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System spectrum conversion from white light interferogram

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Abstract: Capability to simulate the coherence function is important when tuning an interference microscope in an effort to reduce sidelobes in interference signals. The coherence function cannot directly be derived from the light source spectrum since the microscope’s effective spectrum is affected by e.g. spatial coherence effects. We show this by comparing the true system spectrum measured using a spectrometer against the effective system spectrum obtained by Fourier analysis of the interference data. The results show that a modulation function that describes the scattering-induced spatial coherence dampening in the system is needed to correct the observed difference between these two spectra. The validity of this modulation function is further verified by quantifying the arithmetic mean roughness of two specified roughness standards. By providing a spectral transfer function for scattering, our method can simulate a sample specific coherence function, and thus shows promise to increase the quality of interference microscope images.

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References and links

1. Introduction

Scanning white light interferometry (SWLI) allows topographic characterization of materials with nanometer axial resolution and lateral resolution beyond the diffraction limit [1]. Typically SWLI imaging is done in Michelson, Mirau, or Linnik configuration where the sample is illuminated by a broad band light source and where interferograms are acquired by moving the sample by a piezoelectric actuator [2]. White light emitting diodes (LED) have shown feasibility as a light source in SWLI due to their low price and advantage over halogen sources in pulsed operation mode [3]. Unfortunately white LEDs suffer from sidelobes in the interferograms [3,4] causing ghost echoes to appear especially in SWLI imaging of layered structures. A hybrid white light source constructed from several LEDs with different central wavelengths can reduce these sidelobes [3,5,6]. In this approach the LED currents are tuned individually to tailor a hybrid light spectrum that produces nearly a sidelobe-free coherence function. However, the coherence function [2] cannot be directly simulated from the hybrid light spectrum since the coherence function is affected by the imaging system’s transfer function. This transfer function comprises the spectral responsivity of the imaging components and the spatial coherence effects in the system [7].
Claveau et al. [8] describe an experimental method to extract the spectral transfer function of a Linnik interference microscope. In this method an effective sample spectrum is obtained by Fourier analyzing the interference data and the transfer function is then obtained by dividing the effective sample spectrum by one calibrated sample spectrum that is known a priori. The described method allows one to simulate the coherence function for sample geometries similar to the calibration sample, but the method does not correct for spatial coherence dampening caused by scattering from any other sample surface.

Spatial coherence dampening is used in coherence breaking applications: e.g. in laser speckle removal, where the spatial coherence is broken by scattering [9–12] or by employing multiple non-parallel light beams [13]. In an interference microscope non-parallel light beams, where no single angle represents the angle of incidence, are present especially with high numerical aperture (NA) objectives [14]. Scattering, instead, occurs at each rough interface that light encounters in the microscope. The Kirchhoff approximation is a simple and accurate approach to interpret light scattering from rough surfaces [15]. This theory relies on single scattering from a local tangent plane of a surface where the scattering event can be described in a precise manner with Fresnel coefficients. This condition limits the validity of the approximation to flat surfaces (featuring a large radius of curvature compared to the wavelength of the incident light).

In this paper we show that a spatial coherence modulation function based on the Kirchhoff approximation describes the transfer function between the effective and true system spectrum of an SWLI device. To verify the result the arithmetic mean (Ra) roughness of two Rubert roughness standards was measured when taking into account the spatial coherence modulation function. This measurement result was then compared against the nominal Ra roughness of the standards.

2. Theory

2.1 Effective system spectrum conversion from an SWLI signal

White light interferograms comprise incoherently superimposed single-wavelength components of reference and sample reflections. A simplified expression for the intensity of an SWLI signal, i.e., the coherence function, when assuming parallel light beams is [16]

\[
I(\Delta z) = \int \frac{1}{4} S(k) \left[ R_o(k) + R_s(k) \right] + \frac{1}{2} S(k) \sqrt{R_o(k) R_s(k)} \cos(\Delta \phi(k, \Delta z)) \, dk ,
\]

where \( S(k) \) is the system spectrum that comprises the broad band source spectrum and the system transmittance spectra. Here \( R_o(k) \) and \( R_s(k) \) are the reflectance spectra of the reference and sample mirror, \( \Delta \phi(k, \Delta z) \) is the phase difference between the two reflections, \( k \) is the wavenumber of light, and \( \Delta z \) is related to the axial position of the piezoelectric actuator. In Eq. (1) the first term represents a path-length independent DC component whereas the second term represents the interference modulation. In a special case, when neither dispersion nor phase change is present [17], the phases of all wavelength components match which produces maximum constructive interference when the sample plane and the reference plane overlap.

According to the Wiener-Khinchin theorem [18] the interference term in Eq. (1) reveals the power spectrum of the system. The unwanted DC term is removed from the recorded interferogram by subtracting the mean value of the signal. The spectral content of an interference signal is extracted by applying the Fourier transform:

\[
I_{\text{eff}}(\Lambda) = \left| FT \left[ I(\Delta z) \right] \right| ,
\]

where \( I_{\text{eff}}(\Lambda) \) represents an effective power spectral distribution of the captured light as a function of fringe wavelength, \( \Lambda \) [14]. For convenience, wavelengths are used rather than wavenumbers. The interference microscope objective affects the distance between two fringes in the interferogram in such a way that the fringe spacing increases with NA [14]. To change
into optical wavelengths, $\lambda$, the fringe wavelength is corrected for the objective’s NA by applying an aperture function, $\alpha_{\text{NA}}$,

$$\lambda = 2\Lambda\alpha_{\text{NA}} = 2\Lambda \frac{1 + \cos \theta_{\text{max}}}{2} = \Lambda \left(1 + \cos \theta_{\text{max}}\right).$$

This function is based on a paraxial assumption and is therefore valid for low NA objectives [14]. The illumination cone angle $\theta_{\text{max}}$ is related to the NA by $\theta_{\text{max}} = \sin^{-1} \text{NA}$. Other aperture functions have been discussed by Creath [19]. It needs to be noted that the width of the fringe envelope varies with NA. The spatial coherence envelope function can be estimated to be a $sinc$-function that narrows with increasing NA [14]. This broadens the effective spectrum of the interferogram. However, for low NA objectives this effect is negligible and is therefore not accounted for in this paper.

An effective system spectrum is calculated from the Fourier transformed interference data. Since the camera response is uneven across the broad illumination spectrum, the color sensitivity of the camera, $F(\lambda)$, needs to be taken into account. According to this consideration and Eq. (1), the effective system spectrum, $S_{\text{eff}}(\lambda)$, is calculated as

$$S_{\text{eff}}(\lambda) = \frac{2I_{\text{eff}}(\lambda)}{F(\lambda) \sqrt{R_0(\lambda) R_0(\lambda)}}.$$ (4)

### 2.2 Scattering modified coherence function

In an interference microscope any scattering reduces the spatial coherence. The coherent component of the light field scattered from a rough surface is described by a coherence decay factor [20]. This factor describes the light intensity scattered coherently into the specular direction from the rough surface in comparison to light scattering from a perfectly smooth surface. All other light is scattered incoherently. For a random rough surface with Gaussian height distribution the coherent reflectance and transmittance are approximated as [21,22]

$$R_c = R \exp \left[-\left(\frac{2\pi}{\lambda}\right)^2 \left(2\delta n_1 \cos \theta_1\right)^2 \right]$$

$$T_c = T \exp \left[-\left(\frac{2\pi}{\lambda}\right)^2 \left(\delta(n_1 \cos \theta_1 - n_i \cos \theta_i)\right)^2 \right],$$ (5) (6)

where the exponentials describe the spatial coherence modulation. This coherence modulation arises from the surface height distribution that causes the phase of the scattered light under
the integrated area to be distributed. For shorter wavelengths this phase distribution is wider than for longer wavelengths. Thus the scattered light loses coherence as a function of wavelength. In Eq. (5) and (6) \( \delta \) represents the root mean squared (RMS) roughness of the scattering surface, \( n_i \) and \( n_t \) are the refractive indices at the incidence and transmit side of the surface, and \( \theta_i \) and \( \theta_t \) are the incidence and specular refraction angles with respect to the normal of the center plane of the surface profile. The specular refraction angle is described by Snell’s law. Figure 1 depicts the scattering geometry. The above decay factors are based on the Kirchhoff approximation and agreement with numerical simulations and measurements has been shown for scattering in reflection [23,24]. Coherence modulation functions for other surface height distributions than Gaussian have been discussed by Porteus [25].

Scattering modifies the coherence function. Only coherent light can interfere and therefore the interference term is affected by individual coherence decays along the light path. A spatial coherence modulation function, \( \Psi(\lambda) \), that describes the scattering-induced coherence dampening in the entire system is expressed by

\[
\Psi(\lambda) = \exp\left[ -\frac{2\pi}{\lambda} \left( \frac{\delta_{\text{sys}}^2}{\delta_{\text{sys}}^2} + \frac{1}{2} (2\delta_{\text{sys}} n_{\text{inc}} \cos \theta_{\text{max}})^2 + \frac{1}{2} (2\delta_{\text{sys}} n_{\text{trans}} \cos \theta_{\text{max}})^2 \right) \right]
\]

(7)

The effective system roughness, \( \delta_{\text{sys}} \), combines the RMS roughness of the individual interfaces in the system, excluding the sample and reference mirror, weighted by the corresponding refractive indices and scattering angles. The following two terms in the exponential function describe the scattering contributions from sample roughness, \( \delta_{\text{S}} \), and reference mirror roughness, \( \delta_{\text{R}} \). The factors 1/2 in these two terms follow from the square root in the interference term in Eq. (1). For non-immersion objectives the refractive indices corresponding to scattering from the sample and reference mirror, \( n_{\text{S}} \) and \( n_{\text{R}} \), are approximated to be 1. Finally, \( \delta_{\text{tot}} \) combines the effective system roughness, sample roughness, and reference roughness contributions into a total effective roughness. The DC component remains unchanged when both coherent and incoherent light components are fully detected. Considering Fig. 1 this condition is fulfilled by a detector featuring wider clear aperture than the scattered beam diameter. The same applies in SWLI imaging since a large area is illuminated and incoherent light scattered from neighboring surface locations contribute to the detected intensity of small detector pixels. Typical diffraction limited imaging performance indicates negligible diffuse blurring and further supports that the coherent and incoherent light components are completely detected. A scattering modified coherence function is

\[
I(\Delta \omega) = \int \left[ \frac{1}{4} S(k)\left[R_s(k) + R_t(k)\right] + \frac{1}{2} S(k)\Psi(k)\sqrt{R_s(k)R_t(k)} \cos(\Delta \omega(k,\Delta \omega)) \right] dk.
\]

(8)

As the spatial coherence modulation function includes the height distributions of rough surfaces in the system the argument of the cosine function in Eq. (8) is with respect to the center plane of the sample surface.

Since the spatial coherence modulation function depends on \( \lambda \), any scattering in the system affects the shape of the spectrum extracted from the interference signal. A scattering corrected effective system spectrum, \( S_{\text{eff. cor}}(\lambda) \), is now calculated as

\[
S_{\text{eff. cor}}(\lambda) = \frac{2I_{\text{eff.}}(\lambda)}{F(\lambda) \Psi(\lambda) \sqrt{R_s(\lambda) R_t(\lambda)}} = \frac{S_{\text{eff}}(\lambda)}{\Psi(\lambda)}.
\]

(9)
3. Methods

3.1 Measurements for SWLI effective system spectrum

Figure 2(left) depicts the SWLI instrument used in the measurements. White light illumination was produced by a LED (Cree, XM-L2 T5 neutral white, 704 mA forward current) whose spectrum is shown in Fig. 2(right, top). To allow scanning along the optical axis an interferometric objective was mounted onto a piezoelectric actuator (PI, P-721 CDQ). Both Michelson (Nikon CF Plan, 5x/0.13 TI) and Mirau (Nikon CF Plan, 10x/0.30 DI) type interferometric objectives were used. The scanning was done in a stepwise manner with 68.75 nm increments across the sample plane. The intensity data \( I(x, y) \), where \( x \) and \( y \) describe the lateral field position, was recorded without averaging by a camera (Hamamatsu, ORCA-Flash2.8, color sensitivity from the data sheet) at each step during the scan \( \Delta z \). White light interferograms \( I(\Delta z, x, y) \), Fig. 2(right, bottom), were then obtained from the stack of successive camera frames. The lateral pixel size with the 5x Michelson objective was 0.73 µm whereas with the 10x Mirau objective the lateral pixel size was 0.36 µm. To have high fringe contrast the measurements were conducted on a mirror-polished silver sample (Edmund Optics, \( \lambda/20 \) flatness at 632.8 nm). The sample mirror was adjusted to less than 0.003° tilt relative to the reference mirror inside the interferometric objective. The effective system spectra were calculated following the procedure described in section 2.1 from five repeats of 160.4 µm x 160.4 µm area averaged interferograms. Area averaging of interferograms ensures a smooth height distribution under the averaged area and makes the calculated effective system spectrum less sensitive to local defects in the sample. Nonpolarized reflectance spectra of the silver reference and silver sample mirror at \( \theta_{\text{max}} \) illumination cone angle needed for the calculation were obtained from Fresnel equations using complex refractive index data by Rakic et al. [26]. Finally the effective system spectra were normalized to the strongest spectral components.

Fig. 2. Scanning white light interferometer setup (left) and a typical interferogram (right, bottom) using a Cree XM-L2 T5 neutral white LED light source (right, top). Light source spectrum normalized to the strongest spectral component at 448 nm wavelength. Measurements with both Michelson and Mirau (red dashed box) interferometric objectives were conducted. To measure the spectrum of the system the camera was replaced by a spectrometer. Abbreviations: PZT – piezoelectric actuator; \( \Delta z \) – axial scan position; \( x, y \) – lateral field position.
Effective system spectrum measurements were repeated by measuring an aluminum sample mirror using a second SWLI device [27]. A pulsed Cree XM-L U2 cool white LED was used as a light source. A custom made setup employs a Mitutoyo Plan Apo 5x/0.14 objective and a Michelson interferometric arrangement with a silicon reference mirror. The scanning was done in 20 nm steps using a piezoelectric actuator (PI, N-664). White light interferograms were recorded using a Guppy PRO F-046B camera (Allied Vision Technologies, color sensitivity from the data sheet). The reflectance spectra of aluminum and silicon were estimated using complex refractive index data by Rakić et al. [26] and Aspnes and Studna [28], respectively. The effective system spectrum was calculated from five 1.7 µm x 161.0 µm area averaged interferograms and was finally normalized to the strongest spectral component.

3.2 Spectrometer measurements

To compare the effective system spectrum to a true system spectrum, the spectrometer measurements were conducted by replacing the camera in the SWLI system by a calibrated spectrometer (Ocean Optics, HR2000+, calibration light source HL-2000-CAL, ID: 030410235) featuring an integrating sphere (Ocean Optics, FOIS-1), Fig. 2(left). The spectrometer records a scattering modified spectral interferogram [16]

$$I(k) = \frac{1}{4} S(k)[R_n(k) + R_s(k)] + \frac{1}{2} S(k) \Psi(k) \sqrt{R_n(k) R_s(k) \cos(\Delta \phi(k, \Delta z))}. \quad (10)$$

To reduce the interference modulation we moved the sample mirror off focus by 1 mm. Off focusing reduces the fringe contrast since spectrally resolved interferometers exhibit a distance-dependent fringe contrast falloff due to the finite spectral resolution of the spectrometer [29]. According to Eq. (10) the true system spectra were calculated from the DC component as

$$S(\lambda) = \frac{4I(\lambda)}{R_n(\lambda) + R_s(\lambda)}, \quad (11)$$

and were normalized to the highest spectral components. Although featuring approximately 5% uncertainty, the true system spectra are free from coherence effects.

In the repeat SWLI measurement the true system spectrum was measured using a spectrometer Ocean Optics, USB2000+ VIS-NIR-ES.

4. Results

4.1 Comparison between the effective and true system spectrum

Figure 3 shows a comparison between the effective system spectrum (S_eff, blue dashed lines) and the true system spectrum (S, red lines) for the Nikon 5x Michelson setup (top, left), for the Nikon 10x Mirau setup (top, right), and for the custom made 5x Michelson setup (bottom). We tested the feasibility of using the scattering-induced coherence dampening model to correct the shape difference between S_eff and S by fitting a scaled spatial coherence modulation function, N(\lambda), to the S_eff(\lambda) / S(\lambda) data. The scaling coefficient N accounts for the normalizations made to the measured spectra. For the Nikon 5x and 10x setups the fitting was done in the 430 – 665 nm spectral region while for the custom made 5x Michelson setup the fitting was done in the 430 – 600 nm region. Figure 4 shows a fit to the Nikon 5x setup data. Total effective RMS roughness and R^2 coefficient of the model were: (110.8 ± 2.0) nm and 0.94 for the Nikon 5x Michelson setup, (95.1 ± 1.9) nm and 0.93 for the Nikon 10x Mirau setup, and (115.3 ± 2.7) nm and 0.93 for the custom made 5x Michelson setup. The uncertainties represent fitting parameter uncertainty and are quoted at 95% confidence level. In Fig. 3 the scattering corrected effective system spectra are shown by solid blue lines.
Fig. 3. Measured system spectrum of three interferometric setups: Nikon 5x Michelson setup (top, left), Nikon 10x Mirau setup (top, right), and custom made 5x Michelson setup (bottom). S represents the true system spectrum measured using a spectrometer, S_{eff} represents the effective system spectrum converted from interferogram using Fourier analysis, and S_{eff, corr} represents the effective system spectrum corrected for scattering-induced coherence dampening.

Fig. 4. Scaled spatial coherence modulation function fit to S_{eff} / S data for the Nikon 5x Michelson setup. The scaling coefficient N accounts for the normalizations made to the measured spectra. S represents the true system spectrum measured using a spectrometer and S_{eff} represents the effective system spectrum converted from an interferogram using Fourier analysis. Ψ(λ) is the scattering-induced spatial coherence modulation function, δ_{tot} is the total effective RMS roughness, and λ is the wavelength of light.

4.2 Coherence dampening model verification

To verify the validity of the scattering-induced coherence dampening model, we measured the Ra roughness of Rubert 501X and 502X standards using the spatial coherence modulation function. These standards are electroformed nickel replicas from random rough originals with nominal Ra roughness 20 nm for 501X and 30 nm for 502X [30]. A common practice
uncertainty of ± 10% at 95% confidence level is expected for the standards. The reflectance spectrum of nickel was estimated using complex refractive index data by Rakić et al. [26]. The measurements were conducted using the Nikon 5x Michelson setup while the effective system spectrum measured for the silver sample mirror provided a way to calibrate the roughness in the system. The effective system spectra for the roughness specimens were calculated from 160.4 µm x 160.4 µm area averaged interferograms from nine locations on the specimens.

Figure 5 shows a comparison between the silver mirror effective system spectrum ($S_{M, \text{eff}}$, red lines) and the effective system spectrum of the roughness specimens ($S_{RS, \text{eff}}$, blue dashed lines): Rubert 501X (left) and Rubert 502X (right). The shape difference between $S_{RS, \text{eff}}$ and $S_{M, \text{eff}}$ is caused by the RMS roughness of the roughness specimen, $\delta_{RS}$. We note here that the roughness of the silver sample mirror is not compensated for. However, any bias caused by this is assumed to be negligible due to the small roughness, < 1 nm, of the high quality silver mirror [31]. Fitting a scaled spatial coherence modulation function

$$N \exp \left[ -\left( \frac{1}{2} \left( \frac{2\pi}{\lambda} \right)^2 \left( 2\delta_{RS} \cos \theta_{\text{max}} \right)^2 \right) \right]$$

(12)

to the $S_{RS, \text{eff}}(\lambda) / S_{M, \text{eff}}(\lambda)$ data in the 430 – 665 nm spectral region results into an Ra roughness estimate of (20.9 ± 1.8) nm for 501X and (32.9 ± 2.6) nm for 502X, both at 95% confidence level. The conversion from RMS to Ra roughness was done as $Ra = \text{RMS} / 1.11$ [32]. The effective system spectra corrected for the specimen’s RMS roughness are shown in Fig. 5 as solid blue lines.

Fig. 5. Effective system spectrum measured for two Rubert roughness standards using the Nikon 5x Michelson setup: 501X (left) and 502X (right). The effective system spectrum measured for a silver mirror, $S_{M, \text{eff}}$, is used to calibrate the setup for scattering in the system. Finally, $S_{RS, \text{eff}}$ represents the effective system spectrum measured for the roughness specimens whereas $S_{RS, \text{eff, corr}}$ represents the effective system spectrum corrected for the specimen’s RMS roughness.

### 5. Discussion

The effective and true system spectra differ due to the fact that the DC component measured by the spectrometer does not take into account coherence effects. To correct for this difference we applied a spatial coherence modulation function. This function is based on the Kirchhoff approximation and it describes coherence dampening in scattering from a random rough surface. For polished optical components we found the approximation of random roughness feasible. The results show that accounting for the total effective roughness in the system corrects the observed difference in all three setups with moderate success. However, there may be other physical effects which influence the spectrum in similar ways as the exponential decay factor. For example, the chromatic aberration of a microscopic imaging
system underestimates those spectral contributions, for which the reference mirror is not exactly at focus. Consequently, the spectral response of an interferometer depends on the adjusted position of the reference mirror. In other applications of measuring the coherence function, e.g. in Fourier transform spectroscopy [33], phase contrast microscopy [34], and diffraction phase microscopy [35], similar spatial coherence predictions are expected to be applicable.

The total effective roughness ranged in the studied setups from 95.1 nm to 115.3 nm. Assuming approximately 40 scattering interfaces in a typical SWLI system, an average RMS roughness of 20 nm is estimated for the optical components in the studied setups. This value is at least an order of magnitude higher than the typical RMS roughness, < 1 nm, of high quality optical components [31]. One possible explanation for the high average RMS roughness is subsurface damage in the optical components which also contributes to the scattering in transmission. In optical components subsurface damage is generated during grinding and it remains present after polishing although < 1 nm RMS roughness is measured for the polished surface [36]. Subsurface damage can reach 1 µm deep and thus may cause the observed average RMS roughness. To verify the measured total effective roughness one needs to disassemble all optical components and examine roughness of each piece including etching to find hidden subsurface damages. Practically this is unreasonable.

To verify the validity of the spatial coherence modulation function, the Ra roughness of two Rubert random rough standards was measured. The measurement results correspond to the specified Ra roughness within the measurement precision which supports the validity of the proposed method.

Although the most straightforward implication of the scattering-induced spatial coherence modulation function is quantitative roughness evaluation, it also provides a method to simulate parameters of the hybrid light source, i.e., LED current, pulsing frequency, duty cycle [3], in an effort to produce a smooth and narrow coherence function for different sample surfaces. Such a coherence function minimizes ghost echoes and thus increases the quality of SWLI images. In this kind of simulation the total system roughness is needed beforehand which is difficult to achieve for unknown samples. However, an iterative method could overcome this problem: First, a test measurement is done with default parameters to estimate the sample roughness, and second, this result is used as an input in the simulation of hybrid light source parameters.

6. Conclusions

We showed that the effective system spectrum of an interference microscope, which traditionally is quantified from the interferogram using Fourier analysis, differs from the true spectrum measured through the system using a spectrometer. A modulation function that describes the scattering-induced spatial coherence dampening explains the observed difference. Further, its use was verified by measuring Ra roughness of two specified roughness standards. By providing a spectral transfer function for scattering, our method shows promise to simulate coherence function for different sample surfaces which is important when one wants to have high SWLI image quality.

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