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Saara Hämäläinen
University of Helsinki and HECER

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HECER – Helsinki Center of Economic Research, P.O. Box 17 (Arkadiankatu 7), FI-00014 University of Helsinki, FINLAND,
Tel +358-2941-28780, E-mail info-hecer@helsinki.fi, Internet www.hecer.fi

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Abstract

We consider a price search model with gradual information arrival and deadlines to study how consumers search within and across stores during a single search spell. This renders the effects of search costs smooth and allows us to endogenize the intensity of competition in a new way that avoids both Diamond and Bertrand paradoxes.

Firms can commit to any choice complexity levels. They determine the relative numbers of informed and uninformed consumers, which equal in equilibrium. The outcome is thus halfway from Diamond and Bertrand equilibria. Wider price awareness and advertizing improves welfare by discouraging the prominent firm's obfuscation.

JEL Classification: D43, D83

Keywords: time cost, complexity, deadlines, obfuscation, advertising, intrafirm frictions

Saara Hämäläinen

Department of Political and Economic Studies
University of Helsinki
P.O. Box 17 (Arkadiankatu 7)
FI-00014 University of Helsinki
FINLAND

e-mail:saara.hamalainen@helsinki.fi

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1 Introduction

This paper develops a micro foundation for consumer search inside stores and analyzes the effects on welfare and its division when (i) consumers must spend time to learn a price, (ii) each firm can affect how long the learning takes on average, and (iii) consumers have a deadline that limits the time to evaluate competing offers. Welfare and its division are uniquely pinned down.

It is easy to switch from store to store nowadays. In an online search environment, this can be done by a click of a mouse from a search engine's results page. The key frictions of search thereby no longer reside *between stores*, as assumed by brick-and-mortar generation search models where search costs are paid up-front, but *inside stores*. This changes the search environment in two important respects: First, it makes the time costs of information processing in a store the main costs of search.[1] With a fast broadband internet connection, the key determinant of these time costs is not how fast computers process information but how fast consumers can do it.[2] Second, it gives firms a role as controllers of the time cost for their own store. The speed of information processing by browsing consumers can be strongly affected by the way firms lay out information on their websites to their visitors. Like a research article can be either easy to read or incomprehensible, a seller's website can be either straightforward or maze-like depending on how it has been organized.[3] Also, individual price quotations (words) are easier to change than the basic template of the whole website (paper).

To study more closely the effects that arise from these changes, this article develops a novel search model in which there are search frictions only inside stores and not between stores and where firms have control over the intrafirm frictions in their store. To illustrate the underlying idea, one could think, for example, that a firm's total price is composed of several individual components $f_1 + ... + f_n$ or that the product has multiple key attributes $(f_1, ..., f_n)$, that all need to be attended to individually before buying. By changing the dimensionality of the problem, n, a firm can thus choose how complicated consumer search becomes in its store. This easily creates a challenging combinatoric problem.[4]

To simplify, this paper uses therefore a more reduced form approach: we approximate the problem continuously by letting each firm freely commit to the rate θ of the Poisson

[1] For the classic article that introduced the theory of time allocation into economics, emphasizing the opportunity cost of time, see Becker (1965).

[2] By this we refer for example to the limits of cognitive processes of information recognition, understanding, memorizing and retrieval for later use.

[3] Our premise is that the time it takes to find a price depends statistically on information complexity generated by the firm. Data support this. Rubinstein (2007) offers one example in an experimental setup where they present data on response time in problem solving with variable hardness. Reutskaja et al. (2011) use eye tracking to measure the consumer search process in a "supermarket experiment" with changing time pressure and varying numbers of products.

[4] See Spiegler (2016) for a review of endogenous choice complexity and consumer obfuscation and Hämäläinen (2016) for an example of this more complex combinatoric approach to a related problem.

process $P(\theta)$ according to which a consumer finds its price in its store.[5] These observable rates are important to consumers who search under fixed deadlines, which makes them sensitive to both money and time costs. Deadlines could arise from, say, co-ordination with others, convex preferences, contractual clauses, etc. Here, we apply fixed deadlines especially to keep a consumer's dynamic problem simple and smooth so that the relation between frictions, prices, and search remains tractable and mostly continuous.[6]

We thence obtain a setup which resembles brick-and-mortar search models like Varian (1980) but with the important exception that the typically exogenous partition of consumers into informed (who find two prices), uninformed (who find one price), and the rest becomes more refined and completely endogenous: it depends on the order in which consumers are searching the firms and on how complicated the acquisition of information has been made by these firms. Furthermore, additional choice complexity results in reduced trading because fewer consumers discover sufficient information, which here makes them incapable or unwilling to complete a purchase. These crucial margins have been ignored by previous, seminal obfuscation models building on Stahl (1989), like Wilson (2010) and Ellison and Wolitzky (2012), where search costs have also been endogenous.

Our methodology innovation is to show that, by taking the information reducing effects of strategic complexity properly into account, we can endogenize the intensity of competition and clarify the relation between frictions to inefficient consumer outcomes. The incidence of trading failure and the related surplus loss is here a continuous function of information complexity in the market. We can therefore get a clearer picture on the effects of frictions on welfare (trades) and its division (prices) than previous work.

The novel economic discovery of this paper is that firms have incentives to make sure that consumers become informed exactly to the point where equally many of them purchase a product after observing one price and after comparing two prices. This particular mix alleviates price competition the most. The desirable information outcome can be achieved by different means: by making independent search more difficult (by "obfuscation") or by providing consumers with additional prior information (by "advertizing").

Starting with a basic duopoly model, we discover that competitive obfuscation results only in very limited welfare loss: The first key tradeoff is that, if the firm sets too high frictions, it loses the highly profitable prominent role, that fastest service would otherwise grant in the market. Specifically, to gain uncontested market prominence the firm has to have a rate $\theta > 2.3$, that guarantees that 90 % of the consumers who start from there find at least its price; this puts an upper bound on the number of frustrated consumers

[5]We could also talk about the rates at which consumers discover relevant products and gain sufficient information or understanding about the price. "Consumer finds a price" is a shorthand for all that.

[6]To introduce desirable continuity properties in a price competition setup, literature has typically relied on either continuous distributions for heterogeneous characteristics, e.g., Rob (1985) and Stahl (1996), or capacity and mobility constraints with symmetric mixed strategies, e.g., Peters (1984). Here we use deadlines and continuous, gradual information arrival.

who fail to purchase anything. The second tradeoff of a prominent firm is that lower frictions allow more consumers to learn about its price but leave them also more time to evaluate competing offers. This efficiency and extraction tradeoff results in low but positive frictions.

By contrast, the non-prominent firm's incentives are aligned partially with the prominent firm and partially with consumers. We can thus prove that there exist two mirror image equilibria where the numbers of informed consumers and uniformed consumers are the same and price competition at an intermediate level. The unique duopoly outcome lies halfway from Diamond equilibrium (the monopoly case, only uninformed consumers) and Bertrand equilibrium (the competitive case, only informed consumers).

In other words, our new model specification which is motivated by online search yields a natural refinement to the existing consumer search models, that excludes both paradoxes by Diamond and Bertrand. This interior outcome results from to the double-sided commitment that intrafirm frictions give to firms, on the one hand, and that deadlines give to consumers, on the other hand. Commitment on the consumer side helps them to avoid the Diamond paradox whereas commitment on the firm side helps them to avoid the Bertrand paradox. Choice complexity settles at a natural level although no exogenous costs are involved in adjusting complexity. Previously, obfuscation literature (e.g., Wilson (2010) and Ellison and Wolitzky (2012)) has found that firms have incentives to generate search frictions but has not specified how much and what limits this.

We obtain sharp predictions about obfuscation. Both equilibria feature a faster, prominent firm and a slower, non-prominent firm. This noticeable difference guarantees that consumers search efficiently from the former to the latter. Firms optimally readjust their frictions until a clear prominence order arises in the search market. As an important new observation that might help competition authorities to focus their attention on the right key players, we also show that the size of trading surplus is determined by the prominent firm's incentives and its division by the non-prominent firm's incentives. In our Poisson setting, approximately 6 % of the cake is lost. The prominent firm, the non-prominent firm, and consumers divide the remaining surplus in proportions 2:1:1, respectively. We could thus say that the market endogenously settles in a "compromise".

We generalize our model to study comparative statics between obfuscation and advertizing, which apparently serve the opposite roles in controlling information availability. The nature of price awareness turns to be of significance: increase in narrow price awareness (price information about the prominent firm only) increases obfuscation by the prominent firm whereas additional general price information (price information about both firms or the non-prominent firm) decreases obfuscation by the prominent firm and, thus, improves welfare.

To analyze a market with three firms, we extend the symmetric framework in Baye et al. (1992), replacing their continuum of asymmetric equilibria by a finite number;

the uniqueness of equilibrium arises under strong information asymmetry. We find that competition expands information in that the numbers of fully and partially informed consumers exceed the number of uninformed consumers. Vanishing captive demand and more elastic demand from informed consumers imply that the last prominent firm benefits more from a fast service rate.

This paper is organized in the following way. We next discuss most related literature and then lay out the model in Section 3. This section describes optimal search and prices and defines their relation to intrafirm frictions. We also consider the effects of improving search efficiency. The equilibrium is constructed in Section 4 starting from the finding that Bertrand and Diamond equilibria cannot arise in duopolies. We generalize our basic model in Section 5 to consider the effects of advertizing and competition on obfuscation. Using a closely related all-pay-auctions model, we also provide some additional applications of strategic complexity to politics, innovation, and lobbying. Section 6 offers a concluding discussion. Proofs appear in Appendix.

2 Literature

Our paper is closely related to literatures on (i) oligopolistic price competition and equilibrium price dispersion (e.g., Butters (1977), Salop and Stiglitz (1977), Reinganum (1979), Burdett and Judd (1983), Morgan et al. (2004), Baye et al. (2006a), and Baye et al. (2006b)), and (ii) endogenous search frictions, strategic complexity and consumer obfuscation (e.g., Ellison (2005), Gabaix and Laibson (2006), Ireland (2007), Carlin and Manso (2011), Piccione and Spiegler (2012), and Chioveanu and Zhou (2013)).

The nature of equilibria in oligopoly games is known to depend greatly on the strategic variables available in the model. Research suggests that a certain degree of precommitment is crucial to avoid the paradox results by Bertrand (1883) and Diamond (1971).[7] For example, Kreps and Scheinkman (1983) find that a Bertrand setting where capacity is fixed results in a Cournot outcome, where the price is above the competitive price but below the monopoly price. Here time is the limited resource.[8]

Equilibrium price dispersion arises with ex ante identical consumers, for instance, if consumers precommit to sample a fixed number of prices (Burdett and Judd, 1983) or some commit to search at least one price (costly searchers in Stahl (1989)) and others always search all prices (costless shoppers in Stahl (1989)). Particularly, Stahl (1989) observes that depending on the exogenous numbers of shoppers and searchers, equilib-

[7] In the Bertrand case, firms compete their prices down to the marginal cost (result: firms receive no profits) whereas, in the Diamond case, firms elevate their prices up to the monopoly price (result: consumers stay at home).

[8] For the role of limited capacity in avoiding Bertrand outcomes, see also competitive search literature started by Peters (1984). Geromichalos (2014) finds that gradual relaxation of capacity restrictions leads back to the Bertrand case.

rium price distribution spans continuously from the Bertrand case (only shoppers) to the Diamond case (only searchers). Stahl (1996) explores the properties of equilibria with continuously distributed search costs. Optimal search and prices pin down the numbers of consumers who find one price, two prices, etc. The lower part of the search cost distribution determines competition intensity via the number of shoppers. Non-shoppers do not fully participate in Janssen et al. (2005) and higher search costs can thus reduce prices.

Here commitment assumptions are taken to a new level. We demonstrate that with endogenous consumer information and double-sided commitment, to deadlines for consumers and to intrafirm frictions for firms, the unique equilibrium outcome (up to re-indexing of the firms) lies exactly in between Bertrand and Diamond outcomes.[9] To put it another way, our paper introduces commitment to deadlines as an alternative way to avoid the issue of non-existence in Diamond (1971), which finds that in a homogenous environment even the smallest positive search cost eliminates price dispersion and consumer search because of the induced hold-up problem. Because stochastic price discovery in our model always leads to some ex post information differences, price dispersion arises with ex ante homogenous consumers. Our setup thus nests the workhorse search models of Varian (1980) and Stahl (1989) and endogenizes the key parameter, the share of informed consumers.[10]

More obviously, our paper is a study about how consumers search within and across stores, during a single search spell. Search frictions inside stores are analyzed also by Petrikaite (2017) and Hämäläinen (2016). Most other papers treat a firm more like a black box, ignoring the internal structure and consumer incentives in a store.[11] Akin and Platt (2014) analyze search between stores with deadlines and show that more limited ability to recall observed prices can improve consumer welfare. We focus on brief spells of search where it is easy to recall past prices, say, by opening a new browser tab for each firm to compare the best prices when time is up. Their model also belongs to a different model class. Yet, limited recall ability seems like a natural restriction on consumer information and works quite similarly as obfuscation in controlling the availability of price information over search path (obfuscation limits information that could later enter into the consumer's

[9]Note that the deadline could be one second or one minute: anything positive is enough; we only need to tremble a bit away from the costly search assumption.

[10]Giulietti et al. (2014) estimate search frictions in the British electricity market and model the symmetric pricing behavior by entrants. The incumbent's market share ranges from 41 to 64 % and its margins are about twice the lowest margins. Search costs include the cost of estimating annual consumption and that of finding the cheapest supplier. They must be relatively high to rationalize switching behavior: one fifth of consumers switches each year.

[11]However, there is an extensive retailing literature on the effects of store environment on shopping behavior, e.g., Degeratu et al. (2000) and Donovan et al. (1994). Liang and Lai (2002) observe that consumers are more likely to visit well-designed online stores (transparent organization, ease of placing orders, etc.). Kumar et al. (2004) find that complexity of the search task, technology and behavioral factors determine search performance in electronic shopping.

awareness, limited recall ability erases previously observed information from information sets). Future work could take up these ideas.

In the obfuscation literature, the closest articles are Wilson (2010) and Ellison and Wolitzky (2012). In an asymmetric model, Wilson (2010) finds that duopolies have a well known non-obfuscating firm and a less known obfuscating firm. Although it hurts it relative to its peer, one of the firms is willing to assume the non-prominent role because specialization allows the firms to divide the market more peacefully.[12] In a symmetric model, Ellison and Wolitzky (2012) show that, if a consumer's search cost is convex in search time,[13] a firm has an incentive to increase the time the consumer searches inside its store because doing so raises the cost of search for the next store that the consumer might subsequently visit. This makes the hold-up problem stronger.

Although these motives for obfuscation are quite close to ours, previous welfare analysis is restricted. Particularly, neither Wilson (2010) nor Ellison and Wolitzky (2012) endogenizes the availability of information, which is our main concern in this article. Both assume that making prices harder to find has no penalty effect on firms through fewer consumers who are willing or capable of purchasing from them. This lack of penalty is apparently the primary reason why their models could have many equilibria with different welfare properties.

Our more precise analysis shows that equilibrium obfuscation has relatively limited effects on welfare although it could have large competition reducing effects.[14] There has been recently waking interest in such natural limits to equilibrium obfuscation. For instance, Gamp (2016) studies the effects of uninformed purchases (buying without knowing the price nor the match value) on market prices, product design, and obfuscation. Obfuscation encourages uninformed purchases but deteriorates consumers' expectations about their products.[15,16]

[12]There is hence a tendency for firms to differentiate themselves vertically. Motta (1993) shows the incentive is generally strongest under price competition.

[13]With a deadline at $t = q$, search costs feature an extreme form of convexity here: they are 0 for $t < q$ (before the deadline) and ∞ for $t \geq q$ (after the deadline).

[14]Taylor (2017) shows that obfuscation could even increase welfare acting as a sorting device. This sorting role of search costs appears first in Petrikaite (2017).

[15]Our findings can also be juxtaposed with papers about market prominence. If lower search costs or faster consumer service is interpreted as a vertical quality variable, as would be natural, our findings rhyme with those in Armstrong et al. (2009). They show that, without vertical product differences, prominent firm has lower prices and profit (e.g., Rhodes (2011)) but, with them, it is the other way (e.g., Arbatskaya (2007)). We find that the first prominent store is faster and has therefore also higher prices and profit.

[16]Our model has also connections with competitive search models *à* la Peters (1991), Moen (1997), and Burdett et al. (2001): While we analyze a market where firms commit to search frictions that indirectly advertize their price, competitive search models explore a market where firms commit to prices that indirectly advertize their search frictions. In both cases, these frictions are modeled by a Poisson process.

3 Model

There are two firms $i \in \{1, 2\}$ selling identical products and a unit mass of consumers looking for one product each. Both firms have the same constant unit production costs normalized to zero and the consumer valuation for a product is set to one. So far all is standard. We next make two new assumptions relative to previous literature, however.

First, we endow all consumers with a finite time budget for doing their shopping. To simplify, all consumers are assumed have the same deadline equal to unity. That is, there are no search costs before the deadline but infinite costs thereafter.[17] Obviously, this implies that consumers no longer follow a standard static stopping rule.[18] As shopping is costless up to the deadline, all consumers search for better products for a unit of time and then stop to buy the best product they have so far discovered. Our new assumption thus grants them a degree of search commitment and makes consumers less sensitive to hold-up problems in the first store, which usually arise in costly sequential search.[19]

A deadline could arise, for example, because

- Sometimes the consumer actually has a deadline prior to which the purchase has to be made: a flight ticket must be reserved before the plane takes off, a birthday present must be purchased before the party, the ingredients are needed before the dinner is cooked, which should occur before everybody is starving, etc.
- To coordinate with the rest of the society, it is customary to organize one's daily life so that some hours are allocated to work and other hours to more leisurely activities like shopping, etc. With several pressing matters on the to-do list, there is often a limit on how much time can be allocated to completing one purchase.
- With convex preferences, many people enjoy doing different activities in moderate amounts, avoiding extremes: shopping might be a pleasure first but become a nuisance later, say, after two hours of shopping.
- To take a behavioral perspective, consumers with time inconsistent preferences, who tend to shop excessively long from the point of view of their normal selves, may use a deadline to restrict their shopping behavior.

Second, we let firms compete over consumers' restricted time resources. To study firms' basic incentives to make adjustments to their intrafirm search technology, we give them

[17] We explore a resembling setup where the deadline can be either random or fixed in a companion paper (Hämäläinen, 2016). In the first case, consumers search until they are hit by a random deadline shock (they get fed up) whereas, in the second case, consumers search until they hit their fixed deadline (they run out of time). In this paper firms have several potentially interesting products in their store and the expected price consumers get from a firm is decreasing in the time they spend on the firm.

[18] See Weitzman (1979) or Wolinsky (1986).

[19] See Diamond (1971) for a classic example.

full control over the search frictions that consumers face in their store. Specifically, we let each firm i freely commit to any observable intensity $\theta^i \in [0, \infty]$[20] of the Poisson process $P(\theta^i)$, which governs how fast consumers find products in its store: loosely speaking, we thus assume that, if a consumer searches in a given store for a very short time interval $dt = t_1 - t_0 > 0$, her probability of finding the product that the firm has from t_0 to t_1 is $\theta^i dt$. We consider the limiting case where this interval becomes infinitesimal, $dt \to 0$.

In general, we recommend that it is best to view θ^i as a reduced form metaphor of information complexity generated by firm i. In practice, this could involve some aspects of customer service, or refer to price, store, or generally choice complexity (see Spiegel, 2016). For example, one could think that a firm's total expected price is composed of multiple individual components $f_1^i + ... + f_n^i$ (base line price and the add-ons, discounts for special groups, shipping fees, support costs if something goes wrong later, etc.) or that to identify relevant products the consumer needs to check through multiple features $(f_1^i, ..., f_n^i)$ before she can consider buying the product (Is the size and the color right? Is it machine-washable at 60 degrees? Does the brand have a reputation for quality? etc.).[21,22]Assuming here that f_k^i's $(k = 1, ..., n)$ are observed one by one, search complexity depends then on the dimensionality of the search problem, n, and the webpage's navigation parameters, which can render the (shortest) click paths from f_l^i to f_{l+1}^i $(l = 1, ..., n-1)$ either longer or shorter. To capture these related ideas by one model, we apply a parsimonious interpretation which assumes that effects of customer service and information complexity in a firm's store can be continuously approximated by the right choice of θ^i.

We thus assume that a firm can either put sand or oil in the wheels for search inside its store. Nevertheless, since rates θ^i affect the order in which consumers search in different stores, in doing so, the firm has to keep in mind that lower rates are less attractive to consumers and can hence place it at the bottom of their search order. Firms are therefore competing in frictions θ^i and in prices $p^i \in [0, 1]$. Equilibrium pricing strategies are generally randomized.[23] We denote by $F^i \in \Delta[0, 1]$ the price distribution for store i.

Consumer search is then a gradual random process, which takes place in one of the stores at a time. For every point in time $t \in (0, 1)$, a consumer decides whether to search

[20]Note that we have included in the choice set the boundary values of $\theta^i = 0$ (finding the price is almost impossible) and $\theta^i = \infty$ (finding the price is almost immediate). This ascertains that the firm's choice set is not only convex but also compact.

[21]See, e.g., Spiegler (2006) for complexity related and behavioral aspects of search and Gabaix and Laibson (2006) for add-on pricing. In these cases consumers only observe one product attribute or a single price component, f_k^i. In our model, consumers have more stamina in the sense that they search until the price is found.

[22]Even when products are non-homogenous, consumer behavior might be well-described by a 0-1-match value setup where search is about identifying the cheapest product meeting some criteria. Our model corresponds to a simplified case where all firms are known to have exactly one suitable product for each one consumer.

[23]See Lemmata 4 and 2.

in store $i = 1$ or in store $i = 2$. In store $i = 1$, the price, p^1, is found at rate θ^1 whereas, in store $i = 2$, the price, p^2, is found at rate θ^2.[24] A consumer's search cost is zero for $t \leq 1$ (before the deadline) and infinite for $t > 1$ (after the deadline). Consumers can freely move from store to store and recall earlier prices without any costs or delay.[25]

The precise timing is:

1. Firms set rates $\boldsymbol{\theta} = (\theta^1, \theta^2)$, which then become public.

2. Firms choose prices $\mathbf{p} = (p^1, p^2)$, that have to be found.

3. Consumers search optimally from $t = 0$ to $t = 1$ in one store at a time.
   When time is up, $t = 1$, consumers buy the cheapest observed product.

Thus, we have a three stage extensive game with a dynamic program embedded in the final stage, or, equivalently for this case, a two stage game where, first, the firms publicly commit to the frictions and, then, the firms choose their randomized pricing strategies and the consumers select their sequential search strategies.

3.1 Search

The game next is solved by *backwards induction*. Without loss of generality, we assume that $\theta^1 \geq \theta^2$. Thereby, we denote the expected price in the *faster* store by $E[p^1]$ and the expected price at the *slower* store by $E[p^2]$. The expected minimum of both prices is denoted by $E[p_{min}]$.

Heuristically, the problem of a consumer can be captured by the Bellman equation, which gives the value of searching at time t on the condition that the consumer has not found a price yet:

$$V_t = \max_{i=1,2} V_t^i, \tag{1}$$

where

$$V_t^i = \theta^i dt \left((1 - e^{-\theta^{-i}(1-t)}) (1 - E[p_{min}]) + e^{-\theta^{-i}(1-t)} \left(1 - E\left[p^i\right]\right) \right) + (1 - \theta^i dt) V_{t+dt}.$$

[24] Wait times until a price is found are thus drawn from exponential distributions: $Exp(\theta^1)$ and $Exp(\theta^2)$.

[25] This gives our model a slight flavor of a Poisson bandit problem (see Bergemann and Välimäki (2006) for a compact review) where each store represents an "arm". We operate, however, exceptionally without discounting and with a finite time horizon. There is also no usual tradeoff between exploitation and exploration because the arms have a known expected value and they break up after the price is found.

The consumer chooses between searching for price p^1 at store $i = 1$ (found at rate θ^1) and searching for price p^2 at store $i = 2$ (found at rate θ^2). The former gives V_t^1 and the latter yields V_t^2 at time t.

These values are determined by the following basic features of search: If a consumer searches in firm i's store during a short interval dt, the probability that she finds its price is $\theta^i dt$ whereas the complementary probability that no price is found is $1 - \theta^i dt$; the value of continuing the search is V_{t+dt}. When the first price is discovered, the consumer obviously switches immediately to the other firm's store in attempt to find also the other price and continues searching there either until she finds that price or until the deadline arrives. As a result, if the first price is observed at time t, the consumer ends finding exactly one price with probability $e^{-\theta^{-i}(1-t)}$ and exactly two prices with probability $1 - e^{-\theta^{-i}(1-t)}$. In the former case, the consumer buys the product for p^i, which is the only price she has found. In the latter case, the consumer obtains the product for the minimum of p^i and p^{-i}, that is, p_{min}.

To simplify the following analysis, we assume next that, if both stores look equally attractive initially, i.e., if $V_t^1 = V_t^2$ for $t = 0$, one half of the consumers start their search from each of them. Moreover, if no reason to switch the stores arises, i.e., if $V_t^1 = V_t^2$ for $t > 0$, a consumer continues to search in the store where she is at the moment.

To characterize consumer search behavior, it therefore remains to determine only how many consumers start from each firm and whether they have a strict incentive to switch the store at some intermediate time point $t \in (0, 1)$ before their first price discovery. Conveniently, we can show that consumer incentives are basically stationary:

Lemma 1 *Consumers switch the firm only when a price is found.*
(i) If $\theta^i (1 - E[p^i]) > \theta^{-i} (1 - E[p^{-i}])$, *all consumers start from firm* $i = 1, 2$.
(ii) If $\theta^1 (1 - E[p^1]) = \theta^2 (1 - E[p^2])$, *consumers may start from either firm.*

The choice of the first store is thus "myopic". The effect of the deadline vanishes because of the common continuation value $V_{t+dt}^i = V_{t+dt}$ for $i = 1, 2$, before a price is found, and because the probability of observing both prices does not depend on the chosen search order. Note that, if this were a standard bandit problem, the value of each (arm) firm would be proportional to $\theta^i (1 - E[p^i])$.[26]

Thus, whether the consumer first goes to firm $i = 1$ or firm $i = 2$ only depends on expected prices and search frictions but not on how much time is left. Likewise, even if there is a tradeoff between frictions and prices, $\theta^i > \theta^{-i}$ and $E[p^i] > E[p^{-i}]$, consumer attitude towards it remains constant over time $t \in [0, 1]$. Consumers have therefore no incentive to switch between stores before a price is found.

This entails that consumer strategy can be represented by the fraction of consumers,

[26] This together with the terminal condition pins down one solution for the related differential equation. For more on Exponential-Poisson bandit problems, see, e.g., Keller et al. (2005).

s^1, who start from firm $i = 1$. The rest of them, $s^2 = 1 - s^1$, start from firm $i = 2$. The two of these are captured together by $\mathbf{s} = (s^1, s^2)$.

Note that in contrast to the usually exogenous consumer partition as in Varian (1980) and Stahl (1989), the interplay of frictions $\boldsymbol{\theta}$ and consumer strategy $\mathbf{s}$ now partitions the set of consumers *endogenously* into disjoint sets

$$B_0 + B_1 + B_2 + B_{1,2} = 1,$$

where consumers B_0 fail to find any price, consumers B_1 and B_2 ("uninformed consumers" or "captives") find just one of the two prices, p^1 or p^2, and consumers $B_{1,2}$ ("informed consumers" or "shoppers") have time to find both.[27]

The number of consumers observing no price is

$$B_0 = s^1 e^{-\theta^1} + s^2 e^{-\theta^2},$$

and, hence, the number of trades is equal to

$$1 - B_0 = 1 - s^1 e^{-\theta^1} - s^2 e^{-\theta^2}.$$

Given that there are no costs in this game, the number of trades is also the only measure of market efficiency. Notice that full market efficiency, $B_0 = 0$, requires both that all consumers start from the faster firm, $s^1 = 1$ (the right search order starting from the fastest firm, "efficient search"), and that the faster firm serves them immediately, $\theta^1 = \infty$ (no intrafirm frictions in the faster store, "efficient service").

The numbers of captives to each firm are now given by

$$B_1 = s^1 \theta e^{-\theta} \text{ and } B_2 = s^2 \theta e^{-\theta}, \text{ if } \theta = \theta^1 = \theta^2, \tag{2}$$

and

$$\begin{aligned} B_1 &= s^1 \int_0^1 e^{-\theta^2(1-\tau)} \theta^1 e^{-\theta^1 \tau} d\tau = s^1 \frac{\theta^1}{\theta^2 - \theta^1} \left(e^{-\theta^1} - e^{-\theta^2}\right), \\ &= \frac{\theta^1}{\theta^1 - \theta^2} \left(e^{-\theta^2} - B_0\right), \text{ if } \theta^1 \neq \theta^2, \end{aligned} \tag{3}$$

and

[27] By contrast, for example in Ellison and Wolitzky (2012) with two firms, these measures would be $B_0 = 0$, $B_1 = B_2 = \frac{1-\mu}{2}$ and $B_{1,2} = \mu$, where the measure of shoppers is $\mu \in (0, 1)$.

$$B_2 = s^2 \int_0^1 e^{-\theta^1(1-\tau)} \theta^2 e^{-\theta^2 \tau} d\tau = s^2 \frac{\theta^2}{\theta^1 - \theta^2} \left(e^{-\theta^2} - e^{-\theta^1} \right),$$
$$= \frac{\theta^2}{\theta^1 - \theta^2} \left(B_0 - e^{-\theta^1} \right), \text{ if } \theta^2 \neq \theta^1. \tag{4}$$

Above, $e^{-\theta^i \tau}$ is the probability that the consumer does not find p^i during time interval $t \in [0, \tau]$, $\theta^i d\tau$ is the probability that the consumer observes this price exactly at moment $t = \tau$, and $e^{-\theta^{-i}(1-\tau)}$ is the probability that the consumer does not find p^{-i} during time interval $t \in [\tau, 1]$. The shoppers are then just the residual

$$B_{1,2} = 1 - B_0 - B_1 - B_2,$$
$$= 1 - B_0 \left(1 - \frac{\theta^1}{\theta^1 - \theta^2} + \frac{\theta^2}{\theta^1 - \theta^2} \right) - \frac{\theta^1}{\theta^1 - \theta^2} e^{-\theta^2} + \frac{\theta^2}{\theta^1 - \theta^2} e^{-\theta^1},$$
$$= 1 - \frac{\theta^1 e^{-\theta^2} - \theta^2 e^{-\theta^1}}{\theta^1 - \theta^2}, \tag{5}$$

which shows how the effect of market efficiency through $B_0(s^1)$ vanishes. Interestingly, expressions (3) and (4) also suggest that the faster firm and the slower firm could have conflicting incentives regarding efficiency: decreasing market inefficiency $B_0(s^1)$ gives more captives to the faster firm but less captives to the slower firm.

Above notions will be used repeatedly in solving the firm's problem. It is clear from there that $\frac{\partial B_{1,2}}{\partial s^1} = 0$, $\frac{\partial B_0}{\partial s^1} < 0$, $\frac{\partial B_1}{\partial s^1} > 0$ and $\frac{\partial B_2}{\partial s^1} < 0$. In consequence, if consumer search becomes more efficient, the number of shoppers does not change but the number of trades increases and the faster (slower) firm gains more (less) captives.

3.2 Prices

For any consumer partition $\{B_0, B_1, B_2, B_{1,2}\}$, the profit Π^i that firm i obtains has, as standard, a price-insensitive part and a price-sensitive part:

$$\Pi^i(p^i) = \left(B_i + B_{1,2}(1 - F^{-i}(p^i)) \right) p^i.$$

There is inelastic demand B_i from captives who observe one price and elastic demand $B_{1,2}(1 - F^{-i}(p^i))$ from shoppers who compare two prices.

The equilibrium price distribution can now be calculated as in Varian (1980), Stahl (1989), and Ellison and Wolitzky (2012) for symmetric cases (no atoms) and much like in Wilson (2010) for asymmetric cases (one atom). In anticipation of our subsequent findings, we use below the notation which supposes that the faster store has secured more captives, $B_1 \geq B_2$:[28]

Lemma 2 *Consider* $\boldsymbol{\theta} = (\theta^1, \theta^2)$ *and* $\boldsymbol{s} = (s^1, s^2)$ *such that* $B_1 \geq B_2$, $B_1 > 0$, *and* $B_{1,2} > 0$. *Then, there exists a unique equilibrium price distribution* $\boldsymbol{F} = (F^1, F^2)$ *where*

$$F^1(p) = \frac{B_2 + B_{1,2}}{B_{1,2}} - \frac{\Pi^2}{B_{1,2}} \frac{1}{p} \text{ for all } p \in \left[\underline{p}, 1\right),$$

with an atom $\alpha := \frac{B_1 - B_2}{B_1 + B_{1,2}} \leq \underline{p}$ *at the highest price* $\overline{p} = 1$, *and*

$$F^2(p) = \frac{B_1 + B_{1,2}}{B_{1,2}} - \frac{\Pi^1}{B_{1,2}} \frac{1}{p}, \text{ for all } p \in \left[\underline{p}, 1\right].$$

The lowest price is given by $\underline{p} = \frac{B_1}{B_1 + B_{1,2}}$ *and the firms' profits by*

$$\Pi^1 = B_1 \text{ and } \Pi^2 = \underline{p} B_2 + (1 - \underline{p}) B_1 \leq B_1.$$

Observe that both Diamond equilibrium and Bertrand equilibrium could arise in our model, in principle, for suitably chosen frictions $\boldsymbol{\theta}$: if $B_{1,2} = 0$ (no shoppers; this would arise under $\boldsymbol{\theta} = (0, 0)$, or $\boldsymbol{\theta} = (a, 0)$ and $\boldsymbol{\theta} = (0, a)$, for any $a > 0$), both firms use a pure strategy $p^1 = p^2 = 1$ and, if $B_{1,2} > 0$ but $B_1 = B_2 = 0$ (no captives; this would arise under $\boldsymbol{\theta} = (\infty, \infty)$), both firms use a pure strategy $p^1 = p^2 = 0$. Later we prove, however, that two firms never set $\theta^i = 0$ or $\theta^i = \infty$ which could lead to these cases.

Generally, the store with more captives has higher prices and profit. It mixes between offering discount prices, $p^1 < 1$, to compete for shoppers, and setting the monopoly price, $p^1 = 1$, to tax its captives. The other store, who has a smaller number of captives, only charges discount prices, $p^2 < 1$. When a firm gives a discount, its size is a random draw from $\left[\underline{p}, 1\right)$ distributed according to F^1 or F^2. To keep both firms randomizing over the same interval despite different profits, F^1 must thus have an atom at unity.

Automatically, the firms' equilibrium pricing strategies are hardwired so as to let them specialize in different consumer groups. This aligns their payoffs and helps to relax the price competition. To see this payoff alignment more clearly, note that the profit to the high-profit firm, Π^1, equals the number of captives it attracts, B_1, whereas the profit to the low-profit firm, Π^2, is the weighted average of its own captives, B_2, and the other firm's captives, B_1. Furthermore, the weights, $\underline{p} = \frac{B_1}{B_1 + B_{1,2}}$ and $1 - \underline{p} = \frac{B_{1,2}}{B_1 + B_{1,2}}$, could be taken as a measure of how close the market is to Bertrand equilibrium ($\underline{p} = 0$, arises with $B_{1,2} > 0, B_1 = B_2 = 0$) and to Diamond equilibrium ($\underline{p} = 1$, arises with $B_{1,2} = 0, B_1 > 0, B_2 \geq 0$). Near the Bertrand equilibrium ($\underline{p}$ close to zero), the firms have more closely aligned preferences and, near the Diamond equilibrium ($\underline{p}$ close to one), they compete more fiercely. As it later turns out, the outcome that obtains can thus be regarded as a compromise between consumers and both of these firms. We find that in equilibrium $\underline{p} = 1/2$, $B_1 = B_{1,2}$, and $B_2 = 0$.

[28]We reverse the notation in Lemma 2 if $B_2 \geq B_1$.

It is now straightforward to calculate the expected prices for later use:[29]

$$E\left[p^1\right]=\int_{\underline{p}}^{1} pf^1(p)dp+\alpha=\frac{\Pi^2}{B_{1,2}}\ln\left(\frac{1}{\underline{p}}\right)+\alpha \tag{6}$$

$$\geq E\left[p^2\right]=\int_{\underline{p}}^{1} pf^2(p)dp=\frac{\Pi^1}{B_{1,2}}\ln\left(\frac{1}{\underline{p}}\right) \tag{7}$$

$$\geq E\left[p_{min}\right]=\int_{\underline{p}}^{1} p\left(f^2(p)\left(1-F^1(p)\right)+f^1(p)\left(1-F^2(p)\right)\right)dp$$

$$=\frac{2\Pi^1\Pi^2}{B_{1,2}^2}\left(\frac{1-\underline{p}}{\underline{p}}\right)-\frac{B_1\Pi^2+B_2\Pi^1}{B_{1,2}^2}\ln\left(\frac{1}{\underline{p}}\right). \tag{8}$$

Consumer surplus is the average of net utility to captives and shoppers:

$$CS=B_1\left(1-E\left[p^1\right]\right)+B_2\left(1-E\left[p^2\right]\right)+B_{1,2}\left(1-E\left[p_{min}\right]\right).$$

3.3 Improving search efficiency under fixed unobserved frictions

This paper studies competitive obfuscation in a market where consumers search optimally for the best deal, the higher of $\theta^1\left(1-E\left[p^1\right]\right)$ and $\theta^2\left(1-E\left[p^2\right]\right)$. There is no confusion about which deal is the better one because obfuscation is observable and consumers form accurate equilibrium beliefs about prices. This could also be interpreted as the long run limit where each firm's reputation is constant. To understand the implications of the assumption, let us assume for a while that it does not hold. If consumers have either biased information or absolutely no information when they approach the firms, the number of consumers who start from the faster store could be either too large or too small. We next suppose that a fixed share of consumers $s^1\in[0,1]$ begins the search from the faster firm. Proposition 1 analyzes the effects of increasing search efficiency for this alternative case. Such a change in consumer behavior might arise, for example, if the faster firm intensified its advertizing.

Proposition 1 *Suppose the rates θ^1 and θ^2 are fixed such that $\theta^1>\theta^1$. Consider the effects of increasing search efficiency s^1.*

1. *The faster firm's profit Π^1 increases and the slower firm's profit Π^2 decreases as search becomes more efficient.*

2. *The faster firm earns more profit than the slower firm if and only if $s^1>\frac{\theta^2}{\theta^1+\theta^2}$.*

[29] Again, these are $E\left[p^1\right]$ and $E\left[p^2\right]$ assuming $B_1\geq B_2$ whereas reverse notation is needed for $B_2>B_1$.

3. *The expected consumer surplus for average consumer is increasing for* $s^1 < \frac{\theta^2}{\theta^1+\theta^2}$ *and decreasing for* $s^1 > \frac{\theta^2}{\theta^1+\theta^2}$.

The first item shows that, while the intensity of price competition adjusts to render incentives aligned, it does so only up to a limit: the faster firm always benefits and the slower firm always suffers if more consumers start searching from the former rather than the latter. Still, the slower firm can extract a higher profit than the faster firm if consumers are so confused more than $\frac{\theta^2}{\theta^1+\theta^2}$ of them start from its store. This implies that firms' profits converge from $s^1 = 0$ up to $\frac{\theta^2}{\theta^1+\theta^2}$ and diverge from there until $s^1 = 1$. Standard symmetric pricing prevails at $s^1 = \frac{\theta^2}{\theta^1+\theta^2}$. This is the point where price competition is the strongest. The further we move from this point to either direction, the more relaxed competition becomes.

Higher search efficiency may thus have both positive and negative of effects on consumer surplus. As marked before, s^1 reduces B_0, which has an increasing effect on B_1 : $B_1(s^1)' = \frac{\theta^1}{\theta^1-\theta^2} B_0(s^1)' > 0$ and a decreasing effect on B_2 : $B_2(s^1)' = -\frac{\theta^2}{\theta^1-\theta^2} B_0(s^1)' < 0$; s^1 has no effects on $B_{1,2}$. On the positive side, consumers are therefore more likely to find prices for higher s^1, which directly shows up in the consumer surplus via $1 - B_0 = B_1 + B_2 + B_{1,2}$. On the negative side, more efficient search also increases prices through the higher total number of captives, $B_1 + B_2 = \beta(\boldsymbol{\theta}) - B_0$, where $\beta(\boldsymbol{\theta}) = \frac{\theta^1 e^{-\theta^2} - \theta^2 e^{-\theta^1}}{\theta^1-\theta^2} < 1$ is a constant that depends on frictions $\boldsymbol{\theta}$ which are fixed in this exercise.

Yet even more importantly, the increasing effect of search efficiency on $B_1 - B_2$, the positive effect on B_1 and the negative effect on B_2, alter the intensity of price competition in the market. On the one hand, for low levels of $s^1 < \frac{\theta^2}{\theta^1+\theta^2}$, firm $i = 2$ has higher profit but improving search efficiency undermines its relative position: B_2, α and $\underline{p}$ become smaller. On the other hand, for high levels of $s^1 > \frac{\theta^2}{\theta^1+\theta^2}$, firm $i = 1$ has higher profit and improving search efficiency emphasizes its relative position: B_1, α and $\underline{p}$ become larger. Ultimately, the competition relaxing effects of more pronounced firm differences, captured by the non-monotone behavior of α around $s^1 = \frac{\theta^2}{\theta^1+\theta^2}$, turn out to have the largest impact on consumer welfare.

We show in Appendix that consumer surplus can be rewritten simply as

$$CS = (1-\alpha)B_{1,2} = \begin{cases} \frac{B_2-B_1}{B_2+B_{1,2}} B_{1,2}, & \text{for } s^1 < \frac{\theta^2}{\theta^1+\theta^2}, \\ \frac{B_1-B_2}{B_1+B_{1,2}} B_{1,2}, & \text{for } s^1 > \frac{\theta^2}{\theta^1+\theta^2}, \end{cases}$$

which is increasing for $s^1 < \frac{\theta^2}{\theta^1+\theta^2}$ (as firm asymmetry α decreases) and decreasing for $s^1 < \frac{\theta^2}{\theta^1+\theta^2}$ (as firm asymmetry α increases). Figure 1 illustrates the general pattern; the red vertical line captures the symmetric point where $s^1 = \frac{\theta^2}{\theta^1+\theta^2}$. The strictly central position of this red line demonstrates that rational consumers are best off when the other consumers choose the firm somewhat randomly. With ambiguous effect on expected prices, consumers who themselves search optimally can therefore either suffer or benefit from other consumers' confusion.

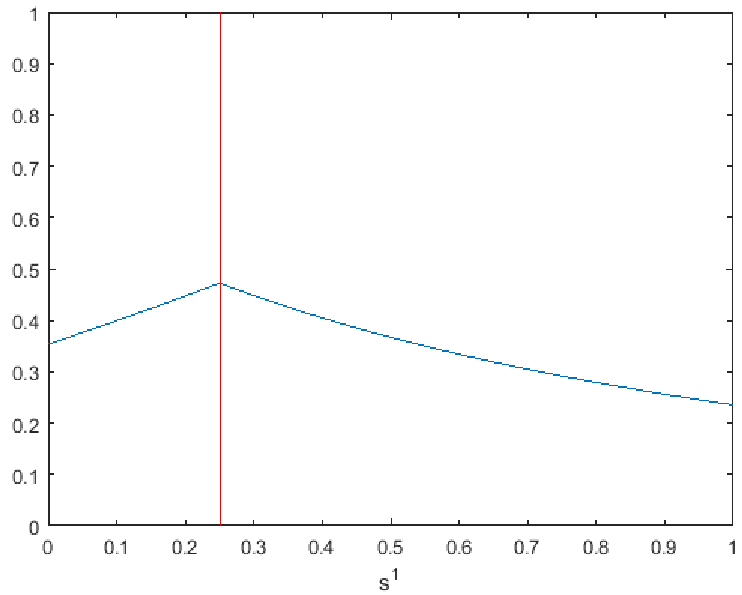


Figure 1: Consumer surplus for different values of search efficiency s^1 for $\boldsymbol{\theta} = (3, 1)$.

Observe that the effects of higher search efficiency in this paper are more general than the effects of higher market participation in Janssen et al. (2005): Both increase the number of costly searchers (here $B_1 + B_2$) while keeping fixed the number of costless shoppers (here $B_{1,2}$). However, in their symmetric equilibria, each firm obtains the same number of costly searchers ($B_1 = B_2$) whereas, in our asymmetric equilibria, the faster firm's share (B_1) increases and the slower firm's share (B_2) decreases. This entails that, in Janssen et al. (2005), additional numbers of uninformed consumers have an increasing effect on prices whereas here their effect could be increasing or decreasing. Moreover, the numbers of captives B_1 and B_2 are non-monotone in the firm's own frictions θ^1 or θ^2 in this paper, which generates new tradeoffs for demand and prices.

We find it somewhat paradox that the equilibrium in competitive obfuscation, that we soon describe, features efficient search, which is here shown to minimize consumer surplus. Seemingly, competition in frictions does more harm to consumers than it benefits them. Here increased competition in one dimension (rates θ) leads to decreased competition in another dimension (prices p), with negative effects on consumer surplus. Ignoring information about intrafirm frictions would thus be best for the average consumer. This might hint to a new way to rationalize consumer inattention to advertizing.

3.4 Fixed point in search and prices with observable frictions

We next move on to analyze observable frictions with optimal prices and search behavior. Based on the earlier analysis, we find importantly that any pair of frictions generates a unique fixed point in search and prices:

Proposition 2 *For any $\boldsymbol{\theta}$, there exists a unique fixed point in search and prices $(\boldsymbol{s}, \boldsymbol{F})$ where $\boldsymbol{F} = \boldsymbol{F}(\boldsymbol{\theta}, \boldsymbol{s})$ and $\boldsymbol{s} = \boldsymbol{s}(\boldsymbol{\theta}, \boldsymbol{F})$. In particular,*

1. *if $\theta^1 \left(1 - E\left[p^1 | (\boldsymbol{\theta}, \boldsymbol{s} = (1,0))\right]\right) \geq \theta^2 \left(1 - E\left[p^2 | (\boldsymbol{\theta}, \boldsymbol{s} = (1,0))\right]\right)$, all consumers start from the same store, $s^1 = 1 - s^2 = 1$, $B_1 > B_2 = 0$ and $E\left[p^1\right] > E\left[p^2\right]$, whereas*

2. *if $\theta^1 \left(1 - E\left[p^1 | (\boldsymbol{\theta}, \boldsymbol{s} = (1,0))\right]\right) < \theta^2 \left(1 - E\left[p^2 (\boldsymbol{\theta}, \boldsymbol{s} = (1,0))\right]\right)$, some consumers start from each store, $s^1 = 1 - s^2 < 1$, $B_1 \geq B_2 > 0$, where $\boldsymbol{s}$ is the unique solution to*

$$\frac{\theta^2}{\theta^1} = \frac{1 - E\left[p^1 | (\boldsymbol{\theta}, \boldsymbol{s})\right]}{1 - E\left[p^2 | (\boldsymbol{\theta}, \boldsymbol{s})\right]} = 1 - \alpha, \tag{9}$$

where $\alpha = \alpha(\boldsymbol{\theta}, \boldsymbol{s})$ is the atom size determined by Lemma 2.

Concerning Proposition 2 note that optimal search behavior (consumer partition B_1, B_2 and $B_{1,2}$) is determined by $\boldsymbol{\theta}$ and $\mathbf{s}$ jointly whereas pricing (distributions F^1 and F^2 and expectations $E\left[p^1\right]$ and $E\left[p^2\right]$) depends on $\boldsymbol{\theta}$ and $\mathbf{s}$ only through consumer partition B_1, B_2, and $B_{1,2}$. This feature enables us to construct a hypothetical price distribution $\mathbf{F}(\boldsymbol{\theta}, \mathbf{s}^0)$ and calculate the expected prices $E\left[p | (\boldsymbol{\theta}, \mathbf{s}^0)\right]$ for each pair of intrafirm frictions and hypothetical consumer strategies $(\boldsymbol{\theta}, \mathbf{s}^0)$ by first deriving the associated consumer partition $B_1(\boldsymbol{\theta}, \mathbf{s}^0), B_2(\boldsymbol{\theta}, \mathbf{s}^0)$, and $B_{1,2}(\boldsymbol{\theta}, \mathbf{s}^0)$, by Equations (2), (3), (4) and (5), and then the distributions of prices they induce $\mathbf{F}(B_1, B_2, B_{1,2})(\boldsymbol{\theta}, \mathbf{s}^0)$ by Lemma 2. The expectations of those price distributions (6), (7) and (8) then prompt some new consumer strategies $\mathbf{s}^1$ by Lemma 1. In a fixed point, the starting strategies $\mathbf{s}^0$ and the finishing strategies $\mathbf{s}^1$ coincide. The existence of a fixed point can be proved by the continuity of expected prices $E\left[\mathbf{p}\right]$ in consumer search $\mathbf{s}$, by spanning from one boundary case $\mathbf{s} = (1, 0)$ (where all consumers start from store $i = 1$) to another $\mathbf{s} = (0, 1)$ (where all consumers start from store $i = 2$). The uniqueness is based on the monotonicity of the problem: firms raise their prices with more captive demand but consumers prefer firms with lower prices.

Proposition 2 has two noteworthy corollaries:

Corollary 1 (Effects of frictions on market prominence) *Lower frictions grant a firm more prominent market position and higher prices and profit: the fastest firm attracts more captives, $B_1 \geq B_2$, which leads to $\Pi^1 \geq \Pi^2$ and $E\left[p^1\right] \geq E\left[p^2\right]$.*

Corollary 2 (Effects of frictions on search efficiency) *Consumers search efficiently, starting from the fastest firm, if the frictions are identical,* $\theta^1 = \theta^2$*, or if they are very different,* $\theta^1 \left(1 - E\left[p^1 | (\boldsymbol{\theta}, \boldsymbol{s} = (1, 0))\right]\right) \geq \theta^2 \left(1 - E\left[p^2 | (\boldsymbol{\theta}, \boldsymbol{s} = (1, 0))\right]\right)$.

To sum up, there are two kinds of candidate equilibria: symmetric and asymmetric ones. If the firms are equally fast, one half of the consumers start from each firm and firms make the same profit using symmetric pricing strategies whereas, if one of the firms is faster than the other, it wins a more prominent market position and has higher prices and profit.

We concentrate next on pure strategies in intrafirm frictions although it is obvious that there could exist also equilibria where the firms mix in frictions.[30] Pure strategies seem more natural, however, because we consider a game in which the frictions are common knowledge at the beginning of the following subgame where pricing and search decision are made.

Note that, by Proposition 2, we can now solve (9) to obtain a closed form expression for $s^1 = 1 - s^2$:

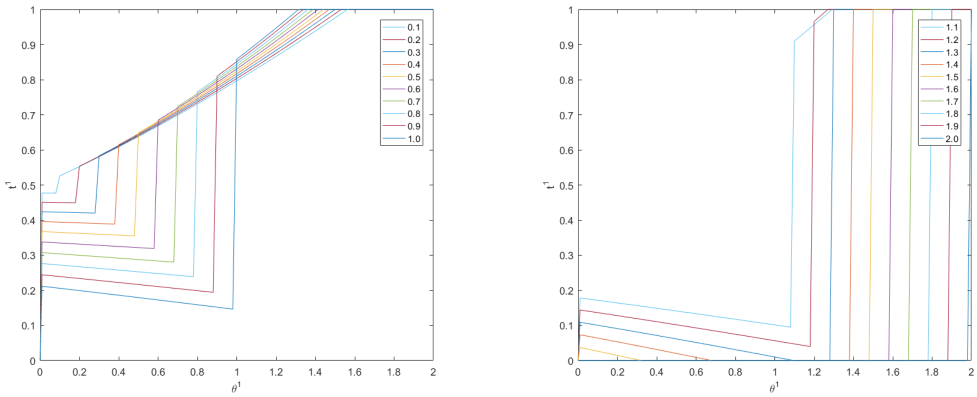
$$\frac{\theta^2}{\theta^1} = 1 - \alpha = \frac{1 - B_0 - B_1}{1 - B_0 - B_2} = \frac{1 - B_0 - B_1}{1 - B_0 - (1-\alpha) B_1},$$

$$\frac{\theta^2}{\theta^1} = \frac{1 - s^1 e^{-\theta^1} - (1 - s^1) e^{-\theta^2} - s^1 \frac{\theta^1}{\theta^1 - \theta^2} \left(e^{-\theta^2} - e^{-\theta^1}\right)}{1 - s^1 e^{-\theta^1} - (1 - s^1) e^{-\theta^2} - s^1 \frac{\theta^2}{\theta^1 - \theta^2} \left(e^{-\theta^2} - e^{-\theta^1}\right)},$$

$$\Longrightarrow s^1 = \begin{cases} \frac{1}{2} \frac{\theta^1 - \theta^2}{\theta^2} \frac{1 - e^{-\theta^2}}{e^{-\theta^2} - e^{-\theta^1}}, \text{ if } \frac{1}{2} \frac{\theta^1 - \theta^2}{\theta^2} \frac{1 - e^{-\theta^2}}{e^{-\theta^2} - e^{-\theta^1}} \in (0,1) \\ 1, \text{ if } \frac{1}{2} \frac{\theta^1 - \theta^2}{\theta^2} \frac{1 - e^{-\theta^2}}{e^{-\theta^2} - e^{-\theta^1}} \geq 1, \\ 0, \text{ if } \frac{1}{2} \frac{\theta^1 - \theta^2}{\theta^2} \frac{1 - e^{-\theta^2}}{e^{-\theta^2} - e^{-\theta^1}} \leq 0. \end{cases} \tag{10}$$

Plotting these for different values of $\boldsymbol{\theta}$ in Figure 2 shows the general pattern. When frictions are identical, one half of the consumers starts from each firm. Otherwise, the faster firm gains a larger share of first timers. Especially, note the jump in their share when a firm improves its service rate starting from a symmetric situation: to keep consumers indifferent between the firms, even a slightest (continuous) reduction in frictions must drastically increase the fraction of consumers starting from the firm. This raises the number of captives it attracts and elevates its profit and prices – by the amount which keeps it on par with its competition, $\theta^1 \left(1 - E\left[p^1\right]\right) = \theta^2 \left(1 - E\left[p^2\right]\right)$, despite its now strictly lower search costs.

[30] The firms could, for example, mix over some interval which contains the set of pure strategy equilibrium frictions $\{1.03, 2.76\}$. In practice, mixing in intrafirm frictions might involve, say, pre-announced campaign content on the firm's site making browsing more sluggish (heavy video content, surveys, new registration requirements, etc.) or, alternatively, making prices easier to observe (advertizing the price in more explicit ways).



(a) (b)

Figure 2: The fraction of consumers who start from store 1 as a function of θ^1: for different values of $\theta^2 \leq 1.0$ (a), for different values of $\theta^2 > 1.0$ (b).

4 Equilibrium obfuscation

This section contains our main results. First, we rule out the existence of Bertrand equilibrium and Diamond equilibrium and, generally, the existence of wholly or nearly symmetric equilibria where a positive number of consumers starts from each firm. We find that all equilibria feature efficient search such that a clear prominence order is generated in the search market. By Proposition 1, this asymmetry is not in the consumer's interest but relates instead to optimal firm behavior. To understand their incentives better, we then move on to describe the problems of the prominent firm and the non-prominent firm and the implications for equilibrium market conditions. We demonstrate that there exists a unique equilibrium pattern, which is invariant to extensions or reductions in consumers' deadlines.

4.1 No Bertrand equilibrium nor Diamond equilibrium

Bertrand equilibrium is avoided here because, even if one firm served all consumers immediately, the other one would benefit from committing to positive frictions, which split consumers into segments with different price information. The best response to infinitely fast competition, $\theta^{-i} = \infty$, is a finite service rate, $\theta^i = ln(2)$. In a duopoly setting, unilateral commitment possibility is sufficient to guarantee that only some consumers find both prices and, therefore, circumvents the price war that would arise if all consumers were informed about two prices. As a result, both firms make positive profits in this game.

Remark 1 *There exists no Bertrand equilibrium, where neither of the firms generates* *any frictions and the market price equals zero.*

To see why Bertrand equilibrium is eliminated, it might be useful to take a look at the game's extensive form once more. Note that our game is *not* equivalent with a strategic game in which firms choose a distribution of prices $\Delta[0,1]$ and a distribution of rates $\Delta[0,\infty]$ simultaneously and consumers choose their search strategies once and for all. In this case, Bertrand equlibrium $(\boldsymbol{\theta},\mathbf{p})=(\infty,\infty;0,0)$ is not eliminated because the other firm still gets all consumers never mind what price or frictions a deviating firm sets. Yet, even in this simultaneous moves modification, Bertrand equilibrium $(\boldsymbol{\theta},\mathbf{p})=(\infty,\infty;0,0)$ would not be robust to perturbations in $p^i=0$ or $\theta^i=0$: both would make a deviation to $p^{-i}>0$ and $\theta^{-i}>0$ profitable.

It is also important that firms cannot later readjust their frictions, when some consumers have already found a price or two.[31] That is, frictions should represent a firm's long-term investment in a particular search technology within its store. This can be motivated by observing that changing the webpage structure typically involves a larger cost than changing prices or search. However, if it was feasible to change the frictions after the other firm has fixed its price, the non-prominent firm would want to maximize its demand by serving immediately all the consumers who visit it after searching the prominent firm. If consumers knew this, they would first visit the non-prominent firm. This would make the prominence order unstable.

Because consumers are committed to searching until their deadline, we can also eliminate Diamond equilibrium as a market outcome. Search commitment guarantees that a firm always receives a positive demand, as a prominent firm or a non-prominent firm, as long as some consumers find its price. Extremely high frictions, on the other hand, would mean that all consumers are uninformed about the price of a firm, which can never be in that firm's best interest. Here uninformed means incapable or unwilling to buy.

Our later analysis shows that firms have no incentives to generate infinite frictions even if the consumers who find no price would buy in random; such non-selective consumers do not exercise much influence over firms. Furthermore, while the symmetric Diamond equilibrium would give each firm 0.5 with randomly buying consumers (interestingly, only slightly more that the unique duopoly profit for the prominent firm $\Pi^1\approx 0.47$), a deviation from $(\theta^i,p^i)=(\infty,1)$ to $(\theta^i,p^i)=(0,1-\epsilon)$ for small ϵ gives the firm almost 1.

Remark 2 *There exists no Diamond equilibrium, where at least one firm generates infinite frictions and the market price equals one.*

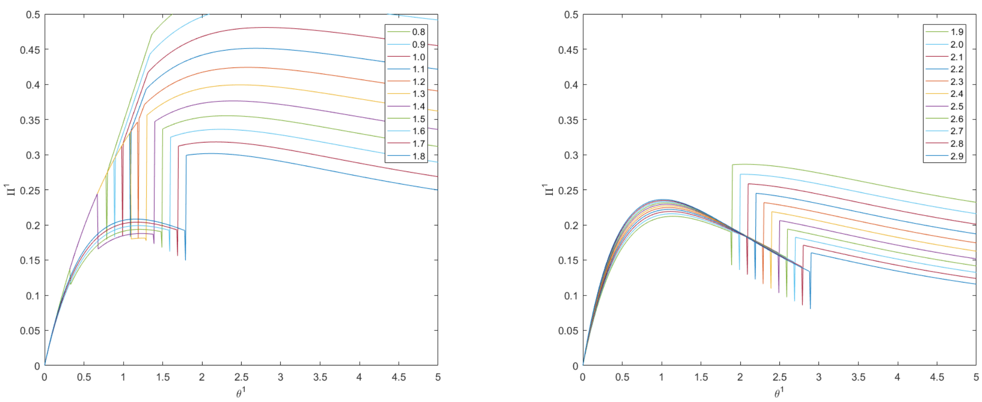
More generally we discover that there exist no symmetric equilibria or asymmetric equilibria where some consumers start from each firm:

[31] The distribution of consumer information changes along the search horizon, which can alter the firms' incentives over time if deadlines are observable. We thus prefer a more natural steady-state interpretation of our model where different consumers can be at different stages of search at the same time.

Lemma 3 *There exists no equlibrium where* $s^1 \in [0, 1)$.

Lemma 3 follows from the idea that we can think that both firms are for their respective parts choosing the value of the ratio $\rho = \theta^1/\theta^2$, keeping fixed the other firm's choice. However, since the faster firm maximizes B_1 and the slower firm maximizes its convex combination, $\underline{p}B_2 + (1 - \underline{p})B_1$ which for $s^1 < 1$ would be $B_2 = \rho^{-1}B_1$, both can never be happy at the same time if $s^1 < 1$ unless they choose the same rates and $\rho = 1$. Otherwise, the faster firm cares only about its captives, B_1, but the slower firm for a product of this, $B_2 = \rho^{-1}B_1$, and they have thus conflicting views about the best ρ.

The conflict of interest cannot be resolved before firms become so different that all consumers start from the faster firm – or remain exactly similar. This of course leaves a small gap in the proof that might allow us to construct a symmetric equilibrium (in the spirit of Ellison and Wolitzky (2012)) in addition to an asymmetric equilibrium (in the spirit of Wilson (2010)). However, as it turns out, firm i's profit has usually at least two local maxima, a higher one (with $s^i = 1$) and a lower one (with $s^i = 0$), and symmetric frictions with $s^i = 0.5$, are dominated by either of these; see Figure 3.



(a) (b)

Figure 3: The profit of firm $i = 1$ as a function of θ^1 for different θ^2.

Recall also that, when one firm is only a bit faster than the other one, the profit to the faster firm is $B_1 = s^1 \frac{\theta^i}{\theta^1 - \theta^2}\left(e^{-\theta^2} - e^{-\theta^1}\right)$ whereas, when both firms are equally fast, it is $0.5\theta^2 e^{-\theta^2}$. While the limit value of the second factor $\frac{\theta^i}{\theta^1-\theta^2}\left(e^{-\theta^2} - e^{-\theta^1}\right)$ is $\theta^2 e^{-\theta^2}$ as $\theta^1 \to \theta^2+$, the limit value of the first one s^1 is not equal to 0.5; see Equation (10). Rather,

$$\lim_{\theta^1 \to \theta^2+} s^1 = \lim_{\theta^1 \to \theta^2+} \frac{1}{2} \frac{\theta^1 - \theta^2}{\theta^2} \frac{1 - e^{-\theta^2}}{e^{-\theta^2} - e^{-\theta^1}} = \frac{1}{2} \frac{1 - e^{-\theta^2}}{\theta^2 e^{-\theta^2}} > \frac{1}{2} \text{ for } \theta^2 > 0.$$

There is thus a jump in the profit when reducing the frictions from a symmetric situation.

Corollary 3 *In any equilibrium,*

1. consumers search efficiently from the faster store to the slower store and

2. firms have a clear prominence order, which is based on intrafirm frictions.

4.2 Unique equilibrium between Diamond and Bertrand

By Lemma 3, any equilibrium where firms use pure strategies for frictions must have a faster, prominent firm and a slower, non-prominent firm. More specifically, all consumers start from the faster firm $i = 1$ and switch to the slower firm $i = 2$ only when they find the price p^1; the slower firm attracts no captives, $B_2 = 0$. By Lemma 2, the profits of the prominent firm and the non-prominent firm are hence given by $\Pi^1 = B_1$ and $\Pi^2 = (1 - \underline{p})B_1 = (1 - \alpha)B_1 = B_1 B_{1,2}/(B_1 + B_{1,2})$. We next describe one by one both firms' best responses in frictions.

4.2.1 Prominent firm's problem

The prominent firm maximizes the following expression:

$$\max_{\theta^1} B_1(\boldsymbol{\theta}) = \max_{\theta^1} \frac{\theta^1}{\theta^1 - \theta^2} \left(e^{-\theta^2} - e^{-\theta^1}\right)$$

Its profit is given by the number of uninformed consumers B_1, who are its captives. As consumers switch the store once they find a price, the prominent firm has a tradeoff between maximizing the number of consumers who find its price (by decreasing its frictions, it increases the inflow from consumer group B_0 to group B_1) and minimizing the number of consumers who find the other firm' price (by increasing its frictions, it decreases the outflow from consumer group B_1 to group $B_{1,2}$).

Note particularly that the factor $\left(e^{-\theta^2} - e^{-\theta^1}\right)$ is the efficiency gain, the difference in the number of trades $1 - e^{-\theta^1} - 1 + e^{-\theta^2}$, that the prominent firm generates by serving its consumers faster than its competitor. However, although this tends to align the prominent firm's private benefits with social benefits, the factor $\frac{\theta^1}{\theta^1 - \theta^2}$, that governs the turnover rate from B_1 to $B_{1,2}$, is decreasing in θ^1 because faster service means that more consumers have time to find also its competitor's price.

Due to these tradeoffs that the prominent firm has, it is optimal for it to generate intermediate frictions, $\theta^1 \in (0, \infty)$. Unfortunately, this implies that the number of trades is suboptimal.

Proposition 3 *There exists no efficient equilibrium, where the prominent firm generates no frictions.*

To emphasize, the intuition for this is that, since the prominent firm cannot reap (bear) the full positive (negative) externality that faster (slower) search has on the consumers, it has no incentive to serve instantaneously every consumer. Thus, a positive welfare loss is always created: search is efficient but service is inefficient.

Though consumers are free to switch the store at any point, we know that in equilibrium they do so only after they have found a price. This entails that the rate at which price information arrives in a store plays also the role of an implicit (endogenous) switching cost. This switching cost benefits the prominent firm, and it has thus no incentive to eliminate it completely.

As the consumers have deadlines, they have more time to discover the other price if the first one is found early on. That intensifies price competition. Therefore, although one store could serve the entire market if it chose to decrease its frictions, it has no incentive to do so because that would also strengthen competition.

4.2.2 Non-prominent firm's problem

The non-prominent firm maximizes the following expression:

$$\max_{\theta^2} \frac{B_1(\boldsymbol{\theta})B_{1,2}(\boldsymbol{\theta})}{B_1(\boldsymbol{\theta})+B_{1,2}(\boldsymbol{\theta})} = \max_{\theta^2} \frac{\frac{\theta^1}{\theta^1-\theta^2}\left(e^{-\theta^2}-e^{-\theta^1}\right)\left(1-e^{-\theta^1}-\frac{\theta^1}{\theta^1-\theta^2}\left(e^{-\theta^2}-e^{-\theta^1}\right)\right)}{1-e^{-\theta^1}},$$

or, equivalently, the product of the other firm's captives and shoppers

$$\max_{\theta^2} B_1(\boldsymbol{\theta})B_{1,2}(\boldsymbol{\theta}) = \max_{\theta^2} \frac{\theta^1}{\theta^1-\theta^2}\left(e^{-\theta^2}-e^{-\theta^1}\right)\left(1-e^{-\theta^1}-\frac{\theta^1}{\theta^1-\theta^2}\left(e^{-\theta^2}-e^{-\theta^1}\right)\right).$$

This formulation demonstrates that the non-prominent firm has an incentive to equalize the numbers of informed consumers and uninformed consumers. Its demand is coming only from shoppers but due to their intensifying effect on competition it wins them over more frequently if the prominent firm has more captives, which makes it raise the price.

The non-prominent firm has thus mixed incentives in choosing the frictions: if it elevates θ^2, the number of informed consumers goes up (its has more potential demand) but, then, the number of uninformed consumers goes down (competition becomes stronger). This clear tradeoff makes it profitable to apply an interior level of search frictions, θ^2.

Proposition 4 *There are equally many informed consumers and uninformed consumers in an equilibrium.*

When the prominence order is clear, the non-prominent firm's choice θ^2 only affects the division of consumers between the informed segment, $B_{1,2}$, and the uninformed segment, B_1 (how many find two prices), but it has no bearing on welfare, $1 - B_0$, which depends instead on the prominent firm's choice θ^1 (how many find one price). The non-prominent firm has thus an incentive to make sure that the outcome is exactly in between Diamond equilibrium and Bertrand equilibrium, as measured by the relative numbers of informed consumers $\frac{B_1}{B_1+B_{1,2}} = \underline{p}$ and uninformed consumers $\frac{B_{1,2}}{B_1+B_{1,2}} = 1 - \underline{p}$ but it does not care about efficiency. Nevertheless, its choices affect also welfare because, to split the consumers into two equally large segments, B_1 and $B_{1,2}$, the non-prominent firm has to apply so high frictions that the prominent firm can keep its frictions quite low while maintaining a substantial captive demand.

The prominent firm's frictions are strategic substitutes to non-prominent firm's frictions and the opposite holds true as well. We verify in Appendix that the maximizer of the prominent firm's problem $\theta^1(\theta^2)^\star$ is decreasing in θ^2 and the maximizer of the non-prominent firm's problem $\theta^2(\theta^1)^\star$ is decreasing in θ^1. Lower information frictions in one store can compensate for higher information frictions in the other store in the optimal management of consumer information. Interestingly, after imposing the equal split of consumers into captives B_1 and shoppers $B_{1,2}$, it turns out that the welfare loss that is generated by the prominent firm's optimally chosen frictions is approximately 6%. The atom size and the lowest price must equal one half $\alpha = \underline{p} = 1/2$, which is later reflected in the surplus sharing in proportions 2:1:1 for the prominent firm, the non-prominent firm and the consumers, respectively.

4.2.3 Fixed point in intrafirm frictions

The firms' reaction curves for intrafirm frictions are presented by Figures 4 and 5. They jump down at $\theta^{-i} \approx 2.33$ from $\theta^i \approx 2.33$ to $\theta^i \approx 1.08$ and they cross each other at $(\theta^1, \theta^2)^\star \approx (2.76, 1.03)$ when $\theta^1 \geq \theta^2$ (the assumed case) and at $(\theta^1, \theta^2)^\star \approx (1.03, 2.76)$ when $\theta^2 \geq \theta^1$ (the inverse case). To secure a prominent market position, the firm must thus have a service rate higher than 2.33, which renders it optimal for its competitor to use a service rate lower than 1.08. This puts a lower bound on the non-prominent firm's frictions and an upper bound on the prominent firm's frictions: the number of trades must exceed $1 - e^{-2.33}$ in equilibrium.

Observe that, while it might look so in Figures 4 and 5, there is no overlap on the diagonal for the (approximative) value range $\theta^i \in (2.0, 2.33)$ because for that range the best response is always a higher rate.[32] This pins down our two equilibrium points and shows that there exists a unique cutoff level for frictions $\theta' \approx 2.33$ such that: if the other firm if slower than the cutoff, $\theta^{-i} < \theta'$, firm i's best response is to become the prominent firm, i.e., $BR_i(\theta^{-i}) > \theta^{-i}$, whereas, if the other firm is faster than the cutoff, $\theta^{-i} > \theta'$, firm i's best response is to become the non-prominent firm, i.e., $BR_i(\theta^{-i}) < \theta^{-i}$. We prove our next main result in Appendix.

[32] E.g., $BR_i(2.00) = 2.00 + \epsilon$, where $\epsilon > 0$ is a small number.

Proposition 5 *Assume without loss of generality that* $\theta^1 \geq \theta^2$*. Then, there exists a unique equilibrium where* $\boldsymbol{\theta}^\star \approx (2.76, 1.03)$*.*

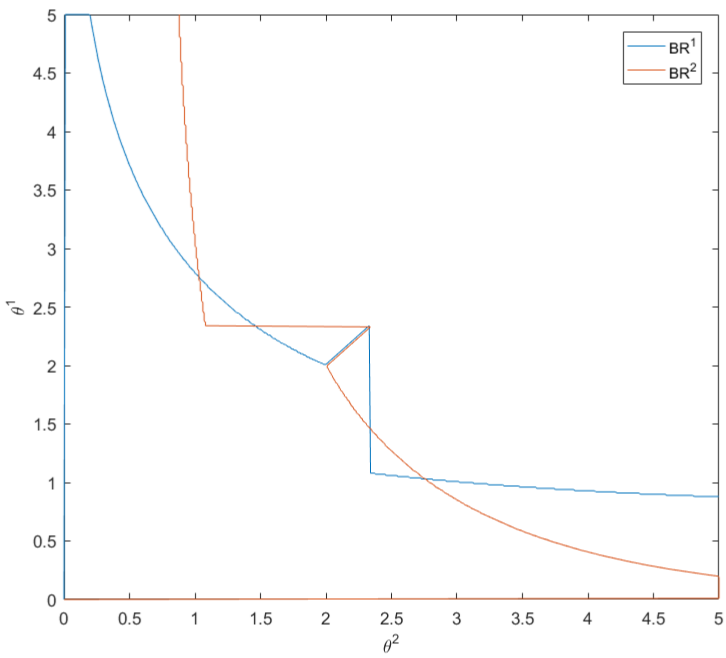


Figure 4: Best response functions for two firms: zoom-in.

Corollary 4 *The equilibrium has the following properties:*

1. *Frictions: there is a prominent firm who sets frictions* $\theta^1 \approx 2.76$ *and a non-prominent firm who sets frictions* $\theta^2 \approx 1.03$*. Before a price is found, the expected wait time in the former is about* 36% *of the total time and the expected wait time in the latter is about* 97% *of the total time. Note that these times can be regarded as endogenous search costs or switching costs.*

2. *Search: The consumers search in the prominent firm until they find their first price quote,* $s^1 = 1$ *and* $s^2 = 0$*. As a result, 47 per cent of the consumers find both prices,* $B_{1,2} \approx 0.47$*, and 47 per cent of the consumers find a price from the prominent firm but not from the non-prominent firm,* $B_1 \approx 0.47$*; 6 per cent of the consumers fail to find any price,* $B_0 \approx 0.06$*.*

3. *Prices: The prominent firm offers the monopoly price* $(p = 1)$ *and a random discount price* $(p < 1)$ *equally often,* $\alpha = 0.5$*; the non-prominent firm always offers a random discount price. Given that a firm offers a discount, the expected discount size is 31 per cent of the monopoly price at either firm; the largest such regularly used discount is 50 per cent,* $\underline{p} = 0.5$*.*

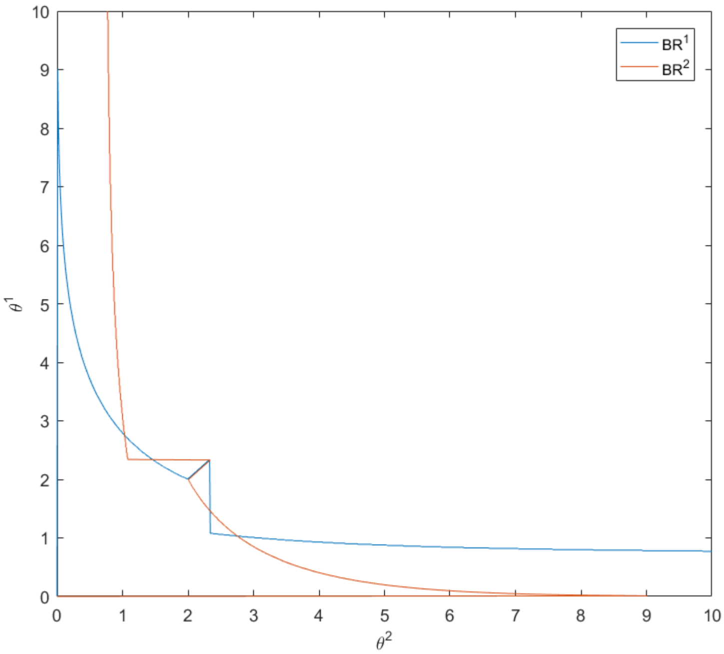


Figure 5: Best response functions for two firms: zoom-out.

4. *Surplus sharing: The prominent firm is earning the double of what the non-prominent firm is earning,* $\Pi^1 = B_1 \approx 0.47, \Pi^2 = \alpha B_{1,2} \approx 0.5 \cdot 0.47$. *The prominent firm obtains half the surplus, the non-prominent firm obtains a quarter and the consumers receive a quarter; 6 per cent of the cake is wasted.*

Corollary 4 can be proved as an elementary calculation that uses the previous finding $\boldsymbol{\theta} \approx (2.76, 1.03)$ and the expressions we derived earlier for $B_i(\boldsymbol{\theta})$, $B_{1,2}(\boldsymbol{\theta})$, $B_0(\boldsymbol{\theta})$, $E[p^1]$ and $E[p^2]$.

It is important to observe that the same robust friction pattern is the unique pure equilibrium even if we lengthen or shorten the deadline. In other words, the outcome is just the same in terms of search, prices, and profit whether the consumers can search for a decade or a minute, as long as the setup is still fitting with our basic model. Particularly, for all choices of consumer deadline, two firms have still an incentive to synchronize their respective frictions such that the numbers of informed consumers and uninformed consumers are kept the same.

Remark 3 *An identical equilibrium outcome arises whatever the deadline* $q < \infty$ *is as long as it is finite: if* (θ^1, θ^2) *is an equilibrium when the search horizon is* $t \in [0, 1]$*, then* $(\frac{\theta^1}{q}, \frac{\theta^2}{q})$ *is an equilibrium when the search horizon is* $t \in [0, q]$*.*

Observe, however, that Bertrand equilibrium would become the unique equilibrium if the deadline was infinity whereas multiple equilibria, including Diamond equilibrium, would arise if the deadline was zero. This again emphasizes the role of deadlines in escaping the paradoxes by Bertrand and Diamond.

Remark 4 *There is a discontinuity in the equilibrium set both at $q = \infty$ and at $q = 0$ because a unique equilibrium arises for any $q \in (0, \infty)$ but, if $q = \infty$, Bertrand equilibrium is the unique equilibrium (consumers find all prices) and, if $q = 0$, any arbitrary prices constitute an equilibrium when (consumers find no prices).*

To summarize, the set of equilibria is invariant to finite translations in the deadline, which is also the only exogenous parameter in our model so far.[33] Bertrand equilibrium is possible only if the consumers are extremely patient and Diamond equilibrium only if the consumers are extremely impatient. Otherwise, duopoly outcome must lie precisely between these extremes in the sense that there must be exactly equally many informed and uninformed consumers.

5 Generalized modeling approach

Although our framework of time cost, strategic complexity and consumer deadlines has several attractive features, some of the findings might perhaps seem overly specific. Moreover, despite the fact that we could solve the base line case with two firms using the Poisson process, its high degree of detail makes it quite cumbersome to work with in extensions. To offer a more tractable framework for applications, this section retains only the main features of our previous approach and generalizes it in important ways.

We are interested especially in the interplay of price obfuscation and price advertizing, which apparently serve the opposite functions in controlling price information: obfuscation restricts and advertizing extends the availability of information in the market. Despite this interest, it is not clear from the start how a firm's obfuscation depends on the firm's advertizing as a part of its marketing mix. To consider their joint effects in a simple basic model, we thus make the following assumptions about consumer search:

- Clear prominence order: All consumers who search for information find the prominent firm's price, p_1, before the non-prominent firm's price, p_2. The prominence order is exogenously given in this case and could be based on, say, relative obfuscation or relative advertizing.

- Partition of consumers: The set of consumers is partitioned into two sets, $A + B = 1$, where A represents consumers who receive their price information (exogenously) from advertizing and B represents consumers who obtain their price information (endogenously) from searching.

- Continuity: B_0, B_1 and $B_{1,2}$ are continuous in (θ^1, θ^2) and sum to $1 - A$.

[33] We do make the distribution assumption that the wait time before a price is found in a store is exponential $Exp(\theta^i)$ and price information thus arrives at Poisson rate θ^i.

- Inefficiency: $B_0(0,\theta^2) = 1 - A$ and $B_0(\theta^1,\theta^2) \to 0$ as $\theta^1 \to \infty$ for all θ^2.
- Uninformed consumers: $B_1(\theta^1, 0) = 1 - A - B_0$ and $B_1(\theta^1,\theta^2) \to 0$ as $\theta^2 \to \infty$ for all $\theta^1 \in (0,\infty)$.
- Informed consumers: $B_{1,2}(\theta^1, 0) = 0$ and $B_{1,2}(\theta^1,\theta^2) \to 1 - A - B_0$ as $\theta^2 \to \infty$ for all $\theta^1 \in (0,\infty)$.
- Unique continuous best response in θ^1: there exists a unique $\theta^1(\theta^2)^\star = \arg\max_{\theta^1} B_1$, which is continuous and decreasing in θ^2; $\theta^1(\theta^2)^\star \to \infty$ as $\theta^2 \to 0$ and $\theta^1(\theta^2)^\star \to \underline{\theta}^1 \in (0,\infty)$ as $\theta^2 \to \infty$.[34]
- Strategic substitutes: B_1 is decreasing in θ^2 and $B_{1,2}$ is increasing in θ^2; B_0 is unaffected by θ^2.
- Unique continuous best response in θ^2: there exists a unique $\theta^2(\theta^1)^\star = \arg\max_{\theta^2} \frac{(A_1+B_1-A_2)(A_{1,2}+B_{1,2})}{A_1+A_{1,2}+B_1+B_{1,2}}$ for all θ^1, A_1, A_2 and $A_{1,2}$; $\theta^2(\theta^1)^\star \to \infty$ as $\theta^1 \to 0$ and $\theta^2(\theta^1)^\star \to \underline{\theta}^2 \in (0,\infty)$ as $\theta^1 \to \infty$.[35]

Note that the game has continuous best responses that cross at least once: an equilibrium always exists with exogenous prominence order. The introduction of exogenous information thus not only reconciles our model with the fact that some consumers have access to alternative information sources but it also serves the purpose of getting rid of the inconvenient discontinuity in the best response functions of obfuscation as depicted by Figures 4 and 5. We can thereby easily proceed to analyze the comparative statics of obfuscation for different levels of advertizing in a given equilibrium. We need not make a claim about the uniqueness of equilibrium obfuscation strategies.

5.1 Obfuscation and advertizing with two firms

Consider a market where firms serve two consumer segments: One consumer segment (e.g., offline-customers) has already received sufficient price information to complete the purchase. This could stem either from advertizing or other forms of pre-existing connections between firms and their customers. The other consumer segment (e.g., online-customers) is still in doubt and is not willing to purchase before receiving more information.

Both segments include consumers who buy for p_1 (A_1 and B_1), consumers who buy for p_2 (A_2 and B_2) and consumers who buy for the lowest of p_1 and p_2 ($A_{1,2}$ and $B_{1,2}$). We can regard A_1 and A_2 as measures of customer loyalty and $A_{1,2}$ as a measure of direct

[34]In our Poisson setting, the best response to zero frictions, $\theta^{-i} = \infty$, was found to be $\theta^i = ln(2)$ whereas the best response to infinite frictions, $\theta^{-i} = 0$, would be monopoly frictions $\theta^i = \infty$.

[35]If the prominent firm is extremely slow, the non-prominent firm has to be extremely fast, to guarantee that equally many consumers have time to discover one price and two prices by the deadline.

exposure to price comparison in the market.[36] Advertizing A may have both *informative* and *persuasive* features: we assume it removes incentives to find more information.

The main insights from our model can be captured by analyzing four distinct cases: total advertizing level, $A = A_1 + A_2 + A_{1,2}$, is fixed but (i) the prominent firm intensifies its advertizing to the other firm's consumers, $dA_1 = -dA_{1,2} > 0$, (ii) the prominent firm's advertizing attracts additional captives from the non-prominent firm, $dA_1 = -dA_2 > 0$, and (iii) the non-prominent firm intensifies its advertizing to the other firm's consumers, $dA_2 = -dA_{1,2} > 0$, or (iv) the prominent firm advertizes more so that the total advertizing level is increased, $dA = dA_1 > 0$. All other cases can be obtained by combining these four cases.

Note first that advertizing has no direct effects on the prominent firm's obfuscation, which is chosen so as to maximize the sum of its offline captives, A_1, and its online captives, B_1. Only the latter number is responsive to obfuscation, the former one is a parameter. This maintains the prominent firm's obfuscation problem independent of advertizing.

$$\max_{\theta^1} A_1 + B_1 = \max_{\theta^1} B_1.$$

Any effect that advertizing might have on the prominent firm's obfuscation must thus come indirectly via the non-prominent firm's obfuscation. The non-prominent firm's obfuscation problem is indeed slightly different now. The additional potential demand from the informed consumers $A_{1,2} + B_{1,2}$ is determined jointly by advertizing and obfuscation.

$$\max_{\theta^2} A_2 + \alpha(A_{1,2} + B_{1,2}) = \max_{\theta^2} \frac{(A_1 + B_1 - A_2)(A_{1,2} + B_{1,2})}{A_1 + A_{1,2} + B_1 + B_{1,2}}.$$

Also the atom α is a bit different now: stronger advertizing asymmetry (higher $A_1 - A_2$) increases the atom and overlapping price advertizing (higher $A_{1,2}$) decreases the atom.

Now, for any given level of prominent firm's obfuscation, the change in B_1 is the negative of the change in $B_{1,2}$. As a result, interior optimum is found at the level where

$$A_1 + B_1 - A_2 = A_{1,2} + B_{1,2} \tag{11}$$

where the lhs is higher when the prominent firm has more captives (this effect comes through higher $\underline{p}$: price competition is then less intensive at the low end of the price distribution) and the rhs is higher when the non-prominent firm has higher potential demand from informed consumers (this effect comes through higher α: price competition is then more intensive at the high end of the price distribution).

By the same logic as earlier in this paper, the non-prominent firm's obfuscation decision can balance Equation 11 by accelerating or decelerating the rate at which uninformed consumers B_1 are transformed into informed consumers $B_{1,2}$. Assuming that advertizing

[36] Like the availability of an extensive price comparison site.

parameters A_1, A_2 and $A_{1,2}$ are set before obfuscation levels are chosen, we obtain the following comparative statics of advertizing on obfuscation:

Proposition 6 *Start from a case where advertizing $A = A_1 + A_2 + A_{1,2}$ is fixed and consider the effects of a marginal or rather small change $\epsilon > 0$ in advertizing on equilibrium obfuscation.*

1. *Suppose the non-prominent firm targets less (informative) advertizing to the other firm's captives: A is fixed and $dA_1 = -dA_{1,2} = \epsilon > 0$. Then, $A_1 + B_1 = A_2 + A_{1,2} + B_{1,2} - 2\epsilon$ in the new equilibrium, which means that the non-prominent firm decreases its obfuscation and the prominent firm increases its obfuscation; the new equilibrium is less efficient than the old one because B_0 is higher.*

2. *Suppose the prominent firm targets less (informative) advertizing to the other firm's captives: A is fixed and $dA_2 = -dA_{1,2} = \epsilon > 0$. Then, $A_1 + B_1 = A_2 + A_{1,2} + B_{1,2}$ in the new equilibrium, which means that nothing changes.*

3. *Suppose the prominent firm targets more (persuasive) advertizing to the other firm's captives: A is fixed and $dA_1 = -dA_2 = \epsilon > 0$. Then, $A_1 + B_1 = A_2 + A_{1,2} + B_{1,2} - 2\epsilon$ in the new equilibrium, which means that the non-prominent firm decreases its obfuscation and the prominent firm increases its obfuscation; the new equilibrium is less efficient than the old one because B_0 is higher.*

4. *Suppose the prominent firm targets more advertizing to previously uninformed consumers: A is higher by the change in A_1 such that $dA = dA_1 = \epsilon > 0$ but A_2 and $A_{1,2}$ stay the same. Then, $A_1 + B_1 = A_2 + A_{1,2} + B_{1,2} - \epsilon$ in the new equilibrium, which means that the non-prominent firm decreases its obfuscation and the prominent firm increases its obfuscation; the new equilibrium could be more or less efficient than the old one because B_0 could be higher or lower.*

This rich set of predictions on advertizing and obfuscation might be empirically testable with appropriate data. Our general result is that the non-prominent firm's obfuscation adjusts to equalize the maximal demands that the prominent firm and the non-prominent firm may have at the monopoly price: respectively, $A_1 + B_1$ and $A_2 + A_{1,2} + B_{1,2}$. Again equilibrium forces thus pin down the relative numbers of informed consumers and uninformed consumers. As in our basic model, when only the prominent firm has captives, the numbers of informed consumers and uninformed consumers are identical. Otherwise, the number of informed consumers is equal to the difference between the prominent firm and the non-prominent firm's captives.

Advertizing by the prominent firm has the same effect whether it is informative (turns captives A_2 into informed $A_{1,2}$) or persuasive (turns captives A_2 into captives A_1) whereas informative advertizing by the non-prominent firm has no effect on equilibrium obfuscation. If the prominent firm releases more information through advertizing it has less of the need to facilitate independent information acquisition by consumers themselves because the non-prominent firm responds to its additional advertizing by putting more effort into helping consumers. Advertizing and obfuscation must thus be strategic complements for the prominent firm when obfuscation by the prominent firm is a strategic substitute to obfuscation by the non-prominent firm.

Our findings are illustrated by Figure 6. It shows an equilibrium as a crossing point between the prominent firm and the non-prominent firm's best response functions. The former stays fixed throughout. The latter shifts to the right if A_1 increases and to the left if $A_2 + A_{1,2}$ increases. To put it differently, narrow price awareness (higher A_1) results in more obfuscation by the prominent firm as the fixed point shifts down and to the right, whereas, general price awareness (higher $A_{1,2}$ or higher A_2) results in less obfuscation by the prominent firm as the fixed point moves up and to the left.

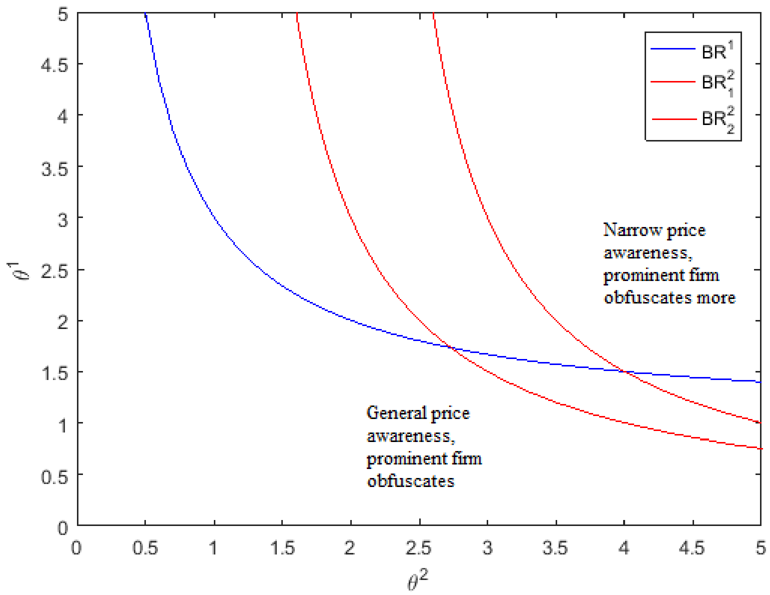


Figure 6: Best response functions with narrow and general price awareness.

5.2 Obfuscation and advertizing with three firms

To illustrate how our setting extends to larger markets, we turn to consider a three firm setup with exogenous advertizing and endogenous obfuscation. The assumption that the market has an exogenously given prominence order is maintained: most consumers start searching from the 1st prominent firm $i = 1$ and then move to the 2nd prominent firm $i = 2$ and to the 3rd prominent firm $i = 3$. To be specific, we introduce the following asymmetry constraint on the consumer partition

$$B_1 > B_2 \geq B_3, B_{1,2} \geq B_{1,3} > B_{3,2}.$$

As before, consumers are partitioned into two sets, $A+B=1$, where consumers A are willing to purchase without searching whereas consumers B are not. With more prices in the market, the information partition of consumers must clearly be more refined now:

$$A = A_1 + A_2 + A_3 + A_{1,2} + A_{2,3} + A_{1,3} + A_{1,2,3},$$
$$B = B_0 + B_1 + B_2 + B_3 + B_{1,2} + B_{2,3} + B_{1,3} + B_{1,2,3}.$$

For instance, consumers $A_{1,3}$ and $B_{1,3}$ purchase for the lowest of prices p_1 and p_3. As each of these consumer groups behaves alike, we denote by C_i the sum $A_i + B_i$ for all $i = 1, 2, 3$. Similarly, $C_{i,j}$ denotes $A_{i,j} + B_{i,j}$, for all $i, j = 1, 2, 3$, and $C_{1,2,3}$ denotes $A_{1,2,3} + B_{1,2,3}$.

$$C = C_0 + C_1 + C_2 + C_3 + C_{1,2} + C_{2,3} + C_{1,3} + C_{1,2,3} = 1.$$

Our model extends quite substantially the symmetric information structures in the literature: for example, in the basic Stahl model with three firms, $C_1 = C_2 = C_3 = \frac{1-\mu}{3}$ and $C_{1,2,3} = \mu$. The model has a unique symmetric equilibrium with two firms but Baye et al. (1992) show that a continuum of asymmetric randomized equilibria arises with more than two firms. The literature has not yet provided a characterization of equilibrium pricing strategies for any general information partition of consumers C. We find that introducing asymmetry and partition completeness strictly refines the set of equilibria in the basic Stahl model with three firms.

Proposition 7 *Assume a weak ordering for market prominence $C_1 > C_2 \geq C_3, C_{1,2} \geq C_{1,3} > C_{3,2}$ and a complete information partition: $C_i > 0$ for all $i = 1, 2, 3$, $C_{i,j} > 0$ for all $i, j = 1, 2, 3$ and $C_{1,2,3} > 0$. There exists a price equilibrium where the profits are*

$$\Pi_1 = C_1 >$$
$$\Pi_2 = C_2 + \alpha C_{1,2} >$$
$$\Pi_3 = C_3 + \alpha C_{1,3}$$

The 1st and the 3rd prominent firms choose their prices randomly from price distributions with interval support $[\underline{p}, 1]$ whereas the 2nd prominent firm uses the interval support $[p', 1]$, where $p' > \underline{p}$. The 1st prominent firm has an atom of size α at the price of one

$$\underline{p} = \frac{C_1}{C_1 + (C_{1,2} + C_{1,3} + C_{1,2,3})},$$
$$\alpha = \frac{C_3 + (C_{1,3} + C_{2,3} + C_{1,2,3})}{C_1 + (C_{1,2} + C_{1,3} + C_{1,2,3})} \frac{C_1}{C_{1,3}} - \frac{C_3}{C_{1,3}}.$$

If we assume instead a strict ordering for market prominence, $C_2 = C_3 = C_{2,3} = 0$, but otherwise maintain our assumptions, the equilibrium price distributions are unique.

Note particularly that Proposition 7 replaces the continuum of asymmetric equilibria with differently sized atoms in Baye et al. (1992) by either a unique or a finite number of asymmetric equilibria because here only one firm can hold an atom in its price distribution. This leaves us with a finite number of candidate equilibria, where different firms may have the atom. If two firms had an atom at the same time, this would create a profitable deviation to either of them. For example, if the 2nd prominent firm had a mass point at unity, then the 1st prominent firm would gain $B_{1,2}$ by deviating from its mass point $p_1 = 1$ to a slightly smaller price $p_1 = 1 - \epsilon$. This shows that the continuum result is special to the symmetric model in in Baye et al. (1992) and not robust to perturbations in information endowments. The authors construct a metagame to eliminate asymmetric equilibria. Our result brings them back – with either uniqueness or finiteness.

Both of our new assumptions, strict prominence order (asymmetric numbers of uninformed consumers, $C_1 > C_2 = C_3 = 0$ and $C_{1,2} \geq C_{1,3} > C_{2,3}$) and complete information partition (presence of partially informed consumers $C_{1,2} > 0$ and $C_{1,3} > 0$), play an important role here. For example, in Example 2 in Baye et al. (1992) where the number of firms is $n > 2$, $k > 2$ firms mix over an interval support and the rest $n - k$ load all the probability mass at one. Consumers are either fully uninformed (know about one price) or fully informed (know all the prices). As a result, none of the mass point firms has an incentive to undercut the price of another such firm because its demand does not jump up unless it wins over all its competitors. By contrast, in our case with a complete information partition, beating one competitor is sufficient to gain a jump in demand. This implies that only one firm, the prominent one, can have an atom at the high end of the price distribution, which elevates the profits of all the other non-prominent firms in the market. The other firms must in a sense realize the benefit from undercutting the atom via their equilibrium pricing behavior already.

In Example 3 in Baye et al. (1992), the atom size can be continuously adjusted but all firms still obtain the same equilibrium profit. This is not possible in our asymmetric equilibria due to the subtle forces that govern mixing behavior when firms earn unequal profits. Here the profits that firms make in equilibrium order the firms (partially or completely) in a strict profit ranking. The higher the profit of a firm, the higher the lowest price that the firm is willing to offer in order to capture all the informed consumers in the market. However, to support optimal mixing, at least two firms must be randomizing their prices in the neighborhood of the lowest price and, to avoid incentives to use it excessively, they must have a shared lowest price. To meet all the requirements, one of the higher profit firms must have an atom at unity to increase the lowest profit to the right level, where both firms' price distributions vanish at the same lowest prices.

A unique equilibrium arises in a market with strict prominence order because only the prominent firm has captives then. It is therefore the only firm who can have an atom α in its equilibrium price distribution and, in doing so, both (i) retain its own positive equilibrium profits (C_1) and (ii) provide positive profits to the other firms ($\alpha C_{1,2}$ and $\alpha C_{1,3}$).

We are now ready to study briefly the effects of competition. We continue to assume a strict exogenous prominence order: all searching consumers start from the 1st prominent firm and, then, a fraction $s^2 > 0$ of them searches from the 2nd to the 3rd prominent firm and a fraction $s^3 = 1 - s^2 > 0$ from the 3rd to the 2nd prominent firm, i.e., $B_1 > B_2 = B_3 = 0, B_{1,2} \geq B_{1,3} > B_{2,3} = 0$.[37] According to Proposition 7, under the reasonable assumption that the 2nd and 3rd prominent firms cannot target advertizing to separate captive consumer segments, i.e., $A_2 = A_3 = 0$, a unique equilibrium price distribution arises. We also introduce the following assumptions for combined effects of obfuscation by the non-prominent firms. They are motivated by the insights from our basic Poisson model. We maintain our previous assumptions about the prominent firm's obfuscation with obvious relevant changes.

Assumption 1 $\frac{\partial C_1(s^i)}{\partial \theta^i} < 0 = \frac{\partial C_1(s^i)}{\partial \theta^j}$ *and* $\frac{\partial C_{1,2,3}(s^i)}{\partial \theta^j} = \frac{\partial C_{1,2,3}(s^j)}{\partial \theta^j} > 0$ *for all* $i, j = 2, 3$ *and* $i \neq j$.

Technically, Assumption 1 entails that partially informed consumers $C_{1,i}$ respond to the ith prominent firm's frictions much like uninformed consumers C_1 respond to the 1st prominent firm's frictions: the numbers of consumers who find two prices are non-monotone in the non-prominent firm's obfuscation while the numbers of consumers who find one price are non-monotone in the prominent firm's obfuscation. The idea is that θ^2 and θ^3 both decrease C_1 and increase $C_{1,2,3}$ by making the second and the third price quote easier to discover. As a result, a lower level of obfuscation by the non-prominent firms' first increases $B_{1,2}$ and $B_{1,3}$ because more consumers become informed about their prices (they move from, say, B_1 to $B_{1,2}$) and then decreases them because more consumers find also the remaining third price (which moves them, say, $B_1, 2$ to $B_{1,2,3}$).

Following Proposition 7, the 1st prominent firm's obfuscation problem can again be written as

[37]Note that competition is likely to reduce obfuscation more if the non-prominent firms must compete over their relative positions after the prominent firm. We do not study that here because also exogenous factors (e.g., the prominence order suggested by the search engine listing) strongly affect the order in which consumers search the non-prominent firms.

$$\max_{\theta^1} C_1 = \max_{\theta^1} B_1.$$

Instead, the 2nd and the 3rd prominent firms' obfuscation decisions are determined respectively by

$$\max_{\theta^2} C_2 + \alpha C_{1,2} = \max_{\theta^2} \frac{C_1(C_{1,3} + C_{2,3} + C_{1,2,3})}{C_1 + C_{1,2} + C_{1,3} + C_{1,2,3}} \frac{C_{1,2}}{C_{1,3}},$$
$$\max_{\theta^3} C_3 + \alpha C_{1,3} = \max_{\theta^3} \frac{C_1(C_{1,3} + C_{2,3} + C_{1,2,3})}{C_1 + C_{1,2} + C_{1,3} + C_{1,2,3}}.$$

The firms' problems show that a market with three firms differs from a market with two firms in important respects. Consider more closely a non-prominent firm $i = 3$ whose price is $p^3 < 1$ (the analysis is identical for $p^2 < 1$). This firm no longer obtains the entire residual demand $1 - C_0 - C_1$ either when $p^3 = \underline{p}$ or when $p^1 = 1$. First, if $p^3 = \underline{p}$, the non-prominent firm only gets $C_{1,3} + C_{1,2,3}$ but not $C_{1,2}$ because these partially informed consumers have not seen its price yet.[38] Second, if $p^1 = 1$, the non-prominent firm only gets $C_{1,3}$ but not $C_{1,2}$ nor $C_{1,2,3}$ because the other non-prominent firm has a lower price with probability one. This is also reflected in the atom size α necessary to keep the firm randomizing in the appropriate way.

Because this atom α synchronizes the incentives of the 1st prominent firm with the 3rd prominent firm – not with the 2nd prominent firm, the incentives of the non-prominent firms usually differ: both of these firms i maximize $C_1(C_{1,3} + C_{1,2,3})\frac{C_{1,i}}{C_{1,3}}$, which captures their selling probability and their demands for the highest price, but for different numbers of "secondary captives", $C_{1,2}$ and $C_{1,3}$.

Additionally, since the firms have different positions on the consumers' path, the ability to affect certain groups of consumers is different for the 2nd and 3rd prominent firms. Lower frictions in the 2nd prominent firm decrease C_1 and $C_{1,3}$ but increase $C_{1,2,3}$ whereas lower frictions in the 3rd prominent firm decrease C_1 and increase $C_{1,3} + C_{1,2,3}$. The demand from informed consumers is thus relatively more elastic for the 3rd prominent firm than for the 2nd prominent firm.

In isolation, the effect has a tendency to make the 2nd prominent firm prefer higher frictions than the 3rd prominent firm to relax price competition from the 1st prominent firm more. Nevertheless, because the frictions of the 2nd prominent firm initially increase and then decrease $C_{1,2}$ but always reduce $C_{1,3}$, there is a counterbalancing effect from its more prominent standing in the market.

Optimal behavior of the 3rd prominent firm pins down the level of price competition α and the relative numbers of uninformed consumers and fully and partially informed

[38]Generally, the same is true also for the prominent firm, which raises the lowest price $\underline{p}$ from $\frac{C_1}{C_1+C_{1,2}+C_{1,3}+C_{2,3}+C_{1,2,3}}$ to $\frac{C_1}{C_1+C_{1,2}+C_{1,3}+C_{1,2,3}}$.

consumers.

Proposition 8 *Consider a market with three firms where* $C_2 = C_3 = C_{2,3} = 0$*. Any equilibrium must feature more informed consumers than uninformed consumers. In particular,* $C_1 = \gamma^3 (C_{1,3} + C_{1,2,3}) < C_{1,2} + C_{1,3} + C_{1,2,3}$ *where* $\gamma^3 < 1$.

In equilibrium, the 1st prominent firm maximizes C_1 and the 3rd prominent firm maximizes αC_1 whereas the 2nd prominent firm has a service rate that is higher than the maximizer of $\max_{\theta^2} \frac{C_{1,2}}{C_{1,2}}$ but lower than the maximizer or $\max_{\theta^2} \alpha C_1$. The frictions of the 2nd prominent firm are thus lower than the 3rd prominent firm prefers in order to maximize its own objective. Nevertheless, we cannot tell from this alone which of the non-prominent firms obfuscates less and we cannot therefore determine for any $s^2 = 1 - s^3$ whether the prominent firm increases its obfuscation. Notwithstanding, the key result of Proposition 8, that there are more fully and partially informed consumers than uninformed consumers, arises under relatively weak assumptions. If we want to have stronger results, we need to strengthen our assumptions; this lies beyond the scope of the paper.

5.3 Applications to politics, innovation, and lobbying

So far we have studied the effects of obfuscation in a price competition setup. This is not the only application where obfuscation-like behavior might arise. As discussed in Baye et al. (1996), the structure of randomized equilibria is essentially identical in an oligopolistic price competition model like Varian (1980) and in all-pay auctions and contests, with applications to promotions, principal-agent-problems etc. We give next some examples of how our model could be rephrased to consider more generally the effects of organizational frictions and institutional complexity.

The simplest way to do so is to consider an all-pay auction, where two players i compete for a prize of value $W(c)$ in a first price sealed bid, all-pay auction by submitting bids x_i. The player who submits the highest bid wins the prize. The players have heterogeneous outside options $W^i(c)$, where $W - W^i > W^i > 0$ and $W^1 > W^2$ and c represents strategic complexity or "frictions", that we soon specify. As demonstrated by Baye et al. (1996), the players randomize their bids over an interval support $[0, \bar{x}]$, possibly with an atom at zero, which leads to a familiar equilibrium structure.

Bidding the highest value $x_i = \bar{x}$ yields the prize net of the outside option $W - W^i$ for both players whereas bidding the lowest value $x_i = 0$ yields W^1 for player $i = 1$ and $W^2 + \alpha(W - 2W^2)$ for player $i = 2$, where α denotes the probability of bidding zero for player $i = 1$. By similar methods as before (i.e., payoff comparisons at $x_i = 0$ and $x_i = \bar{x}$), we find that $\bar{x} = W - 2W^1$ and $\alpha = 1 - \frac{W-2W^1}{W-2W^2}$. Thus, the value of this game for player $i = 1$ is W^1 and that for player $i = 2$ is $W^2 + \alpha(W - 2W^2) = 2W^1 - W^2$. The player's incentives are thereby strongly aligned in that both benefit from an increase in W^1 because it alleviates competition. The value of the prize dissipates in the bidding process. Bidding effort distributions $G^1(x) = \frac{x+W^1-3W^2}{W-2W^2} = \alpha + \frac{x-(W^1+W^2)}{W-2W^1} > G^2(x) = \frac{x}{W-2W^1}$ increase in the sense of first order stochastic dominance in $(W, W^2, -W^1)$ (for player $i = 1$) and in $(W, -W^1)$ (for player $i = 2$). We also assume that $W^1 > 1 + 3W^2 + \frac{(W-W^2)^2}{W-W^1}$ (strong enough asymmetry to support equilibrium behavior and relatively attractive prize to order the bid distributions).

Example 1 (politics and legislative complexity) *Consider a contest between a new party $i = 1$ (wants to change the law) and an old party $i = 2$ (prefers the status quo). The complexity of legislation c determines how large a part of value W is burned in rewriting the law. To obtain a simple model, we can here set $W^1 = c$ (the saved legislative costs) and $W^2 = 0$. Additional complexity has several notable effects: (i) it discourages legislative efforts in general by increasing W^1, (ii) it increases the likelihood of effortless win, α, for the old party, and (iii) it raises the value of the game by W^1, also for the old party.*

Example 2 (innovation and regulatory complexity) *Consider a research and product development contest: the incumbent firm $i = 1$ has a higher outside option than the entrant firm $i = 2$, $W^1 > W^2$. Regulatory complexity in patenting and marketing (the risk of hold-up, litigation, etc.) is given by c and it reduces the value of winning, $W(c)$. Complexity thus decreases the research efforts of both the incumbent and the entrant. However, because the negative effect on $G^1(x)$ is higher than the negative effect on $G^2(x)$ for any x, additional complexity reduces innovation effort more by the incumbent than by the entrant.*[39]

It is easy to see how in a suitable (possibly repeated) metagame the old party might instate a positive level of legislative complexity to deter entry or a consumer protection authority and the entrant might together push for more regulatory complexity. We give one more example from lobbying:

Example 3 (lobbying and the revolving door) *Consider a contest between two lobbyists $i = 1$ (right-lobby) and $i = 2$ (left-lobby) with a revolving door to politics, business, now-profits, etc. The rates of the their revolving doors are given, respectively, by θ^1 (right wing lobbyist goes to private sector) and θ^2 (left wing lobbyist goes to public sector). The outside options are $W^i(\theta^i)$, where a lower rate θ_i reduce the outside options. We assume that frictions are lower for the right-wing lobby than for the left wing lobby: $W^1 > W^2$. Now, lower frictions for the right wing lobby reduce all lobbying by raising W^1, yet, lower frictions for the left wing increase right wing lobbying by raising W^2.*

In addition to their independent value, these examples serve also to show the limits of our exact halfway result, which now fails to arise even though we have seen that it appears in various price competition specifications. The apparent reason for this is that,

[39]The effect can be ambiguous if the value of the prize is small.

in the all-pay auctions we consider, the player's net value is a linear function of choice variable but, in a price competition model, the firm's price has a multiplicative effect on profit. We find that the players' incentives are even more strongly aligned here than in our price competition model. Therefore, both players may rationally advocate complexity even if it elevates only the higher outside option.

6 Concluding discussion

We introduce a new price search model that features endogenous frictions, modeled by the gradual arrival of price information within stores. Intrafirm frictions could refer, for the consumer's part, to the time cost of stochastic cognitive processes involved in understanding price information on an online firm's website. For the firm's part, frictions could represent its long-term investment in a particular search technology inside its store, strategic complexity.

Assuming that consumers are committed to search until their deadlines, we show that there exists a natural level of competition intensity and distribution of consumer information, which could arise from a mix of obfuscation and advertizing. Welfare effects of this optimal control of information are minor but the effects on surplus sharing are dramatic. Wider price awareness and advertizing restrict obfuscation by the prominent firm, whose choices matter the most.

If the prominence order in the market is stable, equilibrium forces push toward the same ultimate information outcome, in which there are equally many uninformed consumers (who find only the first store's price) and informed consumers (who find also the second firm's price) at the time when purchase decisions are made. This apparently robust result is shown to be (i) independent of the deadline length or the search horizon, (ii) invariant to slight changes in the prior distribution of consumers' information and (iii) almost immune to additional competition, which pushes the balance towards more information.[40] Empirical research could perhaps test the finding for shorter and longer search tasks, across older and newer markets with different levels of pre-existing consumer information, etc.

The motivating idea behind this paper has been to move the point of focus from switching and traveling *between stores* onto what goes on *inside stores* and, thus, update price search literature to the internet age. The task has prompted us to analyze more closely the time costs of searching for information in a given store and their relation to store complexity – an idea that has natural connections to exiting literature on obfuscation. This exploration of micro foundations of obfuscation à la Becker (1965), if you will, has also led us to analyze more closely firms' incentives to generate search frictions inside

[40] We specify in Section 5 how exactly the result may have to be modified with advertizing and competition.

their stores and the optimal consumer response in a setup where search is not a discrete decision but a continuous process with positive duration.[41]

As an improvement over previous methods, doing so makes it possible to derive endogenous consumer information and prices as (piecewise) continuous functions of intrafirm frictions because the usual consumer hold-up problem in the first store no longer arises.[42] The problem has been a nuisance of most homogenous-product-and-homogenous-consumer sequential search models[43] in channeling adjustment from extensive to intensive margin; our model engages both margins. Luckily, the results arising from this different modeling approach offer more precision over previous work but do not show any discrepancy.[44] The core ideas are easy to apply out of our model (e.g., by letting consumer partition involve $B_0(\boldsymbol{\theta}) > 0$, that is a function of some variable $\boldsymbol{\theta}$).

We have assumed that search order is either a fixed parameter that we vary or that service frictions are common knowledge in the market. In practice, frictions are not observed, of course, but consumers learn about them during search. This implies that, if a price is not found before some cutoff point in time, the consumer has an incentive to switch over to another firm and maybe come back later. Multi-armed bandit literature[45] is then the appropriate modeling framework. We view our model as a necessary first step towards this fuller analysis.

Our message to competition authorities is that the welfare reducing effects of obfuscation could be quite limited but the effects on consumer surplus drastic. For a fixed level of frictions, a tradeoff between consumer surplus and search efficiency arises and somewhat random store choice is then the best for average consumer. As signs of efficiency in markets with strong prominence differences, regulators should concentrate on (i) low degree of choice complexity in the prominent firm and (ii) easy availability of prior information about competing firms.

[41] For an even more microfounded model, see Hämäläinen (2016), where firms have several product variants in their store. Every product variant, that the consumer attends to, adds its own incremental stochastic time cost on the consumer search process. Both of these papers start from the idea that searching for products has positive time costs even if all the information necessary for making the purchase is right in front of your nose.

[42] The familiar property of "no costly consumer search in equilibrium" must still hold in our model with homogenous consumers. Stahl (1996) shows that the assumption of costless search is necessary for price variance.

[43] But see the example in Ellison and Wolitzky (2012) where costly searchers search sometimes twice.

[44] The extension to more than two firms, nevertheless, demonstrates that intermediate price information is important because it determines the captive demand for those firms who occupy second prominent market positions.

[45] See Keller et al. (2005) for example and Bergemann and Välimäki (2006) for a review of bandit models.

Appendix

PROOF OF LEMMA 1: Search

Step 1: Optimal search

To start, note that a consumer can find either no prices, only firm $i = 1$'s price, only firm $i = 2$'s price, or both prices. In the first case her payoff of course equals zero but in three latter cases her payoffs can be denoted more shortly as follows

$$CS^1 := 1 - E\left[p^1\right], CS^2 := 1 - E\left[p^2\right], \text{ and } CS_{min} := 1 - E\left[p_{min}\right].$$

It is clear that the probability of finding zero prices is minimized and the probability of finding two prices maximized by searching in the faster store until a price is found. If the faster store is also the cheaper one, it is also clearly optimal to start from there.

Now the only unresolved case is thus the one where the faster store has higher prices, i.e., where $\theta^1 > \theta^2$ and $CS^1 < CS^2$. This is also the relevant case here because, as we prove later, in equilibrium this kind of tradeoff between frictions and prices arises.

Note that, as the consumers can switch freely any moment t, their continuation value V_{t+dt} in equation (1) is the same whether the consumer is currently at firm $i = 1$ or at firm $i = 2$. This implies that, to maximize the consumer value, V_t, the consumer should search in the store who is offering the largest marginal descent in consumer value, $\dot{V}_t$:

$$\text{argmax}_i V_t^i = \text{argmin}_i \dot{V}_t^i.$$

Now provided the consumer stays in store i during the next short time interval $[t, t + dt]$, the time derivative of the consumer value can be written as follows:[46]

$$\begin{aligned}
\frac{V_{t+dt} - V_t^i}{dt} &= -\theta^i \Big(e^{-\theta^{-i}(1-t-dt)}(1 - E\left[p^i\right] - V_{t+dt}) \\
&\quad + (1 - e^{-\theta^{-i}(1-t-dt)})(1 - E\left[p_{min}\right] - V_{t+dt}) \Big) \to \\
\dot{V}_t^i &= -\theta^i \Big(e^{-\theta^{-i}(1-t)}(E\left[p_{min}\right] - E\left[p^i\right]) + (1 - E\left[p_{min}\right] - V_t) \Big).
\end{aligned}$$

Obviously, the consumer value is positive, $V_t^i \geq 0$, and the change in consumer value is negative, $\dot{V}_t^i \leq 0$, for any t and i. Otherwise, it would pay off to stay idle.

To sum up what we have, this entails that for any point in time $t \in [0, 1]$ a consumer who has not yet discovered a price chooses store $i = 1$ over store $i = 2$ *iff*

$$\begin{aligned}
&\theta^1 e^{-\theta^2(1-t)}(CS^1 - V_t) + \theta^1(1 - e^{-\theta^2(1-t)})(CS_{min} - V_t) \geq \\
&\theta^2 e^{-\theta^1(1-t)}(CS^2 - V_t) + \theta^2(1 - e^{-\theta^2(1-t)})(CS_{min} - V_t),
\end{aligned} \tag{12}$$

or, *iff*

[46] Observe that the time derivative is well defined as long as the consumer does not change the store at t. Furthermore, even if the consumer does switch the store at t, as long as the consumer does not switch stores infinitely often, we can still use these same expressions which then only refer to the right derivative of consumer's value. It is the right derivative that matters for search incentives.

$$\theta^1 e^{-\theta^2(1-t)}(CS^1 - CS_{min}) + \theta^1(CS_{min} - V_t) \geq$$
$$\theta^2 e^{-\theta^1(1-t)}(CS^2 - CS_{min}) + \theta^2(CS_{min} - V_t). \tag{13}$$

Using these expressions, we proceed by showing that, if a consumer prefers one store over the other at a given point in time, t', this remains her preference order also later, for any $t > t'$, if no price is found.

For the first case, suppose that a consumer prefers firm $i = 1$'s store over firm $i = 2$'s store at time t. That would give us:

$$\theta^1 e^{-\theta^2(1-t)} \left(CS^1 - CS_{min}\right) + \theta^1 \left(CS_{min} - V_t\right)$$
$$-\theta^2 e^{-\theta^1(1-t)} \left(CS^2 - CS_{min}\right) - \theta^2 \left(CS_{min} - V_t\right) \geq 0$$

and

$$\dot{V}_t = -\theta^1 e^{-\theta^2(1-t)} \left(CS^1 - CS_{min}\right) - \theta^1 \left(CS_{min} - V_t\right).$$

To see now whether the consumer's preference for store $i = 1$ over store $i = 2$ becomes stronger or weaker over time, we differentiate (13) with respect to time to obtain

$$\begin{aligned}
&\theta^1\theta^2 e^{-\theta^2(1-t)} \left(CS^1 - CS_{min}\right) - \theta^1 \dot{V}_t \\
&- \theta^1\theta^2 e^{-\theta^1(1-t)} \left(CS^2 - CS_{min}\right) + \theta^2 \dot{V}_t \\
&= \theta^1\theta^2 e^{-\theta^2(1-t)} \left(CS^1 - CS_{min}\right) - \theta^1\theta^2 e^{-\theta^1(1-t)} \left(CS^2 - CS_{min}\right) \\
&+ \left(\theta^1 - \theta^2\right) \left(\theta^1 e^{-\theta^2(1-t)} \left(CS^1 - CS_{min}\right) + \theta^1 \left(CS_{min} - V_t\right)\right) \\
&+ \theta^1\theta^2 \left(CS_{min} - V_t\right) - \theta^1\theta^2 \left(CS_{min} - V_t\right) \\
&= \theta^1 \left(\theta^1 e^{-\theta^2(1-t)} \left(CS^1 - CS_{min}\right) + \theta^1 \left(CS_{min} - V_t\right)\right) \\
&- \theta^1 \left(\theta^2 e^{-\theta^1(1-t)} \left(CS^2 - CS_{min}\right) + \theta^2 \left(CS_{min} - V_t\right)\right) \geq 0.
\end{aligned}$$

For the other case, suppose that a consumer prefers firm $i = 2$'s store over firm $i = 1$'s store at time t. That we have:

$$\theta^1 e^{-\theta^2(1-t)} \left(CS^1 - CS_{min}\right) + \theta^1 \left(CS_{min} - V_t\right)$$
$$-\theta^2 e^{-\theta^1(1-t)} \left(CS^2 - CS_{min}\right) - \theta^2 \left(CS_{min} - V_t\right) \leq 0$$

and

$$\dot{V}_t = -\theta^2 e^{-\theta^1(1-t)} \left(CS^2 - CS_{min}\right) - \theta^2 \left(CS_{min} - V_t\right).$$

Again, to see whether the consumer's preference for store $i = 1$ over store $i = 2$ becomes stronger or weaker over time, we differentiate (13) with respect to time

$$
\begin{aligned}
&\theta^1\theta^2 e^{-\theta^2(1-t)}\left(CS^1 - CS_{min}\right) - \theta^1\dot{V}_t \\
&- \theta^1\theta^2 e^{-\theta^1(1-t)}\left(CS^2 - CS_{min}\right) + \theta^2\dot{V}_t \\
&= \theta^1\theta^2 e^{-\theta^2(1-t)}\left(CS^1 - CS_{min}\right) - \theta^1\theta^2 e^{-\theta^1(1-t)}\left(CS^2 - CS_{min}\right) \\
&+ \left(\theta^1 - \theta^2\right)\left(\theta^2 e^{-\theta^1(1-t)}\left(CS^2 - CS_{min}\right) + \theta^2\left(CS_{min} - V_t\right)\right) \\
&+ \theta^1\theta^2\left(CS_{min} - V_t\right) - \theta^1\theta^2\left(CS_{min} - V_t\right) \\
&= \theta^2\left(\theta^1 e^{-\theta^2(1-t)}\left(CS^1 - CS_{min}\right) + \theta^1\left(CS_{min} - V_t\right)\right) \\
&- \theta^2\left(\theta^2 e^{-\theta^1(1-t)}\left(CS^2 - CS_{min}\right) + \theta^2\left(CS_{min} - V_t\right)\right) \leq 0.
\end{aligned}
$$

Altogether, this implies that the consumers have no incentive to switch before they find a price.[47] In other words, they always prefer to search in the same firm's store from the beginning $t = 0$ up to the deadline $t = 1$ given that no price is found in the meantime. To identify which store this is, note that, at the deadline $t = 1$, consumers prefer firm $i = 1$'s store over firm $i = 2$'s store *iff* the following condition holds:

$$
\theta^1 CS^1 \geq \theta^2 CS^2. \qquad \square
$$

Step 2: Value function

We can now also show how to derive the consumer value function V_t. Based on what we just found in Step 1, it is without loss to assume that all consumers start from store i and switch to store $-i$ only when they find a price. Note first that

$$
\dot{V}_t^i = -\theta^i\left(e^{-\theta^{-i}(1-t)}(CS^1 - CS_{min}) + CS_{min} - V_t\right)
$$

defines a linear first order differential equation

$$
\dot{V}_t^i - \theta^i V_t = -\theta^i\left(e^{-\theta^{-i}(1-t)}(CS^1 - CS_{min}) + CS_{min}.\right).
$$

A solution to the related homogenous equation is

$$
V_t = ce^{\theta^i t},
$$

where c is a constant. To solve the non-homogenous equation, we can use the variation of the constants method in which we let the constants $c(t)$ be dependent on time such that

$$
V_t = c(t)e^{\theta^i t}, \dot{V}_t = c(t)\theta^i e^{\theta^i t} + c'(t)e^{\theta^i t}.
$$

This implies that

$$
\dot{V}_t^i + \theta^i V_t = c'(t)e^{\theta^i t} = -\theta^i\left(e^{-\theta^{-i}(1-t)}(CS^i - CS_{min}) + CS_{min}\right)
$$

and

[47] Note that the derivative $\dot{V}_t$ is well defined in both cases since the consumer has no incentive to switch the firm: by continuity of (1), there exist no kink in V_t unless the consumer changes the store.

$$
\begin{aligned}
c(t) &= -\int \theta^i e^{-\theta^i t} e^{-\theta^{-i}(1-t)} (CS^i - CS_{min}) dt - \int \theta^i e^{-\theta^i t} CS_{min} dt + d, \\
&= \frac{\theta^i}{\theta^i - \theta^{-i}} e^{-\theta^i t} e^{-\theta^{-i}(1-t)} (CS^i - CS_{min}) + e^{-\theta^i t} CS_{min} + d,
\end{aligned}
$$

where d is a constant. The consumer value is thereby given as

$$
V_t = \left(\frac{\theta^i}{\theta^i - \theta^{-i}} e^{-\theta^i t} e^{-\theta^{-i}(1-t)} (CS^i - CS_{min}) + e^{-\theta^i t} CS_{min} + d \right) e^{\theta^i t},
$$

where the constant d is determined by the terminal condition

$$
V_1 = \frac{\theta^i}{\theta^i - \theta^{-i}} (CS^i - CS_{min}) + CS_{min} + d e^{\theta^i} = 0
$$

implying

$$
d e^{\theta^i} = -\frac{\theta^i}{\theta^i - \theta^{-i}} (CS^i - CS_{min}) - CS_{min}.
$$

The general solution to the terminal value problem is given by

$$
\begin{aligned}
V_t = V_t^i &= \frac{\theta^i}{\theta^i - \theta^{-i}} \left(e^{-\theta^{-i}(1-t)} - e^{-\theta^i(1-t)} \right) (CS^i - CS_{min}) + \left(1 - e^{-\theta^i(1-t)} \right) CS_{min} \\
&= B_i^t \left(CS^i - CS_{min} \right) + \left(1 - B_\emptyset^t \right) CS_{min} = B_i^t CS^i + B_{1,2}^t CS_{min},
\end{aligned}
$$

where

$$
\begin{aligned}
B_i^t &= \frac{\theta^i}{\theta^i - \theta^{-i}} \left(e^{-\theta^{-i}(1-t)} - e^{-\theta^i(1-t)} \right), \text{ for } \theta^i \neq \theta^{-i}, \\
B_i^t &= e^{\theta^i(1-t)}, \text{ for } \theta^i = \theta^{-i}, \\
B_{1,2}^t &= \left(1 - e^{-\theta^i(1-t)} \right). \qquad \square
\end{aligned}
$$

PROOF OF LEMMA 2: Prices

Step 1: General form of price distributions

Lemma 4 *Assume $B_i > 0$, either for firm $i = 1$ or firm $i = 2$, and $B_{1,2} > 0$. Then, the following hold true in any equilibrium:*

1. *The firms use randomized pricing strategies: F^1 and F^2.*
2. *Both F^1 and F^2 have the same interval support supp$(F) = \left[\underline{p}, \bar{p}\right]$, where $0 < \underline{p} < \bar{p} = 1$.*
3. *Neither has an atom at $p \in \left[\underline{p}, 1\right)$: $lim_{x \to p-} F^i(x) = F^i(p)$ for all $p < 1$ and $i = 1, 2$.*
4. *If F^1 has an atom at $p = 1$, F^2 has not and, if F^2 has an atom at $p = 1$, F^1 has not.*

We assume in this proof that $B_{1,2} > 0$ (there are shoppers) and $B_1 > 0$ or $B_2 > 0$ (there are captives). We also take $\varepsilon > 0$ to represent some tiny (infinitesimal) number.

First, we analyze three cases to prove by contradiction that both firms mix in equilibrium. In doing so, we make the assumption that one of the firms uses a pure strategy p^i. *Case* 1:

$p^i < \min supp(F^{-i})$. As the demand $B_i + B_{1,2} > 0$ is unchanged as long as p^i stays below $\min supp(F^{-i})$, there is a profitable deviation for firm i from price p^i to price $p^i + \varepsilon$. *Case* 2: $p^i > \max supp(F^{-i})$. As the demand $B_{-i} + B_{1,2} > 0$ is unchanged as long as p^i stays above $\max supp(F^{-i})$, there is a profitable deviation for firm $-i$ from a price $p \in supp(F^{-i})$ to a price $p + \varepsilon$. *Case* 3a: $p^i > 0$ and $p^i \in supp(F^{-i})$. As the demand $B_{-i} + B_{1,2}(1 - F^i(p))$ jumps up at $p = p^i$, there is a profitable deviation for firm $-i$ from price p^{-i} to price $p^{-i} - \varepsilon$. *Case* 3b: $p^i = 0$ and $0 \in supp(F^{-i})$. Note that there are some captive consumers but, as both of the firms use the price zero, both of them are making zero profit. Thus, the firm who has captive consumers has a profitable deviation up from zero to extract some profit from the captive consumers. Altogether, Cases 1, 2, 3a and 3b demonstrate that (i) both stores use randomized pricing strategies and that (ii) both stores' profit and prices are bounded away from zero.

Next, we consider the supports $supp(F^i)$ and $supp(F^{-i})$ of the firm's randomized strategies F^i and F^{-i}. Suppose that $supp(F^i) \neq supp(F^{-i})$. This implies that there is some open set $U \neq \emptyset$ such that, with no loss of generality, $U \subset supp(F^i)$ and $U \cap supp(F^{-i}) = \emptyset$. But now, as the demand is unchanged for all $p^i \in U$ there is a profitable deviation up from the lower prices in U to the higher prices in U. This shows that the firm mix over the same set of prices $supp(F) := supp(F^i) = supp(F^{-i})$.

Last, we examine the support for possible gaps and jumps/atoms and delineate its boundaries. *Gaps*: Suppose the support is not connected but has a gap $\left[\underline{g}, \overline{g}\right] \cup supp(F) = \emptyset$ but for some $\left[\underline{g} - \varepsilon, \underline{g}\right] \cup supp(F) \neq \emptyset$ and $[\overline{g}, \overline{g} + \varepsilon] \cup supp(F) \neq \emptyset$. Then, as the demand is unchanged for all $p \in \left[\underline{g}, \overline{g}\right]$, there is a profitable deviation from some price $p \in \left[\underline{g} - \varepsilon, \underline{g}\right]$ to some price $p \in \left[\underline{g}, \underline{g} + \varepsilon\right]$. *Atoms*: Suppose the strategy F^i is not continuous but has an atom $\alpha^i > 0$ at $p^i_\alpha \in supp(F)$. Then, as the demand from shoppers, $\left(1 - F^i(p)\right) B_{1,2}$, is reduced by α^i at p^i_α, there is a profitable deviation for firm $-i$ from a price p^i_α or some price $p^i_\alpha + \varepsilon$ to some price $p^i_\alpha - \varepsilon$. This implies that there can be an atom at the upper bound only and used by a single firm only; this makes sure the other firm does not use p_α or any $p^i_\alpha + \varepsilon$, from which it would have a profitable deviation. *Bounds*: (i) Consider the highest price $\overline{p}$ the firms use. Note that the firm who has that price is only selling to its captive consumers $B_i > 0$. Hence, there is a profitable deviation up in $\overline{p}$ unless it equals 1. (ii) Consider the lowest price $\underline{p}$ the firms use. As both of the stores make some profit, there is a profitable deviation up in price $\underline{p}$ unless it is bounded away from 0. □

Step 2: Closed form of price distributions

Based on above, we only need to determine the firms' profits Π^i, the lower bound $\underline{p} > 0$ of the support, whether we need an atom $\alpha^i > 0$ at the upper bound $\bar{p} = 1$ of the support for firm $i = 1$ or $i = 2$, and the cumulative distribution functions F^1 and F^2.

Note first that, if firm i uses a price $p = 1 - \epsilon$, which lies just below the upper bound, it sells to its captives with probability one and to the shoppers with probability α^{-i}, which gives the likelihood that firm $-i$ has the price $p = 1$. Evaluated at the upper bound the firm's profit is thus given by $\Pi^i = B_i + \alpha^{-i} B_{1,2}$, for $i = 1, 2$.

Instead, by setting the lowest price $\underline{p}$, the firm can attract both its captives and the shoppers with probability one. Evaluated at the lower bound the firm's profit hence becomes $\Pi^i = (B_i + B_{1,2})\, \underline{p}$, for $i = 1, 2$. As the profit has to be the same over the whole support to sustain randomized pricing strategies, equating

$$\Pi^i = B_i + \alpha^{-i} B_{1,2} = (B_i + B_{1,2})\, \underline{p}$$

for $i = 1$ and $i = 2$ gives us the lower bound

$$\underline{p} = \frac{B_i + \alpha^{-i} B_{1,2}}{B_i + B_{1,2}} = \frac{B_{-i} + \alpha^i B_{1,2}}{B_{-i} + B_{1,2}}.$$

Assuming $B_i \geq B_{-i}$, this is solvable only if $\alpha^i = \frac{B_i - B_{-i}}{B_i + B_{1,2}} \geq 0$ implying $\alpha^{-i} = 0$. To simplify, we hence refer to α^i by the shorter notion α. The profits can thus be written as $\Pi^i = B_i$ and $\Pi^{-i} = B_{-i} + \alpha B_{1,2}$ and the lower bound is $\underline{p} = \frac{B_i}{B_i + B_{1,2}}$.

The cumulative distribution functions F^1 and F^2 can now be obtained in closed-form by observing that the profit has to be invariant everywhere in the support. In particular, if a firm $i = 1, 2$ sets price p, its profit is expressed as follows

$$\Pi^i = \left(B_i + (1 - F^{-i}(p))B_{1,2}\right) p, \text{ for } i = 1, 2,$$

which gives

$$F^i(p) = \frac{B_{-i} + B_{1,2}}{B_{1,2}} - \frac{\Pi^{-i}}{B_{1,2}} \frac{1}{p}, \text{ for } p \leq 1,$$

as required.

Observe also that the profit $\Pi^{-i} \leq \Pi^i$ can be rewritten as a convex combination of firm i's captives and firm $-i$'s captives

$$\Pi^{-i} = B_{-i} + \alpha B_{1,2} = B_{-i} + \frac{B_i - B_{-i}}{B_i + B_{1,2}} B_{1,2}$$

$$\Pi^{-i} = \left(1 - \frac{B_{1,2}}{B_i + B_{1,2}}\right) B_{-i} + \frac{B_{1,2}}{B_i + B_{1,2}} B_i$$

$$\Pi^{-i} = \underline{p} B_{-i} + (1 - \underline{p}) B_i,$$

Also, if we continue still with that last expression we get,

$$\begin{aligned}
\Pi^{-i} &= -\underline{p}\,(B_i - B_{-i}) + B_i \\
&= -\frac{B_i}{B_i + B_{1,2}} (B_i - B_{-i}) + B_i \\
&= \left(1 - \frac{B_i - B_{-i}}{B_i + B_{1,2}}\right) B_i \\
&= (1 - \alpha)\, \Pi^i.
\end{aligned}$$

This expression will be needed a bit later in the paper. □

PROOF OF PROPOSITION 1: Effects of higher search efficiency

By Lemma 2, a firm's profit depends on which of the firms has a higher number of captives. According to Equations (3) and (4), the fastest firm has more captives, $B_1 > B_2$, if and only if

$$\frac{\theta^1}{\theta^1 - \theta^2} \left(e^{-\theta^2} - B_0\right) \geq \frac{\theta^2}{\theta^1 - \theta^2} \left(B_0 - e^{-\theta^1}\right) \Longleftrightarrow B_0 \leq \frac{\theta^1 e^{-\theta^2} - \theta^2 e^{-\theta^1}}{\theta^1 - \theta^2} \Longleftrightarrow s^1 \geq \frac{\theta^2}{\theta^1 + \theta^2}.$$

As a result, when search is less efficient than $s^1 = \frac{\theta^2}{\theta^1 + \theta^2}$, the profits are

$$\Pi^1 = B_1 + \frac{B_2 - B_1}{B_2 + B_{1,2}} B_{1,2} \text{ and } \Pi^2 = B_2$$

but, when search is more efficient than $s^1 = \frac{\theta^2}{\theta^1 + \theta^2}$, the profits become

$$\Pi^1 = B_1 \text{ and } \Pi^2 = B_2 + \frac{B_1 - B_2}{B_1 + B_{1,2}} B_{1,2}.$$

For any given $\boldsymbol{\theta}$, the derivatives of B_1 and B_1 with respect to s^1 are denoted by $B_1'(s^1) > 0$ and $B_1'(s^1) < 0$. This gives the effect on the larger profit, which is B_2 for $s^1 < \frac{\theta^2}{\theta^1+\theta^2}$ and B_1 for $s^1 > \frac{\theta^2}{\theta^1+\theta^2}$. To obtain the effect on the smaller profit, we next differentiate it with respect to s^1.

$$B_1 + \frac{B_2 - B_1}{B_2 + B_{1,2}} B_{1,2}$$

is increasing in $s^1 < \frac{\theta^2}{\theta^1+\theta^2}$ when $B_1 < B_2$ iff

$$B_1' B_2 (B_2 + B_{1,2}) > -B_2' B_{1,2} (B_1 + B_{1,2}). \tag{14}$$

By calculating the derivatives from Equations (3) and (4), we next obtain that

$$\frac{B_1'}{B_2'} = \frac{\theta^1}{\theta^2} =: \rho > 1,$$

which allows us to rewrite (14) as a second order polynomial inequality in $B_{1,2}$

$$B_{1,2}^2 - (\rho B_2 - B_1) - \rho B_2 < 0.$$

The lhs is an upward-opening parabola with a negative and a positive root and the inequality (14) is therefore satisfied for all values of $B_1, 2 \in (0, 1)$ which lie below the positive root located at

$$B_{1,2} = \frac{(\rho B_2 - B_1) + \sqrt{(\rho B_2 - B_1)^2 + 4\rho B_2}}{2} > 1.$$

This shows that Π^1 increases for all values of s^1 and essentially similar analysis demonstrates that Π^2 decreases for all values of s^1.

We turn to consumer surplus and reformulate it:

$$\begin{aligned} CS &= B_1 \left(1 - E\left[p^1\right]\right) + B_2 \left(1 - E\left[p^2\right]\right) + B_{1,2} \left(1 - E\left[p_{min}\right]\right) \\ &= B_1 + B_2 + B_{1,2} - B_1 E\left[p^1\right] - B_2 E\left[p^2\right] - B_{1,2} E\left[p_{min}\right] \\ &= 1 - B_0 - B_1 E\left[p^1\right] - B_2 E\left[p^2\right] - B_{1,2} E\left[p_{min}\right], \end{aligned}$$

where $1 - B_0$ captures the positive welfare effects and $B_1 E\left[p^1\right] - B_2 E\left[p^2\right] - B_{1,2} E\left[p_{min}\right]$ the negative effects on prices. We study it more closely next.

Expected prices differ for $s^1 < \frac{\theta^2}{\theta^1+\theta^2}$ and $s^1 > \frac{\theta^2}{\theta^1+\theta^2}$. We start from the latter case by applying the expected prices derived earlier in (6), (7), and (8)

$$\begin{aligned} & B_1 E\left[p^1\right] + B_2 E\left[p^2\right] + B_{1,2} E\left[p_{min}\right] = \\ & B_1 \left[\frac{\Pi^2}{B_{1,2}} \ln\left(\frac{1}{\underline{p}}\right) + \alpha\right] + B_2 \left[\frac{\Pi^1}{B_{1,2}} \ln\left(\frac{1}{\underline{p}}\right)\right] + B_{1,2} \left[\frac{2\Pi^1\Pi^2}{B_{1,2}^2} \left(\frac{1-\underline{p}}{\underline{p}}\right) - \frac{B_1\Pi^2 + B_2\Pi^1}{B_{1,2}^2} \ln\left(\frac{1}{\underline{p}}\right)\right] = \\ & \alpha B_1 + 2B_2 + \left(\frac{B_1\Pi^2}{B_{1,2}} + \frac{B_2\Pi^1}{B_{1,2}} - \frac{\Pi^2}{B_{1,2}} - \frac{\Pi^2}{B_{1,2}}\right) \ln\left(\frac{1}{\underline{p}}\right) = \alpha B_1 + 2B_2 + 2\alpha B_{1,2} \end{aligned}$$

Joining the results gives a surprisingly simple expression for consumer surplus for $s^1 < \frac{\theta^2}{\theta^1+\theta^2}$

$$B_1 + B_2 + B_{1,2} - (\alpha B_1 + 2B_2 + 2\alpha B_{1,2}) = (1-\alpha)B_1 - B_2 + (1-2\alpha)B_{1,2} =$$
$$B_1 - B_2 - \alpha B_1 + (1-2\alpha)B_{1,2} = (B_1 - B_2)(1 - \frac{B_1}{B_1 + B_{1,2}}) + (1-2\alpha)B_{1,2} =$$
$$(B_1 - B_2)\frac{B_{1,2}}{B_1 + B_{1,2}} + (1-2\alpha)B_{1,2} = \alpha B_{1,2} + (1-2\alpha)B_{1,2} = (1-\alpha)B_{1,2}$$

Given that the case where $s^1 < \frac{\theta^2}{\theta^1+\theta^2}$ is identical, we can now rewrite consumer surplus as

$$CS = (1-\alpha)B_{1,2} = \begin{cases} \left(1 - \frac{B_2 - B_1}{B_2 + B_{1,2}}\right) B_{1,2}, & \text{for } s^1 < \frac{\theta^2}{\theta^1+\theta^2}, \\ \left(1 - \frac{B_1 - B_1}{B_1 + B_{1,2}}\right) B_{1,2}, & \text{for } s^1 > \frac{\theta^2}{\theta^1+\theta^2}. \end{cases}$$

We know that B_1 is increasing and B_2 is decreasing in s^1 whereas $B_{1,2}$ is unaffected by s^1. Thus, consumers surplus is increasing for $s^1 < \frac{\theta^2}{\theta^1+\theta^2}$ and decreasing for $s^1 > \frac{\theta^2}{\theta^1+\theta^2}$. □

PROOF OF PROPOSITION 2: Fixed point in search and prices

Note first that by Lemma 1, if

$$\theta^1 \left(1 - E\left[p^1\right]\right) > \theta^2 \left(1 - E\left[p^2\right]\right).$$

then all consumers start from firm $i = 1$, i.e., $s^1 = 1 - s^2 = 1$, whereas, if

$$\theta^1 \left(1 - E\left[p^1\right]\right) < \theta^2 \left(1 - E\left[p^2\right]\right).$$

then all consumers start from firm $i = 2$, i.e., $s^1 = 1 - s^2 = 0$. Otherwise, if

$$\theta^1 \left(1 - E\left[p^1\right]\right) = \theta^2 \left(1 - E\left[p^2\right]\right).$$

then any $s^1 = 1 - s^2 \in [0,1]$ and $s^2 \in [0,1]$ such that $s^1 = 1 - s^2$ would do.

As discussed in the main text, remember that we can always assign a unique joint price distribution $\mathbf{F} := (F^1, F^2)$ to any frictions inside stores, $\boldsymbol{\theta}$, and fractions of consumers starting from each firm, $\mathbf{s}$. Namely, together $\boldsymbol{\theta}$ and $\mathbf{s}$ generate a unique partition of consumers $\{B_0, B_1, B_2, B_{1,2}\}$, which then in turn gives us a unique joint price distribution $\mathbf{F}$ characterized by Lemma 2; the marginals can be denoted by $F^i(\boldsymbol{\theta}, \mathbf{s}) = F^i(\theta^1, \theta^2, s^1, s^2)$. This notation will be helpful in describing the relationship between frictions $\boldsymbol{\theta}$, search $\mathbf{s}$, and prices $\mathbf{F}$.

Note first that, if $\theta^1 \left(1 - E\left[p^1|(\boldsymbol{\theta}, \mathbf{s} = (1,0))\right]\right) \geq \theta^2 \left(1 - E\left[p^2|(\boldsymbol{\theta}, \mathbf{s} = (1,0))\right]\right)$, then the pair $\mathbf{F}(\boldsymbol{\theta}, \mathbf{s} = (1,0))$ and $\mathbf{s} = (1,0)$ is a fixed point. In other words, the price ratio which would arise if all consumers began from store $i = 1$, $E\left[p^1|(\boldsymbol{\theta}, \mathbf{s} = (1,0))\right] / E\left[p^2|(\boldsymbol{\theta}, \mathbf{s} = (1,0))\right]$, is not too high to discourage consumers from starting from that more expensive firm.

It is also clear that, if we started to increase the fraction s^1, starting from the level $s^\star$ where the firms have equally many captives $B_1 = B_2$ for the given level of frictions and raising s^1 gradually up to one, by continuity of $E\left[p^1|(\boldsymbol{\theta}, \mathbf{s})\right] / E\left[p^2|(\boldsymbol{\theta}, \mathbf{s})\right]$ in $\mathbf{s}$, we must span all the values of $E\left[p^1\right] / E\left[p^2\right]$ between one and $E\left[p^1|(\boldsymbol{\theta}, \mathbf{s} = (1,0))\right] / E\left[p^2|(\boldsymbol{\theta}, \mathbf{s} = (1,0))\right]$.

Hence, if we concentrate on cases where

$$\theta^1 \left(1 - E\left[p^1|(\boldsymbol{\theta}, \mathbf{s} = (1,0))\right]\right) < \theta^2 \left(1 - E\left[p^2|(\boldsymbol{\theta}, \mathbf{s} = (1,0))\right]\right))$$

and $\theta^1 > \theta^2$ for which we would have

$$\theta^1\left(1-E\left[p^1|(\boldsymbol{\theta},(s^\star,1-s^\star))\right]\right)>\theta^2\left(1-E\left[p^2|(\boldsymbol{\theta},(s^\star,1-s^\star))\right]\right),$$

by continuity there necessarily exists a fixed point in search and prices, where $\mathbf{s}$ in between $\mathbf{s}=(1,0)$ and $\mathbf{s}=(s^\star,1-s^\star)$ and the following equality holds

$$\frac{\theta^1}{\theta^2}=\frac{1-E\left[p^2|(\boldsymbol{\theta},\mathbf{s})\right]}{1-E\left[p^1|(\boldsymbol{\theta},\mathbf{s})\right]}. \tag{15}$$

To elaborate on this, if (15) is satisfied, all consumers are indifferent between starting from either firm. Each of them can hence be assigned to any start store. If they are assigned according to $\mathbf{s}$, the firms are willing to price in accordance with $\mathbf{F}(\boldsymbol{\theta},\mathbf{s})$: we have a fixed point.

Observe also that in the symmetric case with $\theta^1=\theta^2$ we have a symmetric fixed point

$$\theta^1\left(1-E\left[p^1|(\boldsymbol{\theta},(s^\star,1-s^\star))\right]\right)=\theta^2\left(1-E\left[p^2|(\boldsymbol{\theta},(s^\star,1-s^\star))\right]\right).$$

For uniqueness, we can rely on the monotonicity of $E\left[p^1|(\boldsymbol{\theta},\mathbf{s})\right]/E\left[p^2|(\boldsymbol{\theta},\mathbf{s})\right]$ in $\mathbf{s}$:

$$\begin{aligned}
\frac{1-E\left[p^2|(\boldsymbol{\theta},\mathbf{s})\right]}{1-E\left[p^1|(\boldsymbol{\theta},\mathbf{s})\right]}=&\frac{1-\frac{\Pi^2}{B_{1,2}}ln\left(\frac{1}{\underline{p}}\right)-\alpha}{1-\frac{\Pi^1}{B_{1,2}}ln\left(\frac{1}{\underline{p}}\right)}\\
=&\frac{1-\frac{\underline{p}B_2+(1-\underline{p})B_1}{B_{1,2}}ln\left(\frac{1}{\underline{p}}\right)-\alpha}{1-\frac{\Pi^1}{B_{1,2}}ln\left(\frac{1}{\underline{p}}\right)}\\
=&\frac{1+\frac{\underline{p}B_1-\underline{p}B_2}{B_{1,2}}ln\left(\frac{1}{\underline{p}}\right)-\frac{B_1}{B_{1,2}}ln\left(\frac{1}{\underline{p}}\right)-\alpha}{1-\frac{\Pi^1}{B_{1,2}}ln\left(\frac{1}{\underline{p}}\right)}\\
=&\frac{1+\alpha\frac{B_1}{B_{1,2}}ln\left(\frac{1}{\underline{p}}\right)-\frac{B_1}{B_{1,2}}ln\left(\frac{1}{\underline{p}}\right)-\alpha}{1-\frac{\Pi^1}{B_{1,2}}ln\left(\frac{1}{\underline{p}}\right)}\\
=&1-\alpha,
\end{aligned}$$

where

$$\frac{\partial\alpha}{\partial s^1}=\frac{\partial}{\partial s^1}\left(\frac{B_1-B_2}{B_1+B_{1,2}}\right)>0,$$

because $\frac{\partial B_1}{\partial s^1}>0$, $\frac{\partial B_2}{\partial s^1}<0$ and $\frac{\partial B_{1,2}}{\partial s^1}=0$; these partials are easy to sign based on (2), (3), (4) and (5). As a result, as we increase s^1, starting from the point $s^\star$ where $B_1=B_2$ holds, all the way up until unity, $E\left[p^1|(\boldsymbol{\theta},\mathbf{s})\right]/E\left[p^2|(\boldsymbol{\theta},\mathbf{s})\right]$ decreases: the fixed point is unique. □

PROOF OF REMARK 1: No Bertrand equilibrium

Bertrand equilibrium requires that both firms choose zero frictions $\boldsymbol{\theta}=(\infty,\infty)$. Yet, both firms gain if one of them deviates to some finite rate θ because it raises their profit up from zero to $\frac{B_iB_{1,2}}{B_i+B_{1,2}}=\left(1-e^{-\theta}\right)e^{-\theta}$ (to the deviator, who has the start share $s^{-i}=0$ due to its positive frictions $\theta^{-i}<\infty$) and $B_i=1-e^{-\theta}$ (to the non-deviator, who gets all starting consumers $s^i=1$ thanks to its markedly lower frictions $\theta^i=\infty$). □

PROOF OF REMARK 2: No Diamond equilibrium

As the consumers always search, Diamond equilibrium requires that at least one of the firms has infinite frictions and is therefore practically out of the market, $\boldsymbol{\theta} = \left(\theta^i, 0\right), \left(0, \theta^{-i}\right)$. Its profit equals zero because it serves nobody. However, for any lower level of frictions, the firm's profit is positive, as a prominent firm $\Pi^i = B_i > 0$ or the non-prominent firm $\Pi^{-i} = (1-\alpha) B_i > 0$. There is hence a profitable deviation to higher $\theta' > 0$. $\square$

PROOF OF LEMMA 3: Efficient search, prominence order

We just proved that for $s^1 < 1$,

$$\begin{aligned}
\frac{\theta^2}{\theta^1} &= 1-\alpha \\
\frac{\theta^2}{\theta^1} &= \frac{B_2 + B_{1,2}}{B_1 + B_{1,2}} \\
\frac{\theta^2}{\theta^1} &= \frac{1 - B_0 - B_1}{1 - B_0 - B_2} \\
\frac{\theta^2}{\theta^1} &= \frac{1 - B_0\left(1 - \frac{\theta^1}{\theta^1 - \theta^2}\right) - \frac{\theta^1}{\theta^1 - \theta^2} e^{-\theta^2}}{1 - B_0\left(1 + \frac{\theta^2}{\theta^1 - \theta^2}\right) + \frac{\theta^1}{\theta^2 - \theta^2} e^{-\theta^1}} \\
\frac{\theta^2}{\theta^1} &= \frac{\theta^1 - \theta^2 + \theta^2 B_0 - \theta^1 e^{-\theta^2}}{\theta^1 - \theta^2 + \theta^2 e^{-\theta^1} - \theta^1 B_0}.
\end{aligned}$$

We can hence solve for B_0 as

$$B_0 = -\frac{1}{2}\frac{\theta^2}{\theta^1}\left(1 - e^{-\theta^1}\right) - \frac{1}{2}\frac{\theta^1}{\theta^2}\left(1 - e^{-\theta^2}\right) + 1.$$

From here on, it is useful to work with the reparametrization $\rho = \frac{\theta^2}{\theta^1}$, which gives

$$B_0 = -\frac{1}{2}\rho\left(1 - e^{-\theta^1}\right) - \frac{1}{2}\rho^{-1}\left(1 - e^{-\theta^2}\right) + 1.$$

Now, since $\frac{\partial \theta^1}{\partial \rho} = -\frac{\theta^1}{\rho}$ and $\frac{\partial \theta^2}{\partial \rho} = \frac{\theta^2}{\rho}$,

$$\frac{\partial B_0}{\partial \rho} = -\frac{1}{2}\left(1 - e^{-\theta^1}\right) + \frac{1}{2}\theta^1 e^{-\theta^1} + \frac{1}{2}\rho^{-1}\left(1 - e^{-\theta^2}\right) + \frac{1}{2}\rho^{-2}\theta^2 e^{-\theta^2}$$

or, returning to the original variables,

$$\frac{\partial B_0}{\partial \rho} = \frac{1}{2}\left(-\left(1 - e^{-\theta^1} - \theta^1 e^{-\theta^1}\right) + \frac{\theta^1}{\theta^2}\left(1 - e^{-\theta^2} + \theta^1 e^{-\theta^2}\right)\right).$$

This is positive for all $\theta^1 \geq \theta^2 > 0$ because

$$\frac{1 - e^{-\theta^1} - \theta^1 e^{-\theta^1}}{\theta^1} < \frac{1 - e^{-\theta^1}}{\theta^1} < \frac{1 - e^{-\theta^2}}{\theta^2} < \frac{1 - e^{-\theta^2} + \theta^1 e^{-\theta^2}}{\theta^2}$$

and the function $\frac{1-e^{-x}}{x}$ is decreasing in x.

We can now revert to $\rho = \frac{\theta^2}{\theta^1} = \frac{1-B_0-B_1}{1-B_0-B_2}$ to solve it for B_1 and B_2 as a function of ρ

$$B_1 = (1-\rho)(1-B_0) + \rho B_2,$$

$$B_2 = \left(1-\rho^{-1}\right)(1-B_0) + \rho^{-1}B_1.$$

Their partials with respect to ρ are given by

$$\frac{\partial B_1}{\partial \rho} = -(1-B_0-B_2) - (1-\rho)\frac{\partial B_0}{\partial \rho} + \rho\frac{\partial B_2}{\partial \rho},$$

$$\frac{\partial B_2}{\partial \rho} = \rho^{-1}(1-B_0-B_1) - \left(1-\rho^{-1}\right)\frac{\partial B_0}{\partial \rho} + \rho^{-1}\frac{\partial B_1}{\partial \rho}.$$

As we can now take $\rho = \rho(\theta^1, \theta^2)$ as a firm's choice variable, the first order conditions are

$$\frac{\partial \Pi^1}{\partial \rho} = 0 \Longleftrightarrow \frac{\partial B_1}{\partial \rho} = 0 \Longleftrightarrow \rho\frac{\partial B_2}{\partial \rho} = 1 - B_0 - B_2 + (1-\rho)\frac{\partial B_0}{\partial \rho} > 0$$

and

$$\frac{\partial \Pi^2}{\partial \rho} = 0 \Longleftrightarrow \frac{\partial (1-\alpha)\Pi^1}{\partial \rho} = 0 \Longleftrightarrow \frac{\partial \rho B_1}{\partial \rho} = 0 \Longleftrightarrow B_1 + \rho\frac{\partial B_1}{\partial \rho} = 0.$$

Both of them cannot be satisfied for the same $\rho \neq 1$ because a firm's profit is positive, $\Pi^1 = B_1 > 0$. This implies that it cannot be optimal for both firms to use such rates θ^1 and θ^2 that $s^1 < 1$. $\square$

PROOF OF PROPOSITION 3: No efficient equilibrium with zero frictions in prominent store

We consider case by case firm i's best response, θ^i, to firm $-i$'s frictions, θ^{-i}.

Case 1: $\theta^{-i} = 0$.

If firm $-i$ is out of the market, firm i acts like a monopolist and serves its consumers instantaneously: $\theta^i = \infty$.

Case 2: $\theta^{-i} \in (0, \infty)$.

First, if the firm chooses an extremely slow rate $\theta^i = 0$ it serves nobody and extracts no profits.[48]

Second, if the firm chooses an extremely fast rate $\theta^i = \infty$ such that $s^i = 1$, the firm's profit is given as[49]

$$\Pi^i = e^{-\theta^{-i}}.$$

Third, if the firm chooses a finite but sufficiently fast rate $\theta^i >> \theta^{-i}$ such that $s^i = 1$, the firm's profit can be written as

$$\Pi^i = \frac{\theta^i}{\theta^i - \theta^{-i}}\left(1 - e^{-(\theta^i - \theta^{-i})}\right)e^{-\theta^{-i}}.$$

It is now easy to show that

$$\frac{\theta^i}{\theta^i - \theta^{-i}}\left(1 - e^{-(\theta^i - \theta^{-i})}\right) > 1$$

[48] For $\theta^i = 0$, $B_{-i} = 1 - e^{-\theta^{-i}}$ and $B_0 = e^{-\theta^{-i}}$ while $B_i = B_{1,2} = 0$.

[49] For $\theta^i = \infty$, $B_i = e^{-\theta^{-i}}$ and $B_{1,2} = 1 - e^{-\theta^{-i}}$ while $B_{-i} = B_0 = 0$.

as long as $\left|\theta^i - 1\right| > \left|\theta^{-i} - 1\right|$.

This implies that, by choosing a large enough finite θ^i, the firm is guaranteed to extract more revenue than by choosing $\theta^i = 0$ or $\theta^i = \infty$.

Case 3: $\theta^{-i} = \infty$.

Note first that, if both firms have an infinite rate, $\theta^i = \infty$, all consumers find all prices and both firms' profits go to zero.

Instead, if firm $-i$ has an infinite rate and firm i has a finite rate, $\theta^{-i} = \infty$ and $\theta^i < \infty$ such that $s^{-i} = 1$, firm i's profit is[50]

$$\Pi^i = \frac{B_{-i} - B_i}{B_{-i} + B_{1,2}} B_{1,2} = e^{-\theta^i}\left(1 - e^{-\theta^i}\right),$$

It is maximized by $\theta^i = ln(2) < \infty$.

It is thus clear from Cases 1, 2 and 3 that $\theta^i = \infty$ cannot arise in equilibrium. $\square$

PROOF OF PROPOSITION 4: Equally many informed consumers and uninformed consumers

For values outside of the boundary where $\frac{\theta^2}{\theta^1} = 1 - \alpha$, the first order condition is

$$\frac{\partial B_1}{\partial \theta^2}(1 - B_0 - B_1) - \frac{\partial B_1}{\partial \theta^2} B_1 = 0$$

where $B_{1,2} = 1 - B_0 - B_1$. The unique solution is thereby given by $B_1 = B_{1,2}$.

Note also that $\frac{\partial B_1}{\partial \theta^2} \leq 0$ because

$$B_1 = \theta^1 e^{-\theta^1} \frac{e^\delta - 1}{\delta}$$

and $\frac{\partial}{\partial \delta}\frac{e^\delta - 1}{\delta} \geq 0$ and $\frac{\partial}{\partial \theta^2}\delta = -1$. $\square$

PROOF OF PROPOSITION 5: Equilibrium obfuscation

Suppose that the other firm has chosen rate θ. We consider separately a firm's the upper best response to $\theta' > \theta$, symmetric response to $\theta' = \theta$ and lower best response $\theta' < \theta$. By Lemma 3, we know already that there exist no symmetric equilibrium where $\theta' = \theta$ or asymmetric equilibrium where $s^1 < 1$. However, to pind down the equilibrium, we need to consider both upper and lower deviations and allow for also the possibility that $s^1 < 1$.

We already know that, if $s^1 = 1$ (Lemma 2),

$$\Pi^1 = B_1 = \frac{\theta^1}{\theta^2 - \theta^1}\left(e^{-\theta^1} - e^{-\theta^2}\right), \Pi^2 = (1 - \alpha)B_1 = \frac{B_1 B_{1,2}}{B_1 + B1,2}. \tag{16}$$

whereas, if $s^1 < 1$, then we have (Prop. 2)

$$\Pi^1 = B_1 = s^1 \frac{\theta^1}{\theta^2 - \theta^1}\left(e^{-\theta^1} - e^{-\theta^2}\right), \Pi^2 = (1 - \alpha)B_1 = \frac{\theta^2}{\theta^1} B_1. \tag{17}$$

We first study the case $s^1 = 1$ to identify a candidate equilibrium and then check that there are no profitable deviations even if we allow for $s^1 < 1$.

[50]Here, $B_{-i} = e^{-\theta^i}$ and $B_{1,2} = 1 - e^{-\theta^i}$ whereas $B_i = B_0 = 0$.

Step 1: Fixed point of a relaxed problem where $s^1 = 1$.

Now the prominent firm's upper best response θ^1 is given by the first order condition

$$\frac{\theta^2}{(\theta^1-\theta^2)^2}\left(e^{-\theta^2}-e^{\theta^1}\right)=\theta^1 e^{-\theta^1}, \tag{18}$$

and the non-prominent firm's lower best response θ^2 is given by the first order condition

$$\frac{\theta^1}{(\theta^1-\theta^2)}\left(e^{-\theta^2}-e^{\theta^1}\right)=\frac{1-e^{-\theta^1}}{2}. \tag{19}$$

Note that the maximizer of the prominent firm's problem $\theta^1(\theta^2)^\star$ is decreasing in θ^2 and the maximizer of the non-prominent firm's problem $\theta^2(\theta^1)^\star$ is decreasing in θ^1. Firms' equilibrium obfuscations are thus strategic substitutes.
Joining these gives the equations

$$\frac{1}{2}\frac{e^{\theta^1}-1}{\theta^1}=\frac{e^{(\theta^1-\theta^2)}-1}{(\theta^1-\theta^2)}=\frac{\theta^1}{\theta^2},$$

that have the unique solution of

$$\boldsymbol{\theta}^\star \approx (2.76, 1.03).$$

This generates the following profits to the prominent firm and the non-prominent firm, respectively,

$$(\Pi^1)^\star \approx \frac{2.76}{2.76-1.03}\left(e^{-1.03}-e^{-2.76}\right)\approx 0.469 \tag{20}$$

$$(\Pi^2)^\star \approx 0.5\frac{2.76}{2.76-1.03}\left(e^{-1.03}-e^{-2.76}\right)\approx 0.234. \tag{21}$$

Step 2: No profitable deviations allowing for any s^1.

Note that in this part we exceptionally allow for the possibility that $\theta^1 < \theta^2$ reversing the usual prominence order: here the faster firm can become the slower firm, and vice versa. In comparing the profits that a firm obtains, a noteworthy observation that can be made from (16) and (17) is that

$$\Pi^1=\begin{cases} B_1=\frac{\theta^1}{\theta^2-\theta^1}\left(e^{-\theta^1}-e^{-\theta^2}\right), & \text{for } s^1=1\\ B_1=s^1\frac{\theta^1}{\theta^2-\theta^1}\left(e^{-\theta^1}-e^{-\theta^2}\right), & \text{for } s^1\in(0,1) \text{ and } \theta^1\geq\theta^2\\ (1-\alpha)B_2=(1-s^1)\frac{\theta^1}{\theta^2-\theta^1}\left(e^{-\theta^1}-e^{-\theta^2}\right), & \text{for } s^1\in(0,1) \text{ and } \theta^1\leq\theta^2\\ (1-\alpha)B_2=(1-\alpha)\frac{\theta^2}{\theta^2-\theta^1}\left(e^{-\theta^1}-e^{-\theta^2}\right), & \text{for } s^1=0 \end{cases}$$

and, similarly,

$$\Pi^2=\begin{cases} B_2=\frac{\theta^2}{\theta^2-\theta^1}\left(e^{-\theta^1}-e^{-\theta^2}\right), & \text{for } s^2=1\\ B_2=s^2\frac{\theta^2}{\theta^2-\theta^1}\left(e^{-\theta^1}-e^{-\theta^2}\right), & \text{for } s^2\in(0,1) \text{ and } \theta^2\geq\theta^1\\ (1-\alpha)B_1=(1-s^2)\frac{\theta^2}{\theta^2-\theta^1}\left(e^{-\theta^1}-e^{-\theta^2}\right), & \text{for } s^2\in(0,1) \text{ and } \theta^2\leq\theta^1\\ (1-\alpha)B_1=(1-\alpha)\frac{\theta^1}{\theta^2-\theta^1}\left(e^{-\theta^1}-e^{-\theta^2}\right), & \text{for } s^2=0 \end{cases}$$

This implies that, when $s^i > 0$, we can approximate the profit of firm i from above by

$$\overline{\Pi}^i(\theta^1, \theta^2) := \frac{\theta^i}{\theta^2 - \theta^1}\big(e^{-\theta^1} - e^{-\theta^2}\big).$$

This upper bound gives the exact profit of firm i when $s^i = 1$ but is strictly greater than it when $s^i \in (0, 1)$. On the other hand, when $s^i = 0$, the profit of firm i is equal to

$$\underline{\Pi}^i(\theta^1, \theta^2) := \frac{B_{-i}B_{1,2}}{B_{-i} + B_{1,2}} = \frac{B_{-i}(1 - e^{-\theta^{-i}} - B_{-i})}{1 - e^{-\theta^{-i}}}$$

where

$$B_{-i} = \frac{\theta^{-i}}{\theta^2 - \theta^1}\big(e^{-\theta^1} - e^{-\theta^2}\big).$$

The maximum profit that firm i can get for any fixed θ^{-i} is hence given by

$$\tilde{\Pi}^i(\theta^{-i}) := \max_{\theta^i}\Big\{\overline{\Pi}^i(\theta^i, \theta^{-i}), \underline{\Pi}^i(\theta^i, \theta^{-i})\Big\}$$

Hence, there are no profitable unilateral deviations from the candidate equilibrium $\boldsymbol{\theta}^\star \approx (2.76, 1.03)$ for either of the firms if the following two conditions are satisfied

$$\Pi^1(\boldsymbol{\theta}^\star) \geq \tilde{\Pi}^1(\theta^{2\star}) \text{ and } \Pi^2(\boldsymbol{\theta}^\star) \geq \tilde{\Pi}^2(\theta^{1\star}).$$

We show this next by studying the local maxima and local minima of $\underline{\Pi}^i(\theta^1, \theta^2)$ and $\underline{\Pi}^i(\theta^1, \theta^2)$, separately, for θ^1 fixed to $(\theta^{1\star}) \approx 2.76$ and then for θ^2 fixed to $(\theta^{2\star}) \approx 1.03$.

(i) We start by considering $\overline{\Pi}^2(\theta^{1\star}, \theta^2)$ to which we refer for concreteness by $\overline{\Pi}^2(2.76, \theta^2)$.[51] We differentiate this with respect to θ^2 to get its first derivative

$$\frac{-2.76}{(\theta^2 - 2.76)^2}\big(e^{-2.76} - e^{-\theta^2}\big) + \frac{\theta^2}{\theta^2 - 2.76}e^{-\theta^2}$$

This is positive for $\theta^2 < \theta^{2\prime} \approx 1.75$ and negative thereafter. There is hence a global maximum, at $\theta^2 = \theta^{2\prime} \approx 1.75$.

(ii) We continue by analyzing $\overline{\Pi}^1(\theta^1, \theta^{2\star})$ to which we refer similarly as above by $\overline{\Pi}^1(\theta^1, 1.03)$. We differentiate this with respect to θ^1 to get its first derivative

$$\frac{-1.03}{(\theta^1 - 1.03)^2}\big(e^{-1.03} - e^{-\theta^1}\big) + \frac{\theta^1}{\theta^1 - 1.03}e^{-\theta^1}$$

This is positive for $\theta^1 < \theta^{1\star} \approx 2.76$ and negative thereafter. There is again a global maximum, at $\theta^1 = \theta^{1\star} \approx 2.76$.

(iii) We move on to $\underline{\Pi}^2(\theta^{1\star}, \theta^2)$ and $\underline{\Pi}^1(\theta^1, \theta^{2\star})$, which have the same general form

$$\frac{B_{-i}B_{1,2}}{B_{-i} + B_{1,2}} = \frac{B_{-i}(1 - e^{\theta^{-i}} - B_{-i})}{1 - e^{\theta^{-i}}}.$$

Both of them are hence first increasing in θ^i (for $B_{-i} < B_{1,2}$) and thereafter decreasing in θ^i (for $B_{-i} > B_{1,2}$). The maximum is given by $B_{-i} = B_{1,2} = (1 - B_0)/2$, which gives us immediately an upper bound for the value attained by function $\underline{\Pi}^i$

[51] By continuity, the difference $\Big|\overline{\Pi}^2(\theta^{1\star}, \theta^2) - \overline{\Pi}^2(2.76, \theta^2)\Big|$ or $\Big|\overline{\Pi}^1(\theta^1, \theta^{2\star}) - \overline{\Pi}^1(\theta^1, 1.03)\Big|$ is small.

$$\underline{\Pi}^i(\theta^i, \theta^{-i}) \leq \frac{B_{-i}^2}{2B_{-i}} = \frac{B_{-i}^2}{2B_{-i}} = \frac{B_{-i}}{2} = \frac{1-e^{-\theta^{-i}}}{4}$$

To complete the proof we observe that firm $i = 1$ has no profitable deviation downwards and nor does firm $i = 1$ have a profitable deviation upwards because

$$\frac{1-e^{-1.03}}{4} \approx 0.321 < 0.469, \text{ and } \frac{1.75}{1.75 - 2.76}\left(e^{-2.76} - e^{-1.75}\right) \approx 0.191 < 0.234.$$

Note also that $2.76\left(1 - E\left[p^1\right]\right) > 1.03\left(1 - E\left[p^2\right]\right)$, which implies that indeed $s^1 = 1$. The expected prices can be calculated based on (6) and (7). □

PROOF OF PROPOSITION 6: Obfuscation and advertizing with two firms

Proposition 6 is written in a self-explanatory way building on previous Lemma 2 and the proofs of Propositions 3, 4 and 5. Only conceptual changes are involved in moving from partition $B = 1$ into partition $A + B = 1$. □

PROOF OF PROPOSITION 7: Prices with three firms

We derive equilibrium profits.

The basic logic in the proof is standard and similar to that in Lemmata 4 and 2. We do not repeat all those steps here. We only sketch the main points where analysis might departure from the standard one.

All the firms must earn equally high profits at $\underline{p} < 1$ and at $\overline{p} = 1$. For each firm who uses the lowest prices $\underline{p}$ this implies that, respectively from the 1st prominent to the 2nd and the 3rd prominent firms,

$$\begin{aligned}
\Pi_1 &= C_1 = (C_1 + C_{1,2} + C_{1,3} + C_{1,2,3})\underline{p}, \\
\Pi_2 &= C_2 + \alpha C_{1,2} = (C_2 + C_{1,2} + C_{2,3} + C_{1,2,3})\underline{p}, \\
\Pi_3 &= C_3 + \alpha C_{1,3} = (C_3 + C_{1,3} + C_{2,3} + C_{1,2,3})\underline{p}.
\end{aligned}$$

The optimality condition of the 1st prominent firm determines the lowest price $\underline{p} = \underline{p}^1$

$$\underline{p}^1 = \frac{C_1}{C_1 + (C_{1,2} + C_{1,3} + C_{1,2,3})},$$

and the optimality condition of the 3rd prominent firm pins down the atom size $\alpha = \alpha^3$

$$\begin{aligned}
\alpha^3 &= \frac{(C_3 + C_{1,3} + C_{2,3} + C_{1,2,3})\underline{p} - C_3}{C_{1,3}}, \\
&= \frac{\frac{C_3+(C_{1,3}+C_{2,3}+C_{1,2,3})}{C_1+(C_{1,2}+C_{1,3}+C_{1,2,3})}C_1 - C_3}{C_{1,3}}, \\
&= \frac{C_3 + (C_{1,3} + C_{2,3} + C_{1,2,3})}{C_1 + (C_{1,2} + C_{1,3} + C_{1,2,3})}\frac{C_1}{C_{1,3}} - \frac{C_3}{C_{1,3}}.
\end{aligned}$$

Note that the atom size is here larger than it would be were it derived from the similar optimality condition for the 2nd or 1st prominent firm

$$\alpha^3 = \frac{(C_3 + C_{1,3} + C_{2,3} + C_{1,2,3})\underline{p}^1 - C_3}{C_{1,3}} >$$
$$\alpha^2 = \frac{(C_2 + C_{1,2} + C_{2,3} + C_{1,2,3})\underline{p}^1 - C_2}{C_{1,2}}.$$

This guarantees that no firm strictly prefers to undercut the lowest prices, which would otherwise constitute a profitable deviation. Particularly, if the 3rd prominent firm has no incentive to deviate below the lowest price, then neither does the 2nd prominent firm

Indeed, here only the 1st prominent and the 3rd prominent firms apply the lowest prices in the neighborhood of $\underline{p}^1$. For this atom size, the 2nd prominent firm earns so much profit that it has no incentive to charge a price below $\underline{p}^2 \in (\underline{p}^1, 1)$, which is given by

$$\Pi_2 = C_2 + \alpha C_{1,2} = (C_2 + C_{1,2} + C_{2,3} + C_{1,2,3})\underline{p}^2.$$

The equilibrium price distributions can now be derived from profit equivalence requirements

$$\Pi_1 = (C_1 + C_{1,2}(1 - F_2(p)) + C_{1,3}(1 - F_3(p)) + C_{1,2,3}(1 - F_2(p))(1 - F_3(p)))p, \text{ for } p \in (\underline{p}^1, 1)$$
$$\Pi_2 = (C_2 + C_{1,2}(1 - F_1(p)) + C_{2,3}(1 - F_3(p)) + C_{1,2,3}(1 - F_1(p))(1 - F_3(p)))p, \text{ for } p \in (\underline{p}^2, 1)$$
$$\Pi_3 = (C_3 + C_{1,3}(1 - F_1(p)) + C_{2,3}(1 - F_2(p)) + C_{1,2,3}(1 - F_1(p))(1 - F_2(p)))p, \text{ for } p \in (\underline{p}^1, 1)$$

For any given price $p \geq p'$, we can use the first equation to solve for $\phi_3 := 1 - F_3(p)$, the second equation to solve for $\phi_1 := 1 - F_1(p)$, and the third equation to solve for $\phi_2 := 1 - F_2(p)$ as

$$\phi_3(\phi_2, p) = \min\left\{\max\left\{\frac{\Pi_1/p - C_1 - C_{1,2}\phi_2}{C_{1,3} + C_{1,2,3}\phi_2}, 0\right\}, 1\right\}$$
$$\phi_1(\phi_3, p) = \min\left\{\max\left\{\frac{\Pi_2/p - C_2 - C_{2,3}\phi_3}{C_{1,2} + C_{1,2,3}\phi_3}, 0\right\}, 1\right\}$$
$$\phi_2(\phi_1, p) = \min\left\{\max\left\{\frac{\Pi_3/p - C_3 - C_{1,3}\phi_1}{C_{2,3} + C_{1,2,3}\phi_1}, 0\right\}, 1\right\}$$

Obviously, ϕ_2 is decreasing in ϕ_1, which is decreasing in ϕ_3, which is decreasing in ϕ_3. This implies that the composite function which is defined by $\phi := \phi_2 \circ \phi_1 \circ \phi_3(\phi_2')$ is decreasing in ϕ_2'. Function ϕ maps a unique ϕ^2 to any $\phi^{2\prime}$ for all $p \in (\underline{p}^2, 1)$ and, when $\phi_2 = 1$, it maps a unique ϕ^3 to any $\phi^{3\prime}$ for all $p \in (\underline{p}^1, \underline{p}^2)$. By continuity of ϕ in p and ϕ_2' or ϕ_3', a unique continuous (almost everywhere) equilibrium price distribution exists if and only if $\phi_2(\phi_1(\phi_3(0))) \geq \phi_2(\phi_1(\phi_3(1)))$ for all $p \in (\underline{p}^2, 1)$. This inequality must clearly hold because ϕ is decreasing in ϕ_2'. At price unity $p = 1$, the unique fixed point is $\phi_2 = 0$, $\phi_3 = 0$ and $\phi_1 = \alpha$.

Note that in certain cases it might be possible to construct an equilibrium where the roles of the 1st and 2nd prominent firms are reversed but the logic is otherwise identical

$$\Pi_1 = C_1 + \alpha^3 C_{1,2} = (C_1 + C_{1,2} + C_{1,3} + C_{1,2,3})\underline{p}^1,$$
$$\Pi_2 = C_2 = (C_2 + C_{1,2} + C_{2,3} + C_{1,2,3})\underline{p}^2,$$
$$\Pi_3 = C_3 + \alpha^3 C_{2,3} = (C_3 + C_{1,3} + C_{2,3} + C_{1,2,3})\underline{p}^2.$$

Again, the optimality condition of the 3rd prominent firm pins down the atom size $\alpha = \alpha^3$

$$\alpha^3 = \frac{(C_3 + C_{1,3} + C_{2,3} + C_{1,2,3})\underline{p}^2 - C_3}{C_{2,3}},$$
$$= \frac{\frac{C_3 + (C_{1,3}+C_{2,3}+C_{1,2,3})}{C_2+(C_{1,2}+C_{2,3}+C_{1,2,3})} C_1 - C_3}{C_{2,3}},$$
$$= \frac{C_3 + (C_{1,3} + C_{2,3} + C_{1,2,3})}{C_2 + (C_{1,2} + C_{2,3} + C_{1,2,3})} \frac{C_1}{C_{2,3}} - \frac{C_3}{C_{2,3}},$$

but the optimality condition of the 2st prominent firm determines the lowest price $\underline{p} = \underline{p}^2$

$$\underline{p}^2 = \frac{C_2}{C_2 + (C_{1,2} + C_{2,3} + C_{1,2,3})}.$$

For this equilibrium type to exist, it must hold that $C_2 > 0$ and $C_{2,3} > 0$, which here implies also $C_1 > 0$ and $C_{1,2} > 0$. □

PROOF OF PROPOSITION 8: Obfuscation and advertizing with three firms

We have assumed that $C_2 = C_3 = C_{2,3} = 0$.

The 1st prominent firm maximizes

$$C_1,$$

and the 3rd prominent firm maximizes

$$\frac{C_1(C_{1,3} + C_{1,2,3})}{1 - C_0},$$

and the 2nd prominent firm maximizes

$$\frac{C_1(C_{1,3} + C_{1,2,3})}{1 - C_0} \frac{C_{1,2}}{C_{1,3}}.$$

We have assumed that a fraction s^2 (s^3) of consumers B follows the path from the 1st prominent firm to the 2nd (3rd) prominent firm and only then to the 3rd (2nd) prominent firm. We refer to these consumers by $B(s^2)$ and $B(s^3)$, respectively, where $s^3 = 1 - s^2 \leq 1/2$ and $B(s^3) \leq B(s^2)$. Naturally,

$$\frac{\partial B_1(s^2)}{\partial \theta^2} + \frac{\partial B_{1,2}(s^2)}{\partial \theta^2} + \frac{\partial B_{1,2,3}(s^2)}{\partial \theta^2} = 0, \quad \frac{\partial B_{1,3}(s^3)}{\partial \theta^2} + \frac{\partial B_{1,2,3}(s^3)}{\partial \theta^2} = 0$$
$$\frac{\partial B_1(s^3)}{\partial \theta^3} + \frac{\partial B_{1,3}(s^3)}{\partial \theta^3} + \frac{\partial B_{1,2,3}(s^3)}{\partial \theta^3} = 0, \quad \frac{\partial B_{1,2}(s^2)}{\partial \theta^3} + \frac{\partial B_{1,2,3}(s^2)}{\partial \theta^3} = 0.$$

We have retained from the Poisson setting the assumption that the initial order of firms does not affect how many consumers find two prices in the end, $\frac{\partial B_{1,2,3}(s^2)}{\partial \theta^i} = \frac{\partial B_{1,2,3}(s^3)}{\partial \theta^i}$ for $i = 2, 3$.

The first order condition of the 1st prominent firm is

$$\frac{\partial B_1}{\partial \theta^1} = 0$$

The first order condition of the 3rd prominent firm is

$$\frac{\partial C_1}{\partial \theta^3}(C_{1,3}+C_{1,2,3})+\left(\frac{\partial C_{1,3}}{\partial \theta^3}+\frac{\partial C_{1,2,3}}{\partial \theta^3}\right)C_1=0$$
$$C_1=\frac{\frac{\partial C_1}{\partial \theta^3}}{-\left(\frac{\partial C_{1,3}}{\partial \theta^3}+\frac{\partial C_{1,2,3}}{\partial \theta^3}\right)}(C_{1,3}+C_{1,2,3})$$
$$C_1=\frac{\frac{\partial C_1}{\partial \theta^3}}{\frac{\partial C_1}{\partial \theta^3}+\frac{\partial C_{1,2}}{\partial \theta^3}}(C_{1,3}+C_{1,2,3})$$

where a higher rate θ^3 reduces both $C_1(s^3)$ and $C_{1,2}(s^2)$ such that $\frac{\frac{\partial C_1}{\partial \theta^3}}{\frac{\partial C_1}{\partial \theta^3}+\frac{\partial C_{1,2}}{\partial \theta^3}}<1$.

The first order condition of the 2nd prominent firm is

$$\left(\frac{\partial C_1}{\partial \theta^2}(C_{1,3}+C_{1,2,3})+\left(\frac{\partial C_{1,3}}{\partial \theta^2}+\frac{\partial C_{1,2,3}}{\partial \theta^2}\right)C_1\right)\frac{C_{1,2}}{C_{1,3}}+C_1(C_{1,2}+C_{1,2,3})\frac{C_{1,2}\frac{\partial C_{1,3}}{\partial \theta^2}-C_{1,2}\frac{\partial C_{1,3}}{\partial \theta^2}}{C_{1,3}^2}=0$$
$$\left(\frac{\partial C_1}{\partial \theta^2}(C_{1,3}+C_{1,2,3})+\left(\frac{\partial C_{1,3}}{\partial \theta^2}+\frac{\partial C_{1,2,3}}{\partial \theta^2}\right)C_1\right)+C_1(C_{1,2}+C_{1,2,3})\left(\frac{\frac{\partial C_{1,2}}{\partial \theta^2}}{C_{1,2}}-\frac{\frac{\partial C_{1,3}}{\partial \theta^2}}{C_{1,3}}\right)=0$$
$$\left(\frac{\partial C_1}{\partial \theta^2}+\frac{\partial C_{1,2}}{\partial \theta^2}\right)C_1=\frac{\partial C_1}{\partial \theta^2}(C_{1,3}+C_{1,2,3})+C_1(C_{1,2}+C_{1,2,3})\left(\frac{\frac{\partial C_{1,2}}{\partial \theta^2}}{C_{1,2}}-\frac{\frac{\partial C_1}{\partial \theta^2}+\frac{\partial C_{1,2}}{\partial \theta^2}}{C_{1,3}}\right)$$
$$C_1=\frac{\frac{\partial C_1}{\partial \theta^2}}{\frac{\partial C_1}{\partial \theta^2}+\frac{\partial C_{1,2}}{\partial \theta^2}}(C_{1,3}+C_{1,2,3})+C_1(C_{1,2}+C_{1,2,3})\left(\frac{\frac{\partial C_{1,2}}{\partial \theta^2}}{C_{1,2}\left(\frac{\partial C_1}{\partial \theta^2}+\frac{\partial C_{1,2}}{\partial \theta^2}\right)}-\frac{\frac{\partial C_1}{\partial \theta^2}+\frac{\partial C_{1,2}}{\partial \theta^2}}{C_{1,3}\left(\frac{\partial C_1}{\partial \theta^2}+\frac{\partial C_{1,2}}{\partial \theta^2}\right)}\right)$$
$$C_1=\frac{\frac{\partial C_1}{\partial \theta^2}}{\frac{\partial C_1}{\partial \theta^2}+\frac{\partial C_{1,2}}{\partial \theta^2}}(C_{1,3}+C_{1,2,3})+C_1(C_{1,2}+C_{1,2,3})\left(\frac{1}{C_{1,2}}\frac{\frac{\partial C_{1,2}}{\partial \theta^2}}{\frac{\partial C_1}{\partial \theta^2}+\frac{\partial C_{1,2}}{\partial \theta^2}}-\frac{1}{C_{1,3}}\right)$$
$$C_1=\frac{\frac{\partial C_1}{\partial \theta^2}}{\frac{\partial C_1}{\partial \theta^2}+\frac{\partial C_{1,2}}{\partial \theta^2}}(C_{1,3}+C_{1,2,3})+C_1(C_{1,2}+C_{1,2,3})\left(\frac{1}{C_{1,2}}\frac{\frac{\partial C_{1,2}}{\partial \theta^2}}{\frac{\partial C_{1,3}}{\partial \theta^2}}-\frac{1}{C_{1,3}}\right)$$

In this equilibrium type it should hold that

$$\frac{C_{1,2}}{C_{1,3}}\geq 1 \text{ and } \frac{\frac{\partial C_{1,2}}{\partial \theta^2}}{\frac{\partial C_{1,3}}{\partial \theta^2}}=\frac{\frac{\partial C_{1,2,3}(s^2)}{\partial \theta^2}+\frac{\partial C_1}{\partial \theta^2}}{\frac{\partial C_{1,2,3}(s^3)}{\partial \theta^2}}\leq 1,$$

This implies that

$$\frac{\frac{\partial C_{1,2}}{\partial \theta^2}}{C_{1,2}}\leq\frac{\frac{\partial C_{1,3}}{\partial \theta^2}}{C_{1,3}}$$

and thus also that

$$\frac{\frac{\partial C_1}{\partial \theta^2}}{\frac{\partial C_1}{\partial \theta^2}+\frac{\partial C_{1,2}}{\partial \theta^2}}\geq\frac{\frac{\partial C_1}{\partial \theta^3}}{\frac{\partial C_1}{\partial \theta^3}+\frac{\partial C_{1,2}}{\partial \theta^3}}.$$

□

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