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A Centralized or a Decentralized Labor Market?

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Abstract

I study a dynamic labor market with homogenous firms and workers. Both types of agents choose between a centralized market and a decentralized search market. Firms have free entry and exit. I consider how bargaining and wage posting in the two types of market affect the equilibrium outcome. For example, if there is bargaining in the centralized market and wage posting in the search market, there exists a centralized market equilibrium, a decentralized market equilibrium, and a mixed market equilibrium where there are agents in both submarkets. If wages are posted in both markets, a search market equilibrium does not exist.

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1 Introduction

A central assumption in labor market search models is that a centralized market does not exist (e.g. Pissarides, 2000). In a centralized market without search frictions it is easy to meet partners, but there is a lot of competition by agents on the same side of the market. Job fairs, academic job markets, and markets governed by employment agencies share features of a centralized market. In a decentralized market job seekers typically can visit one firm at the time. Due to uncoordinated search some employers receive many applicants while some of them receive none. Agents on both sides of the market enjoy some monopoly power because one's partner cannot switch to another partner immediately. Markets where firms place job ads, and workers send applications and travel to interviews, have many features of a decentralized market.

In this paper I ask the following question: Suppose that a centralized market place without search frictions exists, and vacancies and job seekers can choose either it or a decentralized search market. Which market do they choose in equilibrium? I use a discrete-time infinite horizon model where there is a fixed number of homogenous workers. The firms are also homogenous, and their number is determined by free entry and exit. The search market is modeled as an urn-ball process where firms represent urns and workers represent balls, and wages are determined either by Nash bargaining solution or by public posting. In the centralized market all agents on the short side are assumed to be matched, while the agents on the long side are rationed so that each agent has an equal probability of being matched. Wages are determined by Nash bargaining or by public posting.

The central results are (i) If wages are determined by bargaining in both markets, a centralized market equilibrium exists if a firm's share of match surplus is large enough compared to a firm's capital cost. There is also a mixed market equilibrium where a centralized market and a search market coexist. This happens only for a specific value of Nash bargaining parameter, given firms' capital cost. A decentralized market equilibrium does not exist. At very low values of firm's surplus share there is no equilibrium with firms in the economy. (ii) If wages are determined by bargaining in the centralized market but by public posting in the search market, a decentralized market equilibrium exists if a firm's surplus share in the centralized market is either relatively large or small compared to capital cost. For intermediate values of a firm's surplus share a centralized market equilibrium exists. A mixed market equilibrium exists if a firm's surplus share and its capital cost satisfy a specific relation. (iii) If wages are posted in both markets, a decentralized market equilibrium does not exist. A centralized market equilibrium exists, as well as two continua of mixed market equilibrium.

It is known that a search model where wages are determined by Nash bargaining results in a hold-up problem, in addition to coordination problem. As some of the match surplus goes to workers, a fixed bargaining parameter does not give firms a correct incentive to enter the economy, and as a result there are too few firms. But if firms post wages, the hold-up problem disappears. In a centralized market there may be too few or too many firms, depending on the relative magnitude of firms' bargaining power and their capital cost. This makes the Pareto ranking of centralized and decentralized market equilibrium somewhat complicated.

The topic relates closely to clustered markets where, within a cluster, buyers can inspect sellers at a very small cost, and sellers design pricing strategies while facing competition by other sellers. Outside the cluster buyers have higher search costs, and sellers have some degree of monopoly power. The focus is on how search costs and heterogeneity affect buyers' and sellers' choice between the markets. In Fisher and Harrington (1996) there is an endogenous number of sellers who each produce an indivisible piece of goods. They choose either the cluster or periphery, that is, a search market. Sellers set prices in both markets. Then buyers choose an initial location. The cost of entering the cluster is a draw from a probability distribution. Once in the cluster a buyer can sample all sellers at no cost. In periphery buyers sample one firm per period at the same cost per visit. When meeting a seller the type of goods (which is also the buyer's willingness to pay) is drawn from a uniform distribution. Buyers search with recall in both markets, and they can switch between the markets. The cluster survives only if goods are heterogeneous enough. In Neeman and Vulkan (2010) goods are homogenous, but production costs and buyers' willingness to pay are stochastic. Agents cannot switch to another market within a period. In the cluster a Walrasian market-clearing price prevails while agents engage in direct negotiations in the search market. In equilibrium only the cluster exists. Kultti's model (2011) has a fixed number of homogenous sellers and a stochastic number of homogenous buyers. Buyers and sellers choose either a search market or a cluster. Sellers post prices publicly before the number of buyers is realized. It is assumed that in the cluster all agents on the short side are matched. If sellers can choose the market and price, only the cluster survives. Wolinsky (1983) assumes differentiated goods and monopolistic competition. Consumers are imperfectly informed about the characteristics of the goods, and they use a stopping rule. The equilibrium features only a cluster. Miao (2006) considers a model where heterogenous buyers and homogenous sellers choose between a centralized market where market makers publicly post bid and ask prices, and a decentralized market where the terms of trade are determined by Nash bargaining. Opening a centralized market does not necessarily improve social welfare since trading in there is assumed to have a cost, and it makes the decentralized market tighter which makes buyers there worse off.

To my knowledge, only one model (Fisher and Harrington, 1996) results in coexistence of a centralized and a decentralized market. This happens only if the goods are heterogeneous enough and if buyers' knowledge on goods' characteristics and prices is imperfect. In other models, only a cluster exists. There is obviously a need for a model which can explain the co-existence of a centralized and a decentralized market even if the goods (or jobs) are homogenous and/or if buyers (or workers) have perfect information about prices (wages).

The rest of the paper is organized as follows: Section 2 presents some basic ingredients of the models. Section 3 considers a model where wages are determined by bargaining in both markets, and in Section 4 I analyze a model where wages are determined by posting in the search market. In Section 5 I consider a model where wages are posted in both markets. Section 6 considers a static model where the unemployment-vacancy ratio is fixed. Section 7 concludes.

2 Agents and Timing

Time is discrete and extends to infinity. Each agent discounts future at a common factor $\delta \in [0,1]$ per period. There number of workers is L, and we assume that L is large. The number of firms is endogenous and determined by entry and exit. Staying in the economy costs $k \in (0,1)$ for a firm each period whether it is producing or not. Labor is indivisible, and each worker supplies one unit of labor each period, and each firm wishes to employ one worker per period. Each matched firm-worker pair produces a unit output per period. Each pair breaks down with probability $b \in [0,1]$ after a production stage. The separated agents start searching for a new partner, and the others continue producing.

There are two markets where firms and workers can match. The decentralized market is a directed search market (SM) of an urn-ball type where vacancies represent the urns and unemployed represent the balls. Wages are determined by public posting or by Nash bargaining. This market has search friction: some agents on both sides remain unmatched. The centralized market (CM) is like a market place or a monopolistic intermediary market which is assumed to have no search friction: all agents of the smaller population are matched. Wages are determined by Nash bargaining or public posting. Agents choose which market they enter.

In an SM equilibrium all agents are in the search market, and in a CM equilibrium all agents are in the centralized market. In a mixed market equilibrium (MM equilibrium) there are firms and workers in both markets. The CM equilibrium and SM equilibrium are checked against a one-period coalitional deviation to the other market. A market is an equilibrium if there is no coalition where all its members fare at least as well by choosing another market.

Each period consists of a production stage and a matching stage. In the beginning of a matching stage there are u unemployed workers and v vacant firms. Denote $u/v \equiv \theta$. The timing of moves in a matching stage is as follows: 1) Firms choose whether to enter, exit or stay in the economy. Firms which do not exit pay capital cost $k \in (0,1)$. 2) Fraction $\tau \in [0,1]$ of firms locates itself in SM, and fraction $1-\tau$ locates itself in CM. Simultaneously, fraction $\omega \in [0,1]$ of workers go to SM, and fraction $1-\omega$ go to CM. 3) If firms in SM post wages, they do it, knowing the values of u, v, τ and ω.The wages are observed by all agents. 4) In SM, workers choose firms on the basis of observed wages, or at random if bargaining is used. In either case if a firm receives more than one applicant, it hires one at random. At the same, matching takes place in CM such that all agents on the short side form a pair with a random agent on the long side, and agents on the long side are rationed such that each agent has the same probability of being matched. 5) In CM, and in SM if bargaining is applied, a firm receives share $\alpha \in (0,1)$ of the match surplus. 8) All matched pairs start producing, and the unmatched agents wait for the next matching stage.

3 Bargaining in Both Markets

This section considers a model where wages are determined by bargaining in both markets. We apply a standard Nash bargaining model where an agent receives his reservation value plus a share of the match surplus such that the firm's share is α and the worker's share is $1-\alpha$, where α is a parameter. This is equivalent of a procedure where the firm makes a take-it-or-leave-it offer with an exogenous probability $\alpha \in [0,1]$ and the worker makes a take-it-or-leave-it offer with probability $1-\alpha$.

The model can in principle exhibit three kind of equilibrium: In CM equilibrium all agents choose the centralized market, and in SM-equilibrium all agents choose the decentralized search market. In MM equilibrium (MM stands for mixed market) there are agents in both markets. Let V_c and V_s denote a vacancy's expected value in CM and SM, respectively, and let U_c and U_s denote an unemployed worker's expected values. An MM equilibrium exists only if both type of agents are indifferent between the two markets. That is, the expected value of going to CM must be equal to the expected value of going to SM. Then $U_c = U_s \geq 0$, and $V_c = V_s = 0$ by free entry and exit. Which equilibrium prevails depends on parameters α, k, δ and b. I will focus on the relation between α and k, keeping δ and b fixed.

The existence of CM equilibrium and SM equilibrium will be tested by using a one-period coalitional deviation. It is an application of a Nash equilibrium where a deviation by a single agent is replaced by a deviation by a coalition of vacancies and unemployed. This is because a deviation of a single agent into the other market is futile since a match is formed by two agents of different types.

The value functions for matched agents are

$$\begin{aligned} W_c &= w_c + \delta\left((1-b)W_c + bU_c\right), && (1)\\ W_s &= w_s + \delta\left((1-b)W_s + bU_s\right), && (2)\\ J_c &= 1-k-w_c + \delta\left((1-b)J_c + bV_c\right), && (3)\\ J_s &= 1-k-w_s + \delta\left((1-b)J_s + bV_s\right). && (4)\end{aligned}$$

On the first line, W_c is the value for a worker who is just hired in the centralized market. He earns wage w_c for one period. In the beginning of the second period he continues working with probability $1-b$. He becomes unemployed with probability b, goes to back the centralized market and has value U_c. The second line is the value function of a worker who is just hired in the search market. After becoming unemployed he goes to the search market and has value U_s. The third and fourth line depict value functions of a firm which just hired a worker. The firm receives unit output minus capital cost k minus wage w_i, $i = c, s$, depending of which market it hired the worker. The firm continues producing with probability $1-b$. Production ends with probability b, and the firm returns to the centralized market or to the search market and has value V_c or V_s.

First we study the existence of an MM equilibrium and a CM equilibrium if $\alpha < k$. Then we study those equilibria if $\alpha \geq k$. The relative magnitude of α and k determines whether firms or workers form the larger population in the centralized market. Finally we study the existence of SM equilibrium.

Setting $V_i = 0$ gives an often needed expression for a match value: $J_i + k + W_i = \dfrac{1-\delta(1-b)k + \delta b U_i}{1-\delta(1-b)}$, where $i = c, s$. When considering a deviating coalition in analyzing CM equilibrium, we use $J_s^d + k + W_s^d = \dfrac{1-\delta(1-b)k + \delta b U_c}{1-\delta(1-b)}$. On the right-hand side there is U_c because we consider a one-period deviation. When considering an SM equilibrium we have $J_c^d + k + W_c^d = \frac{1-\delta(1-b)k+\delta b U_s}{1-\delta(1-b)}$ for deviators.

3.1 Mixed Market Equilibrium and CM Equilibrium if $\alpha < k$

In MM equilibrium, fraction $\tau \in (0,1)$ of firms and fraction $\omega \in (0,1)$ of workers choose SM, and fractions $1-\tau$ and $1-\omega$ choose CM. Then the ratio of unemployed and vacancies in CM is $\frac{(1-\omega)u}{(1-\tau)v} = \frac{(1-\omega)\theta}{1-\tau} \equiv \phi$. If $\phi > 1$, a worker is matched with probability $1/\phi$, and a firm is matched with probability one. If $\phi \leq 1$, a worker is matched with probability one, and a firm is matched with probability ϕ. The proof below shows that $\phi > 1$ only if $\alpha < k$. In SM, workers choose firms at random with equal probability. Then the number of workers who arrive at a given vacancy is binomially distributed. We assume that u and v are large, and we use a standard method that the binomial distribution is approximated by Poisson distribution, as if $u \to \infty$ and $v \to \infty$. This simplifies the analysis greatly. Let σ be the Poisson term in SM, then $\sigma = \frac{\omega\theta}{\tau}$ where $\theta \equiv \frac{u}{v}$.

Consider first a mixed market where $\phi > 1$. The value functions for unmatched agents are

$$\begin{aligned}
U_c &= \frac{1}{\phi}\left(\alpha\delta U_c + (1-\alpha)(J_c + k + W_c - \delta V_c)\right) + \left(1 - \frac{1}{\phi}\right)\delta U_c, &(5)\\
U_s &= \frac{1-e^{-\sigma}}{\sigma}\left(\alpha\delta U_s + (1-\alpha)(J_s + k + W_s - \delta V_s)\right) + \left(1 - \frac{1-e^{-\sigma}}{\sigma}\right)\delta U_s, &(6)\\
V_c &= -k + \alpha(J_c + k + W_c - \delta U_c) + (1-\alpha)\delta V_c, &(7)\\
V_s &= -k + \left(1-e^{-\sigma}\right)\left(\alpha(J_s + k + W_s - \delta U_s) + (1-\alpha)\delta V_s\right) + e^{-\sigma}\delta V_s. &(8)
\end{aligned}$$

In (5), an unemployed worker is in CM where he meets a firm with probability $1/\phi$. With probability α the firm receives all the match surplus, leaving the worker his continuation value δU_c. With probability $1-\alpha$ the worker gets all the match value $J_c + k + W_c$ minus firm's continuation value δV_c. Notice that we include k in the match value in order to not count it twice, because the firm already paid it before they matched. That is, k is a sunk cost. With probability $1 - 1/\phi$ a worker remains unemployed and continues in CM. Notice also that $\alpha\delta U_c + (1-\alpha)(J_c + k + W_c - \delta V_c) = \delta U_c + (1-\alpha)(J_c + k + W_c - \delta V_c - \delta U_c)$. That is, a worker gets his continuation value plus share $1-\alpha$ of match surplus.

In (6) a worker is in SM where he meets a firm, and he is chosen by it with probability $(1-e^{-\sigma})/\sigma$. The firm and worker divide the surplus in shares α and $1-\alpha$. With probability $1-$ $(1-e^{-\sigma})/\sigma$ the worker is not recruited and he goes back to search market and gets continuation value δU_s. In (7) a firm in CM pays capital cost k and meets a worker. They divide the surplus in shares α and $1-\alpha$. In (8) a firm in SM pays capital cost k and meets a worker with probability $1-e^{-\sigma}$, and they share the surplus. With probability $e^{-\sigma}$ a firm does not get any applicants, and it goes to SM in the next period.

An MM equilibrium exists if firms and workers are indifferent between the markets, firms have zero values, workers have positive values, and the Poisson term in SM is positive. That is, an MM equilibrium exists if $U_c = U_s \geq 0$, $V_c = V_s = 0$, and $\sigma > 0$.

Proposition 1 *(i) If $\alpha < k$, an MM equilibrium exists if and only if $\alpha = \frac{(1-\delta(1-b))k}{1-\delta(1-b)k}$.*
(ii) In an MM equilibrium $\tau = \omega$.

Proof. (i) The value functions above give $U_c = U_s$ and $V_c = V_s$ only if $1 - e^{-\sigma} = 1$ and $\sigma = \phi(1-e^{-\sigma})$. Both the equations hold only if $\sigma \to \infty$ and $\phi \to \infty$. Equation (5) gives $U_c = \frac{1-\alpha}{1-\delta} \frac{1-\delta(1-b)k}{(1-\delta(1-b))\phi + \delta(1-b)(1-\alpha)}$ if $V_c = 0$ which holds by (7) if $U_c = \frac{\alpha - k + \delta(1-b)k(1-\alpha)}{\delta(1-\delta)(1-b)\alpha}$. Equating the solutions for U_c gives $\phi = \frac{\delta(1-b)(1-\alpha)k}{\alpha - k + \delta(1-b)k(1-\alpha)}$. Then $\phi \to \infty$ if $\alpha = \frac{(1-\delta(1-b))k}{1-\delta(1-b)k}$. Equation (6) gives $U_s = \frac{1-\alpha}{1-\delta} \frac{(1-\delta(1-b)k)(1-e^{-\sigma})}{\delta(1-b)(1-\alpha)(1-e^{-\sigma}) + \sigma(1-\delta(1-b))}$ if $V_s = 0$ which holds by (8) if $U_s = \frac{\alpha(1-\delta(1-b)k)(1-e^{-\sigma}) - (1-\delta(1-b))k}{\alpha\delta(1-\delta)(1-b)(1-e^{-\sigma})}$. Equating the solutions for U_s gives $\sigma \to \infty$ and $\alpha = \frac{(1-\delta(1-b))k}{1-\delta(1-b)k}$. Notice that $\frac{(1-\delta(1-b))k}{1-\delta(1-b)k} < k$.(ii) Equation $\frac{1}{\phi} = \frac{1-e^{-\sigma}}{\sigma}$ can be written, using $\theta = \frac{\tau\sigma}{\omega}$, as $(1-\tau)\omega = (1-\omega)\tau(1-e^{-\sigma})$ where $\sigma \to \infty$. This gives $\tau = \omega$. ■

Because $\sigma \equiv \frac{\omega\theta}{\tau}$, $\phi \equiv \frac{(1-\omega)\theta}{1-\tau}$, and in a mixed market equilibrium $\tau = \omega$, then $\sigma = \theta = \phi$, and then $\sigma \to \infty$ and $\phi \to \infty$ only if $\theta \to \infty$. Then $v \to 0$ because the upper bound of u is L. In a mixed market equilibrium $U_c = U_s = 0$.The expected utility of unemployed workers is zero because in both markets finding a partner is almost impossible. A firm's probability of finding a partner approaches one in both markets, but having $\alpha = \frac{(1-\delta(1-b))k}{1-\delta(1-b)k}$ drives a vacancy's value to zero. As $v \to 0$, then $\hat{u} \to L$ in a steady state, and the total output approaches zero. The relative size of CM and SM is indeterminate: result $\tau = \omega$ only tells that the fractions of firms and workers that choose SM are equal. We also find that if $\delta = 0$, an MM equilibrium does not exist if $\alpha < k$.

Next we study if a CM equilibrium exists if $\alpha < k$. In this case it follows, as shown in the proof below, that a firm matches in CM with probability one, and a worker matches with probability $1/\theta < 1$. The value functions for unmatched workers and firms in CM are

$$U_c = \frac{1}{\theta}(\alpha\delta U_c + (1-\alpha)(J_c + k + W_c - \delta V_c)) + \left(1 - \frac{1}{\theta}\right)\delta U_c, \tag{9}$$

$$V_c = -k + \alpha(J_c + k + W_c - \delta U_c) + (1-\alpha)\delta V_c. \tag{10}$$

In (9) a worker in CM is chosen by a firm with probability $v/u = 1/\theta$, and they share the surplus. With probability $1 - 1/\theta$ he is not chosen and goes back to CM in the beginning of the next period and has thus continuation value δU_c. In (10) a firm pays k and receives applicants. He chooses one at random and they share the surplus.

Proposition 2 *If $\alpha < k$, a CM equilibrium where $\theta \nrightarrow \infty$ exists if and only if* $\alpha > \dfrac{(1-\delta(1-b))k}{1-\delta(1-b)k}$.

Proof. Setting $V_c = 0$ equation (9) gives $U_c = \dfrac{1-\alpha}{1-\delta}\dfrac{1-\delta(1-b)k}{(1-\delta(1-b))\theta+\delta(1-b)(1-\alpha)}$, and equation (10) gives $U_c = \dfrac{\alpha-k+\delta(1-b)(1-\alpha)k}{\delta(1-\delta)(1-b)\alpha}$. Equating the solutions gives $\theta = \dfrac{\delta(1-b)(1-\alpha)k}{\alpha-k+\delta(1-b)(1-\alpha)k} > 1$ if $\alpha < k$, and $\theta \nrightarrow \infty$ if $\dfrac{(1-\delta(1-b))k}{1-\delta(1-b)k} < \alpha < k$. Suppose a coalition of workers and firms deviate for one period to SM so that the Poisson term in SM is $\tilde{\sigma}$. The value function for a deviating firm is, following (8) above, $V_s^d = -k + \left(1-e^{-\tilde{\sigma}}\right)\left(\alpha\left(J_s^d + k + W_s^d - \delta U_c\right) + (1-\alpha)\delta V_c\right) + e^{-\tilde{\sigma}}\delta V_c$. Setting $V_c = 0$ and using $J_s^d + k + W_s^d = \dfrac{1-\delta(1-b)k+\delta b U_c}{1-\delta(1-b)}$ we have

$V_s^d = -k + \left(1-e^{-\tilde{\sigma}}\right)\alpha\left(\dfrac{1-\delta(1-b)k-\delta(1-\delta)(1-b)U_c}{1-\delta(1-b)}\right)$. Using

$U_c = \dfrac{\alpha-k+\delta(1-b)(1-\alpha)k}{\delta(1-\delta)(1-b)\alpha}$ gives $V_s^d = -ke^{-\tilde{\sigma}} < 0$. That is, it does not pay a firm to participate in a deviating coalition, and then a CM equilibrium exists. ■

Firms match with probability one, therefore $V_c = J_c = 1-k-w_c+\delta((1-b)J_c + bV_c)$. Setting $V_c = 0$ gives $w_c = 1-k$. If $\delta = 0$, a CM equilibrium does not exist if $\alpha < k$ because condition $\dfrac{(1-\delta(1-b))k}{1-\delta(1-b)k} < \alpha < k$ becomes $k < \alpha < k$.

3.2 Mixed Market Equilibrium and CM Equilibrium if $\alpha \geq k$

In the previous case $\phi > 1$ was supported iff $\alpha < k$. If $\alpha \geq k$, then we have $\phi \leq 1$. Then in CM a worker matches with probability one, and a firm matches with probability $\phi \leq 1$. The value functions for unmatched agents are

$$\begin{aligned}
U_c &= \alpha\delta U_c + (1-\alpha)(J_c + k + W_c - \delta V_c), &(11)\\
U_s &= \frac{1-e^{-\sigma}}{\sigma}(\alpha\delta U_s + (1-\alpha)(J_s + k + W_s - \delta V_s)) + \left(1-\frac{1-e^{-\sigma}}{\sigma}\right)\delta U_s, &(12)\\
V_c &= -k + \phi(\alpha(J_c + k + W_c - \delta U_c) + (1-\alpha)\delta V_c) + (1-\phi)\delta V_c, &(13)\\
V_s &= -k + \left(1-e^{-\sigma}\right)(\alpha(J_s + k + W_s - \delta U_s) + (1-\alpha)\delta V_s) + e^{-\sigma}\delta V_s, &(14)
\end{aligned}$$

with the now familiar interpretations.

Proposition 3 *An MM equilibrium does not exist if $\alpha \geq k$.*

Proof. Setting $V_c = V_s = 0$, equations (11) and (12) give $U_c = U_s$ only if $1-e^{-\sigma} = \sigma$ which holds iff $\sigma = 0$.Then $\omega = 0$ or $\theta = 0$. If $\omega = 0$, SM has no workers. If $\theta = 0$, then $u = 0$, which is possible only if all workers match in CM, which means that SM has no workers. ■

Next we study whether a CM equilibrium exists. Assuming $\alpha \geq k$ gives $\theta \leq 1$ as shown in the proof below. Then a worker matches in CM with probability one, and a firm matches with probability $\theta \leq 1$. The value functions for unmatched agents are

$$U_c = \alpha\delta U_c + (1-\alpha)(J_c + k + W_c - \delta V_c), \tag{15}$$
$$V_c = -k + \theta(\alpha(J_c + k + W_c - \delta U_c) + (1-\alpha)\delta V_c) + (1-\theta)\delta V_c. \tag{16}$$

Again, the only difference to the case $\alpha < k$ is that a worker matches with probability one and a firm matches with probability $\theta \leq 1$.

Proposition 4 *A CM equilibrium exists if $\alpha \geq k$.*

Proof. It suffices to show that it is not profitable for a worker to participate in a deviating coalition. Setting $V_c = 0$ equation (15) gives $U_c = \frac{1-\alpha}{1-\delta}\frac{1-\delta(1-b)k}{1-\delta(1-b)\alpha}$, and (16) gives $U_c = \frac{(1-\delta(1-b)k)\alpha\theta - (1-\delta(1-b))k}{\delta(1-\delta)(1-b)\alpha\theta}$. Equating the solutions gives $\theta = \frac{(1-\delta(1-b)\alpha)k}{(1-\delta(1-b)k)\alpha}$. Then $\theta < 1$ if $\alpha > k$, and $\theta = 1$ if $\alpha = k$. Suppose a coalition of firms and workers deviates for one period to SM where the Poisson term is $\tilde{\sigma}$. The value function for a deviating worker is $U_s^d = \frac{1-e^{-\tilde{\sigma}}}{\tilde{\sigma}}\left(\alpha\delta U_c + (1-\alpha)\left(J_s^d + k + W_s^d - \delta V_c\right)\right) + \left(1 - \frac{1-e^{-\tilde{\sigma}}}{\tilde{\sigma}}\right)\delta U_c$. Setting $V_c = 0$ and using $J_s^d + k + W_s^d = \frac{1-\delta(1-b)k + \delta b U_c}{1-\delta(1-b)}$ and $U_c = \frac{1-\alpha}{1-\delta}\frac{1-\delta(1-b)k}{1-\delta(1-b)\alpha}$ gives $U_s^d = \frac{1}{\tilde{\sigma}}\frac{1-\alpha}{1-\delta}\frac{1-\delta(1-b)k}{1-\delta(1-b)\alpha}\left((1-\delta)\left(1-e^{-\tilde{\sigma}}\right) + \delta\tilde{\sigma}\right)$. Then $U_s^d - U_c = \frac{1-\alpha}{\tilde{\sigma}}\frac{1-\delta(1-b)k}{1-\delta(1-b)\alpha}\left(1-\tilde{\sigma}-e^{-\tilde{\sigma}}\right) < 0$ if $\tilde{\sigma} > 0$. That is, it does not pay to a worker to participate in a deviating coalition, thus a CM equilibrium exists. ■

Next we solve the equilibrium wage. The value function for a matched firm is $J_c = 1-k-w_c+\delta((1-b)J_c + bV_c)$. Setting $V_c = 0$ gives $J_c = \frac{1-k-w_c}{1-\delta(1-b)}$.The value function for an unmatched firm is $V_c = -k + \theta(J_c + k) + (1-\theta)\delta V_c$. Setting $V_c = 0$ gives $J_c = \frac{k(1-\theta)}{\theta}$. Then $\frac{1-k-w_c}{1-\delta(1-b)} = \frac{k(1-\theta)}{\theta}$ gives $w_c = 1-\delta(1-b)k - \frac{1}{\theta}(1-\delta(1-b))k$. Using $\theta = \frac{(1-\delta(1-b)\alpha)k}{(1-\delta(1-b)k)\alpha}$ gives $w_c = \frac{(1-\alpha)(1-\delta(1-b)k)}{1-\delta(1-b)\alpha}$.

3.3 SM Equilibrium

Consider next whether a decentralized market equilibrium exists. The Poisson term in SM is θ. The value functions for unmatched agents are

$$U_s = \frac{1-e^{-\theta}}{\theta}(\alpha\delta U_s + (1-\alpha)(J_s + k + W_s - \delta V_s)) + \left(1 - \frac{1-e^{-\theta}}{\theta}\right)\delta U_s, \tag{17}$$
$$V_s = -k + \left(1-e^{-\theta}\right)(\alpha(J_s + k + W_s - \delta U_s) + (1-\alpha)\delta V_s) + e^{-\theta}\delta V_s. \tag{18}$$

They are replications of (12) and (14) except that the Poisson term is $\theta \equiv u/v$ instead of $\sigma \equiv \frac{\omega\theta}{\tau}$. Setting $V_s = 0$, equation (17) gives

$$U_s = \frac{1-\alpha}{1-\delta} \frac{(1-\delta(1-b)k)\left(1-e^{-\theta}\right)}{\delta(1-b)(1-\alpha)(1-e^{-\theta}) + (1-\delta(1-b))\theta}, \tag{19}$$

and (18) gives

$$U_s = \frac{(1-\delta(1-b)k)\,\alpha\left(1-e^{-\theta}\right) - (1-\delta(1-b))\,k}{\alpha\delta(1-\delta)(1-e^{-\theta})(1-b)}. \tag{20}$$

Equating the solutions gives

$$\alpha = \frac{\left((1-\delta(1-b))\,\theta + \delta(1-b)\left(1-e^{-\theta}\right)\right)k}{(1-e^{-\theta})\left(\delta(1-b)\,k + \theta\,(1-\delta(1-b)\,k)\right)}, \tag{21}$$

which determines the equilibrium value of θ implicitly as a function of α, b, δ and k.

The equilibrium is checked against a coalitional deviation. Suppose a coalition of μv firms and ηu workers can deviate to CM for one period. In CM all agents on the short side match. An SM-equilibrium does not exist if there is a coalition where its members fare at least as well as in SM.

Proposition 5 *An SM equilibrium does not exist.*

Proof. Suppose $\eta u > \mu v$. Then in CM a firm matches with probability one, and a worker matches with probability $\phi^d = \mu/\eta\theta < 1$. The value function for a deviating firm is $V_c^d = -k + \alpha\left(J_c^d + k + W_c^d - \delta U_s\right) + (1-\alpha)\,\delta V_s$. A worker's continuation value is U_s because the coalition deviates for one period only. Setting $V_s = 0$ and using $J_c^d + k + W_c^d = \frac{1-\delta(1-b)k + \delta b U_s}{1-\delta(1-b)}$ and (20) gives $V_c^d = \frac{ke^{-\theta}}{1-e^{-\theta}} > 0$. The value function for a deviating worker is $U_c^d = \phi^d\left(\alpha\delta U_s + (1-\alpha)\left(J_c^d + k + W_c^d - \delta V_s\right)\right) + \left(1-\phi^d\right)\delta U_s$. Then, setting $V_s = 0$, we have

$$U_c^d - U_s = \left(\phi^d\alpha\delta + \frac{\phi^d(1-\alpha)\,\delta b}{1-\delta(1-b)} + \left(1-\phi^d\right)\delta - 1\right)U_s + \frac{\phi^d(1-\alpha)(1-\delta(1-b)k)}{1-\delta(1-b)}.$$

Using (20) and (21) gives $U_c^d - U_s = \frac{e^{-\theta} + \theta\phi^d - 1}{1-e^{-\theta}} \frac{1-k-(1-\delta(1-b)k)\,e^{-\theta}}{\delta(1-b)(1-e^{-\theta}) + (1-\delta(1-b))\,\theta}$ where $1-k-(1-\delta(1-b)k)\,e^{-\theta} > 0$ because $\alpha < 1$.Then $U_c^d \geq U_s$ if $\phi^d \geq \frac{1-e^{-\theta}}{\theta}$, that is, if a worker's matching probability in CM is larger than in SM. This is satisfied if $\mu/\eta \geq 1-e^{-\theta}$. This holds together with $\eta u > \mu v$ if $1-e^{-\theta} \leq \mu/\eta < \theta$. A deviating coalition exists for all $\theta > 0$. ■

One can show that there are also coalitions where $\eta u < \mu v$ or $\eta u = \mu v$ such that $U_c^d > U_s$ and $V_c^d > 0$. There is a CM equilibrium if $\alpha > \frac{(1-\delta(1-b))\,k}{1-\delta(1-b)\,k}$, and an MM equilibrium if $\alpha = \frac{(1-\delta(1-b))\,k}{1-\delta(1-b)\,k}$. An SM equilibrium does not exist. If $\alpha <$

$\dfrac{(1-\delta(1-b))\,k}{1-\delta(1-b)\,k}$, there is no equilibrium with firms in the economy. Figure 1 depicts the equilibria.

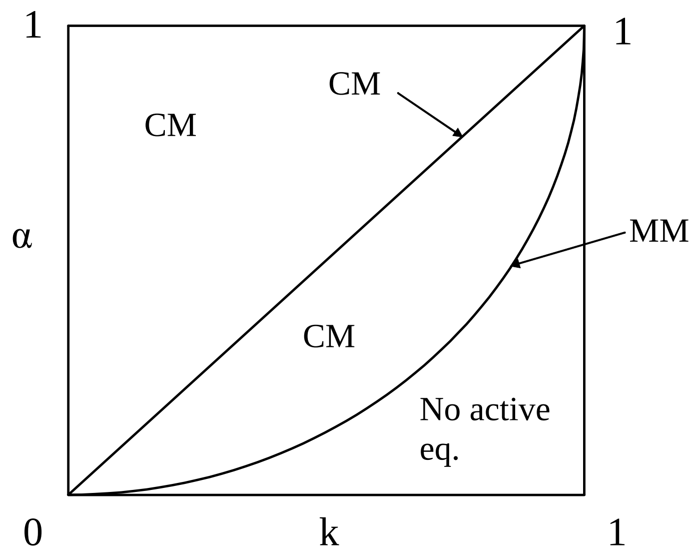

Figure 1: Equilibria when wages are determined by bargaining in the decentralized market

3.4 Efficiency

The total net output per worker is $Q = \dfrac{(L-\hat{u})(1-k)-\hat{v}k}{L}$, where $\hat{u}$ and $\hat{v}$ are the numbers of unemployed and vacancies during a production period, that is, between the matching stages. Given that a CM equilibrium exists, the net production of the economy is a function of parameters α, k, δ and b. A larger α induces more firms into the economy, leading to a higher employment and total production, but it also increases the total capital cost. A smaller k also induces more firms and increases employment, at a lower capital cost per firm-worker pair. Consider a planner who takes k, δ and b as given and chooses α in order to maximize total net output per worker.

Proposition 6 *The total net output per worker is maximal if $\alpha = k$.*

Proof. Consider cases $\alpha < k$, $\alpha > k$ and $\alpha = k$.

(i) Let $\alpha < k$, then $\theta > 1$ in a CM equilibrium. A worker matches with probability $1/\theta$, and a firm matches with probability one. The number of unemployed in the beginning of a matching stage is $u = \hat{u} + b(L-\hat{u})$, where $b(L-\hat{u})$ is the number of workers that becomes unemployed after a production period. In a steady state $u\dfrac{1}{\theta} = b(L-\hat{u})$. Then $(\hat{u} + b(L-\hat{u}))\dfrac{1}{\theta} = b(L-\hat{u})$ gives $\hat{u} = \dfrac{(\theta-1)\,bL}{1+b(\theta-1)}$. Using $\theta = \dfrac{\delta(1-b)(1-\alpha)\,k}{\alpha-k+\delta(1-b)(1-\alpha)\,k}$ gives $\hat{u} = \dfrac{(k-\alpha)\,bL}{(1-b)(\alpha-k+\delta(1-\alpha)\,k)}$. Because firms match with probability one, then $\hat{v} = 0$, and the net production per worker in steady state is $Q \equiv \frac{1}{L}(L - \hat{u})(1-k) = \frac{(1-k)(\alpha - k + \delta(1-b)(1-\alpha)k)}{(1-b)(\alpha - k + \delta(1-\alpha)k)}$. Then we have $\frac{\partial Q}{\partial \alpha} = \frac{\delta bk}{1-b}\frac{(1-k)^2}{((1-\delta)k - (1-\delta k)\alpha)^2} > 0$.

(ii) Let $\alpha > k$, then $\theta < 1$ in a CM equilibrium. A firm matches with probability θ, and a worker matches with probability one. During a production period $\hat{v} > 0$ and $\hat{u} = 0$. Then $v = \hat{v} + b(L - \hat{u}) = \hat{v} + bL$. In a steady state $\theta v = bL$, then $\theta(\hat{v} + bL) = bL$ gives $\hat{v} = \frac{bL}{\theta} - bL$. The net production per worker in steady state is $Q \equiv \frac{1}{L}(L(1-k) - \hat{v}k) = \frac{1}{L}\left(L(1-k) - \left(\frac{bL}{\theta} - bL\right)k\right) = 1 - (1-b)k - \frac{bk}{\theta}$. Using $\theta = \frac{(1-\delta(1-b)\alpha)k}{(1-\delta(1-b)k)\alpha}$ gives $Q = \frac{(1-k)(1-\delta(1-b)\alpha) - b(\alpha - k)}{1-\delta(1-b)\alpha} > 0$ because $\alpha < \frac{1-(1-b)k}{\delta(1-b)(1-k)+b}$. We have $\frac{\partial Q}{\partial \alpha} = -\frac{(1-\delta(1-b)k)b}{(1-\delta(1-b)\alpha)^2} < 0$.

(iii) Let $\alpha = k$, then $\theta = 1$,and then $\hat{u} = \hat{v} = 0$, and then $Q = 1 - k$ which is the largest Q possible. If $\alpha \neq k$ then $\theta \neq 1$, and then either $\hat{u} > 0$ or $\hat{v} > 0$, and then $Q < 1 - k$. ■

4 Bargaining in CM and Wage Posting in SM

In this section a match surplus is shared by Nash bargaining solution in CM, but in SM firms post wages publicly. I apply a result in Kultti (1999) which shows that in a large market wage posting is utilitywise equivalent to an auction where a firm gets its reservation value if no workers or only one worker visits it. If at least two workers visit the firm, the firm gets the value of the match minus the reservation value of the worker. The equivalence result simplifies the analysis considerably.

Consider for a moment a static model where the reservation values are zero. Assume that there is a fixed number of identical firms, and the workers observe them all. Each worker chooses one firm at random. The number of workers arriving a given firm is then binomially distributed. It is assumed that the numbers of firms and workers are large, and the binomial distribution is approximated by Poisson distribution. Denote the unemployment-vacancy ratio by σ. Then the probability that a firm receives no applicants is $e^{-\sigma}$, the probability that it receives one applicant is $\sigma e^{-\sigma}$, and the probability that it receives at least two applicants is $1 - e^{-\sigma} - \sigma e^{-\sigma}$. The probability for a worker of being the only applicant to a firm it contacted is $e^{-\sigma}$, and then $1 - e^{-\sigma}$ is the probability that the worker has at least one rival applicant. Setting the match value to unity the expected utility of a firm is $V = 1 - e^{-\sigma} - \sigma e^{-\sigma}$, and the utility of a worker is $U = e^{-\sigma}$.

Suppose then that firms attract workers by posting wages publicly so that all agents observe them. Workers choose between firms based on wages. A larger wage is balanced against larger probability of receiving at least one applicant. Consider a symmetric Nash equilibrium where each firm posts w. Following Kultti (1999)[1] we have $w =$

[1] The idea of the proof is to consider a subset of firms that deviates by posting w' instead of w posted

$\sigma e^{-\sigma}/\left(1-e^{-\sigma}\right)$. The expected utility of an unemployed worker is the probability of being hired times the wage. As the former equals $\left(1-e^{-\sigma}\right)/\sigma$, the expected utility of a worker is $U = w\left(1-e^{-\sigma}\right)/\sigma = e^{-\sigma}$. The expected utility of a firm is $V = \left(1-e^{-\sigma}\right)(1-w)$ where the first term is the probability that a firm meets at least one worker. Using the equation for wage above we have $V = 1-e^{-\sigma}-\sigma e^{-\sigma}$. The expected utilities are thus the same as in the auction model. In the rest of the paper I assume that the number of firms is determined by entry and exit such that the expected value of a vacancy is zero. Also, the model is dynamic.

Equations (1) – (4) hold here, too. As in the previous model, in a mixed market equilibrium fraction $\tau \in (0,1)$ of firms and fraction $\omega \in (0,1)$ of workers choose SM, and fractions $1-\tau$ and $1-\omega$ choose CM. In SM the Poisson term is $\sigma = \omega\theta/\tau$. In CM, let $\phi \equiv (1-\omega)\theta/(1-\tau)$. If $\phi > 1$, a worker is matched with probability $1/\phi$, and a firm is matched with probability one. If $\phi \leq 1$, a worker is matched with probability one, and a firm is matched with probability ϕ.

4.1 Mixed Market Equilibrium and CM Equilibrium if $\alpha < k$

Let us first study a mixed market where in CM a worker matches with probability $1/\phi < 1$ and a firm matches with probability one. This is possible only if $\alpha < k$ as shown in the first proof below. We label this equilibrium as MM1. The value functions for unmatched agents are

$$
\begin{aligned}
U_c &= (1/\phi)\left(\alpha\delta U_c + (1-\alpha)\left(J_c + k + W_c - \delta V_c\right)\right) + (1-1/\phi)\,\delta U_c, && (22)\\
U_s &= e^{-\sigma}\left(J_s + k + W_s - \delta V_s\right) + \left(1-e^{-\sigma}\right)\delta U_s, && (23)\\
V_c &= -k + \alpha\left(J_c + k + W_c - \delta U_c\right) + (1-\alpha)\,\delta V_c, && (24)\\
V_s &= -k + \left(e^{-\sigma} + \sigma e^{-\sigma}\right)\delta V_s + \left(1-e^{-\sigma}-\sigma e^{-\sigma}\right)\left(J_s + k + W_s - \delta U_s\right). && (25)
\end{aligned}
$$

The value functions for agents in CM, (22) and (24), are the same as (5) and (7). Equation (23) gives an unemployed worker's value function in SM. He contacts a firm, and he is the only applicant with probability $e^{-\sigma}$, and he receives the match value minus firm's reservation value δV_s. With probability $1-e^{-\sigma}$ he has at least one rival, and he gets his reservation utility δU_s. In (25) a firm in SM pays capital cost k, and receives no applicants or just one applicant with probability $e^{-\sigma}+\sigma e^{-\sigma}$, then it gets its reservation value δV_s. With probability $1-e^{-\sigma}-\sigma e^{-\sigma}$ the firm receives at least two applicants and consequently gets all the match value minus the worker's reservation value δU_s. A mixed market equilibrium exists if and only if $U_c = U_s \geq 0$, $V_c = V_s = 0$, $\phi > 1$, and $\sigma > 0$.

Lemma 1 *If $\alpha < k$, a mixed market equilibrium exists if and only if $\alpha = 1-e^{-\sigma}-\sigma e^{-\sigma}$ where the value of σ is determined by*

$$\frac{1-e^{-\sigma}-\sigma e^{-\sigma}}{1-\delta(1-b)\,\sigma e^{-\sigma}} = k. \qquad (26)$$

by all other firms, and a subset of workers who choose the deviating firms. Then we let the sizes of the subsets to approach zero.

Proof. Setting $V_c = 0$ equation (22) gives $U_c = \frac{1-\alpha}{1-\delta}\frac{1-\delta(1-b)k}{\delta(1-b)(1-\alpha)+(1-\delta(1-b))\phi}$, and (24) gives $U_c = \frac{\alpha - k + \delta(1-b)(1-\alpha)k}{\delta(1-\delta)(1-b)\alpha}$. The solutions are equal iff $\phi = \frac{\delta(1-b)(1-\alpha)k}{\alpha - k + \delta(1-b)(1-\alpha)k}$. Then $\phi > 1$ if $\frac{(1-\delta(1-b))k}{1-\delta(1-b)k} < \alpha < k$. Setting $V_s = 0$ equation (23) gives $U_s = \frac{e^{-\sigma}(1-\delta(1-b)k)}{(1-\delta)(1-\delta(1-b)(1-e^{-\sigma}))}$, and (25) gives $U_s = \frac{1-k-(1-\delta(1-b)k)e^{-\sigma}(1+\sigma)}{\delta(1-\delta)(1-b)(1-e^{-\sigma}(1+\sigma))}$. Equating the solutions gives (26) .Setting $V_c = V_s = 0$ and $U_c = U_s$, equations (24) and (25) give $1 - e^{-\sigma} - \sigma e^{-\sigma} = \alpha$. This together with (26) determines the equilibrium. ■

A firm is indifferent between CM and SM only if the match surplus times a firm's probability of receiving it are the same in both markets. The match surpluses are equal only if workers are indifferent between the markets. The latter holds together with free entry of firms only if $\alpha = 1 - e^{-\sigma} - \sigma e^{-\sigma}$. The larger k the larger must α be in MM1 equilibrium: We have $\frac{d\sigma}{dk} > 0$ by $\frac{1-e^{-\sigma}-\sigma e^{-\sigma}}{1-\delta(1-b)\sigma e^{-\sigma}} = k$, and $\partial(1-e^{-\sigma}-\sigma e^{-\sigma})/\partial\sigma > 0$. Then $\frac{d\alpha}{dk} = \frac{\partial(1-e^{-\sigma}-\sigma e^{-\sigma})}{\partial\sigma}\frac{d\sigma}{dk} > 0$. Equation (26) gives $\sigma = 0$ if $k = 0$ and $\sigma \to \infty$ if $k = 1$. Equation $1 - e^{-\sigma} - \sigma e^{-\sigma} = \alpha$ gives $\alpha = 0$ if $\sigma = 0$ if and $\alpha = 1$ if $\sigma \to \infty$. Then the equilibrium locus (α, k) starts at $(0,0)$ and ends at $(1,1)$.

Proposition 7 *There is a continuum of mixed market equilibria where* $w_c = 1 - k, \omega = \frac{\tau\sigma}{1-\tau+\sigma} < \tau, \phi = 1 + \sigma$, *and* $\theta = 1 - \tau + \sigma$, *where the value of* σ *is determined by* (26).

Proof. *Using* (26) *and* $\alpha = 1 - e^{-\sigma} - \sigma e^{-\sigma}$ *we have* $\phi = \frac{\delta(1-b)(1-\alpha)k}{\alpha - k + \delta(1-b)(1-\alpha)k} = 1 + \sigma$. *Then* $\sigma > 0$ *if* $\frac{(1-\delta(1-b))k}{1-\delta(1-b)k} < \alpha < k$. *Matching identity* $(1-\omega)u\frac{1}{\phi} + \omega u\frac{1-e^{-\sigma}}{\sigma} = (1-\tau)v + \tau v(1-e^{-\sigma})$ *and* $\phi = 1+\sigma$ *give* $\theta = \frac{\sigma(1+\sigma)(1-\tau e^{-\sigma})}{\sigma + \omega(1-e^{-\sigma}-\sigma e^{-\sigma})}$. *Also,* $\theta = \frac{\tau\sigma}{\omega}$. *The two equations for* θ *give* $\omega = \frac{\tau\sigma}{1-\tau+\sigma} < \tau$. *Then* $\theta = 1 - \tau + \sigma$. ■

Consider then a CM equilibrium where a worker matches with probability $1/\theta < 1$, and a firm matches with probability one. This happens if $\alpha < k$, as shown in the first proof below. We label this equilibrium as CM1. The value functions for unmatched agents are

$$U_c = (1/\theta)(\alpha\delta U_c + (1-\alpha)(J_c + k + W_c - \delta V_c)) + (1 - 1/\theta)\delta U_c, \quad (27)$$
$$V_c = -k + \alpha(J_c + k + W_c - \delta U_c) + (1-\alpha)\delta V_c, \quad (28)$$

which are as(22) and (24) where ϕ is replaced by θ. Setting $V_c = 0$ and using $J_c + k + W_c = \frac{1-\delta(1-b)k + \delta b U_c}{1-\delta(1-b)}$, equation (27) gives $U_c = \frac{1-\alpha}{1-\delta}\frac{1-\delta(1-b)k}{\delta(1-b)(1-\alpha)+(1-\delta(1-b))\theta}$, and (28) gives $U_c = \dfrac{\alpha - k + \delta(1-b)(1-\alpha)k}{\delta(1-\delta)(1-b)\alpha}$. The solutions are equal if $\theta = \dfrac{\delta(1-b)(1-\alpha)k}{\alpha - k + \delta(1-b)(1-\alpha)k}$. Then $\theta > 0$ if $\alpha > \dfrac{(1-\delta(1-b))k}{1-\delta(1-b)k}$, and $\theta > 1$ if $\alpha < k$.

Proposition 8 *If $\alpha < k$, a CM equilibrium exists if $\alpha > \tilde{\alpha}$ where $\tilde{\alpha} = 1 - e^{-\tilde{\sigma}_0} - \tilde{\sigma}_0 e^{-\tilde{\sigma}_0}$ where the value of $\tilde{\sigma}_0$ is determined by $\dfrac{1 - e^{-\tilde{\sigma}_0} - \tilde{\sigma}_0 e^{-\tilde{\sigma}_0}}{1-\delta(1-b)\tilde{\sigma}_0 e^{-\tilde{\sigma}_0}} = k$.*

Proof. Assume a group of μv firms and ηu workers can deviate to SM for one period. The Poisson term in SM is $\tilde{\sigma} = \eta\theta/\mu$. The value functions for deviating agents are $U_s^d = e^{-\tilde{\sigma}}\left(J_s^d + k + W_s^d - \delta V_c\right) + \left(1 - e^{-\tilde{\sigma}}\right)\delta U_c$ and $V_s^d = -k + \left(e^{-\tilde{\sigma}} + \tilde{\sigma}e^{-\tilde{\sigma}}\right)\delta V_c + \left(1 - e^{-\tilde{\sigma}} - \tilde{\sigma}e^{-\tilde{\sigma}}\right)\left(J_s^d + k + W_s^d - \delta U_c\right)$. A deviating group exists only if $U_s^d \geq U_c$ and $V_s^d \geq 0$. Setting $V_c = 0$ and using $J_s^d + k + W_s^d = \dfrac{1-\delta(1-b)k + \delta b U_c}{1-\delta(1-b)}$ and $U_c = \dfrac{\alpha - k + \delta(1-b)(1-\alpha)k}{\delta(1-\delta)(1-b)\alpha}$, we have $V_s^d = \dfrac{k}{\alpha}\left(1 - e^{-\tilde{\sigma}} - \tilde{\sigma}e^{-\tilde{\sigma}} - \alpha\right)$, and $U_s^d - U_c = \dfrac{\left(1-\delta(1-b)\left(1-e^{-\tilde{\sigma}}\right)\right)k - (1-\delta(1-b)k)\alpha}{\delta(1-b)\alpha}$. We have $V_s^d \geq 0$ if (i) $1 - e^{-\tilde{\sigma}} - \tilde{\sigma}e^{-\tilde{\sigma}} \geq \alpha$, and $U_s^d \geq U_c$ if (ii) $\dfrac{\left(1-\delta(1-b)\left(1-e^{-\tilde{\sigma}}\right)\right)k}{1-\delta(1-b)k} \geq \alpha$. The LHS of (i) increases in $\tilde{\sigma}$, and the LHS of (ii) decreases in $\tilde{\sigma}$. Let $\tilde{\alpha} = 1 - e^{-\tilde{\sigma}_0} - \tilde{\sigma}_0 e^{-\tilde{\sigma}_0} = \dfrac{\left(1-\delta(1-b)\left(1-e^{-\tilde{\sigma}_0}\right)\right)k}{1-\delta(1-b)k}$. Suppose $\alpha > \tilde{\alpha}$. Then $V_s^d \geq 0$ iff $\tilde{\sigma} \geq \tilde{\sigma}_1$ where $\tilde{\sigma}_1$ satisfies $1 - e^{-\tilde{\sigma}_1} - \tilde{\sigma}_1 e^{-\tilde{\sigma}_1} = \alpha$, and $U_s^d \geq U_c$ if $\tilde{\sigma} \leq \tilde{\sigma}_2$ where $\tilde{\sigma}_2$ satisfies $\dfrac{\left(1-\delta(1-b)\left(1-e^{-\tilde{\sigma}_2}\right)\right)k}{1-\delta(1-b)k} = \alpha$. But $\tilde{\sigma}_2 < \tilde{\sigma}_1$ because $\alpha > \tilde{\alpha}$.Then $V_s^d \geq 0$ and $U_s^d \geq U_c$ cannot both hold, and then a CM equilibrium exists. If $\alpha < \tilde{\alpha}$, then $V_s^d \geq 0$ if $\tilde{\sigma} \geq \tilde{\sigma}_3$ where $\tilde{\sigma}_3$ satisfies $1 - e^{-\tilde{\sigma}_3} - \tilde{\sigma}_3 e^{-\tilde{\sigma}_3} = \alpha$, and $U_s^d \geq U_c$ if $\tilde{\sigma} \leq \tilde{\sigma}_4$ where $\tilde{\sigma}_4$ satisfies $\dfrac{\left(1-\delta(1-b)\left(1-e^{-\tilde{\sigma}_4}\right)\right)k}{1-\delta(1-b)k} = \alpha$. We have $\tilde{\sigma}_3 < \tilde{\sigma}_4$ because $\alpha < \tilde{\alpha}$.Then all $\tilde{\sigma} \in (\tilde{\sigma}_3, \tilde{\sigma}_4)$ give $V_s^d > 0$ and $U_s^d > U_c$.Then if $\alpha < \tilde{\alpha}$, a deviating group exists, and a CM equilibrium does not exist. Finally, $1 - e^{-\tilde{\sigma}_0} - \tilde{\sigma}_0 e^{-\tilde{\sigma}_0} = \dfrac{\left(1-\delta(1-b)\left(1-e^{-\tilde{\sigma}_0}\right)\right)k}{1-\delta(1-b)k}$ gives $\dfrac{1 - e^{-\tilde{\sigma}_0} - \tilde{\sigma}_0 e^{-\tilde{\sigma}_0}}{1-\delta(1-b)\tilde{\sigma}_0 e^{-\tilde{\sigma}_0}} = k$. If $\alpha = \tilde{\alpha}$, a mixed market equilibrium exists. ■

If α is smaller than k but large enough, there is no group of firms and workers such that all its members can benefit from choosing the decentralized market instead of the centralized market. If one group benefits, the other group necessarily looses. The lower bound $\tilde{\alpha}$ of a firm's surplus share in CM equals the probability that a deviating firm receives at least two applicants in SM.

In CM equilibrium we have $\dfrac{1 - e^{-\tilde{\sigma}_0} - \tilde{\sigma}_0 e^{-\tilde{\sigma}_0}}{1-\delta(1-b)\tilde{\sigma}_0 e^{-\tilde{\sigma}_0}} = k$, and in MM equilibrium we have $\dfrac{1 - e^{-\sigma} - \sigma e^{-\sigma}}{1-\delta(1-b)\sigma e^{-\sigma}} = k$. Then $\tilde{\sigma}_0 = \sigma$. Then the locus (α, k) which determines MM equilibrium determines also the lower boundary for α in CM equilibrium.

4.2 Mixed Market Equilibrium and CM Equilibrium if $\alpha \geq k$

Consider first a mixed market equilibrium where $\alpha > k$. Then a firm matches in CM submarket with probability $\phi < 1$ as shown in the first proof below, and a worker matches with probability one. We label this equilibrium as MM2. The value functions for unmatched agents are

$$U_c = \alpha\delta U_c + (1-\alpha)(J_c + k + W_c - \delta V_c), \quad (29)$$
$$U_s = e^{-\sigma}(J_s + k + W_s - \delta V_s) + \left(1 - e^{-\sigma}\right)\delta U_s, \quad (30)$$
$$V_c = -k + \phi(\alpha(J_c + k + W_c - \delta U_c) + (1-\alpha)\delta V_c) + (1-\phi)\delta V_c, \quad (31)$$
$$V_s = -k + \left(e^{-\sigma} + \sigma e^{-\sigma}\right)\delta V_s + \left(1 - e^{-\sigma} - \sigma e^{-\sigma}\right)(J_s + k + W_s - \delta U_s), \quad (32)$$

with the familiar interpretations from the case $\alpha < k$. A mixed market equilibrium exists if $V_c = V_s = 0$, $U_c = U_s \geq 0$, $0 < \phi < 1$,and $\sigma > 0$.

Lemma 2 *A mixed market equilibrium exists if and only if $\alpha = 1 - e^{-\sigma}$ where the value of σ is determined by* (26).

Proof. Assume first $\alpha > k$. Setting $V_c = 0$ equation (29) gives $U_c = \frac{1-\alpha}{1-\delta}\frac{1-\delta(1-b)k}{1-\delta(1-b)\alpha}$, and (31) gives $U_c = \frac{(1-\delta(1-b)k)\alpha\phi - (1-\delta(1-b))k}{\delta(1-\delta)(1-b)\alpha\phi}$. The solutions to U_c are equal iff $\phi = \frac{(1-\delta(1-b)\alpha)k}{(1-\delta(1-b)k)\alpha}$. Then $\phi < 1$ if $\alpha > k$. Setting $V_s = 0$ equation (30) gives $U_s = \frac{(1-\delta(1-b)k)e^{-\sigma}}{(1-\delta)(1-\delta(1-b)(1-e^{-\sigma}))}$, and (32) gives

$$U_s = \frac{1 - e^{-\sigma} - \sigma e^{-\sigma} - (1 - \delta(1-b)(1+\sigma)e^{-\sigma})k}{\delta(1-\delta)(1-b)(1 - e^{-\sigma} - \sigma e^{-\sigma})}.$$

Equating the solutions to U_s gives (26), and then $U_s = \frac{e^{-\sigma}}{(1-\delta)(1-\delta(1-b)\sigma e^{-\sigma})}$. Setting $V_c = V_s = 0$ and $U_c = U_s$, equations (29) and (30) give (ii) $\alpha = 1 - e^{-\sigma}$. Then assume $\alpha = k$. Then (26) and $\alpha = 1 - e^{-\sigma}$ hold only if $\delta(1-b) = \frac{1}{1-e^{-\sigma}} > 1$, and then a mixed market equilibrium does not exist. ■

Because $\alpha = 1 - e^{-\sigma}$ and $\frac{1 - e^{-\sigma} - \sigma e^{-\sigma}}{1 - \delta(1-b)\sigma e^{-\sigma}} = k$, a mixed market equilibrium exists if α and k are on locus $k = \frac{\alpha + (1-\alpha)\ln(1-\alpha)}{1 + \delta(1-b)(1-\alpha)\ln(1-\alpha)}$. The equilibrium locus (α, k) begins at $(0,0)$ and ends at $(1,1)$, and $\frac{dk}{d\alpha} = \frac{\delta(1-b)\alpha - (1-\delta(1-b))\ln(1-\alpha)}{(1+\delta(1-b)(1-\alpha)\ln(1-\alpha))^2} > 0$.

Proposition 9 *There is a continuum of mixed market equilibria where*

$$w_c = \frac{1-\alpha}{1 + \delta(1-b)(1-\alpha)\ln(1-\alpha)}, \quad \omega = \frac{-\tau\alpha\ln(1-\alpha)}{(1-\tau)\alpha + (1-\tau-\alpha)\ln(1-\alpha)} > \tau,$$
$$\phi = 1 + \frac{1}{\alpha}(1-\alpha)\ln(1-\alpha), \text{ and } \theta = 1 - \tau + \frac{1}{\alpha}(1-\tau-\alpha)\ln(1-\alpha).$$

Proof. Because a worker matches with probability one in CM, then $U_c = \dfrac{w_c}{1-\delta}$. Equalizing this with $U_s = \dfrac{e^{-\sigma}}{(1-\delta)(1-\delta(1-b)\sigma e^{-\sigma})}$ gives $w_c = \dfrac{e^{-\sigma}}{1-\delta(1-b)\sigma e^{-\sigma}}$. Using (26) and $\alpha = 1-e^{-\sigma}$ we have $\phi = \dfrac{(1-\delta(1-b)\alpha)k}{(1-\delta(1-b)k)\alpha} = \dfrac{1-e^{-\sigma}-\sigma e^{-\sigma}}{1-e^{-\sigma}}$. Matching identity $(1-\omega)u + \omega u\dfrac{1-e^{-\sigma}}{\sigma} = (1-\tau)v\phi + \tau v(1-e^{-\sigma})$ and $\phi = \dfrac{1-e^{-\sigma}-\sigma e^{-\sigma}}{1-e^{-\sigma}}$ give $\theta = \dfrac{(1-\tau)\sigma(1-e^{-\sigma}-\sigma e^{-\sigma}) + \tau\sigma(1-e^{-\sigma})^2}{(1-\omega)\sigma(1-e^{-\sigma}) + \omega(1-e^{-\sigma})^2}$. Also, $\theta = \dfrac{\tau\sigma}{\omega}$. Then

$$\omega = \frac{\tau\sigma(1-e^{-\sigma})}{1-e^{-\sigma}-\sigma e^{-\sigma}+\tau(\sigma+e^{-\sigma}-1)} > \tau, \text{and } \theta = \frac{1-e^{-\sigma}-\sigma e^{-\sigma}+\tau(\sigma+e^{-\sigma}-1)}{1-e^{-\sigma}}.$$

Using $\alpha = 1-e^{-\sigma}$ we end up with the proposition above. ■

In a CM equilibrium where $\alpha > k$, a worker matches with probability one, and a firm matches with probability $\theta < 1$, as shown below. Let us label this equilibrium as CM2. The value functions for unmatched agents are

$$U_c = \alpha\delta U_c + (1-\alpha)(J_c + k + W_c - \delta V_c), \quad (33)$$
$$V_c = -k + \theta(\alpha(J_c + k + W_c - \delta U_c) + (1-\alpha)\delta V_c) + (1-\theta)\delta V_c. \quad (34)$$

Setting $V_c = 0$, equation (33) gives $U_c = \dfrac{1-\alpha}{1-\delta}\dfrac{1-\delta(1-b)k}{1-\delta(1-b)\alpha}$, and (34) gives $U_c = \dfrac{(1-\delta(1-b)k)\alpha\theta - (1-\delta(1-b))k}{\delta(1-\delta)(1-b)\alpha\theta}$. Equating the solutions gives $\theta = \dfrac{(1-\delta(1-b)\alpha)k}{(1-\delta(1-b)k)\alpha}$. If $\alpha > k$, then $\theta < 1$.

Proposition 10 *A CM equilibrium exists iff $\alpha < 1-e^{-\hat{\sigma}}$ where the value of $\hat{\sigma}$ is given by $\dfrac{1-e^{-\hat{\sigma}}-\hat{\sigma}e^{-\hat{\sigma}}}{1-\delta(1-b)\hat{\sigma}e^{-\hat{\sigma}}} = k$.*

Proof. Assume μv firms and ηu workers form a group which deviates to SM for one period. The Poisson term in the deviating market is $\sigma = \eta\theta/\mu$. The value functions for deviators are $U_s^d = e^{-\sigma}\left(J_s^d + k + W_s^d - \delta V_c\right) + (1-e^{-\sigma})\delta U_c$ and $V_s^d = -k + (e^{-\sigma}+\sigma e^{-\sigma})\delta V_c + (1-e^{-\sigma}-\sigma e^{-\sigma})\left(J_s^d + k + W_s^d - \delta U_c\right)$. A deviating group exists only if $U_s^d \geq U_c$ and $V_s^d \geq 0$. Setting $V_c = 0$ and using $J_s^d + k + W_s^d = \dfrac{1-\delta(1-b)k + \delta b U_c}{1-\delta(1-b)}$ and $U_c = \dfrac{1-\alpha}{1-\delta}\dfrac{1-\delta(1-b)k}{1-\delta(1-b)\alpha}$ we have $V_s^d = -k + (1-e^{-\sigma}-\sigma e^{-\sigma})\dfrac{1-\delta(1-b)k}{1-\delta(1-b)\alpha}$. We have $\dfrac{\partial V_s^d}{\partial\alpha} > 0$, and $\dfrac{\partial V_s^d}{\partial\sigma} > 0$. Then $V_s^d > 0$ if $\alpha > \dfrac{k-(1-\delta(1-b)k)(1-e^{-\sigma}-\sigma e^{-\sigma})}{\delta(1-b)k}$. Also, $U_s^d - U_c = (\alpha + e^{-\sigma} - 1)\dfrac{1-\delta(1-b)k}{1-\delta(1-b)\alpha}$. We have $\dfrac{\partial\left(U_s^d - U_c\right)}{\partial\alpha} > 0$ and $\dfrac{\partial\left(U_s^d - U_c\right)}{\partial\sigma} < 0$. Then $U_s^d > U_c$ if $\alpha > 1-e^{-\sigma}$. Let $\hat{\sigma}$ satisfy $\dfrac{k-(1-\delta(1-b)k)\left(1-e^{-\hat{\sigma}}-\hat{\sigma}e^{-\hat{\sigma}}\right)}{\delta(1-b)k} = 1-e^{-\hat{\sigma}}$. This gives $\dfrac{1-e^{-\hat{\sigma}}-\hat{\sigma}e^{-\hat{\sigma}}}{1-\delta(1-b)\hat{\sigma}e^{-\hat{\sigma}}} = k$. Then $V_s^d = 0$ and $U_s^d = U_c$ at $\hat{\sigma}$ if $\alpha = 1-e^{-\hat{\sigma}}$, and $V_s^d > 0$ and $U_s^d > U_c$ at $\hat{\sigma}$ if $\alpha > 1 - e^{-\hat{\sigma}}$. Suppose $\alpha > 1 - e^{-\hat{\sigma}}$. Then $V_s^d > 0$ and $U_s^d > U_c$ if $\sigma \in (\sigma_1, \sigma_2)$ where $\sigma_1 < \hat{\sigma} < \sigma_2$ and where $V_s^d = 0$ at σ_1 and $U_s^d = U_c$ at σ_2. Then a profitable deviating coalition exists. Suppose $\alpha < 1 - e^{-\hat{\sigma}}$. Then $U_s^d > U_c$ if $\sigma < \sigma_3$, and $V_s^d > 0$ if $\sigma > \sigma_4$ where $\sigma_3 < \sigma_4$. Then a profitable deviating coalition does not exist. ■

Equations (26) and $\alpha = 1 - e^{-\sigma}$ give $k = \dfrac{\alpha + (1-\alpha)\ln(1-\alpha)}{1 + \delta(1-b)(1-\alpha)\ln(1-\alpha)}$ where RHS increases in α. We have $V_s^d > 0$ and $U_s^d > U_c$ only if α is large enough. Then $V_s^d > 0$ and $U_s^d > U_c$ only if $k < \dfrac{\alpha + (1-\alpha)\ln(1-\alpha)}{1 + \delta(1-b)(1-\alpha)\ln(1-\alpha)}$. Then a CM2 equilibrium exists if $k > \dfrac{\alpha + (1-\alpha)\ln(1-\alpha)}{1 + \delta(1-b)(1-\alpha)\ln(1-\alpha)}$. A CM2 equilibrium exists also if $\alpha = k$ because $\alpha > \dfrac{\alpha + (1-\alpha)\ln(1-\alpha)}{1 + \delta(1-b)(1-\alpha)\ln(1-\alpha)}$.

4.3 SM Equilibrium

Consider next a decentralized market equilibrium. The Poisson parameter in SM is $u/v \equiv \theta$. The value functions for unmatched agents are

$$U_s = e^{-\theta}(J_s + k + W_s - \delta V_s) + \left(1 - e^{-\theta}\right)\delta U_s, \tag{35}$$

$$V_s = -k + \left(e^{-\theta} + \theta e^{-\theta}\right)\delta V_s + \left(1 - e^{-\theta} - \theta e^{-\theta}\right)(J_s + k + W_s - \delta U_s). \tag{36}$$

Setting $V_s = 0$ and using $J_s + k + W_s = \dfrac{1 - \delta(1-b)k + \delta b U_s}{1 - \delta(1-b)}$, (35) gives

$$U_s = \frac{e^{-\theta}}{1-\delta}\frac{1 - \delta(1-b)k}{1 - \delta(1-b)(1-e^{-\theta})}, \tag{37}$$

and (36) gives

$$U_s = \frac{1 - k - (1 - \delta(1-b)k)(1+\theta)e^{-\theta}}{\delta(1-\delta)(1-b)(1 - e^{-\theta} - \theta e^{-\theta})}. \tag{38}$$

Equating the solutions for U_s yields $\dfrac{1 - e^{-\theta} - \theta e^{-\theta}}{1 - \delta(1-b)\theta e^{-\theta}} = k$. This determines the equilibrium value of θ.

Proposition 11 *An SM equilibrium exists if $\alpha < 1 - e^{-\theta} - \theta e^{-\theta}$ or $\alpha > 1 - e^{-\theta}$, where the value of θ is determined by $\dfrac{1 - e^{-\theta} - \theta e^{-\theta}}{1 - \delta(1-b)\theta e^{-\theta}} = k$.*

Proof. Suppose that a coalition of ηu workers and μv firms can deviate for one period to CM. We study cases $\eta u > \mu v$, $\eta u < \mu v$, and $\eta u = \mu v$, and we use $J_c^d + k + W_c^d = \dfrac{1 - \delta(1-b)k + \delta b U_s}{1 - \delta(1-b)}$ and $\dfrac{1 - e^{-\theta} - \theta e^{-\theta}}{1 - \delta(1-b)\theta e^{-\theta}} = k$.(i) Suppose $\eta u > \mu v$. Then in CM a firm matches with probability one, and a worker matches with probability $\phi^d = \mu/\eta\theta < 1$. The value functions for deviating agents are $U_c^d = \phi^d\left(\alpha\delta U_s + (1-\alpha)\left(J_c^d + k + W_c^d - \delta V_s\right)\right) +$

$\left(1-\phi^d\right)\delta U_s$ and $V_c^d = -k+\alpha\left(J_c^d+k+W_c^d-\delta U_s\right)+(1-\alpha)\,\delta V_s$. Setting $V_s=0$ and using (37) and the above condition for k gives $V_c^d = \dfrac{\alpha+e^{-\theta}+\theta e^{-\theta}-1}{1-\delta(1-b)\,\theta e^{-\theta}} < 0$ if $\alpha < 1-e^{-\theta}-\theta e^{-\theta}$. Setting $V_s=0$ and using (37) gives $U_c^d - U_s = \dfrac{\left(\phi^d(1-\alpha)-e^{-\theta}\right)(1-\delta(1-b)\,k)}{1-\delta(1-b)\left(1-e^{-\theta}\right)}$ < 0 if $\alpha > 1-\dfrac{e^{-\theta}}{\phi^d}$. Using $\phi^d=\mu/\eta\theta$ we have $U_c^d < U_s$ if $\alpha > 1-\dfrac{\eta}{\mu}\theta e^{-\theta}$. Because $\mu/\eta\theta<1$, then $1-e^{-\theta} > 1-\dfrac{\eta}{\mu}\theta e^{-\theta}$. Then $U_c^d<U_s$ if $\alpha>1-e^{-\theta}$. Then a deviating coalition does not exist if $\alpha<1-e^{-\theta}-\theta e^{-\theta}$ or $\alpha>1-e^{-\theta}$. (ii) Suppose $\eta u<\mu v$. Then in CM a worker matches with probability one, and a firm matches with probability $\phi^d=\eta\theta/\mu<1$. The value functions for deviating agents are $U_c^d=\alpha\delta U_s+(1-\alpha)\left(J_c^d+k+W_c^d-\delta V_s\right)$ and $V_c^d=-k+\phi^d\left(\alpha\left(J_c^d+k+W_c^d-\delta U_s\right)+(1-\alpha)\,\delta V_s\right)+\left(1-\phi^d\right)\delta V_s$. Setting $V_s=0$ and using (37) gives $U_c^d-U_s=\dfrac{(1-\delta(1-b)\,k)\left(1-\alpha-e^{-\theta}\right)}{1-\delta(1-b)\left(1-e^{-\theta}\right)}<0$ if $\alpha>1-e^{-\theta}$. Setting $V_s=0$ and using (37) and the condition for k gives $V_c^d=\dfrac{\alpha\phi^d+e^{-\theta}+\theta e^{-\theta}-1}{1-\delta(1-b)\,\theta e^{-\theta}}<0$ if $\alpha\phi^d<1-e^{-\theta}-\theta e^{-\theta}$. Because $\phi^d<1$, the latter holds if $\alpha<1-e^{-\theta}-\theta e^{-\theta}$. Here, too, a deviating coalition does not exist if $\alpha<1-e^{-\theta}-\theta e^{-\theta}$ or $\alpha>1-e^{-\theta}$. (iii) Suppose $\eta u=\mu v$. Then both firms and workers match in CM with probability one. The value functions of deviating agents are $U_c^d=\alpha\delta U_s+(1-\alpha)\left(J_c^d+k+W_c^d-\delta V_s\right)$ and $V_c^d=-k+\alpha\left(J_c^d+k+W_c^d-\delta U_s\right)+(1-\alpha)\,\delta V_s$. Setting $V_s=0$ and using (37) gives $U_c^d-U_s=\dfrac{(1-\delta(1-b)\,k)\left(1-\alpha-e^{-\theta}\right)}{1-\delta(1-b)\left(1-e^{-\theta}\right)}<0$ if $\alpha>1-e^{-\theta}$. Setting $V_s=0$ and using (37) and the condition for k gives $V_c^d=\dfrac{\alpha+e^{-\theta}+\theta e^{-\theta}-1}{1-\delta(1-b)\,\theta e^{-\theta}}<0$ if $\alpha<1-e^{-\theta}-\theta e^{-\theta}$. Again, a deviating coalition does not exist if $\alpha<1-e^{-\theta}-\theta e^{-\theta}$ or $\alpha>1-e^{-\theta}$. By (i) - (iii) a deviating coalition cannot exist if $\alpha<1-e^{-\theta}-\theta e^{-\theta}$ or $\alpha>1-e^{-\theta}$, and then an SM equilibrium exists. ■

Notice that using $\dfrac{1-e^{-\theta}-\theta e^{-\theta}}{1-\delta(1-b)\,\theta e^{-\theta}}=k$ condition $\alpha>1-e^{-\theta}$ is equivalent to $k<\dfrac{\alpha+(1-\alpha)\ln(1-\alpha)}{1+\delta(1-b)(1-\alpha)\ln(1-\alpha)}$. Notice that if $\alpha<1-e^{-\theta}-\theta e^{-\theta}$, then $\alpha<k$ because $\alpha<1-e^{-\theta}-\theta e^{-\theta}=\left(1-\delta(1-b)\,\theta e^{-\theta}\right)k<k$. If $\alpha>1-e^{-\theta}$, then $\alpha>k$ because $\alpha>1-e^{-\theta}=\left(1-\delta(1-b)\,\theta e^{-\theta}\right)k+\theta e^{-\theta}>k$. An SM equilibrium exists either if a firm's probability of receiving at least two applicants in SM is larger than its surplus share in CM, or if a worker's probability of being the sole applicant to a particular firm is larger than his surlpus share in CM. If $\alpha<k$, the locus of an MM equilibrium is also the lower boundary of CM equilibrium and the upper boundary of SM equilibrium. If $\alpha>k$, the locus of MM equilibrium is also the upper boundary of CM equilibrium and the lower boundary of SM equilibrium. Figure 2 depicts the equilibria.

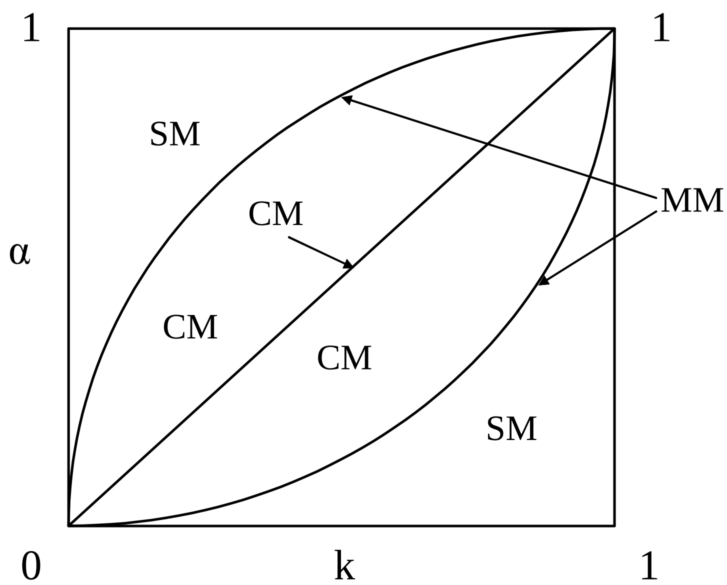

Figure 2: Equilibria when wages are determined by posting in the decentralized market

4.4 Efficiency

In this section we compare the total net output per worker, Q, in different equilibria. We have $Q = \frac{(L-\hat{u})(1-k)-\hat{v}k}{L}$. We fix δ, b, and k, and we let α change. As α increases from zero to one, the equilibrium changes from SM to MM1 to CM to MM2 and finally back to SM.

In a CM equilibrium Q equals that in the model where there is bargaining in both markets: If $\alpha < k$, then $Q_{CM1} = \frac{(1-k)(\alpha - k + \delta(1-b)(1-\alpha)k)}{(1-b)(\alpha - k + \delta k(1-\alpha))}$. If $\alpha > k$ then $Q_{CM2} = \frac{(1-k)(1-\delta(1-b)\alpha) - b(\alpha - k)}{1-\delta(1-b)\alpha}$. If $\alpha = k$, then $Q_{CM3} = 1-k$, which is the highest possible output. In an SM equilibrium there are both unemployed and vacancies during a production period. In the beginning of a matching stage the number of unemployed is $u = \hat{u} + b(L-\hat{u})$, and the number of vacancies is $v = \hat{v} + b(L-\hat{u})$,where $\hat{u}$ and $\hat{v}$ are the numbers of unemployed and vacancies in a production period. Then $\hat{v} = v - b(L-\hat{u})$. Also, $\theta \equiv \frac{u}{v} = \frac{\hat{u}+b(L-\hat{u})}{\hat{v}+b(L-\hat{u})}$ gives $\hat{v} = \frac{\hat{u}+(1-\theta)b(L-\hat{u})}{\theta}$. The equations for $\hat{v}$ give $\hat{u} = \frac{v\theta - bL}{1-b}$. In a steady state $u\frac{1-e^{-\theta}}{\theta} = b(L-\hat{u})$, which gives $\hat{u} = L - \frac{1}{b}\left(1-e^{-\theta}\right)v$. Equating the solutions for $\hat{u}$ gives $v = \frac{bL}{(1-b)(1-e^{-\theta})+b\theta}$, and then $\hat{u} = \frac{v\theta - bL}{1-b}$ gives $\hat{u} = \frac{\left(\theta + e^{-\theta} - 1\right)bL}{(1-b)(1-e^{-\theta})+b\theta}$. Then $\hat{v} = v - b(L-\hat{u}) = \frac{e^{-\theta}bL}{(1-b)(1-e^{-\theta})+b\theta}$. Plugging the solutions for $\hat{u}$ and $\hat{v}$ into $Q = \frac{(L-\hat{u})(1-k)-\hat{v}k}{L}$

yields $Q_{SM} = \frac{(1-k)\left(1-e^{-\theta}\right) - bke^{-\theta}}{(1-b)\left(1-e^{-\theta}\right)+b\theta}$. Using $\frac{1-e^{-\theta}-\theta e^{-\theta}}{1-\delta(1-b)\theta e^{-\theta}} = k$ yields $Q_{SM} = \frac{\left(1-e^{-\theta}\right)e^{-\theta}\left(1-b+(1-\delta(1-b))\theta\right)+b\theta e^{-2\theta}}{\left((1-b)\left(1-e^{-\theta}\right)+b\theta\right)\left(1-\delta(1-b)\theta e^{-\theta}\right)}$.

In an MM equilibrium we have, like in an SM equilibrium, $\hat{v}_i = \frac{\hat{u}_i + (1-\theta_i)\,b\,(L-\hat{u}_i)}{\theta_i}$ and $\hat{u}_i = \frac{v_i\theta_i - bL}{1-b}$, where $i=1$ denotes MM1-equilibrium, and $i=2$ denotes MM2 equilibrium.

(i) In MM1 firms match with probability one in CM submarket. In a steady state $\tau v_1\left(1-e^{-\sigma}\right)+(1-\tau)\,v_1 = b\,(L-\hat{u}_1)$, which gives $\hat{u}_1 = \frac{1}{b}\left(bL - v_1 + v_1\tau e^{-\sigma}\right)$. Equating the solutions for $\hat{u}_1$ gives $v_1 = \frac{bL}{b\theta_1+(1-b)\left(1-\tau e^{-\sigma}\right)}$.Then $\hat{u}_1 = \frac{\left(\theta_1+\tau e^{-\sigma}-1\right)bL}{b\theta_1+(1-b)\left(1-\tau e^{-\sigma}\right)}$ and $\hat{v}_1 = \frac{\tau e^{-\sigma}bL}{b\theta_1+(1-b)\left(1-\tau e^{-\sigma}\right)}$. Using $\theta_i = \frac{\tau\sigma}{\omega_i}$ and $\omega_1 = \frac{\tau\sigma}{1-\tau+\sigma}$ we have $Q_{MM1} = \frac{(1-k)\left(1-\tau e^{-\sigma}\right)-bk\tau e^{-\sigma}}{(1-b)\left(1-\tau e^{-\sigma}\right)+b\,(1-\tau+\sigma)}$.

(ii) In MM2 firms match with probability $\phi < 1$ in the CM submarket. In a steady state $\tau v_2\left(1-e^{-\sigma}\right)+(1-\tau)\,v_2\phi = b\,(L-\hat{u}_2)$, which gives $\hat{u}_2 = \frac{1}{b}\left(bL+\tau v_2\left(\phi+e^{-\sigma}-1\right)-v_2\phi\right)$. Equating the solutions for $\hat{u}_2$ gives $v_2 = \frac{bL}{b\theta_2+(1-b)\left(\tau\left(1-e^{-\sigma}\right)+\phi(1-\tau)\right)}$. Then $\hat{u}_2 = \frac{\left(\theta_2-\phi+\tau\left(\phi+e^{-\sigma}-1\right)\right)bL}{b\theta_2+(1-b)\left(\tau\left(1-e^{-\sigma}\right)+\phi(1-\tau)\right)}$, and using this gives $\hat{v}_2 = \frac{\left((1-\tau)(1-\phi)+\tau e^{-\sigma}\right)bL}{b\theta_2+(1-b)\left(\tau\left(1-e^{-\sigma}\right)+\phi(1-\tau)\right)}$. Using the solutions for $\hat{u}_2$ and $\hat{v}_2$ gives $Q_{MM2} = \frac{\left(\tau\left(1-e^{-\sigma}\right)+\phi(1-\tau)\right)(1-k)-\left((1-\tau)(1-\phi)+\tau e^{-\sigma}\right)bk}{(1-b)\left(\tau\left(1-e^{-\sigma}\right)+\phi(1-\tau)\right)+b\theta_2}$.

Using $\theta_i = \frac{\tau\sigma}{\omega_i}$, $\phi = \frac{1-e^{-\sigma}-\sigma e^{-\sigma}}{1-e^{-\sigma}}$, and $\omega_2 = \frac{\tau\sigma\left(1-e^{-\sigma}\right)}{1-e^{-\sigma}-\sigma e^{-\sigma}+\tau\left(\sigma+e^{-\sigma}-1\right)}$ we have $Q_{MM2} = \frac{\left(1-e^{-\sigma}\right)(1-k)-e^{-\sigma}\left((1-\tau)\sigma+\tau\left(1-e^{-\sigma}\right)\right)\left(1-(1-b)k\right)}{1-e^{-\sigma}-\sigma e^{-\sigma}+\tau\left(\sigma+e^{-\sigma}-1\right)\left(e^{-\sigma}+b\left(1-e^{-\sigma}\right)\right)}$.

Then $Q_{MM2}-Q_{MM1} = \frac{b\sigma(1-\tau)\left(1-e^{-\sigma}-\sigma e^{-\sigma}-k\left(1-(1-b)\sigma e^{-\sigma}\right)\right)}{\left(1-e^{-\sigma}-\sigma e^{-\sigma}+\tau\left(\sigma+e^{-\sigma}-1\right)\left(e^{-\sigma}+b\left(1-e^{-\sigma}\right)\right)\right)\left((1-b)\left(1-\tau e^{-\sigma}\right)+b(1-\tau+\sigma)\right)}$.

Using (26) gives $Q_{MM2}-Q_{MM1}$

$$= \frac{(1-\tau)(1-\delta)\,b\,(1-b)\,\sigma^2 e^{-\sigma}\left(1-e^{-\sigma}-\sigma e^{-\sigma}\right)}{\left(1-\delta(1-b)\sigma e^{-\sigma}\right)\begin{pmatrix} 1-e^{-\sigma}-\sigma e^{-\sigma} \\ +\tau\left(\sigma+e^{-\sigma}-1\right)\left(e^{-\sigma}+b\left(1-e^{-\sigma}\right)\right)\end{pmatrix}\left((1-b)\left(1-\tau e^{-\sigma}\right)+b(1-\tau+\sigma)\right)} > 0.$$

Remark 1 $Q_{MM2} > Q_{MM1}$.

The net outputs in the different equilibria have the following order: $Q_{MM1} < Q_{SM} < Q_{MM2} < Q_{CM2}$. Also, $Q_{CM1} = Q_{SM}$ if $\alpha = \tilde{\alpha}$, $Q_{CM1} < Q_{SM}$ if $\alpha < \tilde{\alpha}$, and $Q_{CM1} > Q_{SM}$

if $\alpha > \tilde{\alpha}$. We use the facts that in an MM equilibrium (26) holds, and in SM equilibrium $\frac{1-e^{-\theta}-\theta e^{-\theta}}{1-\delta(1-b)\theta e^{-\theta}} = k$. Then σ in an MM equilibrium equals θ in an SM equilibrium.

(i) We have $Q_{SM} > Q_{MM1}$:

$Q_{SM} - Q_{MM1} = \frac{(1-\tau)b(1-e^{-\sigma}-\sigma e^{-\sigma}-(1-(1-b)\sigma e^{-\sigma})k)}{((1-b)(1-e^{-\sigma})+b\sigma)((1-b)(1-\tau e^{-\sigma})+b(1-\tau+\sigma))}$. Using (26) gives

$$Q_{SM} - Q_{MM1} = \frac{(1-\tau)(1-\delta)b(1-b)\sigma e^{-\sigma}(1-e^{-\sigma}-\sigma e^{-\sigma})}{(1-\delta(1-b)\sigma e^{-\sigma})((1-b)(1-e^{-\sigma})+b\sigma)((1-b)(1-\tau e^{-\sigma})+b(1-\tau+\sigma))} > 0.$$

(ii) We have $Q_{MM2} > Q_{SM}$:

$Q_{MM2} - Q_{SM} = \frac{(1-\tau)b(\sigma+e^{-\sigma}-1)(1-e^{-\sigma}-\sigma e^{-\sigma}-k(1-(1-b)\sigma e^{-\sigma}))}{\left(\begin{array}{c}1-e^{-\sigma}-\sigma e^{-\sigma}+\tau(\sigma+e^{-\sigma}-1)\\(e^{-\sigma}+b(1-e^{-\sigma}))\end{array}\right)((1-b)(1-e^{-\sigma})+b\sigma)}$.

Using (26) yields

$$Q_{MM2} - Q_{SM} = \frac{(1-\tau)(1-\delta)b(1-b)\sigma e^{-\sigma}(\sigma+e^{-\sigma}-1)(1-e^{-\sigma}-\sigma e^{-\sigma})}{(1-\delta(1-b)\sigma e^{-\sigma})\left(\begin{array}{c}1-e^{-\sigma}-\sigma e^{-\sigma}+\tau(\sigma+e^{-\sigma}-1)\\(e^{-\sigma}+b(1-e^{-\sigma}))\end{array}\right)((1-b)(1-e^{-\sigma})+b\sigma)} > 0.$$

(iii) We have $Q_{CM2} > Q_{MM2}$:

First, $\frac{\partial Q_{CM2}}{\partial\alpha} = \frac{-b(1-\delta(1-b)k)}{(1-\delta(1-b)\alpha)^2} < 0$. Let $Q_{l2} \equiv \lim_{\alpha\to 1-e^{-\sigma}} Q_{CM2}$ where $\frac{1-e^{-\sigma}-\sigma e^{-\sigma}}{1-\delta(1-b)\sigma e^{-\sigma}} = k$. That is, Q_{l2} is the net output in CM2 if α approaches the MM2 locus. Then $Q_{l2} = \frac{(1-k)(1-\delta(1-b)(1-e^{-\sigma}))-b(1-e^{-\sigma}-k)}{1-\delta(1-b)(1-e^{-\sigma})} < Q_{CM2}$. Then we have

$$Q_{l2} - Q_{MM2} = \frac{((1-k)(1-\delta(1-b)(1-e^{-\sigma}))-b(1-e^{-\sigma}-k))(1-e^{-\sigma}-\sigma e^{-\sigma}+\tau(\sigma+e^{-\sigma}-1)(e^{-\sigma}+b(1-e^{-\sigma}))) - \left(\begin{array}{c}(1-e^{-\sigma})(1-k)-e^{-\sigma}((1-\tau)\sigma+\tau(1-e^{-\sigma}))\\(1-(1-b)k)\end{array}\right)(1-\delta(1-b)(1-e^{-\sigma}))}{(1-\delta(1-b)(1-e^{-\sigma}))(1-e^{-\sigma}-\sigma e^{-\sigma}+\tau(\sigma+e^{-\sigma}-1)(e^{-\sigma}+b(1-e^{-\sigma})))}.$$

Using (26) yields

$$Q_{l2} - Q_{MM2} = \frac{\tau(1-\delta)b(1-b)\sigma e^{-\sigma}(1-e^{-\sigma})(\sigma+e^{-\sigma}-1)}{(1-\delta(1-b)\sigma e^{-\sigma})\left(\begin{array}{c}1-e^{-\sigma}-\sigma e^{-\sigma}\\+\tau(\sigma+e^{-\sigma}-1)(e^{-\sigma}+b(1-e^{-\sigma}))\end{array}\right)} > 0.$$

Then $Q_{CM2} > Q_{MM2}$.

(iv) We have $Q_{CM1} = Q_{SM}$ if $\alpha = \tilde{\alpha}$, and $Q_{CM1} < (>)\, Q_{SM}$ if $\alpha < (>)\, \tilde{\alpha}$, where $\tilde{\alpha} \in (1-e^{-\sigma}-\sigma e^{-\sigma}, k)$. We have $\frac{\partial Q_{CM1}}{\partial\alpha} = \frac{\delta bk(1-k)^2}{(1-b)((1-\delta)k-(1-\delta k)\alpha)^2} > 0$. Let $Q_{l1} \equiv \lim_{\alpha\to 1-e^{-\sigma}-\sigma e^{-\sigma}} Q_{CM1}$ where $\frac{1-e^{-\sigma}-\sigma e^{-\sigma}}{1-\delta(1-b)\sigma e^{-\sigma}} = k$. That is, Q_{l1} is the net output in CM1 if

α approaches the MM1 locus. Then $Q_{l1} = \dfrac{(1-k)(1-k-(1-\delta(1-b)k)(1+\sigma)e^{-\sigma})}{(1-b)(1-k-(1+\sigma)e^{-\sigma}(1-\delta k))} < Q_{CM1}$. Using $\theta = \sigma$ we have $Q_{SM} = \dfrac{(1-k)(1-e^{-\sigma}) - bke^{-\sigma}}{(1-b)(1-e^{-\sigma}) + b\sigma}$. Using (26) gives $Q_{l1} - Q_{SM} = \dfrac{(1-\delta)b(1-b)\sigma e^{-\sigma}(e^{-\sigma} + \sigma e^{-\sigma} - 1)}{(1+b\sigma)(1-\delta(1-b)\sigma e^{-\sigma})((1-b)(1-e^{-\sigma}) + b\sigma)} < 0$. Because Q_{CM1} increases in α, and $\lim_{\alpha \to k} Q_{CM1} = 1-k > Q_{SM}$, there exists $\tilde{\alpha} \in (1-e^{-\sigma} - \sigma e^{-\sigma}, k)$ such that $Q_{CM1} = Q_{SM}$ if $\alpha = \tilde{\alpha}$, and $Q_{CM1} < (>)\, Q_{SM}$ if $\alpha < (>)\ \tilde{\alpha}$.

If $\alpha < 1 - e^{-\theta} - \theta e^{-\theta}$, a decentralized market equilibrium exists, and there is a coordination problem but no hold-up problem. If $\alpha = 1 - e^{-\theta} - \theta e^{-\theta}$, a mixed market equilibrium exists, and both sources of inefficiency exist. The total net output per worker is smaller than in SM equilibrium. If $1 - e^{-\theta} - \theta e^{-\theta} < \alpha < k$, a centralized market equilibrium exists and there is no coordination problem, but there is a hold-up problem: there are too few firms. The total net production per worker can be larger or smaller than in SM, depending on the value of α. If $k < \alpha < 1 - e^{-\theta}$, a centralized market equilibrium exists and there is no hold-up problem, instead there are vacancies but no unemployed in the production period. The total net production per worker is larger than in SM. If $\alpha = 1 - e^{-\theta}$, a mixed market equilibrium exists with both sources of inefficiency, but the total net output per worker is larger than in SM equilibrium. If $\alpha > 1 - e^{-\theta}$ then the equilibrium is SM with coordination problem only.

4.5 A Static Model with Bargaining in CM and Wage Posting in SM

I briefly present the results of a static model where $\delta = 0$. The qualitative difference from the dynamic model is that if $\alpha < k$, a centralized market equilibrium does not exist, but a decentralized market equilibrium exists. (i) If $\alpha < k$, a CM equilibrium exists only if $1 - e^{-\tilde{\sigma}_0} - \tilde{\sigma}_0 e^{-\tilde{\sigma}_0} < \alpha < k = 1 - e^{-\tilde{\sigma}_0} - \tilde{\sigma}_0 e^{-\tilde{\sigma}_0}$ which fails to hold. (ii) If $\alpha > k$, a CM equilibrium exists if $k < \alpha < \tilde{\alpha} = 1 - e^{-\tilde{\sigma}_0}$ where the value of $\tilde{\sigma}_0$ is determined by $1 - e^{-\tilde{\sigma}_0} - \tilde{\sigma}_0 e^{-\tilde{\sigma}_0} = k$. Equation $\tilde{\alpha} = 1 - e^{-\tilde{\sigma}_0}$ gives $\tilde{\sigma}_0 = -\ln(1-\tilde{\alpha})$. Then we have $\tilde{\alpha} + (1-\tilde{\alpha})\ln(1-\tilde{\alpha}) = k$, and a CM equilibrium exists if $k < \alpha < \tilde{\alpha}$ where $\tilde{\alpha} + (1-\tilde{\alpha})\ln(1-\tilde{\alpha}) = k$. (iii) A CM equilibrium exists if $\alpha = k$. (iv) An SM equilibrium exists if $\alpha < 1 - e^{-\theta} - \theta e^{-\theta}$ or $\alpha > 1 - e^{-\theta}$ where the value of θ satisfies $1 - e^{-\theta} - \theta e^{-\theta} = k$. Then an SM equilibrium exists if $\alpha < k$ or if $\alpha > \tilde{\alpha}$ where $\tilde{\alpha}$ satisfies $\tilde{\alpha} + (1-\tilde{\alpha})\ln(1-\tilde{\alpha}) = k$. (v) An MM equilibrium does not exist if $\alpha < k$ because $\alpha = 1 - e^{-\sigma} - \sigma e^{-\sigma}$ and $1 - e^{-\sigma} - \sigma e^{-\sigma} = k$. If $\alpha > k$, an MM equilibrium exists if $\alpha = 1 - e^{-\sigma}$ where σ satisfies $1 - e^{-\sigma} - \sigma e^{-\sigma} = k$. Then an MM equilibrium exists if $\alpha + (1-\alpha)\ln(1-\alpha) = k$.

5 Wage Posting in CM and in SM

In this section we consider a model where wages are posted in both types of market. We show first that there are two continua of mixed market equilibria. Then we study centralized market equilibrium and decentralized market equilibrium and show that the latter does not exist.

5.1 Mixed Market Equilibria

Like before, we study separately the cases where either firms or workers match with probability one in the centralized submarket.

(i) Assume that in CM submarket a worker matches with probability $1/\phi < 1$ and a firm matches with probability one. The value functions for unmatched agents are

$$\begin{aligned} U_c &= \frac{1}{\phi} W_c + \left(1 - \frac{1}{\phi}\right) \delta U_c, & (39)\\ V_c &= J_c, & (40)\\ U_s &= e^{-\sigma}\left(J_s + k + W_s - \delta V_s\right) + \left(1 - e^{-\sigma}\right) \delta U_s, & (41)\\ V_s &= -k + \left(e^{-\sigma} + \sigma e^{-\sigma}\right) \delta V_s + \left(1 - e^{-\sigma} - \sigma e^{-\sigma}\right)\left(J_s + k + W_s - \delta U_s\right). & (42)\end{aligned}$$

We have $V_c = J_c$ because a firm in CM matches with probability one. The value functions for matched agents are

$$\begin{aligned} W_c &= w_c + \delta\left(\left(1-b\right) W_c + bU_c\right), & (43)\\ J_c &= 1 - k - w_c + \delta\left(\left(1-b\right) J_c + bV_c\right), & (44)\\ W_s &= w_s + \delta\left(\left(1-b\right) W_s + bU_s\right), & (45)\\ J_s &= 1 - k - w_s + \delta\left(\left(1-b\right) J_s + bV_s\right). & (46)\end{aligned}$$

Setting $V_c = V_s = 0$ gives $W_c = \dfrac{w_c + \delta b U_c}{1 - \delta\left(1-b\right)}$, $J_c = \dfrac{1 - k - w_c}{1 - \delta\left(1-b\right)}$, $W_s = \dfrac{w_s + \delta b U_s}{1 - \delta\left(1-b\right)}$, and $J_s = \dfrac{1 - k - w_s}{1 - \delta\left(1-b\right)}$. Also,

$$J_i + k + W_i = \frac{1 - \delta\left(1-b\right) k + \delta b U_i}{1 - \delta\left(1-b\right)}, i = c, s. \tag{47}$$

Using $V_c = J_c = \dfrac{1 - k - w_c}{1 - \delta\left(1-b\right)}$ and setting $V_c = 0$ gives $w_c = 1 - k$. Setting $V_s = 0$ equations (41) and (42) give (26) .Equations (39) and (43) give, using $w_c = 1 - k$, that $U_c = \dfrac{1-k}{\left(1-\delta\right)\left(\delta\left(1-b\right) + \left(1 - \delta\left(1-b\right)\right)\phi\right)}$. Equation (41) gives, using $J_s + k + W_s = \dfrac{1 - \delta\left(1-b\right) k + \delta b U_s}{1 - \delta\left(1-b\right)}$, that $U_s = \dfrac{e^{-\sigma}}{1-\delta} \dfrac{1 - \delta\left(1-b\right) k}{1 - \delta\left(1-b\right)\left(1 - e^{-\sigma}\right)}$. Setting $U_c = U_s$ gives $\phi = \dfrac{1 - \left(1 + \delta\left(1-b\right) e^{-\sigma}\right) k}{e^{-\sigma}\left(1 - \delta\left(1-b\right) k\right)}$. Then using (26) yields $\phi = 1 + \sigma$.

Proposition 12 *If firms in CM set $w_c = 1 - k$, there is a continuum of mixed market equilibria where $\omega_1 = \dfrac{\tau\sigma}{1 - \tau + \sigma} < \tau$ and $\phi = 1 + \sigma$, where the value of σ is determined by* (26).

Proof. In the above we showed that in a mixed market equilibrium $\phi = 1+\sigma$ where the value of σ is determined by (26). Using $\phi = \frac{(1-\omega_1)\theta}{1-\tau}$ and $\theta = \frac{\tau\sigma}{\omega_1}$ gives $\omega_1 = \frac{\tau\sigma}{1-\tau+\sigma} < \tau$. ■

The result is the same as in the model where wages are determined by bargaining in CM.

(ii) Assume then that in CM submarket a worker matches with probability one, and a firm matches with probability $\phi < 1$. The value functions for matched agents are (43)-(46). The value functions for unmatched agents are

$$
\begin{aligned}
U_c &= W_c, &(48)\\
V_c &= -k+\phi(J_c+k)+(1-\phi)\delta V_c, &(49)\\
U_s &= e^{-\sigma}(J_s+k+W_s-\delta V_s)+\left(1-e^{-\sigma}\right)\delta U_s, &(50)\\
V_s &= -k+\left(e^{-\sigma}+\sigma e^{-\sigma}\right)\delta V_s+\left(1-e^{-\sigma}-\sigma e^{-\sigma}\right)(J_s+k+W_s-\delta U_s). &(51)
\end{aligned}
$$

Set $V_c = V_s = 0$. Equations (43) and (48)give $U_c = \frac{w_c}{1-\delta}$. Equations (50) and (47) give $U_s = \frac{e^{-\sigma}}{1-\delta}\frac{1-\delta(1-b)k}{1-\delta(1-b)(1-e^{-\sigma})}$, and (51) and (47) give $U_s = \frac{1-k-(1-\delta(1-b)k)(1+\sigma)e^{-\sigma}}{\delta(1-\delta)(1-b)(1-e^{-\sigma}-\sigma e^{-\sigma})}$. Setting the solutions for U_s equal we have (26), and then $U_s = \frac{e^{-\sigma}}{(1-\delta)(1-\delta(1-b)\sigma e^{-\sigma})}$. Setting $U_c = U_s$ gives $w_c = \frac{e^{-\sigma}}{1-\delta(1-b)\sigma e^{-\sigma}} < 1-k$. Setting $V_c = 0$ gives $\phi = \frac{k}{J_c+k}$. Using $J_c = \frac{1-k-w_c}{1-\delta(1-b)}$, (26), and $w_c = \frac{e^{-\sigma}}{1-\delta(1-b)\sigma e^{-\sigma}}$ gives $\phi = \frac{1-e^{-\sigma}-\sigma e^{-\sigma}}{1-e^{-\sigma}} < 1$.

Proposition 13 *If firms in CM set $w_c < 1-k$, there is a continuum of mixed market equilibria where $w_c = \frac{e^{-\sigma}}{1-\delta(1-b)\sigma e^{-\sigma}}$, $\omega_2 = \frac{\tau\sigma(1-e^{-\sigma})}{1-e^{-\sigma}-\sigma e^{-\sigma}+\tau(\sigma+e^{-\sigma}-1)} > \tau$, $\phi = \frac{1-e^{-\sigma}-\sigma e^{-\sigma}}{1-e^{-\sigma}}$, and $\theta = \frac{1-e^{-\sigma}-\sigma e^{-\sigma}+\tau(\sigma+e^{-\sigma}-1)}{1-e^{-\sigma}}$, where the value of σ is given by* (26).

Proof. Same as in the case where there is bargaining in CM. ■

(iii) Assume that in CM submarket firms and workers match with probability one. The value functions for matched agents are (43)- (46), and the value functions for unmatched agents are (48), (50), (51), and $V_c = J_c$. Set $V_c = V_s = 0$. Then $w_c = 1-k$, $U_c = \frac{1-k}{1-\delta}$, and $U_s = \frac{e^{-\sigma}}{1-\delta}\frac{1-\delta(1-b)k}{1-\delta(1-b)(1-e^{-\sigma})}$. Equations (41) and (42) give (26). Setting $U_c = U_s$ gives $k = 1-e^{-\sigma}$. This together with (26) yields $\frac{1}{1-e^{-\sigma}} = \delta(1-b)$ which cannot hold since $\frac{1}{1-e^{-\sigma}} > 1$ and $\delta(1-b) < 1$. This leads to

Proposition 14 *There is no mixed market equilibrium where* $\phi = 1$.

We collect these findings in

Theorem 1 *There exists two continua of mixed market equilibrium (i) A high wage equilibrium where* $w_{c1} = 1 - k$, $\phi = 1 + \sigma$, $\omega_1 = \frac{\tau\sigma}{1-\tau+\sigma}$, *and* $\theta = 1 - \tau + \sigma$. *(ii) A low wage equilibrium where* $w_{c2} = \frac{e^{-\sigma}}{1-\delta(1-b)\sigma e^{-\sigma}}$, $\phi = \frac{1-e^{-\sigma}-\sigma e^{-\sigma}}{1-e^{-\sigma}}$, $\omega_2 = \frac{\tau\sigma(1-e^{-\sigma})}{1-e^{-\sigma}-\sigma e^{-\sigma}+\tau(\sigma+e^{-\sigma}-1)}$, *and* $\theta = \frac{1-e^{-\sigma}-\sigma e^{-\sigma}+\tau(\sigma+e^{-\sigma}-1)}{1-e^{-\sigma}}$, *where* $\omega_1 < \tau < \omega_2$. *In all mixed market equilibria the value of* σ *is determined by* (26).

If $\phi > 1$, then w_c is larger than when $\phi < 1$. This seems first counterintuitive because as ϕ is the worker-firm ratio in CM, a large worker-firm ratio should imply a low wage. But if $\phi > 1$, a firm gets a worker with probability one, and then the free entry condition implies $w_c = 1 - k$. If $\phi < 1$, a firm gets a worker with probability less than one, and then the free entry condition implies that the wage should be smaller than $1 - k$. Wages $w_{c1} = 1 - k$ and $w_{c2} = \frac{e^{-\sigma}}{1-\delta(1-b)\sigma e^{-\sigma}}$ correspond to the lower and upper MS loci, respectively, if bargaining is used in CM. If firms post wages in CM, this is equivalent to making Nash bargaining parameter endogenous. If $w_c = 1 - k$, this corresponds to $\alpha = 1 - e^{-\sigma} - \sigma e^{-\sigma}$. If $w_c < 1 - k$, then $w_c = \frac{e^{-\sigma}}{1-\delta(1-b)\sigma e^{-\sigma}}$. This corresponds to $\alpha = 1 - e^{-\sigma}$.

5.2 CM Equilibrium

We divide the analysis of a centralized market equilibrium in two parts depending on whether $\theta > 1$ or $\theta < 1$.

(i) Suppose $\theta > 1$.

In CM a worker matches with probability $1/\theta < 1$, and a firm matches with probability one. The value functions for matched agents are (43) and (44), and the value functions for unmatched agents are

$$V_c = J_c = \frac{1-k-w_c}{1-\delta(1-b)}, \quad (52)$$

$$U_c = \frac{1}{\theta}W_c + \left(1 - \frac{1}{\theta}\right)\delta U_c. \quad (53)$$

Then $V_c = 0$ gives $w_c = 1 - k$, and (43) and (53) give

$$U_c = \frac{1-k}{(1-\delta)(\delta(1-b)+(1-\delta(1-b))\theta)}. \quad (54)$$

Assume a group of μv firms and ηu workers can deviate to SM for one period. The Poisson term in the market of deviators is $\sigma = \eta\theta/\mu$. The value functions for deviating agents are

$$U_s^d = e^{-\sigma}\left(J_s^d + k + W_s^d - \delta V_c\right) + \left(1 - e^{-\sigma}\right)\delta U_c, \tag{55}$$
$$V_s^d = -k + \left(e^{-\sigma} + \sigma e^{-\sigma}\right)\delta V_c + \left(1 - e^{-\sigma} - \sigma e^{-\sigma}\right)\left(J_s^d + k + W_s^d - \delta U_c\right). \tag{56}$$

Setting $V_c = 0$ and using $J_s^d + k + W_s^d = \dfrac{1 - \delta(1-b)k + \delta b U_c}{1 - \delta(1-b)}$ and (54)we have

$$V_s^d = -k + \left(1 - e^{-\sigma} - \sigma e^{-\sigma}\right)\left(\frac{\delta(1-b)k + (1 - \delta(1-b)k)\theta}{\delta(1-b) + (1 - \delta(1-b))\theta}\right), \tag{57}$$

and $\dfrac{\partial V_s^d}{\partial \sigma} > 0$ and $\dfrac{\partial V_s^d}{\partial \theta} > 0$. For workers we have

$$U_s^d - U_c = \frac{(1 - \delta(1-b)k)\theta e^{-\sigma} + \left(1 + \delta(1-b)e^{-\sigma}\right)k - 1}{\delta(1-b) + (1 - \delta(1-b))\theta}, \tag{58}$$

and $\dfrac{\partial\left(U_s^d - U_c\right)}{\partial \sigma} < 0$ and $\dfrac{\partial\left(U_s^d - U_c\right)}{\partial \theta} = \dfrac{(1-k)\left(1 - \delta(1-b)\left(1 - e^{-\sigma}\right)\right)}{\left(\delta(1-b) + (1 - \delta(1-b))\theta\right)^2} > 0$. Then $V_s^d = 0$ if $\theta = \dfrac{\delta(1-b)k(1+\sigma)e^{-\sigma}}{\left(1 - e^{-\sigma} - \sigma e^{-\sigma}\right)(1 - \delta(1-b)k) - (1 - \delta(1-b))k}$, and $U_s^d = U_c$ if $\theta = \dfrac{1 - k - \delta(1-b)ke^{-\sigma}}{(1 - \delta(1-b)k)e^{-\sigma}}$.Then $V_s^d = 0$ and $U_s^d = U_c$ at $\hat{\sigma}$ if $\dfrac{\delta(1-b)k(1+\hat{\sigma})e^{-\hat{\sigma}}}{\left(1 - e^{-\hat{\sigma}} - \hat{\sigma}e^{-\hat{\sigma}}\right)(1 - \delta(1-b)k) - (1 - \delta(1-b))k} = \dfrac{1 - k - \delta(1-b)ke^{-\hat{\sigma}}}{(1 - \delta(1-b)k)e^{-\hat{\sigma}}}$, and then $\dfrac{1 - e^{-\hat{\sigma}} - \hat{\sigma}e^{-\hat{\sigma}}}{1 - \delta(1-b)\hat{\sigma}e^{-\hat{\sigma}}} = k$. Using this we have $\hat{\theta} = 1 + \hat{\sigma} > 1$.That is, $V_s^d = 0$ and $U_s^d = U_c$ at $\left(\hat{\sigma}, \hat{\theta}\right)$. If $\theta > \hat{\theta}$, then $V_s^d > 0$ and $U_s^d > U_c$ at $\hat{\sigma}$.Then there exists $\sigma_1 < \hat{\sigma}$ such that $V_s^d(\sigma_1) = 0$, and $\sigma_2 > \hat{\sigma}$ such that $U_s^d(\sigma_2) = U_c$. Then $V_s^d \geq 0$ and $U_s^d \geq U_c$ if $\theta > \hat{\theta}$ and $\sigma \in [\sigma_1, \sigma_2]$.

Proposition 15 *A CM equilibrium exists (does not exist) if* $\theta < (\geq) 1 + \hat{\sigma}$ *where* $\hat{\sigma}$ *satisfies* $\dfrac{1 - e^{-\hat{\sigma}} - \hat{\sigma}e^{-\hat{\sigma}}}{1 - \delta(1-b)\hat{\sigma}e^{-\hat{\sigma}}} = k$.

Proof. If $\theta \geq 1 + \hat{\sigma}$ where $\hat{\sigma}$ satisfies $\dfrac{1 - e^{-\hat{\sigma}} - \hat{\sigma}e^{-\hat{\sigma}}}{1 - \delta(1-b)\hat{\sigma}e^{-\hat{\sigma}}} = k$, then we find $\sigma \in [\sigma_1, \sigma_2]$ such that $V_s^d \geq 0$ and $U_s^d \geq U_c$. If $\theta < 1 + \hat{\sigma}$, then either $V_s^d < 0$ or $U_s^d < U_c$ or both. ■

(ii) Suppose $\theta < 1$.

In CM a worker matches with probability one, and a firm matches with probability θ. The value functions for matched agents are (43) and (44), and for unmatched agents we have

$$U_c = W_c \tag{59}$$
$$V_c = -k + \theta(J_c + k) + (1 - \theta)\delta V_c \tag{60}$$

We have $U_c = \frac{w_c}{1-\delta}$. Setting $V_c = 0$ gives $J_c = \frac{1-k-w_c}{1-\delta(1-b)}$, and then (60) gives $w_c = 1 - \left(\delta(1-b) + \frac{1}{\theta}(1-\delta(1-b))\right) k < 1-k$. That is, if firms set $w_c < 1-k$, then $\theta = \frac{(1-\delta(1-b))k}{1-w_c-\delta(1-b)k} < 1$ in equilibrium. Solving for U_c yields

$$U_c = \frac{(1-\delta(1-b)k)\theta - (1-\delta(1-b))k}{(1-\delta)\theta}. \tag{61}$$

Notice that $\frac{\partial U_c}{\partial \theta} = \frac{(1-\delta(1-b))k}{(1-\delta)\theta^2} > 0$, given that $\theta < 1$. This is because w_c increases in θ, which results from firms' free entry. The larger θ the higher the probability that a firm recruits a worker, and the higher must w_c be to satisfy $V_c = 0$.

Assume again that μv firms and ηu workers form a group which deviates to SM for one period. The value functions for deviating agents are (55) and (56). Setting $V_c = 0$ and using $J_s^d + k + W_s^d = \frac{1-\delta(1-b)k+\delta b U_c}{1-\delta(1-b)}$ and (61) we have

$$V_s^d = -k + \left(1-e^{-\sigma}-\sigma e^{-\sigma}\right)\frac{1}{\theta}\left(\delta(1-b)k + (1-\delta(1-b)k)\theta\right). \tag{62}$$

We have $\frac{\partial V_s^d}{\partial \sigma} > 0$, and $\frac{\partial V_s^d}{\partial \theta} = -\frac{k}{\theta^2}\delta(1-b)(1-e^{-\sigma}-\sigma e^{-\sigma}) < 0$.

Setting $V_c = 0$ yields $U_s^d - U_c = e^{-\sigma}\left(J_s^d + k + W_s^d\right) + (1-e^{-\sigma})\delta U_c - U_c =$ $e^{-\sigma}\left(\frac{1-\delta(1-b)k+\delta b U_c}{1-\delta(1-b)}\right) + (1-e^{-\sigma})\delta U_c - U_c$, and using (61) results in

$$U_s^d - U_c = \frac{1}{\theta}\left(k - (\theta + (1-\theta)\delta(1-b)k)\left(1-e^{-\sigma}\right)\right). \tag{63}$$

We have $\frac{\partial\left(U_s^d - U_c\right)}{\partial \sigma} < 0$, and $\frac{\partial\left(U_s^d - U_c\right)}{\partial \theta} = -\frac{k}{\theta^2}(1-\delta(1-b)(1-e^{-\sigma})) < 0$.

We have $V_s^d = 0$ if $\theta = \frac{\delta(1-b)k(1-e^{-\sigma}-\sigma e^{-\sigma})}{k-(1-\delta(1-b)k)(1-e^{-\sigma}-\sigma e^{-\sigma})}$, and $U_s^d = U_c$ if θ $= \frac{(1-\delta(1-b)(1-e^{-\sigma}))k}{(1-\delta(1-b)k)(1-e^{-\sigma})}$. Then $V_s^d = 0$ and $U_s^d = U_c$ at $\hat{\sigma}$ if $\frac{\delta(1-b)k\left(1-e^{-\hat{\sigma}}-\hat{\sigma}e^{-\hat{\sigma}}\right)}{k-(1-\delta(1-b)k)(1-e^{-\hat{\sigma}}-\hat{\sigma}e^{-\hat{\sigma}})} = \frac{\left(1-\delta(1-b)\left(1-e^{-\hat{\sigma}}\right)\right)k}{(1-\delta(1-b)k)(1-e^{-\hat{\sigma}})}$, and then $\frac{1-e^{-\hat{\sigma}}-\hat{\sigma}e^{-\hat{\sigma}}}{1-\delta(1-b)\hat{\sigma}e^{-\hat{\sigma}}} = k$. Using this we have $\hat{\theta} = \frac{1-e^{-\hat{\sigma}}-\hat{\sigma}e^{-\hat{\sigma}}}{1-e^{-\hat{\sigma}}} < 1$.That is, $V_s^d = 0$ and $U_s^d = U_c$ at $\left(\hat{\sigma}, \hat{\theta}\right)$. If $\theta < \hat{\theta}$, then $V_s^d > 0$ and $U_s^d > U_c$ at $\hat{\sigma}$.Then there exist $\sigma_1 < \hat{\sigma}$ such that $V_s^d(\sigma_1) = 0$, and $\sigma_2 > \hat{\sigma}$ such that $U_s^d(\sigma_2) = U_c$. Then $V_s^d \geq 0$ and $U_s^d \geq U_c$ if $\theta \leq \hat{\theta}$ and $\sigma \in [\sigma_1, \sigma_2]$.

Proposition 16 *A CM equilibrium exists (does not exist) if* $\theta > (\leq) \frac{1-e^{-\hat{\sigma}}-\hat{\sigma}e^{-\hat{\sigma}}}{1-e^{-\hat{\sigma}}}$ *where* $\hat{\sigma}$ *satisfies* $\frac{1-e^{-\hat{\sigma}}-\hat{\sigma}e^{-\hat{\sigma}}}{1-\delta(1-b)\hat{\sigma}e^{-\hat{\sigma}}} = k$.

Proof. If $\theta \leq \frac{1-e^{-\hat{\sigma}}-\hat{\sigma}e^{-\hat{\sigma}}}{1-e^{-\hat{\sigma}}}$ where $\hat{\sigma}$ satisfies $\frac{1-e^{-\hat{\sigma}}-\hat{\sigma}e^{-\hat{\sigma}}}{1-\delta(1-b)\hat{\sigma}e^{-\hat{\sigma}}} = k$, we find $\sigma \in [\sigma_1, \sigma_2]$ such that $V_s^d \geq 0$ and $U_s^d \geq U_c$. If $\theta > \frac{1-e^{-\hat{\sigma}}-\hat{\sigma}e^{-\hat{\sigma}}}{1-e^{-\hat{\sigma}}}$ then either $V_s^d < 0$ or $U_s^d < U_c$ or both. ■

We collect the above results as

Theorem 2 *Let $\hat{\sigma}$ satisfy $\frac{1-e^{-\hat{\sigma}}-\hat{\sigma}e^{-\hat{\sigma}}}{1-\delta(1-b)\hat{\sigma}e^{-\hat{\sigma}}} = k$. A CM equilibrium exists if $\frac{1-e^{-\hat{\sigma}}-\hat{\sigma}e^{-\hat{\sigma}}}{1-e^{-\hat{\sigma}}} < \theta < 1+\hat{\sigma}$. A CM equilibrium does not exist if $\theta \geq 1+\hat{\sigma}$ or $\theta \leq \frac{1-e^{-\hat{\sigma}}-\hat{\sigma}e^{-\hat{\sigma}}}{1-e^{-\hat{\sigma}}}$.*

Proof. If $\theta > 1+\hat{\sigma}$ where $\hat{\sigma}$ satisfies $\frac{1-e^{-\hat{\sigma}}-\hat{\sigma}e^{-\hat{\sigma}}}{1-\delta(1-b)\hat{\sigma}e^{-\hat{\sigma}}} = k$, then we find $\sigma \in (\sigma_1, \sigma_2)$ such that $V_s^d > 0$ and $U_s^d > U_c$.If $\theta \leq 1+\hat{\sigma}$, then either $V_s^d \leq 0$ or $U_s^d \leq U_c$. If $\theta < \frac{1-e^{-\hat{\sigma}}-\hat{\sigma}e^{-\hat{\sigma}}}{1-e^{-\hat{\sigma}}}$ where $\hat{\sigma}$ is determined by $\frac{1-e^{-\hat{\sigma}}-\hat{\sigma}e^{-\hat{\sigma}}}{1-\delta(1-b)\hat{\sigma}e^{-\hat{\sigma}}} = k$, then there exists $\sigma \in (\sigma_1, \sigma_2)$ such that $V_s^d > 0$ and $U_s^d > U_c$. If $\theta \geq \frac{1-e^{-\hat{\sigma}}-\hat{\sigma}e^{-\hat{\sigma}}}{1-e^{-\hat{\sigma}}}$ then either $V_s^d \leq 0$ or $U_s^d \leq U_c$. ■

5.3 SM Equilibrium

The value functions for unmatched agents are (35) and (36). The value of U_s is given by (37) and (38), and making the solutions equal yields $\frac{1-e^{-\theta}-\theta e^{-\theta}}{1-\delta(1-b)\theta e^{-\theta}} = k$. This determines the equilibrium value of θ. Suppose that a coalition of ηu workers and μv firms can deviate for one period to CM where all agents on the short side match. Suppose $\eta u > \mu v$, then $\phi \equiv \frac{\eta u}{\mu v} > 1$. In CM a worker matches with probability $1/\phi < 1$, and a firm matches with probability one. The value functions for unmatched agents are $U_c^d = (1/\phi) W_c^d + (1-(1/\phi))\delta U_s$, and $V_c^d = J_c^d$. For the matched agents in CM we have $W_c^d = w_c^d + \delta\left((1-b) W_c^d + bU_s\right)$, and $J_c^d = 1-k-w_c^d + \delta\left((1-b) J_c^d + bV_s\right)$. Setting $V_s = 0$ gives $V_c^d = \frac{1-k-w_c^d}{1-\delta(1-b)} > 0$ if $w_c^d < 1-k$. Using $W_c^d = \frac{w_c^d + b\delta U_s}{1-\delta(1-b)}$ gives $U_c^d - U_s = \frac{1}{\phi}\frac{w_c^d+\delta bU_s}{1-\delta(1-b)} - \left(1-\delta\left(1-\frac{1}{\phi}\right)\right)U_s > 0$ if $w_c^d > (1-\delta)\left(\delta(1-b)+(1-\delta(1-b))\phi\right)U_s$. Using (37) gives $U_c^d > U_s$ if $w_c^d > \frac{e^{-\theta}(1-\delta(1-b)k)\left(\delta(1-b)+(1-\delta(1-b))\phi\right)}{1-\delta(1-b)(1-e^{-\theta})}$. Using $\frac{1-e^{-\theta}-\theta e^{-\theta}}{1-\delta(1-b)\theta e^{-\theta}} = k$ we have $U_c^d > U_s$ if $w_c^d > \frac{e^{-\theta}\left(\delta(1-b)+(1-\delta(1-b))\phi\right)}{1-\delta(1-b)\theta e^{-\theta}}$, and $V_c^d > 0$ if $w_c^d < \frac{e^{-\theta}(1+\theta-\delta(1-b)\theta)}{1-\delta(1-b)\theta e^{-\theta}}$. That is, $V_c^d > 0$ and $U_c^d > U_s$ only if

$\dfrac{e^{-\theta}\left(\delta\left(1-b\right)+\left(1-\delta\left(1-b\right)\right)\phi\right)}{1-\delta\left(1-b\right)\theta e^{-\theta}} < w_c^d < \dfrac{e^{-\theta}\left(1+\theta-\delta\left(1-b\right)\theta\right)}{1-\delta\left(1-b\right)\theta e^{-\theta}}$. This holds only if $\phi < 1+\theta \Leftrightarrow \dfrac{\eta}{\mu} < 1+\dfrac{1}{\theta}$. Together with $\phi = \dfrac{\eta\theta}{\mu} > 1$, there exists w_c^d such that $V_c^d > 0$ and $U_c^d > U_s$ if $\dfrac{1}{\theta} < \dfrac{\eta}{\mu} < 1+\dfrac{1}{\theta}$. We can state

Proposition 17 *An SM equilibrium does not exist because there is a coalition $(\eta u, \mu v)$ such that $V_c^d > 0$ and $U_c^d > U_s$.*

5.4 Efficiency

Let us compare the net production per worker, $Q = \dfrac{\left(L-\hat{u}\right)\left(1-k\right)-\hat{v}k}{L}$, in the different equilibria. Notice that in mixed market equilibrium we have, like in an SM-equilibrium, $\hat{v} = \dfrac{\hat{u}+\left(1-\theta\right)b\left(L-\hat{u}\right)}{\theta}$ and $\hat{u} = \dfrac{v\theta - bL}{1-b}$.

(i) In a high wage MM equilibrium firms match with probability one in CM submarket. In a steady state $\left(1-\tau\right)v+\tau v\left(1-e^{-\sigma}\right) = b\left(L-\hat{u}_h\right)$, which gives $\hat{u}_h = \dfrac{1}{b}\left(bL - v + v\tau e^{-\sigma}\right)$. Setting $\hat{u}_h = \dfrac{v\theta_h - bL}{1-b}$ gives $v = \dfrac{bL}{b\theta_h+\left(1-b\right)\left(1-\tau e^{-\sigma}\right)}$.Then $\hat{u}_h = \dfrac{\left(\theta_h+\tau e^{-\sigma}-1\right)bL}{b\theta_h+\left(1-b\right)\left(1-\tau e^{-\sigma}\right)}$ and $\hat{v}_h = \dfrac{\tau e^{-\sigma}bL}{b\theta_h+\left(1-b\right)\left(1-\tau e^{-\sigma}\right)}$. Using $\theta_h = \dfrac{\tau\sigma}{\omega_h} = 1-\tau+\sigma$ gives $\hat{u}_h = \dfrac{\left(\sigma-\tau\left(1-e^{-\sigma}\right)\right)bL}{b\left(1-\tau+\sigma\right)+\left(1-b\right)\left(1-\tau e^{-\sigma}\right)}$ and $\hat{v}_h = \dfrac{\tau e^{-\sigma}bL}{b\left(1-\tau+\sigma\right)+\left(1-b\right)\left(1-\tau e^{-\sigma}\right)}$. Then we have $Q_h = \dfrac{\left(1-k\right)\left(1-\tau e^{-\sigma}\right)-bk\tau e^{-\sigma}}{\left(1-b\right)\left(1-\tau e^{-\sigma}\right)+b\left(1-\tau+\sigma\right)}$.

(ii) In a low wage MM equilibrium firms match with probability $\phi < 1$ in the CM submarket. In a steady state $\left(1-\tau\right)v\phi+\tau v\left(1-e^{-\sigma}\right) = b\left(L-\hat{u}_l\right)$, which gives $\hat{u}_l = \dfrac{1}{b}\left(bL+v\tau\left(\phi+e^{-\sigma}-1\right)-v\phi\right)$. Setting $\hat{u}_l = \dfrac{v\theta_l - bL}{1-b}$ gives

$v = \dfrac{bL}{b\theta_l+\left(1-b\right)\left(\tau\left(1-e^{-\sigma}\right)+\phi\left(1-\tau\right)\right)}$. Then $\hat{u}_l = \dfrac{\left(\theta_l-\phi+\tau\left(\phi+e^{-\sigma}-1\right)\right)bL}{b\theta_l+\left(1-b\right)\left(\tau\left(1-e^{-\sigma}\right)+\phi\left(1-\tau\right)\right)}$,

and $\hat{v}_l = \dfrac{\left(\left(1-\tau\right)\left(1-\phi\right)+\tau e^{-\sigma}\right)bL}{b\theta_l+\left(1-b\right)\left(\tau\left(1-e^{-\sigma}\right)+\phi\left(1-\tau\right)\right)}$. Using $\theta_l = \dfrac{\tau\sigma}{\omega_l} = \dfrac{1-e^{-\sigma}-\sigma e^{-\sigma}+\tau\left(\sigma+e^{-\sigma}-1\right)}{1-e^{-\sigma}}$ and $\phi = \dfrac{1-e^{-\sigma}-\sigma e^{-\sigma}}{1-e^{-\sigma}}$ gives

$$\hat{u}_l = \frac{\tau\left(1-e^{-\sigma}\right)\left(\sigma+e^{-\sigma}-1\right)bL}{b\left(1-e^{-\sigma}-\sigma e^{-\sigma}+\tau\left(\sigma+e^{-\sigma}-1\right)\right)+\left(1-b\right)\left(\tau\left(1-e^{-\sigma}\right)^2+\left(1-e^{-\sigma}-\sigma e^{-\sigma}\right)\left(1-\tau\right)\right)}$$

and

$$\hat{v}_l = \frac{\left(\left(1-\tau\right)\sigma e^{-\sigma}+\tau e^{-\sigma}\left(1-e^{-\sigma}\right)\right)bL}{b\left(1-e^{-\sigma}-\sigma e^{-\sigma}+\tau\left(\sigma+e^{-\sigma}-1\right)\right)+\left(1-b\right)\left(\tau\left(1-e^{-\sigma}\right)^2+\left(1-e^{-\sigma}-\sigma e^{-\sigma}\right)\left(1-\tau\right)\right)}.$$

Then we have $Q_l = \dfrac{\left(1-e^{-\sigma}\right)\left(1-k\right)-e^{-\sigma}\left(\left(1-\tau\right)\sigma+\tau\left(1-e^{-\sigma}\right)\right)\left(1-\left(1-b\right)k\right)}{1-e^{-\sigma}-\sigma e^{-\sigma}+\tau\left(\sigma+e^{-\sigma}-1\right)\left(e^{-\sigma}+b\left(1-e^{-\sigma}\right)\right)}$.

Using (26) gives $Q_h = \dfrac{(1-\delta(1-b)\sigma e^{-\sigma})(1-\tau e^{-\sigma}) - (1-e^{-\sigma}-\sigma e^{-\sigma})(1-\tau(1-b)e^{-\sigma})}{(1-\delta(1-b)\sigma e^{-\sigma})((1-b)(1-\tau e^{-\sigma}) + b(1-\tau+\sigma))}$,

and $Q_l = \dfrac{\begin{array}{c}(1-e^{-\sigma}-\sigma e^{-\sigma})e^{-\sigma}(1+(1-b)(1-\delta)\sigma) \\ +\tau e^{-\sigma}(e^{-\sigma}+\sigma-1)(1-(1-b)(1-e^{-\sigma}-(1-\delta)\sigma e^{-\sigma}))\end{array}}{(1-\delta(1-b)\sigma e^{-\sigma})\left(\begin{array}{c}1-e^{-\sigma}-\sigma e^{-\sigma} \\ +\tau(\sigma+e^{-\sigma}-1)(e^{-\sigma}+b(1-e^{-\sigma}))\end{array}\right)}$.

Remark 2 *$Q_l > Q_h$, given the value of τ.*

Proof. We have $Q_l > Q_h$ if $(L-\hat{u}_l)(1-k) - \hat{v}_l k - (L-\hat{u}_h)(1-k) + \hat{v}_h k = \hat{u}_h - \hat{u}_l + k(\hat{v}_h - \hat{u}_h + \hat{u}_l - \hat{v}_l) > 0$. Using the above solutions for $\hat{u}_h$, $\hat{v}_h$, $\hat{u}_l$ and $\hat{v}_l$ we have $\hat{u}_h - \hat{u}_l + k(\hat{v}_h - \hat{u}_h + \hat{u}_l - \hat{v}_l)$

$$= \frac{(1-\tau)\sigma(1-e^{-\sigma}-\sigma e^{-\sigma}-(1-(1-b)\sigma e^{-\sigma})k)bL}{(b(1-\tau+\sigma)+(1-b)(1-\tau e^{-\sigma}))\left(\begin{array}{c}b(1-e^{-\sigma}-\sigma e^{-\sigma}+\tau(\sigma+e^{-\sigma}-1)) \\ +(1-b)\left(\tau(1-e^{-\sigma})^2+(1-e^{-\sigma}-\sigma e^{-\sigma})(1-\tau)\right)\end{array}\right)}$$

where the denominator is positive. Using (26) the numerator is $\dfrac{(1-\tau)(1-\delta)(1-b)b\sigma^2 e^{-\sigma}(1-e^{-\sigma}-\sigma e^{-\sigma})L}{1-\delta(1-b)\sigma e^{-\sigma}} > 0$, and then $Q_l > Q_h$. ■

We notice that $\dfrac{\partial Q_l}{\partial \tau} = \dfrac{(1-\delta)b(1-b)\sigma e^{-\sigma}(1-e^{-\sigma})\left((1-e^{-\sigma})^2 - \sigma(1-\sigma e^{-\sigma}-e^{-2\sigma})\right)}{\begin{array}{c}(1-\delta(1-b)\sigma e^{-\sigma}) \\ ((1-e^{-\sigma}-\sigma e^{-\sigma}+\tau((\sigma+e^{-\sigma}-1)(b(1-e^{-\sigma})+e^{-\sigma}))))^2\end{array}}$

< 0, and $\dfrac{\partial Q_h}{\partial \tau} = \dfrac{(1-\delta)b(1-b)\sigma e^{-\sigma}(1-e^{-\sigma}-\sigma e^{-\sigma})}{(1-\delta(1-b)\sigma e^{-\sigma})((1-b)(1-\tau e^{-\sigma})+b(1-\tau+\sigma))^2} > 0$.

(iii) Consider a CM equilibrium where $\dfrac{1-e^{-\sigma}-\sigma e^{-\sigma}}{1-e^{-\sigma}} < \theta < 1$. A firm matches with probability θ, and a worker matches with probability one. During a production period there are vacancies but no unemployed workers. Then $\hat{u} = 0$. Then $v = \hat{v} + b(L-\hat{u}) = \hat{v} + bL$. In a steady state $\theta v = bL$, then $\theta(\hat{v}+bL) = bL$ gives $\hat{v} = bL\left(\dfrac{1}{\theta}-1\right)$. The net production per worker is $Q_{CM1} = \dfrac{1}{L}(L(1-k)-\hat{v}k) = 1-(1-b)k - \dfrac{bk}{\theta}$. Because $\theta > \dfrac{1-e^{-\sigma}-\sigma e^{-\sigma}}{1-e^{-\sigma}}$, then $Q_{CM1} > 1-(1-b)k - \dfrac{bk(1-e^{-\sigma})}{1-e^{-\sigma}-\sigma e^{-\sigma}}$. Using (26) gives $Q_{CM1} > \dfrac{e^{-\sigma}(1+(1-\delta)(1-b)\sigma)}{1-\delta(1-b)\sigma e^{-\sigma}}$. Using the solution for Q_l above we have

$$\frac{e^{-\sigma}(1+(1-\delta)(1-b)\sigma)}{1-\delta(1-b)\sigma e^{-\sigma}} - Q_l = \frac{\tau(1-\delta)b(1-b)\sigma e^{-\sigma}(1-e^{-\sigma})(\sigma+e^{-\sigma}-1)}{(1-\delta(1-b)\sigma e^{-\sigma})\left(\begin{array}{c}1-e^{-\sigma}-\sigma e^{-\sigma} \\ +\tau(\sigma+e^{-\sigma}-1)(e^{-\sigma}+b(1-e^{-\sigma}))\end{array}\right)}$$

> 0. Then $Q_{CM1} > Q_l > Q_h$.

(iv) Consider a CM equilibrium where $1 < \theta < 1+\sigma$. A firm matches with probability one, and a worker matches with probability $1/\theta$. Then $\hat{v} = 0$. Also, $u = \hat{u} + b(L-\hat{u})$. In a steady state $v = b(L-\hat{u}) \Leftrightarrow u/\theta = b(L-\hat{u}) \Leftrightarrow (\hat{u}+b(L-\hat{u}))/\theta = b(L-\hat{u})$ which gives $\hat{u} = \dfrac{b(\theta-1)L}{1+b(\theta-1)}$. Then $Q_{CM2} = \dfrac{(L-\hat{u})(1-k)}{L} = \dfrac{1-k}{1+b(\theta-1)}$. Using (26) gives

$$Q_{CM2} = \frac{1-\delta(1-b)\sigma e^{-\sigma} - (1-e^{-\sigma}-\sigma e^{-\sigma})}{(1+b(\theta-1))(1-\delta(1-b)\sigma e^{-\sigma})}.$$

Let us first compare Q_{CM2} and Q_h. We have $Q_{CM2} > (<)\, Q_h$ if $\theta < (>)\, \bar{\theta}_h$ $= \frac{\sigma(1-\tau+\sigma)(1-\delta(1-b)) + (1+\sigma)(1-\tau(1-b)e^{-\sigma}) - \tau b}{1-\tau b + (1-\delta(1-b))\sigma - \tau(1-b)(1+(1-\delta)\sigma)e^{-\sigma}}$, where $\bar{\theta}_h \in (1, 1+\sigma)$ because the numerator is larger than $(1-\tau+\sigma)(1+(1-\delta(1-b))\sigma) > 0$, the denominator is larger than $\sigma(1-(1-b)(\delta+(1-\delta)e^{-\sigma})) > 0$, and numerator minus denominator is equal to $\sigma(1-\tau+\sigma+\delta(1-b)(\tau(1-e^{-\sigma})-\sigma)) > \sigma(1-\tau+\sigma)(1-\delta(1-b))$ > 0. Also, $1+\sigma-\bar{\theta}_h = \frac{\tau(1-\delta)(1-b)\sigma(1-e^{-\sigma}-\sigma e^{-\sigma})}{1-\tau b+(1-\delta(1-b))\sigma-\tau(1-b)(1+(1-\delta)\sigma)e^{-\sigma}} > 0$.We have $\frac{\partial\bar{\theta}_h}{\partial\tau} = \frac{(1-\delta)(1-b)\sigma(1+(1-\delta(1-b))\sigma)(e^{-\sigma}+\sigma e^{-\sigma}-1)}{(1-\tau b+(1-\delta(1-b))\sigma-\tau(1-b)(1+(1-\delta)\sigma)e^{-\sigma})^2} < 0$.

Compare then Q_{CM2} and Q_l. We have $Q_{CM2} > (<)\, Q_l$ if $\theta < (>)\, \bar{\theta}_l$ $= 1 + \frac{\sigma(1-e^{-\sigma}-\sigma e^{-\sigma}+\tau(\sigma+e^{-\sigma}-1)(1-\delta(1-b)(1-e^{-\sigma})))}{(1+(1-\delta)(1-b)\sigma)(1-e^{-\sigma}-\sigma e^{-\sigma}) + \tau(\sigma+e^{-\sigma}-1)(1-(1-b)(1-e^{-\sigma}-(1-\delta)\sigma e^{-\sigma}))}$ where $\bar{\theta}_l \in (1, 1+\sigma)$:

Clearly, $\bar{\theta}_l > 1$. Also, $\bar{\theta}_l < 1+\sigma$ if $\frac{1-e^{-\sigma}-\sigma e^{-\sigma}+\tau(\sigma+e^{-\sigma}-1)(1-\delta(1-b)(1-e^{-\sigma}))}{(1+(1-\delta)(1-b)\sigma)(1-e^{-\sigma}-\sigma e^{-\sigma}) + \tau(\sigma+e^{-\sigma}-1)(1-(1-b)(1-e^{-\sigma}-(1-\delta)\sigma e^{-\sigma}))} <$ 1 which holds because numerator minus denominator is equal to $(1-\delta)(1-b)(e^{-\sigma}+\sigma e^{-\sigma}-1)(\sigma(1-\tau)+\tau(1-e^{-\sigma})) < 0$.We have

$$\frac{\partial\bar{\theta}_l}{\partial\tau} = \frac{\begin{array}{c}(1-\delta)(b-1)\sigma(1-e^{-\sigma})(1+(1-\delta(1-b))\sigma)\\ (1-\sigma-(2-\sigma^2)e^{-\sigma}+(1+\sigma)e^{-2\sigma})\end{array}}{\left(\begin{array}{c}(1+(1-\delta)(1-b)\sigma)(1-e^{-\sigma}-\sigma e^{-\sigma})\\ +\tau(\sigma+e^{-\sigma}-1)(1-(1-b)(1-e^{-\sigma}-(1-\delta)\sigma e^{-\sigma}))\end{array}\right)^2} > 0$$

Then, if $1 < \theta < 1+\sigma$, there exists $\bar{\theta}_h \in (1, 1+\sigma)$ such that $Q_{CM2}(\bar{\theta}_h) = Q_h$. Then $Q_{CM2} > Q_h$ if $\theta < \bar{\theta}_h$, and $Q_{CM2} < Q_h$ if $\theta > \bar{\theta}_h$. Also, $\frac{\partial\bar{\theta}_h}{\partial\tau} < 0$. There exists also $\bar{\theta}_l \in (1, 1+\sigma)$ such that $Q_{CM2}(\bar{\theta}_l) = Q_l$. Then $Q_{CM2} > Q_l$ if $\theta < \bar{\theta}_l$, and $Q_{CM2} < Q_l$ if $\theta > \bar{\theta}_l$. Also, $\frac{\partial\bar{\theta}_l}{\partial\tau} > 0$. Collect the results in

Remark 3 *We have* $\bar{\theta}_l < \bar{\theta}_h$. *If* $\theta < \bar{\theta}_l$, *then* $Q_h < Q_l < Q_{CM2}$. *If* $\bar{\theta}_l < \theta < \bar{\theta}_h$, *then* $Q_h < Q_{CM2} < Q_l$. *If* $\theta > \bar{\theta}_h$, *then* $Q_{CM2} < Q_h < Q_l$.

Proof. We showed that $Q_h < Q_l$. If $\theta < \bar{\theta}_l$, then $Q_h < Q_l < Q_{CM2}$, and if $\theta > \bar{\theta}_l$, then $Q_{CM2} < Q_l$. If $\theta < \bar{\theta}_h$, then $Q_h < Q_{CM2}$, and if $\theta > \bar{\theta}_h$, then $Q_{CM2} < Q_h < Q_l$. Suppose $\bar{\theta}_h < \bar{\theta}_l$. Then, if $\bar{\theta}_h < \theta < \bar{\theta}_l$, we have $Q_{CM2} < Q_h < Q_l$ and $Q_h < Q_l < Q_{CM2}$ which cannot hold. Therefore $\bar{\theta}_l < \bar{\theta}_h$, and the rest of the result follows. ■

Using the results above we have

Proposition 18 *If* $\frac{1-e^{-\sigma}-\sigma e^{-\sigma}}{1-e^{-\sigma}} < \theta < 1$, *then* $Q_{CM} > Q_l > Q_h$ *for given* τ *and* σ. *If* $1 < \theta < 1+\sigma$, *then (i)* $Q_h < Q_l < Q_{CM2}$ *if* $\theta < \bar{\theta}_l$, *(ii)* $Q_h < Q_{CM2} < Q_l$ *if* $\bar{\theta}_l < \theta < \bar{\theta}_h$, *and (iii)* $Q_{CM2} < Q_h < Q_l$ *if* $\theta > \bar{\theta}_h$, *where* $1 < \bar{\theta}_l < \bar{\theta}_h < 1+\sigma$.

That is, a centralized market equilibrium is more efficient than a mixed market equilibrium if $\theta < 1$. If $1 < \theta < 1 + \sigma$, a centralized market equilibrium is more efficient than the mixed market equilibria if θ is small enough. If θ is large enough, a centralized market equilibrium is less efficient than either of the mixed market equilibria.

6 A Static Model with a Fixed Number of Firms and Price Posting in Both Markets

Consider a static model where u and v are fixed. This is equivalent to setting $\delta = 0$ and $b = 1$, and setting θ as a parameter. We set $k = 0$ as capital cost is not needed to regulate the number of firms. The model is the same as Kultti (2011) except that the number of unemployed (corresponding to buyers in Kultti's article) is deterministic, not stochastic. We show that mixed market equilibria exist.

6.1 Mixed Market Equilibrium

We solve the (τ, ω) relation in a mixed market equilibrium. As before, we study three cases which differ in the u/v ratio in CM.

(i) There are more workers than firms in CM submarket, and then firms there match with probability one, and workers match with probability $p = \dfrac{1-\tau}{(1-\omega_1)\theta} < 1$. The value functions for unmatched agents are $U_c = pw_{c1}$, $U_s = e^{-\sigma_1}$, $V_c = 1 - w_{c1}$, and $V_s = 1 - e^{-\sigma_1} - \sigma_1 e^{-\sigma_1}$, where $\sigma_1 = \dfrac{\omega_1\theta}{\tau}$. Then $V_c = V_s$ if $w_{c1} = (1+\sigma_1)\, e^{-\sigma_1}$, and then $U_c = U_s$ if $p = \dfrac{1}{1+\sigma_1}$. Using $p = \dfrac{1-\tau}{(1-\omega_1)\theta}$ and $\sigma_1 = \dfrac{\omega_1\theta}{\tau}$ equation $p = \dfrac{1}{1+\sigma_1}$ gives $\omega_1 = \dfrac{\tau}{\theta}(\tau+\theta-1) < \tau$, and $\dfrac{d\omega_1}{d\tau} > 0$. If $\theta \geq 1$, then $\omega_1 > 0$ if $\tau > 0$. If $\theta < 1$, then $\omega_1 > 0$ if $\tau > 1-\theta$. There is a continuum of mixed market equilibria where $(\tau, \omega_1) = \left(\tau, \dfrac{\tau}{\theta}(\tau+\theta-1)\right)$, where $\omega_1 < \tau$. We have $\dfrac{\partial\omega_1}{\partial\tau} > 0$, and $\dfrac{\partial\omega_1}{\partial\theta} = \dfrac{1}{\theta^2}\tau(1-\tau) > 0$.

We have $\sigma_1 = \theta + \tau - 1$.Then $w_{c1} = (1+\sigma_1)\, e^{-\sigma_1} = (\theta+\tau)\, e^{-(\theta+\tau-1)}$, and $\dfrac{\partial w_{c1}}{\partial\theta} = \dfrac{\partial w_{c1}}{\partial\tau} = -\sigma_1 e^{-\sigma_1} < 0$. That is, if firms choose τ and w_{c1} such that $w_{c1} = (\theta+\tau)\, e^{-(\theta+\tau-1)}$ where $\tau > 1-\theta$, then $\omega_1 = \dfrac{\tau}{\theta}(\tau+\theta-1)$ gives $U_c = U_s$ and $V_c = V_s$. We have $w_{c1} = (1+\sigma_1)\, e^{-\sigma_1} > e^{-\sigma_1} > e^{-\theta}$ because $\omega_1 < \tau$.

Proposition 19 *There is a continuum of mixed market equilibria where*

$$w_{c1} = (\theta+\tau)\, e^{-(\theta+\tau-1)} > e^{-\theta},\ \omega_1 = \frac{\tau}{\theta}(\tau+\theta-1) < \tau,\ \textit{and}\ \frac{d\omega_1}{d\tau} > 0.$$

(ii) There are more firms than workers in CM submarket, and then workers there match with probability one, and firms match with probability $q = \dfrac{(1-\omega_2)\theta}{1-\tau} < 1$. The value functions for unmatched agents are $U_c = w_{c2}$, $U_s = e^{-\sigma_2}$, $V_c = q(1-w_{c2})$, and $V_s = 1 - e^{-\sigma_2} - \sigma_2 e^{-\sigma_2}$. Then $U_c = U_s$ and $V_c = V_s$ if $w_{c2} = e^{-\sigma_2} = 1 - \frac{1}{q}\left(1 - e^{-\sigma_2} - \sigma_2 e^{-\sigma_2}\right)$, which gives $q = \frac{1 - e^{-\sigma_2} - \sigma_2 e^{-\sigma_2}}{1 - e^{-\sigma_2}}$. This, together with $q = \frac{(1-\omega_2)\theta}{1-\tau}$ and $\theta = \frac{\tau\sigma_2}{\omega_2}$ gives $\omega_2 = \frac{\tau\sigma_2\left(1 - e^{-\sigma_2}\right)}{1 - e^{-\sigma_2} - \sigma_2 e^{-\sigma_2} + \tau\left(\sigma_2 + e^{-\sigma_2} - 1\right)} > \tau$. Because $\frac{\partial q}{\partial \omega_2} = \frac{\theta e^{-\sigma_2}\left(\sigma_2 + e^{-\sigma_2} - 1\right)}{\tau\left(1 - e^{-\sigma_2}\right)^2} > 0$, there exists a unique ω_2 for given (θ, τ). Equation $\frac{1 - e^{-\sigma_2} - \sigma_2 e^{-\sigma_2}}{1 - e^{-\sigma_2}} = \frac{(1-\omega_2)\theta}{1-\tau}$ gives $\frac{d\omega_2}{d\tau} > 0$.

Using $\omega_2 > \tau$ we have $w_{c2} < e^{-\theta}$. Also, $w_{c2} = e^{-\sigma_2}$ and $\sigma_2 = \frac{\omega_2\theta}{\tau}$ give $\omega_2 = \frac{-\tau \ln w_{c2}}{\theta}$. The solutions for ω_2 give $\tau = \frac{(1-\theta)\left(1 - e^{-\sigma_2}\right) + e^{-\sigma_2}\ln w_c}{1 - e^{-\sigma_2} + \ln w_c}$. Using $w_{c2} = e^{-\sigma_2}$ gives $\tau = \frac{(\theta - 1)(1 - w_{c2}) - w_{c2}\ln w_{c2}}{w_{c2} - \ln w_{c2} - 1}$.

Proposition 20 *There is a continuum of mixed market equilibria where* $w_c < e^{-\theta}$, $\tau = \frac{(\theta - 1)(1 - w_{c2}) - w_{c2}\ln w_{c2}}{w_{c2} - \ln w_{c2} - 1}$, $\omega_2 = \frac{-\tau \ln w_c}{\theta} > \tau$, *and* $\frac{d\omega_2}{d\tau} > 0$.

(iii) There are equally many firms and workers in CM

The value functions for unmatched agents are $U_c = w_{c3}$, $U_s = e^{-\sigma_3}$, $V_c = 1 - w_{c3}$, and $V_s = 1 - e^{-\sigma_3} - \sigma_3 e^{-\sigma_3}$. Then $V_c = V_s$ gives $w_{c3} = (1 + \sigma_3)e^{-\sigma_3}$, and $U_c = U_s$ gives $w_{c3} = e^{-\sigma_3}$. Then $V_c = V_s$ and $U_c = U_s$ only if $(1 + \sigma_3)e^{-\sigma_3} = e^{-\sigma_3}$. This holds only if $\sigma_3 = 0$ which holds only if $\omega_3 = 0$ or $\theta = 0$. This means that a mixed market equilibrium does not exist.

Kultti (2011) analyzes an otherwise similar setting, except that the number of moving agents (buyers in Kultti's article) is stochastic, and the stayers (sellers in Kultti's article) choose locations and prices before the number of buyers is realized. He finds that a mixed market equilibrium does not exist. In p. 17 he writes: "One can understand the result [non-existence of a mixed market equilibrium] even without stochastic demand. Assume that there is a fixed number of buyers and sellers. Assume further that half of them are in each market. In the non-clustered market there may be local under- or overdemand, and consequently the number of trades is smaller than in the clustered market where all the possible trades are realised. If the buyers in both markets are to do equally well they must get a larger share of realised trades in the non-clustered market than in the clustered market. But then the sellers in the non-clustered market necessarily fare worse than in the clustered market. Now, if the sellers are allowed to reallocate themselves some of them go to the clustered market."

Kultti's claim is premature. He only says that it is not true that in equilibrium *one half* of the buyers and *one half* of the sellers are in each market, and he concludes that the two markets do not coexist. I showed that there are two continua of mixed market equilibria. In one continuum, there are more workers than firms in the centralized market, and then $\omega < \tau$. In the other one, the centralized market has more firms than workers, and $\omega > \tau$. If there are equally many workers and firms in CM, a mixed market equilibrium does not exist.

6.2 CM Equilibrium

We check the existence of a centralized market equilibrium against a coalitional deviation to a decentralized market. The value functions for deviating agents are $U_s^d = e^{-\sigma}$ and $V_s^d = 1 - e^{-\sigma} - \sigma e^{-\sigma}$, where $\sigma = \eta u / \mu v = \eta\theta/\mu$, where ηu and μv are the sizes of deviating groups of workers and firms, respectively.

(i) Assume $\theta > 1$. Then in CM a firm matches with probability one, and a worker matches with probability $1/\theta < 1$. The value functions for unmatched agents in CM are $U_c = w_c/\theta$ and $V_c = 1 - w_c$ where $w_c \leq 1$. Then $U_s^d > U_c$ if $\theta e^{-\sigma} > w_c$, and $V_s^d > V_c$ if $(1+\sigma) e^{-\sigma} < w_c$. A coalitional deviation to SM is profitable if $(1+\sigma) e^{-\sigma} < w_c < \theta e^{-\sigma}$. If firms set $w_c < (1+\sigma) e^{-\sigma}$, there is no profitable deviation. But because a firm hires a worker with probability one anyhow, conditional that no deviation takes place, the equilibrium wage is $w_c = 0$.

(ii) Assume $\theta < 1$.

In CM a worker matches with probability one, and a firm matches with probability θ. The value functions for unmatched agents in CM are $U_c = w_c$, and $V_c = \theta(1 - w_c)$, where $w_c \leq 1$. Then $U_s^d > U_c$ if $e^{-\sigma} > w_c$, and $V_s^d > V_c$ if $w_c > 1 - \frac{1}{\theta}(1 - e^{-\sigma} - \sigma e^{-\sigma})$. There is a profitable deviation to SM if $1 - \frac{1}{\theta}(1 - e^{-\sigma} - \sigma e^{-\sigma}) < w_c < e^{-\sigma}$. As $1 - \frac{1}{\theta}(1 - e^{-\sigma} - \sigma e^{-\sigma})$ and $e^{-\sigma}$ decrease in σ, and $1 - \frac{1}{\theta}(1 - e^{-\sigma} - \sigma e^{-\sigma}) < (>) e^{-\sigma}$ if $\sigma > (<) \tilde{\sigma}$ where $\tilde{\sigma}$ satisfies $\frac{1 - e^{-\tilde{\sigma}} - \tilde{\sigma} e^{-\tilde{\sigma}}}{1 - e^{-\tilde{\sigma}}} = \theta$, a profitable deviation exists whenever $w_c < e^{-\tilde{\sigma}}$. Suppose all firms choose a uniform wage $w_c \in \left(e^{-\sigma^*}, 1\right)$. If firm i chooses $w_{ci} = w_c + \varepsilon_i$ where $\varepsilon_i > 0$, it hires a worker with probability one. It is indifferent between w_c and w_{ci} if $\varepsilon_i = (1-\theta)(1-w_c)$. Then firm i benefits if $\varepsilon_i \in (0, (1-\theta)(1-w_c))$. By this logic, a firm gains if it chooses a wage a bit larger than the rest of the firms. As all firms do this, wages increase until all firms pay $w_c = 1$.

(iii) Assume $\theta = 1$.

In CM firms and workers match with probability one. Then $U_c = w_c$ and $V_c = 1 - w_c$ where $w_c \leq 1$. Then $U_s^d > U_c$ if $e^{-\sigma} > w_c$, and $V_s^d > V_c$ if $1 - e^{-\sigma} - \sigma e^{-\sigma} > 1 - w_c$. There is a profitable deviation to SM if $(1+\sigma) e^{-\sigma} < w_c < e^{-\sigma}$ which cannot hold. Any $w_c \in [0, 1]$ is compatible with equilibrium.

We gather the above results in

Proposition 21 *A CM equilibrium exists. If $\theta > 1$, the equilibrium wage is $w_c = 0$. If $\theta < 1$, the equilibrium wage is $w_c = 1$. If $\theta = 1$, the equilibrium wage is $w_c \in [0, 1]$.*

6.3 SM Equilibrium

The value functions for unmatched agents are $U_s = e^{-\theta}$, and $V_s = 1 - e^{-\theta} - \theta e^{-\theta}$. Consider a coalition of ηu workers and μv firms which can deviate to CM. Suppose $\eta u > \mu v$, then $\phi \equiv \eta\theta/\mu > 1$. In CM a worker matches with probability $1/\phi < 1$, and a firm matches with probability one. The value functions for unmatched agents are $U_c^d = w_c/\phi$, and $V_c^d = 1 - w_c$. Then $U_c^d > U_s$ if $w_c > \phi e^{-\theta}$, and $V_c^d > V_s$ if $w_c < (1+\theta) e^{-\theta}$. A profitable deviation exists if $\phi e^{-\theta} < w_c < (1+\theta) e^{-\theta}$. Condition $\phi e^{-\theta} < (1+\theta) e^{-\theta}$ holds if $\frac{\eta}{\mu} < 1 + \frac{1}{\theta}$. Together with $\phi > 1$, a profitable deviation exists if $\frac{1}{\theta} < \frac{\eta}{\mu} < 1 + \frac{1}{\theta}$. Then there exist $w_c \in \left(\phi e^{-\theta}, (1+\theta) e^{-\theta}\right)$. That is, there is a profitable deviation for all $\theta > 0$, and a decentralized market equilibrium does not exist.

7 Conclusion

I have shown that a labor market with homogenous firms and workers can be purely centralized, or purely decentralized, and that both markets can coexist. The outcome depends on the way wages are determined, and on the relative magnitude of capital cost and firms' bargaining power. If wages are determined by bargaining in both markets, a decentralized market equilibrium does not exist. Both markets coexist only for a specific combination of capital cost and firms' bargaining power. If the latter is relatively large, a centralized market equilibrium exists. If wages are posted in the search market but there is bargaining in the centralized market, a relatively equal sharing of match surplus in the centralized market leads to a centralized market equilibrium. If the match surplus is shared relatively unequally, a decentralized market equilibrium exists. For specific combinations of bargaining parameter and capital cost both markets coexist.

I considered also a model where wages are posted in both markets. Then a decentralized market equilibrium does not exist. A centralized market equilibrium exists if the unemployment-vacancy ratio in the matching stage is not very small or very large. There are two continua of mixed market equilibria. Finally I showed that a static model with a fixed unemployment-vacancy ratio produces a centralized market equilibrium and two continua of mixed market equilibria, but no decentralized market equilibrium exists.

The real world labor markets share features of centralized and decentralized markets. Therefore it would be interesting to build a model where a centralized market and a search market coexist for a large range of parameter values. In the present model agents can visit only one market in a period, and therefore the two markets coexist only if both firms and workers are indifferent between the markets. If there is wage bargaining in the centralized market, the indifference holds only for a specific relation between capital cost and firms' bargaining power. The need for indifference is removed if for example workers can visit both markets in the same period. This is left for future work.

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