China’s Economic Transition

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The aim of this study is to understand the fundamental features of China’s economic transition since 1992. In order to do so, the central features of China’s transition are reviewed, most notably the main economic reforms, the firm-level resource reallocation, productivity differences between state-owned and private enterprises, moderate wage growth and rising income inequality, financial market imperfections and the central macroeconomic indicators: accumulation of foreign surplus and high aggregate investment and savings rates. A growth model consistent with China’s growth experience is built to give a clear qualitative explanation to China’s puzzling phenomena: Why does a country accumulate a foreign surplus despite of high domestic rate of return to capital? Why does a country’s rate of return to capital remain high in spite of a high investment rate?

The cornerstones of the model are heterogeneity in productivity, reallocation of resources and asymmetric financial imperfections. The enterprise sector is divided into private and state-owned enterprises. Private enterprises are more productive, but due to the discrimination by the financial sector they must rely on internal savings, while state-owned enterprises are less productive, but survive in equilibrium due to better access to external financing. If the entrepreneurial savings are large enough, private enterprises gradually outgrow state-owned enterprises. Financial integration of state-owned firms and labor mobility sustains the rate of return for both types of firms during the transition. Moreover, the aggregate rate of return to capital increases due to the composition effect.

The accumulation of foreign surplus originates from the financial imperfections. The wage earners deposit their savings to the banks, which in turn, can either invest to domestic enterprises or in foreign bonds. As the transition progresses the volume of high-productive financially constrained enterprises increase while the volume of low-productive externally financed enterprises decrease. Hence as the volume of state-owned enterprises decrease, a higher amount of domestic savings is invested into foreign assets by the financial intermediaries causing the foreign surplus to increase. After the transition is over, the economy is dominated by private enterprises and capital accumulation is subject to diminishing return to capital.

The main contradictions with China’s experience are frictionless labor market, financial market laissez-faire environment and the prediction that state-owned enterprises fully fades from the economy. Despite of these simplifications, the model gives a clear qualitative explanation to China’s puzzling phenomena of sustained return to capital and growing foreign surplus. The simplifications allow the model to focus on the main differences between E and F firms, that is to say the heterogeneity in productivity and asymmetric financial imperfections.
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Notations for the models

\( U \) Utility

\( c \) Consumption

\( \beta \) The discount factor

\( \theta \) The intertemporal elasticity of substitution in consumption \( c_t \)

\( v \) The exogenous population growth rate

\( N \) Agents (workers) with no entrepreneurial skills

\( \mu N \) Agents (entrepreneurs) with entrepreneurial skills

\( \chi \) Extra efficiency unit per worker

\( \psi \) The share of output that E firm managers can steal in case of delegation

\( y \) Output

\( k \) Capital

\( \alpha \) The output elasticity of capital

\(^1\) For simplicity: the time subscripts are omitted from the list of notations. In general, the time subscripts used in the model are period \( t \), past period \( t - 1 \), next period \( t + 1 \), and period at the end of the transition \( T \). Moreover, without few exceptions, the following subscripts are also omitted from the list of notations: F, E, and W; referring for financially integrated firms, entrepreneurial firms and workers, respectively. For clarity: in some cases the above subscripts are marked as superscripts.
$n \quad$ Labor

$A \quad$ Technology parameter

$z \quad$ Exogenous growth rate of technology parameter, $A$

$w \quad$ Wage

$s \quad$ Savings

$R^d \quad$ Interest factor paid by the intermediaries

$L \quad$ Lagrangian

$\lambda \quad$ The Lagrange multiplier

$\zeta^w \quad$ Refers to the term $\left( \left( 1 + \beta^{-\theta} R^{(d)} 1-\theta \right)^{-1} \right)$ appearing in the expression of the optimal savings for workers

$m \quad$ Managerial compensation for young entrepreneurs

$R \quad$ Interest factor on foreign bonds

$R^l \quad$ The lending rate to the domestic firms

$\xi \quad$ Iceberg cost, into which banks are exposed when lending to firms

$\rho_E \quad$ The rate of return to capital for E firms

$\chi \quad$ Largest extra efficiency unit per worker ($\chi > \chi$) for the transition not to occur
\( l^E \) Bank loans for E firms

\( \eta \) The share that entrepreneurs can promise to repay to the banks

\( \zeta^E \) Refers to the term \( \left( 1 + \beta^{-\theta} \left( \frac{(1-\eta)\rho_E R^l (1-\theta)}{R^l - \eta \rho_E} \right)^{1-\theta} \right)^{-1} \) appearing in the expression for the optimal savings for entrepreneurs

\( \kappa \) The capital per effective unit of labor

\( K \) Aggregate capital

\( \hat{\chi} \) Largest extra efficiency unit per worker \( (\chi > \hat{\chi}) \) for the employment share for E firms not to grow over time

\( \gamma_{K_E} \) Refers to the term appearing in the expression for the equilibrium dynamics of total capital of E firms during the transition, that is,

\[
\frac{K_{Et+1}}{K_{Et}} = 1 + \gamma_{K_E} = \frac{R^l}{R^l - \eta \rho_E} \left( 1 + \beta^{-\theta} \left( \frac{(1-\eta)\rho_E R^l}{R^l - \eta \rho_E} \right)^{1-\theta} \right)^{-1} \psi \rho_E \frac{1-\psi}{1-\alpha}
\]

\( \gamma_{K_F} \) Refers to the term appearing in the expression for the equilibrium dynamics of total capital of F firms during the transition, that is,

\[
\frac{K_{Ft+1}}{K_{Ft}} = 1 + \gamma_{K_F} = (1 + \beta) \left( 1 + \psi \left( \frac{1-\frac{N_{E0} (1+\psi) t^{1+1}}{N_0} - \frac{N_{E0} (1+\psi) t^{1+1}}{1+\psi}} {1-\frac{N_{E0} (1+\psi) t^{1+1}}{N_0} - \frac{N_{E0} (1+\psi) t^{1+1}}{1+\psi}} \right) \right)
\]

\( B \) Foreign bonds in assets of the banks

\( Y \) Gross domestic product
\( S \)  
Aggregate savings

\( \rho \)  
The rate of return to capital

\( \kappa^*_E \)  
Steady state value of capital per effective unit of labor for \( E \) firms

\( \rho^*_E \)  
The rate of return to capital in steady state for \( E \) firms
1. Introduction

Understanding the fundamental sources of China’s economic transition since 1992 is the main focus of this paper. Since the late 1970s, there have been only few phenomena as globally significant as China’s remarkable economic transition, which I will in this study henceforth simply refer as to the transition. During 1978, at the beginning of China’s transition, the state-owned enterprises accounted for more than three quarters of China’s industrial output, and by 2014, the state-owned enterprises yielded only one quarter (Zhu 2012). In only a few decades, China has transitioned itself from a poor economy dominated by state-owned enterprises into a dynamic and increasingly market-driven economy.

The main features of China’s transition alongside with astonishing economic growth have been a growing momentum of the private sector and markets, the reallocation of resources within the enterprise sector, sustained returns on capital despite of high capital accumulation, a large productivity growth, raising inequality and moderate wage growth, high savings and investment rates, and the accumulation of an enormous trade surplus. Sectoral swifts within the manufacturing sector have been the main drivers of productivity growth that is to say mainly the reallocation of labor and capital from low-productive state-owned enterprises to high-productive private enterprises (see Hsieh and Klenow 2009 and Brandt, Van Biesebrock, and Zhang 2012).

Besides China’s astonishing economic growth, it is also remarkable how the main features of China’s transition are in contradiction to several core predictions of conventional neoclassical theories. Based on a closed economy growth model, the rate of return to capital would decline due to a high investment rate. Despite of the high investment rate in China, the rate of return to capital has remained high. In an open economy growth model, high domestic return to capital would assume a high capital inflow, and hence a
large foreign deficit rather than surplus. Standard models also predict that capital would flow into the most productive counterparties. In China’s growth experience, financial intermediaries have been investing in low-productive state-owned enterprises rather than high-productive private enterprises.

This thesis has been made as an assignment for *Tekes – Finnish Funding Agency for Technology and Innovation*. In order to comprehensively grasp the patterns of China’s transition, I construct a dynamic growth model consistent with China’s transitional characteristics. The benchmark model was introduced in an article *Growing like China* in the American Economic Review in February 2011 by Zheng Song, Kjetil Storesletten and Fabrizio Zilibotti. The model accounts for China’s high growth rate, high return to capital and accumulation of foreign surplus. The cornerstones of the theory are heterogeneity in productivity between the state-owned and private enterprises, reallocation of resources within the enterprise sector and asymmetric financial imperfections. The reallocation of labor and capital from the state-owned enterprises to private enterprises creates the two specific features of China’s transition: sustained returns to capital and a large foreign surplus.

Private enterprises are financially constrained due to financial imperfections, but possess higher productivity than state-owned enterprises, which are externally financed. Hence private enterprises must rely on entrepreneurial savings to finance their investments. On the other hand, despite that state-owned enterprises have lower productivity, they can survive in the equilibrium due to the favoring of the financial intermediaries that allows a better access to external financing. Because of the inequality of the financial sector, the growth of the private enterprises is being limited to entrepreneurial savings. If the entrepreneurial savings are large enough, the
private enterprises will step by step outgrow the state-owned enterprises from the market.

The model explains the features which are in contradiction with the neoclassical growth theories. During the transition phase nor private or state-owned enterprises are subject to diminishing return to capital. Sustained returns to capital originates from the labor mobility within the enterprise sector, low growth rate and from the financial preference of low-productive enterprises. Moreover, the aggregate rate of return to capital increases rather due to a composition effect. Sustained returns to capital is a feature that is in effect only during the transition. Once the transition phase is over capital accumulation is subject to diminishing return to capital.

The accumulation of the foreign surplus originates from financial imperfections. The wage earners deposit their savings to the banks, which in turn, can either invest to domestic firms or in foreign bonds. As the transition progresses the volume of high-productive financially constrained firms increase while the volume of low-productive externally financed firms decrease. Hence, a higher amount of domestic savings are invested in foreign assets by the financial intermediaries, which in turn, causes the foreign surplus to increase.

This thesis is motivated by several questions. A popular explanation for China’s foreign surplus is China’s exchange rate manipulation (see Goldstein and Lardy 2009). This thesis gives an alternative explanation to China’s foreign surplus. Difficulties working with the Chinese data are widely recognized (see Brandt, Van Biesbroeck and Zhang 2014). The method of understanding China’s transition through a theoretical framework allows to concentrate to the idiosyncrasies of China’s transition and gives a comprehensive general view over the whole phenomenon. Besides of how well does the model represent China’s transition, this thesis addresses the following questions: Why does a country accumulate a foreign surplus
despite of high domestic rate of return? Why does a country’s rate of return to capital remain high in spite of high investment rates?

The structure of the thesis is organized as follows: In chapter two, I review the theoretical and empirical works related to China’s transition and to the model presented. In chapter three, I present a narrative interpretation with empirical evidence of China’s transition. In chapters four to seven, I construct a growth model that captures China’s transition presented in chapter three. In chapter four, I present the basic structure of the model: preferences, enterprise sector, how the wages, savings and investments are formed in the model, the profit maximizations of both state-owned and private firms and the entrepreneurs’ investment problem. In chapter five, I characterize the equilibrium dynamics during the transition. In chapter six, I present the country’s growing foreign surplus along with the country’s increasing gross saving rate and gross investment rate during the transition. In chapter seven, I present the post-transition equilibrium. Chapter eight concludes. I discuss the main results and assumptions of the model and the relationship of the model and China’s transition.
2. Research overview

In this chapter, I briefly overview earlier and recent most significant theoretical and empirical works that have influenced to the model presented in this thesis.

China’s huge saving rates and macroeconomic imbalances has been widely recognized, but a huge part of the academic research focuses often only to the saving rates (e.g. Ma and Wang 2010 and Kraay 2000) or to the connection of trade surplus and exchange rates (e.g. Goldstein, M. and Lardy, N. 2009). This thesis is a part of the literature that offer understanding for both saving rates and external imbalances in China (e.g. Yang 2012). In order to find the reason why China is accumulating a large foreign surplus, I underline, as do most papers, China’s high savings rate. Yang (2012) argues that the main reasons for China’s high savings rate and current account surplus are the combination of policies, institutions and structural distortions. Following the same explanation, Kraay (2000) emphases the role of imperfect financial sector as one of the factors for China’s high saving account. Interestingly, Kraay highlights the banking sectors’ poor-performing loans to state-owned enterprises when addressing the concerns created by the growth in deposits in the banking system creates.

Yang (2012) also underlined the national income identity, which clarifies the link between domestic and international linkages; the national savings not invested or consumed domestically are invested abroad. Given the high rate of return to capital in China the following puzzle arises: Why are the domestic savings not invested domestically? Yang argues that the inefficient financial system has failed to channel domestic savings to high return investments or consumption loans, the excess savings have been invested by Chinese banks mainly in low-yielding U.S. government bonds.
The model in this thesis has been influenced by the recent literature claiming that resource misallocation creates the low aggregate total factor productivity (TFP) particularly in developing economies (see Restuccia and Rogerson 2008; Hsieh and Klenow 2009 and Brandt, Van Biesebrock and Zhang 2012). Especially Hsieh and Klenow (2009) and Brandt et al. (2012), who have both focused on China, presented how productivity growth has been affected by resource reallocation in the manufacturing sector.

Although in the model presented in this thesis the main focus is in the reallocation within the enterprise sector, it has, in some respects, similar mechanism as in the Lewis (1954) model. Lewis model is based on the idea of dual economy; the focus of the Lewis’ model is overpopulated countries (i.e. developing economies), where the development process is explained by the movement of low-productive underemployed workers from traditional sectors to modern sectors. Compared with the modern sector, the attributes of the traditional sector are that it has lower living standards (i.e. subsistence sector) and the marginal product of workers is negligible (or even negative). Vice versa the modern sector (i.e. capitalist sector) has higher output per worker. At the beginning of the development process, due to the unlimited supply of labor, the modern sector can expand without raising wages, which creates higher returns to capital. In conclusion, the expansion of modern sector creates growth. (Gollin 2014.)

Moving in similar areas as in the model presented in this thesis, giving explanation to the East Asian Miracle, Ventura (1997) provided a model, where he explains how economies characterized by high saving rates are not under the influence of diminishing returns. Unlike in the model of this thesis, where TFP grows in each industry and countries suffer from initial inefficiency, the intuition in Ventura’s model goes as follows: capital accumulation is not subject to decreasing returns, because as the capital stock increases, resources are reallocated from labor-intensive to capital-
intensive sector, and hence the demand for capital raises. Preventing the fall of prices, external trade moves the excess production of capital-intensive goods into exports.

From the point of view of trade imbalances, the model presented in this thesis is related to the model by Antrás and Caballero (2009). Although the mechanism is different, in the model presented in this thesis, less-efficient firms can survive due to a better access to the credit markets. Antrás and Caballero showed that when trade frictions are large, capital will run from the less financially developed economy to the financially developed economy. They also show that the capital movement will increase the comparative disadvantage sector in the financially developed economy, shrinking the comparative advantage sector in the less financially developed economy, hence reducing the trade flows across economies. A financially underdeveloped economy is able to allocate resources in sectors in which financial frictions are less problematic. Vice versa, if trade frictions are small, then capital will run from the financially developed economy to the comparative advantage sector of the financially underdeveloped economy, and boost the trade flows across economies.

The inconsistency between the standard neoclassical growth model’s predictions and the empirical evidence of the capital flows across developing countries is well documented by Gourinchas and Jeanne (2013). Based on the standard open economy neoclassical growth model, countries with high marginal product of capital and higher productivity growth should invest and attract more foreign capital. Gourinchas and Jeanne showed that there is a net capital outflow rather than inflow in fast-growing developing countries. The growth model presented in this thesis gives a rationalization to the above mentioned discrepancy phenomenon, also known as the allocation puzzle.

Buera and Shin (2016) gives an alternative approach on explaining why capital tends to flow out instead of floating in to the economies characterized
with fast-growing productivity. Similarly to the model presented in this thesis, they highlight the role of domestic financial distortions. In their model the heterogeneity of producers and the distortions of domestic financial markets are in the key role of explaining the joint dynamics of the rise of TFP and the directions of capital flows. The efficient reallocation of resources, caused by economic reforms, rises TFP, but due to financial market distortions, the saving rates expands unlike investment rates, which reacts with a lag creating the capital to flow out of the country. In explaining the rise of domestic saving rates, Buera and Shin highlight agents who save to cover the costs of becoming an entrepreneur.

The model has similarities to the model made by Caselli and Gennaioli (2013), where they examined the corporate-arrangements between rich and poor countries. They presented how the poor countries rely more on dynastic family firms, where ownership and control are passed from generation to another. They show that dynastic management reduces firm’s TFP if the heir of the family has no talent in managerial skills. Although in the model of this thesis the managers of private firms have the opposite talents (i.e. superior management skills), the similarities are indisputable.

China’s structural transformation has been examined a in growing amount of literature. Dekle and Vandenbroucke (2012) created a two-sector neoclassical growth model to examine quantitatively China’s structural transformation between 1978 and 2003. Despite their focus on the reallocation of resources from the agricultural sector to the non-agricultural sector, they found that the sectoral productivity growth and gradual reduction of the Chinese government had a notable role on the Chinese transformation.

Focusing more on the financial frictions between private and state-owned enterprises, Curtis (2016) presented a model with two sectors in production to represent China’s financial frictions between private and state sectors. As
in the model presented in this thesis, the expansion of private firms and the reduction of state-owned firms contributes to the growth in TFP. Also, with similar assumptions, private firms are more productive and financially constrained while state-owned firms are less-productive and not financially constrained. The model created by Curtis gives additional insights of China’s TFP dynamics.

The microfoundations of the model presented in this thesis have been influenced by the model created by Acemoglu, Aghion, Lelarge, Reenen and Zilibotti (2007). They analyzed the relationship of decentralization decision of firms and the diffusion of new technologies. As in the model of this thesis, firms can choose between centralized or decentralized control. Decentralized control delegates the decision making authority to a manager who possesses superior information. However, the manager can use this advantage to make decisions that are in contradiction with the principals’ preferences.

Moreover, the model presented in this thesis acts as a benchmark model for further extensions and modifications. Song, Storesletten and Zilibotti (2014) modified the model by removing the laissez-faire environment by adding capital controls and regulations of the financial system to study how they affect to the key measures such as the growth in wages and productivity and trade surplus. In their modification, the economy is a non-monetary small semi-open economy. Consumers demand two goods that are produced by domestic firms or abroad. F firms have access to external financing. Contrary to the original model, domestic savers, firms and banks cannot access the international credit market due to the capital controls and only the government can hold positive or negative debt positions with rest of the world.

Also, Storesletten and Zilibotti (2014) modified the model by introducing labor market frictions in addition to financial frictions. In this extension, labor costs differ between E and F firms due to a labor wedge affecting E
firms. Prior to the economic reforms, the wedge is assumed to be immense, so that there is no employment in E firms. Once the economic reforms begin, the wedge starts to diminish. The transition is ignited by the change in the labor wedge in E firms. Moreover, in their extension, banks do not lend to E firms. In the original model, E firms can borrow from the banks up to a limit. Similarly to the original model, F firms survive due to better access to external funding and during the transition E firms out-grow F firms.
3. Features of China’s economic transition

In this chapter, I present a narrative interpretation of China’s economic transition along with empirical evidence. The main focus of this thesis is post-1992, but due to the fundamental understanding of China’s transition, I start the examination from 1978. First I shortly go through the main macroeconomic trends of China’s transition in a chronological fashion. In the second part I examine the resource reallocation within the enterprise sector, productivity differences between state-owned and private firms, wage structure, and the rising inequality. In the third part I examine the financial sector imperfections along with institutions. Finally I examine accumulation of foreign imbalances and aggregate savings and investment rates.

3.1 Economic reforms since 1978

China’s first economic reforms started in December 1978, when the government announced a general policy of reform and opening up the society. The economic reforms occurred more in an experimental and gradual way and at least initially there was no grand design. (Zhu 2011.)

China has separated its economy into two segments: rural and urban. Labor mobility between rural and urban sectors was extremely slight before the economic reforms started in the late 1970s. The reason why there was de facto no rural to urban mobility was due to the household registration practice so-called hukou system. The economic reforms for rural and urban sectors began in different periods. (Meng 2012.)
The economic reforms started from the agricultural sector, influenced by several food crises in the past. The early reforms accelerated the role of township and village-level governments, e.g. township and village enterprises (TVEs), which created the rural industrial enterprises. As mentioned earlier, the first rural reforms occurred gradually; the reforms were completed in 1984. In the agricultural sector, the TFP grew 5.62 percent per year between 1978 and 1984, and as a result, agricultural output grew by 47 percent during the same period. Also, during the same period, the total employment of agriculture fell from 69 percent into 50 percent. (Zhu 2011.)

Alongside with the agricultural reforms, China’s government established Special Economic Zones (SEZs), which had an important role in China’s transition. The establishment of SEZs opened up the economy to the foreign direct investments and to the distribution of technological knowledge. Characteristics of SEZs were lower corporate tax, subtraction of custom duty, lower land prices and flexibility in financial contracts and labor. Owing to the initial success of SEZs, they progressively expanded; starting from four SEZs, in 1984 SEZs expanded to 14 coastal cities, later to three delta areas and two provinces, and lastly gradually in 1992, 1998 and 2005, SEZs expanded to first to inland capitals and then to medium-sized cities. (Storesletten and Zilibotti 2014.)

As a result of the increased productivity due to the rural economic reforms between 1978 and 1984, the rural unemployment also increased. By the mid-1980s the rural unemployment had become a growing problem. The first solution for the rural unemployment issue was to encourage to establish TVEs. After the establishment of SEZs, the demand for services and unskilled labor rose in SEZs and started a limited rural-urban migration. However, during the 1980s and early 1990s, the rural to urban migration was remarkably restricted. After the mid-1990s demand for unskilled labor rose
significantly due to the economic growth in cities and especially after China joined to the World Trade Organization (WTO) in 2001, the demand bursted and hence migration restrictions were notably attenuated. In 1990, 1997 and 2009 there were 25 million, 37 million and 145 million rural migrant workers in cities, respectively. (Meng 2012.)

During the mid-1980s urban labor market reforms began. The reforms were mild before the following two events occurred: in the 1980s the urban economy experienced notable unemployment as the people who were sent to the rural areas to work during the Cultural Revolution in the 1960s started to return to the cities in the 1980s. In response, the government started to encourage self-employment. After these events, the urban hukou labor markets began to reform. (Meng 2012.)

In the mid-1990s more than 40 percent of state-owned enterprises were making losses (Meng 2012). However, besides the direct economic impact, the SEZs reinforced the economic reforms and hardened the policies of the opening up of China’s markets. In the 1980s the majority of the country still remained under the influence of the centralized planning system. A new stage of reforms began during Deng Xiaoping’s Southern Tour in 1992, where the paramount leader emphasized deeper economic reforms (Storesletten and Zilibotti 2014). This lead to momentum in 1997, when the 15th Congress of the Communist Party of China officially placed state-owned enterprise reforms and endorsed the development of private enterprises. Following the new set of policies, private enterprises grew promptly (Zhu 2012). A new policy called “Hold on the large, let go of the small” was introduced. The purpose of the new policy was state-sector restructuring; the aim was to retain 1 000 largest state-owned enterprises and to force the smaller state-owned enterprises to compete in the marketplace or to declare bankruptcy. Examining the industrial output value it can be seen that the state and collective sector’s share decreased gradually; in 1990 the share was
over 90 percent, in 1997 the share had decreased into 70 percent and in 2008 the share was only 30 percent (Meng 2012).

In 1978 only 0.02 percent of urban hukou labor force was self-employed. Practically all urban labor force was working in either state or collective sectors (Meng 2012). The features throughout the state sector were lifetime employment and benefits to urban workers. From 1995 to 2001, during the state-sector economic reforms, employment in state-owned enterprises fell from 113 million to 67 million and in the urban collective sector from 31.5 million to 12.9 million. During the same period of the total 43 million workers who were registered as laid-offs 34 million were from the state sector (Giles, Park and Cai 2006).

The pre-WTO and post-WTO reforms reduced trade barriers and tariffs, liberalized trade and foreign direct investment regimes and improved overall business environment (Branstetter and Lardy 2006). Zhu (2012) presented that the average annual TFP growth rates for state and non-state sectors were 5.50 percent and 3.67 percent between 1998 and 2007, respectively. Both privatization and trade liberalization improved the productivity of state and non-state sectors.

China’s enterprise and financial sector reforms have occurred in gradual pace. The Chinese stock market growth after 1991 is an essential part of the financial development. The development of Chinese stock market is highly connected with enterprise sector reforms and must be looked at the context of partial privatization of SOEs. SOE reforms in the 1980s aimed to decentralize the managerial decision rights of central government in SOEs. In the 1990s, as part of the partial privatization, SOEs issued minority shares for individual investors; in 1990 Shanghai stock market reopened and in 1991 Shenzhen stock market opened enabling individual investors to trade their minority shares on the recently developed stock markets. (Hsu and Simon 2016.)
As part of the WTO agreement the foreign entry into China’s financial sector began in 2006, but the foreign bank activity remains vastly restricted (Hsu and Simon 2016). In 2013 the Shanghai Free Trade Zone was established as a part of the financial liberalization (Storesletten and Zilibotti 2014). The goal of the Shanghai Free Trade Zone is to ease and boost business (Hsu and Simon 2016).

3.2 Resource reallocation, productivity differences and income inequality

There are two fundamental differences between Chinese domestic private enterprises (DPEs) and state-owned enterprises (SOEs) that are the focus points of this thesis. The first one is the productivity difference within the enterprise sector; DPEs tend to have higher TFP than SOEs (see Hsieh and Klenow 2009; Brandt et al. 2012). The second is the discrimination in the Chinese financial system; due to financial imperfections DPEs tend to have inferior access to external credit than do SOEs (see Allen, Qian, and Qian 2005; Poncet and Steingrass 2011; Guariglia, Liu, and Song 2011). I will examine the financial imperfections in the next section.

Hsieh and Klenow (2009) provided quantitative evidence of how resource misallocation has impacted the aggregate TFP and how the reallocation within the manufacturing sector has had a wide effect in productivity growth in China. They used microdata on manufacturing plants to measure the potential volume caused by the misallocation of labor and capital in China in comparison of the United States. They found out that reallocation within manufacturing sector has increased the productivity efficiency in China by two percent per year from 1998 to 2005. Moreover, they estimated that if the resources would be reallocated to the level where marginal products would equate to the observed levels of the United States, TFP in China would
increase from 30 to 50 percentages. If capital would accumulate alongside with the aggregate TFP gains, the output would increase approximately twice as much.

Brandt et al. (2012) estimated that between 1998 and 2007 the TFP growth in China’s firm-level manufacturing firms was by average 2.58 percent for gross output production function and by average 7.96 percent for a value-added production function. The panel of firms included approximate 90 percent of gross output in manufacturing. Their results also indicated that more than two thirds of the total TFP growth was due to the dynamic force of creative destruction, i.e. in China’s case, the enormous entry of new firms with the characteristics of higher average productivity and growth rates and the exit of inefficient firms. Moreover, the results confirmed the findings in Hsieh and Klenow (2009), stating that in the Chinese manufacturing sector the aggregate TFP growth is constrained by the small volume of efficiency-enhancing input reallocation between active firms, i.e. resource misallocation.

Figure 1 shows the labor reallocation from SOEs to DPEs: it points out the conversion of the employment ratio between SOEs and DPEs within the enterprise sector. Based on the annual firm-level NBS surveys, the solid line expresses the proportion of DPEs as a percent of DPEs and SOEs. In 1998 the proportion was only 4 percent, in 2007 the proportion had reached to 56 percent, and by 2011 the proportion had exceeded 60 percent. For coherence, the dotted line shows also conversion of the employment ratio with a broader measure of private and state-owned enterprises: it takes into account also the foreign enterprises (FEs) and collectively owned enterprises (COEs) so that the measurement is as follows: the employment proportion of DPEs and FEs as a percent of DPEs, FEs, SOEs and COEs. Both lines show the same pattern: the huge decrease of SOEs and increase of DPEs. This is consistent with the events described in the previous section.
Figure 1: China’s DPE employment shares in the manufacturing sector between 1998 and 2011

Source: Storesletten and Zilibotti (2014), p. 339.²

It has been widely recognized that DPEs tend to be on average more productive than SOEs. Hsieh and Klenow (2009) showed in their estimation that the revenue-productivity gap between DPEs and SOEs is 41.5 percent. Song et al. (2011) present a productivity difference of 9 percent on average between DPEs and SOEs between 1998 and 2007 in their measurement of profitability. They measured the ratio between total profits and the book value of fixed assets.³ Also based on different TFP calculations Islam, Dai, and Sakamoto (2006) and Dollar and Wei (2007) found that SOEs had lower productivity than DPEs.

² Storesletten and Zilibotti (2014) used the data from various issues of the China Statistical Yearbook.
³ Song et al. (2011) used the data from various issues of CSY.
Based on the calculations by Bai, Hsieh, and Qian (2006) the aggregate return to capital in China was approximately 20 percent in 1979 and 25 percent in 1992. Between 1993 and 1998, the aggregate rate of return to capital fell from approximately 25 percent to 20 percent. Between 1998 and 2005, the aggregate rate of return was roughly around 20 percent, despite the huge increase in the investment rate at the same time (8 percent point increase). However, they show that since the early 1990s the rate of return to capital in primary (agricultural) and tertiary (services) sectors has been decreasing while the secondary (construction, mining and manufacturing) sector has increased and reached more than 30 percent in 2003. Liu and Siu (2011) estimated that SOEs have significantly lower return on capital investments than non-SOE. Their findings stated that internal rate of return of DPEs is 10 percent higher than SOEs.5

Moderate wage growth has been one of the features of China’s transition. The real annual earnings for urban hukou workers grew from 3 880 yuan in 1988 to 19 674 yuan in 2009 (Meng 2012). Despite of the continuous wage growth, the actual wage growth has been de facto moderate during this time. Ge and Yang (2014) estimated by using a sample of national Urban Household Surveys that between 1992 and 2007 the annual growth of wages in the manufacturing sector was 7.6 percent, while the annual GDP growth rate per capita during the same period was 9.7 percent. Ge and Yang identifies in their decomposition analysis that the main sources of wage growth in China have been higher wage for basic labor, increased returns on human labor and the rise in state-sector wage premiums. Moreover, Ge and Yang showed that while the average real wage in urban areas grew by 202 percent, the wage growth is not only explained by the growth in wage for unskilled labor; at the same period, the growth for middle-school educated

4 Bai, Hsieh, and Qian (2006) used market prices, not constant prices, in calculating the capital stock.
5 The data Liu and Siu (2011) used is based on Industrial Survey data collected by NBS.
workers grew 135 percent, while the growth for college-educated workers was 240 percent. Also the employment change between 1992 and 2007 was -16.7 percent for middle-school educated workers and 16.9 percent for college-educated workers. This implies that the share of educated labor has clearly risen.

Meng (2012) presented that the Gini coefficient for annual wages grew from 0.26 in 1988 to 0.38 in 2009. Amongst the urban population the income inequality has escalated substantially. The wage structure has equally had notable changes. During the 1990s, the difference between the state sector’s and private sector’s wages were substantial; the state sector paid notably less than the private sector, but the ratio reversed in the 2000s as a part of the governments struggle against corruption. Meng (2012) presented that since 2002, the state sector has been paying along with other benefits 20 percent higher wages than private sector. In China, the moderate wage growth along with the increasing level of entrepreneurs has affected to income inequality (Storesletten and Zilibotti 2014).

3.3 Asymmetric financial imperfections and institutions

China’s financial sector is highly dominated by a small number of state-owned banks, therefore the industry is moderately oligopolistic. Private banks possess only a small role in China’s financial system. China’s four largest banks, all state-owned, compromise 93 percent of the banking sector’s market capital and 40-50 percent of the bank loan market share. The foreign subsidiaries control only approximately 1 percent of the market share. Hence the four largest banks have more market power over the financial industry than do thousands of other financial institutions. (Hsu and Simon 2016.)
The four largest banks are not as efficient or profitable as other types of banks. After 2007, the financing constraints have created a shadow banking sector. This sector gives alternative methods of financing to institutions that are discriminated by the formal banking sector. Through the shadow banking sector non-state firms can acquire loans, but risks in the shadow banking sector are higher than in the formal banking sector. (Hsu and Simon 2016)

The Chinese stock market is dominated by state-owned firms. The equity value is the second largest in the world, but the market prices are inefficient as the price bubble of 2015 indicated. The capital controls deteriorate the interaction between the Chinese financial sector and the international financial sector. (Hsu and Simon 2016) In a nutshell, the Chinese financial sector continues to be underdeveloped.

Alongside the financial system, the legal system and institutions both remain underdeveloped. Allen, Qian, and Qian (2005) clarified that China’s investor protection systems, corporate governance, accounting standards, rule of law, corruption and quality of government are all underdeveloped. They also find that a notable imperfection in the banking sector is the amount of non-performing loans from the four largest banks; a significant fraction of these loans were made for SOEs from political or non-economic reasons.

Allen et al. (2005) presented that domestic bank loans, firms’ self-fundraising, state budget and foreign direct investment important financing sources from which bank loans and self-fundraising are the most important sources. Between 1994 and 2002 self-fundraising accounted for approximately 60 percent of the total funds raised by the private sector. Self-

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6 For simplicity in the model presented in this thesis the government is assumed not to have an active role in altering the exchange or interest rates. The focus of this thesis is not to examine the exchange rate policy, but to give an alternative explanation for China’s macroeconomic imbalances. See Yang (2012) for more detailed analysis why China’s exchange rate policy is not amongst the most important factors causing China’s current account surplus.
fundraising consist of internal finance, capital raised from friends and family of the managers and founders, and capital raised in form of loans and private equity. In credit markets private enterprises face severe discrimination. Ending up with similar conclusions Poncet and Steingrass (2011) examined the magnitude of credit constraints by ownership type using a micro-level data set between 1998 and 2005. They also find that DPEs are severely credit constrained, while SOEs and FEs are not. Similarly, Guariglia, Liu and Song (2011) find evidence of credit discrimination against private firms by using firm-level data between 2000 and 2007.

3.4 Accumulation of net foreign surplus and aggregate savings and investment rates

Based on World Bank data, the foreign reserves of China were 25 billion USD in 1992, 147 billion USD in 1997, 1 546 billion USD in 2007 and 3 405 billion USD in 2015. Also the current account surplus of China was 6 billion USD in 1992, 37 billion USD in 1997, 421 billion USD in 2008 (at the peak) and 331 billion USD in 2015 (The World Bank 2016). China’s enormous current account and net foreign surplus has been widely recognised and researched by economists (e.g. Yu 2007; Yang 2012). Even though there is a vast amount of research around China’s external imbalances, the attempts to discover the causes of the trade surplus have not reach a consensus.

Gourinchas and Jeanne (2013) showed that on average there is a net capital outflow rather than inflow (i.e. accumulating a foreign surplus) in countries characterized by fast TFP growth. In order to explain why capital tends to flow out instead of an inflow in to economies characterized by fast TFP growth, Buera and Shin (2016) highlight the role of domestic financial
distortions. They presented that TFP rises due to the efficient reallocation of resources. Financial market distortions cause the saving rates to expand. Investment rates react with a lag thus creating the capital to flow out of the country and accumulating a foreign surplus.

Figure 2: Foreign reserves and domestic bank deposits minus domestic loans and between 1992 and 2011 in China


Song et al. (2011) offer a structural explanation for the accumulation of foreign reserves and China’s financial frictions. As presented in figure 2, the timing of accumulation of foreign reserves follows closely with the difference between deposits and loans. Moreover, as presented in figure 1,

7 Storesletten and Zilibotti (2014) used the data from various issues of the China Statistical Yearbook.
the growth of DPEs (and the decrease of SOEs) follows the same pattern. The intuition comes from national identity: decrease of financially constrained SOEs causes the foreign surplus to increase due to a higher amount of domestic savings being invested in foreign assets.

Figure 3: Aggregate savings and investment rates between 1992 and 2008

![Aggregate savings and investment rates](image)


As presented in figure 3, the large aggregate investment rates have been outweighed by even higher aggregate savings rate. After 1997 the aggregate savings rate has explicitly remained in a higher level. Yang (2012) and afterwards Storesletten and Zilibotti (2014) showed that the decomposition of aggregated savings and investment rates revealed that the largest net supplier of savings was the households sector. Moreover, since 1992, the net demand of external funds from firms (corporate investments minus savings) has had a declining trend. This indicates that a larger proportion of corporate investments has been financed through retained earnings rather than from the financial sector, i.e. from household savings which are mediated through financial sector (Storeletten and Zilibotti 2014).

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8 Yang (2012) used the data from Statistical Yearbook of China (NBS 2009).
Song et al. (2011) brings up a clarifying statistical relationship between the reallocation process in manufacturing and accumulation of foreign surplus and productivity growth. The progression of the accumulation of foreign reserves and the transition from SOE to DPE follows a similar pattern; around the year 2000 both of them accelerated (also seen from figures 1 and 2). Song et al. also pointed out that the net surplus, i.e. savings minus investments, across provinces offers the same pattern in the cross section. Provinces, which were characterized with large growth in the DPE employment share, had uniformly a larger net surplus. Moreover, they point out that provinces, where the employment share has grown in DPEs, the labor productivity has also grown faster.
4. Basic structure of the growth model

In the following four chapters, I construct a dynamic exogenous growth model developed by Song, Storesletten, and Zilibotti (2011). The growth theory is consistent with the features of China’s transition presented in previous chapter. The microfoundations which allow the heterogeneous productivity to exist in equilibrium follows Acemoglu, Aghion, Lelarge, Reenen and Zilibotti (2007). The main features of the growth theory are the firm-level heterogeneity in productivity, reallocation of resources, and financial sector’s inequality. Also one of the key prediction of the model is the growing foreign surplus during the transition.

Following the common custom, uppercase characters are aggregate variables and lowercase characters are firm-level or per capita variables. All notations used in the model are listed and explained at the beginning of this thesis. Also, for simplicity, the time subscripts are omitted from the model when it creates no confusion. I present the model in four parts at the following order: first I present the environment of the model, second the equilibrium during the transition, third the accumulation of foreign surplus, national savings and investments, and financial development and fourth the post-transition equilibrium.

In this chapter I present the model’s environment by first presenting the preferences of the agents. Then I present the enterprise sector and production technologies along with the microfoundations of the theory which explains the heterogeneous productivity between state-owned and private firms. In the third part of this chapter, I examine how the wages, savings, and investments are formed in the model and finally I examine the profit maximizations of both state-owned and private firms and the contract between entrepreneurs and banks, i.e. the entrepreneurs’ investment problem.
4.1 Preferences

In the model, the economy is modeled by overlapping generations (OLG) of agents who live two-periods: in the first period, the agents work and are referred to be young, and in the second period, the agents consume savings and are referred to be old. In order to model the preferences of agents, the preferences are presented as a time-separable utility function as follows:

\[
U_t = \frac{c_{1t}^{1-\frac{1}{\theta}} - 1}{1 - \frac{1}{\theta}} + \beta \frac{c_{2t}^{1-\frac{1}{\theta}} - 1}{1 - \frac{1}{\theta}},
\]

where \(c_{1t}\) is the consumption in the first period, \(c_{2t}\) is the consumption in the second period, \(\beta\) is referred to be the discount factor and \(\theta\) is counted as the intertemporal elasticity of substitution in consumption \(c_t\). In the model, the concentration is on the case where \(\theta \geq 1\) so that the rate of return is non-decreasing for the agents’ savings.

Population growth rate, which is assumed to be exogenous, is represented by \(v\). This encases the demographic changes in China, especially urbanization and the rural to urban migration movement. The population growth in group \(N_t\) is expressed as follows:

\[
N_{t+1} = (1 + v)N_t.
\]

The heterogeneous skills that agents possess are inherited from parents to children. Each set consists of two different groups of agents: in group \(N_t\) agents are counted as workers with no entrepreneurial skills and in group \(\mu N_t\) agents are counted as entrepreneurs with entrepreneurial skills. This
division of agents into two separate groups is essential for the creation of the enterprise sector where the division follows the same logic.

### 4.2 Enterprise sector and production

In the model, the enterprise sector is divided into two categories. The first category is entrepreneurial firms, which is referred as E firms. This category is owned by old entrepreneurs (defined in the previous section), who are residual claimants of the profits; they hire only their own children as managers (i.e. young entrepreneurs). As mentioned in the previous section, the managerial skills, which are superior in E firms, are transferred from parents to children (i.e. from old entrepreneurs to young entrepreneurs). This allows E firms to use more productive technologies, and hence, be referred as high-productivity firms.

The second category is state-owned and financially integrated firms, which are referred as F firms. They are owned by intermediaries (i.e. centralized organization, to be defined subsequently in this section), which acts as a negative advantage for F firms and causes a productivity drop. Therefore, F firms are referred as low-productivity firms. However, financial and contractual imperfections cause a preferential position for F firms, which have full access to mainly state-owned banks. In the model, international financial markets and banks are assumed to be perfectly integrated. F firms are assumed to be competitive profit-maximizing firms and act as standard neoclassical firms.

Two of the key assumptions of the theory are (1) the financial sector’s inequality and (2) heterogeneous productivity across firms. As a result, E firms have a higher productivity but they are financially constrained, and this allows F firms, which are financially integrated but have a lower productive, to survive in equilibrium. Hence, low-productivity firms survive in
equilibrium in consequence of having access to external funds. Moreover, the labor market is frictionless and competitive.

The following microfoundations, which allow firms with heterogeneous productivity to exist in equilibrium, follows Acemoglu, Aghion, Lelarge, Reenen and Zilibotti (2007). Firms can choose between two modes of production which behaves differently for E and F firms (to be defined later). In the first production mode, firms can hire a manager to delegate the decision making authority. In the second production mode firms can retain the direct control and keep the strategic decision making authority in house.

If firms choose to delegate and hire managers, the benefits are higher productivity, and hence, delegation will lead to greater TFP. This can be interpreted by the superior information that the manager possesses. This advantage can be expressed by $\chi$ extra efficiency units per worker. Note that in order delegating authority to gain higher productivity compared to firms with centralized control we presume that $\chi > 1$.

The production mode differences between E and F firms creates an agency problem that arises from delegation. The managers have incentive to steal a positive share $\psi$ of the firm’s output. This self-interest driven behavior can be prevented by paying managers a compensation $m_t$ that would be at least as large as the amount that the managers could steal. F firms are not as efficient in monitoring the managers as E firms. This leads to the assumption that managers can steal a share of $\psi < 1$ of E firms’ output. In the case of F firms, due to weak corporate governance of F firms, the managers end up stealing all output if F firms choose to delegate. Therefore it is optimal for E firms to choose delegation whereas F firms choose always centralized control. Hence, F firms would not exist in equilibrium without their better access to the financial markets.

The production function for F firms is as follows:
where the output for F firm’s is given as $y_{Ft}$, the capital for F firm’s is given as $k_{Ft}$, the labor for F firm’s is given as $n_{Ft}$, the technology parameter is given as $A_t$ (i.e. $A_t$ is a TFP parameter), and the output elasticities of capital and labor are given as $\alpha$ and $1-\alpha$, respectively. Following the same logic with the inclusion of extra efficiency units per worker, the production function for E firms is as follows:

\begin{equation}
\begin{split}
y_{Et} &= k_{Et}^{\alpha}(\chi A_t n_{Et})^{1-\alpha},
\end{split}
\end{equation}

where the output is given as $y_{Et}$, capital is given as $k_{Et}$, labor is given as $n_{Et}$, and the extra efficiency unit per worker is given as $\chi$. The technology and output elasticity parameters are the same for both firm types. After one period, the capital depreciates fully. Notice that for E firms the manager’s contribution leads to an extra efficiency units per worker $\chi$, but for F firms the manager’s contribution is included in $n_F$ and the manager is therefore equivalent to workers. The technology $A$ grows at an exogenous rate $z$. Hence, the technological progress evolves according to an exogenous law of motion

\begin{equation}
A_{t+1} = (1+z)A_t.
\end{equation}
4.3 Wages, savings of the workers and banks

I now examine the interpretation between young workers’ and entrepreneurs’ savings and banks. For young workers and banks the relationship goes as follows: young workers earn a wage $w_t$ which can be used either on consumption or savings. The savings young workers deposit to competitive intermediaries, i.e. banks, which in turn, yields the interest factor $R^d$ for the workers.

As in a standard two-period OLG model, where agents work in period 1 and live off savings in period 2, the budget constraint for period 1 is as follows:

$$c^{W}_{1t} + s^{W}_{t} = w_t,$$

where $c^{W}_{1t}$ is the workers’ consumption in period 1 and $s^{W}_{t}$ are the workers’ savings in period 1. The budget constraint for period 2 is as follows:

(4) $$c^{W}_{2t} = R^d s^{W}_{t},$$

where, equivalently, $c^{W}_{2t}$ is the workers’ consumption in period 2. Hence, combining the budget constraints in both periods, one gets the intertemporal budget constraint

(5) $$c^{W}_{1t} + \frac{c^{W}_{2t}}{R^d} = w_t,$$
where $R^d$ is the interest factor paid by the competitive intermediaries for agents’ savings. The conduct of equation (5) is presented in Appendix 1.

The optimal savings for workers is the solution to the following maximization equation, where young workers maximize their utility, $U_t$, (1):

$$
\max_{c_{1t}^w, c_{2t}^w} \frac{c_{1t}^{(w)1-\frac{1}{\theta}} - 1}{1 - \frac{1}{\theta}} + \beta \frac{c_{2t}^{(w)1-\frac{1}{\theta}} - 1}{1 - \frac{1}{\theta}}
$$

subject to an intertemporal budget constraint (5). The Lagrange function corresponding to young workers’ maximization problem is

$$
\mathcal{L} = \frac{c_{1t}^{(w)1-\frac{1}{\theta}} - 1}{1 - \frac{1}{\theta}} + \beta \frac{c_{2t}^{(w)1-\frac{1}{\theta}} - 1}{1 - \frac{1}{\theta}} + \lambda \left( w_t - c_{1t}^w + \frac{c_{2t}^w}{R^d} \right),
$$

where $\lambda$ is the Lagrange multiplier. The maximization problem yields that the optimal savings for young workers are

$$
s_t^w = \xi^w w_t,
$$

where

$$
(6) \quad \xi^w = \left( 1 + \beta^{-\theta} R^d (1-\theta) \right)^{-1}.
$$

The construct of equation (6) is presented in Appendix 2.

The investment relation differs for young entrepreneurs: the managerial compensation $m_t$, which the young entrepreneurs earn, can be invested either in their family business, i.e. E firms or alternatively in bank deposits. I
examine the relationship and intuition between banks and entrepreneurs more closely subsequently.

In the model banks can either invest the collected savings from workers to

1) foreign bonds which yield a gross return of $R$ or

2) loans to domestic firms.

In Walrasian equilibrium, i.e. competitive equilibrium, the following assumption must hold:

$$R^d = R = R^l,$$

where $R^d$ is the gross return on domestic loans, $R$ is the gross return on foreign bonds and $R^l$ is the lending rate to domestic firms. Given this assumption, in competitive equilibrium, the gross return on domestic loans equals to the gross return on foreign bonds, which equals to the lending rate to domestic firms. Lending to the firms exposure banks to an iceberg cost $\xi$, which consists of bureaucracy costs, operational expenses, etc., and can be seen as intermediation efficiency’s inverse measurement. Note that lending rate for domestic firms is as follows:

$$R^l = \frac{R}{(1 - \xi)},$$

and therefore subject to an iceberg cost $\xi$. However, an iceberg cost has no role in this part of the analysis, and hence it is assumed to be $\xi = 0$. In order to examine the financial development more closely, $\epsilon$ would become important, but for simplicity and without loss of generality, it is being assumed to be zero (to be discussed more in section 6.4).
Various contractual imperfections between the banks and entrepreneurs cause hindrances to their relationship. In the model, due to the not verifiable output of E firms, the entrepreneurs can only promise to the banks to repay a share $\eta$ from the net profits of second period. If we would assume that entrepreneurs could commit to repay, it would change the dynamics of the whole model; then F firms would hire old entrepreneurs and all firms would be managed by entrepreneurs.

4.4 Profit maximizations and the contract between entrepreneurs and banks

I now examine the profit maximizations for both F and E firms. In the F firms’ case, the profit maximization states the following: lending rate for domestic firms, $R^l$, equals to the marginal product of capital. Equivalently wages equal to the marginal product of labor. Taking the partial derivative with respect to capital from F firms’ production function (2) gives the marginal product of capital

$$\frac{\partial y_{Ft}}{\partial k_{Ft}} = \alpha k_{Ft}^{\alpha-1}(A_t n_{Ft})^{1-\alpha} = 0.$$ 

Hence, as being stated earlier, $R^l$ equals to the marginal product of capital

$$R^l = \alpha k_{Ft}^{\alpha-1}(A_t n_{Ft})^{1-\alpha},$$

which yields
(9) \[ n_{Ft} = \left( \frac{R^l}{\alpha k_{Ft}^{\alpha-1} A_t^{1-\alpha}} \right)^{\frac{1}{1-\alpha}}. \]

The construct of equation (9) is presented in Appendix 3. Equivalently, taking partial derivatives with respect to labor from F firms’ production function gives the marginal product of labor

\[ \frac{\partial y_{Ft}}{\partial n_{Ft}} = k_{Ft}^\alpha (1 - \alpha) A_t (A_t n_{Ft})^{-\alpha} = 0. \]

Equivalently, as stated earlier, wages equal to the marginal product of labor

(10) \[ w_t = k_{Ft}^\alpha (1 - \alpha) A_t (A_t n_{Ft})^{-\alpha}. \]

Plugging (9) into (10) yields the equilibrium wage

(11) \[ w_t = (1 - \alpha) \left( \frac{\alpha}{R^l} \right)^{\frac{\alpha}{1-\alpha}} A_t. \]

The construct of equilibrium wage is presented in Appendix 4.

In E firms’ case, we need to take into account the fundamental differences between firm attributes. E firms are owned by old entrepreneurs with \( \chi \) extra efficiency units per worker and with capital \( k_{Et} \). The value function is created by diminishing the costs of E firms, which are managerial compensation and wages for workers, from equation (3). Hence the value of an E firm is the solution to the following maximization equation:
(12) \[ \Xi(k_{Et}) = \max_{m_t, n_{Et}} \{ k_{Et}^a (\chi A_t n_{Et})^{1-\alpha} - m_t - w_t n_{Et} \} \].

This is subject to the managerial incentive constraint

\[ m_t \geq \psi k_{Et}^a (\chi A_t n_{Et})^{1-\alpha}, \]

where, as stated before, \( m_t \) is seen as the managerial compensation, and \( w_t \) as the wage for workers, implied by the arbitrage in the labor markets, as defined earlier in (11). Note that the following assumption must stand:

\[ m_t > w_t. \]

Hence, payment to the manager must outweigh the workers’ wage rate. As stated earlier, the managers in F firms make no discretionary decisions. Hence they are not subject to any incentive constraint and thus the managers earn the same wage as workers. As a result there are no separation for wages for F firms’ managers and workers. By redesigning the managerial incentive constraint as follows:

(13) \[ m_t = \psi k_{Et}^a (\chi A_t n_{Et})^{1-\alpha}, \]

the managerial incentive constraint is now binding as the optimal contract implicates. Plugging equilibrium wage (11) and managerial incentive constraint (13) into equation (12) yields the following maximization problem:
\[
\max_{n_{Et}} \left\{ k_{Et}^\alpha (\chi A_t n_{Et})^{1-\alpha} - (\psi k_{Et}^\alpha (\chi A_t n_{Et})^{1-\alpha}) - \left(1 - \alpha \right) \left( \frac{\alpha}{R_t^{1/\alpha}} A_t \right) \right\}.
\]

Taking the first order conditions with respect to \( n_E \) yields

\[(14) \quad n_{Et} = \left( (1 - \psi) \chi \right)^{1/\alpha} \left( \frac{R_t^{1/\alpha}}{\chi A_t} \right)^{1-\alpha} k_{Et}^\alpha (\chi A_t n_{Et})^{1-\alpha}. \]

The construct of equation (14) is presented in Appendix 5. Plugging the equations (13), (11), and (14) into equation (12) yields

\[(15) \quad \Xi(k_{Et}) = k_{Et}^\alpha \chi A_t \left( (1 - \psi) \chi \right)^{1/\alpha} \left( \frac{R_t^{1/\alpha}}{\chi A_t} \right)^{1-\alpha} k_{Et}^\alpha (\chi A_t n_{Et})^{1-\alpha} - \left(1 - \alpha \right) \left( \frac{\alpha}{R_t^{1/\alpha}} A_t \right) \left( (1 - \psi) \chi \right)^{1/\alpha} \left( \frac{R_t^{1/\alpha}}{\chi A_t} \right)^{1-\alpha} k_{Et}^\alpha (\chi A_t n_{Et})^{1-\alpha} - \left(1 - \alpha \right) \left( \frac{\alpha}{R_t^{1/\alpha}} A_t \right)^{1-\alpha} k_{Et}^\alpha (\chi A_t n_{Et})^{1-\alpha}. \]

Modifying the equation (15) yields the value of E firm
(16) 

$$\Xi(k_{Et}) = (1 - \psi)\chi^{1-\alpha}R^l k_{Et} \equiv \rho_E k_{Et},$$

where E firms rate of return to capital is denoted to $\rho_E$. The construct of equation (16) is presented in Appendix 6. Assumption 1 verifies that the rate of return to capital for E firms is higher than the lending rate for domestic firms:

(17) 

$$\rho_E > R^l.$$

**Assumption 1**

*For the transition to occur, a sufficiently large difference in productivity between E and F firms is essential:*

(18) 

$$\chi > \bar{\chi} \equiv \left(\frac{1}{1 - \psi}\right)^{1-\alpha}.$$

Assumption 1 is necessary for E firms; if the assumption would not exist, the model would not work and there would not be any E firms in equilibrium. The assumption indicates that E firms favor delegation over centralization. Given the assumption, it is also ideal for young entrepreneurs to invest in the family business.

I next derive the contract between banks and entrepreneurs. E firms can finance their operations by internal savings or by lending from the banks.
Key assumption here is that the rate of return to capital for E firms is higher than the lending rate for domestic firms, as stated in equation (17). The composition of E firms’ capital stock is as follows:

\[(19) \quad k_{Et} = s_{t-1}^E + l_{t-1}^E,\]

where \(s_{t-1}^E\) refers to the savings of the young entrepreneurs and \(l_{t-1}^E\) refers to the bank loan. The entrepreneur’s incentive-compatibility constraint can be presented as follows:

\[(20) \quad R^l l^E \leq \eta \rho_E (s^E + l^E),\]

where, as stated before, \(R^l\) is the lending rate for domestic firms, \(\eta\) is the share that entrepreneurs can promise to repay to the banks and \(\rho_E\) is the rate of return to capital for E firm. Equation (17) holds, i.e. \(\rho_E > R^l\). Hence the incentive-compatibility constraint is binding if and only if

\[\eta < \frac{R^l}{\rho_E},\]

which is assumed to be true. Using equation (20), the share of bank invested investments for entrepreneurs can be presented as follows:

\[(21) \quad \frac{l_E}{l_E + s_E} = \frac{\eta \rho_E}{R^l}.\]

The construct of equation (21) is presented in Appendix 7.
Consider, next, the entrepreneur’s investment problem. The maximization problem can be presented by finding optimal choices of \( l_E \) and \( s_E \) that maximize the discounted utility function subject to

\[
(22) \quad c_{1t}^E = m - s_E,
\]

\[
(23) \quad c_{2t}^E = \rho_E (l_E + s_E) - R^t l_E,
\]

and the incentive-compatibility constraint (21). Using (21) to substitute \( l_E \) from equation (23), the consumption in period 2 can be expressed as follows:

\[
(24) \quad c_{2t}^E = \frac{(1 - \eta) \rho_E R^t}{R^t - \eta \rho_E} s_E.
\]

The construct of equation (24) is presented in Appendix 8. After substituting \( l_E \), and plugging the equations (22) and (24) into equation (1), the entrepreneur’s investment maximization problem is as follows:

\[
(25) \quad \max_{s_E} \frac{(m - s_E)^{1 - \frac{1}{\theta}} - 1}{1 - \frac{1}{\theta}} + \beta \frac{\left(\frac{(1 - \eta) \rho_E R^t}{R^t - \eta \rho_E} s_E\right)^{1 - \frac{1}{\theta}} - 1}{1 - \frac{1}{\theta}}.
\]

The solution to the maximization problem yields that the optimal savings are

\[
(26) \quad s_E = \zeta^E m_t,
\]
where $\zeta^E$ is defined as follows:

(27) \[ \zeta^E \equiv \left( 1 + \beta^{-\theta} \left( \frac{(1 - \eta) \rho_E R^l}{R^l - \eta \rho_E} \right)^{1-\theta} \right)^{-1}. \]

The construct of equation (26) is presented in Appendix 9.
5. **Equilibrium dynamics during the transition**

In this chapter, I characterize the equilibrium dynamics in the course of the transition. First I present how E firms have a lower capital-output ratio than do F firms. In the second and third part of this chapter I present the equilibrium dynamics first to E firms and then to F firms. Note that at the time of the transition, positive employment is assumed to occur for both F and E firms. In the last part of this chapter, I present how the GDP per worker increases along with the average rate of return to capital during the transition.

As Song et al. (2011) points out the financial frictions affect the flow of capital by obstructing entrepreneurial firms, which are more productive than F firms, from borrowing external funds from banks. This is essential for the model to function. Without financial frictions entrepreneurs could borrow external funding without barriers. This would cause the transition to occur instantly and only E firms would be in equilibrium. With the financial frictions in the model the transition phase will occur in a gradual pace.

### 5.1 Capital-output ratios

The financial system imperfections favor F firms and discriminate E firms, which are pressed to choose a lower capital-output ratio than F firms. Hence in equilibrium, E firms have a lower capital-output ratio than do F firms. To present this, as in common case (i.e. in this model for F firms), I first denote the capital per effective unit of labor as follows:
(28) \[ \kappa_F \equiv \frac{k_F}{(A_F n_F)'}, \]

where technology is given as \( A_F \), labor is given as \( n_F \) and capital is given as \( k_F \) for \( F \) firms. Note that for \( E \) firms capital per effective unit of labor must be expressed as follows:

(29) \[ \kappa_E \equiv \frac{k_E}{(\chi A_E n_E)'}, \]

where one needs to take into account the extra efficiency unit per worker \( \chi \).\(^9\) As presented in equation (8), for \( F \) firms, the lending rate for domestic firms \( R^l \) equals to the marginal product of capital. Therefore by plugging equation (28) into (8), the capital per effective unit of labor for \( F \) firms is as follows:

(30) \[ \kappa_F = \left( \frac{\alpha}{R^l} \right)^{\frac{1}{1-\alpha}}. \]

The construct for equation (30) is presented in Appendix 10. \( F \) firms operate as standard neoclassical firms. Hence, as in open-economy growth models, since \( \kappa_F \) is constant, the equilibrium wage for workers in \( F \) firms, which is presented in equation (11), grows at the rate of technical progress

\(^9\) In the original model by Song et al. (2011), the capital by effective unit of labor was presented only as equation (28) describes. Without losing consistency, I added the equation (29) to clarify the differences between \( E \) and \( F \) firms.
$$w_{t+1} = (1 + z)w_t.$$  

By following the similar process than in constructing the equation (30), plugging equations (29) and (30) into equation (14) yields the capital-output ratio for E firms

$$\kappa_E = \kappa_F \left((1 - \psi)\chi\right)^{-\frac{1}{\alpha}}.$$  

The construct of equation (32) is presented in Appendix 1. After defining the capital-output ratios for both E and F firms, along with Assumption 1, the analysis leads to Lemma 1:

**Lemma 1**

*The capital-output ratio for E firms is lower than capital-output ratio for F firms: $\kappa_E < \kappa_F$, since*

$$\frac{k_E}{y_E} = \kappa_E^{1-\alpha} < \kappa_F^{1-\alpha} = \frac{k_F}{y_F}.$$  

*Moreover, E firms have a lower labor-output ratio than F firms if Assumption 1 is in effect (i.e. $\chi > \chi \equiv \left(\frac{1}{1-\psi}\right)^{1-\alpha}$):*
\[
\frac{k_F}{n_F} = \frac{\kappa_F A}{\kappa_E \chi A} = \left(\frac{\chi}{\chi_A}\right)^{\frac{1-\alpha}{\alpha}}
\]

The construct of equations (33) and (34) are presented in Appendixes 12 and 13, respectively.

5.2 Equilibrium dynamics for E firms

Owing to constant return to scale, lower-case individual-firm variables can be replaced by upper-case aggregate variables. Hence, aggregation gives the following equations

\[
\kappa_{Et} \equiv \frac{K_{Et}}{(\chi_A t N_{Et})}
\]

and

\[
N_{Et} = N_t - N_{Et}.
\]

Since \(\kappa_{Et}\) is constant, by using simple algebra, equation (35) can be expressed as follows:

\[
N_{Et} = \frac{K_{Et}}{(\chi A_t \kappa_{Et})}.
\]

\(^{10}\) I present the correct representation of equation (34). A typo found from the Appendix of Song et al. (2011) is fixed.
Plugging this into equation (36) yields

\begin{equation}
N_{Ft} = N_t - \frac{K_{Et}}{(\chi A_t K_{Et})},
\end{equation}

where \(K_{Et}\) is given by (32). Before starting to examine the equilibrium dynamics more closely, it is clarifying to look at the three main properties of the model:

i. \(K_{Et}\) and \(A_t\) are considered as state variables. The fact that \(K_{Ft}\) is not a state variable stems from equation (30).

ii. For both E and F firms, the capital per effective unit of labor, i.e. \(\kappa_E\) and \(\kappa_F\), is constant.

iii. Entrepreneurial savings are considered to be linear in \(K_{Et}\) in period \(t\), i.e. \(K_{Et+1}\).

The three main properties imply that for E firms the capital, employment and output grows at a constant rate in the course of the transition. In order to continue to the subsequent Lemma, I modify the equation (19) by moving it from period \((t - 1)\) to period \(t\), so that the capital for the next period for E firms is as follows:

\begin{equation}
k_{Et+1} = s_t^E + l_t^E.
\end{equation}

Replacing \(l_t^E\) by modifying and plugging equation (21) into equation (38) and then replacing \(s_t^E\) by plugging equation (26) into equation (38) yields
(39) \[ k_{Et+1} = \left( \frac{R^l}{R^l - \eta \rho^E_t} \right) \zeta^E m_t, \]

where \( \zeta^E \) is determined in equation (27). The construct of equation (39) is presented in Appendix 14. Using equation (13) to replace \( m_t \), and aggregating over all entrepreneurs gives the following:

(40) \[ K_{Et+1} = \left( \frac{R^l}{R^l - \eta \rho^E_t} \right) \zeta^E \psi \kappa^E \chi A_t N_{Et},^{11} \]

The construct of equation (40) is presented in Appendix 15. This leads to the equation (41) and to the following lemma by dividing both sides of (40) by \( K_{Et} \) and using equations (32) and (30) to replace \( \kappa_E \) (i.e. using the equilibrium expression of \( \kappa_E \)):

**Lemma 2**

*In the course of the transition, given \( K_{Et} \) and \( A_t \), E firms’ equilibrium dynamics of total capital is as follows:*

(41) \[ \frac{K_{Et+1}}{K_{Et}} = 1 + \gamma \kappa^E, \]

*and accordingly, E firms’ equilibrium dynamics of total employment is given as follows:*

\[ \]

\[^{11} \text{I present the correct representation of equation (40). In Appendix of Song et al. (2011), } \chi \text{ was missing from the equation. The missing } \chi \text{ is essential in order to calculate the standard neoclassical law of motion in equation (54) in chapter 7 (i.e. Post-Transition Equilibrium).} \]
\[ \frac{N_{Et+1}}{N_{Et}} = \frac{(1 + \gamma_{K_E})}{(1 + z)} \equiv 1 + \nu_E, \]

where

\[ \frac{1 + \gamma_{K_E}}{R^l} = \frac{R^l}{R^l - \eta \rho_E} \left( 1 + \beta^{-\theta} \left( \frac{(1 - \eta) \rho_E R^{l(1-\theta)}}{R^l - \eta \rho_E} \right)^{-1} \right)^{-1} \frac{\psi \rho_E}{1 - \psi \alpha}. \]

where as in equation (16) the rate of return to capital for E firms is

\[ \rho_E = (1 - \psi) \frac{1}{\alpha} \frac{(1 - \alpha)}{\alpha} R^l \]

and as in equation (7), the lending rate for domestic firms is

\[ R^l = \frac{R}{(1 - \xi)}. \]

Note that there exists such \( \hat{\chi} = \hat{\chi}(\beta, \chi, \psi, \eta, \alpha, v, z, R, \xi) < \infty \) so that, for E firms, if and only if \( \chi > \hat{\chi} \), the employment share (i.e. \( \frac{N_E}{N} \)) grows over time (i.e. \( \nu_E > v \)). Note that E firms’ employment share grows if \( z \) and \( v \) are sufficiently small or if \( \beta \) and \( \eta \) are sufficiently large, ceteris paribus. \( \hat{\chi} \) is increasing in \( z \) and \( v \), and decreasing in \( \beta \) and \( \eta \).

The proof of Lemma 2 is presented in Appendix 16. Also, the definition for \( \hat{\chi} \) is presented the Appendix 16. The constant growth rate of \( K_{Et} \) depends on two facts: Firstly, the savings and earnings of young entrepreneurs are
proportional to the profits of E firms. Secondly, E firms’ rate of return to capital is constant.

In order to demonstrate this, the following assumptions need to be made: during the transition, suppose that the technological growth rate $z$ is zero, and by equation (31), the wage for workers would remain constant. The inequality between workers and entrepreneurs rises from the fact that the managerial compensation, $m_t$, depends on the output of E firms and grows proportionally along with the output. Since the entrepreneurial savings are used to finance the investments in E firms, and wages, which are constant (depending on the growth rate of $z$), escape a falling return to investment, the growing earning inequality is necessary for the transition to take place; the entrepreneurial investments would not rise, if young entrepreneurs would not earn any rents but instead only $w_t$.

Note the following condition: the growth rate is hump-shaped in $\psi$. In order to view this, the expression of $\rho_E$ from equation (16) needs to be plugged into equation (43). One can see that

i. if the value of $\psi$ is small, i.e. entrepreneurial rents are minor, the young entrepreneurs are considered as poor and the investments are low; and

ii. if the value of $\psi$ is large, it impacts to E firms’ profitability and growth ($\rho_E$) negatively.

The theory has an interesting phenomenon: it can predict a miscarried take-off. The TFP gap (i.e. $\chi^{1-\alpha}$) is required to be large in the conditions of $\chi$ in Assumption 1 and in Lemma 2 (i.e. $\chi > \chi$, and $\chi > \tilde{\chi}$, respectively). In general, only one of the conditions will be binding. A miscarried take-off can be created in the following situation: In the beginning, both of the conditions are satisfied, but for instance due to fall in $\beta$, the entrepreneurial saving rate
\( \zeta^E \) in equation (27) drops. Then after the shock the conditions would state the following:

\[
\hat{\kappa}^\ast (\cdot, \beta) > \chi > \underline{\chi}.
\]

In this case, the employment share of E firms would decrease while the investments of E firms would increase over time.

## 5.3 Equilibrium dynamics for F firms

I next present the equilibrium dynamics for F firms. During the transition, the equilibrium dynamics for F firms can be presented as follows:

(44) \[ K_{Ft} = \kappa_F A_t (N_t - N_{Et}), \]

where, residually, the workers not being employed by E firms are employed by F firms. Moreover, \( K_F \) adjusts to the optimal capital-labor ratio. As the transition proceeds, the following conditions occur:

i. the growth rate of \( K_F \) declines when the employment share of E firms continues to increase; and

ii. the aggregate capital accumulation is hump-shaped during the transition, hence \( K_F \) grows when E firms’ employment share is small, and as the transition advances, \( K_F \) growth rate declines until it turns negative.

The decline in the growth rate of \( K_F \) follows the subsequent equation:
From this, one can observe that

\[
\frac{d}{dt} \left( 1 + \gamma_{K_{Ft}} \right) = (1 + z) \frac{N_{E0}}{N_0} \left( \frac{1 + \nu_E}{1 + v} \right)^t \left( \ln \frac{1 + \nu_E}{1 + v} \right) (v - \nu_E) \left( 1 - \left( \frac{1 + \nu_E}{1 + v} \right)^t \right)^{-2} < 0 \text{ iff } \nu_E > v.
\]

The construct of equation (45) is presented in Appendix 17. As presented earlier, the aggregate capital accumulation is hump-shaped in the course of the transition; at the beginning, when the proportion of E firms is small and only a small share of labor is employed by E firms, \( K_{Ft} \) grows at a positive but decreasing rate. This requires the population growth rate or technological growth rate to be positive \( (v > 0 \text{ or } z > 0, \text{ equivalently}) \). Along the transition \( K_{Ft} \) grows at a decreasing rate and ultimately turns negative.

### 5.4 GDP per worker and average rate of return

In the course of the transition, the growth rate of GDP per worker increases, provided that the condition \( \chi > \hat{\chi} \) holds. The aforesaid condition implies that the direction of resource reallocation is from less efficient F firms to efficient E firms. Hence this affects positively to the growth rate of GDP per worker, which is presented as follows:
\[
\frac{Y_t}{N_t} = \frac{Y_{Et} + Y_{El}}{N_t} = k_F^\alpha \left( 1 + \frac{\psi}{1 - \psi} \frac{N_{El}}{N_t} \right) A_t.
\]

The construct of equation (46) is presented in Appendix 18.

Moreover, provided that condition \( \chi > \hat{\chi} \) holds, the average rate of return to capital increases in the course of the transition. Note that the rates of return to capital in both E and F firms are constant. Due to a composition effect, the average rate of return to capital is increasing in the economy. Note that owing to the condition that rate of return in E firms is higher than in F firms, the increasing average rate of return indicates that the share of E firms’ capital stock is increasing. The average rate of return during the transition is presented as follows:

\[
\rho_t = \frac{\rho_E K_{Et} + \rho_F K_{Ft}}{K_{Et} + K_{Ft}} = \frac{R^t}{1 - \left( 1 - \chi \left( (1 - \psi) \chi \right)^{-\frac{1}{\alpha}} \right) \frac{N_{El}}{N_t}}.
\]

The construct of equation (47) is presented in Appendix 19.
6. Foreign surplus, national savings and investments and financial development

In this chapter, I present the country’s growing foreign surplus during the transition, along with the country’s increasing gross saving rate and decreasing gross investment rate. These are the focal points of the theory and they are consistent with the empirical evidence presented in chapter 3. First I present the accumulation of foreign surplus then the increasing national saving rate. In the third part of this chapter I present the decreasing national investment rate. In the last part I examine the role of financial development in the model.

6.1 Accumulation of foreign surplus

Consider, next, the relation of country’s financial system and the accumulation of foreign surplus. The balance sheet of banks can be presented as follows:

\[
K_{Ft} + \frac{\eta_p}{R_t} K_{Et} + B_t = \gamma w_{t-1} N_{t-1},
\]

where, following common custom, the banks’ assets are on the left side, while the banks’ liabilities are on the right side. The assets consist of \( K_{Ft} \) which refers to the loans to the F firms (F firms are financially unconstrained, hence the capital of F firms consists solely of loans from the banks), \( \frac{\eta p}{R_t} K_{Et} \) which refers to the loans for the E firms (this originates from equation (21), E firms are financially constrained, hence the capital is a
mixed combination of savings and loans), and $B_t$ which refers to the foreign bonds. Liabilities consist of $\zeta^W w_{t-1} N_{t-1}$ which refers to the savings of workers. After defining the banks’ balance sheet, equation (48) leads to the following lemma:

**Lemma 3**

*Foreign surplus of the economy is presented as*

\[
B_t = \left( \zeta^W \frac{(1 - \alpha) \kappa_F^{-1} - 1 + (1 - \eta) \frac{N_{Et}}{N_t}}{(1 + z)(1 + v)} \right) \kappa_F A_t N_t.
\]

*In the course of the transition, the foreign surplus of the economy per efficiency unit $\left( \frac{B_t}{A_t N_t} \right)$ increases as $E$ firms’ employment share $\left( \frac{N_{Et}}{N_t} \right)$ increases. At the end of the transition, in period $T$, the country's foreign surplus is as follows:*

\[
B_T = \left( \zeta^W (1 - \alpha) \kappa_F^{-1} \left( \frac{N_{Et}}{N_t} \right) \kappa_F A_T N_T. \right)
\]

*At the end of the transition, when all workers are employed by $E$ firms, the country’s foreign surplus will inevitably be positive if $E$ firms remain enough credit constrained, i.e. $\eta$ is small.*

The proof of Lemma 3 is presented in Appendix 20.

Note that the growth rates of a country’s foreign surplus and GDP can create the following phenomenon: if the growth rate of GDP is less than the growth rate of foreign surplus, then the foreign surplus-to-GDP ratio grows over time as follows:
\[
B_t \frac{\zeta^W}{Y_t} = \frac{(1 - \alpha)k_F^{\theta-1}}{(1 + z)(1 + v)} - 1 + (1 - \eta)\frac{N_{EL}}{N_t}k_F^{1-\alpha},
\]

which is increasing with \(\frac{N_{EL}}{N_t}\) if:

\[
\frac{\psi}{1 - \eta(1 + \psi)} < \frac{\alpha(1 + v)(1 + z)1 + \beta^{-\theta}R^{1-\theta}}{(1 - \alpha)R^l}.
\]

The set of parameters is nonempty that satisfy this condition along with Assumption 1 and the condition of Lemma 2. The construct of equation (41) is presented in Appendix 21. Note that in order for this to occur, credit market discrimination and contractual imperfections needs to be serious enough, i.e. \(\psi\) and \(\eta\) would be sufficiently small.

In summary, the country’s growing foreign surplus is caused by the contract of financially integrated F firms; more employment is being reallocated to productive E firms, hence the demand for domestic loans declines along with the decline of F firms, the banks must now focus more towards the foreign bonds. This link can be seen from equation (48). In the model, there lies the potential that E firms would increase their demand for domestic loans, but due to the financial sector’s inequality this would be negligible.

### 6.2 Gross domestic saving rate

In the course of the transition, the country’s gross saving rate grows over time. Gross domestic saving rate is presented as follows:

\[
\frac{S_t}{Y_t} = \frac{\zeta^Ww_tN_t + \xi^Em_t}{Y_t}.
\]
The growth in aggregate saving rate is due to the following circumstances: In F firms’ case, the workers save a fraction of $\zeta^W$ of their earnings which is a constant share $(1 - \alpha)$ of the output. In E firms’ case, the workers of E firms save a fraction of $\zeta^W (1 - \alpha)(1 - \psi)$ of the output and young entrepreneurs save a share of $\psi \zeta^E$. The saving rate of the output equals the following:

$$(1 - \alpha)\zeta^W + \alpha \psi \zeta^E + (1 - \alpha)\psi(\zeta^E - \zeta^W).$$

Since $\zeta^E \geq \zeta^W$, the saving rate of the output out of E firms exceeds the saving rate out of the output of F firms. This is due to the following: The equation (17) states that $\rho^E > R^l$ and the intertemporal elasticity of substitution is restricted to $\theta \geq 1$. Hence the saving rate of young entrepreneurs (defined in equation (27)) is higher than the saving rate of workers (defined in equation (9)), i.e. $\zeta^E \geq \zeta^W$.

6.3 Gross domestic investment rate

The prediction that the aggregate investment rate falls in the course of the transition is being examined in this and in the following sections (6.3). Note that since the growing nature of aggregate saving rate during the transition, the model does not depend on the decreasing investment rate at this point.

For simplicity, $z$ and $v$ are assumed to be zero. Hence, the employment, which is reallocated from F firms to E firms, needs less capital. Aggregate investments decrease in the course of the transition; this result generalizes to positive $z$ and $v$, respectively. Proposition 1 compresses the focal results thus far:

**Proposition 1**

Consider, next, that $\chi > \max \left\{ \chi, \tilde{\chi} \right\}$. Thus, the equilibrium employment in the course of the transition is divided by E and F firms as follows:
\( N_{Et} = \frac{K_{Et}}{(A_t \kappa_F (1 - \psi) \frac{1}{\alpha} \chi^{\frac{(1-\alpha)}{\alpha}})} \)

and

\( N_{Et} = N_t - N_{Et}, \)

where \( \kappa_F \) is presented in equation (30), and \( A_t \) and \( K_{Et} \) are predetermined in period \( t \).

The rate of return to capital in \( F \) firms is \( \rho_F = R^t \). Accordingly, the rate of return to capital in \( E \) firms is \( \rho_E = (1 - \psi) \frac{1}{\alpha} \chi^{\frac{1-\alpha}{\alpha}} R^t \), as expressed in equation (16). Hence, the rate of return to capital is higher in \( E \) firms than in \( F \) firms. Moreover, the rate of return to capital is constant in \( E \) and \( F \) firms.

Lemma 2 shows that \( E \) firms’ capital and employment grows over time. Lemma 3 shows that the stock of foreign assets per efficiency unit grows over time. Equation (50) shows that the foreign surplus-to-GDP ratio grows over time, if \( \psi \) and/or \( \eta \) are sufficiently small, hence contractual imperfections and/or asymmetric credit market are serious enough.

The construct of equation (51) is presented in Appendix 22.

### 6.4 Financial development

In the standard neoclassical growth model, the investment rate falls because of capital deepening. The model of interest predicts also that in the course of the transition the investment rates fall. However, the falling investment rates are not caused by capital deepening, instead falling investment rates are
caused by the composition effect. As the transition proceeds, E firms expand in detriment of F firms; E firms are financially constrained and have a lower capital-output ratio than F firms which are financially unconstrained.

As presented in section 3.4, the Chinese empirical evidence is against a falling investment rate. However, there is a possibility to moderate the theory so that there would be capital deepening for both E and F firms at the course of the transition by reducing the financial frictions. Note that the lending rate for domestic firms is presented in equation (7) as follows:

\[ R^l = \frac{R}{(1 - \xi)}. \]

Although the iceberg cost (i.e. \( \xi \)) has been assumed to be zero so far, it has a significant role in order to construct financial development into the theory: Financial development is modelled to the theory by decreasing \( \xi \) as the transition proceeds. The decrease of \( \xi \) causes the lending rate for domestic firms also to decrease. Decrease in both iceberg cost and lending rate cause the increase in wages in both F and E firms. Equivalently, the capital-output ratios in both F and E firms increase.

The decrease in \( \xi \) can have significant impacts on the model: it can alter the investment rate’s tendency to fall and hence increase the average rate of return to capital. This would slow down the transition in the following ways:

i. the comparative advantage of F firms increases as the wages increase,
ii. entrepreneurs increases the savings in order to attract F firms’ workers and
iii. the rate of return to capital in E firms, \( \rho_E \), reduces along with the entrepreneurs’ saving rate.
The reduction of $\xi$ is not the only method to incorporate financial development into the model. One way would be the model entrepreneurs’ better credit market access. In order to do this would be to reduce $\eta$, which incorporates the share that entrepreneurs promise to repay to the banks from the net profits of the second period. The reduction of $\eta$ would accelerate the transition, but would not have an effect to neither wages nor capital intensity of $E$ firms (i.e. $\kappa_E$). However, as shown in section 3.3, the Chinese empirical evidence does not clearly support of DPE’s improved access to the credit market.
7. Post-Transition Equilibrium

In this chapter I present the post-transition equilibrium. As mentioned earlier, in period T, after the transition phase is over, the economy is dominated by E firms and all F firms have perished, hence all workers are employed by E firms. Also, once the transition phase ends, the theory follows and predicts by the dynamics of the standard OLG-model.

So far the intertemporal elasticity of substitution has been restricted to $\theta \geq 1$. Next, assume a case of log preferences, i.e. $\theta \to 1$. Now, the aggregate capital stock is as follows:

\begin{equation}
K_{Et+1} = \left( \frac{\beta}{1 + \beta} \right) \left( \frac{R^t}{(R^t - \eta \rho_{Et})} \right) \mu t.
\end{equation}

Under the log preferences, also the equilibrium wage is given by

\[ w_t = A_t (1 - \alpha)(1 - \psi) \kappa_{Et}^\alpha, \]

rate of return to capital is given by

\begin{equation}
\rho_t = \rho_{Et} = \alpha (1 - \psi) \kappa_{Et}^{\alpha - 1},
\end{equation}

output is given by

\[ Y_t = A_{Et} N_t \kappa_{Et}^\alpha, \]

and foreign balance is given by
\[
\frac{B_t}{A_t N_t} = \frac{\beta}{1 + \beta} \frac{w_t}{A_t N_t} = \frac{\beta}{1 + \beta} \chi (1 - \alpha)(1 - \psi) \kappa_{Et}^g.
\]

Substituting the equilibrium expression of \(m_t\) and equation (53) into equation (52) gives the standard neoclassical law of motion

\[
(54) \quad \kappa_{Et+1} = \frac{\beta}{1 + \beta} \frac{\psi}{(1 + z)(1 + v)} \frac{R^l}{R^l - \eta \alpha (1 - \psi) \kappa_{Et}^{g-1}} \kappa_{Et}^g. \tag{12}
\]

If

\[
(55) \quad \alpha (1 - \eta) (1 - \psi) > \frac{\beta}{1 + \beta} \frac{\psi R}{(1 + z)(1 + v)},
\]

then the standard neoclassical law of motion given in (54) ultimately converges to a steady state

\[
(56) \quad \kappa_E^* = \left( \frac{\beta}{1 + \beta} \frac{\psi}{(1 + z)(1 + v)} + \frac{\eta \alpha (1 - \psi)}{R} \right)^{\frac{1}{1 - \alpha}}.
\]

Note that in the steady state the condition \(R^l = R\) holds and the rate of return to capital in steady state is as follows:

---

12 The missing \(\chi\) found from equation (40) is required in to construct of equation (54).
\( \rho^*_E = \frac{\alpha (1 - \psi)}{\beta \frac{\psi}{1 + \beta (1 + z)(1 + v)} + \frac{\eta \alpha (1 - \psi)}{R}} \)

The construct of equations (52) and (54) is presented in Appendix 23, the construct of equation (56) is presented in Appendix 24 and the construct of equation (57) is presented in Appendix 25. According to condition (55), the steady state rate of return to capital is larger than gross return to foreign bonds, i.e. \( \rho^*_E > R \). Entrepreneurs do not find it appealing to invest in foreign bonds. Stated otherwise: entrepreneurs never invest in foreign bonds. If this would not be the case, entrepreneurs would deposit part of their savings in banks.

In the converging path, the rate of return to capital falls; capital deepening causes the rate of return to capital to fall to \( R^l \) or the capital per effective unit of labor converges to a steady state where the rate of return to capital is larger than the \( R^l \). However, in the converging path, the foreign surplus, wages per effective unit and output per effective unit all increases.
8. Conclusions

This chapter concludes this thesis by answering the key research questions. It also presents the results and main elements of the model built. I discuss the main assumptions and results of the model along with the features of China’s transition presented in chapter three and point out the contradictions between the model and empirical evidence.

In this thesis, I reviewed the central features of China’s transition, most notably the resource reallocation from the state-owned sector to the private sector, productivity differences between the state-owned and private enterprises, moderate wage growth and rising income inequality, asymmetric financial imperfections and the central macroeconomic indicators: accumulation of foreign surplus and high aggregate investment and saving rates. With the aid of a theoretical model, I characterized the nature of China’s transition. Moreover, while I was calculating the model through, few inconsistencies were found and subsequently recognized by professors Song, Storesletten and Zilibotti. The inconsistencies found are corrected in this thesis (see the footnotes from pages 44, 46 and 60).

In the model E firms refer to the private enterprises, and accordingly, F firms refer to the state-owned enterprises. The main assumptions of the model are firm-level heterogeneity in productivity and asymmetric financial market discrimination. E firms are more productive, but due to the discrimination of the financial sector they must rely on entrepreneurial savings, while F firms are less productive, but survive in the equilibrium due to the external financing. If the entrepreneurial savings are large enough E firms gradually outgrow F firms. During the transition phase both enterprise types have sustained returns to capital due to labor mobility and financial integration of F firms. Low wage growth sustains the rate of return to capital. Interestingly, the aggregate rate of return to capital increases due to composition effect.
The wage earners deposit their earnings into banks, which can either invest into domestic firms or foreign bonds. As the volume of F firms decrease the financial sector invests a higher amount into foreign bonds causing the foreign surplus to increase. After the transition is over, the economy is dominated by E enterprises and capital accumulation is subject to diminishing return to capital.

As presented in chapter three, the following features that characterize the difference between private and state-owned enterprises are well documented: SOEs depend more on external financing due to their better access to state-owned banks and hence credit markets, DPEs are financially discriminated and rely more on self-funding and SOEs are weak in corporate governance. Moreover, during the transition phase, the model behaves similar to China’s growth experience presented in chapter three: the rate of return to capital does not fall in the course of the transition despite of high investment and growth in production; Lemma 1 indicates a lower capital to output and capital to labor ratios for E firms, which stems from higher TFP and limitations to access external funding of the E firms; E firms have a higher rate of return to capital than do F firms; factor reallocation from E firms to F firms during the transition phase; factor reallocation causes external imbalances as financially integrated firms diminish while financially constrained firms increase and the inequality between raising entrepreneurs’ earnings and the moderate wages growth of workers.

However, as presented in chapter three, the following features are in contradiction with China’s growth experience: frictionless labor market, financial market laissez-faire environment, competitive F firms and the prediction that F firms fully disappears from the economy. As presented in chapter three, the labor market in China is characterized by huge frictions (i.e. hukou system), but the labor market is assumed to be frictionless and competitive in the model. However, the absence of the labor market frictions
does not affect the model’s qualitative predictions. Insertion of labor market frictions into the model would affect the rate of reallocation and wage growth. Moreover, in contradiction to the evidence presented in section 3.2, the capital controls deteriorate the interaction between the Chinese financial sector and the international financial sector, but in the model the banks are perfectly integrated in international financial markets. For simplicity, the government is assumed to have no role in monetary policy (i.e. altering the exchange or interest rates). The contradictions of labor market and laissez-faire environment of financial market are confronted in the extensions by Storesletten and Zilibotti (2014) and Song, Stroresletten and Zilibotti (2014), respectively.

The assumption that F firms are competitive stems from the enterprise sector reforms started in the 1990s. The purpose was not to privatize the whole economy, but to retain the largest state-owned enterprises and force the smaller state-owned enterprises to either compete in the marketplace or to default. Hence F firms are assumed to be competitive profit-maximizing firms. However, this assumption blocks the other institutional features that characterize SOEs: market power and the objectives of SOEs and their managers that may be fundamental in China’s paradigm. Also, the prediction that SOEs fully disappear from the economy is not supported by data.

To confront the above mentioned phenomena, Song et al. (2011) extended the model into a two-sector environment, where industries are divided into capital- and labor-intensive industries. In the model of this thesis, E and F firms differ in productivity, but produce the same good. In the extension, E firms have an endogenous comparative advantage in labor-intensive industries due to the credit-market discrimination. The comparative advantage generates E firms to specialize in labor-intensive industries. The transition advances in stages: in the first stage both firms are investing into labor-intensive sector, but only F firms are investing in the capital-intensive
sector. As the transition advances, F firms first retreat into the capital-intensive sector and gradually vanish also from the capital-intensive sector as E firms start to invest into both sectors. Finally the economy is as in post-transition equilibrium (chapter 7), where only E firms exists. Song et al. (2011) also extended the two-sector model into a model, where the capital-intensive sector is monopolized by a large F firm while the labor-intensive sector is competitive. As the transition proceeds, labor-intensive sector’s productivity increases as does demand for the capital-intensive goods along with the profit of the monopolist.

Despite a number of simplifications, the model gives a clear qualitative explanation to China’s puzzling phenomena of sustained return to capital and growing foreign surplus. The simplifications allow the model to focus on the main differences between E and F firms, that is to say the heterogeneity in productivity and asymmetric financial imperfections. Overall, the model presented captures the main features of China’s transition. This is not only done in qualitative fashion, but also quantitatively: Song et al. (2011) showed that a calibrated version of the model is also consistent with the features of China’s growth story. The model successfully mimics much of China’s growth experience, most notably the raising inequality between the workers and the entrepreneurs, firm-level productivity differences, reallocation of resources from low-productivity enterprises to high-productivity enterprises, high return to capital despite of high capital accumulation, financial market imperfections and accumulation of foreign surplus.
9. References


Appendices

Appendix 1. The conduct of equation (5)

Period 1 budget constraint is as follows:

\[ c_{1t}^W + s_t^W = w_t. \]

Period 2 budget constraint is as follows:

\[ c_{2t}^W = R^d s_t^W. \]

\[ s_t^W = \frac{c_{2t}^W}{R^d}. \]

Placing this into period 1 budget constraint, one gets the intertemporal budget constraint

\[ c_{1t}^W + \frac{c_{2t}^W}{R^d} = w_t. \]
Appendix 2. The construct of equation (6)

The Lagrange function is as follows:

\[
\mathcal{L} = \frac{c_{1t}^{(W)} - 1}{1 - \frac{1}{\sigma}} + \beta \frac{c_{2t}^{(W)} - 1}{1 - \frac{1}{\bar{\sigma}}} + \lambda \left( w_t - c_{1t}^W + \frac{c_{2t}^W}{R^d} \right).
\]

The first-order conditions (FOCs) are

\[
\frac{\partial \mathcal{L}}{\partial c_{1t}^W} = c_{1t}^{(W)} \frac{1}{\sigma} - \lambda = 0 \iff \lambda = c_{1t}^{(W)} \frac{1}{\sigma},
\]

\[
\frac{\partial \mathcal{L}}{\partial c_{2t}^W} = \beta c_{2t}^{(W)} \frac{1}{\bar{\sigma}} - \frac{\lambda}{R^d} = 0 \iff \lambda = R^d \beta c_{2t}^{(W)} \frac{1}{\bar{\sigma}},
\]

\[
\frac{\partial \mathcal{L}}{\partial \lambda} = w_t - c_{1t}^W + \frac{c_{2t}^W}{R^d} = 0.
\]

Combining the first two gives

\[
c_{1t}^{(W)} \frac{1}{\sigma} = R^d \beta c_{2t}^{(W)} \frac{1}{\bar{\sigma}}
\]

\[
c_{1t}^W = R^{(d) - \theta} \beta^{-\theta} c_{2t}^W.
\]

Inserting this into third FOC yields

\[
w_t - \left( R^{(d) - \theta} \beta^{-\theta} c_{2t}^W \right) - \frac{c_{2t}^W}{R^d} = 0
\]

\[
R^{(d) - \theta} \beta^{-\theta} c_{2t}^W \frac{c_{2t}^W}{R^d} = w_t
\]
\[
c_{2t}^W \left( R^{(d)\theta} \beta^{-\theta} + \frac{1}{R^d} \right) = w_t
\]

\[
c_{2t}^W \left( \frac{R^d R^{(d)\theta} \beta^{-\theta} + 1}{R^d} \right) = w_t
\]

\[
c_{2t}^W = w_t \left( \frac{R^{(d)1-\theta} \beta^{-\theta} + 1}{R^d} \right)^{-1}.
\]

Plugging this into period 2 budget constraint, (4), gives the optimal savings

\[
s_t^W = \frac{w_t \left( \frac{R^{(d)1-\theta} \beta^{-\theta} + 1}{R^d} \right)^{-1}}{R^d}
\]

\[
s_t^W = \frac{w_t \left( \frac{R^d}{R^{(d)1-\theta} \beta^{-\theta} + 1} \right)}{R^d}
\]

\[
s_t^W = \frac{w_t R^d}{R^d \left( \frac{R^{(d)1-\theta} \beta^{-\theta} + 1}{R^d} \right)}
\]

\[
s_t^W = w_t \left( \frac{1}{R^{(d)1-\theta} \beta^{-\theta} + 1} \right)
\]

\[
s_t^W = w_t \left( 1 + \beta^{-\theta} R^{(d)1-\theta} \right)^{-1}
\]

\[
s_t^W = \zeta^W w_t,
\]

where \( \zeta^W = \left( 1 + \beta^{-\theta} R^{(d)1-\theta} \right)^{-1} \).
Appendix 3. The construct of equation (9)

As stated earlier, the lending rate for domestic firms, $R_l$, is equal to the marginal product of capital. With simple algebra one obtains the following:

$$R_l = \alpha k_{Ft}^{\alpha-1} (A_t n_{Ft})^{1-\alpha}$$

$$\frac{R_l}{n_{Ft}^{1-\alpha}} = \alpha k_{Ft}^{\alpha-1} A_t^{1-\alpha}$$

$$\frac{n_{Ft}^{1-\alpha}}{R_l} = \frac{1}{\alpha k_{Ft}^{\alpha-1} A_t^{1-\alpha}}$$

$$n_{Ft}^{1-\alpha} = \frac{R_l}{\alpha k_{Ft}^{\alpha-1} A_t^{1-\alpha}}$$

$$n_{Ft} = \left(\frac{R_l}{\alpha k_{Ft}^{\alpha-1} A_t^{1-\alpha}}\right)^{\frac{1}{1-\alpha}}.$$
Appendix 4. The construct of equation (11)

Equivalently, as stated before, wages equals to the marginal product of labor. Substituting $n_{ft}$ by plugging the equation (9) into the formula yields the equilibrium wages as follows:

$$w_t = k_{ft}^\alpha (1 - \alpha) A_t \left( A_t \left( \frac{R_l}{\alpha k_{ft}^{\alpha-1} A_t^{1-\alpha}} \right)^{\frac{1}{1-\alpha}} \right)^{-\alpha}$$

$$w_t = k_{ft}^\alpha (1 - \alpha) A_t^\alpha \left( \frac{R_l}{\alpha k_{ft}^{\alpha-1} A_t^{1-\alpha}} \right)^{-\frac{\alpha}{1-\alpha}}$$

$$w_t = k_{ft}^\alpha (1 - \alpha) A_t^{1-\alpha} \left( \frac{1}{\alpha k_{ft}^{\alpha-1} A_t^{1-\alpha}} \right)^{\frac{1}{1-\alpha}}$$

$$w_t = k_{ft}^\alpha (1 - \alpha) A_t^{1-\alpha} (\alpha k_{ft}^{\alpha-1} A_t^{1-\alpha})^\frac{\alpha}{1-\alpha}$$

$$w_t = k_{ft}^\alpha (1 - \alpha) A_t^{1-\alpha} k_{ft}^{\alpha-1} A_t^{1-\alpha} \left( \frac{\alpha}{R_l} \right)^{\frac{\alpha}{1-\alpha}}$$

$$w_t = (1 - \alpha) A_t^{1-\alpha} \frac{1-\alpha}{1-\alpha+\alpha(1-\alpha)} k_{ft}^{\alpha-1} A_t^{1-\alpha} \left( \frac{\alpha}{R_l} \right)^{\frac{\alpha}{1-\alpha}}$$

$$w_t = (1 - \alpha) A_t^{1-\alpha} k_{ft}^{0} \left( \frac{\alpha}{R_l} \right)^{\frac{\alpha}{1-\alpha}}$$

$$w_t = (1 - \alpha) \left( \frac{\alpha}{R_l} \right)^{\frac{\alpha}{1-\alpha}} A_t.$$
Appendix 5. The construct of equation (14)

Plugging (11) and (13) into (12) gives

\[
\max_{n_{Et}} \left\{ k^a_{Et} (\chi A_t n_{Et})^{1-\alpha} - (\psi k^a_{Et} (\chi A_t n_{Et})^{1-\alpha}) - \left(1 - \alpha \right) \left(\frac{\alpha}{R^l}\right)^{\frac{1}{1-\alpha}} A_t \right\} n_{Et} \right). 
\]

Taking the FOC with respect to \(n_{Et}\) gives

\[
k^a_{Et} (1 - \alpha) (\chi A_t) (\chi A_t n_{Et})^{-\alpha} - \psi k^a_{Et} (1 - \alpha) (\chi A_t) (\chi A_t n_{Et})^{-\alpha}
- (1 - \alpha) \left(\frac{\alpha}{R^l}\right)^{\frac{1}{1-\alpha}} A_t = 0
\]

\[
n_{Et}^{-\alpha} = \frac{(1 - \alpha) \left(\frac{\alpha}{R^l}\right)^{\frac{1}{1-\alpha}} A_t}{(k^a_{Et} (1 - \alpha) (\chi A_t)(\chi A_t)^{-\alpha} - \psi k^a_{Et} (1 - \alpha) (\chi A_t)(\chi A_t)^{-\alpha})}
\]

\[
n_{Et} = \left(\frac{(1 - \alpha) \left(\frac{\alpha}{R^l}\right)^{\frac{1}{1-\alpha}} A_t}{(k^a_{Et} (1 - \alpha) (\chi A_t)(\chi A_t)^{-\alpha} - \psi k^a_{Et} (1 - \alpha) (\chi A_t)(\chi A_t)^{-\alpha})}\right)^{-\frac{1}{\alpha}}
\]

\[
n_{Et} = \frac{(1 - \alpha)^{-\frac{1}{\alpha}} \left(\frac{\alpha}{R^l}\right)^{-\frac{1}{1-\alpha}} A_t^{-\frac{1}{\alpha}}}{k^a_{Et}^{-1} (1 - \alpha)^{-\frac{1}{\alpha}} (\chi A_t)^{-\frac{1}{\alpha}} A_t - \psi \left(\frac{1}{\alpha} k^a_{Et}^{-1} (1 - \alpha)^{-\frac{1}{\alpha}} (\chi A_t)^{-\frac{1}{\alpha}} A_t \right)}
\]

\[
n_{Et} = \frac{1}{ \left(\frac{\alpha}{R^l}\right)^{\frac{1}{1-\alpha}} A_t^{\frac{1}{\alpha}}} \frac{\chi A_t}{k^a_{Et} (\chi A_t)^{\frac{1}{\alpha}}} - \frac{1}{\psi\alpha k^a_{Et} (\chi A_t)^{\frac{1}{\alpha}}}
\]
\[ n_{Et} = \frac{1}{\left(\frac{\alpha}{R^l}\right)^{\frac{1}{1-\alpha}} A_t^{\frac{1}{\alpha}}} \left( k_{Et}(\chi A_t)^{\frac{1}{\alpha}} - \psi^{\frac{1}{\alpha}} k_{Et}\frac{1}{\chi A_t^{\frac{1}{\alpha}}} \right) \]

\[ n_{Et} = \frac{k_{Et} \chi^{\frac{1}{\alpha}} - \psi^{\frac{1}{\alpha}} k_{Et} \chi^{\frac{1}{\alpha}}}{\left(\frac{\alpha}{R^l}\right)^{\frac{1}{1-\alpha}} \chi A_t} \]

\[ n_{Et} = \frac{((1 - \psi) \chi)^{\frac{1}{\alpha}} k_{Et}}{\left(\frac{\alpha}{R^l}\right)^{\frac{1}{1-\alpha}} \chi A_t} \]

\[ n_{Et} = \frac{((1 - \psi) \chi)^{\frac{1}{\alpha}} \left( R^l \right)^{\frac{1}{1-\alpha}} k_{Et} \chi A_t}{\alpha}. \]
Appendix 6. The construct of equation (16)

After plugging the equations (13), (11), and (14) into equation (12) and modifying the equation as follows yields to the value of E firm:

$$
\Xi(k_{Et}) = k_{Et}^\alpha \left( \chi A_t \left( \frac{1}{(1 - \psi) \chi} \left( R^l \frac{1}{1-\alpha} \frac{k_{Et}}{\chi A_t} \right) \right)^{1-\alpha} \right.
$$

$$
- \left( \psi k_{Et}^\alpha \left( \chi A_t \left( \frac{1}{(1 - \psi) \chi} \left( R^l \frac{1}{1-\alpha} \frac{k_{Et}}{\chi A_t} \right) \right)^{1-\alpha} \right) \right.
$$

$$
- \left( 1 - \alpha \left( \frac{\alpha}{R^l} \right)^{1-\alpha} A_t \right) \left( (1 - \psi) \chi \left( R^l \frac{1}{1-\alpha} \frac{k_{Et}}{\chi A_t} \right) \right)^{1-\alpha}
$$

$$
\Xi(k_{Et}) = (1 - \psi) k_{Et}^\alpha (\chi A_t)^{1-\alpha} \left( \chi A_t \left( \frac{1}{(1 - \psi) \chi} \left( R^l \frac{1}{1-\alpha} \frac{k_{Et}}{\chi A_t} \right) \right)^{1-\alpha} \right.
$$

$$
- \left( 1 - \alpha \left( \frac{\alpha}{R^l} \right)^{1-\alpha} A_t \right) \left( (1 - \psi) \chi \left( R^l \frac{1}{1-\alpha} \frac{k_{Et}}{\chi A_t} \right) \right)^{1-\alpha}
$$

$$
\Xi(k_{Et}) = (1 - \psi) k_{Et}^\alpha (\chi A_t)^{1-\alpha} (1 - \psi)^{1-\alpha} \left( \chi A_t \left( \frac{1}{\chi A_t} \right) \left( R^l \frac{1}{1-\alpha} \frac{k_{Et}}{\chi A_t} \right) \right)^{1-\alpha}
$$

$$
- \left( 1 - \alpha \left( \frac{\alpha}{R^l} \right)^{1-\alpha} A_t \right) \left( (1 - \psi) \chi \left( R^l \frac{1}{1-\alpha} \frac{k_{Et}}{\chi A_t} \right) \right)^{1-\alpha}
$$

$$
\Xi(k_{Et}) = (1 - \psi) \left( \frac{\alpha+1-\alpha}{\alpha} \right) \left( \frac{\alpha}{R^l} \right)^{1-\alpha} A_t \left( (1 - \psi) \chi \left( R^l \frac{1}{1-\alpha} \frac{k_{Et}}{\chi A_t} \right) \right)^{1-\alpha}
$$

$$
- \left( 1 - \alpha \left( \frac{\alpha}{R^l} \right)^{1-\alpha} A_t \right) \left( (1 - \psi) \chi \left( R^l \frac{1}{1-\alpha} \frac{k_{Et}}{\chi A_t} \right) \right)^{1-\alpha}
$$

$$
\Xi(k_{Et}) = (1 - \psi) \left( \frac{\alpha+1-\alpha}{\alpha} \right) \left( \frac{\alpha}{R^l} \right)^{1-\alpha} (1 - \psi) \chi ^{1-\alpha} \left( \frac{R^l}{\alpha} \right)^{1-\alpha} \frac{k_{Et}}{\chi A_t}
$$

$$
- \left( 1 - \alpha \left( \frac{\alpha}{R^l} \right)^{1-\alpha} (1 - \psi) \chi ^{1-\alpha} \left( \frac{R^l}{\alpha} \right)^{1-\alpha} \frac{k_{Et}}{\chi A_t} \right)^{1-\alpha}
$$
\[ \Xi(k_{Et}) = (1 - \psi) \frac{1}{\alpha} \frac{1-\alpha}{\alpha} k_{Et} \left( \frac{R^l}{\alpha} \right) \]

\[ - (1 - \psi) \frac{1}{\alpha} \frac{1-\alpha}{\alpha} k_{Et} (1 - \alpha) \left( \frac{\alpha}{R^l} \right) \frac{1}{1-\alpha} \frac{R^l}{\alpha} \]

\[ \Xi(k_{Et}) = \left( (1 - \psi) \frac{1}{\alpha} \frac{1-\alpha}{\alpha} k_{Et} \right) \left( \frac{R^l}{\alpha} \right) - (1 - \alpha) \left( \frac{\alpha}{R^l} \right) \frac{1}{1-\alpha} \frac{R^l}{\alpha} \]

\[ \Xi(k_{Et}) = \left( (1 - \psi) \frac{1}{\alpha} \frac{1-\alpha}{\alpha} k_{Et} \right) \left( \frac{R^l}{\alpha} \right) - \left( (1 - \alpha) \frac{1-\alpha}{\alpha^1} \frac{\alpha}{R^l} R^l \right) \frac{1}{1-\alpha} \frac{R^l}{\alpha} \]

\[ \Xi(k_{Et}) = \left( (1 - \psi) \frac{1}{\alpha} \frac{1-\alpha}{\alpha} k_{Et} \right) \left( \frac{R^l}{\alpha} \right) - \left( (1 - \alpha) \frac{1-\alpha}{\alpha^1} \frac{\alpha}{R^l} R^l \right) \frac{1}{1-\alpha} \frac{R^l}{\alpha} \]

\[ \Xi(k_{Et}) = \left( (1 - \psi) \frac{1}{\alpha} \frac{1-\alpha}{\alpha} k_{Et} \right) \left( \frac{R^l - (1 - \alpha) R^l}{\alpha} \right) \]

\[ \Xi(k_{Et}) = \left( (1 - \psi) \frac{1}{\alpha} \frac{1-\alpha}{\alpha} k_{Et} \right) \left( \frac{R^l - R^l + \alpha R^l}{\alpha} \right) \]

\[ \Xi(k_{Et}) = (1 - \psi) \frac{1}{\alpha} \frac{1-\alpha}{\alpha} k_{Et} R^l. \]
Appendix 7. The construct of equation (21)

Using simple algebra, the incentive-compatibility constraint can be presented as follows:

\[ R^l l^E \leq \eta \rho_E (s^E + l^E) \]

\[ \frac{R^l l^E}{(s^E + l^E)} \leq \eta \rho_E \]

\[ \frac{l^E}{(s^E + l^E)} \leq \frac{\eta \rho_E}{R^l} \]

\[ \frac{l^E}{(s^E + l^E)} = \frac{\eta \rho_E}{R^l}. \]
Appendix 8. The construct of equation (24)

By using simple algebra, equation (21) can be expressed as follows:

\[
R^l l^E = \eta \rho_E (s^E + l^E)
\]

\[
l^E (R^l - \eta \rho_E) = \eta \rho_E s^E
\]

\[
l^E = \frac{\eta \rho_E s^E}{R^l - \eta \rho_E}.
\]

Plugging \(l^E\) into equation (23) gives

\[
c_2 = \rho_E \left( \frac{\eta \rho_E s^E}{R^l - \eta \rho_E} + s_E \right) - R^l \left( \frac{\eta \rho_E s^E}{R^l - \eta \rho_E} \right)
\]

\[
c_2 = \frac{\eta \rho_E^2 s^E}{R^l - \eta \rho_E} + \frac{\rho_E s_E (R^l - \eta R^l)}{R^l - \eta \rho_E} - \frac{\eta \rho_E s^E R^l}{R^l - \eta \rho_E}
\]

\[
c_2 = \frac{\rho_E s_E (R^l - \eta \rho_E s^E R^l)}{R^l - \eta \rho_E}
\]

\[
c_2 = \frac{(1 - \eta) \rho_E R^l}{R^l - \eta \rho_E} s_E.
\]
Appendix 9. The construct of equation (26)

\[
\max_{\gamma_E} \left( \frac{m - \gamma_E}{1 - \frac{1}{\theta}} \right)^{1 - \frac{1}{\theta}} - 1 + \beta \left( \frac{\left(1 - \eta\right) \rho_E R^l}{R^l - \eta \rho_E} \gamma_E \right)^{1 - \frac{1}{\theta}} - 1.
\]

Taking the FOC with respect to \(\gamma_E\) gives

\[
-(1 - \frac{1}{\theta}) \left( m - \gamma_E \right)^{1 - \frac{1}{\theta}} + \beta \left( \frac{\left(1 - \eta\right) \rho_E R^l}{R^l - \eta \rho_E} \gamma_E \right)^{1 - \frac{1}{\theta}} = 0
\]

\[
-(m - \gamma_E)^{1 - \frac{1}{\theta}} + \beta \left( \frac{\left(1 - \eta\right) \rho_E R^l}{R^l - \eta \rho_E} \gamma_E \right)^{1 - \frac{1}{\theta}} = 0
\]

\[
-m + \gamma_E + \beta^{-\theta} \left( \frac{\left(1 - \eta\right) \rho_E R^l}{R^l - \eta \rho_E} \gamma_E \right)^{-\theta} \left( \frac{\left(1 - \eta\right) \rho_E R^l}{R^l - \eta \rho_E} \gamma_E \right) \left( \frac{1}{(R^l - \eta \rho_E)^{-\theta}} \right) = 0
\]

\[
-m + \gamma_E + \beta^{-\theta} \left( \frac{\left(1 - \eta\right) \rho_E R^l}{R^l - \eta \rho_E} \gamma_E \right)^{-\theta} \left( \frac{\left(1 - \eta\right) \rho_E R^l}{R^l - \eta \rho_E} \gamma_E \right) \left( \frac{1}{(R^l - \eta \rho_E)^{-\theta}} \right) = 0
\]

\[
-m + \gamma_E + \left( \frac{\beta^{-\theta} \left( \frac{\left(1 - \eta\right) \rho_E R^l}{R^l - \eta \rho_E} \gamma_E \right)^{-\theta} \left( \frac{\left(1 - \eta\right) \rho_E R^l}{R^l - \eta \rho_E} \gamma_E \right) \left( \frac{1}{(R^l - \eta \rho_E)^{-\theta}} \right)}{\left( \frac{\left(1 - \eta\right) \rho_E R^l}{R^l - \eta \rho_E} \gamma_E \right)^{1 - \theta}} \right) = 0
\]

\[
-m + \gamma_E + \left( \frac{\beta^{-\theta} \left( \frac{\left(1 - \eta\right) \rho_E R^l}{R^l - \eta \rho_E} \gamma_E \right)^{1 - \theta}}{(R^l - \eta \rho_E)^{1 - \theta}} \gamma_E \right) = 0
\]

\[
\gamma_E \left( 1 + \frac{\beta^{-\theta} \left( \frac{\left(1 - \eta\right) \rho_E R^l}{R^l - \eta \rho_E} \gamma_E \right)^{1 - \theta}}{(R^l - \eta \rho_E)^{1 - \theta}} \right) = m
\]
\[ s_E = m \left( 1 + \beta^{-\theta} \left( \frac{(1 - \eta)\rho_E R^l}{R^l - \eta \rho_E} \right)^{1-\theta} \right)^{-1}. \]
Appendix 10. The construct of equation (30)

Modifying the equation (8) gives

\[ R^l = \alpha k_{Fl}^{-1} (A_t n_{Fl})^{1-\alpha} \]

\[ R^l = \alpha \left( \frac{A_t n_{Fl}}{k_{Fl}} \right)^{1-\alpha} \]

\[ \frac{R^l}{\alpha} = \left( \frac{A_t n_{Fl}}{k_{Fl}} \right)^{1-\alpha} \]

\[ \frac{\alpha}{R^l} = \left( \frac{k_{Fl}}{A_t n_{Fl}} \right)^{1-\alpha} \]

\[ \left( \frac{\alpha}{R^l} \right)^{1-\alpha} = \frac{k_{Fl}}{A_t n_{Fl}} \]

Now plugging equation (28) into the equation above gives equation (30):

\[ \kappa_F = \left( \frac{\alpha}{R^l} \right)^{1-\alpha} \]

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Appendix 11. The construct of equation (32)

Modifying the equation (14) as follows gives

\[ n_{Et} = ((1 - \psi)\chi)^{\frac{1}{\alpha}} \left( \frac{R^l}{\alpha} \right)^{\frac{1}{1-\alpha}} \frac{k_{Et}}{\chi A_t} \]

\[ 1 = ((1 - \psi)\chi)^{\frac{1}{\alpha}} \left( \frac{R^l}{\alpha} \right)^{\frac{1}{1-\alpha}} \frac{k_{Et}}{\chi A_t n_{Et}} \]

\[ \frac{1}{k_{Et}} = ((1 - \psi)\chi)^{\frac{1}{\alpha}} \left( \frac{R^l}{\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1}{\chi A_t n_{Et}} \]

\[ \frac{\chi A_t n_{Et}}{k_{Et}} = ((1 - \psi)\chi)^{\frac{1}{\alpha}} \left( \frac{R^l}{\alpha} \right)^{\frac{1}{1-\alpha}} \]

\[ \frac{k_{Et}}{\chi A_t n_{Et}} = ((1 - \psi)\chi)^{\frac{1}{\alpha}} \left( \frac{\alpha}{R^l} \right)^{\frac{1}{1-\alpha}}. \]

Then plugging the equations (29) and (30) into the equation above yields

\[ \kappa_{Et} = \kappa_{Ft} ((1 - \psi)\chi)^{\frac{1}{\alpha}}. \]
Appendix 12. The construct of equation (33)

In order to clarify equation (33), I first calculate the left-side and then the right-side of the equation.

\[
(33) \quad \frac{k_{Et}}{y_{Et}} = \kappa_{Et}^{1-\alpha} < \kappa_{Ft}^{1-\alpha} = \frac{k_{Ft}}{y_{Ft}}.
\]

By first plugging equation (3) into the left-side of equation (33) and using the expression of capital per effective unit of labor for E firms from equation (29) yields

\[
\frac{k_{Et}}{y_{Et}} = \frac{k_{Et}}{k_{Et}^{\alpha}(\chi A_t n_{Et})^{1-\alpha}} = \frac{k_{Et}^{1-\alpha}}{(\chi A_t n_{Et})^{1-\alpha}} = \left(\frac{k_{Et}}{\chi A_t n_{Et}}\right)^{1-\alpha} = \kappa_{Et}^{1-\alpha}.
\]

Similarly, plugging the equation (2) into the right-side of equation (33) and using the expression of capital per effective unit of labor for F firms from equation (28) yields

\[
\frac{k_{Ft}}{y_{Ft}} = \frac{k_{Ft}}{k_{Ft}^{\alpha}(A_t n_{Ft})^{1-\alpha}} = \frac{k_{Ft}^{1-\alpha}}{(A_t n_{Ft})^{1-\alpha}} = \left(\frac{k_{Ft}}{A_t n_{Ft}}\right)^{1-\alpha} = \kappa_{Ft}^{1-\alpha}.
\]

Hence

\[
\frac{k_{Et}}{y_{Et}} = \kappa_{Et}^{1-\alpha} < \kappa_{Ft}^{1-\alpha} = \frac{k_{Ft}}{y_{Ft}}.
\]
Appendix 13. The construct of equation (34)

First by plugging the expressions of $n_F$ and $n_E$ from equations (9) and (14) into the left-side of equation (34) yields the following:

\[
\frac{k_F}{n_F} = \frac{k_E}{n_E}
\]

\[
= \frac{k_F}{k_E} \left( \frac{R^l}{\alpha k_{Ft} A^{1-\alpha}} \right)^{1-\alpha} \frac{1}{\left(1 - \psi \chi \right)^{1-\alpha} \left(\frac{R^l}{\alpha} \right)^{1-\alpha} k_{Et} A_t \chi A_t}
\]

\[
= \frac{k_F}{k_{Et} A_t} \left( \frac{R^l}{\alpha} \right)^{1-\alpha} \frac{1}{\left(\frac{R^l}{\alpha} \right)^{1-\alpha} A_t} \chi A_t \left(1 - \psi \chi \right)^{1-\alpha}
\]

\[
= \frac{\left( \frac{\alpha}{R^l} \right)^{1-\alpha} k_{Ft} A_t}{\left( \frac{\alpha}{R^l} \right)^{1-\alpha} \left(1 - \psi \chi \right)^{1-\alpha}}
\]

Now using the expressions of $\kappa_{Ft}$ and $\kappa_{Et}$ from equations (30) and (32) yields

\[
= \frac{\kappa_{Ft} A_t}{\kappa_{Et} \chi A_t}
\]

Now by plugging the equation (32) yields
\[
\frac{\kappa_{Ft} A_t}{\kappa_{Ft}((1 - \psi)\chi)^{\frac{1}{\alpha}}\chi A_t} = \frac{1}{((1 - \psi)\chi)^{\frac{1}{\alpha}}\chi} = \frac{1}{(1 - \psi)^{\frac{1}{\alpha}}}^{\frac{1}{1+\alpha}} = \chi^{-\frac{1}{\alpha}}^{(-1+\alpha)} \frac{1}{(1 - \psi)^{-\frac{1}{\alpha}}} = \frac{1}{\chi^{\frac{1}{\alpha}}} \cdot \frac{1}{\left(\frac{1}{1 - \psi}\right)^{\frac{1}{\alpha}}}
\]

Now using the equation (18) from Assumption 1 (i.e. \( \chi > \chi \equiv \left(\frac{1}{1 - \psi}\right)^{\frac{1}{1-\alpha}} \)) yields

\[
\left(\frac{\chi^{\frac{1}{\alpha}}}{\chi}\right)
\]

since

\[
\frac{1-\alpha}{\chi^{\frac{1}{\alpha}}} = \left[\left(\frac{1}{1 - \psi}\right)^{\frac{1}{1-\alpha}}\right]^{\frac{1-\alpha}{\alpha}} = \left(\frac{1}{1 - \psi}\right)^{\frac{(1-\alpha)}{(1-\alpha)^{\frac{1}{\alpha}}}} = \left(\frac{1}{1 - \psi}\right)^{\frac{1}{\alpha}}
\]

Hence
\[
\frac{k_F}{n_F} = \frac{\kappa_F}{\kappa_E} \frac{A}{\chi A} = \left(\frac{\chi}{\chi'}\right)^{1-\alpha}.
\]
Appendix 14. The construct of equation (39)

Plugging the modified equation (21) (see Appendix 8) into equation (38) yields

\[
(38) \quad k_{Et+1} = s_t^E + l_t^E
\]

\[
k_{Et+1} = s_t^E + \frac{\eta \rho_E s_t^E}{R_i - \eta \rho_E}
\]

\[
k_{Et+1} = s_t^E \left(1 + \frac{\eta \rho_E}{R_i - \eta \rho_E}\right)
\]

\[
k_{Et+1} = s_t^E \left(\frac{R_i - \eta \rho_E + \eta \rho_E}{R_i - \eta \rho_E}\right)
\]

\[
k_{Et+1} = s_t^E \left(\frac{R_i}{R_i - \eta \rho_E}\right)
\]

Next, by plugging \(s_t^E\) from equation (26) yields

\[
(39) \quad k_{Et+1} = \left(\frac{R_i}{R_i - \eta \rho_E}\right) \zeta^E m_t.
\]
Appendix 15. The construct of equation (40)

Plugging equation (13) by replacing $m_t$ yields

\[
k_{Et+1} = \left( \frac{R^l}{R^l - \eta \rho^E_t} \right) \zeta^E m_t
\]

\[
k_{Et} = \left( \frac{R^l}{R^l - \eta \rho^E_t} \right) \zeta^E \psi(k_{Et})^\alpha (\chi A_t n_{Et})^{1-\alpha}.
\]

Then, aggregating over all entrepreneurs yields

\[
K_{Et+1} = \left( \frac{R^l}{R^l - \eta \rho^E_t} \right) \zeta^E \psi(K_{Et})^\alpha (\chi A_t N_{Et})^{1-\alpha}
\]

\[
K_{Et+1} = \left( \frac{R^l}{R^l - \eta \rho^E_t} \right) \zeta^E \psi(K_{Et})^\alpha (\chi A_t N_{Et})^{-\alpha} (\chi A_t N_{Et})
\]

\[
K_{Et+1} = \left( \frac{R^l}{R^l - \eta \rho^E_t} \right) \zeta^E \psi \left( \frac{K_{Et}}{\chi A_t N_{Et}} \right)^\alpha (\chi A_t N_{Et}).
\]

Using the expression of $\kappa_{Et}$ from equation (29) yields

\[
K_{Et+1} = \left( \frac{R^l}{R^l - \eta \rho^E_t} \right) \zeta^E \psi \kappa_{Et}^\alpha (\chi A_t N_{Et}).
\]
Appendix 16. Proof of Lemma 2

(40) \[ K_{Et+1} = \left( \frac{R^l}{R^l - \eta \rho_t^E} \right) \zeta^E \psi \kappa_E^g \chi A_N N_{Et}. \]

Substituting $\zeta^E$ by plugging equation (27) into equation (40) yields

\[ K_{Et+1} = \left( \frac{R^l}{R^l - \eta \rho_t^E} \right) \left( 1 + \beta^{-\theta} \left( \frac{(1 - \eta) \rho_E R^l}{R^l - \eta \rho_E} \right)^{1-\theta} \right)^{-1} \psi \kappa_E^g \chi A_N N_{Et}. \]

Dividing the both side of the equation by $K_{Et}$ yields

\[ \frac{K_{Et+1}}{K_{Et}} = \left( \frac{R^l}{R^l - \eta \rho_t^E} \right) \left( 1 + \beta^{-\theta} \left( \frac{(1 - \eta) \rho_E R^l}{R^l - \eta \rho_E} \right)^{1-\theta} \right)^{-1} \psi \kappa_E^g \chi A_N N_{Et}. \]

Using the expression of $\kappa_{Et}$ by plugging equation (32) yields

\[ \frac{K_{Et+1}}{K_{Et}} = \left( \frac{R^l}{R^l - \eta \rho_t^E} \right) \left( 1 + \beta^{-\theta} \left( \frac{(1 - \eta) \rho_E R^l}{R^l - \eta \rho_E} \right)^{1-\theta} \right)^{-1} \psi \left( \frac{\alpha}{R^l} \right)^{\frac{1}{1-\alpha}} (1 - \psi) \chi^{\frac{1}{1-\alpha}} \frac{\chi A_N N_{Et}}{K_{Et}}. \]
\[
\frac{K_{Et+1}}{K_{Et}} = \left(\frac{R^l}{R^l - \eta \rho_E^l}\right)
\cdot \left(1 + \beta^{-\theta} \left(\frac{(1 - \eta) \rho_E R^l}{R^l - \eta \rho_E}\right)^{1-\theta}\right)^{-1} \psi \left(\frac{\alpha}{R^l}\right)^{\frac{\alpha}{1-\alpha}} \psi (1 - \psi)^{-1} \chi A_t N_{Et} \frac{\chi A_t N_{Et}}{K_{Et}}
\]

\[
\frac{K_{Et+1}}{K_{Et}} = \left(\frac{R^l}{R^l - \eta \rho_E^l}\right)
\cdot \left(1 + \beta^{-\theta} \left(\frac{(1 - \eta) \rho_E R^l}{R^l - \eta \rho_E}\right)^{1-\theta}\right)^{-1} \psi \left(\frac{\alpha}{R^l}\right)^{\frac{\alpha}{1-\alpha}} \frac{A_t N_{Et}}{\kappa_{Et} \chi A_t N_{Et}}
\]

Using the modified equation (35) to substitute \(K_{Et}\) yields

\[
\frac{K_{Et+1}}{K_{Et}} = \left(\frac{R^l}{R^l - \eta \rho_E^l}\right)
\cdot \left(1 + \beta^{-\theta} \left(\frac{(1 - \eta) \rho_E R^l}{R^l - \eta \rho_E}\right)^{1-\theta}\right)^{-1} \psi \left(\frac{\alpha}{R^l}\right)^{\frac{\alpha}{1-\alpha}} \frac{A_t N_{Et}}{\kappa_{Et} \chi A_t N_{Et}}
\]

Again, using the expression of \(\kappa_{Et}\) by plugging equation (32) yields

\[
\frac{K_{Et+1}}{K_{Et}} = \left(\frac{R^l}{R^l - \eta \rho_E^l}\right)
\cdot \left(1 + \beta^{-\theta} \left(\frac{(1 - \eta) \rho_E R^l}{R^l - \eta \rho_E}\right)^{1-\theta}\right)^{-1} \psi \left(\frac{\alpha}{R^l}\right)^{\frac{\alpha}{1-\alpha}} \frac{1}{\kappa_{Et} \chi}
\]
\[
\frac{K_{Et+1}}{K_{Et}} = \left( \frac{R^l}{R^l - \eta \rho _E} \right)
\]

\[
\cdot \left( 1 + \beta^{-\theta} \left( \frac{(1 - \eta) \rho _E R^l}{R^l - \eta \rho _E} \right)^{1-\theta} \right)^{-1} \psi \left( \frac{\alpha}{\alpha^{1-\alpha} R^l(1 - \psi) (1 - \psi)^{1 - \alpha}} \right)
\]

\[
\cdot \left( 1 - \eta \rho _E R^l \right)^{1-\theta} \left( 1 - \psi \right) \left( \frac{\alpha}{\alpha^{1-\alpha} (1 - \psi)^{1 - \alpha}} \right)
\]

\[
\frac{K_{Et+1}}{K_{Et}} = \left( \frac{R^l}{R^l - \eta \rho _E} \right)
\]

\[
\cdot \left( 1 + \beta^{-\theta} \left( \frac{(1 - \eta) \rho _E R^l}{R^l - \eta \rho _E} \right)^{1-\theta} \right)^{-1} \psi \left( \frac{\alpha}{\alpha^{1-\alpha} R^l(1 - \psi) (1 - \psi)^{1 - \alpha}} \right)
\]

\[
\cdot \left( 1 - \eta \rho _E R^l \right)^{1-\theta} \left( 1 - \psi \right) \left( \frac{\alpha}{\alpha^{1-\alpha} (1 - \psi)^{1 - \alpha}} \right)
\]

\[
\frac{K_{Et+1}}{K_{Et}} = \left( \frac{R^l}{R^l - \eta \rho _E} \right)
\]

\[
\cdot \left( 1 + \beta^{-\theta} \left( \frac{(1 - \eta) \rho _E R^l}{R^l - \eta \rho _E} \right)^{1-\theta} \right)^{-1} \psi \left( \frac{\alpha}{\alpha^{1-\alpha} R^l(1 - \psi) (1 - \psi)^{1 - \alpha}} \right)
\]

\[
\cdot \left( 1 - \eta \rho _E R^l \right)^{1-\theta} \left( 1 - \psi \right) \left( \frac{\alpha}{\alpha^{1-\alpha} (1 - \psi)^{1 - \alpha}} \right)
\]

\[
\frac{K_{Et+1}}{K_{Et}} = \left( \frac{R^l}{R^l - \eta \rho _E} \right)
\]
\[
\left(1 + \beta^{-\theta} \left(\frac{(1 - \eta) \rho_E R^l}{R^l - \eta \rho_E}\right)^{1-\theta}\right)^{-1} \left(\frac{\psi}{1 - \psi}\right) \left(\frac{(1 - \psi)^{\frac{1}{\alpha}} \chi^\alpha R^l}{\alpha^{1-\alpha} \lambda^1}\right)
\]

\[
\frac{K_{E_{t+1}}}{K_{E_t}} = \left(\frac{R^l}{R^l - \eta \rho_E}\right)
\]

\[
\left(1 + \beta^{-\theta} \left(\frac{(1 - \eta) \rho_E R^l}{R^l - \eta \rho_E}\right)^{1-\theta}\right)^{-1} \left(\frac{\psi}{1 - \psi}\right) \left(\frac{(1 - \psi)^{\frac{1}{\alpha}} \chi^\alpha R^l}{\alpha^{1-\alpha} \lambda^1}\right)
\]

\[
\frac{K_{E_{t+1}}}{K_{E_t}} = \left(\frac{R^l}{R^l - \eta \rho_E}\right)
\]

Now, one needs to recall that condition \(v_E > \nu\) is equivalent to

\[
\frac{R^l}{R^l - \eta \rho_E} \left(1 + \beta^{-\theta} \left(\frac{(1 - \eta) \rho_E R^l}{R^l - \eta \rho_E}\right)^{1-\theta}\right)^{-1} \left(\frac{\psi}{1 - \psi}\right) \left(\frac{(1 - \psi)^{\frac{1}{\alpha}} \chi^\alpha R^l}{\alpha^{1-\alpha} \lambda^1}\right) > (1 + \nu)(1 + z).
\]

In order to proceed the proof of lemma 2, one needs to use the following fact:

\[
\frac{R^l - \eta \rho_E}{R^l \rho_E}
\]

\[
= \frac{R^l}{R^l \rho_E} - \frac{\eta \rho_E}{R^l \rho_E}
\]

\[
= \frac{1}{\rho_E} - \frac{\eta}{R^l}
\]

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Next, plugging in the expression of $\rho_E$ from equation (16) gives

$$\frac{1}{\frac{1}{(1 - \psi)\bar{x}^{\frac{1-\alpha}{\alpha}}} R^l} - \frac{\eta}{R^l}$$

$$= \frac{1}{\frac{1}{(1 - \psi)\bar{x}^{\frac{1-\alpha}{\alpha}}} R^l} - \frac{\eta}{R^l}$$

$$= \frac{1}{R^l} \left( \frac{1}{\frac{1}{(1 - \psi)\bar{x}^{\frac{1-\alpha}{\alpha}}} - \eta} \right)$$

$$= \frac{1}{R^l} (1 - \psi) - \frac{1}{\bar{x}^{\frac{1-\alpha}{\alpha}}} - \eta.$$

First, by rearranging the condition $v_E > v$ and using the above defined fact gives

$$\frac{R^l}{R^l - \eta \rho_E} \left( 1 + \beta^{-\theta} \left( \frac{(1 - \eta) \rho_E R^l}{R^l - \eta \rho_E} \right)^{1-\theta} \right)^{-1} \frac{\psi \rho_E}{1 - \psi} \alpha > (1 + v)(1 + z)$$

$$\frac{R^l}{R^l - \eta \rho_E} \left( 1 + \beta^{-\theta} (1 - \eta)^{1-\theta} \left( \frac{\rho_E R^l}{R^l - \eta \rho_E} \right)^{1-\theta} \right)^{-1} \frac{\psi \rho_E}{1 - \psi} \alpha(1 + v)(1 + z)$$

$$> 1$$

$$\frac{\rho_E R^l}{R^l - \eta \rho_E} \left( 1 + \beta^{-\theta} (1 - \eta)^{1-\theta} \left( \frac{\rho_E R^l}{R^l - \eta \rho_E} \right)^{1-\theta} \right)^{-1} \frac{\psi \rho_E}{1 - \psi} \alpha(1 + v)(1 + z)$$

$$> 1$$

$$\frac{\rho_E R^l}{R^l - \eta \rho_E} \left( 1 + \beta^{-\theta} (1 - \eta)^{1-\theta} \left( \frac{\rho_E R^l}{R^l - \eta \rho_E} \right)^{1-\theta} \right)^{-1} \frac{\psi \rho_E}{1 - \psi} \alpha(1 + v)(1 + z)$$

$$> 1$$
\[
\left( \frac{1}{R_l} (1 - \psi)^{\frac{1}{\alpha}} \chi^{-(\frac{1}{\alpha})} - \eta \right)^{-1} \\
\cdot \left( 1 + \beta^{-\theta} (1 - \eta)^{1-\theta} \left( \frac{1}{R_l} (1 - \psi)^{\frac{1}{\alpha}} \chi^{-(\frac{1}{\alpha})} - \eta \right)^{-1} \right)^{-1} \\
\cdot \frac{\psi}{1 - \psi \alpha(1 + \nu)(1 + z)} > 1 \\
\frac{\psi}{1 - \psi \alpha(1 + \nu)(1 + z)} > \\
\left( \frac{1}{R_l} (1 - \psi)^{\frac{1}{\alpha}} \chi^{-(\frac{1}{\alpha})} - \eta \right) \\
\cdot \left( 1 + \beta^{-\theta} (1 - \eta)^{1-\theta} \left( \frac{1}{R_l} (1 - \psi)^{\frac{1}{\alpha}} \chi^{-(\frac{1}{\alpha})} - \eta \right)^{-1} \right)^{-1} \\
\cdot \frac{\psi}{1 - \psi \alpha(1 + \nu)(1 + z)} > \\
\left( \frac{1}{R_l} (1 - \psi)^{\frac{1}{\alpha}} \chi^{-(\frac{1}{\alpha})} - \eta \right) \\
\cdot \left( 1 + \beta^{-\theta} (1 - \eta)^{1-\theta} \left( \frac{1}{R_l} (1 - \psi)^{\frac{1}{\alpha}} \chi^{-(\frac{1}{\alpha})} - \eta \right)^{-1} \right)^{-1} \\
\frac{\psi}{1 - \psi \alpha(1 + \nu)(1 + z)} > \left( \frac{1}{R_l} (1 - \psi)^{\frac{1}{\alpha}} \chi^{-(\frac{1}{\alpha})} - \eta \right) \\
+ \beta^{-\theta} (1 - \eta)^{1-\theta} \left( \frac{1}{R_l} (1 - \psi)^{\frac{1}{\alpha}} \chi^{-(\frac{1}{\alpha})} - \eta \right)^{\theta}. 
\]
One can see that the right-side of the equation is monotonically decreasing in \( \chi \) and the left-side is constant. Since right-side tends to \( \infty \) (0) as \( \chi \to 0 \) (\( \infty \)), a unique \( \hat{\chi} \) exists such that:

\[
\frac{\psi}{1 - \psi \alpha (1 + v)(1 + z)} > \left( \frac{1}{R_l (1 - \psi)^{-\frac{1}{\alpha}} \hat{\chi}^{-\frac{1-\alpha}{\alpha}}} - \eta \right) + \beta^{-\theta} (1 - \eta)^{1-\theta} \left( \frac{1}{R_l (1 - \psi)^{-\frac{1}{\alpha}} \hat{\chi}^{-\frac{1-\alpha}{\alpha}}} - \eta \right)^{\theta}.
\]

Hence, the condition \( v_E > v \) is satisfied when \( \chi > \hat{\chi} \). This leads to the following results:

i. The right-side is decreasing in \( \beta, \eta \) and \( R_l \). The inequality holds for sufficiently large \( \beta, \eta \) and \( R_l \).

ii. Left-side side is decreasing in \( v \) and \( z \). Hence, the condition \( v_E > v \) is satisfied for sufficiently small \( v \) and \( z \).
Appendix 17. The construct of equation (45)

Plugging equation (36) into equation (44) yields

\[ K_{Ft} = \kappa_F A_t N_{Ft} \]

Now, plugging this into equation (45) yields

\[ \frac{K_{Ft+1}}{K_{Ft}} = \frac{\kappa_F A_{Ft+1} N_{Ft+1}}{\kappa_F A_{Ft} N_{Ft}}. \]

Now, plugging equation (36) yields

\[ = \frac{\kappa_F A_{Ft+1} N_{t+1} - N_{Et+1}}{\kappa_F A_{Ft} N_t - N_{Et}}. \]

Next, recall that \( A_{t+1} = (1 + z)A_t \). Plugging this into the equation yields

\[ = \frac{(1 + z)A_{Ft} N_{t+1} - N_{Et+1}}{A_{Ft} N_t - N_{Et}} \]

\[ = (1 + z) \left( \frac{N_{t+1} - N_{Et+1}}{N_t - N_{Et}} \right) \]

Now, recall that \( N_{t+1} = (1 + v)N_t \). Equivalently, plugging this into the equation yields

\[ = (1 + z) \left( \frac{(1 + v)N_t - N_{Et+1}}{N_t - N_{Et}} \right) \]
\[= (1 + z) \left( \frac{(1 + v) - \frac{N_{Et+1}}{N_t}}{1 - \frac{N_{Et}}{N_t}} \right)\]

\[= (1 + z)(1 + v) \left( \frac{1 - \frac{N_{Et+1}}{(1 + v)N_t}}{1 - \frac{N_{Et}}{N_t}} \right)\]

\[= (1 + z)(1 + v) \left( \frac{1 - \left[ \frac{(1 + v_E)}{(1 + v)} \right]^{t+1} \frac{N_{E0}}{N_0}}{1 - \left[ \frac{(1 + v_E)}{(1 + v)} \right]^t \frac{N_{E0}}{N_0}} \right).\]

Note that for \(N_{Et}\):

\[N_{Et} = N_{E0}(1 + v_E)^t,\]

\[N_{Et+1} = N_{E0}(1 + v_E)^{t+1},\]

\[N_{Et+2} = N_{E0}(1 + v_E)^{t+2},\]

\[\vdots\]

Equivalently, note that for \(N_t\):

\[N_t = N_0(1 + v_E)^t,\]

\[N_{t+1} = N_0(1 + v_E)^{t+1},\]

\[N_{t+2} = N_0(1 + v_E)^{t+2},\]

\[\vdots\]
Appendix 18. The construct of equation (46)

First, by defining the expressions of $Y_{ft}$ and $Y_{et}$ one can move into the construction of equation (46). The aggregated $Y_{ft}$ is defined as

$$Y_{ft} = K_{ft}^\alpha (A_t N_{ft})^{1-\alpha}.$$

Then, plugging the equation (36) yields

$$Y_{ft} = K_{ft}^\alpha (A_t (N_t - N_{et}))^{1-\alpha}.$$

$$Y_{ft} = K_{ft}^\alpha (A_t (N_t - N_{et}))^{-\alpha} (A_t (N_t - N_{et})).$$

$$Y_{ft} = \left( \frac{K_{ft}}{A_t (N_t - N_{et})} \right)^\alpha (A_t (N_t - N_{et})).$$

Using the aggregated expression of equation (28) yields

$$Y_{ft} = \kappa_{ft}^\alpha (A_t (N_t - N_{et})).$$

The aggregated $Y_{et}$ is similarly defined as

$$Y_{et} = K_{et}^\alpha (\chi A_t N_{et})^{1-\alpha}$$

$$Y_{et} = K_{et}^\alpha (\chi A_t N_{et})^{-\alpha} (\chi A_t N_{et}).$$

$$Y_{et} = \left( \frac{K_{et}}{\chi A_t N_{et}} \right)^\alpha (\chi A_t N_{et}).$$

Using the aggregated expression equation (29) yields

$$Y_{et} = \kappa_{et}^\alpha (\chi A_t N_{et}).$$
Now, plugging equation (32) gives the following:

\[
Y_{Et} = \left( \kappa_{Ft} \left( (1 - \psi) \chi \right)^{-\frac{1}{\alpha}} \right)^{\alpha} (\chi A_t N_{Et})
\]

\[
Y_{Et} = \kappa_{Ft}^\alpha (1 - \psi) \chi^{-1} (\chi A_t N_{Et})
\]

\[
Y_{Et} = \frac{\kappa_{Ft}^\alpha \chi A_t N_{Et}}{(1 - \psi) \chi}
\]

\[
Y_{Et} = \frac{\kappa_{Ft}^\alpha A_t N_{Et}}{1 - \psi}.
\]

Next, by using the above expressions for \(Y_{Ft}\) and \(Y_{Et}\) in order to create the equation (46) gives

\[
\frac{Y_t}{N_t} = \frac{Y_{Ft} + Y_{Et}}{N_t}
\]

\[
= \frac{\kappa_{Ft}^\alpha \chi A_t (N_t - N_{Et}) + \kappa_{Ft}^\alpha A_t N_{Et}}{1 - \psi}
\]

\[
= \kappa_{Ft}^\alpha A_t \left( \frac{(N_t - N_{Et}) + \frac{N_{Et}}{1 - \psi}}{N_t} \right)
\]

\[
= \kappa_{Ft}^\alpha A_t \left( \frac{N_t - N_{Et}}{N_t} + \frac{N_{Et}}{(1 - \psi) N_t} \right)
\]

\[
= \kappa_{Ft}^\alpha A_t \left( 1 + \frac{N_{Et}}{N_t} \left( 1 + \frac{1}{(1 - \psi)} \right) \right)
\]
\[ = \kappa_{Ft}^{g} A_{t} \left( 1 + \frac{N_{Et}}{N_{t}} \left( \frac{- (1 - \psi) + 1}{(1 - \psi)} \right) \right) \]

\[ = \kappa_{Ft}^{g} A_{t} \left( 1 + \frac{\psi}{1 - \psi} \frac{N_{Et}}{N_{t}} \right). \]
Appendix 19. The construct of equation (47)

First, modifying the equation (35) yields

$$\kappa_{Et} = \frac{K_{Et}}{(\chi A_t N_{Et})}$$

$$K_{Et} = \kappa_{Et} \chi A_t N_{Et}.$$  

Now, plugging the expression of $\kappa_{E}$ from equation (32) gives

$$K_{Et} = \kappa_{F}((1 - \psi)\chi)^{-\frac{1}{\alpha}}\chi A_t N_{Et}.$$  

Next, plugging the above equation along with equation (44) into (47) yields

$$\rho_{t} = \frac{\rho_{E}K_{Et} + \rho_{F}K_{Et}}{K_{Et} + K_{Ft}}$$

$$\rho_{t} = \frac{\rho_{E}\kappa_{F}((1 - \psi)\chi)^{-\frac{1}{\alpha}}\chi A_t N_{Et} + \rho_{F}\kappa_{F}A_t(N_{t} - N_{Et})}{\kappa_{F}((1 - \psi)\chi)^{-\frac{1}{\alpha}}\chi A_t N_{Et} + \kappa_{F}A_t(N_{t} - N_{Et})}.$$  

Using the definition of $\rho_{E}$ from equation (16) yields

$$\rho_{t} = \frac{\left((1 - \psi)^{\frac{1}{\alpha}}\chi^{1-\frac{1}{\alpha}} R^{l}\right)\kappa_{F}((1 - \psi)\chi)^{-\frac{1}{\alpha}}\chi A_t N_{Et} + \rho_{F}\kappa_{F}A_t(N_{t} - N_{Et})}{\kappa_{F}((1 - \psi)\chi)^{-\frac{1}{\alpha}}\chi A_t N_{Et} + \kappa_{F}A_t(N_{t} - N_{Et})}.$$  

Using the fact that $\rho_{F} = R^{l}$ gives
\[
\rho_t = \frac{(1 - \psi) \frac{1}{\alpha} \frac{1 - \alpha}{R^l} \kappa_F((1 - \psi)\chi)^{-\alpha/\alpha} A_t N_{E_t} + R^l \kappa_F A_t (N_t - N_{E_t})}{\kappa_F((1 - \psi)\chi)^{-\alpha/\alpha} A_t N_{E_t} + \kappa_F A_t (N_t - N_{E_t})}
\]

\[
= \frac{(1 - \psi)\chi^{-\alpha/\alpha} R^l \kappa_F A_t N_{E_t} + R^l \kappa_F A_t (N_t - N_{E_t})}{\kappa_F A_t (1 - \psi)\chi^{-\alpha/\alpha} + \kappa_F A_t (N_t - N_{E_t})}
\]

\[
= \frac{\kappa_F A_t (R^l N_{E_t} + R^l (N_t - N_{E_t}))}{\kappa_F A_t (N_{E_t}((1 - \psi)\chi)^{-\alpha/\alpha} + (N_t - N_{E_t}))}
\]

\[
= \frac{R^l N_{E_t} + R^l N_t - R^l N_{E_t}}{N_{E_t}((1 - \psi)\chi)^{-\alpha/\alpha} + (N_t - N_{E_t})}
\]

\[
= \frac{R^l N_t}{N_{E_t}((1 - \psi)\chi)^{-\alpha/\alpha} + (N_t - N_{E_t})}
\]

\[
= \frac{R^l N_t}{N_t \left(1 + \frac{N_{E_t}}{N_t}((1 - \psi)\chi)^{-\alpha/\alpha} - \frac{N_{E_t}}{N_t}\right)}
\]

\[
= \frac{R^l}{1 + \frac{N_{E_t}}{N_t}((1 - \psi)\chi)^{-\alpha/\alpha} - \frac{N_{E_t}}{N_t}}
\]

\[
= \frac{R^l}{1 + \frac{N_{E_t}}{N_t}((1 - \psi)\chi)^{-\alpha/\alpha} - 1}
\]

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\[
\frac{R^t}{1 - \left(1 - \chi((1 - \psi)\chi)^{-\frac{1}{\alpha}}\right) \frac{N_{Et}}{N_t}}.
\]
Appendix 20. The proof of Lemma 3

Plugging the modified equation (35) (i.e. \( K_{Et} = \kappa E \chi t N_{Et} \)) and the equation (32) (i.e. \( K_{Fr} = \kappa F A_t (N_t - N_{Et}) \)) into the equation (48) yields

\[
K_{Fr} + \frac{\eta \rho_E}{R^l} K_{Et} + B_t = \zeta^w w_{t-1} N_{t-1}
\]

\[
\kappa F A_t (N_t - N_{Et}) + \frac{\eta \rho_E}{R^l} \kappa E \chi A_t N_{Et} + B_t = \zeta^w w_{t-1} N_{t-1}
\]

\[
B_t = \zeta^w w_{t-1} N_{t-1} - \kappa F A_t (N_t - N_{Et}) - \frac{\eta \rho_E}{R^l} \kappa E \chi A_t N_{Et}
\]

\[
B_t = \left( \frac{\zeta^w w_{t-1} N_{t-1}}{A_t N_t} - \kappa F \left( \frac{N_t - N_{Et}}{N_t} \right) - \frac{\eta \rho_E}{R^l} \kappa E \chi \frac{N_{Et}}{N_t} \right) A_t N_t
\]

Modifying and plugging the equation (11) in past time form (i.e. \( w_{t-1} \)) yields

\[
B_t = \left( \frac{\zeta^w (1 - \alpha) \kappa^g E A_{t-1} N_{t-1}}{A_t N_t} - \kappa F \left( 1 - \frac{N_{Et}}{N_t} \right) - \frac{\eta \rho_E}{R^l} \kappa E \chi \frac{N_{Et}}{N_t} \right) A_t N_t
\]

\[
B_t = \left( \frac{\zeta^w (1 - \alpha) \kappa^g E A_{t-1} N_{t-1}}{A_t N_t} - \kappa F \left( 1 - \frac{N_{Et}}{N_t} \right) - \frac{\eta \rho_E}{R^l} \kappa E \chi \frac{N_{Et}}{N_t} \right) A_t N_t
\]

Next, one knows that

\[
\frac{A_{t-1} N_{t-1}}{A_t N_t} = \frac{1}{(1 + z)(1 + v)}
\]

By plugging the above expression into the expression for \( B_t \) yields:
\[ B_t = \left( \frac{\zeta^w (1 - \alpha) \kappa_{Et}^a}{(1 + z)(1 + v)} - \kappa_F \left( 1 - \frac{N_{Et}}{N_t} \right) \right) - \kappa_F \left( 1 - \frac{N_{Et}}{N_t} \right) - \eta \frac{\rho_E \kappa_{Et} \chi N_{Et}}{N_t} A_t N_t. \]

Plugging the definition of \( \rho_E \) from equation (16) yields

\[ B_t = \left( \frac{\zeta^w (1 - \alpha) \kappa_{Et}^a}{(1 + z)(1 + v)} - \kappa_F \left( 1 - \frac{N_{Et}}{N_t} \right) - \eta \frac{(1 - \psi) \frac{1}{\alpha} \frac{1 - \alpha}{\chi} \frac{1 - \alpha}{\alpha} \kappa_E \chi N_{Et}}{R^l} \right) A_t N_t. \]

Using equation (32) to substitute \( \kappa_E \) yields

\[ B_t = \left( \frac{\zeta^w (1 - \alpha) \kappa_{Et}^a}{(1 + z)(1 + v)} - \kappa_F \left( 1 - \frac{N_{Et}}{N_t} \right) \right) - \frac{N_{Et}}{N_t} \eta \frac{(1 - \psi) \frac{1}{\alpha} \frac{1 - \alpha}{\alpha} \kappa_E \chi (1 - \psi) \chi}{\chi} A_t N_t. \]

\[ B_t = \left( \frac{\zeta^w (1 - \alpha) \kappa_{Et}^a}{(1 + z)(1 + v)} - \kappa_F \left( 1 - \frac{N_{Et}}{N_t} \right) - \frac{N_{Et}}{N_t} \eta \kappa_F (1 - \psi) \frac{1}{\alpha} \frac{1 - \alpha}{\alpha} \chi \right) A_t N_t. \]

\[ B_t = \left( \frac{\zeta^w (1 - \alpha) \kappa_{Et}^a}{(1 + z)(1 + v)} - \kappa_F \left( 1 - \frac{N_{Et}}{N_t} \right) \right) - \frac{N_{Et}}{N_t} \eta \kappa_F (1 - \psi) \frac{1}{\alpha} \frac{1 - \alpha}{\alpha} \chi, \]
\[ B_t = \left( \xi^W (1 - \alpha) \kappa_{Et}^{\frac{\alpha-1}{2}} \frac{N_{Et}}{N_t} \right) \kappa_{Et} A_t N_t. \]
Appendix 21. The construct of equation (41)

Using the expressions (49) and (46) for $B_t$ and $Y_t$, respectively, gives

\[
\frac{B_t}{Y_t} = \left( \frac{\zeta^w (1 - \alpha) \kappa_F^{\alpha - 1}}{(1 + z)(1 + \nu)} - 1 + (1 - \eta) \frac{N_{EL}}{N_t} \right) \kappa_{Ft} A_t N_t \\
\frac{\kappa_{Ft}^\alpha A_t N_t}{\left( 1 + \frac{\psi}{1 - \psi} \frac{N_{EL}}{N_t} \right)^2}
\]

\[
= \left( \frac{\zeta^w (1 - \alpha) \kappa_F^{\alpha - 1}}{(1 + z)(1 + \nu)} - 1 + (1 - \eta) \frac{N_{EL}}{N_t} \right) \kappa_{Ft} \\
\frac{\kappa_{Ft}^\alpha}{\left( 1 + \frac{\psi}{1 - \psi} \frac{N_{EL}}{N_t} \right)^2}
\]

\[
= \left( \frac{\zeta^w (1 - \alpha) \kappa_F^{\alpha - 1}}{(1 + z)(1 + \nu)} - 1 + (1 - \eta) \frac{N_{EL}}{N_t} \right) \kappa_{Ft}^{1 - \alpha} \\
\frac{1}{\left( 1 + \frac{\psi}{1 - \psi} \frac{N_{EL}}{N_t} \right)^2}.
\]
Appendix 22. The construct of equation (51)

Substituting $\kappa_{Et}$ by plugging equation (32) into equation (35) yields

\[
N_{Et} = \frac{K_{Et}}{\chi A_t \kappa_{Et}}
\]

\[
N_{Et} = \frac{K_{Et}}{\left(\chi A_t \kappa_{F} \left(1 - \psi\right) \chi \right)^{-\frac{1}{\alpha}}}
\]

\[
N_{Et} = \frac{K_{Et}}{\left(A_t \kappa_{F} \left(1 - \psi\right) \chi \right)^{-\frac{1}{\alpha}}}
\]
Appendix 23. The construct of equations (52) and (54)

Substituting equation (27) into equation (40) yields

\[ K_{Et+1} = \left( \frac{R^l}{R^l - \eta \rho_{Et}^E} \right) \left( 1 + \beta^{-\theta} \left( \frac{(1 - \eta) \rho_{Et}^E}{R^l - \eta \rho_{Et}^E} \right)^{1-\theta} \right)^{-1} \psi \kappa_{Et}^{\alpha} \chi A_t N_{Et}. \]

Let \( \theta \to 1 \). In this case:

\[ K_{Et+1} = \left( \frac{R^l}{R^l - \eta \rho_{Et}^E} \right) (1 + \beta^{-1})^{-1} \psi \kappa_{Et}^{\alpha} \chi A_t N_{Et} \]

\[ K_{Et+1} = \left( \frac{R^l}{R^l - \eta \rho_{Et}^E} \right) \left( \frac{\beta + 1}{\beta} \right)^{-1} \psi \kappa_{Et}^{\alpha} \chi A_t N_{Et} \]

\[ K_{Et+1} = \left( \frac{R^l}{R^l - \eta \rho_{Et}^E} \right) \left( \frac{\beta}{\beta + 1} \right) \psi \kappa_{Et}^{\alpha} \chi A_t N_{Et}. \]

Now, one can observe that the equation is similar to the equation (52) (by the use of aggregated equation (13)):

\[ (52) \quad K_{Et+1} = \left( \frac{\beta}{1 + \beta} \right) \left( \frac{R^l}{R^l - \eta \rho_{Et}^E} \right) m_t. \]

Next, by continuing the modification of the earlier equation yields

\[ K_{Et+1} = \left( \frac{R^l}{R^l - \eta \rho_{Et}^E} \right) \left( \frac{\beta}{\beta + 1} \right) \psi \kappa_{Et}^{\alpha} \chi A_t N_{Et} \]

\[ \frac{K_{Et+1}}{(1 + z)(1 + v)} = \left( \frac{R^l}{R^l - \eta \rho_{Et}^E} \right) \left( \frac{\beta}{\beta + 1} \right) \left( \frac{\psi}{(1 + z)(1 + v)} \right) \kappa_{Et}^{\alpha} \chi A_t N_{Et} \]
\[
\frac{K_{E_{t+1}}}{\chi A_t (1 + z) N_{E_{t}} (1 + v)} = \left( \frac{\beta}{\beta + 1} \right) \left( \frac{\psi}{(1 + z)(1 + v)} \right) \left( \frac{R^l}{R^l - \eta \rho^E_t} \right) \kappa_{E_{t+1}}^\alpha.
\]

Next, note that the aggregated capital per effective unit of labor of E firms is expressed as in equation (29) as follows:

\[
\kappa_E = \frac{K_E}{\chi A_E N_E}.
\]

Hence \( \kappa_{E+1} \) is as follows:

\[
\kappa_{E_{t+1}} = \frac{K_{E_{t+1}}}{\chi A_t (1 + z) N_{E_{t}} (1 + v)}.
\]

Now, by plugging \( \kappa_{E_{t+1}} \) into the equation yields

\[
\kappa_{E_{t+1}} = \left( \frac{\beta}{\beta + 1} \right) \left( \frac{\psi}{(1 + z)(1 + v)} \right) \left( \frac{R^l}{R^l - \eta \rho^E_t} \right) \kappa_{E_{t+1}}^\alpha.
\]

Finally, by plugging equation (53) to substitute \( \rho^E_t \) yields the equation (54):

\[
\kappa_{E_{t+1}} = \left( \frac{\beta}{\beta + 1} \right) \left( \frac{\psi}{(1 + z)(1 + v)} \right) \left( \frac{R^l}{R^l - \eta \alpha(1 - \psi) \kappa_{E_{t+1}}^\alpha} \right) \kappa_{E_{t+1}}^\alpha.
\]

(54)

\[
\kappa_{E_{t+1}} = \frac{\beta \psi R^l}{1 + \beta(1 + z)(1 + v) R^l - \eta \alpha(1 - \psi) \kappa_{E_{t+1}}^\alpha} \kappa_{E_{t+1}}^\alpha.
\]
Appendix 24. The construct of equation (56)

Note that in steady state $\kappa_{E_{t+1}} = \kappa_E = \kappa_E^*$ and $R^l = R$. Hence the equation (54) is as follows:

$$\kappa_E^* = \left( \frac{\beta}{\beta + 1} \right) \left( \frac{\psi}{(1 + z)(1 + \upsilon)} \right) \left( \frac{R}{R - \eta \left( \alpha(1 - \psi) \kappa_E^{(\alpha-1)} \right)} \right) \kappa_E^{(\alpha)}$$

$$\kappa_E^* \left( R - \eta \left( \alpha(1 - \psi) \kappa_E^{(\alpha-1)} \right) \right) = \left( \frac{\beta}{\beta + 1} \right) \left( \frac{\psi}{(1 + z)(1 + \upsilon)} \right) R \kappa_E^{(\alpha)}$$

$$\kappa_E^* R - \eta \alpha (1 - \psi) \kappa_E^{(\alpha)} = \left( \frac{\beta}{\beta + 1} \right) \left( \frac{\psi}{(1 + z)(1 + \upsilon)} \right) R \kappa_E^{(\alpha)}$$

$$\kappa_E^{(*)^{1-\alpha}} R - \eta \alpha (1 - \psi) = \left( \frac{\beta}{\beta + 1} \right) \left( \frac{\psi}{(1 + z)(1 + \upsilon)} \right) R$$

$$\kappa_E^{(*)^{1-\alpha}} R = \left( \frac{\beta}{\beta + 1} \right) \left( \frac{\psi}{(1 + z)(1 + \upsilon)} \right) R + \eta \alpha (1 - \psi)$$

$$\kappa_E^{(*)^{1-\alpha}} = \left( \frac{\beta}{\beta + 1} \right) \left( \frac{\psi}{(1 + z)(1 + \upsilon)} \right) + \frac{\eta \alpha (1 - \psi)}{R}$$

$$\kappa_E^* = \left[ \left( \frac{\beta}{\beta + 1} \right) \left( \frac{\psi}{(1 + z)(1 + \upsilon)} \right) + \frac{\eta \alpha (1 - \psi)}{R} \right]^{1-\alpha}. $$
Appendix 25. The construct of equation (57)

By plugging equation (56) into equation (53) in a steady state yields

\[
\rho_E^* = \alpha(1 - \psi) \left[ \left( \frac{\beta}{\beta + 1} \right) \left( \frac{\psi}{(1 + z)(1 + v)} \right) + \frac{\eta \alpha (1 - \psi)}{R^l} \right]^{\frac{1}{1 - \alpha}}
\]

\[
\rho_E^* = \alpha(1 - \psi) \left[ \left( \frac{\beta}{\beta + 1} \right) \left( \frac{\psi}{(1 + z)(1 + v)} \right) + \frac{\eta \alpha (1 - \psi)}{R^l} \right]^{\frac{1}{1 - \alpha}}
\]

\[
\rho_E^* = \alpha(1 - \psi) \left[ \left( \frac{\beta}{\beta + 1} \right) \left( \frac{\psi}{(1 + z)(1 + v)} \right) + \frac{\eta \alpha (1 - \psi)}{R^l} \right]^{-1}
\]

\[
\rho_E^* = \frac{\alpha (1 - \psi)}{\frac{\beta}{1 + \beta + z(1 + v)} + \frac{\eta \alpha (1 - \psi)}{R}}
\]