The Macroeconomic Effects of Fiscal Policy in a Small Open Economy under a Flexible Exchange Rate Regime

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1 Introduction

This licentiate’s thesis analyzes the macroeconomic effects of fiscal policy in a small open economy under a flexible exchange rate regime, assuming that the government spends exclusively on domestically produced goods. The motivation for this research comes from the observation that the literature on the new open economy macroeconomics\(^1\) (NOEM) has focused almost exclusively on two-country global models and the analyses of the macroeconomic effects of fiscal policy on small economies are almost completely ignored. In choosing a small country setting, this thesis focuses on a simpler framework, but it brings in interesting insights into the effects of fiscal policy in open economies.

After the seminal paper "Exchange Rate Dynamics Redux” (1995) by Obstfeld and Rogoff, the profession started to use the term new open economy macroeconomics to describe research on open economy macroeconomics that integrates typical Keynesian features, imperfect competition and nominal rigidities, into a dynamic general equilibrium framework. These new open economy macroeconomic models are based on the assumption that nominal rigidities and market imperfections in a dynamic general equilibrium framework better describe the reality. Furthermore, the adoption of a dynamic utility-theoretic approach by the NOEM literature allows an explicit utility-based welfare analysis of fiscal policy, and can thus be used as the basis for the design of optimal fiscal policy.

This thesis presents two analytically tractable models to study the positive and normative effects of permanent balanced-budget fiscal expansion in a small open economy under a flexible exchange rate regime. As stressed by Sarno (2001), the NOEM literature has shown that the effects of economic policies in this framework have turned out to be sensitive to the specification of preferences. As mentioned, the

\(^{1}\) See Lane (2001b) for an excellent survey on the NOEM literature.
NOEM literature neglects the study of the effects of fiscal policy in small open economies. Therefore, this thesis aims at filling in this gap by illustrating how the macroeconomic effects of fiscal policy in a small open economy depend on the specification of preferences. In this thesis I present two extensions to the small open economy model contained in the Appendix to Obstfeld and Rogoff (1995). The aim of the first model is to analyze the positive and normative effects of fiscal policy, making use of a model that exploits the idea of modelling private and government consumption as substitutes in private utility. The model offers simple and intuitive predictions on how fiscal expansion affects the economy, and it clearly demonstrates how the effects depend on the marginal rate of substitution between private and government consumption. The aim of the second model is to analyze the relationship between fiscal policy, output, the exchange rate and the current account. The focus of this model is to study how the effects of fiscal policy on macroeconomic variables depend on the elasticity of substitution between traded and nontraded goods.

The rest of the paper is organized as follows. Section 2 provides an overview of some of the NOEM models that address fiscal policy issues. Section 3 introduces the model that makes use of the idea of modelling private and government consumption as substitutes in private utility. This section demonstrates how the macroeconomic effects of fiscal policy depend on the marginal rate of substitution between private and government consumption. Section 4 lays out the model which is used to analyze the effects of fiscal policy on output, the current account and the exchange rate. This section illustrates how the effects of fiscal policy depend on the elasticity of substitution between traded and nontraded goods. Section 6 briefly summarizes the thesis and provides conclusions.
2 Fiscal Policy in the New Open Economy Macroeconomics

Starting with the celebrated article by Obstfeld and Rogoff (1995), the field of open economy macroeconomics has witnessed the development of a new generation of economic models, generally referred as new open economy macroeconomic models\(^2\). These models integrate typical Keynesian features, imperfect competition and nominal rigidities, into a dynamic general equilibrium framework. The main attention in the NOEM literature has been on the analysis of monetary shocks, and the analysis of fiscal shocks have received, at least relative to monetary shocks, little attention. For example, in an excellent survey article on the NOEM literature Lane (2001b) points this out, also pointing out a reason why the analysis of monetary shocks have received so much attention in the NOEM literature. “In describing the findings of this research program [NOEM], I focus exclusively on the analysis of monetary shocks. This reflects the emphasis in the literature, for the role of nominal rigidities is mostly starkly illustrated in the case of monetary shocks and it is this kind of disturbance that flexible-price models are least well-equipped to handle.” (Lane 2001, 236; italicization added)

However, an explicitly optimizing, general equilibrium framework also provides a very potential framework for analysing fiscal policy issues. Since new open economy macroeconomic models use solid microfoundations for intertemporal choice, equilibrium conditions are derived from optimal individual behaviour on part of consumers and producers, and due to the general equilibrium framework, equilibrium conditions are consistent with the simultaneous clearing of all markets. For these reasons, the new models have much stronger theoretical foundations than traditional Keynesian models, and in contrast with models in the flexible-price framework, the new paradigm has one feature that gives them an edge over flexible-price models: the

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\(^2\) For more detailed analysis on new open economy macro models, see survey articles. An excellent survey by Lane (2001b) focuses completely on monetary policy issues. Ganelli and Lane (2002) survey more recent developments in the NOEM literature and also contributions that address fiscal policy issues, while Coutinho (2003) focuses completely on fiscal policy issues.
presence of nominal rigidities. In addition, the adoption of an explicit utility function gives an opportunity to investigate the welfare effects of fiscal policy, and the analysis can thus be used as the basis for the design of optimal fiscal policy.

The purpose of the remainder of the section is twofold. First, it tries to provide an overview of some of the NOEM models that address fiscal policy issues. Second, it pays attention to the facts how government spending can be introduced into NOEM models and how fiscal shocks affect economies. I cover two models in more detail, the OR model in which government spending is assumed to bring no utility and a model by Ganelli (2003) in which government spending affect utility in a non-separable way.

### 2.1 The Obstfeld-Rogoff Model

As mentioned, Obstfeld and Rogoff (1995) effectively initiated the new paradigm. In this section I briefly outline the main features and results of the textbook version of the OR model (Obstfeld – Rogoff 1996). They developed a perfect-foresight two-country general equilibrium model. In their model the world is populated by a continuum of infinitely-lived agents (indexed by z) that are both consumers and monopolistic producers of a continuum of differentiated goods. The home agents are on the interval [0,n], while the foreign agents are on the interval (n,1]. The OR model focuses on the case where there is no home bias in government spending, but government spending is assumed to be pure waste and does not affect private utility at all. In addition, prices are sticky, in the producer’s currency, as they are set one period in advance and fully flexible after the period. It follows from this assumption that it takes one period to reach the new steady state after a fiscal shock hits the economy.

In the model, the lifetime utility of representative home agent z is given by\(^3\)

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\(^3\) I present here the 1996 version of the OR model. The difference between the 1995 article version and the 1996 textbook version (with respect to fiscal issues) is the functional form of real money balances in
\[ U_t = \sum_{s=t}^{\infty} \beta^{t-s} \left[ \log C_s + \chi \log \left( \frac{M_s}{P_s} \right) - \frac{\kappa}{2} y_s(z)^2 \right]. \]

In this equation, \( U_t \) denotes utility at time \( t \), \( \beta \) (0 < \( \beta \) < 1) is the discount factor, \( C \) is the consumption index (a composite of consumption of individual goods) defined as

\[
C = \left( \int_0^1 c(z)^{\theta-1} dz \right)^{\frac{\theta}{\theta-1}},
\]

where \( c(z) \) is consumption of good \( z \), and \( \theta \) (\( \theta > 1 \)) is the elasticity of substitution between varieties. In the utility function \( \chi \) is the parameter, \( M_s \) is nominal money balances held by the agent at time \( s \) and \( P_s \) is the price level. The last term captures the disutility of work effort, \( y_s(z) \) is the output of good \( z \) and \( \kappa \) is the parameter. (Obstfeld – Rogoff 1995, 661)

Government spending is introduced into the model as a composite of goods, aggregated in the same way as for private consumption with same elasticity of substitution. Accordingly,政府 spending is given by

\[
G = \left( \int_0^1 g(z)^{\theta-1} dz \right)^{\frac{\theta}{\theta-1}},
\]

which indicates that there is no home bias in government spending. In this framework the representative agent faces the constant-elasticity demand curve for his output

\[
y_t^\theta(z) = \left( \frac{P_t(z)}{P_t} \right)^{\theta} \left( C_t^h + G_t^h \right),
\]

which indicates that demand for good \( z \) depends on its relative price, the elasticity of demand with respect to relative price, and aggregate private and government expenditures. (Obstfeld – Rogoff 1995, 700)

As mentioned, prices are sticky in the short run (for one period) and fully flexible after the period. In the short run, output is entirely demand-determined. A permanent

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the utility function. The 1995 version assumes \( \chi(M/P)^{1-\varepsilon} / (1-\varepsilon) \), where \( \varepsilon \) is the elasticity of money demand. However, the 1996 version assumes a logarithmic function.
balanced-budget rise in government spending then reduces short-run relative (relative to the foreign country) consumption, depreciates the nominal exchange rate and increases relative output. With no home bias in government spending, the rise in government spending increases demand for both domestic and foreign goods, while the tax bill falls entirely on the domestic agents. The rise in government spending increases domestic incomes (output), but not enough to offset the rise in taxes. Consequently, the rise in government spending leads to an immediate fall in relative consumption. Since money demand is a positive function of consumption (not a function of gross income), lower money demand requires a rise in the price level and a depreciation of the domestic currency. The depreciation induces the expenditure-switching effect, raising demand for domestic goods and lowering demand for foreign good, further rising relative output in the home country. When the agents smooth consumption and short-run income rises by more than long-run income, the home country runs a current account surplus in the short-run. In addition, the rise in government spending leads to a fall on the real interest rate in anticipation of lower consumption in the future. (Obstfeld – Rogoff 1995, 704–705)

After one period all price are fully flexible and the economy reaches the new steady state. In the steady state, when the economy is again on the labour supply curve, the permanent rise in government spending induces a rise in relative home output and a fall in relative consumption. As in the short run, higher taxes bring about a negative wealth effect for the home agents inducing the agents to cut their consumption and also increase their labour supply. Since the agents supply more labour, output rises and consequently home consumption falls by less than the rise in government spending. The current account surplus in the short run makes the agents wealthier and this higher wealth leads to some reduction in work effort and output. The former effect dominates, and the rise in government spending decreases steady-state relative home consumption and increases output, and therefore allows private consumption to fall by less than the rise in government spending. (Obstfeld – Rogoff 1995, 703)
The welfare effect of fiscal policy is the sum of short-run change in utility and the discounted present value of the steady state change in utility. Since the utility function depends positively on private consumption, negatively on output and government spending does not bring utility in this model, the above analysis suggests that fiscal expansion decreases domestic welfare, but it increases it abroad. The main reason for the beggar-thyself nature of fiscal expansion is that the increase in world demand falls on both home and foreign goods, but the taxes that finance it are borne entirely by the home agents. Obviously, the welfare effects are sensitive to the way in which government spending affects private utility. (Obstfeld – Rogoff 1995, 706) But on the other hand, “the positive results that follow [from the pure waste case] would be the same if government spending entered separably into preferences.” (Obstfeld – Rogoff 1996, 700).

Although, the Obstfeld-Rogoff model is a two-country global model it can be used for the analysis of small economies by taking the limit of the two-county model as the other country’s relative size goes to zero (Obstfeld – Rogoff 1995, 688). It is apparent that a small country’s expansion has no effect on the world interest rate or world output. On the other hand, the other effects of fiscal expansion are the same in the small-country case as in the two-country global model.

Ganelli (2005) introduces home bias in the OR model. With complete home bias in government spending, the domestic government consumption index is given by

$$G = \left( \int_0^\infty g(z) \frac{\theta-1}{\theta} dz \right)^{\frac{\theta}{\theta-1}}.$$  

When domestic temporary fiscal expansion stimulates demand for home goods only, a rise in government spending increases domestic output on a one-to-one basis, while the nominal exchange rate, home and foreign consumption are unaffected in the short run. This is a consequence of the fact that, when home government spending falls exclusively on domestically produced goods the rise in government spending increases domestic incomes (output). Thus the wealth costs (taxes) of this policy for the domestic agents are perfectly offset by the gains from the stimulation of domestic
demand. Therefore, there is no effect on domestic consumption. And since money demand is a function of (unchanged) consumption, fiscal expansion has no effect on the exchange rate and the interest rate. With no effect on foreign demand and the exchange rate, the introduction of home bias in government spending isolates the economy with respect to fiscal policy.

A number of modifications of and extensions to the OR model has been done. However, virtually all the models in the NOEM literature that address fiscal policy issues concentrate on the simple case where government spending does not affect private utility at all. Only two NOEM models with utility enhancing government spending have been published so far: Corsetti and Pesenti (2001) in which government spending is assumed to affect private utility in an additively separable way and Ganelli (2003) in which government spending affects utility in a non-separable way.

2.2 NOEM Models with Useful Government Spending

Corsetti and Pesenti (2001) introduce useful government spending into the NOEM framework by assuming that government spending affect utility in an additively separable way. However, advantages of the introduction of useful government spending in an additively separable way into are modest. As mentioned, positive results change only by having government spending entering directly in the utility function. On the other hand, the welfare effects of fiscal policy can be totally different if government spending affects utility in an additively separable way. Nonetheless, Corsetti and Pesenti did not specify the functional form of individual utility from government spending, and probably for this reason, they did not analyse the effects of fiscal expansion on domestic welfare.

Ganelli (2003) presents an extension to the OR model which introduces utility enhancing government spending by modelling private and government consumption as substitutes in private utility. Otherwise the model by Ganelli maintained all other
assumptions of the OR model. The main contribution of the model is that of studying the consequences of the introduction of the useful government spending on output and consumption multipliers. The intertemporal utility function is given by

$$U_i = \sum_{s=0}^{\infty} \beta^s \left[ \log(C_i + \alpha G_s) + \chi \log \left( \frac{M_s}{P_s} \right) - \frac{k}{2} y_i(c)^2 \right],$$

where $\alpha$ is the marginal rate of substitution between private and government consumption. The preceding formulation implies a direct crowding-out effect of government spending on private consumption when government provides goods that are substitutes for private consumption. A salient feature of the introduction utility enhancing government spending in a non-separable way is that the first-order conditions change and consequently both the positive and normative effect of fiscal expansion change relative to the pure waste case ($\alpha = 0$). (Ganelli 2003, 90–91)

As in the OR model, a permanent rise in government spending reduces consumption, depreciates the nominal exchange rate and increases output. Maintaining the assumption of no home bias in government spending, the policy makes the home agents poorer. Therefore, they react by reducing their consumption. When private and government consumption are substitutes for private consumption is bigger than in the pure waste case due to direct crowding-out. An odd consequence of government spending directly entering the utility function is that private money demand is not only a positive function on private consumption but also on government consumption ($\alpha G$). Consequently, the positive effect on money demand (due to a rise in government spending) partially offsets the negative effect (due to a fall in consumption). Therefore, the introduction of useful government spending implies a depreciation of the home currency that is diminished as $\alpha$ increases. When $\alpha = 1$ the effects perfectly offset each other and the effect on the nominal exchange rate is zero. Due to the exchange rate movement the effect of the rise in government spending on output is positive. However, this effect is decreasing in $\alpha$. The exchange rate change and direct crowding-out induce that the rise in government spending causes a smaller positive impact on output compared to the pure waste case. In addition, as in the OR model, the economy runs a current account surplus in the short run. (Ganelli 2003, 94–97)
In the long run, as in the OR model, higher taxes bring about a negative wealth effect for the home agents inducing the agents to cut their consumption and increase labour supply. The long-run effects on consumption and output go in the same direction as the short-run ones, but in the long run consumption fall by less than in the short run. The fact that government and private consumption are substitutes has a crowding-out effect on private consumption, as consumption falls by more than in the pure waste case. The effect of a rise in government spending on output is positive but decreasing in $\alpha$. Thus, the introduction of useful government spending implies a smaller positive impact on output compared to the pure waste case. As in the OR model the current account surplus in the short run makes the agents wealthier in the long run, which leads to some reduction in labour effort. (Ganelli 2003, 99–100)

The above analysis shows that the introduction of useful government spending reduces consumption and output compared to the pure waste case. These effects must be weighted against the fact that government spending now directly affects private utility. The direct increase in utility from government spending and a positive effect on leisure more than offset the negative welfare effect arising from the reduction in consumption due to direct crowding-out. Therefore, the introduction of utility enhancing government spending unambiguously increases welfare compared to the pure waste benchmark. Whether the introduction of utility enhancing government spending is strong enough to reverse the beggar-thyself result depends on the specific parameter values. (Ganelli 2003, 100–101)

The NOEM literature has focused almost exclusively on two-country global models. As emphasised by Lane (2001, 256) advantages of this approach are that it highlights international transmission channels, allows interest rates and asset prices to be determined in international capital markets and permits the analysis of international policy interdependence. But he also pointed out that: “However, these benefits come at the price of considerable model complexity and may not be compelling importance for the analysis of small open economies.”
3 The Effects of Fiscal Policy: The Role of Useful Government Spending

The purpose of this section is to examine how permanent balanced-budget fiscal expansion affects a small open economy under a flexible exchange rate regime, making use of a model with utility enhancing government spending. The main motivation for this research comes from the observation that most models in the NOEM literature assume that government spending brings no utility, which can be interpreted as an oversimplified assumption. The main focus of this section is to examine how the effects of fiscal expansion depend on the marginal rate of substitution between private and government consumption. The second purpose of the section is to implement a detailed utility-based welfare analysis of fiscal policy. This section attempts to fill in the gap in the literature by analyzing the effects of fiscal expansion in a small open economy in a framework where fiscal policy shocks can have a direct crowding-out effect on private consumption.

As mentioned in the introduction section, the model I present in this section combines the small open economy model contained in the Appendix to Obstfeld and Rogoff (1995) with the idea of modelling private and government consumption as substitutes in private utility. As emphasized by Ganelli (2003), one drawback of the recent NOEM literature is that government spending is normally assumed to be pure waste or to affect private utility in an additively separable way. The idea of specifying preferences in a non-separable way and modelling private and government consumption as substitutes in private utility is advantageous: in this framework government consumption can have a direct crowding-out effect on private consumption. The idea of viewing government consumption as substitute for private consumption was pioneered by Bailey (1971, Section 9), who studied the direct crowding-out effect in the IS-LM model. The topic was also studied in the IS-LM model by Buiter (1977). An important contribution to the topic was done by Barro (1981 & 1989), who studied direct crowding-out in the neoclassical approach to fiscal policy. Studies that address
direct crowding-out also include Heijdra and Ligthart (1997), who used a static closed economy model with imperfect competition. In the Real Business Cycle framework (perfect competition, flexible price-models), the topic has been studied by Roche (1996) and Finn (1998). Making use of the NOEM framework, Ganelli (2003) studied the consequences of modelling private and government consumption as substitutes on consumption and output multipliers and welfare in a two-country global model.

3.1 A Model

In this section, I develop a perfect-foresight general equilibrium model to analyze the effects of fiscal policy. Specifying preferences, technology, demand functions and budget constraints starts the presentation of the model, after which needed first-order conditions are derived. Then we solve for an initial (zero government spending) steady state. To study the dynamic effects of fiscal policy, a log-linear approximation around the initial steady state is used. Since prices are sticky in the short run (by assumption), and flexible in the long run, the solution allows for distinguishing the impact of the first-period and the steady-state effects of fiscal expansion. This creates an opportunity to examine both the short-run and the long-run effects of fiscal expansion.

3.1.1 Market Structure and Preferences

The assumption about imperfect competition is a pivotal factor of the model. Firstly, the effects on fiscal expansion in an imperfectly competitive economy are important and interesting in the sense that the equilibrium in imperfectly competitive economies is not Pareto-efficient. For that reason, it is possible that expansive fiscal policy brings output closer to the social optimum and increases welfare. Secondly, in the imperfectly competitive equilibrium prices are above the social optimum and policies that increase output are more likely to be a welfare increasing compared to in perfectly competitive economies. Thirdly, in imperfect competition prices are above marginal costs, and it is
profitable to meet unexpected demand at the present price. This serves as a justification of the assumption of demand-determined output in the short-run.

As mentioned, I consider a small-country model in which the nontraded goods sector is monopolistically competitive, with an elastic labour supply and prices sticky in the short-run. The home country is populated by a continuum of consumer-producers (agents) that are indexed by \( z \in [0,1] \). As producers, they all produce, using their own labour as input differentiated perishable nontraded goods which are also indexed by \( z \). As consumers, they consume all goods produced in the home country. The traded goods sector has a single homogeneous output that is priced in the competitive world market. As in Obstfeld and Rogoff (1995), the output of tradables is exogenous as the agents are endowed with a constant quantity of tradables in each period.

Preferences are specified in such a way that the model combines the idea that government consumption affects utility in a non-separable way. A representative home agent maximizes his/her intertemporal utility function that depends positively on private consumption, (per-capita) government spending and real balances and, negatively, on output because of the disutility cost of producing it. The lifetime utility of agent \( z \) is given by

\[
U_t = \sum_{s=1}^{\infty} \beta^{t-s} \left[ \gamma \log C_{T,s} + (1 - \gamma) \log (C_{N,s} + \alpha G_{N,s}) + \chi \log \left( \frac{M_s}{P_s} \right) - \frac{\kappa}{2} y_{N,s}(z)^2 \right],
\]

where \( U_t \) denotes utility at time \( t \), \( \beta \ (0 < \beta < 1) \) is the discount factor, \( C_{T,s} \) is consumption of tradables at time \( s \), and \( \gamma \) is the share of tradables in total consumption. The variable \( C_{N,s} \) is the private nontraded goods consumption index (a composite of differentiated varieties of nontraded goods), defined as

\[
C_N = \left[ \int_0^1 c(z) \frac{\theta-1}{\theta} dz \right]^{\frac{\theta}{\theta-1}},
\]

where \( \theta \ (> 1) \) is the elasticity of substitution between varieties of nontraded goods (and also the price elasticity of demand of a single good \( z \)) and \( c(z) \) is consumption of good \( z \). Per capita government consumption, \( G_{N_t} \), is the government consumption index that...
is aggregated in the same manner as private nontraded goods consumption, and with the same elasticity of substitution\(^4\)

\[ G_N = \left[ \int_0^1 g(z) \frac{\theta - 1}{\theta} \, dz \right]^{\frac{\theta}{\theta - 1}}, \]

where \(g(z)\) is government consumption of good \(z\). In equation (1), \(\alpha (0 \leq \alpha \leq 1)\) is the marginal rate of substitution between private and government nontraded goods consumption, and it therefore reflects the utility that consumers get from government consumption. In the utility function (1), \(\chi (> 0)\) is the parameter and \(M_s\) is nominal money balances held by the agent at time \(s\). \(P_s\) is the consumption-based price index (defined below). The last term in (1) captures the disutility of work effort, \(y_s(z)\) is the output of good \(z\), and \(\kappa (> 0)\) is the parameter. The production function can take the form \(y = AL^{\delta}\), where \(L\) stands for labour input, \(A\) technology and \(\eta < 1\).

The consumption-based price index, defined as the minimum cost of purchasing one unit of private composite consumption, is

\[ P = \frac{P_T^\delta P_N^{1-\delta}}{\delta^\delta (1-\delta)^{1-\delta}}. \]

In this equation, \(\delta\) and \(1 - \delta\) denote the shares of private consumption of traded and nontraded goods in total private consumption, respectively. The variable \(P_T\) is the price of tradables. The law of one price holds in tradables, as we assume that there are no costs or impediments to trade between the home country and the world market. For simplicity, we can normalize the exogenously determined world price of tradables (the foreign currency price of it) to unity, which then implies \(P_T = E\), where \(E\) denotes the nominal exchange rate, defined as the home currency price of foreign currency. Hence, the price of tradables is proportional to the nominal exchange rate. The variable \(P_N\) in equation (3) is the nontraded goods price index. It follows from equation (2) that the nontraded goods price index is

\[^4\text{The assumption that private and government consumption have the same elasticity of demand rules out the possibility that a rise in government spending would change the elasticity of total demand.}\]
\[ P_N = \left( \int_0^1 p_N(z)^{1-\theta} \, dz \right)^{\frac{1}{1-\theta}}, \]

where \( p(z) \) is the price of nontraded good \( z \).

The preferences assumed here imply that the producers of nontraded goods face the downward-sloping (constant-elasticity-of-substitution) demand curves

\[ y_N(z) = \left( \frac{p_N(z)}{P_N} \right)^{\theta} \left( C_N^A + G_N^A \right), \]

where superscript \( A \) is used to denote aggregates. This equation indicates that demand for a good depends on its relative price, the elasticity of demand with respect to relative price and aggregate (per-capita) nontraded goods expenditures.

### 3.1.2 Budget Constraints and Utility Maximization

To complete the specification of the individual’s problem, we need to write down budget constraints faced by the private agents. In every period, the representative individual is subject to the budget constraint

\[ P_{T,t} B_t + M_t = P_{T,t} (1 + r) B_{t-1} + M_{t-1} + p_{N,t} (z) y_{N,t}(z) + P_{T,t} G_{T,t} - P_{T,t} C_{T,t} - P_{N,t} \tau_t, \]

where \( B_t \) denotes the stock of riskless real bonds (denominated in tradables) held by the agent entering period \( t + 1 \), \( M_t \) is the agent’s money balances entering period \( t + 1 \), \( r \) denotes the constant world net interest rate earned on bonds between periods \( t - 1 \) and \( t \), \( G_{T,t} \) is exogenously given quantity of tradables and \( \tau_t \) is per capita taxes denominated in units of nontraded goods.

The government is assumed to provide nontraded goods, free of charge, to the agents. The governmentally provided goods can be utility enhancing to the agents; however, as modelling assumes that individual utility depends on per-capita government consumption these goods cannot be “public goods” in sense of being nonrival goods.
(Barro 1981, 1090). As noted by Obstfeld and Rogoff (1995, 628–629), Ricardian equivalence holds in this framework, thus nothing is lost by assuming that all government spending is financed by non-distortionary taxes and seigniorage. The government budget constraint, expressed in per capita terms and in units of nontraded goods, can be written as

$$G_{N,t} = \tau_t + \frac{M_t - M_t^{-1}}{P_{N,t}}.$$

The representative home agent maximizes the utility function (1) subject to the budget constraint specified in equation (5). In this framework, an increase in nontraded output lowers its price and this has to be taken into account. As shown in Appendix A, the first-order conditions for the maximization problem of the representative agent can be written (the indexes denoting the agents are dropped):

$$(6) \quad \beta(1+r)C_{T,t} = C_{T,t+1},$$

$$(7) \quad C_{N,t} + \alpha G_{N,t} = \left(1 - \frac{\gamma}{\gamma}\right) \frac{P_{T,t}}{P_{N,t}} C_{T,t},$$

$$(8) \quad \left(1 - \frac{\gamma}{\gamma}\right) \frac{\gamma^1}{\theta} = \left(1 - \frac{\gamma}{\gamma}\right) \left(\frac{\theta - 1}{\theta}\right) \left(C_{N,t}^A + G_{N,t}^A\right) \left(C_{N,t} + \alpha G_{N,t}\right)^{-1}$$ and

$$(9) \quad \frac{M_t}{P_t} = \frac{X}{(1 - \gamma)} \left(1 + i\right) \left(C_{N,t} + \alpha G_{N,t}\right),$$

where $i$ is the nominal interest rate defined by the Fisher identity$^5$

$$1 + i = (1 + r) \frac{P_{T,t+1}}{P_{T,t}}.$$

Equation (6) is the standard consumption Euler equation. It implies that the representative agent smooth consumption of tradables independently of nontraded goods production or consumption. For simplicity, I assume equality of the discount rate and the world interest rate, which then implies that the optimal time path for tradables consumption is perfectly flat. Equation (7) governs the optimal allocation of total consumption spending between tradables and nontraded goods. The ratio of the

$^5$ Note that, because the price of tradables also denotes the nominal exchange rate the Fisher identity implies uncovered interest parity.
marginal utilities of tradables to nontraded goods equals the relative price of tradables to nontraded goods. Thus, the time path of nontraded goods consumption is tilted by changes in the relative price of tradables to nontraded goods. Equation (8) is the labour-leisure trade-off condition that ensures that the marginal disutility of producing an extra unit of a nontraded good is equal to the marginal utility from consuming the added revenue that the extra unit of the nontraded good brings. The equation indicates that labour supply is a positive function of aggregate government expenditure, but it is also a negative function of government consumption. The reason why labour supply is a negative function of government consumption is the following: An increase in $\alpha G$ reduces the marginal utility of private consumption, inducing the agents to substitute into leisure out of work. Equation (9) is the money market equilibrium condition which indicates that demand for real balances is a positive function of private and government consumption (instead of being only a function of gross income) and a negative function of the interest rate. Also according to the money market equilibrium condition (see equation (A6)) the agents must get same utility from spending money today (consuming a unit of the consumption good) or from holding money today and using today’s money for consumption tomorrow (at $t + 1$). The optimal amount of money balances is chosen such that the marginal utility of private consumption equals the marginal utility of real balances. This fact can be used to understand why demand for real balances is a positive function of government consumption (follows Ganelli 2003): An increase in $\alpha G$ reduces the marginal utility of private consumption, inducing the agents to substitute private consumption with real balances. Therefore, demand for real balances is a positive function of government consumption.

3.1.3 A Symmetric Steady State Equilibrium

In order to solve the model I am going to use a log-linear approximation around a steady state. On the choice of the steady state I follow the OR model. Firstly, it is convenient to log-linearize the model around a steady state in which initial net foreign asset and government expenditure are both zero ($B_0 = 0$ and $G_0 = 0$, where zero
subscripts denote the initial steady state), and all prices are fully flexible and all exogenous variables are constant. Secondly, the endowment of tradables can be normalized so that the relative price of nontraded goods in terms of tradables is unity. Thirdly, in a symmetric equilibrium, all agents set the same price, and consume and produce the same amount of the differentiated good in the economy. In addition, we assume that all agents consume the same amount of governmentally provided goods. Then the equilibrium on the market for nontraded goods imply

\[ y_N(z) = C_N(z) + G_N(z) = C_N^A + G_N^A. \]

From the Euler equation for tradables consumption follows that the intertemporal profile of tradables consumption is flat. And given that the output of tradables is constant and initial holding of net foreign asset is zero, these assumptions imply that consumption of tradables is constant in every period. Thus shocks to the output or consumption of nontraded goods do not have spillover effects on tradables consumption. Consequently, the economy has a balanced current account and fiscal policy does not lead to international redistribution of wealth through current account imbalances.

In the initial zero government spending steady state \( C_{N,t} = y_{N,t} \), thus the first-order condition governing each the agents’ optimal choice of the output of nontraded goods, equation (8), then implies that the steady-state output of nontraded goods is given by (as in the small-country OR model)

\[ \bar{y}_{N,0} = \left( \frac{(1-\gamma)(\theta-1)}{\theta \kappa} \right)^{\frac{1}{2}}. \]

Because each agent has monopoly power over the nontraded good she/he produces, the home output is suboptimally low in the decentralised competitive equilibrium. Too see this note that a benevolent social planner (SP) would maximize total utility from the output (consumption) of nontraded goods

\[ \max_{\bar{y}_N} \left( 1 - \gamma \right) \log(\bar{y}_N) - \frac{\kappa}{2} \bar{y}_N^2. \]
Implying that
\[ \bar{y}_N^{dp} = \left(\frac{1 - \gamma}{\kappa}\right)^{\frac{1}{2}} \geq \left[\frac{(1 - \gamma)(\theta - 1)}{\theta \kappa}\right]^{\frac{1}{2}} = \bar{y}_{N,0}. \]

This clearly indicates that the output level in the decentralised competitive equilibrium is below the output level realized a benevolent social planner. Hence, it is possible that fiscal expansion can increase output closer to the social optimum and increase welfare. In addition, the preceding equations show that the degree of monopolistic distortion determines by how much the output level the initial steady state is below the socially optimal output level. As the elasticity of demand increases, the differentiated nontraded goods become closer substitutes, and consequently the monopoly power decreases.

### 3.2 The Effects of Fiscal Expansion

Now I turn to analysis of the effects of unanticipated permanent balanced-budget fiscal expansion. Assume that prices of nontraded goods sticky, as they are set one period in advance and fully flexible after the period. It follows from this assumption that it takes one period to reach the new steady state after a fiscal shock hits the economy. The next step is to solve for the steady-state effects of a fiscal shock when prices are flexible and all variables except government spending are constant.

#### 3.2.1 The Steady State Effects of Fiscal Expansion

As mentioned above, as the next step, I use a log-linear approximation of the model around the initial (zero government spending) steady state. Each variable are expressed in deviations from the initial steady state, and the short run and the long run deviations are denoted as follows
\[ \hat{x} = \frac{x_t - x_0}{x_0} \quad \text{and} \]
\[
\hat{X} = \frac{x_{t+1} - x_o}{x_0}.
\]

Since initial steady-state government spending is zero, government spending is normalized by initial consumption of nontraded goods.

Taking logarithmic transformation and then the first-order Taylor expansion \( f'(x) (x - x_0) \) from the goods-market equilibrium condition and from the labour-leisure trade-off condition (8) using above mentioned definitions one can get

\[
\hat{\gamma}_N = \hat{\xi}_N + \hat{\zeta}_N \quad \text{and}
\]

\[
(\theta + 1) \hat{\gamma}_N = (1 - \theta) \hat{\xi}_N + (1 - \alpha \theta) \hat{\zeta}_N.
\]

These equations can be solved to yield

\[
\hat{\xi}_N = \left(\frac{1 + \alpha}{2}\right) \hat{\gamma}_N \quad \text{and}
\]

\[
\hat{\gamma}_N = \left(\frac{1 - \alpha}{2}\right) \hat{\gamma}_N.
\]

The interpretation of equations (11) and (12) is the following: First, permanent balanced-budget fiscal expansion raises the steady-state output of nontraded goods (unless \( \bar{y} = 1 \)) and crowds out private consumption. The steady-state output increases as the agents respond to fiscal expansion by substituting into work out of leisure. Consequently, private consumption falls by less than the rise in government spending.

Second, when the governmentally provided goods are utility enhancing to the agents (that is \( \bar{y} > 0 \), instead of \( \bar{y} = 0 \)), the consumption and output multipliers are reduced by an amount that is increasing in \( \bar{y} \). It implies that the higher the substitutability is between private and government consumption (higher value of \( \bar{y} \)), (i) the bigger is the crowding-out effect on private consumption (ii) and the smaller is the positive impact on output (as in Ganelli 2003). The effects clearly indicate the logic of direct crowding-out: when the government provides goods that are substitutes for private consumption, the fall in consumption is bigger than in the pure waste case. If the marginal rate of substitution between private and government consumption is less than one, the economy reaches an equilibrium which corresponds to lower private
consumption and higher output levels relative to the initial steady-state allocation. However, if government consumption is perfect substitute with private consumption ($\alpha = 1$) a rise in government spending does not affect the output of nontraded goods but it indeed crowds out private consumption on a one-to-one basis.

As I showed in Section 3.3, the initial steady-state output of nontraded goods is inefficiently low because of the monopolistic distortion. A rise in government spending which induces an increase in labour supply (requires $\alpha < 1$) consequently brings the output level of nontraded goods closer to the social optimum. A rise in government spending can thus abate the distortion caused by monopolistic competition. However, the flip side of the coin is that a rise in government spending crowds out private consumption, and as private consumption falls by less than $\alpha G_N$ increases, “effective consumption” of nontraded goods (that is $C_N + \alpha G_N$) falls. Effective consumption, which was already suboptimally low, is thus driven even lower and farther away from its social optimal level.

To solve for $\hat{P}_N$, one can substitute equation (11) into the log-linearized money market equilibrium condition to yield

$$\hat{P}_N = \left( 1 - \frac{\alpha}{2} \right) \hat{G}_N. \tag{13}$$

According to equation (13), fiscal expansion raises the nontraded goods price index, if governmentally provided goods are not perfect substitutes with private consumption. Higher government spending leads to an outward shift in the demand curve facing the agents, therefore allowing them to raise their prices. The preceding equation implies a role for the marginal rate of substitution between private and government consumption: the smaller the substitutability is the greater is the positive impact on the nontraded goods price index. In addition, the rise in the nontraded goods price index is proportional to that in the output of nontraded goods.

If $\alpha$ is less than one, a rise in the nontraded goods price index is necessary is order to maintain the money market equilibrium. In this case, a rise in government spending
crowds out private consumption by more than $\alpha G$ raises, thus “effective consumption” of nontraded goods deceases, which lowers money demand. The reduction in “effective consumption” of nontraded goods then implies that in order to maintain equilibrium in the money market, money demand must increase, requiring a rise in the nontraded goods price index. This rise increases money demand, causing equilibrium in the money market.

As mentioned in Section 3.1.3, tradables consumption is constant in every period implying that $\hat{C}_T = \hat{C}_T^* = 0$. Thus the log-linearized version of equation (7), which governs the optimal allocation of total consumption spending between tradables and nontraded goods, is

$$\hat{C}_N + \alpha \hat{G}_N^* = \hat{P}_T^* - \hat{P}_N^*.$$

Substituting equations (11) and (13) into the preceding equation yield to $\hat{P}_T^* = 0$. The startling implication of this equation is that fiscal expansion does not affect the nominal exchange rate (the price of tradables is proportional to the exchange rate). From the allocation of total consumption spending between tradables and nontraded goods, it is known that in an optimal case, the ratio of the marginal utilities of tradables to nontraded goods equals the relative price of tradables to nontraded goods. Consumption of tradables does not change, which implies that the marginal utility of consumption of tradables is constant over time. If $\alpha$ is less than one, the reduction in “effective consumption” increases the marginal utility of consumption of nontraded goods. Hence, in order to maintain the optimal allocation of total consumption spending, an adjustment in the relative price ratio is needed. As mentioned, because of the increase in demand for nontraded goods it is optimal for the agents to increase their prices. Since the price of tradables remains constant, the relative price of nontraded goods increases. As a result, the rise in the nontraded goods price index and the reduction in “effective consumption” of nontraded goods guarantee that the ratio of the marginal utilities equals the relative price ratio without an adjustment in the price of tradables. However, if $\alpha$ is one, "effective consumption" does not change, which
implies that the marginal utility of "effective consumption" does not change. Therefore, an adjustment in the relative price is not needed.

One can use the log-linear version of the price index equation (3), with \( \hat{P}_r \) unchanged and \( \hat{P}_N \) given by equation (13), to yield the steady-state change in the consumption-based price index

\[
\hat{P} = (1 - \delta) \left( \frac{1 - \alpha}{2} \right) \hat{G}_N.
\]

The preceding equation clearly indicates that the rise in the consumption-based price index is determined by the rise in the nontraded goods price index and the share of private consumption of nontraded goods in total private consumption.

### 3.2.2 Short Run Equilibrium Response to Fiscal Expansion

The next step is to solve for the short-run effects of fiscal expansion when prices in the nontraded goods sector are sticky. The assumption of sticky prices introduces a typical Keynesian feature in the model: output is entirely demand determined (for small enough expansion) in the period following a fiscal shock. Since output is demand-determined the labour-leisure trade-off equation (8) does not hold. To solve for the short-run effects of a fiscal shock on private consumption and output one can log-linearize equation (7) and the money market equilibrium condition (9), keeping in mind that \( \hat{P}_r = 0 \), to yield

\[
\hat{C}_N + \alpha \hat{G}_N = \hat{P}_r \quad \text{and} \quad \hat{C}_N + \alpha \hat{G}_N = -\frac{\beta}{1 - \beta} \hat{P}_r.
\]

Together these equations imply that \( \hat{P}_r \) must be zero. A rise in government spending raises money demand, thus in order to maintain equilibrium in the money market, money demand must fall. Demand for money falls, when the prices of nontraded goods are sticky, if the nominal interest rate rises and/or consumption decreases. Since the
nominal interest rate is defined by the Fisher identity and the nominal rate is unchanged in the steady state, the rise in nominal interest rate means an appreciation of the currency. Thereby, in order to balance money demand with supply, consumption must fall and/or the currency must appreciate. The latter is, however, inconsistent with the optimal allocation of total consumption spending. In the short run, when both tradables consumption and the prices of nontraded goods are constant, private consumption has to decrease and/or the price of tradables raise. Otherwise the ratio of the marginal utilities of tradables to nontraded goods does not equal their relative price ratio. The only means to both allocate total consumption spending optimally and to maintain the money market equilibrium simultaneously is a fall in private consumption with an unchanged exchange rate. The following equation, therefore, has to hold

\[ \hat{C}_N = -\alpha \hat{G}_N. \]  

(14)

Combining this with the log-linear demand equation one can get

\[ \hat{y}_N = (1 - \alpha) \hat{G}_N. \]  

(15)

Equations (14) and (15) imply that the short-run effects on consumption and output of fiscal expansion go in the same direction as the steady state effects, if \(0 < \alpha < 1\): Fiscal expansion increases short-run output and crowds out private consumption. Comparing equation (12), which gives the steady-state rise in output, with equation (15), one can see that the rise in output is higher in the short run. On the other hand from equations (11) and (14), one can see that the crowding-out effect is smaller in the short run. Equations (14) and (15) strengthen the argument that when government spending is useful, the output and consumption multipliers are reduced by an amount that is increasing in \(\alpha\): The higher the substitutability between private and government consumption, (i) the smaller is the positive effect on output (ii) and the bigger is the crowding-out effect on private consumption. A rise in government spending leaves the exchange rate unaffected, due to the reasons discussed above, and thereby it does not cause crowding-out or crowding-in through exchange rate changes. The expansionary effect of fiscal policy is offset only to the extent that governmentally provided goods are substitutes for private consumption. There are no other effects on private consumption – only the direct crowding-out effect.
The model presented here yields important insights into the effectiveness of fiscal policy in small open economies. In the case when government spending is pure waste, it can be seen from equation (15), that the “balanced budget multiplier” is exactly one. This result is in sharp contrast with the result derived from the Mundell-Fleming model. In the Mundell-Fleming model, the effectiveness of fiscal policy on output in a small open economy depends primarily on whether the exchange rate is flexible or fixed. In a flexible exchange rate regime, a rise in government spending tends to increase money demand raising the interest rate, thus capital inflows attracted by the higher interest rate appreciate the exchange rate. This appreciation induces an expenditure-switching effect which causes complete crowding-out, and consequently fiscal policy becomes ineffective and the “balanced budget multiplier” becomes zero. In this model, the effectiveness of fiscal policy is independent of the exchange rate regime. This effectiveness is determined by the marginal rate of substitution between private and government consumption, and only in the case when private and government consumption are perfect substitutes is fiscal policy unable to influence output. In this case fiscal policy only affects the composition of aggregate demand, but not the level of output.

In summary, a permanent rise in government spending generates a short-run increase in the output of nontraded goods (unless $\alpha \neq 1$), but it leaves the current account and the nominal exchange rate unaffected. Since the price of tradables is unchanged and the prices of nontraded goods are sticky, the consumption-based price index remains in the pre-shock level. After one period, the prices of nontraded goods are adjusted and a rise in government spending raises the consumption-based price index (unless $\alpha = 1$). In the steady state, when the economy returns to the labour supply curve, output decreases relative to the short run output, and consequently the crowding-out effect is bigger in the steady state (again, unless $\alpha = 1$). In the case when private and government consumption are perfect substitutes a rise in government spending only affects the composition of aggregate demand (private consumption is reduced on a one-to-one basis), but leaves all other macroeconomic variables unaffected.
3.2.3 Welfare Effects of Fiscal Expansion

As mentioned, one advantage of the NOEM framework is that it allows an explicit utility-based welfare analysis of fiscal policy, and the analysis can thus be used as the basis for the design of optimal fiscal policy. Analyzing the welfare effects of fiscal policy is important, because unless economic models embody meaningful welfare criteria, they can yield misleading policy prescriptions even for the problems they were designed to address (Obstfeld – Rogoff 1995, 625).

According to the utility function (1), fiscal policy can affect welfare by changing private consumption, government consumption, real balances and output. As usual in the NOEM literature, I focus on the real component of the utility function, neglecting the welfare effect of real balances. The welfare effect of fiscal policy is the sum of short-run change in utility and the discounted present value of the steady-state change in utility. Since the economy reaches the steady state after one period, the differentiated utility function is (see Appendix B)

\[ \Delta U_t^R = \gamma \hat{C}_t + (1 - \gamma) \left( \hat{C}_N + \alpha \hat{G}_N \right) - \kappa \delta^{l,0} \hat{y} + \frac{\beta}{1 - \beta} \left[ \gamma \hat{S}_t + (1 - \gamma) \left( \hat{C}_N + \alpha \hat{G}_N \right) - \kappa \delta^{l,0} \hat{y} \right]. \]

As shown in Appendix B, by substituting into (16) steady state output equation (9) and the multipliers (11), (12), (14) and (15) derived, that a rise in government spending does not affect welfare if \( \alpha = 1 \) and causes a fall in welfare if \( \alpha < 1 \). In the case when governmentally provided goods are partial substitutes with private consumption the negative welfare effect results from three factors. Firstly, the increase in output, both in the short run and in the steady state, leads to a decrease in welfare. Secondly, in the short run the rise in government consumption crowds out private consumption, which decreases welfare. However, the fall in welfare caused by the crowding-out effect is perfectly offset by the positive welfare effect that the rise in government spending affects. Thirdly, expansive fiscal policy leads to a higher crowding-out effect in the steady state than in the short run, and in the long-run the negative welfare effect that crowding-out causes is larger than the positive welfare effect that higher government
spending brings. All in all, the main reason for the beggar-thyself result of fiscal expansion is that private consumption is reduced while government consumption is increased: the total utility is lowered since one unit of government consumption gives less utility than one unit of private consumption. Furthermore, the agents have to work more to produce less utility enhancing government consumption.

Maintaining the assumption of \( \alpha < 1 \), the welfare effects also indicate a role for the marginal rate of substitution between private and government consumption. This is a consequence of three factors. The higher the substitutability, (i) the bigger is the direct increase in utility, (ii) the smaller is the rise in output (iii) and the bigger is the direct crowding-out effect on private consumption. The first two effects indicate that the smaller the substitutability is the more a rise in government spending decreases welfare. However, the third effect means the opposite. It could be shown that the first two effects more than offset the third one. Consequently, the less fiscal policy decreases welfare the higher is the marginal rate of substitution between private and government consumption. This result is consistent with Ganelli (2003), who claimed that the introduction of a positive \( \alpha \) unambiguously raises welfare compared to the pure waste case.

However, in the case when private and government consumption are perfect substitutes a rise in government spending does not lower welfare. In this case a rise in government spending does not raise the output of nontraded goods and the direct increase in utility from government spending perfectly offsets the negative welfare effect caused by direct crowding-out. The result that fiscal expansion is never welfare-improving is in contrast with Ganelli (2003), who concluded that the introduction of utility enhancing government spending reverses the beggar-thyself welfare result on the specific parameter values. A rise in welfare happens if the direct increase in utility from government spending more than offsets the negative effects caused by decreased private consumption and increased output. In this model, the positive welfare effect that higher government spending brings is never enough to more offset the negative welfare effect. Therefore, if a government in a small open economy desires to
maximize the welfare of the representative agent, it should not implement fiscal expansion.

4 Fiscal Policy, the Exchange Rate and the Current Account

The primary focus of this section is to examine how the effects of fiscal expansion depend on the elasticity of substitution between traded and nontraded goods. To address this research agenda I develop a simple model in which the elasticity of substitution between traded and nontraded goods is not restricted to a particular value even if the model is numerically solved. Relatively few studies have examined the effects of fiscal policy in calibrated versions of NOEM models. For example, Kim and Roubini (2004, 11) point out that “[t]he effects of fiscal policy on the current account and the real exchange rate in calibrated versions of these NOEM models are still waiting to be analyzed.”

One of the biggest advantages of the NOEM framework, pioneered by Obstfeld and Rogoff (1995, 1996 Section 10), is that it “incorporates the price rigidities essential to explain exchange rate behavior without sacrificing the insights of the intertemporal approach to the current account” (Obstfeld – Rogoff 1995, 624). The model presented in this section uses the strengths of the NOEM framework by deriving the short-run and long-run effects of fiscal expansion not only on the exchange rate and the current account but also on several other macroeconomics variables. The model builds on the monetary policy model developed by Lane (2001a). His model extended the small-country model contained in the Appendix to Obstfeld and Rogoff (1995) by introducing a utility function that is non-separable between tradables and nontradables consumption. The main advantage of this specification of preferences is that in this framework economic shocks to the nontraded goods sector affect tradables consumption and consequently the current account. This framework, therefore, is well equipped to study the effects of fiscal policy on the optimal time path of consumption and external borrowing.
4.1 A Model

4.1.1 Market Structure and Preferences

The basic structure of this model is identical to the one presented in Section 3. The only difference between these two models is the intertemporal utility function which is now given by

$$U_t = \sum_{s=2}^{\infty} \beta^{t-s} \left[ \frac{\sigma}{\sigma - 1} C_s^{\frac{\sigma - 1}{\sigma}} + \frac{\chi}{1 - \epsilon} \left( \frac{M_s}{P_s} \right)^{1 - \epsilon} - \frac{\kappa}{2} \left( y_{N,s}(z) \right)^2 \right],$$

where

$$C_t = \left[ \gamma^{\theta} C_{T,t}^{\theta} + (1 - \gamma)^{\frac{1}{\theta}} C_{N,t}^{\frac{1}{\theta}} \right]^{\frac{\theta}{\theta - 1}}.$$

In the utility function (17) $U_t$ stands for utility at time $t$, $\beta$ ($0 < \beta < 1$) is the discount factor. The first term in (17) is the utility for consumption, where $C$ is the overall consumption index that aggregates consumption of traded and nontraded goods and $\sigma$ is the intertemporal elasticity of aggregate consumption. In equation (18) $C_{T,t}$ is consumption of tradables at time $t$, $\gamma$ is the share of tradables in total consumption, $C_{N,t}$ is the private nontraded goods consumption index (to be defined below) and $\theta$ is the elasticity of substitution between traded and nontraded goods. The second term in the utility function (17) reflects the utility for holding real balances, where $\chi$ is the positive parameter, $M_s$ is nominal money balances held by the agent at time $s$, $P_s$ is the consumption-based price index (to be defined below) and $\epsilon$ is the money demand parameter. The last term captures the disutility the agent experiences in having to produce output, where $y_s(z)$ is the output of nontraded good $z$ and $\kappa$ the positive parameter.

The overall consumption index, given by (18), aggregates consumption of traded and nontraded goods. As mentioned, $C_{T,t}$ is consumption of tradables. The variable $C_{N,t}$ is the private nontraded goods consumption index, a CES aggregator of quantities of different nontraded goods consumed:
\begin{equation}
C_N = \left( \int_0^1 c(z)^{\frac{\mu}{\mu-1}} dz \right)^{\frac{\mu}{\mu-1}},
\end{equation}

where \( c(z) \) is consumption of good \( z \) and \( \mu (> 1) \) denotes the elasticity of substitution between varieties of nontraded goods (the parameter also denotes the price elasticity of demand of good \( z \)). Government expenditures are assumed to be pure waste and not to affect private utility. Per capita government consumption, \( G_N \), is the government consumption index that is aggregated in the same manner as private nontraded goods consumption, and with the same elasticity of substitution

\begin{equation}
G_N = \left( \int_0^1 g(z)^{\frac{\mu}{\mu-1}} dz \right)^{\frac{\mu}{\mu-1}},
\end{equation}

where \( g(z) \) is government consumption of good \( z \).

Home tradables are perfect substitutes with foreign-produced tradables, and the foreign currency price of tradables is exogenously determined in the world market. There are no costs or impediments to trade between the home country and the world market, and thus the law of one price holds in tradables. The foreign currency price of tradables can be normalized to unity, which then implies \( P_T = E \), where \( P_T \) is the domestic currency price of tradables and \( E \) is the nominal exchange rate, defined as the home currency price of the foreign currency. The price of tradables, therefore, also stands for the nominal exchange rate.

Given the level of aggregate consumption, the optimal allocation of expenditures between traded and nontraded goods is given by

\begin{equation}
C_T = \gamma \left( \frac{P_T}{P} \right)^{-\theta} C
\end{equation}

\begin{equation}
C_N = (1 - \gamma) \left( \frac{P_N}{P} \right)^{-\theta} C.
\end{equation}

In the preceding equations, \( P \) denotes the consumption-based price index and \( P_N \) denotes the nontraded goods price index. The preceding equations imply that the
demands for goods are proportional to aggregate consumption with a proportionality coefficient that is an isoelastic function of the ratio of the goods’ price to the consumption-based price index. The consumption-based price index, defined as the minimum expenditure required to purchase one unit of aggregate consumption, is given by

\[ P = [p_{P}^{\rho} + (1 - \gamma)P_{N}^{1 - \theta}]^{\frac{1}{\theta}}. \]

The nontraded goods price index, defined as the minimum expenditure required to purchase one unit of a basket of nontraded goods, is given by

\[ P_{N} = \left( \int_{0}^{1} P_{N}(z)^{1 - \mu} \,dz \right)^{\frac{1}{\mu}}, \]

where \( p(z) \) denotes the price of nontraded good \( z \).

Making use of the constant-elasticity of substitution nontraded goods consumption index, equation (19), and adding up private and government demands yields the demand curve. The total demand for each nontraded good, therefore, is given by

\[ y_{N}(z) = \left( \frac{P_{x}(z)}{P_{N}} \right)^{\gamma_{N}} \left( C_{N}^{A} + G_{N}^{A} \right), \]

where superscript \( A \) denotes aggregates. This equation simply shows that that the demand for each nontraded good depends on its relative price, the elasticity of demand, and aggregate private and government (per-capita) expenditures.

### 4.1.2 Budget Constraints and Optimality Conditions

The intertemporal budget for the representative agent is written, in nominal terms, as

\[ P_{T,j}B_{j} + M_{t} = P_{T,j}(1 + r)B_{t-1} + M_{t-1} + p_{N,t}(z) y_{N,t}(z) \]

\[ \quad + P_{T,j} \tilde{y}_{T,j} - P_{T} C_{t} - P_{N,t} r_{t}, \]

where \( B_{t} \) denotes the stock of riskless real bonds (denominated in tradables) held by the agent entering period \( t + 1 \). \( M_{t} \) is the agent’s money balances entering period \( t + 1 \), \( r \) denotes the constant world net interest rate earned on bonds between periods \( t - 1 \) and
t, \( y_{T,t} \) is exogenously given quantity of tradables and \( \tau \) denotes per capita taxes (in units of nontraded goods).

The government finances its purchases through lump-sum taxes and seigniorage. Under these assumptions, the government budget constraint, expressed in per capita terms and in units of nontraded goods, can be written as

\[
G_{N,t} = \tau_t + \frac{M_t - M_{t-1}}{P_{N,t}}.
\]

The representative agent solves an intertemporal maximization problem, choosing the levels of consumption, money holding, bond holding and the output of nontraded goods that maximizes the discounted lifetime utility. To solve the first-order conditions for the representative agent, equation (23) is used to eliminate \( p_{N,t}(x) \) from (24), and the utility function (17) is maximized subject to the resulting budget constraint. The optimal behaviour of the representative agent is characterized by the following optimality conditions (see Appendix C and note that the indexes denoting the agents are dropped):

\[
\frac{C_{T,t+1}}{C_{T,t}} = \left( \frac{P_{t}/P_{T,t}}{P_{T,t+1}/P_{T,t+1}} \right)^{\sigma - \theta}.
\]

\[
\frac{C_{N,t}}{C_{T,t}} = \left( \frac{1 - \gamma}{\gamma} \right) \left( \frac{P_{N,t}}{P_{T,t}} \right)^{-\theta}.
\]

\[
\kappa^\mu = C^{1/\sigma} \left( \frac{P_{N,t}}{P_{T,t}} \right) \left( \frac{\mu - 1}{\mu} \right) \left( C_{N,t}^A + G_{N,t}^A \right)^{1/\gamma}.
\]

\[
\frac{M_t}{P_t} = \left[ \chi C^{1/\sigma} \left( \frac{1 + i}{i} \right) \right]^{1/\tau}.
\]

where \( i \) is the nominal interest rate defined by the Fisher identity

\[
1 + i = (1 + r) \frac{P_{T,t+1}}{P_{T,t}}.
\]
Since the price of tradables also denotes the nominal exchange rate, the Fisher identity implies uncovered interest parity. Equation (25) is the Euler equation governing the optimal *intertemporal* allocation of tradables consumption. As noted by Dornbusch (1983), the relevant real interest rate, for a small country with a nontraded goods sector, is not the world interest rate but the interest rate stated in terms of the domestic consumption basket. For example, if the consumption-based price index is relative to the price of tradables is temporarily low relative to its future ratio the consumption-based real interest rate is also temporarily low.\(^6\) This favours short-run over long-run consumption and raises short-run consumption with elasticity \(\sigma\). However, as the consumption-based price index rises consumption of tradables becomes relatively dearer, and consequently consumption of tradables falls as a fraction of aggregate consumption with elasticity \(\theta\) [recall equation (20)]. The interplay between \(\sigma\) and \(\theta\) determines whether consumption of tradables raises or drops. Equation (26) governs the optimal *intragtemporal* allocation of expenditures between traded and nontraded goods. The optimal allocation of expenditures depends on the openness of the economy, the relative price ratio and the elasticity of substitution between traded and nontraded goods. Equation (27) is the labour-leisure trade-off condition. It states that the marginal disutility of producing an extra unit of a nontraded good is equal to the marginal utility from consuming the added revenue that the extra unit of the nontraded good brings. Equation (28) is the money market equilibrium condition, which shows that the demand for real balances is a positive function of aggregate consumption and a negative function of the interest rate. It also shows that the demand of real money balances is influenced by the consumption elasticity of money demand \((1/c)\).

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\(^6\) In this case, the relative price of tradables in terms of the consumption-based price index falls in the long run. Then one unit of tradables borrowed at today has relatively much purchasing power in terms of aggregate consumption today and costs a little in terms of aggregate consumption upon the repayment of the loan next period. Since the loan adds more consumption today than it costs to repay tomorrow with the falling relative price of tradables the real interest rate in terms of aggregate consumption (the consumption-based real interest rate) is below the world interest rate. This argument directly follows Dornbusch (1983, 145).
4.1.3 The Current Account

The current account is defined as the sum of the trade balance and the services balance. The trade balance is here the difference between the output of tradables and their consumption. The services account is here the product of the stock of real bonds and the world net interest rate earned on bonds. As standard in the literature, we consider the steady state in which the initial stock of net foreign asset is zero. The short-run current account identity, therefore, can be written as

\[(29) \quad B_t = y_{t, T} - C_{t, T} \cdot \]

Since the optimal intertemporal consumption of tradables is tilted by changes in the prices and the output of tradables is constant current account behaviour can be non-zero. This implies that fiscal expansion can generate current account imbalances, and the economy (the representative agent) either accumulates net foreign assets or issues foreign bonds in response to fiscal expansion.

Assume that the prices of nontraded goods are set one period in advance and that the economy reaches the new steady state after one period. The steady-state current account equation can be written as

\[(30) \quad rB_t = C_{t, T+1} - C_{t, 0} \cdot \]

The current account imbalances in the short run determine the stock of net foreign assets in the steady state. Should an economic shock induce a current account deficit in the short run the economy must run a current account surplus in the steady state in order to finance its net foreign debt services. On the other hand, should the economy accumulate net foreign assets in the short run it uses interests earned on bonds for steady-state consumption of tradables.
4.1.4 A Symmetric Steady-State Equilibrium

The model is log-linearized around the flexible price steady state, in which all exogenous variables are constant and the initial stock of net foreign asset and government spending are both zero. In addition, we assume a symmetric equilibrium, in which all agents consume and produce the same amount of all differentiated nontraded goods and all prices are equal. In this symmetric equilibrium, equation (23) implies that the demand for nontraded goods is given by

\[
y_N = C_N + G_N.
\]

As in Lane (2001a), the endowment of tradables is normalized such that the relative price of nontraded goods in terms of tradables is unity. In this symmetric steady state \( y_{N,1} = (1-\gamma)C_t \), therefore the labour-leisure trade-off condition (27), can be solved to yield the steady-state output of nontraded goods

\[
y_{N,0} = \left(\frac{\mu-1}{\mu \kappa}\right)^{\frac{\sigma}{\sigma+1}} (1-\gamma)^{\frac{1}{1+\sigma}}.
\]

This equation implies that due to monopolistic competition in the nontraded goods sector the output of nontraded goods is suboptimally low in the decentralised competitive equilibrium. As the elasticity of demand increases, the differentiated nontraded goods become closer substitutes, and consequently the monopoly power decreases.

4.1.5 The Log-Linear Version of the Model

As mentioned, the model is log-linearized around a symmetric steady state which was characterized above. The next step is to derive log-linear versions of all of the model's key equations. Each variable is expressed in percentage deviations from the initial (zero government spending) steady state. In the short run, nominal prices of nontraded
goods are predetermined: they are set one period in advance and can be adjusted fully after one period. It follows from this assumption that it takes one period to reach the new steady state after a fiscal shock hits the economy.

The variables whose initial steady-state value is zero, government spending and foreign bond holdings, are normalized by appropriate initial consumptions: government spending is normalized by consumption of nontraded goods and net foreign assets by consumption of tradables.

We begin with the aggregate consumption index, equation (18). The log-linearized versions of it in the short run and in the steady state, respectively, are

(32) \[ \hat{C} = \gamma \hat{C}_t + (1 - \gamma) \hat{C}_N \] and
(33) \[ \hat{C} = \gamma \hat{C}_t + (1 - \gamma) \hat{C}_N. \]

The short-run stickiness of the prices of nontraded goods means that \( \hat{P}_N = 0 \). Hence, the log-linearized versions of the consumption-based price index (22) are

(18) \[ \hat{P} = \gamma \hat{P}_t \] and
(19) \[ \hat{P} = \gamma \hat{P}_t + (1 - \gamma) \hat{P}_N. \]

The log-linearized versions of the demand curve for the representative agent, equation (31), are

(36) \[ \hat{y}_N = \hat{C}_N + \hat{G}_N \] and
(37) \[ \hat{y}_N = \hat{C}_N + \hat{G}_N. \]

The log-linearized version of the optimal intertemporal consumption of tradables, equation (25), is

(38) \[ \hat{C}_t - \hat{C}_T = (\sigma - \theta)(\hat{p} - \hat{P}_T) - (\sigma - \theta)(\hat{P} - \hat{P}_T). \]
This equation illustrates that to the extent that shocks to the nontraded goods sector affect the relative price ratio \((P, P_T)\) over time, they also affect the optimal time path of consumption of tradables and consequently the current account.

The log-linearized versions of the optimal allocation of expenditures between traded and nontraded goods link changes in consumption of traded and nontraded goods. Equation (26) takes the log-linear forms

\[
\hat{C}_N - \hat{C}_T = \theta \hat{P}_T \quad \text{and} \\
\hat{C}_N - \hat{C}_T = -\theta \left( \hat{P}_N - \hat{P}_T \right).
\]

The assumption of sticky prices introduces a typical Keynesian feature into the model: Output becomes entirely demand-determined for a small enough rise in government spending. The labour-leisure trade-off condition, therefore, is required to hold only in the steady-run. Together, the log-linearized versions of equation (27) and the optimized relationship between \(C_N\) and \(C\) [equation (21)] imply

\[
\frac{\mu + 1}{\mu} \hat{y}_N = \left( 1 - \frac{\theta}{\sigma} \left( \hat{P}_N - \hat{P} \right) \right) - \left( \frac{1}{\sigma} - \frac{1}{\mu} \right) \hat{C}_N + \frac{1}{\mu} \hat{G}_N.
\]

The log-linearized versions of the money-demand equation (28), making use of the optimized relationship between \(C_T\) and \(C\) [equation (20)], can be written as

\[
-\hat{\epsilon} \hat{P} = \frac{1}{\sigma} \hat{C}_T + \frac{\theta}{\sigma} \left( \hat{P}_T - \hat{P} \right) + \frac{1}{r} \left( \hat{P}_T - \hat{P} \right) \quad \text{and} \\
-\hat{\epsilon} \hat{P} = \frac{1}{\sigma} \hat{C}_T + \frac{\theta}{\sigma} \left( \hat{P}_T - \hat{P} \right).
\]

In equation (44) the real interest rate and the discount rate are tied down by the familiar condition

\[
\rho = \frac{1 - \beta}{\beta}.
\]

Finally, the current account equations (29) and (30), given the constant endowment of tradables, take the log-linear forms
Equations (44) and (45) together imply that an increase (decrease) \( \hat{B} \) in per-capita net foreign assets increases (decreases) steady-state consumption of tradables by the amount \( \hat{C}_r \) since the output of tradables is exogenous.

Equations (32) – (45) fully describe the equilibrium dynamics of the model. Having laid out the equations of the model, we now turn to the analysis of the effects of fiscal expansion.

4.2 The Effects of Fiscal Expansion: A Special Case

The log-linear equations would allow us to solve for closed-form solutions for the short-run and steady-state effects of fiscal expansion. However, a numerical solution of the model can be used to illustrate the effects of fiscal expansion. Nonetheless, we, for a start, solve for an analytical solution of the model in a simple special case. To simplify the analysis we assume a logarithmic utility for consumption and real money balances, which corresponds to \( \varepsilon = \sigma = 1 \). In addition, we assume that the elasticity of substitution between traded and nontraded goods is also unity (\( \theta = 1 \)).

In the case where \( \theta = \sigma = 1 \), as pointed out by Lane (2001a), the utility function is log-separable in consumption of traded and nontraded goods. Equation (38) reveals that in this case the optimal intertemporal profile of tradables consumption is perfectly flat. Since the output of tradables is constant and initial net foreign assets are zero the economy has always a balanced current account regardless of shock to the output or consumption of nontraded goods.

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7 Lane (2001a) used this solution technique to solve for the effects of an exogenous rise in the money supply.
Together the steady-state market clearing condition for nontraded goods (37) and the labour-leisure trade-off condition (41) imply that

$$\hat{C}_N = -\frac{1}{2} \hat{G}_N$$ and

$$\hat{G}_N = \frac{1}{2} \hat{G}_N.$$  

The steady-state output of nontraded goods increases as the agents respond to a rise in government spending by substituting into work out of leisure. Consequently, private consumption falls by less than the rise in government spending.

Substitution (46) into the log-linearized version of the money demand function that makes use of the optimized relationship between $C_N$ and $C$ [equation (21)] yields to

$$\hat{P}_N = \frac{1}{2} \hat{G}_N.$$  

The preceding equation indicates that a rise in government spending raises the nontraded goods price index. Higher government spending leads to an outward shift in the demand curve facing the agents, therefore allowing them to raise their prices. Furthermore, the rise in the price index is proportional to that in the output of nontraded goods.

Substituting equations (46) and (48) into equation (40) yields to

$$\hat{P}_T = 0.$$  

The startling implication of this equation is that a rise in government spending does not affect the nominal exchange rate in the steady state (the price of tradables also denotes the nominal exchange rate). The economic intuition behind this result is the following: the allocation of total consumption spending between traded and nontraded goods implies that in an optimal case the ratio of marginal utilities of traded and nontraded goods equals the relative price of tradables in terms of nontraded goods. Consumption of tradables does not change, consequently the marginal utility of

\[\text{The money demand function can be now written as } \frac{M}{P_T} = \frac{\chi}{(1-\gamma)} \frac{P_{NX}}{P_t} \left(\frac{1+i}{i}\right)^{C_{NJ}}.\]
tradables consumption is constant. The fall in nontraded goods consumption increases the marginal utility of nontraded goods consumption. Therefore, an adjustment in the relative price ration is needed in order to maintain the allocation of total consumption in optimum. As shown, a rise in government spending raises the price of nontraded goods and crowds out nontraded goods consumption. These effects guarantee that the ratio of marginal utilities equals the relative price ratio without an adjustment in the price of tradables.

Together, the rise in the nontraded goods price index and the unaffected price of tradables have two consequences. Firstly, the steady-state change in the consumption-based price index (35) is determined by the rise in the nontraded goods price index and the share of nontraded goods in total consumption. Secondly, a rise in government spending appreciates the equilibrium real exchange rate which is defined as the price of tradables in terms of nontraded goods. Defined this way the equilibrium real exchange rate represents an internal terms of trade measuring how much of nontraded goods must be given up for one unit of tradables in the steady state. Since a rise in government spending appreciates the equilibrium real exchange rate, it thus improves the economy’s steady-state terms of trade.

The next step is to solve for the short-run effects of a rise in government spending. Substituting the market clearing condition (36) into the money market equilibrium condition that makes use of the optimized relationship between \( C_N \) and \( C \) yields to

\[
\hat{C}_N = 0.
\]

Substituting this equation into (36) yields to

\[
\hat{y}_N = \hat{G}_N.
\]

This equation clearly shows that a rise in government spending increases the output of nontraded goods. Furthermore, in this special case the “balanced budget multiplier” is exactly one in the short run. Since a rise in government spending increases output on a one-to-one basis it does not crowd out private consumption, as equation (50) illustrates.
Since consumption of traded and nontraded goods are both unaffected, equation (39) shows that
\[ \hat{P}_t = 0. \]

A rise in government spending, as the preceding equation brings out, does not affect the nominal exchange rate in the short run. This is a consequence of two factors. Firstly, a rise in government spending does not affect money demand. The money demand function (28) shows that in the case where \( \varepsilon = 1 \) the short-run money demand is proportional to aggregate consumption. Since aggregate consumption does not change the unaffected money demand leaves the nominal exchange rate unaffected. Secondly, since neither traded nor nontraded goods consumption changes the ratio of marginal utilities of traded and nontraded goods equals the relative price ratio without an adjustment in the price of tradables.

### 4.3 The Effects of Fiscal Expansion: The General Case

#### 4.3.1 The Calibration of the Model

The log-linear equations would allow us to solve for closed-form solutions of the model also in the general case. Alternatively, a numerical solution of the model can be used to illustrate, in a tractable way, the effects of a rise in government spending. Seven short-run and seven steady-state variables are to be determined. Fourteen equations that jointly determine them are (32) – (45). In order to solve the model numerically, it can be written in the matrix form \( Ax = B \), where the matrix \( A \) (14 x 14) contains the structural parameters of the equations, the vector \( x \) (14 x 1) contains the endogenous variables of the equations and the vector \( B \) (14 x 1) contains the exogenous shock (a rise in government spending). In this case, the model can be solved by using linear algebra, as the solution of the model can then be written as \( x = A^{-1}B \).

In order to solve the model numerically, values for six parameters are required: the intertemporal elasticity of aggregate consumption, the share of tradables in total
consumption, the consumption elasticity of money demand, the elasticity of substitution between traded and nontraded goods, the elasticity of substitution between varieties of nontraded goods and the real interest rate. We focus attention on how the effects of fiscal expansion depend on the marginal rate of substitution between traded and nontraded goods. This parameter, therefore, is not restricted to a particular value, but it is analyzed how the solution of the model depends on this parameter value. We let this elasticity of substitution to be between 0.4 and 4. In the calibration, we assume a logarithmic utility for consumption, which corresponds to $\sigma = 1$. This is a standard assumption, and one that would render the model compatible with a balanced growth path if trend technological progress was introduced (see e.g. King, Plosser and Rebelo 1988). Stockman and Tesar (1995) estimated that nontraded goods make up about half of output, and thus $\gamma$ is set to 0.5. Mankiw and Summers (1986) estimated the consumption elasticity of money demand ($1/\varepsilon$ in this model) to be very close unity, so it is chosen a value $\varepsilon = 1$. The elasticity of substitution between varieties of nontraded goods is set to 6, which implies a 20 percent mark-up in the steady state. This is consistent with the mark-up estimated by Rotemberg and Woodford (1992) and it is widely used in related work. Finally, the real interest rate is chosen to be 4 percent. (Print of the Mathematica file that solves the baseline case of the model is at end this thesis.)

4.3.2 The Effects of a Permanent Rise in Government Spending

Figures 1 and 2 illustrate the effects of a 1 percent rise in government spending (relative to initial consumption of nontraded goods). In all diagrams, the horizontal axis marks the elasticity of substitution between traded and nontraded goods and the

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9 There is limited evidence on this elasticity of substitution in the empirical macroeconomics literature. Mendoza (1991) estimate this elasticity of substitution to be 0.74, Ostry and Reinhart (1992) report estimates in the range of 0.66 to 1.3 (for developing countries) and Stockman and Tesar (1995) find an estimate as low as 0.44.

10 In the case of a permanent increase $\hat{G}_N = \hat{G}_N = 1$. 
vertical axis marks the variable’s percentage deviation from the initial steady state.\textsuperscript{11} To illuminate how the interplay between $\sigma$ and $\theta$ influences the current account and nominal exchange rate responses to a rise in government spending, three cases can be distinguished to consider: (i) $\theta = \sigma = 1$, (ii) $\theta > \sigma = 1$ and (iii) $\theta < \sigma = 1$.

In case (i) the solution of the model corresponds to the special case that was presented in Section 4. Figure 1 indicates the result that a rise in government spending affects nothing other than the output of nontraded goods on a one-to-one basis leaving all other variables unaffected in the short run. As mentioned, in the case where $\theta = \sigma = 1$ the utility function is log-separable in consumption of traded and nontraded goods. In this case the optimal intertemporal profile of tradables consumption is perfectly flat, as the intra- and intertemporal substitution effects cancel out each others. Therefore, although a rise in government spending affects the relative price ratio ($P, P_T$) in the steady state this price ratio change has to influence on consumption of traded and consequently on the current account. As noted previously, in the case where $\epsilon = 1$ the short-run money demand is proportional to aggregate consumption, therefore the unchanged money demand leaves the nominal exchange rate unaffected. Figure 2 illustrates that in the steady state a 1 percent rise in government spending, among others, increases the output of nontraded goods by a half percent and raises the nontraded goods price index by a half percent as equations (47) and (48) indicate, respectively.

In case (ii), as can be seen from Figure 1, a rise in government spending increases nontraded goods consumption and production, decreases tradables and aggregate consumption, depreciates the nominal exchange rate and induces a current account surplus in the short run. Since the short-run money demand is proportional to aggregate consumption a fall in aggregate consumption tends to lower money demand

\textsuperscript{11} As noted previously, the model is log-linearized around the steady state, in which net foreign assets holdings is zero and the change in net foreign assets is normalized by consumption of tradables. The current account diagram, therefore, shows by how much the current account changes relative to initial consumption of tradables. In addition, the real exchange rate is defined as the relative price of tradables in terms of nontraded goods.
requiring a depreciation of the nominal exchange rate in order to maintain equilibrium in the money market. This depreciation and the sticky prices in the nontraded goods sector imply that the relative price of tradables rises, which encourages the agents to switch their consumption towards nontraded goods. The strength of this effect depends on the intratemporal elasticity of substitution between traded and nontraded goods. On the other hand, since the aggregate price level relative to the price of tradables is currently low relative to its future ratio, the consumption-based real interest rate is temporarily low. This low consumption-based real interest rate induces the agents to switch consumption from the future to the present. The strength of this effect depends on the intertemporal elasticity of substitution. The intra- and intertemporal substitution effects on short-run consumption of tradables pull in opposite directions. Since $\theta > \sigma$, the intratemporal substitution effect wins out and consequently consumption of tradables decreases. This reduction in consumption of tradables in turn induces a short-run current account surplus, which implies a permanent improvement in the economy’s net foreign assets. In the steady state this entails a permanent services balance surplus, which is used to finance a trade balance deficit. This trade balance deficit allows consumption of tradables to remain permanently above the endowment of tradables. Nonetheless, the raise in steady-state consumption of tradables is fairly small.

In case (iii), a rise in government spending, contrary to the previous case, appreciates the nominal exchange rate, increases tradables and aggregate consumption and generates a current account surplus in the short run. Interestingly, a one percent rise in government spending increases the output of nontraded goods by more than one percent in spite of the appreciation of the nominal exchange rate. Increased aggregate consumption raises money demand, which tends to raise the interest rate. An appreciation of the nominal exchange rate, therefore, is required to balance money demand and supply. This appreciation raises the relative price of nontraded goods, which favours substitution from traded to nontraded goods. However, this negative effect on consumption of nontraded goods is more than offset by the positive effect. As in the previous case, also in this case the aggregate price level relative to the price of
tradables is currently low relative to its future value. The consumption-based real interest rate, therefore, is temporarily low, which induces the agents to switch consumption from the steady state to the short run thus also increasing consumption of nontraded goods. Since $\theta$ is now low, implying little substitutability in consumption between traded and nontraded goods, the relative strength of the intertemporal substitution is low. The intratemporal effect, therefore, dominates increasing consumption of nontraded goods in spite of the appreciation of the nominal exchange rate. From the above discussion, it should be clear that the intratemporal and intertemporal substitution effects increase consumption of tradables thereby generating a short-run current account deficit. This in turn induces a permanent reduction in net foreign assets. In the steady state this entails a permanent services balance deficit, which must be financed by a trade balance surplus. In order to achieve a trade balance surplus, consumption of tradables must remain permanently below the endowment of tradables.

Figure 2 illustrates that a rise in government spending raises the steady-state output of nontraded goods. Output raises as the agents respond to a rise in government spending by substituting into work out of leisure. There can be, in some cases, negative effects on labour supply, as explained in a moment, but they are more then offset by the positive effects. Consequently, consumption of nontraded goods falls by less than the rise in government spending. As stressed by Lane (2001a), net foreign assets have effects on the level of desired consumption of nontraded goods and on the optimal labour supply, and these effects on the output of nontraded goods pull in opposite directions. Firstly, due to the nonseparability between traded and nontraded goods consumption the change in steady-state consumption of tradables affect the desired consumption of nontraded goods. For example, in the case where $\theta < \sigma$ the declined steady-state consumption of tradables induces a decline in desired consumption of nontraded goods, which tends to lower the output of nontraded goods. However, this effect plays only a minor role here since output raises by the most in the case where this effects tends to reduce output. Secondly, short-run current account imbalances have a wealth effect on the optimal labour supply: As equation (27) shows, higher
consumption induces a reduction in labour supply. Therefore, if the economy accumulated net foreign assets in the short run, higher wealth leads to some reduction in labour supply. For this reason, output raises by less than in the case where the current account remained in balance in the short run. On the other hand, if a rise in government spending generated a current account deficit, lower wealth leads to some increase in labour supply and output.

As Figure 2 illustrates, a rise in government spending appreciates the equilibrium real exchange rate and raises the nontraded goods price index in the steady state. Higher government spending leads to an outward shift in the demand curve facing the agents, therefore allowing them to raise their prices. Furthermore, this rise in the price index is proportional to the rise in the output of nontraded goods. In the steady state, as before, a rise in government spending appreciates the nominal exchange rate appreciates if $\theta < \sigma$. Indeed, the nominal exchange rate jumps immediately to its steady-state level despite the stickiness of the prices of nontraded goods in the short run. The equilibrium real exchange rate was defined as the price of tradables in terms of nontraded goods. As Figure 2 illustrates, a rise in government spending appreciates the equilibrium real exchange rate, improving the economy’s steady-state terms of trade. The change in the equilibrium real exchange rate is required to lead the agents to revise their consumption allocation between traded and nontraded goods in a consistent way. Since the steady-state trade balance needs to change to reach a particular value, the equilibrium exchange rate has to change accordingly. It has to appreciate adequately to induce the agents to change their consumption allocation in a way consistent with the required change in the steady-state trade balance.

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12 As later shown, exchange overshooting (undershooting) takes place if $\varepsilon$ is bigger (smaller) than one.
4.3.3 The Effects of a Temporary Rise in Government Spending

We now turn to examining the effects of a temporary rise in government spending. A temporary rise in government spending is assumed to last for one period, and as before, the price of nontraded goods are sticky in short run and the economy reaches the new steady state after one period. A temporary rise in government spending can have effects on the steady state, because of induced wealth changes through short-run current account imbalances. If fiscal policy induced short-run wealth changes, these changes would to affect the optimal labour supply and output in the steady state. Consequently, fiscal policy would affect the economy well beyond the time frame of a temporary rise in government spending.

Surprisingly, a one percent temporary rise in government spending raises the short-run output of nontraded goods by one percent (for all values of the elasticity of substitution between traded and nontraded goods consumption) but it leaves all other endogenous variables unaffected both in the short run and in the steady state. Therefore, even though a temporary rise in government spending induces a tilt into the time profile of aggregate demand, it does not introduce a tilt in the time profile of output net of government consumption. It is interesting to notice that the effects of a rise in government spending in the short run differ largely depending on whether a rise in government spending is permanent or temporary.

The intuition behind the result, that a temporary rise in government spending affects nothing other than the short-run output of nontraded goods, is rather straightforward. Together unaffected consumptions of traded and nontraded goods imply that the unchanged pressure on money demand leaves also the nominal exchange rate unaffected. With the nominal exchange rate been unchanged and the price of

\[ \hat{G}_N = 1 \text{ and } \hat{G}_N = 0 . \]

13 In the case of a temporary increase
nontraded goods fixed, the relative price ratio \((P, P_T)\) also remains constant in the short run. Again, this relative price ratio remains constant also in the steady state. A temporary rise in government spending does not affect this price ratio either today or tomorrow and consequently the optimal intertemporal profile of consumption of tradables is perfectly flat for all values of the elasticity of substitution between traded and nontraded goods. The constant consumption of tradables has two implications. Firstly, with no effect on consumption of tradables the assumption that the government spends exclusively on nontraded goods isolates the shock to the nontraded goods sector and thus the short-run output of nontraded goods raises on a one-to-one basis. Secondly, a temporary rise in government spending does not induce short-run current account imbalances that would affect the optimal labour supply and output in the steady state. Fiscal policy, therefore, does not affect the economy beyond the time frame of a temporary rise in government spending.

The finding, that a temporary rise in government spending affects nothing other than the short-run output of nontraded goods, can be interpreted as providing favourable evidence for the effectiveness of fiscal policy as a stabilization tool. Temporary changes in government spending, that last no longer than price rigidities and are used to purchase only nontraded goods, are an effective stabilization tool when used wisely and timely in response to undesired fluctuations in output. Undesired fluctuations in domestic output can be perfectly offset by an opposite change in government spending. Most significantly, this can be done without causing any side-effects.

4.3.4 Sensitivity Analysis: The Role of Openness and Money Demand

In this section, we explore to what extent the effects of a rise in government spending represented above may be sensitive to the calibration of two central parameters characterizing the small open economy: the degree of openness and the consumption elasticity of money demand. To begin with, we can discover that changing these parameters does not influence the effects of temporary fiscal expansion.
Figure 3 illustrates the consequences of changing the share of tradables in total consumption to 0.2. As can be seen from Figure 3, a rise in government spending induces a smaller effect on the output of nontraded goods and a larger effect on tradables consumption in the more closed economy in the short run. The main reason behind these changes is the change in the nominal exchange rate. The greater change in the nominal exchange rate causes greater intratemporal consumption switching which increases tradables consumption and decreases nontraded goods consumption relative to the baseline case. Due to the change in tradables consumption, the current account alters by less in the more open economy. However, one should recall that the current account is normalized by initial consumption of traded goods. Hence, a rise in government spending influences the current account, relative to total consumption, by more in the more open economy. Since the share of tradables in total consumption is low, the wealth effect due to current account imbalances induces a smaller effect on the optimal labour supply than in the baseline case. Figure 4 also illustrates that the degree of openness has little influence on the equilibrium real exchange rate.

Finally, we explore the consequences of changing the consumption elasticity of money demand. This elasticity is critical for the response of the nominal exchange rate. Helliwell, Conkerline and Lafrance (1990) estimate a large number of money demand elasticises for G7 countries. The estimates of money demand elasticises suggests that $\epsilon > 1$, and thus we change to this parameter to 1.5.

Figures 4 and 5 illustrate the consequences of changing the consumption elasticity of money demand. It emerges from the Figures that this parameter has the biggest influence on the nominal exchange rate and the short-run output (and consumption) of nontraded goods. In this case, the nominal exchange rate depreciates if $\theta > 1.5$. In general, the nominal exchange rate always depreciates in the short run if $\epsilon > 0$, and appreciates if $\epsilon < 0$. Due to the smaller rise in the output of nontraded goods aggregate consumption is lower than in the baseline case. This has its own effect on money demand; however, the main reason for the different exchange rate response to a rise in
government spending is the altered consumption elasticity of money demand. The higher $\varepsilon$ induces higher demand for real money balances, and the nominal exchange has to change accordingly to balance money demand and supply. The change in the nominal exchange rate leads up to intratemporal consumption substitution. Anyway, the main reason for the lower rise in output is the increased demand for real money balances which thus decreases consumption of nontraded goods.

Figures 4 and 5 also illustrate that the change in the consumption elasticity of money demand has only a modest effect on the current account and an infinitesimal effect on steady-state output and the real equilibrium exchange rate. Due to the small change in tradables consumption the current account changes only by little relative to the baseline case. Thus, the change in wealth causes only an infinitesimal effect on the optimal labour supply in the steady state. Although the consumption elasticity of money demand affects the nominal exchange rate in the steady state it has only an infinitesimal effect on the real equilibrium exchange rate. In addition, monetary equilibrium requires overshooting of the nominal exchange rate. Generally, overshooting takes place if $\varepsilon > 1$, which is the same overshooting condition as in the small-country monetary policy model by Obstfeld-Rogoff (1995, Appendix; 1996, 689-694).

As the analysis above shows, temporary and permanent changes in government spending have different effects in the short run. In the baseline case ($\varepsilon = 1$), a permanent rise in government spending raises output more than a temporary one (unless $\theta = 1$), where as in the case of $\varepsilon = 1.5$ the opposite result is more likely. In closed economy models, Hall (1980) argue that temporary changes in government spending have larger effects than permanent ones, as against e.g. Aiyagari, Christiano and Eichenbaum (1992) and Baxter and King (1993) find the opposite result. In the above-mentioned models, the main reason behind the result that the effects of permanent changes have larger effects is that they cause a larger increase in investment in the short run. In this model, the optimal consumption response alone explains why permanent changes might have bigger effects that temporary ones.
It is also worth observing that the findings on the output effects of a rise in government spending are rather consistent with the range of multipliers obtained using a variety of macroeconometric models. Hemming, Kell and Mahfouz (2002) survey the empirical literature on the effectiveness of fiscal policy. They conclude that “[t]he range of estimated short-run multipliers is wide, (…), but most expenditure multipliers are in the range 0.6 to 1.4.” The surveyed results also support the view that long-run multipliers are smaller that short-run multipliers.

5 Summary and Concluding Remarks

This thesis presents two models to study the macroeconomic effects of permanent balanced-budget fiscal expansion in a small open economy under a flexible exchange rate regime. This thesis illustrates that the macroeconomic effects of fiscal policy are sensitive to the exact specification of preferences. The first model attempts to fill in the gap in the literature by analyzing the effects of fiscal expansion in a small open economy in a framework where fiscal policy shocks can have a direct crowding-out effect on private consumption. It demonstrates how the macroeconomic effects of fiscal policy largely depend on the marginal rate of substitution between private and government spending. The second model shows how the effects depend on the elasticity of substitution between traded and nontraded goods consumption.

In the first model, fiscal expansion raises the output of nontraded goods and crowds out private nontraded goods consumption. The magnitudes of the effects depend on the time horizon and the marginal rate of substitution between private and government spending. The rise in output is bigger in the short run, when prices are sticky and output demand-determined, whereas the crowding-out effect is bigger in the long run. The higher the substitutability between private and government consumption, (i) the bigger is the crowding out effect on private consumption (ii) and the smaller is the positive effect on output. These results are consistent with Ganelli (2003), who showed that the introduction of useful government spending tends to reduce consumption and
output relative to the pure waste case. The detailed utility-based welfare analysis shows that fiscal expansion does not induce a rise in domestic welfare, even if government spending directly affects private utility. This result is in contrast with Ganelli (2003), who concluded that the introduction of utility enhancing government spending reverses the beggar-thyself welfare result on the specific parameter values.

The second model of this thesis demonstrates how the effects of fiscal policy depend on the substitutability between traded and nontraded goods. One advantage of the fully dynamic model is that it allows fiscal policy to induce tilts into the time profile of output and relative prices. The model brings in important insights into the effects of fiscal policy in small open economies under flexible exchange rates. This study reveals that, under a specific parameterization, a one percent permanent rise in government spending increases the output of nontraded goods by at least one percent in the short run. Moreover, it is shown that fiscal expansion increases output by more than one percent also in the situation where the nominal exchange rate appreciates. Notwithstanding the appreciation of the nominal exchange rate, output rises because a change in relative prices induces consumers to choose a profile of consumption that is tilted towards the present. Open economy models in which a rise in government spending does not lead to any tilting into the time profile of relative prices and the consumption-based real interest rate do not allow for this effect. It is also shown that permanent fiscal expansion is the most expansionary in the case where the long-run real exchange rate appreciates the most. In addition, it is demonstrated that the sign of the current account response to permanent fiscal expansion depends on the interplay between the intratemporal elasticity of aggregate consumption and the intratemporal elasticity of substitution between traded and nontraded goods.

Finally, it is interesting to perceive that the short-run effects of a rise in government spending differ largely depending on whether a rise government spending is permanent or temporary. The finding, that a temporary rise in government spending affects nothing other than the short-run output of nontraded goods, can be interpreted as providing favourable evidence for the effectiveness of fiscal policy as a stabilization
tool. Temporary changes in government spending are an effective stabilization tool when used wisely and timely in response to undesired fluctuations in domestic output. Undesired fluctuations in output can be perfectly offset by an opposite change in government spending without causing any side-effects.
References


Appendix A. Solving for the First-Order Conditions

Equation (3) can be manipulated to yield

\[
(p_N(z))^{-\theta} = P_N^{-\theta} \frac{y_N(z)}{C_N^A + G_N^A} \Rightarrow p_N(z) = y_N(z)^{-\theta} \left( C_N^A + G_N^A \right)^{-\theta} P_N.
\]

This equation is substituted into the budget constraint (5), and then one can write the appropriate Lagrangean as (the indexed denoting the agents are dropped)

\[
L = \sum_{s=1}^{\infty} \beta^{s-t} \left[ \gamma \log C_{T,s} + (1 - \gamma) \log (C_{N,s} + \alpha G_{N,s}) + \chi \log \left( \frac{M_s}{P_s} \right) - \frac{\kappa}{2} y_{N,s}^2 \right] - \sum_{s=1}^{\infty} \lambda_s \left( P_{T,s} B_s + M_s - P_{T,s} (1 + r) B_{s-1} + M_{s-1} + P_{N,s} y_{N,s}^{\theta-1} \left( C_{N,s}^A + G_{N,s}^A \right)^{\frac{1}{\theta}} \right) - P_{T,s} C_{T,s} - P_{N,s} C_{N,s} - P_{T,s} C_{T,s}.
\]

\[
\frac{\partial L}{\partial C_{T,s}} = 0 \Rightarrow \gamma \frac{1}{C_{T,s}^A} - \lambda_s P_{T,s} = 0 \Rightarrow \lambda_s = \frac{\gamma}{C_{T,s} P_{T,s}} \quad (A1)
\]

\[
\frac{\partial L}{\partial C_{N,s}} = 0 \Rightarrow (1 - \gamma) \frac{1}{C_{N,s}^A + \alpha G_{N,s}} - \lambda_s P_{N,s} = 0 \Rightarrow \lambda_s = \frac{1 - \gamma}{(C_{N,s}^A + \alpha G_{N,s}) P_{N,s}} \quad (A2)
\]

\[
\frac{\partial L}{\partial y_{N,s}} = 0 \Rightarrow -\kappa y_{N,s} + \lambda_s \left( \frac{\theta - 1}{\theta} \right) y_{N,s}^{\theta-1} \left( C_{N,s}^A + G_{N,s}^A \right)^{\frac{1}{\theta}} P_{N,s} = 0 \quad (A3)
\]

\[
\frac{\partial L}{\partial B_t} = 0 \Rightarrow -\lambda_s P_{T,s} + \beta \lambda_s P_{T,s+1} (1 + r) = 0 \quad (A4)
\]

\[
\frac{\partial L}{\partial M} = 0 \Rightarrow \chi \frac{1}{P_s} P_t - \lambda_s + \beta \lambda_{s+1} = 0 \quad (A5)
\]

Substituting (A1) into (A4) yields

\[
(6) \quad \beta (1 + r) C_{T,s} = C_{T,s+1}.
\]

Combining (A2) and (A1) yields

\[
\frac{1 - \gamma}{(C_{N,s}^A + \alpha G_{N,s}) P_{N,s}} = \frac{\gamma}{C_{T,s} P_{T,s}} \Rightarrow \left( \frac{1 - \gamma}{\gamma} \right) P_{T,s} C_{T,s} = C_{N,s}^A + \alpha G_{N,s} \quad (7)
\]

And because

\[
\frac{\partial U / \partial C_T}{\partial U / \partial (C_N + \alpha G_N)} = \frac{P_T}{P_N}, \text{ the ratio of the marginal utilities of tradables and nontraded goods equals the relative price ratio.}
\]
Substituting (A2) into (A3) yields
\[ \kappa_{N,t} = \frac{1-\gamma}{(C_{N,t} + \alpha G_{N,t})} \left( \frac{\theta - 1}{\theta} \right)^{-\frac{1}{\theta}} (C_{N,t}^A + G_{N,t}^A) P_{N,t} \]  
and
\[ \theta_{N,t}^{\frac{\theta+1}{\theta}} = \left( 1 - \gamma \right) \left( \frac{\theta - 1}{\theta} \right)^{-\frac{1}{\theta}} (C_{N,t}^A + G_{N,t}^A) \left( C_{N,t} + \alpha G_{N,t} \right)^{-1}. \]

Equation (A5) can be written by using (A1) and multiplying by \( P_T \) to yield
\[ \frac{\gamma}{C_{T,t}} = \chi \left( \frac{M}{P_t} \right)^{-\frac{1}{\gamma}} \left( \frac{P_{T,t}}{P_t} \right) + \beta \left( \frac{P_{T,t}}{P_{T,t+1}} \right) \frac{\gamma}{C_{T,t+1}}. \]

We can interpret this to be:
\[ \frac{\partial U}{\partial M} = \frac{\partial U}{\partial C_{T,t}} \frac{1}{C_{T,t}} + \beta \frac{\partial U}{\partial C_{T,t+1}} \frac{1}{P_{T,t+1}} = 0 \quad \Rightarrow \quad P_{T,t} \]

(A6) \[ \frac{\partial U}{\partial C_T} = \frac{\partial U}{\partial (M/P)} \frac{P_{T,t}}{P} + \beta \frac{\partial U}{\partial C_{T,t+1}} \frac{P_{T,t}}{P_{T,t+1}}, \]
where first term on the right-hand side is utility from holding money, second term is utility from using today's money for tomorrow's consumption and the term on the left-hand side is utility from spending money. From equation (A4) one can get \( \frac{\lambda_i P_{T,t}}{P_{T,t+1}(1+r)} = \beta \lambda_{t+1}, \) plugging this into (A5) yields
\[ \chi \frac{1}{M_{it}} = \lambda_i \left( 1 - \frac{P_{T,t}}{P_{T,t+1}(1+r)} \right) = 0. \]
And by using the Fisher parity
\[ 1 + i = (1 + r) \frac{P_{T,t+1}}{P_{T,t}}, \] and equation (A2) one can get
\[ \chi \frac{1}{M_{it}} = \frac{1-\gamma}{P_{N,t} \left( C_{N,t} + \alpha G_{N,t} \right)} \left( \frac{i}{1+i} \right). \] Multiplying this by \( \frac{P_t}{\chi} \) yields
\[ \frac{P_t}{M_{it}} = \frac{(1-\gamma) \left( \frac{i}{1+i} \right) C_{N,t} + \alpha G_{N,t}}{P_{N,t}} \]

(9) \[ \frac{M_{it}}{P_t} = \frac{\chi \left( 1+i \right) C_{N,t} + \alpha G_{N,t}}{(1-\gamma) \left( \frac{i}{i} \right) C_{N,t} + \alpha G_{N,t}}. \]
Appendix B. Deriving the Welfare Effects of Fiscal Expansion

Log-linearizing the utility function (1) one can get (ignoring real balances)

\[ \Delta U_T^R = \gamma \hat{C}_T + (1 - \gamma) \left( \hat{C}_N + \alpha \hat{\kappa} \right) - \kappa \bar{y}_{N,0} \hat{y} + \beta \left[ \gamma \hat{C}_T + (1 - \gamma) \left( \hat{C}_N + \alpha \hat{\kappa} \right) - \kappa \bar{y}_{N,0} \hat{y} \right] + \beta^2 \left[ \gamma \hat{C}_T + (1 - \gamma) \left( \hat{C}_N + \alpha \hat{\kappa} \right) - \kappa \bar{y}_{N,0} \hat{y} \right] + \ldots \]

Note that \( \sum_{i=1}^{\infty} \beta^i = \frac{\beta}{1 - \beta} \) and \( \kappa \bar{y}_{N,0} \hat{y} \Rightarrow \kappa \bar{y}_{N,0} \hat{y} \). Then the differentiated utility function can be written as

\[ \Delta U_T^R = \gamma \hat{C}_T + (1 - \gamma) \left( \hat{C}_N + \alpha \hat{\kappa} \right) - \kappa \bar{y}_{N,0} \hat{y} + \frac{\beta}{1 - \beta} \left[ \gamma \hat{C}_T + (1 - \gamma) \left( \hat{C}_N + \alpha \hat{\kappa} \right) - \kappa \bar{y}_{N,0} \hat{y} \right] \]

Dividing this equation by \( \gamma \) and substituting into this equations (9), (11), (12), (13) and (14) one can get

\[ \Delta U_T^R = \left( \frac{1 - \gamma}{\gamma} \right) \left( -\alpha \hat{G}_N + \alpha \hat{\kappa} \right) - \kappa \left[ \frac{(1 - \gamma)(\theta - 1)}{\gamma \theta \kappa} \right] \left( 1 - \alpha \right) \hat{G}_N \]

\[ + \frac{\beta}{1 - \beta} \left[ \left( \frac{1 - \gamma}{\gamma} \right) \left( -\alpha \hat{G}_N + \alpha \hat{\kappa} \right) - \kappa \left[ \frac{(1 - \gamma)(\theta - 1)}{\gamma \theta \kappa} \right] \left( 1 - \alpha \right) \hat{G}_N \right] \Rightarrow \]

\[ \Delta U_T^R = -(1 - \alpha) \left( \frac{(1 - \gamma)(\theta - 1)}{\gamma \theta} \right) \hat{G}_N + \frac{\beta}{1 - \beta} \left[ \left( \frac{(1 - \gamma)(\alpha - 1)}{2 \gamma} \right) \left( 1 - \alpha \right) \hat{G}_N \right] \leq 0 \]
Appendix C. Solving for the Optimality Conditions

To solve the first-order conditions for the representative agent, we use (23) to eliminate \( p_{N,t}(z) \) from (24), and then maximize the utility function (17) subject to the resulting budget constraint. From equation (23) we can get

\[
p_{N}(z) = y_{N}(z) \left( \frac{1}{\sigma} \right)^{\mu} P_{N} \left( C_{N}^{A} + G_{N}^{A} \right).
\]

By substituting this equation into the budget constraint (24), we can thus write the Lagrangean as (the indexed denoting agents are dropped):

\[
L = \sum_{s=1}^{\infty} \beta^{r-t} \left[ \left( \frac{\sigma}{\sigma - 1} C_{s}^{\sigma - 1} + \frac{\chi}{1 - \epsilon} \left( \frac{M_{s}}{P_{t}} \right)^{\epsilon} \right) - \frac{\kappa}{2} y_{N,t}(z)^{2} \right] - \sum_{s=1}^{\infty} \beta^{r-t} \lambda_{t}[P_{T,t}(1+r)B_{t-1} + M_{t-1} + p_{N,t}(z) y_{N,t}(z) + P_{T,t} \bar{y}_{T,t} - P_{t} C_{t} - P_{N,t} \tau_{t} - P_{T,t} B_{t} - M_{t}]
\]

\[
\frac{\partial L}{\partial C_{t}} = 0 \Rightarrow C_{t}^{\frac{1}{\sigma}} = \lambda_{t} P_{t} \Rightarrow \lambda_{t} = \frac{C_{t}^{\frac{1}{\sigma}}}{P_{t}} \quad \text{(C1)}
\]

\[
\frac{\partial L}{\partial y_{N,t}} = 0 \Rightarrow -\kappa y_{N,t} + \lambda_{t} \left( \frac{\mu - 1}{\mu} \right) y_{N,t}^{\frac{1}{\mu}} \left( C_{N}^{A} + G_{N}^{A} \right)^{\frac{1}{\mu}} = 0 \quad \text{(C2)}
\]

\[
\frac{\partial L}{\partial B_{t}} = 0 \Rightarrow -\lambda_{t} P_{T,t} + \beta \lambda_{t+1} P_{T,t+1} (1+r) = 0 \quad \text{(C3)}
\]

\[
\frac{\partial L}{\partial M} = 0 \Rightarrow \chi \left( \frac{M_{t}}{P_{t}} \right)^{\epsilon} \frac{1}{P_{t}} - \lambda_{t} + \beta \lambda_{t+1} = 0 \quad \text{(C4)}
\]

Substituting (C1) into (C2) yields to

\[
\kappa y_{N,t} = C_{t}^{\frac{1}{\sigma}} \frac{P_{N,t}}{P_{t}} \left( \frac{\mu - 1}{\mu} \right) y_{N,t}^{\frac{1}{\mu}} \left( C_{N}^{A} + G_{N}^{A} \right)^{\frac{1}{\mu}} \Rightarrow
\]

\[
\kappa y_{N,t}^{\frac{\mu}{\mu + 1}} = C_{t}^{\frac{1}{\sigma}} \frac{P_{N,t}}{P_{t}} \left( \frac{\mu - 1}{\mu} \right) C_{N}^{A} \left( C_{N}^{A} + G_{N}^{A} \right)^{\frac{1}{\mu}} \quad \text{(27)}
\]

Equation (C3) can be written as \( \lambda_{t} P_{T,t} = \beta \lambda_{t+1} P_{T,t+1} (1+r) \), which then implies

\[
\frac{\lambda_{t}}{\lambda_{t+1}} = \frac{P_{T,t+1}}{P_{T,t}} \Rightarrow \frac{C_{t}^{\frac{1}{\mu}}}{C_{t+1}^{\frac{1}{\mu}}} \frac{P_{T,t+1}}{P_{T,t}} = \frac{P_{t}}{P_{t+1}} \Rightarrow \frac{\left( \frac{C_{t}}{C_{t+1}} \right)^{\frac{1}{\sigma}}}{\left( \frac{P_{t}}{P_{t+1}} \right)^{\frac{1}{\sigma}}} = \left( \frac{P_{t}}{P_{t+1}} \right)^{\frac{1}{\sigma}} \Rightarrow
\]
\[
\left( \frac{C_i}{C_{i+1}} \right) = \left( \frac{P_i / P_{T,i}}{P_{i+1} / P_{T,i+1}} \right)^{-\sigma}
\]

Equation (20) implies that
\[
\frac{C_{T,i}}{C_{T,i+1}} = \frac{\gamma(P_{T,i}/P_i)}{\gamma(P_{T,i+1}/P_{i+1})} \frac{C_i}{C_{i+1}}
\]
Substituting this into the preceding equation yields
\[
\frac{C_{T,i}}{C_{T,i+1}} = \left( \frac{P_i / P_{T,i}}{P_{i+1} / P_{T,i+1}} \right)^{\sigma} \Rightarrow \frac{C_{T,i}}{C_{T,i+1}} = \left( \frac{P_i / P_{T,i}}{P_{i+1} / P_{T,i+1}} \right)^{\sigma-\sigma} \quad (25)
\]

Equation (C3) can be written as
\[
\frac{\lambda_i P_{T,i}}{P_{T,i+1}(1+r)} = \lambda_{i+1} P_{T,i+1} \cdot \text{Substituting this into (C4) yields}
\]
\[
\chi \left( \frac{M_i}{P_i} \right)^{1/r} = \lambda_i \left( 1 - \frac{P_i}{P_{T,i+1}(1+r)} \right). \text{Making use of the Fisher parity}
\]
\[
1 + i = (1 + r) \frac{P_{T,i}}{P_{T,i+1}}, \quad \text{we write this equation in form}
\]
\[
\chi \left( \frac{M_i}{P_i} \right)^{1/r} = \frac{1}{P_i} \Rightarrow \frac{P_i}{\chi} \Rightarrow \left( \frac{M_i}{P_i} \right)^{-r} = \left( \frac{i}{1+i} \right) \Rightarrow \left( \frac{1+i}{i} \right)^{1/r}
\]
\[
\frac{M_i}{P_i} = \left[ \chi C_i^{1/r} \left( \frac{1+i}{i} \right) \right]^{1/r} \quad (28)
\]
Figure 1. The effects of a permanent rise in government spending. The horizontal axis marks the elasticity of substitution between traded and nontraded goods and the vertical axis marks the variable’s percentage deviation from the initial steady state.
Figure 2. The effects of a permanent rise in government spending
Figure 3. Sensitivity analysis, the role of openness

The solid line $\gamma = 0.5$, the dashed line $\gamma = 0.2$
Figure 4. Sensitivity analysis, the role of money demand

The solid line $\varepsilon = 1$, the dashed line $\varepsilon = 1.5$
Figure 5. Sensitivity analysis, the role of money demand

The solid line $\varepsilon = 1$, the dashed line $\varepsilon = 1.5$