Abstract: Firstly we describe a research project on problem solving implemented in 2010–13 in the Department of Teacher Education at the University of Helsinki. But we are especially concentrating on the results of one background study in the project – pupils’ drawings in a mathematics lesson. Pupils’ drawings seem to be a powerful method to gather information from small children. With the aid of drawings one may investigate different topics in children’s thinking. Here we focus on pupils’ and teachers’ communication, the emotional atmosphere of the class, and the types of work used in class. The drawing studies offer three different channels to pupils’ conceptions in problem solving.

Key words: problem solving, mathematics lesson, pupils’ drawings, research tool

In Finland, there is a nine-year comprehensive school where all children study in heterogeneous groups, including in mathematics. Teaching in schools is regulated by the national curriculum (National Board of Education, 2004). The national curriculum emphasizes the importance of creating a learning environment having an open, encouraging, easygoing, and positive atmosphere, and that the responsibility to maintain this environment belongs to both the teacher and the pupils. Teaching mathematics in elementary grades is usually concentrated on the use of textbooks. Details on mathematics teaching in Finland can be found, for example, in the book by Pehkonen, Ahtee, and Lavonen (2007). Here we aim to clarify third graders’ (about nine years old) conceptions of mathematics and mathematics teaching for problem solving through their drawings.

Pupils’ conceptions are considered from the viewpoint of classroom communication, emotional atmosphere, and types of work. It is important to grasp what is happening in Finnish schools, because pupils’ attitudes in mathematics get increasingly worse after Grade 3 (Tuohilampi, Hannula, Laine, and Metsämuuronen 2014, p. 285).

The background study on pupils’ drawings came up surprisingly as an interesting new method to gather and interpret data on pupils’ mathematics-
related conceptions in a research project on problem solving. When we have presented our results at international conferences, many researchers from different countries have been interested in implementing the same task in their own country.

Therefore, we focus here on pupils’ drawings. The purpose of this study was to explore what can be said, based on the pupils’ drawings, about how third graders experience their mathematics lessons. In particular, we were curious about what communication on mathematics is like between the teacher and her pupils as well as the pupils’ emotional position.

In order that the teacher can be successful in guiding pupils’ problem solving lesson, he/she should be aware of pupils’ mathematical conceptions. These form basis for teaching problem solving. As background information for the drawing study, we first present the research project on problem solving from which our results come.

THE RESEARCH PROJECT

The three-year research project was run in the Department of Teacher Education at the University of Helsinki in the years 2010–13. The objectives of the project were to clarify the development of pupils’ and teachers’ mathematical understanding and problem-solving skills during 3 years – from Grade 3 to Grade 5 – when open problems were used regularly once a month. It was a joint comparative research project with Chile, and we aimed to parallel the Finnish and Chilean teaching practices in mathematics.

Data gathering

For the research project, we selected two groups of Grade 3 classes: one experimental group and one control group, for a total of 10 teachers. The experimental classes were from cities surrounding Helsinki (Vantaa, Espoo, and Kirkkonummi), and the control classes were from Helsinki. In both groups same background studies were implemented, but in the experimental group once a month there was an additional lesson on open problem solving that two researchers videotaped. In the control group, only initial and final measurements (background studies) were implemented.

Background studies

For the background studies of the research project (the initial measurements in 2010), we used different methods to uncover pupils’ mathematics-related beliefs and mathematical knowledge: a questionnaire of pupils’ conceptions of mathematics, pupils’ drawings in a mathematics lesson, a test on pupils’ mathematics knowledge and problem-solving skills, and a postal survey of teachers’ conceptions of problem solving.

1 The research project was financed by the Academy of Finland (Project 135556).
Implementation of the project

In the experimental group, the teachers taught on average one mathematics lesson per month dealing with open-ended problems, and for all other lessons they used their own teaching method. The teachers in the control group applied only their conventional methods for mathematics teaching. Data were gathered from pupil and teacher questionnaires, pupils’ drawings, teacher interviews, classroom observations and field-notes during the implementation of the open-problem-solving lessons, videotaped work, thinking-aloud protocols, and videotaped discussions.

We never expected that such a short interference in teaching (only once a month an open-problem lesson, and everything else conventional teaching) would result in a big change in mathematical knowledge or teaching habits. But we anticipated that when the teachers and pupils experienced open mathematics teaching (open problem solving), that would offer them an idea on an alternative way of teaching that might, with time, help them to change their understanding of mathematics and its teaching.

The experimental tasks used in the project were open problems where either the starting situation or the ending situation or both contained some additional options. Therefore, the problems did not have one definite answer, but they might have many different answers depending on the auxiliary conditions the solver put forward. Thus, solving these problems required that the solver must combine in a new way the information already familiar to him or her.

These tasks were introduced beforehand in the experimental group teachers’ and the researchers’ monthly joint meetings. In the meetings, the teachers helped us provide a proper wording and presentation mode for the tasks. But finally every teacher pondered for herself the implementation of the task in her own teaching group and gave us her lesson plan before the experimental lesson. Altogether 20 different open problems were dealt with in 3 years; they were on various topics of elementary mathematics: arithmetic, combinatorics, and geometry.

There are some published papers that describe the research project in more detail (e.g., Pehkonen et al., 2013).

Here we will restrict our discussion to the Finnish part of the project, and more specially to one of the background studies: The Pupils’ Drawing Task.

PUPILS’ DRAWINGS

The research study we are describing in this chapter is on pupils’ drawings and what we can gain from the drawings. We have selected the dealing with three different aspects in the drawings: (1) communication, (2) emotional atmosphere, and (3) types of work. The order of these aspects is chronological. We began about 5 years ago our research on drawings with the case of classroom

As an aspect of the analysis, we developed an a priori coding scheme to be applied to all the drawings that focused on aspects of communication in the drawings. There is another classification for pupils’ emotional position. Thus, our research question was as follows:

> What can we reveal via pupils’ drawings on mathematics teaching in their class?

And we developed from this question three more specific subquestions:

1. How do the teacher and the pupils communicate with each other as seen in third graders’ drawings?
2. What kind of emotional atmosphere in a mathematics lesson can be seen in third graders’ drawings?
3. How can we identify the type of work done during a mathematics lesson as seen in third graders’ drawings?

THEORETICAL FRAMEWORK

Here we consider the main concepts of our drawings study at a theoretical level with the help of the existing literature. The most important concept is the use of drawings as a source for information. Another important concept is the emotional atmosphere in class.

**Pupils’ drawings as research tools**

Drawing is an alternative form of expression for children. Barlow, Jolley, and Hallam (2010) have noted that freehand drawings help children recall and express more details about events they illustrate. Drawings tend to facilitate the recalling of events that are unique, interesting, or emotional, but not routine events or isolated bits of information that are not part of a narrative. Pupils’ drawings open an holistic way to evaluate and monitor pupils’ understandings of classroom climate, where there are several facets of communication and types of work.

Ruffell, Mason, and Allen (1998), in addition to Bragg (2007), challenge the written questionnaire as a method for studying children, as children do not necessarily understand the words and statements used in a questionnaire in the way that the researcher has intended. In an ideal situation, the child under study has an opportunity to verbalise his or her own concepts. According to Hannula...
Pehkonen, Ahtee & Laine (2007), it is not easy to get linguistically rich responses from young children. Thus, it is a challenge to develop for teaching mathematics research methods that would be suitable for young children and that take the children’s views into account.

Many researchers (e.g., Ludlow, 1999; Tikkanen, 2008; Remesal, 2009; Dahlgren & Sumpter 2010) emphasize that one way to evaluate the teaching of mathematics is to ask pupils to draw a picture about the lesson: Pupils who have received teacher-centered teaching often draw a blackboard and a teacher in front of the class. The pictures less often include references to communication between pupils. The drawings also tell us about beliefs, attitudes and feelings that have to do with mathematics. When pupils have taken part in the kind of teaching that activates them, they produce pictures that emphasize activities and communication between pupils.

Drawings help pupils to overcome the difficulties in disclosing their thoughts, feelings and opinions to an adult researcher (Zambo & Zambo, 2006). According to Weber and Mitchell (1996), pupils’ classroom drawings form rich data to study children’s conceptions on teaching. Pupils’ drawings have made an alternative and complementary contribution to conventional research methods by conveying their images about mathematics, mathematics teaching, their teacher, and their peers and classrooms in mathematics lessons.

Both meaning making and interpretation have a central role in analysing drawings. According to Blumer (1986), the meanings given by the pupils to various situations and things guide their actions, how they interpret different situations and what they include in their drawings. Giving meaning is a continuous process, which in this study takes place particularly in the social context of the mathematics lesson. Different pupils will find different meanings in the same situations and things. The meanings may have to do with physical objects, such as the classroom blackboard or a desk; social interaction, such as working alone or in a group; or with abstract matters, such as the concepts of mathematics or the feelings that are elicited by teaching. The methods used in teaching organise both the actions between the teacher and the pupils as well as the actions between pupils. Based on the influences of the teaching, the pupil may evaluate himself or herself as poor and his or her classmates as good in mathematics.

Tikkanen (2008) compared Finnish and Hungarian fourth graders’ experiences with mathematics teaching. The data consisted of pupils’ drawings and narratives. Three types of classrooms were identified according to their mathematical contents and the style of narration. Regardless of teaching methods, most of the pupils had a positive attitude towards mathematics and a positive self-concept.
In the framework of motivation theory, Dahlgren and Sumpter (2010) compared second and fifth graders’ conceptions of mathematics and mathematics teaching via drawings with a written questionnaire in Sweden. All pupils presented mathematics teaching as an individual activity with a focus on the textbook. Most of the second graders had a positive attitude toward mathematics, whereas a larger proportion of the fifth graders had a negative one.

Rolka and Halverscheid (2011) analysed fifth and sixth graders’ drawings, texts, and interviews, for studying their mathematical worldviews. They tried to classify the drawings into the three categories proposed by Ernest (1991): instrumentalist view, Platonist view, or problem-solving view. Their conclusion was the following: “Considering the picture alone as the data source for extracting the underlying mathematical worldview is related to a large amount of subjectivity in interpretation and will certainly not allow for an unambiguous classification” (Rolka & Halverscheid, 2011, p. 522). Therefore, they considered also text and interviews in order to study pupils’ mathematical worldviews.

In our research project we used as background tests besides pupils’ drawings a questionnaire of pupils’ conceptions of mathematics, and a test on pupils’ mathematics knowledge and problem-solving skills.

**On the emotional atmosphere in a classroom**

Teachers have a central role in advancing the affective atmosphere and social interaction in their class. Harrison, Clarke, and Ungerer (2007) summarize that a positive teacher-pupil relation advances both pupils’ social accommodation and their orientation to school, and it is thus an important foundation for the pupils’ academic career in the future.

Evans, Harvey, Buckley, and Yan (2009) define three complementary components of classroom atmosphere: (1) academic, referring to pedagogical and curricular elements of the learning environment; (2) management, referring to discipline styles for maintaining order; and (3) emotional, the affective interactions within the classroom. In this study, we concentrate on the last component; that is, the emotional atmosphere, which can be noticed, for example, as an emotional relation between the pupils and the teacher. The state of the pupils’ emotional atmosphere is an important background factor in problem solving.

The emotional atmosphere within the classroom can be regarded either from the viewpoint of individuals in the class (psychological dimension) or from the viewpoint of a community (social dimension). While the individual perspective looks at the individual experiences in the class, the social perspective looks at the class more holistically with a focus on social interaction, communication, and norms. Furthermore, a distinction can be made between two temporal aspects of affect: state and trait. *State* refers to the emotional atmosphere in a specific moment in the class while *trait* refers to a more stable condition or
property (cf. Hannula, 2011). In this study our perspective is holistic, connecting pupils’ individual dimensions.

Different affective dimensions can be studied also using social level concepts at the level of community; that is, of a classroom. Rapidly changing affective states include, for instance, a social interaction connected to a certain situation, communication related to this, and the emotional atmosphere present in the classroom. When similar situations happen repeatedly in a classroom, pupils may form more stable affective traits typical to a certain classroom. Social norms (Cobb & Yackel, 1996), social structures and atmosphere in a classroom are such traits. Pupils will “learn” that, during mathematics lessons, homework is always checked in the same way, and a certain norm is developed. When also other parts of the mathematics lesson happen repeatedly in the same kind of atmosphere, the atmosphere may become general and include all mathematics lessons, possibly also lessons of other subjects.

METHODS

The results of the study are based on pupils’ drawings that were gathered in autumn 2010 in Greater Helsinki. The teacher of the class gave the following task instruction to her pupils, who worked independently, and then she collected the drawings for the researchers.

The third graders in question (about 9 years old) came only from the nine teachers in the Greater Helsinki area. The drawings from the class of one teacher had to be put aside, since the pupils had produced them in pairs.

The drawing task:

“Draw your teaching group, the teacher and the pupils in a mathematics lesson. Use balloons for speech and thought to describe conversation and thinking. Mark the pupil that represents you by writing on it ME.”

The drawings by 133 pupils were analyzed, of which there were 72 from boys and 61 from girls. About two thirds of the pupils had added into their drawings some balloons for speech and thought. Thus, they enabled us to investigate communication between the teacher and the pupils as well as among the pupils.

Pupils have marked in many drawings the pupils’ and the teacher’s faces. With the help of those facial expressions, we were able to conclude how the pupil who did the drawing has experienced the emotional atmosphere in class. Thus deciding in each drawing the pupil’s attitude the person who did the drawing, with the help of facial expressions, we can add up the emotional atmosphere in the whole class.

Since this chapter is a combound of three substudies, they are dealt with separately. And therefore, the methods used in each substudy are presented apart later on in detail in a proper place.
RESULTS

Communication

We wanted to find out how the pupils experienced the kind of communication used in problem solving during mathematics lessons. Thus, we sought answers to the following question:

*How do the teacher and the pupils communicate with each other as seen in third graders’ drawings?*

The starting point of the classification of pupil drawings was the analysis method developed by Tikkanen (2008) in her doctoral dissertation. According to this method, a drawing as data source for observation can be divided into content categories. A content category means the phenomenon on which data is gathered. We have chosen here the following as content categories: (1) teacher’s communication, (2) pupils’ communication.

For analysis the content categories were operationalized into the following subcategories:

(1) Teacher’s communication: gives instructions; keeps order; teaches; gives feedback; observes quietly while the pupils work.

(2) Pupils’ communication: a pupil makes/ asks/ or thinks a remark in connection to teaching; a pupil asks for help; pupils discuss with each other; a pupil makes/ or thinks an improper remark.

Two researchers classified the pupils’ drawings, and in the case of a difference of opinion, both researchers reexamined and discussed the drawing in question together. All the drawings \((N = 133)\) were carefully classified. The evaluation of agreement was elicited by calculating the classifiers’ differences.

The method of analysing the drawing was a mixed method, and it can be classified as inductive content analysis (Patton, 2002), as we were trying to describe the situation in the drawing without letting our own interpretations influence it. Each drawing was carefully examined in order to find all subcategories of the main content category. In every content category, the last subcategory was “not recognizable”. The agreement between two classifiers in all subcategories was very good; that is, over 90% (range 91% – 95%).

In many drawings, one can see only stick figures; in some cases hands are beginning from the head, and in some drawings there are only pupil desks representing pupils. However, some of the third graders were very talented in drawing, and then in the pupils’ drawing one can see several details. The example in Figure 1 of the pupils’ drawings is very informative. In speaking bubbles, the pupils present their memory pictures about mathematics lessons and
their atmosphere. But the pupils’ method of presenting a saying (loud or whispering) and thinking is not always consequent.

**Figure 1.** An example of a pupil’s drawing; its analysis is below.

An interpretation of the drawing in Figure 1 is presented using the content categories previously described. In many content classes there is not only one feature, but many (cf. the second content category): 1. Teacher’s communication (the teacher observes quietly). 2. Pupils’ communication (pupils make remarks in connection to teaching; pupils ask for help).

**Results**

In this study, we tried to answer the research question with the help of the drawing analysis. It is helpful to notice that in the categories of the classification, the frequency is larger than the number of the pupils, since in many drawings one can find several features.

Firstly, we will deal with the content category "Teacher’s communication” (cf. Table 1). Since in the drawings of many pupils there were several indicators, the total frequency was 145. This totality is divided rather uniformly between several factors. In the parenthesis we give first the absolute frequency and then the relative frequency in a percentage.
In "Teacher’s communication" the mode value (36; 25%) is "teaches" that contains both a teacher’s own questions and expository teaching. But the frequencies are almost as large in the subcategories "follows quietly pupils’ working" (33; 23%) and "not recognizable" (28; 19%). Thus most of the pupils convey an impression that a teacher asks questions and delivers knowledge in mathematics lessons. There are many drawings where the teacher is not drawn at all.

Secondly, we take the content category "Pupils’ communication" (cf. Table 1). Since in the drawings there were several indicators, the totality is here 191. The largest frequency is in the subcategory "a pupil makes/ asks/ or thinks a remark in connection to teaching" (65; 34 %). The next largest frequency (48; 25 %) is in the subcategory "a pupil makes or thinks an improper remark". The frequencies of the rest of the three subcategories are under half of the maximum frequency. Therefore, we could say that in the drawings, pupils’ communication is a compound of pupils’ remarks where the largest share form the remarks connected to teaching or learning of mathematics, but there are also a great many improper remarks.

Table 1. The relative frequencies in the content categories:

<table>
<thead>
<tr>
<th>A teacher’s communication (%)</th>
<th>Pupils’ communication (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>teaches</td>
<td>25</td>
</tr>
<tr>
<td>follows quietly</td>
<td>23</td>
</tr>
<tr>
<td>maintains order</td>
<td>13</td>
</tr>
<tr>
<td>gives orders</td>
<td>10</td>
</tr>
<tr>
<td>gives feed-back</td>
<td>10</td>
</tr>
<tr>
<td>not recognizable</td>
<td>19</td>
</tr>
</tbody>
</table>

In Table 1, there are relative frequencies of communication in class. If we select from the content categories the most popular one (the mode category), we receive from the third graders’ communication in mathematics lessons the following prototypic picture: According to the pupils’ drawings, a teacher’s communication consists mainly of teaching (25%). It is interesting to notice that also another quarter of the pupils’ experiences (23%) carry another idea of teaching: The teacher quietly follows her pupils’ working. Pupils’ communication is clearly connected with teaching (34%).

Conclusion

No negative attitude to the teacher could be found in these drawings. This finding is different from the results of the study by Picker and Berry (2000). They found that pupils often ask teachers for help. Of course, a teacher
commands somewhat when maintaining order. The interaction between a teacher and pupils seem to be positive in the drawings, and that is important, since pupils are in cooperation with their teacher for about 4–5 lessons during a school day.

Altogether two-thirds (67%) of the third graders produced drawings where the pupils’ thinking, speaking, and action can be seen. According to Tikkanen (2008) mathematics lessons seem to contain many actions that pupils include in their drawings from working environment to mathematics drawn in the blackboard and in the speech bubbles.

In the published paper (Pehkonen, et al. 2011), the communication results are presented more in detail.

**Emotional atmosphere**

In the second substudy we wanted to find out what kind of emotional atmosphere the pupils convey in their drawings of a mathematics lesson. Thus, we sought answers to the following question:

*What kind of emotional atmosphere in a mathematics lesson can be seen in third graders’ drawings?*

In this study, we were concentrating only on the holistic evaluation of the emotional atmosphere in a classroom, which is based on all the pupils’ and the teacher’s moods seen in a drawing as well as on the pupils’ speech and thought bubbles in the picture. The pupils’ mood and the teacher’s mood are determined from the form of the mouth (smiling, neutral, sad/angry, not visible) and on their utterances or thoughts. The emotional atmosphere was classified as one of the following:

1) positive (all persons smile or think positively, some part can be neutral);
2) ambivalent (positive and negative), if at least one contradicting (positive or negative) facial or other expression is found in the drawing;
3) negative (all persons are sad or angry or think negatively; some can be neutral);
4) neutral (all facial or other expressions are neutral);
5) unidentifiable (when it is impossible to see any facial or other expressions).

In order to get an overview of the emotional atmosphere of the whole class, we made a summary of the holistic evaluation of the individual pupils’ drawings. It is important to notice that we were interested in the general atmosphere during mathematics lessons and not in any specific feelings toward mathematics activities.

*Results*
The emotional atmosphere in a mathematics lesson is taken as an entirety that consists of the pupils’ and the teacher’s facial expressions and their utterances or thoughts in the drawings. The observations are classified using the scale: positive, ambivalent, negative, neutral, and unidentifiable. The result of the analysis is presented in Table 2.

Table 2. Emotional atmosphere in a mathematics lesson in third grade (frequency; percent).

<table>
<thead>
<tr>
<th></th>
<th>positive</th>
<th>ambivalent</th>
<th>negative</th>
<th>neutral</th>
<th>unidentifiable</th>
</tr>
</thead>
<tbody>
<tr>
<td>total (133)</td>
<td>50 (38%)</td>
<td>44 (33%)</td>
<td>13 (10%)</td>
<td>20 (15%)</td>
<td>6 (5%)</td>
</tr>
</tbody>
</table>

The mode value of the emotional atmosphere in mathematics lessons was classified as positive, since it can be seen in 50 (38%) of the drawings. For example, the drawing in Figure 2 (Appendix 1) was classified as positive because all the pupils as well as the teacher are smiling. Furthermore, both the teacher’s and the pupils’ speech or thought bubbles are either positive or neutral.

The number of the pupils (44; 33%) who portrayed the emotional atmosphere in their mathematics lesson as ambivalent is almost the same as the number of pupils who described it as positive. An example of an ambivalent case is presented in Figure 3 (Appendix 1). The pupils are sitting in rows, and there are both positive and negative facial expressions in the drawing.

A tenth of the pupils pictured the emotional atmosphere as negative; that is, they drew sad or angry faces or the speech bubbles contained negative (or neutral) thoughts. In Figure 4 (Appendix 1) there is an example of a drawing showing a negative emotional atmosphere.

In 15% of the drawings the emotional atmosphere was classified as neutral because the persons’ facial or other expressions were neither positive nor negative. Pupils and teachers were normally talking only about mathematical tasks as in Figure 5 (Appendix 1).

As a summary, we can conclude that the mode value of the emotional atmosphere in the pupils’ drawings of mathematics lessons is positive in 50 cases (38%), where both the teacher and all the pupils are smiling (or some of them are neutral) or thinking positively or neutrally (cf. Table 2). A third of the pupils have drawn the emotional atmosphere in the classroom as ambivalent, which means that in their drawings there is at least one person whose facial expression is sad or angry or who says (or thinks) something that is interpreted as negative. The difference between the positive and ambivalent subcategories is not large, as the latter category contains also the drawings in which among many smiling pupils there is at least and perhaps only one pupil showing a sad face. It can thus be said that in these third graders’ drawings the principal mood in mathematics lessons is positive.
Next we looked at a classroom-specific emotional atmosphere in the mathematics lessons found in the third graders’ drawings from the classes of nine different teachers. We made a summary of the holistic evaluation of the individual pupils’ drawings in order to get an overview of the emotional atmosphere of the whole class. The summary of emotional atmosphere in the different classrooms is presented in Table 3.

Table 3. The distribution of emotional atmosphere in mathematics lesson in the nine classes (frequency; percent).

<table>
<thead>
<tr>
<th></th>
<th>Positive</th>
<th>Ambivalent</th>
<th>Negative</th>
<th>Neutral</th>
<th>Unidentifiable</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8; 53%</td>
<td>4; 27%</td>
<td>3; 20%</td>
<td>0; 0%</td>
<td>0; 0%</td>
</tr>
<tr>
<td>B</td>
<td>7; 50%</td>
<td>1; 7%</td>
<td>1; 7%</td>
<td>3; 22%</td>
<td>2; 14%</td>
</tr>
<tr>
<td>C</td>
<td>9; 47%</td>
<td>7; 37%</td>
<td>2; 11%</td>
<td>0; 0%</td>
<td>1; 5%</td>
</tr>
<tr>
<td>D</td>
<td>8; 44%</td>
<td>6; 33%</td>
<td>0; 0%</td>
<td>2; 11%</td>
<td>2; 11%</td>
</tr>
<tr>
<td>E</td>
<td>4; 25%</td>
<td>2; 13%</td>
<td>1; 6%</td>
<td>9; 56%</td>
<td>0; 0%</td>
</tr>
<tr>
<td>F</td>
<td>5; 29%</td>
<td>4; 24%</td>
<td>0; 0%</td>
<td>8; 47%</td>
<td>0; 0%</td>
</tr>
<tr>
<td>G</td>
<td>2; 12%</td>
<td>5; 29%</td>
<td>5; 29%</td>
<td>4; 24%</td>
<td>1; 6%</td>
</tr>
<tr>
<td>H</td>
<td>4; 36%</td>
<td>5; 46%</td>
<td>1; 9%</td>
<td>1; 9%</td>
<td>0; 0%</td>
</tr>
<tr>
<td>I</td>
<td>2; 33%</td>
<td>4; 67%</td>
<td>0; 0%</td>
<td>0; 0%</td>
<td>0; 0%</td>
</tr>
<tr>
<td>average (133 pupils)</td>
<td>49; 37%</td>
<td>38; 29%</td>
<td>13; 10%</td>
<td>27; 1%</td>
<td>6; 4%</td>
</tr>
</tbody>
</table>

Even though the emotional atmosphere in pupils’ drawings on mathematics lessons is mostly positive in the total data (cf. Table 2), there are large differences among the different classrooms. It is possible to look at the mode of the emotional atmosphere in every classroom (cf. Table 3) but it is important to notice that this mode does not reveal the whole truth. The profiles of the emotional atmospheres also vary widely within the classrooms.

In this study, our summary is that the emotional atmosphere in these mathematics classes at Grade 3 seems to be mainly positive, although there are big differences between classes. More details on the study of emotional atmosphere can be read in the published paper (Laine et al., 2013).

**Types of working**

As said earlier, a drawing gives a “snapshot” of how the pupil who did the drawing has experienced his or her teacher’s and his or her classmates’ activities during mathematics lessons. Here our aim was to find out how the pupils saw what type of work is done in mathematics lessons. Therefore, we
needed to create a method to analyse young pupils’ drawings in order to find answers to the following question:

*How can we identify the type of work done during a mathematics lesson as seen in third graders’ drawings?*

The concepts of teacher-centeredness vs. pupil-centeredness are actually rather complicated ideas, and they contain a wide range of meanings (cf. Neumann, 2013).

**Data analysis**

Since our aim was to develop a research method, we made several experiments to elaborate the range of teacher-centeredness vs. pupil-centeredness used in the classroom as seen in the pupils’ drawings. After several trials, we listed together from the drawings in one classroom all possible teachers’ and pupils’ activities during mathematics lessons as well as the types of work used in the classrooms as seen in the pupils’ drawings.

Two researchers then completed these lists by going through all the third graders’ drawings at hand ($N = 133$). In this way, three different lists were formed, but we deal here only with the third one: Types of work during a mathematics lesson as seen in the pupils’ drawings. The final list is given in Appendix 2.

This list contains three different types of work: namely, Independent work, Group work, and Work with the teacher in charge. When pupils are working independently they seem to be solving by themselves problems from the textbook or those that the teacher has written on the blackboard or given as a spreadsheet.

When pupils are working with the teacher in charge, the teacher is teaching – for example, asking questions to the whole class, or all pupils seem to be concentrating on the same task. In the case of group work, the pupils are discussing their tasks with their classmates, and the teacher is more a supporter than a foreman. In the case of “Impossible to say” there is no indication of pupils’ work. Furthermore, we have also listed whether the pupils are sitting alone beside their tables or in pairs or bigger groups.

**Examples**

Here we point out Figures 2–5 (Appendix 1) as examples that will illustrate the coding in the category Types of work. Additionally we use Figure 1 as a model example. In Appendix 2 are the categories in Types of work. For example, the abbreviation TW12 refers to the second subcategory in the content category TW1 (pupils are working independently).

In Figure 2 (Appendix 1), there are 18 pupils sitting in groups. The smiling teacher is sitting behind her desk and praises them, saying, “*I am very satisfied with my pupils.*” The tasks on which the pupils are working are from the textbook. Almost all the pupils are working on these tasks at their desks. Thus we can say that the pupils are working independently (TW12).
In Figure 3 (Appendix 1), the teacher is standing beside the blackboard and asks questions. The pupils are sitting by themselves and working with the teacher in charge (TW31).

In Figure 4 (Appendix 1), seven pupils are sitting in pairs and working independently (TW12), but the teacher is maintaining order.

In Figure 5 (Appendix 1), the teacher stands beside the blackboard and questions the pupils. Therefore, the situation in the drawing is classified as working with the teacher in charge (TW31).

Furthermore, the type of work in Figure 1 seems to be group work (TW2). The teacher is sitting quietly, and the nine pupils are working in two groups.

Figures 2–5 show that the organization of pupils’ desks does not indicate the type of work.

**Results**

In Table 4 one can see the distribution of the three different types of work – independent work, group work, and work with the teacher in charge – found in the third graders’ drawings.

<table>
<thead>
<tr>
<th>TW</th>
<th>Description</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>TW1</td>
<td>Independent work</td>
<td>57%</td>
</tr>
<tr>
<td>TW11</td>
<td>sitting alone</td>
<td>32%</td>
</tr>
<tr>
<td>TW12</td>
<td>sitting in a group</td>
<td>68%</td>
</tr>
<tr>
<td>TW2</td>
<td>Group work</td>
<td>3%</td>
</tr>
<tr>
<td>TW3</td>
<td>Work with the teacher in charge</td>
<td>26%</td>
</tr>
<tr>
<td>TW32</td>
<td>sitting alone</td>
<td>51%</td>
</tr>
<tr>
<td>TW32</td>
<td>sitting in a group</td>
<td>49%</td>
</tr>
<tr>
<td>TW4</td>
<td>Impossible to say</td>
<td>13%</td>
</tr>
</tbody>
</table>

According to the pupils’ drawings the most usual type of work during the mathematics lesson is independent work (57%) even though the pupils are more frequently sitting in groups (68%) than in rows (32%). This coincides with the traditional image according to which pupils are solving the given tasks mostly from the textbook by themselves. Next comes the type of work in which the teacher is standing in front of the classroom and pointing at the task on the blackboard (26%). In this case there is no difference in how the pupils are sitting, whether in rows or groups.

Only in four drawings (3%) could we find the pupils doing group work; that is the different groups had different tasks, the pupils were discussing together, and the teacher was going around giving advice and guiding the work. However, there were altogether 19 drawings (13%) in which it was impossible to say what
the type of work in the classroom was like. For example, in one drawing there were only three girls talking together.

Discussion and conclusions

The main aim of this study is to introduce a method to analyse teachers’ and pupils’ activities from young pupils’ drawings. With the help of the list we collected from third graders’ drawings that it is possible to identify types of work during a mathematics lesson. We started this research in order to find out which method – teacher- or pupil-centered (see, e.g., Thomas, Pederson, & Finson, 2001) – the teachers use more in their mathematics lessons.

However, we came to the conclusion that it is impossible to decide from the drawings whether they show teacher- or pupil-centeredness because these drawings are snapshots from a certain event though perhaps quite usual situations during mathematics lessons. For example, work with the teacher in charge certainly belongs to every teacher’s repertoire when she or he is introducing new topics.

A paper that describes the study more in detail has been submitted to an international journal (cf. Ahtee et al., submitted).

CONCLUSIONS

The drawings collected contain rich information from which we have selected only a small part for our purpose. The instruction given to the pupils was quite open, thus there is large variability in the drawings.

Here we first provide a summary of the results, in order to answer the research questions. Secondly, we discuss the reliability of the drawing study.

Pupils’ drawings reveal important information on pupils’ behaviour that is difficult to obtain from young children using more conventional methods (cf. Weber & Mitchell, 1996; Pehkonen et al., 2011). Especially by connecting words and images the pupils who did the drawings reflect their feelings and attitudes towards their teacher, other pupils, and situations. They also express the group values that are prevalent within their specific environment. Thus the method developed in this study gives us a tool to find out how young pupils see teachers’ activities as well as pupils’ activities in mathematics lessons. Therefore, it gives researchers and school authorities the possibility to see what is happening in classrooms. It also gives the opportunity to compare, for example, different grades, different systems, and even different countries.

Summary of results

The first research question was ”How do the teacher and the pupils communicate with each other as seen in third graders’ drawings?” In about a half of the drawings, the pupils convey that a teacher teaches (25%) and quietly
follows the class (23%). This finding is understandable, since that is the reason for teachers to be with pupils in the classroom. Usually for some part of a lesson the teacher teaches a new topic or questions the old knowledge. And a part of the lesson is dedicated to the pupils’ independent work (practicing new tasks), and therefore, the teacher follows the class quietly.

The second research question was “What kind of emotional atmosphere in a mathematics lesson can be seen in third-graders’ drawings?” In these third graders’ drawings the mode value of the emotional atmosphere in the mathematics lesson was positive. This finding matches the result for learning outcomes in mathematics in the beginning of the third grade (cf. Huisman, 2006); namely, that third graders’ collective attitude towards studying mathematics is fairly positive. However, it seems possible to obtain more information on this many-sided question with the aid of pupils’ drawings (e.g., Kearney & Hyle, 2004).

The third research question was ”How can we identify the type of work used during a mathematics lesson as seen in third graders’ drawings?” According to the drawings by the third graders, pupils are working independently twice as often as working with the teacher in charge. So it seems fair to conclude that third grade teaching seems to be fairly teacher-centered.

**On an enlargement of the drawing study**

As we realized the possibilities of drawings to reveal pupils’ conceptions on mathematics teaching and learning, we began actively to enlarge our data-base. Our aim is to develop an international comparison project on pupils’ drawings of mathematics lessons. Thus, we will be able to compare mathematics teaching in different countries, in order to single out similarities and differences.

Today we have, beside Finnish and Chilean drawings, pupils’ drawings from the United States (Georgia), Germany (Sachsen-Anhalt), and England. There is even a journal paper based on the U.S. material; that is, a bilateral comparison between Finland and Georgia, USA (cf. Hart, Pehkonen, & Ahtee, 2014). Furthermore we are expecting to receive comparative drawing material from Albania, and Italy. The size of each sample is about 100–200 drawings, and all are from third graders.

**On reliability**

In thinking about the reliability and validity of drawings in mathematics, Stiles, Adkinson, Sebben, and Tamashiro (2008) concluded that drawings enable stronger and more personal expressions than an opinion about statements in a questionnaire, such as “I like mathematics.” In this way, pupils may draw hearts if they love mathematics, or an assault rifle to destroy mathematics to convey not liking it. In addition, Dahlgren and Sumpter (2010) consider pupils’
drawings to be a reliable method for assessing pupils’ concepts about mathematics teaching.

It is evident that in pupils’ drawings, there are many kinds of influences. These drawings were made in the beginning of the third grade (September 2010). When evaluating a teacher’s effect in this study, one has to take into account that the third graders made their drawings at the beginning of a new school year when they had gone to school for only one month after the summer holiday. On the one hand, the pupils’ conceptions of mathematics lessons had been affected mainly by the two previous school years. Thus they might have been thinking about their teacher in Grades 1 or 2. On the other hand, pupils’ affective conditions and properties affect how they interpret different situations during mathematics lessons (Hannula, 2011).

Additionally, many third graders seem to have difficulties in drawing, and therefore, they might concentrate on drawing only situations that are easy to draw for them. To overcome these difficulties, the teacher might ask a pupil to explain his or her drawing. Another solution could be a whole class discussion on drawings. Ruffell & al. (1998) support these solutions, since they emphasize that the child under study should have an opportunity to verbalise his or her own concepts.

Endnotes

As a summary, drawings seem to be a versatile way to collect information about emotional atmosphere in mathematics lessons (see also Harrison et al., 2007). The method offers a single teacher the possibility to obtain and evaluate information about how his or her pupils experience mathematics and mathematics lessons. And the method can also hint to which features the teacher should pay more attention, how the teaching should be developed. Furthermore, it is fairly easy to open such a ”window” on pupils’ thinking in a lesson without much additional work by the teacher.

Pupils’ drawings reveal important information as to what kind of view the pupils have extracted from their lessons. Especially by connecting words and images, the pupils who did the drawings reflect their feelings and attitudes towards their teacher, other pupils, and situations. When all the pupils’ drawings in a classroom or a random sample in a country are collected, it is possible to obtain a view that is prevalent in this specific environment. Thus by analysing the pupils’ drawings, we can find out how young pupils see their mathematics lessons.

REFERENCES

mathematics lesson as seen in young pupils’ drawings. *Mediterranean Journal for Research in Mathematics Education.*


APPENDIX 1: Four examples of drawings

Figure 2. A positive emotional atmosphere; the type of work is independent.

Figure 3. An example of an ambivalent emotional atmosphere; the type of work is the teacher in charge.
Figure 4. An example of a negative emotional atmosphere; the type of work is independent.
Figure 5. An example of a neutral emotional atmosphere; the type of work is the teacher in charge.
**Appendix 2**: The type of work during a mathematics lesson as seen in the pupils’ drawings

<table>
<thead>
<tr>
<th>Code</th>
<th>Title</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>TW1</td>
<td>Working independently</td>
<td>Pupils are solving the same problem or working with different tasks in their own pace. “May I go and check?”</td>
</tr>
<tr>
<td>TW11</td>
<td>pupils are sitting by themselves</td>
<td></td>
</tr>
<tr>
<td>TW12</td>
<td>pupils are sitting in pairs or in bigger groups</td>
<td></td>
</tr>
<tr>
<td>TW2</td>
<td>Working in groups</td>
<td>Pupils are working in pairs or bigger groups. The groups may have different tasks. The teacher does not have a central role.</td>
</tr>
<tr>
<td>TW3</td>
<td>Work with the teacher in charge</td>
<td>All the pupils are thinking about the same part of the task</td>
</tr>
<tr>
<td>TW31</td>
<td>pupils are sitting by themselves</td>
<td></td>
</tr>
<tr>
<td>TW32</td>
<td>pupils are sitting in pairs or in bigger groups</td>
<td></td>
</tr>
<tr>
<td>TW4</td>
<td>Impossible to say</td>
<td>It is impossible to conclude the type of work.</td>
</tr>
<tr>
<td>TW41</td>
<td>pupils are sitting by themselves</td>
<td></td>
</tr>
<tr>
<td>TW42</td>
<td>pupils are sitting in pairs or in bigger groups</td>
<td></td>
</tr>
</tbody>
</table>

Extra remark: In some cases there were clearly two different types of work in the same classroom, and then they were both accepted.