</td>
<td>1.799</td>
<td>0.023</td>
<td>2.0</td>
<td>0.12</td>
<td>22</td>
</tr>
<tr>
<td>30.11.00 E15</td>
<td>7.9</td>
<td>1825</td>
<td>1.708</td>
<td>0.024</td>
<td>2.0</td>
<td>0.12</td>
<td>22</td>
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</tbody>
</table>
spectra less than 5000. At least an indication of the resonance peak should be seen in these spectra.

Fig. 8 shows these spectra normalized by the frequency corresponding to waves with phase speed equal to the wind speed. The average depth in the subfetch was used as the depth and wind speed was measured at point L. In our previous study of this subject (Kahma, 1979) the sea wind estimates were used, because at that time it was suspected that the wind measurements at point L might not be representative of the conditions at the wave profile. The final results were essentially the same, only the number of the spectra in which the waves could reach the wind speed was reduced.

5.2.1. The spectra with no significant two-peak structure

The spectra in groups 1, 2 and 5 show little energy in the normalized frequency range 0.8 ... 1.2, where the resonance peak should be. A very small low-frequency peak can be seen in most of the spectra in groups 1 and 2, especially at point D; this peak will be discussed in detail in section 5.2.3. In the spectra from point B the main peak starts to rise fairly near to the predicted resonance frequency, but it is clear from Fig. 8a that only an insignificantly small resonance peak could be hidden by the main peak in group 1. In group 2 there are two
Fig. 8. Spectra normalized by the frequency $\omega_1$ of waves with phase speed equal to the wind speed, a group 1, b group 5, c group 2, d groups 3 and 4.
spectra in which the dimensionless fetch $X$ exceeds 5000, and therefore there is more energy in the frequency range $0.8 \ldots 1.2$; even so the low-frequency peak which could be hidden under the main peaks of these two spectra is an order of magnitude too small compared with that predicted by Eq. (2). In group 2 there is also one clear two-peaked spectrum, but the phase speed of the low-frequency peak is only $0.75 U_{10}$. The predicted phase speed from (5) for an ordinary high-frequency peak is $0.62 U_{10}$ and therefore this low-frequency peak can hardly be considered as a resonance peak.

Since R/V Aranda was in the wave measuring area during the measurements of group 5, the absence of swell could be observed visually. In this data no low-frequency peaks can be seen.

In conclusion, in 51 of the 72 spectra in which the phase speed of the waves can reach the wind speed there is no significant peak which could be interpreted as a resonance peak.

5.2.2. The two-peaked spectra

The spectra of groups 3 and 4 contain a considerable amount of energy at the predicted resonance frequency. Some of them have a clear two-peak structure, e.g. the spectrum from point C Oct 19 04.15 ... 04.30, Fig. 9b. However, the two-peak structure is not clear in all of the spectra and especially in group 3 there are spectra in which there is no statistically significant minimum which would divide the spectrum into two peaks.

There are in principle at least three possible ways to define the two-peak parameters from this kind of spectra. The first is to choose the two most significant peaks; this method was used in Kahma (1979). The second, which is used in this study, is to choose the high-frequency peak first, so that it is consistent with the other spectra which were measured close to that time. The third is to use the relations (3) and (5) to determine the frequency ranges in which the peaks should be and to calculate the parameters $\alpha_i$ and $\alpha_2$. Results from this third method can hardly be considered meaningful, when we take into account the observed differences between relations from different experiments (cf. Fig. 11).

The two-peak parameters $c_i/U_{10}$, $\omega_i$, $\phi_i$ and $\sigma_2$ of groups 3 and 4 are shown in Fig. 10. Our data agree with the two-peak fetch relations (2) ... (5) considerably less well than the data KSC quote. The scattering is much larger also when the spectra in which the two-peak structure is not clear are not taken into account. This is surprising, since the variations in the wind speed are only slightly larger than in group 1 in which the scattering of $\omega_m$ and $\sigma$ is very small (Fig. 11).

Although this scattering is taken into account the parameters $\phi_i$ and $\sigma_2$ show systematically higher values than those predicted by KSC. The possibility that a larger wind speed would remove the difference was considered, but the necessary wind speed was found to be so high that the depth would be insufficient for waves to reach the wind speed.

The relations (2) ... (5) given in KSC therefore do not hold for our data. However, the presentation in Fig. 10 does not yet exclude the possibility of the two-peak growth, and, on the other hand, it should be noted that the relation for $\sigma_2$ given in KKS is fairly close to our data. When the development of the spectrum with fetch in an individual profile is considered, more features appear which disagree with the two-peak growth. The growth of $\omega_i$ with the fetch in the simultaneous spectra is not linear but increases with fetch. This can
Fig. 9. Examples of spectra in groups 3 and 4. The arrow denotes the frequency of waves with phase speed equal to the wind speed.
Fig. 10. The dependence of the parameters of groups 3 and 4 on the fetch. The lines represent the relation given in KSC (1976) Eqs. (2) . . . (5). Data from KSC is added to the figures.
be understood as an effect of the insufficient depth, but it then means that the observed growth rate at the end of the fetch would correspond to deep-water growth. Our data indicates therefore an even larger growth for \( \sigma_i \) than Fig. 10 shows; this means that the resonance peak should dominate at all fetches, if the depth is sufficient throughout.

During a period of constant wind speed \( \sigma_i \) was found to decrease smoothly to about half of its initial value in 16 hours. This change was consistent at all points along the fetch. This is inconsistent with both the Phillips theory and the hypothesis in KSC. No similar change was observed in the high-frequency peak.

From Fig. 9 it can be seen that the low-frequency peak in the spectra from point D is near the position of the low-frequency peak in the spectra from points B and C. In most cases this peak is even slightly displaced towards higher frequencies. Point D is, however, in such shallow water that waves with phase speed equal to the wind speed have lower frequencies than at points C and B.

Fig. 11. The dependence of parameters of groups 1 and 5 on the fetch. Solid lines are based on this data; dashed lines on the JONSWAP data.
These properties of the low-frequency peak are typical for swell, i.e. waves which are not generated by the local wind. Fig. 12 shows an example of the properties of swell in this bottom topography. This time the direction of the waves was observed visually from an aircraft.

As mentioned in the beginning of this chapter, we analysed the data from all Finnish weather stations along the Gulf of Bothnia. During the period Oct 10 ... Oct 16 no wind observations were reported which could indicate swell to the profile area. From Oct 17 wind directions of 170° were observed in Utö and Mariehamn, about 130 km SSW from the profile. This supports the explanation of swell in the spectra of group 3 and 4. It also gives an example of how difficult it is to be sure of the absence of swell in open sea experiments.

Fig. 12. Spectra which were measured when swell from the north was observed from aircraft.
5.2.3. The small low-frequency peak

In Fig. 8c a small low-frequency peak can be seen in some of the spectra. This same peak of 0.12 Hz exists also in the spectra of group 1 during a long period; examples are given in Fig. 13. The peak appeared on Oct 13 at 4h and, with its frequency constant, gradually increased with time, reached its maximum energy on Oct 16 and then slowly decreased and disappeared under the larger peak discussed in the preceding section. No indications of instrument failure were found, and we believe that this peak is real.

Although the frequency of this peak is very close to the resonance frequency in deep water, its properties are not consistent with the Phillips theory. The growth of \( \sigma_z \) is not linear with fetch and is most rapid at the beginning of the fetch where the waves cannot reach the wind speed. Fig. 13 shows that at point D the low-frequency peak contains one third of the energy of the high-frequency peak, but at point B only one tenth.

The time history of the peak is not correlated to the wind speed; the energy of the peak at point D increases an order of magnitude in the same interval in which the total energy changes less than 30 %.

The origin of this peak is not certain but one probable explanation is that waves generated by the same global wind field in the northern part of the Baltic proper have spread through the Åland Sea to the profile area. The gradual growth of this peak is in accordance with the slow change in the wind direction from 80° to 120°.
5.3 DISCUSSION

The conclusion reached in section 5.2 is that in conditions in which the measurements were made, no peak was observed which could be interpreted as a peak caused by the Phillips resonance mechanism. Table 2 shows the two-peak parameters for each spectrum. In 72 of the 91 spectra an incomplete resonance peak at least should appear and in 23 spectra the parameters were $\bar{h} \geq 2$, $q \leq 0.1$ and $X_1/X_0 \geq 0.5$, which are so close to ideal that a considerable resonance peak should appear if the two-peak hypothesis were valid.

The spectra divide into the previously mentioned groups as follows:

<table>
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<th>group</th>
<th>number of spectra</th>
<th>considerable two-peak structure observed</th>
<th>at least an incomplete resonance peak should appear</th>
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</table>

The best two-peaked spectra should therefore appear in group 1 in which no two-peaked spectra were observed. On the other hand two-peaked spectra were observed under conditions in which the reasons suggested in KSC should prevent or at least limit the growth of the resonance peak (point D group 4, see table 2).

The examination of our profile data showed that it is very difficult to find conditions in which one could be certain, on the basis of the wind data, that no swell was present. Although a careful analysis of the wind field was made, e.g., in the experiment by Barnett and Wilkerson (1967), the explanation for the two-peak structure should probably be looked for in the initial conditions of the wave field. This is supported by a comment in the summary of KSC, where it is pointed out that in reservoirs the resonance peak normally does not appear.

The growth of the dimensionless wave component with dimensionless fetch was approximately exponential in this experiment (Kahma 1981) and no indication of a linear growth were observed. Our results therefore support the calculations based on Phillips' theory (Phillips, 1977; Snyder, Dobson, Elliot and Long 1981) which indicate that direct atmospheric forcing is negligible in the principal stage of wave development.

6. CONCLUDING REMARKS

The development of the wave spectrum with fetch in steady wind conditions was studied by a profile of consecutive wave buoys. Data were obtained from a period of exceptionally steady offshore wind. The two-peak structure observed in the spectra in the end of the period has been discussed. It was found that these spectra were not consistent with the hypothesis (Krylov, Strekalov and Cypluhin, 1976) that in suitable conditions the Phillips resonance mechanism could, together with the Miles mechanism, generate waves having a two-peaked spectrum. A
considerable part of our data was obtained in conditions comparable to those during which two-peak wave growth has previously been reported. However, most of those of our spectra which were measured in such conditions had very little energy at the frequencies at which the resonance peak should appear, and our data contained several spectra in which the absence of a considerable resonance peak could not be understood by any of the explanations given in the two-peak hypothesis. Although weather maps did not indicate it, swell seemed to be the most probable explanation of the observed two-peak structure. The same was found to be true with the one very small low-frequency peak observed.

Our result therefore support the calculations based on recent data about the turbulent pressure fluctuations (Snyder, Dobson, Elliott and Long, 1981) which indicate that the linear growth by the Phillips resonance mechanism is negligible at the principle stage of wave development.

Acknowledgements

I am indebted to Prof. S. A. Kitaigorodskii for his lectures and discussions during his visits to Finland and to Dr. Pentti Mäkki, deputy Director of the Institute of Marine Research, for valuable advice and encouragement.

I wish to thank Mr. J. Holmström of the Shipbuilding Laboratory of Helsinki University of Technology for cooperation in the field experiments, the National Board of Public Roads and Waterways for contributing equipment for the field experiments, and Mr. H. Söderman for his efficient help in the field work. I further wish to thank Mr. J. Saarinen for his contribution to the data processing.
LIST OF SYMBOLS

\( c \) \hspace{0.5cm} \text{phase speed}

\( c_{\text{max}} \) \hspace{0.5cm} \text{maximum phase speed in shallow water} \quad (= (gh)^{1/2})

\( c_i \) \hspace{0.5cm} \text{phase speed of waves having frequency} \; \omega_i

\( F(k) \) \hspace{0.5cm} \text{energy spectrum with respect to wave number} \; k

\( F_h \) \hspace{0.5cm} \text{modification of the Phillips equation in shallow water}

\( f_i \) \hspace{0.5cm} \text{peak frequency of peak} \; i \quad (f_i = \omega_i / 2\pi)

\( \tilde{f}_i \) \hspace{0.5cm} \text{dimensionless} \; f_i \; \text{scaled by} \; u_0

\( g \) \hspace{0.5cm} \text{acceleration due to gravity}

\( H_{\text{sv}} \) \hspace{0.5cm} \text{significant wave height}

\( H_{\text{av}} \) \hspace{0.5cm} \text{average height of the highest one-fourth of the waves}

\( h \) \hspace{0.5cm} \text{water depth}

\( h \) \hspace{0.5cm} \text{dimensionless water depth} \; h = gh/U^2_{10}

\( \bar{h} \) \hspace{0.5cm} \text{average water depth in the subfetch}

\( \bar{h} \) \hspace{0.5cm} \text{dimensionless average water depth}

\( i \) \hspace{0.5cm} \text{subscript}

\( k \) \hspace{0.5cm} \text{wavenumber vector}

\( k \) \hspace{0.5cm} \text{modulus of} \; k

\( \tilde{k} \) \hspace{0.5cm} \text{dimensionless} \; k

\( p \) \hspace{0.5cm} \text{turbulent pressure fluctuations}

\( q \) \hspace{0.5cm} \text{dimensionless parameter to estimate the conditions for two-peak growth}

\( r \) \hspace{0.5cm} \text{relative position vector}

\( t \) \hspace{0.5cm} \text{time}

\( U_{\infty} \) \hspace{0.5cm} \text{wind speed at the upper boundary of the boundary layer}

\( U_{10} \) \hspace{0.5cm} \text{wind speed at 10 m height}

\( u_* \) \hspace{0.5cm} \text{friction velocity}

\( X \) \hspace{0.5cm} \text{fetch}

\( \tilde{X} \) \hspace{0.5cm} \text{dimensionless fetch scaled by} \; U_{10}

\( \tilde{X} \) \hspace{0.5cm} \text{dimensionless fetch scaled by} \; u_0

\( x \) \hspace{0.5cm} \text{position vector}

\( X_3 \) \hspace{0.5cm} \text{subfetch in the deep end of the fetch}

\( \lambda \) \hspace{0.5cm} \text{wave length}

\( \rho_a \) \hspace{0.5cm} \text{density of air}

\( \rho_w \) \hspace{0.5cm} \text{density of water}

\( \sigma \) \hspace{0.5cm} \text{square root of the total variance of the wave spectrum.}

\( \sigma \) \hspace{0.5cm} \text{dimensionless} \; \sigma, \text{scaled by} \; U_{10}

\( \sigma_i \) \hspace{0.5cm} \text{square root of the variance of peak} \; i

\( \tilde{\sigma}_i \) \hspace{0.5cm} \text{dimensionless} \; \sigma_i, \text{scaled by} \; U_{10}

\( \tilde{\sigma}_i \) \hspace{0.5cm} \text{dimensionless} \; \sigma_i, \text{scaled by} \; u_0

\( \tau, \tau', \tau'' \) \hspace{0.5cm} \text{time lags}

\( \varphi \) \hspace{0.5cm} \text{direction of wave components with reference to the wind}

\( \Omega(k, \tau', \tau'') \) \hspace{0.5cm} \text{wavenumber spectrum of turbulent pressure fluctuations}
\( \omega \) (angular) frequency \( 2\pi f \)

\( \tilde{\omega} \) dimensionless (angular) frequency

\( \omega_m \) frequency of the main peak of the spectrum

\( \tilde{\omega}_m \) dimensionless frequency of the main peak of the spectrum

\( \omega_i \) peak frequency of peak \( i \)

\( \tilde{\omega}_i \) dimensionless \( \omega_i \)

\( \omega_{\text{min}} \) (angular) frequency at which the high-frequency peak begins to rise

\( \Pi(k, \omega) \) three-dimensional spectrum of the turbulent pressure fluctuations

\( \mu \) coupling coefficient of the Miles mechanism

REFERENCES


ABSTRACT

Nitrogen fixation by heterocystic blue-green algae was studied (by the acetylene reduction method) in the Gulf of Bothnia in 1978 and 1979. The measurements in 1978 and 1979 showed that nitrogen fixation was negligible in the Bothnian Bay and that the level of nitrogen fixation was markedly lower (1/10) in the central and southern Bothnian Sea than in the northern Baltic Proper and the Gulf of Finland.

INTRODUCTION

Vigorous blooms of nitrogen-fixing blue-green algae are a regular annual phenomenon in the Baltic Proper and the Gulf of Finland. One essential factor promoting such mass occurrences seems to be the excess of inorganic phosphorus in relation to nitrogen (e.g. Niemi 1979, Rinne et al. 1980). Areas with upwelling phosphorus-rich Baltic deep water are especially important for the development of blooms (Jansson 1978, Rinne et al. 1979).

The N:P ratio in the northern Gulf of Bothnia is much higher than in the Baltic proper (e.g. Voipio 1976, Pietikäinen et al. 1978, Alasaarela 1979a, Niemi 1979). This is a result on the one hand of the influx of phosphorus-poor Baltic surface water to the Åland Sea and the Archipelago Sea and further to the Gulf of Bothnia, and on the other to the great amount of river water, which is markedly richer in nitrogen than in phosphorus. Moreover, good oxygen conditions in the whole water column and the presence of enough ferric ions promotes the sedimentation of phosphorus and thus decreases the phosphorus content of the pelagial in the northern areas (Voipio 1969, 1976, Niemistö et al. 1978). Blue-green algal blooms in the southern Bothnian Sea have been observed (Jansson & Nyqvist 1977, Rinne et al. 1980). No blooms have been reported from the open part of the Bothnian Bay (Alasaarela 1979b).
In our previous papers (Niemi 1979, Rinne et al. 1980) we have suggested that the high inorganic N:P ratio in the northern Gulf of Bothnia is an unfavourable factor in the development of blue-green algal blooms fixing molecular nitrogen. Information on nitrogen fixation in the open sea area of the Gulf of Bothnia was first given at the Conference of Baltic Oceanographers in Leningrad (Rinne et al. 1980). The present study is a continuation of the MERININNI project, which has been studying the nitrogen fixation in the open Baltic Sea since 1975 (Rinne et al. 1976, 1978, 1979, 1980). The aim of our present study is to establish the occurrence and level of nitrogen fixation in different parts of the Gulf of Bothnia.

Fig. 1. Sampling stations in 1978 and 1979.
STUDY AREA, MATERIAL AND METHODS

The study area is shown in Fig. 1. The cruises were made in the Gulf of Bothnia on r/v Aranda from 7.8. to 9.8.1978 and from 1.8. to 3.8.1979. Material from an earlier cruise made in the Uusikaupunki archipelago on 12.8.1975 is also given. (Table 3).

The hydrographical and chemical analyses were made on board using the standard methods of the Institute of Marine Research (Koroleff 1976, 1979).

The nitrogen fixation of blue-green algae was measured by the acetylene reduction technique (Burris 1972), which is described in detail in Vuorio (1977), Vuorio et al. (1978), Rinne et al. (1978, 1979). Samples for the acetylene reduction measurements were usually taken with a 10-l water sampler and immediately concentrated through a 25-μm plankton net to about 100/1. They were divided into five subsamples. Four 5-ml subsamples were injected into 12-ml serum-stoppered bottles, the rest of the sample being preserved for plankton counting. Acetylene was injected into the four 12-ml bottles to an acetylene concentration of 20 %.

The reaction was stopped in one of the four bottles by adding formalin immediately after acetylene injection. The other three were incubated for 2 hours at 18°C and in an illumination of 5000 lx. The reaction was then stopped by adding formalin.

The ethylene concentration was measured with a Carlo Erba Fractovap gas chromatograph. One mole of ethylene produced corresponds to 2/3 mole of NH₃ produced (Klucas 1969). The phytoplankton was analysed from the net samples, which were preserved in Keefe’s solution and counted by Utermöhl technique (for details, see Melvasalo et al. 1973). Only the heterocystic blue-green algae and their heterocysts were counted.

HYDROGRAPHY AND WEATHER CONDITIONS

It was very windy during the 1978 cruise (7.8. — 9.8.). At no time during the cruise did the temperature reach the annual maximum, which occurred on about 1st August. The summer of 1979 was generally colder than the year before. Even so, temperatures passed the annual maximum during the cruise.

These meteorological conditions are reflected in the hydrographic conditions. In 1978 the thermocline was found at a depth of 20 to 30 m. The cooling of the air temperature during the 1978 cruise was reflected in slight instability, the warmest water lying somewhat below the surface at several stations (Fig 2).

Fig. 2. Temperature and salinity in the Gulf of Bothnia during the cruise 7.8. — 9.8.1978.
In 1979 the thermocline was sharp and situated nearer the surface, at a depth of 10 to 15 m. The surface temperature was 1 to 2 degrees warmer than in 1978 (Fig 3). The salinity, which dropped from about 6% in the Bothnian Sea to less than 3.5% in the Bothnian Bay, was similar during both cruises.

**NITROGEN AND PHOSPHORUS**

The ratio of nitrogen to phosphorus available for phytoplankton is most marked in late winter before the onset of the vernal phytoplankton production. The inorganic N:P ratio (w/w) increases from the northern Baltic Proper through the Åland Sea and the Bothnian Sea up to the Bothnian Bay. In the Baltic Proper the ratio is ca. 3-4, in the Bothnian Sea ca. 5, but in the Bothnian Bay 12—18 (e.g. Nehring et al. 1969, Voipio 1976, Pietikäinen et al. 1978, Alasaarela 1979a, Niemi 1979). In Fig. 4 the mean concentrations of total phosphorus and nitrate nitrogen are given as mean values of the period 1962—1975 (Pietikäinen et al. 1978). In the northern Bothnian Bay there is a scarcity of phosphorus but a surplus of nitrate nitrogen. The same trends in the distribution of inorganic nitrogen and phosphorus were formed throughout the area during both cruises (Table 1). The phosphate and nitrate concentration were low in the trophogenic layer except in the Bothnian Bay where nitrate still occurred.
Fig. 4. Means of observations of phosphorus and nitrate nitrogen made in summers 1962 — 1975 (Pietikäinen et al. 1978).

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TABLE 1. 1. Name of station. 2. Date. 3. Depht to bottom (m). 4. Surface temperature. 5. Surface salinity. 6. Total nitrogen (µg · dm⁻³) as a mean of 0—25 m. 7. Total phosphorus (µg · dm⁻³) as a mean of 0—25 m. 8. N:P ratio (weight per weight). 9. Biomass of heterocystic blue-green algae (mg/m²). 10. Number of heterocysts × 10⁶ · m⁻². 11. Acetylene reduction (µM C₂H₂ · m⁻² · 2h⁻¹).
BLUE-GREEN ALGAE AND NITROGEN FIXATION

The nitrogen-fixing blooms of blue-green algae in the Baltic Sea consist mainly of three heterocystic species (Rinne et al. 1978): *Aphanizomenon flos-aquae* and *Nodularia spumigena* and to a lesser degree *Anabaena lemmermannii* (sometimes *A. baltica*, cf. Niemi & Hällfors 1974). In 1978 *Aphanizomenon* dominated in the Bothnian Sea. Both *Aphanizomenon* and *Nodularia* occurred in the Bothnian Bay but only in small abundances. In 1979 most blue-green algae occurred in the central part of the Bothnian Sea (st. F-26). *Aphanizomenon* and *Nodularia* were found in equal abundances. *Anabaena* was very scarce in the Gulf of Bothnia.

In the central and northern part of the Bothnian Sea and in the Bothnian Bay the vertical maximum of *Aphanizomenon* occurred at 10—20 m. The highest vertical abundances of *Nodularia*, however, were found near the surface. The results of the acetylene reduction

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### TABLE 2

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<td>16</td>
<td>1.2</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>US-5</td>
<td>0.20</td>
<td>25</td>
<td>1.2</td>
<td>0.7</td>
<td>0.22</td>
<td>0</td>
<td>0.03</td>
<td>26</td>
<td>1.5</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>SR-5</td>
<td>0.07</td>
<td>38</td>
<td>3.4</td>
<td>4.7</td>
<td>1.4</td>
<td>0.1</td>
<td>0.03</td>
<td>43</td>
<td>4.8</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td>IU-2</td>
<td>1.7</td>
<td>4.5</td>
<td>0.85</td>
<td>3.7</td>
<td>1.4</td>
<td>1.8</td>
<td>0.06</td>
<td>9.8</td>
<td>2.9</td>
<td>0.54</td>
<td></td>
</tr>
<tr>
<td>X-1</td>
<td>0.08</td>
<td>6.4</td>
<td>0.61</td>
<td>0.1</td>
<td>0.01</td>
<td>0.05</td>
<td>6.5</td>
<td>0.67</td>
<td>0.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LL-9</td>
<td>0.53</td>
<td>100</td>
<td>5.1</td>
<td>1.9</td>
<td>0.33</td>
<td>0.6</td>
<td>0.13</td>
<td>110</td>
<td>5.7</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>LL-9</td>
<td>0.67</td>
<td>88</td>
<td>4.6</td>
<td>4.8</td>
<td>1.2</td>
<td>1.7</td>
<td>0.56</td>
<td>94</td>
<td>6.3</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>LL-11</td>
<td>4.0</td>
<td>84</td>
<td>8.7</td>
<td>22</td>
<td>6.6</td>
<td>2.2</td>
<td>1.3</td>
<td>110</td>
<td>17</td>
<td>0.22</td>
<td></td>
</tr>
</tbody>
</table>

| 1979            |   |   |   |   |   |   |   |    |    |
| BO-3            | 0 | 0.2 | 0 | 0 | 0 | 0 | 0.01 | 0.2 | 0.01 |
| F16             | 0 | 0.4 | 0.02 | 0.1 | 0.02 | 0 | 0.03 | 0.5 | 0.04 |
| F18             | 0 | 3.5 | 0.16 | 0.2 | 0.03 | 0 | 0.03 | 3.6 | 0.19 |
| F26             | 3.4 | 46 | 4.4 | 61 | 16 | 0.2 | 0.04 | 110 | 21 | 0.12 |
| SR-5/Män       | 31 | 3.3 | 24 | 6.9 | 0.9 | 0.25 | 56 | 10 |
| SR-5/Män       | 23 | 2.2 | 4.2 | 6.2 | 1.2 | 1.4 | 1.5 | 10 |
| F64             | 0.38 | 24 | 2.6 | 6.0 | 0.8 | 0.15 | 45 | 8.8 | 0.02 |
| F69             | 0.90 | 19 | 2.6 | 6.7 | 2.7 | 0.83 | 42 | 10 | 0.03 |
| F71             | 1.5 | 66 | 7.2 | 25 | 8.4 | 5.5 | 1.3 | 96 | 17 | 0.05 |
| LL-12           | 6.4 | 160 | 22 | 16 | 4.3 | 12 | 4.2 | 190 | 31 | 0.23 |
measurements and the total number of heterocysts showed the same geographical distribution as did the total biomass of the algae mentioned (Table 2). Nitrogen fixation values between 0.5 (st. US-5) and 3.4 (st. F-26) \( \mu \text{MC}_2\text{H}_2\text{m}^{-3}(2h)^{-1} \) (dimension expressed according to the incubation time used) were measured. The same level was found in the Åland Sea. The studies carried out in this area in 1975 (Rinne et al. 1978) showed the same level of nitrogen fixation (10 \( \mu \text{MC}_2\text{H}_2\text{m}^{-3}(2h)^{-1} \)).

In the Quark and in the Bothnian Bay no marked nitrogen fixation was observed or it was negligible (0—0.2 \( \mu \text{MC}_2\text{H}_2 \cdot \text{m}^{-3}(2h)^{-1} \)). The nitrogen fixation in the surface layer of the archipelago waters outside the town of Uusikaupunki, which are loaded with phosphorus-rich industrial discharges, was measured at 6 stations in 1975 (Table 3). At that time the three heterocystic species mentioned occurred in the area. Their biomass varied between 150 and 33 mg m\(^{-3}\). The highest acetylene reduction (10 \( \mu \text{MC}_2\text{H}_2 \cdot \text{m}^{-3}(2h)^{-1} \)) was obtained at the innermost station, where the biomass was highest and where the influence of the phosphorus-rich discharge was strongest. The lowest acetylene reduction and biomass values were obtained at the outermost stations. The heterocystic activity in the Uusikaupunki archipelago varied from ca. 0.3 to 0.6 pM C\(_2\text{H}_2\) (2h heterocyst) (Table 3). In this area the low N:P ratio near the industrial discharges seems to promote nitrogen-fixing blooms (see Häkkilä 1980).

DISCUSSION

In coastal waters, nitrogen fixation by planktonic, heterocystic blue-green algae has been studied by Rinne (1976), Vuorio (1977) and Vuorio et al. (1978) in eutrophicated waters off Helsinki in the Gulf of Finland. Brattberg (1975, 1977) has conducted similar studies in polluted waters off Stockholm (northern Baltic Proper). Lindahl et al. (1978, 1980) have studied nitrogen fixation in the Askö area, in the outer Stockholm archipelago and in the Öregrund archipelago (Åland Sea).

In the Baltic open sea areas pelagic nitrogen fixation has been studied by Hübel & Hübel (1976a, b) in the southern Baltic Sea, and in the northern Baltic Proper, Gulf of Finland, Åland Sea and Gulf of Bothnia by the MERININNI group (Rinne et al. 1976, 1978, 1979, 1980).

Blooms of heterocystic blue-green algae are natural phenomena in the open Baltic Sea. Observations of blooms were made back in the last century in the Baltic Sea (Pouchet & deGuerne 1885). The dominant algae, Aphanizomenon and Nodularia, are favoured by phosphorus but they are more or less independent of the amount of available nitrogen (e.g. Melin & Lindahl 1973, Horstmann 1975, Rinne & Tarkiainen 1975). Blooms of heterocystic blue-green algae fixing molecular nitrogen are also typical of natural lake waters with a phosphorus excess (Whitton 1973, Fogg 1975: 127). Thus upwelling phosphorus-rich water promotes the development of blooms of such algae (Jansson 1978).

TABLE 3. Sampling stations and surface values of nitrogen fixation, number of heterocysts, heterocystic activity, biomass of heterocystic blue-green algae, percental share of Aphanizomenon, Nodularia and Anabaena of total heterocystic blue-green algae and N:P ratio on August 12, 1975 in Uusikaupunki archipelago.

<table>
<thead>
<tr>
<th>Station</th>
<th>N-fixation</th>
<th>Heterocysts</th>
<th>Heterocystic activity</th>
<th>Biomass</th>
<th>Aphanizomenon %</th>
<th>Nodularia %</th>
<th>Anabaena %</th>
<th>N:P ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu$MC$_2$H$_2$m$^{-2}$24h$^{-1}$</td>
<td>$10^6$/m$^3$</td>
<td>pMC$_2$H$_2$/2h/het.</td>
<td>mg m$^{-3}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.8</td>
<td>3.4</td>
<td>6.6</td>
<td>3.5</td>
<td>8.0</td>
<td>10.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.4</td>
<td>6.1</td>
<td>18.0</td>
<td>6.3</td>
<td>16.0</td>
<td>30.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.57</td>
<td>0.56</td>
<td>0.37</td>
<td>0.56</td>
<td>0.50</td>
<td>0.33</td>
<td></td>
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<tr>
<td>45</td>
<td>38</td>
<td>130</td>
<td>33</td>
<td>83</td>
<td>150</td>
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<tr>
<td>61</td>
<td>68</td>
<td>81</td>
<td>80</td>
<td>81</td>
<td>75</td>
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<tr>
<td>36</td>
<td>21</td>
<td>12</td>
<td>10</td>
<td>8</td>
<td>14</td>
<td></td>
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<tr>
<td>3</td>
<td>11</td>
<td>7</td>
<td>10</td>
<td>11</td>
<td>11</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>28.6</td>
<td>15.6</td>
<td>7.1</td>
<td>8.2</td>
<td>5.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The low inorganic N:P ratio of nutrients available for phytoplankton production probably remains at a low level (during the summer stratification).

The nitrogen and phosphorus discharge from the land probably has some influence on the nutrient level in the open sea areas. The estimated discharge of total nitrogen and phosphorus to the Gulf of Finland and the Gulf of Bothnia has a N:P ratio of 10:1 and 15:1, respectively (Finnish-Soviet Working Group on the Protection of the Gulf of Finland 1979, National Environment Protection Board, Sweden and National Board of Waters, Finland 1979). Input from the atmosphere is not included in the estimates of the Gulf of Finland, but if it were, it would further increase the high N:P ratio (cf. Nehring & Wilde 1979). These inputs must increase the N:P ratio, i.e. they will not promote blooms causing nitrogen fixation. The essential contribution of phosphorus must come from the upwelling Baltic deep water rich in phosphorus (Niemi 1979).

The absence of blooms of heterocystic species and nitrogen fixation from the northern Gulf of Bothnia is apparently connected with the high inorganic N:P ratio in that area. Even in summer there is an excess of nitrate nitrogen in the euphotic layer of the Bothnian Bay (Volpio 1976, Pietikäinen et al. 1978, Alasaarela 1979a). The scarce phosphate in that area is rapidly consumed by the vernal phytoplankton immediately after the break-up of the ice in late May. However, low temperature may prevent development of blooms. According to Hübel et Hübel (1976 a, b), Nodularia cannot grow effectively without high temperatures.

In eutrophicated coastal areas in the Bothnian Bay blue-green algal blooms have been caused off Oulu by the non-heterocystic Oscillatoria agardhii (Alasaarela 1979b) and off Haparanda by the heterocystic Anabaena (Nauwerck 1978). Such blooms in polluted areas may be promoted by other factors, e.g. higher temperatures. Nevertheless as is shown in the Uusikaupunki area where eutrophication is caused by phosphorus-rich industrial wastes, the most important factor still seems to be the N:P ratio. This is corroborated by studies off the city of Helsinki (Rinne & Tarkiainen 1975).

Nitrogen fixation in the Gulf of Bothnia plays a conspicuously less important role in the nitrogen budget of the sea, than that noted in the earlier data collected during the MERININNI project, which concentrated mainly on the northern Baltic Proper and the Gulf of Finland. We roughly estimate that the annual nitrogen fixation in the Bothnian Bay is only 22 tons (<0.1 % of the total nitrogen input to the Bothnian Bay), and in the Bothnian Sea 1500—5000 tons (5—15 % of the nitrogen input). These values are smaller than the fixed values for the northern and central Baltic Proper; these were estimated at some 10^6 tons in 1974, which is the same order of magnitude as the land-based input of nitrogen in the area (cf. Rinne et al. 1978, 1979).

Blue-green algae fixing molecular nitrogen also occur in the littoral ecosystem of the Baltic Sea. Using the acetylene reduction method, Hübel & Hübel (1974 a, b, 1976a, b) have shown marked nitrogen fixation in littoral blue-green algal communities (Rivularia atra, Calothrix scopulorum, Anabaena torulosa) in eutrophicated coastal waters in the southern Baltic Sea. These species are also common in the northern Baltic archipelago waters. The epilithic Calothrix scopulorum inhabits the large areas covering splash zones in the archipelagos; hence, quantitatively, it is probably of considerable importance in the littoral nitrogen budget (cf. Niemi 1976, 1979). The occurrence of these species and their effect on the nitrogen budget have, however, not been studied in the Gulf of Bothnia.
REFERENCES


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FLUORIDE DISTRIBUTION ALONG CHLORINITY GRADIENTS IN BALTIC SEA WATERS

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30122 Venezia, Italy

ABSTRACT

Fluoride concentration was determined colorimetrically in coastal and inland waters of Finland and Sweden, focusing on the relationship between fluoride concentration and chlorinity. A positive linear regression could be established between the two parameters ($F_{\text{mg kg}^{-1}} = 0.1576 + 0.0560 \text{ Cl \%}_0$). Fluoride concentration varied from 0.14 mg dm$^{-3}$ in waters with a chlorinity of 1.10 %o to 0.69 mg dm$^{-3}$ in waters with a chlorinity of 8.83 %o. The $F/\text{Cl}$ ratio ($\times 10^5$) changed from 3000 in the lakes to 7.70 in the brackish water areas with the highest chlorinity. The fluoride levels and the $F/\text{Cl}$ ratios were similar or slightly lower than those reported in the literature for some of the same areas in previous years. The $F/\text{Cl}$ ratios recorded in the Pojoviken were among the highest ever found in brackish water.

INTRODUCTION

The distribution of fluoride in marine ecosystems is scarcely known. This element is toxic if present in excessive amounts (National Academy of Sciences, 1971). Considerable amounts of fluoride can be contained in waste water discharged from certain types of chemical factories as well as from aluminium smelters (Martin & Taft, 1975; Wright & Davison, 1975) and from uranium hexafluoride conversion plants producing nuclear fuel (Comitato Nazionale per l’Energia Nucleare, 1976; Dall’Aglio, 1976).

In the countries bordering the Baltic Sea many factories are likely to discharge fluoride into the sea. Data on fluoride concentration and the fluoride-chlorinity relationship in the Baltic have been published by Kremling (1969, 1970) and Kullenberg & Sen Gupta (1973).

The predisposition of certain marine animals for accumulation of fluoride and in particular the ability of barnacles to reflect differences in fluoride content of seawater have been demonstrated in recent papers (Hemens & Warwick, 1972; Wright & Davison, 1975; Barbaro et al., 1978).

The objective of our investigations was to provide data on fluoride levels in water and benthic organisms (barnacles and mussels) in the coastal waters of Finland, which could serve as a reference for future studies, especially with regard to a possible increase in contamination levels. For comparison, other Scandinavian waters (Swedish coasts, including the Kattegat, and Finnish lakes) were also sampled. In the present paper only the fluoride concentration in the water is dealt with. The data regarding the organisms will be published later.
Fig. 1. Sampling stations in Finland and Sweden (A) and in the fjord-like Pojoviken (B) in July and August 1977.

MATERIALS AND METHODS

Samples of surface waters (depth: 1 m) were collected in the following areas: in Finland, on the coast of the Baltic (Fig. 1A, station 1—11), in the fjord-like Pojoviken (Fig. 1B, station a—f) and in two of the main lakes, the Kallavesi (Fig. 1A, station 12), and the Oulujärvi (station 13); in Sweden, on the coast of the Kattegat (Fig. 1A, station 19 and 20) and in Lake Mälaren (station 14—18).

The sampling stations were mainly chosen with regard to the horizontal chlorinity gradient. This gradient was followed geographically considering stations between the Kattegat and the inner part of the Baltic and also inland waters. In addition, the gradient from the open sea to the inner parts of Pojoviken was studied. Only in one case (station "Ajax", southeast of the Zoological Station of Tvärminne; Fig. 1A, station 9) the vertical chlorinity gradient was considered, water samples being taken from the surface (1 m) to the bottom (70 m), at intervals of 10 m.

Sampling was carried out from July to August 1977. Water samples were taken with a Van Dorn bottle and stored in polyethylene bottles for a maximum of one week. The analyses were carried out in unfiltered water.

Fluoride was determined colorimetrically with the lanthanumalizarin complexone method (Greenhalgh & Riley, 1961), measurements of optical density being made with a Beckman DB-G spectrophotometer. Chlorinity was determined according to the Mohr-Knudsen method and pH with a Beckman Zeromatic pH meter. All samples were run in duplicate.

For the fluoride/chlorinity (F/Cl) ratio also the concentrations of F⁻ were expressed in g kg⁻¹ instead of mg dm⁻³ and, to facilitate comparison with literature data, the ratio was expressed in units × 10⁵. For Cl⁻, 900 is used instead of g kg⁻¹ in order to abbreviate the quotations.

RESULTS

Mean surface temperature of the water at the stations in Finland and Sweden in August was 16.4±1.4 °C (1); pH ranged from 7.3 to 8.4 in brackish water and from 6.2 to 6.4 in freshwater (Table 1).

(1) Variation of a parameter is always indicated as 95 % confidence limits.
Table 1. Temperature (t), pH, fluoride concentration (F), chlorinity (Cl) and fluoride/chlorinity ratio (F/Cl) in unfiltered surface water sampled in Finland and Sweden (August 1977; Fig. 1A)

<table>
<thead>
<tr>
<th>Station</th>
<th>t (°C)</th>
<th>pH</th>
<th>F (mg dm(^{-3}))(^*)</th>
<th>Cl (‰)</th>
<th>F/Cl (10(^5) g kg(^{-1})/‰)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gulf of Bothnia</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 Ajos (Kemi)</td>
<td>14.5</td>
<td>7.4</td>
<td>0.14</td>
<td>1.10</td>
<td>12.73</td>
</tr>
<tr>
<td>2 Raahø</td>
<td>8.5</td>
<td>7.4</td>
<td>0.19</td>
<td>1.84</td>
<td>10.33</td>
</tr>
<tr>
<td>3 Pörkenäs (Pietarsaari)</td>
<td>12.6</td>
<td>7.6</td>
<td>0.23</td>
<td>2.11</td>
<td>10.90</td>
</tr>
<tr>
<td>4 Bergø (Vaasa)</td>
<td>15.2</td>
<td>7.9</td>
<td>0.29</td>
<td>2.84</td>
<td>10.21</td>
</tr>
<tr>
<td>5 Lankooori (Pori)</td>
<td>19.4</td>
<td>8.3</td>
<td>0.32</td>
<td>3.24</td>
<td>9.88</td>
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<td>Archipelago Sea</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>6 Lillmälö (Turku)</td>
<td>17.3</td>
<td>8.1</td>
<td>0.35</td>
<td>3.53</td>
<td>9.92</td>
</tr>
<tr>
<td>7 Mariehamn (Åland)</td>
<td>—</td>
<td>7.3</td>
<td>0.34</td>
<td>3.56</td>
<td>9.55</td>
</tr>
<tr>
<td>8 Hitis (Hanko)</td>
<td>—</td>
<td>8.0</td>
<td>0.34</td>
<td>3.59</td>
<td>9.47</td>
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<td>Gulf of Finland</td>
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</tr>
<tr>
<td>9 Ajax (Hanko)</td>
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<td>8.4</td>
<td>0.33</td>
<td>3.29</td>
<td>10.03</td>
</tr>
<tr>
<td>10 Helsinki</td>
<td>17.3</td>
<td>8.1</td>
<td>0.33</td>
<td>2.86</td>
<td>11.54</td>
</tr>
<tr>
<td>11 Tuskas (Kotka)</td>
<td>18.2</td>
<td>7.8</td>
<td>0.28</td>
<td>2.15</td>
<td>13.02</td>
</tr>
<tr>
<td>Lakes of Finland</td>
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<td></td>
</tr>
<tr>
<td>12 Kallavesi (Kuopio)</td>
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<td>6.3</td>
<td>0.06</td>
<td>0.01</td>
<td>600.00</td>
</tr>
<tr>
<td>13 Oulujärvi (Kajaani)</td>
<td>17.5</td>
<td>6.4</td>
<td>0.03</td>
<td>0.01</td>
<td>300.00</td>
</tr>
<tr>
<td>Lake Mälaren (Sweden)</td>
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<td>14 Kvävsund</td>
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<td>0.28</td>
<td>0.01</td>
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<td>18.6</td>
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<td>0.30</td>
<td>0.01</td>
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<td>—</td>
<td>6.4</td>
<td>0.37</td>
<td>0.02</td>
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<td>18 Södertälje</td>
<td>16.1</td>
<td>7.3</td>
<td>0.35</td>
<td>2.98</td>
<td>11.74</td>
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<td>Kattegat</td>
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</tr>
<tr>
<td>19 Helsingör</td>
<td>17.0</td>
<td>7.5</td>
<td>0.42</td>
<td>4.74</td>
<td>8.86</td>
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<td>20 Getterö (Varberg)</td>
<td>17.0</td>
<td>8.0</td>
<td>0.69</td>
<td>8.83</td>
<td>7.70</td>
</tr>
</tbody>
</table>

\* At very low chlorinities mg dm\(^{-3}\) = mg kg\(^{-1}\)

In the Baltic Sea the fluoride concentration ranged from 0.14 mg dm\(^{-3}\) in waters with a chlorinity of 1.10 ‰ to 0.69 mg dm\(^{-3}\) in waters with a chlorinity of 8.83 ‰. The concentration of this element increased gradually with chlorinity, i.e. from the innermost stations of the Gulfs of Bothnia and Finland towards the Kattegat. The lowest F/Cl ratios (7.70—8.86) were found in the Kattegat and the highest in the inner stations of the Baltic (Gulf of Bothnia: 12.73; Gulf of Finland: 13.02). Fluoride levels of the Finnish lakes of Oulujärvi and Kallavesi (0.03—0.06 mg dm\(^{-3}\)) were about 10 times lower than those found in Lake Mälaren in Sweden (0.28—0.37 mg dm\(^{-3}\)); in this lake the values for the F/Cl ratio were of the order of 10\(^3\).

Mean temperature of the surface waters sampled in Pojoviken (Table 2) was 14.1±0.8 °C in July and 18.4±0.5 °C in August. A gradient from the most internal station (a) to the seaward station (j) was recorded for all other parameters considered in July as well as in August. pH ranged from 7.5 to 8.2, the higher values referring to the seaward stations. The fluoride concentration varied between a minimum of 0.20 mg dm\(^{-3}\) at station a having the
Table 2. Temperature (t), pH, fluoride concentration (F), chlorinity (Cl) and fluoride/chlorinity ratio (F/Cl) in unfiltered surface water sampled in Pojoviken (1977; Fig. 1B)

<table>
<thead>
<tr>
<th>Station</th>
<th>Month</th>
<th>t (°C)</th>
<th>pH</th>
<th>F (mg dm(^{-3}))</th>
<th>Cl (‰)</th>
<th>F/Cl (10(^5) g kg(^{-1})/‰)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a Pohjankuru</td>
<td>July</td>
<td>15.4</td>
<td>7.5</td>
<td>0.20</td>
<td>0.69</td>
<td>28.99</td>
</tr>
<tr>
<td></td>
<td>August</td>
<td>18.6</td>
<td>7.6</td>
<td>0.24</td>
<td>0.79</td>
<td>30.38</td>
</tr>
<tr>
<td>b Sällvik</td>
<td>July</td>
<td>14.0</td>
<td>7.6</td>
<td>0.27</td>
<td>1.46</td>
<td>18.49</td>
</tr>
<tr>
<td></td>
<td>August</td>
<td>18.7</td>
<td>7.9</td>
<td>0.24</td>
<td>0.90</td>
<td>26.67</td>
</tr>
<tr>
<td>c Tammisaari</td>
<td>July</td>
<td>13.3</td>
<td>7.6</td>
<td>0.29</td>
<td>1.67</td>
<td>17.37</td>
</tr>
<tr>
<td></td>
<td>August</td>
<td>18.6</td>
<td>8.0</td>
<td>0.24</td>
<td>1.01</td>
<td>23.76</td>
</tr>
<tr>
<td>d Källviken</td>
<td>July</td>
<td>13.7</td>
<td>7.8</td>
<td>0.33</td>
<td>2.13</td>
<td>15.49</td>
</tr>
<tr>
<td></td>
<td>August</td>
<td>19.0</td>
<td>8.1</td>
<td>0.26</td>
<td>1.45</td>
<td>17.93</td>
</tr>
<tr>
<td>e Hermansö</td>
<td>July</td>
<td>13.4</td>
<td>8.0</td>
<td>0.35</td>
<td>3.25</td>
<td>10.77</td>
</tr>
<tr>
<td></td>
<td>August</td>
<td>17.9</td>
<td>8.1</td>
<td>0.31</td>
<td>2.67</td>
<td>11.61</td>
</tr>
<tr>
<td>f Tvärminne</td>
<td>July</td>
<td>14.6</td>
<td>7.9</td>
<td>0.35</td>
<td>3.36</td>
<td>10.42</td>
</tr>
<tr>
<td></td>
<td>August</td>
<td>17.8</td>
<td>8.2</td>
<td>0.34</td>
<td>3.16</td>
<td>10.76</td>
</tr>
</tbody>
</table>

The minimum chlorinity (0.69 ‰) and a maximum of 0.35 mg dm\(^{-3}\) at station f with the highest chlorinity (3.36 ‰). The gradient is more strongly reflected by the F/Cl ratio, for which the minimum, at the seaward station, was 10.42 and the maximum, at the innermost station of the fjord, 30.38. The mean F/Cl ratio of Pojoviken (18.55 ± 4.65) was significantly higher (P < 0.05) than that of the other stations (11.05 ± 0.83) with a chlorinity similar to that of the fjord (0.69—3.36 ‰).
Table 3. Temperature (t), pH, fluoride concentration (F), chlorinity (Cl) and fluoride/chlorinity ratio (F/Cl) in unfiltered water sampled from the surface to the bottom at Ajax station (1977; Fig. 1A, station 9)

<table>
<thead>
<tr>
<th>Depth m</th>
<th>Month</th>
<th>t (°C)</th>
<th>pH</th>
<th>F (mg dm(^{-3}))</th>
<th>Cl (%/oo)</th>
<th>F/Cl (10(^5) g kg(^{-1})/%/oo)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>July</td>
<td>12.7</td>
<td>8.0</td>
<td>0.35</td>
<td>3.49</td>
<td>10.03</td>
</tr>
<tr>
<td></td>
<td>August</td>
<td>16.4</td>
<td>8.4</td>
<td>0.33</td>
<td>3.29</td>
<td>10.03</td>
</tr>
<tr>
<td>10</td>
<td>July</td>
<td>12.7</td>
<td>7.9</td>
<td>0.35</td>
<td>3.47</td>
<td>10.09</td>
</tr>
<tr>
<td></td>
<td>August</td>
<td>15.4</td>
<td>8.2</td>
<td>0.33</td>
<td>3.27</td>
<td>10.09</td>
</tr>
<tr>
<td>20</td>
<td>July</td>
<td>10.7</td>
<td>7.8</td>
<td>0.36</td>
<td>3.57</td>
<td>10.08</td>
</tr>
<tr>
<td></td>
<td>August</td>
<td>12.6</td>
<td>8.0</td>
<td>0.34</td>
<td>3.45</td>
<td>9.86</td>
</tr>
<tr>
<td>30</td>
<td>July</td>
<td>3.7</td>
<td>7.5</td>
<td>0.37</td>
<td>3.94</td>
<td>9.39</td>
</tr>
<tr>
<td></td>
<td>August</td>
<td>5.8</td>
<td>7.7</td>
<td>0.36</td>
<td>3.82</td>
<td>9.42</td>
</tr>
<tr>
<td>40</td>
<td>July</td>
<td>2.5</td>
<td>7.5</td>
<td>0.40</td>
<td>4.23</td>
<td>9.46</td>
</tr>
<tr>
<td></td>
<td>August</td>
<td>2.4</td>
<td>7.6</td>
<td>0.38</td>
<td>4.11</td>
<td>9.25</td>
</tr>
<tr>
<td>50</td>
<td>July</td>
<td>2.4</td>
<td>7.5</td>
<td>0.40</td>
<td>4.25</td>
<td>9.41</td>
</tr>
<tr>
<td></td>
<td>August</td>
<td>2.2</td>
<td>7.5</td>
<td>0.39</td>
<td>4.33</td>
<td>9.01</td>
</tr>
<tr>
<td>60</td>
<td>July</td>
<td>2.2</td>
<td>7.5</td>
<td>0.41</td>
<td>4.26</td>
<td>9.62</td>
</tr>
<tr>
<td></td>
<td>August</td>
<td>2.2</td>
<td>7.5</td>
<td>0.40</td>
<td>4.41</td>
<td>9.07</td>
</tr>
<tr>
<td>70</td>
<td>July</td>
<td>2.3</td>
<td>7.5</td>
<td>0.41</td>
<td>4.30</td>
<td>9.53</td>
</tr>
<tr>
<td></td>
<td>August</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

At the Ajax station (Table 3) the thermocline was found between 20 and 30 m both in July (10.7—3.7 °C) and in August (12.6—5.8 °C). Fluoride concentration increased with chlorinity: the minimum value of 0.33 mg dm\(^{-3}\) was found at the surface at a chlorinity of 3.29 %/oo and the maximum of 0.41 mg dm\(^{-3}\) near the bottom at a chlorinity of 4.30 %/oo. The mean F/Cl ratio for the surface layers (10.03 ± 0.09) was higher than for the deep layers (9.35 ± 0.15).

The relationship between F/Cl ratio and chlorinity is illustrated in Fig. 2, where the difference between the values of Pojoviken and those of the other coastal areas of the Baltic (chlorinity ~0.02 %/oo) is clearly recognizable.

For the chlorinity range from 0.02 to 8.83 %/oo the regression of the fluoride concentration on chlorinity was calculated:

\[
F \left[ \text{mg kg}^{-1}\right] = 0.1576 + 0.0560 \text{Cl} \left[\% /\text{oo}\right]
\]

The regression coefficient (0.0560) was found to be significantly different from 0 (P <0.001).

**DISCUSSION**

The data reported in the present paper for Baltic waters sampled along horizontal and vertical gradients of chlorinity showed fluoride concentrations increasing from the stations with low chlorinities towards the stations with higher chlorinity and also from the surface water to the near-bottom water; indeed, a positive linear relationship between fluoride
content and chlorinity was found. Similar conditions have been reported by Kremling (1969): a fluoride minimum of 0.122 mg kg\(^{-1}\) (Cl, 1.213 °/oo) for the Gulf of Bothnia and a maximum of 0.750 mg kg\(^{-1}\) (Cl, 10.981 °/oo) for a station near the Kattegat. Kullenberg & Sen Gupta (1973) found fluoride concentrations of 0.6 and 0.4 mg dm\(^{-3}\) (Cl, 3.1 °/oo), recorded in June and September respectively, for surface waters sampled in the Finnish coast waters of the Gulf of Bothnia; these concentrations are higher than those reported in the present paper for comparable stations, probably as a consequence of seasonal variations. All cited authors pointed out that fluoride concentrations increased in the deeper waters with increasing chlorinity.

Kullenberg & Sen Gupta (1973) recorded large variations of fluoride in Swedish rivers (0.090–0.605 mg dm\(^{-3}\)), the minimum being higher than the fluoride content reported in the present paper for Finnish lakes.

The F/Cl ratio decreased with increasing chlorinity from the highest values (10\(^3\)) found in the freshwater of the lakes to a minimum value of 7.70 recorded in the Kattegat (Cl, 8.83 °/oo). A similar hyperbolic relationship was found in the northern part of the Lagoon of Venice, influenced in the mainland portion by a considerable river runoff and in the more seaward portion by the waters of the Adriatic sea (Fig. 3; Francescon & Barbaro, in press). These findings are in good agreement with those of other authors. Kremling (1969, 1970) and Kullenberg & Sen Gupta (1973) found that this ratio was much higher in the Baltic than in the open ocean. Windom (1971, coastal and estuarine areas of Georgia, U.S.A.) reported F/Cl ratios to approach open ocean values at chlorinities above 10–11 °/oo, and Greenhalgh & Riley (1963) and Brewer et al. (1970) published values for the F/Cl ratio of oceanic areas which are of the same level as those for chlorinities < 10 °/oo in figure 3 of the present paper.

The mean F/Cl ratio found in the Pojoviken was significantly higher than that of all other stations of the Baltic with a similar chlorinity range; the maximum ratio of 30.38 (Cl, 0.79 °/oo) was recorded in the innermost portion of the fjord. This could perhaps be due to...
the presence of certain metal industries in this area. A similarly high F/Cl ratio (38.71), but
determined at a lower chlorinity (0.434 °/oo), has been found in the Baltic only in the inner
part of the Gulf of Finland (Kremling, 1969)(2). In the Northern Lagoon of Venice, a ratio
of the same order of magnitude (41.38; Cl, 0.87 °/oo) was recorded in one of the two
estuarine areas examined, i.e. near the river with the higher fluoride level (Francescon &
Barbaro, in press). For coastal and estuarine areas of the Atlantic, at chlorinities below
0.8 °/oo, Windom (1971, Fig. 2) reported a maximum F/Cl ratio of only about 16.4. It thus
appears that the F/Cl ratios recorded in the Pojoviken were among the highest ever found
for brackish water.

The fluoride levels (and the F/Cl ratios) in the areas of the Baltic Sea sampled for this
study were thus, on the whole, similar, or sometimes even lower, than those found in some
of the same areas in previous years (Kremling, 1969, 1970; Kullenberg & Sen Gupta, 1973).
Such a comparison is not possible for the Pojoviken since data for previous years are not
available.

ACKNOWLEDGEMENTS

The authors are indebted to Professor R. Kristoffersson, Head of the Zoological Station Tvärminne, for putting at
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particular Manager Dr. K. J. Purasjoki and Amanuensis Mr. Eero Aro for continuous assistance and availability.
Dr. M. Bilio, Director of the S.I. VAL. CO. Fish Culture Research Institute in Comacchio (Italy), kindly corrected
the manuscript and provided useful comments.

(2) The relatively high (F/Cl ratio in this area could be due to the discharge of the Kymi-River containing
fluoride from the Rapakivigranite present in the catchment area of this river (National Board of Waters, 1978).
REFERENCES


COMPARISON OF THE SAMPLING EFFICIENCY OF TWO VAN VEEEN GRABS

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ABSTRACT

Efficiency in sampling the macrozoobenthos on a soft bottom in the Baltic Sea was compared between two van Veen grabs, one with about 5% and the other with about 45% of the upper side composed of windows covered with 0.5 mm net. Judged from the numbers of the most abundant species, the amphipods Pontoporeia femorata and P. affinis, the efficiency of the grab with large windows was about 50% higher than that of the grab with small windows. For the polychaete Harmothoe sarsi the difference was even greater. The differences were statistically significant in respect of both these individual species and the total fauna. No differences in sampling efficiency were observed in the case of the isopod Mesidotea entomon and the lamellibranch Macoma baltica. The size distributions of the Pontoporeia species were similar in the material of the two grabs. The results show that the comparability of different zoobenthos investigations can be strongly affected by the construction of the grabs.

INTRODUCTION

The van Veen grab has been recommended by the Baltic Marine Biologists (BMB) as the main tool for sampling the benthic macrofauna in the Baltic Sea (Dybern et al. 1976). One of the chief disadvantages of this gear is the restricted flow of water through the grab when it is lowered, which causes a strong shock wave in front of the grab. This shock wave is diminished by fitting the upper side of the grab with net-covered windows. The influence of the size of these windows has now been studied by comparing a van Veen grab equipped with very small windows, and a grab with enlarged windows.

The present study was prompted by the results obtained in intercalibrating the zoobenthos methods used by the Institute of Marine Research, Helsinki and the Askö Laboratory, University of Stockholm at Askö in June 1974 (Ankar et al. 1979). The substantial differences found between the results were mainly attributed to differences in the size of the grab windows and in sieving techniques.
MATERIAL AND METHODS

The grabs were both made by the Keturi metal workshop, Helsinki. They differed mainly in the size of the windows in their upper sides (Fig. 1). The windows of the old grab (van Veen 3) covered only 5% of the upper side, while the corresponding value for the new grab (van Veen 4) was 47%. The biting areas of the grabs were the same, but the volumes of the grabs differed slightly.

The sampling was carried out on R/V Aranda on 25 September 1975 at Tvärminne Storfjärd, Gulf of Finland. The depth at the sampling site was 39 m. The bottom sediment consists of soft mud and has a strong smell of hydrogen sulphide. The bottom fauna at this locality is composed mainly of the crustaceans *Pontoporeia femorata* and *P. affinis*. Twelve samples were taken with each grab. After every three samples the grabs were changed, and after every six samples the ship was moved, to avoid digging in the same places. All the samples were sieved through 1 mm and 0.6 mm sieves and the sieving residues containing the animals were stored in 4% buffered formalin. The 0.6 mm fractions were dyed with Rose Bengal. Abundance values were determined for the different species, and the size distributions for the *Pontoporeia* species.

RESULTS

The sediment volume sampled varied between 13 and 19 litres. On average, the old grab was filled to about 70% and the new one to nearly 90%.
Since the abundance values in the 0.6 mm fractions were too small to be used separately (see Table 1), the results of the 1 mm and 0.6 mm fractions were pooled for statistical comparisons. The species composition and the abundance values are given in Table 1. Species atypical of this bottom, or not quantitatively sampled have been omitted in the statistical comparisons.

TABLE 1. Comparison of the abundance values of the benthic macrofauna obtained in a van Veen grab with small windows on the upper side (van Veen 3) and in a grab with enlarged windows (van Veen 4).

<table>
<thead>
<tr>
<th>Species</th>
<th>1 mm fraction</th>
<th></th>
<th>1 + 0.6 mm fraction</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ind./sample</td>
<td>X</td>
<td>SD</td>
<td>ind./sample</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Harmothoe sarsi</td>
<td>van Veen 3</td>
<td>10</td>
<td>4.3</td>
<td>+50%</td>
</tr>
<tr>
<td></td>
<td>van Veen 4</td>
<td>15</td>
<td>5.2</td>
<td></td>
</tr>
<tr>
<td>Mesidotea entomon</td>
<td>van Veen 3</td>
<td>10</td>
<td>7.1</td>
<td>+10%</td>
</tr>
<tr>
<td></td>
<td>van Veen 4</td>
<td>11</td>
<td>5.9</td>
<td></td>
</tr>
<tr>
<td>Pontoporeia femorata</td>
<td>van Veen 3</td>
<td>68</td>
<td>11.2</td>
<td>+59%</td>
</tr>
<tr>
<td></td>
<td>van Veen 4</td>
<td>108</td>
<td>14.1</td>
<td></td>
</tr>
<tr>
<td>Pontoporeia affinis</td>
<td>van Veen 3</td>
<td>74</td>
<td>23.7</td>
<td>+58%</td>
</tr>
<tr>
<td></td>
<td>van Veen 4</td>
<td>117</td>
<td>26.8</td>
<td></td>
</tr>
<tr>
<td>Macoma baltica</td>
<td>van Veen 3</td>
<td>3</td>
<td>2.3</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>van Veen 4</td>
<td>3</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>van Veen 3</td>
<td>164</td>
<td>36.6</td>
<td>+54%</td>
</tr>
<tr>
<td></td>
<td>van Veen 4</td>
<td>253</td>
<td>39.2</td>
<td></td>
</tr>
<tr>
<td>Nereis diversicolor</td>
<td>van Veen 3</td>
<td>+</td>
<td></td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>van Veen 4</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>-</td>
<td>2</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>van Veen 4</td>
<td>-</td>
<td>2</td>
<td></td>
</tr>
<tr>
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<td>-</td>
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</tr>
<tr>
<td></td>
<td>van Veen 4</td>
<td>+</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mytilus edulis</td>
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<td>1</td>
<td>2.6</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>van Veen 4</td>
<td>+</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Membranipora crustulenta</td>
<td>van Veen 3</td>
<td>+</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>van Veen 4</td>
<td>+</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

+= less than 0.5 ind./sample

The grab with larger windows collected significantly more individuals of the amphipods *Pontoporeia femorata* and *P. affinis* and the polychaete *Harmothoe sarsi* (Mann—Whitney U-test, Conover 1971). No difference was found between the grabs in the abundance of the isopod *Mesidotea entomon* and the lamellibranch *Macoma baltica*, but the numbers of *M. baltica* were not high enough to allow a proper comparison.

The chi-square test was used to determine whether there were significant differences in the size distributions of *Pontoporeia femorata* and *P. affinis*. It was expected that the size distributions in the material collected with the new grab were the same as in the material collected with the old grab. As might be expected from Figure 2, no significant differences between the two grabs could be proved.
Fig. 2. The percentage size distributions of *Pontoporeia femorata* and *P. affinis* obtained in a van Veen grab with small windows (van Veen 3) and a grab with large windows (van Veen 4). The values at the top of the bars show the number of individuals per square metre.

**DISCUSSION**

As was expected from the intercalibration study at Askö (Ankar et al. 1979) the grab with large windows had a much higher sampling efficiency. The differences in the abundance values of the two *Pontoporeia* species and the total values were much greater, however, in the Tvärminne material than in the Askö material, while the differences in the abundance of *Harmothoe sarsi* were of the same order of magnitude. In the Tvärminne material the sampling efficiency was not observed to differ with the size of the *Pontoporeia* individuals,
TABLE 2. The abundance values (ind./m²) of *Pontoporeia femorata*, *P. affinis* and *Harmothoe sarsi* obtained in van Veen grabs with different sized windows in Tvärminne and Askö.

<table>
<thead>
<tr>
<th></th>
<th>1'st year class</th>
<th>2'nd year class</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>window size</td>
<td>window size</td>
<td>window size</td>
</tr>
<tr>
<td></td>
<td>small</td>
<td>big</td>
<td>difference</td>
</tr>
<tr>
<td><strong>Pontoporeia femorata</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Askö</td>
<td>523</td>
<td>651</td>
<td>128 (+24%)</td>
</tr>
<tr>
<td>Tvärminne</td>
<td>196</td>
<td>331</td>
<td>135 (+69%)</td>
</tr>
<tr>
<td><strong>P. affinis</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Askö</td>
<td>2313</td>
<td>3202</td>
<td>889 (+38%)</td>
</tr>
<tr>
<td>Tvärminne</td>
<td>385</td>
<td>595</td>
<td>210 (+55%)</td>
</tr>
<tr>
<td><strong>Harmothoe sarsi</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Askö</td>
<td>88</td>
<td>162</td>
<td>74 (+84%)</td>
</tr>
<tr>
<td>Tvärminne</td>
<td>113</td>
<td>200</td>
<td>87 (+77%)</td>
</tr>
</tbody>
</table>

while at Askö only small-sized individuals were sampled more efficiently with the grab with large windows (Table 2).

The most plausible explanation of the differences between the results of the two investigations is variation in the vertical distribution of the animals in the sediment, due to differences in the sediment or stage of development of the animals or some other factor. According to Ankar (1977), in the Askö area the small individuals live higher up in the sediment than the big ones, and are thus more exposed to the shock wave. No similar information is available from the Tvärminne area, but there can hardly have been any distinct vertical differentiation in the size classes of the *Pontoporeia* species, since the size distributions were similar in the samples taken with the two grabs, even though the grab with enlarged windows had a smaller shock wave.

The comparison may be affected by the fact that although the community structure was almost similar in Tvärminne and Askö, the population density at Tvärminne was only half that at Askö. The influence of the different sampling seasons is difficult to estimate. In Tvärminne sampling was carried out in September and at Askö in June, and thus the *Pontoporeia* individuals were considerably bigger in the Tvärminne material, especially the young ones.

Comparison with the results of the Askö study showed that, although an increase of the water flow through the grab clearly improves sampling efficiency, this improvement is not constant. When the shock wave in front of the grab is diminished by enlarging the windows in the upper side of the grab, the degree of improvement in sampling efficiency depends on at least the following factors:

— the nature of the bottom at the sampling locality.
— the community structure, i.e. the size of the species and their penetration into the sediment.
— the sampling season.

The present study shows that besides being impaired by differences in sieving techniques (see Ankar et al. 1979), the comparability of the results of different zoobenthos investigations is also clearly affected by differences in the construction of the grab. More detailed standardization of the grabs is therefore needed. Maximal sampling efficiency is of course desirable, but as regards monitoring, a maximal degree of standardization is perhaps even more important.
REFERENCES


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