SPIN AND RELATIVISTIC MOTION
OF BOUND STATES

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Preface

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Matti Järvinen
Abstract

The wave functions of moving bound states may be expected to contract in the direction of motion, in analogy to a rigid rod in classical special relativity, when the constituents are at equal (ordinary) time. Indeed, the Lorentz contraction of wave functions is often appealed to in qualitative discussions. However, only few field theory studies exist of equal-time wave functions in motion. In this thesis I use the Bethe-Salpeter formalism to study the wave function of a weakly bound state such as a hydrogen atom or positronium in a general frame. The wave function of the $e^-e^+$ component of positronium indeed turns out to Lorentz contract both in $1+1$ and in $3+1$ dimensional quantum electrodynamics, whereas the next-to-leading $e^-e^+\gamma$ Fock component of the $3+1$ dimensional theory deviates from classical contraction.

The second topic of this thesis concerns single spin asymmetries measured in scattering on polarized bound states. Such spin asymmetries have so far mainly been analyzed using the twist expansion of perturbative QCD. I note that QCD vacuum effects may give rise to a helicity flip in the soft rescattering of the struck quark, and that this would cause a nonvanishing spin asymmetry in $\ell p^\uparrow \rightarrow \ell' + \pi + X$ in the Bjorken limit. An analogous asymmetry may arise in $pp^\uparrow \rightarrow \pi + X$ from Pomeron-Odderon interference, if the Odderon has a helicity-flip coupling. Finally, I study the possibility that the large single spin asymmetry observed in $pp^\uparrow \rightarrow \pi(x_F,k_\perp) + X$ when the pion carries a high momentum fraction $x_F$ of the polarized proton momentum arises from coherent effects involving the entire polarized bound state.
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List of included papers

The articles included in this thesis are:

[I] M. Järvinen,  
“Lorentz contraction of bound states in 1+1 dimensional gauge theory,”  

[II] M. Järvinen,  
“Hydrogen atom in relativistic motion,”  

[III] P. Hoyer and M. Järvinen,  
“Soft rescattering in DIS: Effects of helicity flip,”  
JHEP 10, 080 (2005), [hep-ph/0509058].

[IV] P. Hoyer and M. Järvinen,  
“Single spin asymmetry at large $x_F$ and $k_{\perp}$,”  
JHEP 02, 039 (2007), [hep-ph/0611293].
Chapter 1
Introduction

Gauge field theory accurately describes the strong and electroweak interactions of Nature. Quantum Electro Dynamics (QED) is the fundamental theory of electromagnetic interactions. It was formulated in its final form in the late 1940s as the first quantum theory that consistently includes special relativity. The theory thus respects the Lorentz symmetry of special relativity and also the gauge symmetry which is familiar from classical electromagnetism. This fixes the basic structure of the theory to a rather simple form: QED can be essentially defined through its Lagrangian density

\[ \mathcal{L}_{\text{QED}} = \bar{\psi} \left( i \partial - e A - m_e \right) \psi - \frac{1}{4} \left( \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \right) \left( \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu} \right) \]  

(1.1)

where \( \psi \) is the electron field and \( A^{\mu} \) is the photon field. Yet solving the theory, e.g., finding exact solutions of bound states and scattering amplitudes, is difficult.

Perturbation theory is a basic tool to obtain analytical results from a quantum field theory. In perturbative QED any observable is expressed as a power series in the charge (squared) of the electron, the \( e \) in the first term of (1.1). In the case of QED we are lucky, because the coupling constant \( \alpha = e^2 / 4\pi \simeq 1/137 \) is small and the series expansion converges well. Thus very accurate predictions can be made and tested experimentally. The most accurate tests show agreement between theory and experiment to eleven significant digits. While perturbation theory is straightforward for, e.g., scattering problems, bound states are more difficult to describe in terms of a perturbation series. Bound states poles do not appear at any finite order in perturbation theory, but result from the divergence of an infinite sum of diagrams.

The first topic covered in this thesis is the relativistically moving hydrogen atom (or positronium). The spectrum of the hydrogen atom and its wave functions in the Center of Mass (CM) frame are usually discussed (to leading order in \( \alpha \)) in the first course in quantum mechanics. It is thus surprising that a thorough study of such a weakly coupled system in a moving frame cannot be found in the literature. In the CM frame the hydrogen atom wave functions satisfy the Schrödinger equation. When the atom is boosted to a large velocity, comparable to the speed of light, the
internal motion of the state stays nonrelativistic, but the overall motion becomes relativistic. Hence nonrelativistic quantum mechanics cannot be used anymore and one has to resort to the fully relativistic framework of QED.

In general, finding the wave functions and energies of relativistic bound states is a demanding task. The states may contain sizeable contributions from Fock states that have a large number of particles. Thus relativistic bound states are described by an infinite chain of coupled wave functions. The first and most widely used method for solving bound states in field theory was developed by Salpeter and Bethe in the early 1950s [1, 2]. The Bethe-Salpeter equation (BSE) is an integral equation for a Lorentz covariant, two-particle (Bethe-Salpeter) wave function. The equation involves an integration kernel that in perturbation theory is given by the sum of infinitely many Feynman diagrams.

However, it is well known that for weakly coupled, nonrelativistic states and in the CM frame, the rather complicated Bethe-Salpeter equation reduces to the Schrödinger equation. In the weak-coupling limit $\alpha \to 0$ only one (Coulomb) photon exchange contributes to the kernel and the equation becomes solvable in terms of elementary functions. The Lorentz covariant Bethe-Salpeter wave function equals the usual hydrogen atom wave function when the times of the constituents are set to be equal.

As the concept of equal time is frame dependent, there are several options for fixing the relative time between the constituents in a moving system. A boost covariant method is to use the light front wave functions (see [3] for a review) where the constituents have equal light front time $x^+ = x^0 + x^3$. The concept of equal light front time is boost invariant (for boosts in the $x^3$ direction), which may be easily verified using standard Lorentz transformation rules. The equal-light-front-time wave functions are particularly convenient if the theory is quantized on the light front [4]. They are also relevant for interpreting experimental data. For instance, in deep inelastic scattering a high-energy (virtual) photon (with $q^- = q^0 - q^3$ large) probes the constituents of the proton at equal light front time [5–7].

In order to study quantum effects on the usual classical picture involving an external observer, e.g., to see phenomena like Lorentz contraction, a different quantization scheme needs to be used. Lorentz contraction of the wave function is expected when the (ordinary) times of the constituents are set equal in a frame where the atom is in relativistic motion, corresponding to the instant form quantization of the theory. Highly contracted “pancakes” are often drawn when high-energy nucleons or nuclei are discussed at a qualitative level. The theoretical framework, such as the quantization scheme, which seems to be crucial for the study of contraction, is seldom referred to. In fact, few studies of Lorentz contraction in any bound state are to be found in the literature. To my knowledge Lorentz contraction has been investigated only in some approximations and in model theories [8–14].

I study [I, II] a relativistically moving hydrogen atom in terms of wave functions where the constituents have equal (ordinary) time. The Bethe-Salpeter equation simplifies considerably in the weak-coupling limit also in the case of the moving
hydrogen atom. Systematically expanding at small $\alpha$ one may verify that the wave function satisfies a modified Schrödinger equation. The energy spectrum transforms according to the standard Lorentz transformation rules while the equal-time wave function of the lowest (electron-proton) Fock state Lorentz contracts as expected from classical special relativity. This contraction is nontrivial and probably holds only in the weak-coupling limit. I demonstrate this by calculating the wave function of the next-to-leading ($epr\gamma$) Fock state with one additional (transverse) photon. This component does not have the classical contraction property.

Articles III and IV of this thesis deal with strong interaction phenomena governed by Quantum Chromo Dynamics (QCD). QCD is a generalization of QED with a non-Abelian gauge symmetry. In analogy to perturbative QED, standard perturbative QCD assumes that the degrees of freedom of the Lagrangian, quarks and gluons, are physically observable. The fact that free quarks and gluons are not seen experimentally implies that QCD perturbation theory works differently from QED. This may be a consequence of the true vacuum of QCD being a condensate of quarks and gluons. It remains a central challenge to describe the mechanism of color confinement and the formation of hadrons using QCD.

The structure of the proton is probed in Deep Inelastic Scattering (DIS, $\ell_p \to \ell' + X$). In DIS a highly virtual photon scatters from the proton constituents, partons, providing information on their character and distribution. DIS thus involves “soft” low-energy physics in the form of the internal dynamics of the proton, but also “hard” physics, the high-energy scattering between the photon and the partons. In the theoretical description of hadron scattering the soft and hard parts need to be combined. QCD factorization is the standard framework that connects the two regimes. The short-distance, high-energy subprocess can be evaluated in terms of quarks, gluons and perturbation theory due to the smallness of the running coupling constant of QCD at large energies. The long distance dynamics, which determines how the partons form the target proton, cannot be described using perturbation theory, but it is instead parametrized using universal parton distribution functions.

DIS can also be used to probe the spin-dependent structure of hadrons. Spin has been the subject of many experiments and theoretical studies during the last few decades. The transverse Single Spin Asymmetry (SSA) in DIS has gained much interest recently. A transverse SSA means that the cross section depends on the direction of the (transverse) spin of one of the initial or final particles. The measure for the size of the asymmetry is the analyzing power

$$A_N = \frac{2\sigma(\uparrow) - \sigma(\downarrow)}{\sigma(\uparrow) + \sigma(\downarrow)}$$

where the arrows refer to the direction of the measured spin.

A nonzero SSA requires a quark helicity flip which in a hard subprocess is proportional to the small current quark mass of perturbative QCD [15]. Hence a sizeable $A_N$ suggests that the helicity flip may occur in the soft part of the process which is described in terms of parton distributions and fragmentation functions in the standard QCD factorization framework. Helicity-flip parton distributions
are absent in standard collinear factorization, but arise in generalized schemes, where one considers the transverse motion of partons or next-to-leading corrections in the hard scattering scale. Such schemes have been widely studied during the last fifteen years (see, e.g., [16, 17]). In particular, the recently observed SSA’s in polarized Semi-Inclusive Deep Inelastic Scattering [18, 19] (SIDIS, \(\ell p \rightarrow \ell' \pi X\)) have been investigated in the transverse-momentum-dependent scheme by parametrizing the new spin-dependent distribution or fragmentation functions. While the fits are acceptable, the large number of parameters needed to describe the new distributions makes predicting the results of forthcoming experiments demanding.

The physical understanding of the QCD dynamics that generates the asymmetries can be improved via model calculations. In the scalar diquark model of SIDIS suggested by Brodsky, Hwang, and Schmidt [20], the spectators of the hard interaction are modelled by a pointlike scalar diquark and QCD is replaced by an Abelian theory. The model is useful for studying effects which arise when the struck quark rescatters on target spectators. In the original work it was demonstrated how coherent rescattering can lead to a SSA at leading twist, i.e., at leading order in the inverse virtuality of the exchanged photon \(1/Q\).

In [III] I consider the possibility that the rescattering flips the helicity of the struck quark, using the model of Brodsky, Hwang, and Schmidt. This does not occur in perturbative QCD, but since rescattering is soft helicity-flip contributions may arise from nonperturbative effects such as instantons [21–24]. The helicity flip leads to a leading twist asymmetry which has the same angular dependence as the previously known Collins fragmentation effect [25]. The dynamics is, however, different: the helicity-flip rescattering is coherent with the hard subprocess, whereas fragmentation effects are incoherent.

In SIDIS (\(\ell p \rightarrow \ell' \pi X\)) the hard scale \(Q\) is provided by the virtuality of the photon, whereas the transverse momentum of the pion is small. In \(p^\dagger p \rightarrow \pi X\) on the other hand the hard scale of the reaction is given by the transverse momentum \(k_\perp\) of the pion. Large (up to 40 %) asymmetries have been measured in \(p^\dagger p \rightarrow \pi X\) [26–29] and in \(pp \rightarrow \Lambda^\dagger X\) [30–36]. Such large asymmetries were not expected in QCD since \(A_N\) is suppressed in the underlying hard subprocesses. The factorization based approaches also predict that the asymmetry decreases with \(k_\perp\), \(A_N \propto \Lambda_{\text{QCD}}/k_\perp\), while the measured \(A_N\) rather seems to increase with \(k_\perp\).

I pay special attention to the fact that the largest asymmetries in \(p^\dagger p \rightarrow \pi(x_F, k_\perp) + X\) have been observed at high \(k_\perp\) and at high \(x_F\). At large \(x_F\) several quarks from the same hadron may participate in the scattering event. The coherence of soft and hard subprocesses at large \(k_\perp\) and fixed \((1 - x_F)k_\perp^2\) then naturally explains the large size of the asymmetries [IV]. This is a novel mechanism for single spin asymmetries.

This thesis is organized as follows. In Chapter 2 I review the Bethe-Salpeter formalism. This framework is then applied to the case of relativistically moving positronium and the results of the first two publications are summarized in Chapter 3. In Chapter 4 the atomic wave functions are used to build a toy model of DIS.
In Chapter 5 I review the available experimental results and earlier theoretical work on SSA’s in SIDIS and in $p^+_p \rightarrow \pi X$. In Chapter 6 I go on to study the asymmetries in both processes using the model of article III of this thesis. Chapter 7 reviews high-$x_F$ coherence effects and the results of article IV. Conclusions and outlook are given in Chapter 8.
Chapter 2

Bound states in field theory and the Bethe-Salpeter formalism

In this chapter I briefly review the definitions of bound states in field theory, their wave functions and the Bethe-Salpeter formalism. I show how the weak-coupling limit leads to the familiar nonrelativistic results in the center of mass frame.

Bound states in quantum field theory are defined in the same way as in nonrelativistic quantum mechanics: they are eigenstates of the Hamiltonian. However, in moving from nonrelativistic to relativistic quantum mechanics several complications arise that make solving the eigenstates a difficult task. In special relativity the concept of time is frame dependent, and, e.g., particle scattering from an external source may involve pair creation in the intermediate state (see Fig. 2.1). Lorentz covariance requires that the Hamiltonian of an interacting field theory can change particle number. Thus bound states, in general, involve Fock states with arbitrary numbers of particles. Instead of a single wave function, the state is described by an infinite number of wave functions each of which depends on all the momenta and spins of the constituents of the corresponding Fock state. The eigenvalue equation of the Hamiltonian is a complicated integral equation connecting wave functions of Fock states with different numbers of particles (see, e.g., [3]).

The Bethe-Salpeter formalism [1] is an alternative formulation of the bound

![Diagram](image)

Figure 2.1: The two time ordered diagrams for double scattering of an electron from an external source. The diagram to the right involves a Fock state with an extra $e^-e^+$ pair. The contributions of individual diagrams are frame dependent and only the sum is Lorentz covariant. Time flows to the right.
state problem. The Bethe-Salpeter equation is an integral equation for a two-particle Fock state amplitude, the Bethe-Salpeter wave function. It is defined for the meson case as a projection on a quark-antiquark state

\[
\psi_{P,\alpha\beta}(p) = \int d^4x \langle \Omega | T \{ \bar{q}_\beta(0)q_\alpha(x) \} | \psi, P \rangle \exp(ip \cdot x) \tag{2.1}
\]

where \( P \) is the three momentum of the bound state, \( p \) is the momentum of the quark, \( |\psi\rangle \) is the bound state, \( |\Omega\rangle \) is the vacuum state of the theory, and \((\alpha, \beta)\) are Dirac indices. Note that the Lorentz covariant definition (2.1) necessarily depends on the relative energy \( p^0 \) (or relative time in coordinate space) of the constituents.

![Diagram](image)

**Figure 2.2:** The Bethe-Salpeter equation. \( \psi \) is the covariant Bethe-Salpeter wave function, \( K \) is the two-particle irreducible interaction kernel, and \( S \) is the full two-particle propagator. The arrows indicate the direction of momenta.

The wave function (2.1) satisfies the Bethe-Salpeter Equation (BSE)

\[
\psi(p) = S(p) \int \frac{d^4q}{(2\pi)^4} K(p, q) \psi(q) \tag{2.2}
\]

where the Dirac indices were suppressed. The equation is diagrammatically represented in Fig. 2.2. In this form the complexity of the bound state is hidden in the two-particle propagator \( S \), and, in particular, in the interaction kernel \( K \). The propagator \( S \) is the product of two full one-particle propagators, including all corrections in perturbation theory. The interaction kernel \( K \) is two-particle irreducible, meaning that it cannot be split into two parts by cutting two fermion lines. Hence, in perturbation theory, \( K \) includes diagrams of arbitrary complexity and the contributions of many-particle Fock states are hidden in \( K \). The infinite number of integral equations with simple interactions in the Hamiltonian method can be thus transformed to a single integral equation with a highly complicated kernel.

\footnote{An illustrative proof of the equation is found by writing an iterative equation for the four-quark Green function, and then studying the residue contribution at the bound state pole [1,37,38].}
2.1 Reduction to the Schrödinger equation

In the center of mass frame the BSE reduces to the usual Schrödinger equation in the weak-coupling limit \( \alpha \to 0 \). The equal-time wave function \( \varphi(p) \) is defined as

\[
\varphi(p) = \int \frac{dp^0}{2\pi} \psi(p)|_{p=0}
\]

(2.3)

where the momentum integral fixes the relative time \( t \) between the constituents of the Bethe-Salpeter wave function (2.1) to \( t = 0 \). The function \( \varphi(p) \) is expected to reduce to the usual nonrelativistic spin independent (hydrogen atom) wave function. Next I will sketch how this result follows by considering (2.2) in the limit \( \alpha \to 0 \) in Abelian gauge theory (QED) when the constituent masses are equal.

The small \( \alpha \) limit of \( K \) and \( S \) is found using perturbation theory. The kernel \( K \) includes at least one-gluon exchange and is thus proportional to \( \alpha \). Hence, to have contributions of the same order in \( \alpha \) on both sides of the BSE, one needs factors of \( \alpha \) from other sources than the coupling constants. It turns out that these factors are due to the scaling of the internal momenta and the binding energy \( \Delta E \) of the bound state (the scaling is also manifest in the final result, the Schrödinger equation)

\[
p^i \sim \alpha m
\]

\[
\Delta E \sim P^2/m \sim \alpha^2 m
\]

(2.4)

Using (2.4) one may check that only single (Abelian) gluon exchange contributes to the leading kernel \( K \), and \( S \) may be replaced by a product of free quark propagators. Then (2.2) becomes (in Feynman gauge)

\[
\psi(p) = -ie^2 \frac{\hat{p} + m}{p^2 - m^2 + i\varepsilon} \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 + i\varepsilon} \gamma^\mu \psi(p - q) \gamma^\mu \frac{\hat{p} - P + m}{(P - p)^2 - m^2 + i\varepsilon} .
\]

(2.5)

In the nonrelativistic limit, the weakness of the photon exchange forces the quarks to be almost on mass-shell. In the complex \( p^0 \) plane, the “forward-moving” poles of the quark propagators in (2.5) at \( p^0 = E_p - i\varepsilon \), \( p^0 = M - E_p + i\varepsilon \) need to coincide within the accuracy \( \alpha^2 m \), giving

\[
2E_p = M + \mathcal{O}(\alpha^2 m)
\]

\[
\Delta E = M - 2m = \mathcal{O}(\alpha^2 m)
\]

(2.6)

where \( M \) is the bound state mass and \( E_p = \sqrt{p^2 + m^2} \). In particular

\[
p^2 - m^2 + i\varepsilon \simeq 2m(p^0 - E_p + i\varepsilon)
\]

\[
(P - p)^2 - m^2 + i\varepsilon \simeq -2m(p^0 - M + E_p - i\varepsilon)
\]

(2.7)

\[\text{One might as well use, \textit{e.g.}, the light front wave function where the constituents have equal light front time } x^+ = x^0 + x^3. \text{ There is no difference in the wave functions since the constituents move slowly } v \sim \alpha \text{ in the weak-coupling limit and in the CM frame.}\]
and since the estimates (2.4) need to hold both before and after the gluon exchange one finds
\[ q^0 \sim \alpha^2 m; \quad q^i \sim \alpha m \] (2.8)
so that \( q^2 \simeq -q^2 \). The Dirac structure can be considered to leading order in \( \alpha \).
One can replace
\[ (p + m)\gamma^\mu \simeq m(1 + \gamma^0)\gamma^\mu \simeq 2m\delta^\mu_0 + \gamma^\mu(-p + m) \rightarrow 2m\delta^\mu_0 \] (2.9)
where the term \( \gamma^\mu(-p + m) \) is negligible as seen by iterating the equation (2.5) [one then gets a small term \((-p + m)(p + \gamma + m))\]. Similarly the antiquark Dirac structure simplifies to
\[ \gamma^\mu(p - P + m) \simeq \gamma^\mu m(1 - \gamma^0) \rightarrow -2m\delta^\mu_0 . \] (2.10)

Inserting the above estimates in (2.5) gives
\[ \psi(p) = \frac{i e^2}{(p^0 - E_p + i\varepsilon)(p^0 - M + E_p - i\varepsilon)} \int \frac{d^3q}{(2\pi)^3} \frac{1}{q^2} \varphi(p - q) \] (2.11)
\[ \varphi(p) = -\frac{4\pi\alpha}{M - 2E_p} \int \frac{d^3q}{(2\pi)^3} \frac{1}{q^2} \varphi(p - q) . \] (2.12)

After a multiplication by \( M - 2E_p \simeq \Delta E - p^2/m \) one finally finds the Schrödinger equation
\[ \left( \Delta E - \frac{p^2}{m} \right) \varphi(p) = -4\pi\alpha \int \frac{d^3q}{(2\pi)^3} \frac{1}{q^2} \varphi(p - q) . \] (2.13)

Choosing a suitable normalization one can write the solution for the equal-time wave function as
\[ \varphi(p) = \sum_{s_1,s_2} \frac{u(0,s_1)\bar{v}(0,s_2)}{2m} \chi_{s_1,s_2} \Phi_{NR}(p) \] (2.14)
where \( \chi_{s_1,s_2} \) is a constant spinor and \( \Phi_{NR} \) is the usual nonrelativistic scalar wave function. Note that since the Dirac matrices were considered only to the leading power of \( \alpha \) in the above calculation, (2.14) holds only for the leading components of the spinors. The result (2.12), (2.14) reflects the spin independence of the non-relativistic approximation.

It is also well known how other equations that have been used to study the hydrogen atom spectrum relate to the BSE. The Dirac equation is obtained from the BSE in the limit \( m_e/m_p \rightarrow 0 \) after summing all “crossed graph” contributions to \( K \) [39, 40]. Radiative corrections (gluon and quark loops) are not included in the Dirac equation. For later use I present another approximation of the BSE, the Breit equation [41] which reads in the CM frame
\[ [H_0^a(p) + H_0^a(-p) + V - M] \psi = 0 \] (2.15)
where
\[ H_0^{a,b}(p) = \alpha^{a,b} \cdot p + \beta^{a,b}m \] (2.16)
are the free Dirac Hamiltonians of the constituents, \( M \) is the bound state mass, and \( V \) is the interaction potential which includes the Coulomb potential and possibly some spin-dependent (retarded) interaction. Here the masses \( m \) of the constituents were set to be equal for simplicity. The Breit equation is one of the first attempts to describe relativistic effects in a two-particle bound state. It correctly includes the largest subleading corrections to the Schrödinger equation.

Higher order corrections to the nonrelativistic limit (2.12) can be evaluated using “better” approximate equations such as (2.15), or by directly expanding the BSE in \( \alpha \). However, the two energy scales of (2.4) make the expansions in \( \alpha \) rather tricky. Moreover, the \( O(\alpha^3) \) correction to the kernel \( K \) contains an infinite number of diagrams [2] whence accurate predictions require other methods (see, e.g., [42–45]).
Chapter 3

Relativistically moving bound states

In this chapter I study bound states in motion. After discussing general aspects, I review previous work on wave functions of moving bound states. I then discuss the exactly solvable 't Hooft model as an example. Finally I present the main results of my own work on the hydrogen atom.

Lorentz contraction is often appealed to in qualitative descriptions of high-energy hadronic and nuclear collisions. However, there are few studies in the literature on the deformation of the wave functions of systems moving with relativistic velocity. It is clear that truly relativistic bound states such as the proton cannot be solved in a general frame as the task is yet too difficult even in the CM frame. However, it is surprising that weakly coupled QED states such as the hydrogen atom or positronium have not been studied in a general frame long ago.

In classical special relativity Lorentz contraction is a direct consequence of the transformation formulae. Let a rod lie at rest with the two ends at $x^3 = 0$ and at $x^3 = L$. Let us then consider the rod in a frame where it is moving with a large velocity $\beta$ in the $x^3$ direction. The boost to such a frame is given by $x' = \Lambda x$ with

$$\Lambda = \begin{pmatrix} \gamma & \beta \gamma \\ \beta \gamma & \gamma \end{pmatrix}$$

(3.1)

where $\gamma = 1/\sqrt{1 - \beta^2}$ and the nonrelevant $x^1, x^2$ elements were suppressed. The tracks of the two endpoints in the moving frame are obtained from $x'^3 = (\Lambda^{-1}x')^3 = 0, L$, giving

$$x'^3 |_{x^3=0} = \beta(x'^0)$$
$$x'^3 |_{x^3=L} = \beta(x'^0) + L/\gamma.$$  

(3.2)

The rod thus moves with the velocity $\beta$ in the positive $x^3$ direction, and its length seems to be contracted by the $\gamma$ factor to $L' = L/\gamma$. Due to the linearity of the Lorentz transformations it is clear that also any other rigid, moving object contracts
linearly by the $\gamma$ factor in the boost direction. Such a contraction is called “classical Lorentz contraction” in the following.

In the usual instant form of quantization bound states are conveniently described in terms of equal-time wave functions, which may be expected to have contraction properties similar to the classical rod. Boosting such wave functions is, however, nontrivial. The reason is essentially the absence of an absolute time in special relativity. Translation in time does not commute with boosts: $[H, K^i] \neq 0$, i.e., the boosts are dynamical [4]. Thus boosting is as complicated as solving the bound states directly in the new frame. This fact may be contrasted with the simple, covariant transformation rule of the Bethe-Salpeter wave function (2.1) (here in coordinate space)

$$\psi_{P'}(x') = S(\Lambda) \psi_{P=0}(x) S^{-1}(\Lambda)$$

where $x' = \Lambda x$, $P' = \Lambda P$, and $S(\Lambda)$ is the usual representation of $\Lambda$ in Dirac space. Setting $x^0 = 0$ in (3.3) one obtains the equal-time wave function on the right hand side, but then $x'^0 \neq 0$ and the left hand side is not the desired wave function. The transformation formula between two equal-time wave functions depends on the dynamics of the system.

In light front quantization the situation is quite different. Boosts in the $x^3$ direction are kinematical, $[H, K^3] = 0$, and consequently light front wave functions are boost invariant when expressed in terms of the light front momentum fractions $x_i = p_i^+ / P^+$, where $p_i^+ = p_0^i + p_3^i$ is the constituent momentum and $P^+$ the total momentum of the states. Hence the (classical) Lorentz contraction is not a property of the light front wave functions even though they are typically most directly related to high-energy scattering cross sections.

It is generally accepted that the light front picture emerges in the infinite momentum limit [46, 47] of the usual instant form quantization. Consequently the Bethe-Salpeter equal-time wave function

$$\varphi_P(p) = \int \frac{dp^0}{2\pi} \psi_P(p)$$

should obey

$$\lim_{|P| \to \infty} \varphi_P(p_{\parallel}, p_{\perp})|_{p_{\parallel} = x|P|} \propto \phi^{LF}(x, p_{\perp}) .$$

The existence of the limit (3.5) requires that $p_{\parallel} \sim |P|$ for high $|P|$, which is consistent with the expectation of classical contraction $p_{\parallel} / M \sim \gamma = \sqrt{1 + P^2 / M^2}$. However, the connection between the infinite momentum frame and the rest frame wave functions, anticipated by the classical contraction picture, can only be studied by solving the deformation of the wave function for all values of $|P|$.

### 3.1 Review of Lorentz contraction studies

I review now previous studies of Lorentz contraction in moving systems. As stressed above such studies require the instant form of quantization. The theoretical frame-
work should also be relativistically covariant. Note that Lorentz covariance is not
 guaranteed for an ad hoc relativistic equation. E.g., Artru has shown [48] that the
eigenvalues of the relativistic 1+1 dimensional two-body “Schrödinger” equation
\[
\left[ \sqrt{p_1^2 + m_1^2} + \sqrt{p_2^2 + m_2^2} + V - E \right] \psi = 0
\] (3.6)
do not obey \( E = \sqrt{M^2 + \mathbf{P}^2} \) for any potential \( V = V(|x_1 - x_2|) \).
A Lorentz covariant quantum theory must in addition to the Hamiltonian have
all the other nine Poincaré generators with correct commutators [38]. A field
theory is conveniently defined in terms of an explicitly Lorentz covariant Lagrangian
density. Then the Poincaré generators may be derived from the Lagrangian and
the Lorentz covariance at the quantum level may be checked in a straightforward
manner. Fully covariant bound states include, however, Fock states with any num-
ber of particles. In most of the examples discussed below, an ad hoc restriction to
two-particle Fock states is made, neglecting all pair creation diagrams. Then the
Lorentz covariance is only approximate: it is recovered in the weak-coupling limit,
which typically removes Fock states with more than two particles. In some special
cases it is possible to construct fully relativistic dynamics with only two particles.
For a physical theory this means in practice, that the dynamical boost operators \( K^i \)
can be constructed for a given Hamiltonian. Two examples of such theories [11, 12]
are discussed below.

I am aware of only one previous study that treats the equal-time wave functions
of moving states in a physical field theory. The wave function of a moving hydrogen
atom was considered by Brodsky and Primack [8, 9] in the late sixties in a study
of the interaction of the atom with a background electromagnetic field. They start
from the CM frame where the QED Bethe-Salpeter equation is approximated by the
Breit equation (2.15). The solution for a moving atom is obtained by transforming
the CM solution as in (3.3). The dependence on relative time on the left hand
side of (3.3) can be neglected as an approximation for small boosts, whence (3.3)
directly gives a transformation rule for the equal-time wave functions. The result
reads
\[
\psi_p(x) = \frac{M + E}{2M} \int \frac{d^3p}{(2\pi)^{3/2}} \frac{E_p + m}{2E_p} \left( \frac{1 + \frac{\sigma_a^p + P}{M + E} \omega_a}{\frac{\sigma_a^p + P}{M + E} + \omega_a} \right) \otimes \left( \frac{1 - \frac{\sigma_b^p + P}{M + E} \omega_b}{\frac{\sigma_b^p + P}{M + E} + \omega_b} \right) \phi(p) \chi_{SM} \exp [ip \cdot \bar{x}]
\] (3.7)
where \( E = \sqrt{M^2 + \mathbf{P}^2} \) is the bound state energy, \( \bar{x} \) is the Lorentz contracted
coordinate vector
\[
\bar{x}_\perp = x_\perp; \quad \bar{x}_\parallel = \gamma x_\parallel ,
\] (3.8)
the operators \( \omega_{a,b} \) are defined by
\[
\omega_{a,b} = (2m + V/2 + \Delta E/2)^{-1} \sigma_{a,b} \cdot \mathbf{p} ,
\] (3.9)
\( \sigma_{a,b} \) operate on the constant nonrelativistic \( 2 \otimes 2 \) spinor \( \chi_{SM} \) (given by a Clebsch-Gordan composition of the constituent spinors), and the scalar wave function \( \phi(p) \) satisfies a “Pauli” two-body equation

\[
[\sigma_a \cdot p \omega_a + \sigma_b \cdot p \omega_b + V + \Delta E] \phi \chi_{SM} = 0
\]

that reduces to the Schrödinger equation for small \( \alpha \). The result (3.7) thus contains the classical contraction by \( \gamma \) but is only valid for small boosts. It also contains some subleading contributions in \( \alpha \), including the leading nontrivial spin structure. Note the similarity of the spin structure with that of the free Dirac spinors

\[
u(p,s) = \sqrt{E_p + m} \left( \frac{1}{E_p + m} \right) \chi_s.
\]

Equal-time wave functions of moving systems have also been studied in various models and approximation schemes, which restrict to 1+1 dimensions and/or to two-particle Fock states. In [10,11] a system of two fermions interacting via a zero range \( \delta \)-function potential is studied in 1+1 dimensions. The system is described by the 1+1 dimensional version of (2.15) where \( V \) is the \( \delta \)-potential (in coordinate space) that is multiplied with different Dirac structures, corresponding to vector, scalar, or pseudoscalar exchange. The boost operator \( K \) can be explicitly constructed and thus the model is exactly covariant [11]. The exact covariance is lost for potentials of any other form. The wave functions are simple exponential functions that have the classical Lorentz contraction property, and the energy spectrum transforms in boosts as \( E = \sqrt{M^2 + P^2} \).

In addition to the \( \delta \)-potential, Glöckle and Nogami consider a counterexample to the classical Lorentz contraction [12]. In the Bakamjian-Thomas model [49] the interaction between two spinless particles may be introduced via a relative momentum-dependent interaction term in the bound state mass operator. Then the system is manifestly Lorentz covariant. Glöckle and Nogami consider certain interaction terms that give simple wave functions, and classical Lorentz contraction is not observed. In particular, the wave functions seem also to contract perpendicularly to the boost direction. The authors continue with a more natural, field theory based model of a two-nucleon bound state bound by meson exchange. Their treatment that includes only the two-particle Fock state is Lorentz covariant only in the weak-coupling limit. They find numerically that the energy relation \( E = \sqrt{M^2 + P^2} \) and the classical Lorentz contraction of wave functions hold approximately.

Hoyer [13] considers a two-body meson bound state equation that is similar to (2.15) but has a linear potential \( V(x) = c|x| \). In the special case of the linear potential the transformation rule between the equal-time wave functions of different frames can be found in the \( x_\perp = 0 \) plane. The Lorentz covariance of the spectrum \( E = \sqrt{M^2 + P^2} \) can also be verified but the result reduces to the classical Lorentz contraction only in the weak-coupling limit.

In a more recent paper Schön and Thies [14] discuss the frame dependence of meson wave functions in the Gross-Neveu model. The Gross-Neveu model [50] is a
1+1 dimensional field theory for \( N \) massless fermions with \( U(N) \) symmetry and a pointlike quartic interaction term. It is usually considered in the limit of large \( N \). Schön and Thies solve the meson wave functions in a general frame in the Gross-Neveu model using Hartree-Fock and relativistic random phase approximations for large \( N \). Their treatment shows the evolution for the model wave function of a relativistic state from the CM frame to the infinite momentum frame. No classical contraction is observed.

### 3.2 The ’t Hooft model

The above review indicates how difficult it is to handle truly relativistic systems that involve an arbitrary number of particles. Wave functions are obtained only by neglecting the contributions of many-particle Fock states. However, there is an important example of a fully relativistic field theoretical model that provides a simple two-body equation for meson wave functions, namely the ’t Hooft model [51,52]. This is the large \( N \) limit of 1+1 dimensional QCD where \( N \) is the number of colors (see [53,54] for reviews). I study now this model in more detail. As we shall see, its simplicity is a consequence of the fact that Fock states with more than two particles do not couple to the meson. It is striking that the decoupling of the higher Fock states only occurs at the light front. While the instant and front forms have been found to produce equivalent meson spectra numerically [55], many-particle Fock states are present in the instant form quantization whence the meson structure cannot be described in terms of a single wave function.

The model has some peculiar features. To leading order in \( 1/N \) all Green functions are given by the sum of planar diagrams [51] in perturbation theory. Moreover, in axial gauges there are no gluon self-interactions and gluons do not propagate in time, they only contribute via an instantaneous Coulomb interaction. In the absence of gluon self-interactions the planar interaction kernel \( K \) of the Bethe-Salpeter equation (Fig. 2.2) includes only a single gluon exchange. Then the only nontrivial term in the BSE is the full quark propagator.

The original study [52] uses the light front picture and the axial gauge \( A^a_1 = A^{a0} + A^{a1} = 0 \). Then the gluon interaction is instantaneous in light front time \( x^+ \) and the quark self-energy can be solved explicitly. The Bethe-Salpeter equation simplifies to an exact two-body equation for the equal-light-front-time wave function

\[
\mu^2 \phi(x) = \left[ \frac{M_1^2}{x} + \frac{M_2^2}{(1-x)} \right] \phi(x) - \frac{g^2}{2\pi} P \int_0^1 dy \frac{1}{(x-y)^2} \phi(y) \tag{3.12}
\]

where \( x \) is the momentum fraction carried by the quark, \( P \) denotes the principal value prescription [52,56], \( \mu \) is the meson mass, and \( M_i \) are the renormalized quark masses. The equation (3.12) can easily be solved numerically.

In the instant form picture the gluons are also instantaneous in the axial gauge \( A^{a1} = 0 \), but the quark self-energy cannot be found explicitly as in the light front case. Instead, the self-energy satisfies a complicated set of integral equations [57],
which must be solved before studying the Bethe-Salpeter wave function. An apparent inconsistency of these equations is studied and removed in [58]. The equivalence of the energy spectra in the two pictures is verified numerically in [55]. Hanson et al. [59] study the meson spectrum in the instant form picture and in a general frame by restricting to the two-particle Fock state. They find numerically an approximate covariance \( E \simeq \sqrt{M^2 + (P^1)^2} \).

As mentioned above, the simplicity of the light front picture in the ’t Hooft model is due to the absence of pair production. Both quarks move only forward in light front time whence only two-particle Fock states contribute to the meson. This fact is certainly known to the experts (see, e.g., [14, 58]) but seldom stressed in the literature. The Fock state structure is best seen in the light front time ordered formalism, which is used in the discussion below.

\[
\Sigma = \begin{array}{c}
\end{array} 
= \begin{array}{c}
S 
\end{array} 
= \begin{array}{c}
\end{array}
\]

Figure 3.1: The quark self-energy in the light front ’t Hooft model. \( \Sigma \) is the (amputated) self-energy, \( S \) is the full quark propagator in the model and the dashed line is the instantaneous Coulomb interaction. The direction of light front time \( x^+ \) is to the right.

In \((x^+, p^+)\) space and in the \(A^{a+} = 0\) gauge the free quark and gluon propagators are given by

\[
S_F(x^+, p^+) = \theta(x^+ p^+) \ \text{sgn}(p^+) \ \exp \left( -i \frac{m^2}{2p^+} x^+ \right); \\
D(x^+, p^+) = \delta(x^+) \frac{i}{(p^+)^2},
\]

respectively. The \( P \) in the instantaneous gluon propagator indicates that the principal value prescription is used to regulate the infrared divergence. Due to the \( \theta(x^+ p^+) \) factor in (3.13) a quark with \( p^+ > 0 \) (\( p^+ < 0 \)) only propagates forward (backward) in light front time. The same holds for the full quark propagator for which the self-energy corrections exponentiate to

\[
S(x^+, p^+) = \theta(x^+ p^+) \ \text{sgn}(p^+) \ \exp \left[ -i \left( \frac{m^2}{2p^+} + \Sigma(p^+) \right) x^+ \right].
\]

where I used the fact that the quark self-energy is instantaneous \([\Sigma(x^+, p^+) = \delta(x^+) \ \Sigma(p^+), \text{ see Fig. 3.1}]\). Thus \( S \) includes only the one-quark Fock state and instantaneous corrections. Moreover, the radiative corrections do not contribute to
the instantaneous part of the full quark propagator that appears in Fig. 3.1: from (3.14) we see that

\[ S(0, p^+) = S_F(0, p^+) \] (3.15)

so the quark self-energy is in fact given by the lowest order radiative correction as indicated in Fig. 3.1.

\[ \begin{array}{cccc}
  & & & \\
  & P^+ - p^+ & & \\
  & & & \\
 X^+ & X^+ - Y^+ & & \\
  & & & \\
  & & & \\
\end{array} \]

Figure 3.2: The vanishing backward-moving part in the light front bound state equation \((Y^+ < 0)\).

As noted above the kernel \(K\) of the BSE only includes single gluon exchange in the \(\text{'t} \text{Hooft}\) model. Integrating over the light front energy \(p^-\) one obtains a bound state equation for the equal-light-front-time wave functions that contains both forward and backward-moving quark-antiquark propagators. However, the backward-moving term vanishes (see Fig. 3.2) as the necessarily negative momenta \(p^+\) and \(P^+ - p^+\) of the quark and the antiquark, respectively, cannot add up to a positive total momentum \(P^+\). Thus the backward-moving quarks decouple from the equation and the meson only includes the quark-antiquark Fock state.

The situation is drastically different in the instant form where the wave function is frame dependent. With the gauge choice \(A^a_1 = 0\) the gluon stays instantaneous, but the backward-moving quarks do not decouple. Hence the equal (ordinary) time wave functions involve Fock states with an arbitrary number of quark-antiquark pairs as intermediate states. This precludes an explicit solution of the full quark propagator in the instant form, whence the BSE becomes complicated. However, also in the instant form the extra Fock states disappear in the weak-coupling limit. Then one obtains similar simplifications as in the light front \(\text{'t} \text{Hooft}\) model, as demonstrated in the following section.

3.3 The moving hydrogen atom

I review here the main results of the first two papers [I, II] of this thesis, where the wave function of the hydrogen atom is found in a general frame. The internal motion of the atom is nonrelativistic, but the overall motion may be relativistic and consequently the field theoretical Bethe-Salpeter approach needs to be used. The weak-coupling expansion of the Bethe-Salpeter equation is done using “old-fashioned”
time ordered perturbation theory where the relevant Fock state components and the internal dynamics are explicitly seen.

In [I] the weak-coupling limit of the Bethe-Salpeter wave function is evaluated in 1 + 1 dimensional QED. The 1 + 1 dimensional case is particularly simple as in the \( A^1 = 0 \) gauge there are no time derivatives of the photon fields in the Lagrangian. Thus the photon field does not propagate in time and there only is an instantaneous Coulomb interaction between the constituents at the same instant of time as in the 't Hooft model. The leading component of the wave function in the QED coupling \( \alpha \) is seen to Lorentz contract in the classical fashion, and the Lorentz covariance of the energy spectrum \( E = \sqrt{M^2 + (P^1)^2} \) is verified in the weak-coupling limit. These results are also valid in 1 + 1 dimensional QCD and in the 't Hooft model, which are similar to 1 + 1 dimensional QED in the weak-coupling limit.

In [II] the derivation of [I] is generalized to the 3 + 1 dimensional physical QED, i.e., to the hydrogen atom or positronium. In 3 + 1 dimensions transverse, propagating photons are present which adds new features to the problem. The lowest, \( e^- e^+ \), Fock state contracts again classically to leading order in \( \alpha \). The wave function of the next-to-leading \( e^- e^+ \) Fock state is also evaluated and seen not to contract classically. This implies together with the various examples discussed in the previous section that classical contraction is not a general property of bound states in field theory.

![Figure 3.3](image)

**Figure 3.3:** Typical structure of the hydrogen atom dynamics for \( P \neq 0 \) in time ordered perturbation theory and Coulomb gauge. \( \varphi_P \) is the equal-time wave function, the dashed exchange represents the instantaneous Coulomb photon interaction, the wavy lines are transverse photons, and time flows to the right. The dashed vertical cuts indicate time slices, and the \( \Delta E \)'s are the corresponding energy differences.

I present now some details of the more general 3+1 dimensional calculation. The constituent masses \( m \) are set equal for simplicity. As in the CM frame one-photon exchange dominates the bound state equation for \( P \neq 0 \). When the BSE (Fig. 2.2) is iterated, the leading bound state ladder typically looks like that of Fig. 3.3 with Fock states of only two fermions. The uncertainty relation implies that the lifetime of a Fock state is the inverse of the energy difference between the bound state and
Fock state energies. The energy difference for the free $e^- e^+$ propagation ($\Delta E_F$) and for transverse photon exchange ($\Delta E_I$) indicated in Fig. 3.3 are

$$\begin{align*}
\Delta E_F & \sim \alpha^2 m \gamma^{-1}; \\
\Delta E_I & \sim \alpha m \gamma^{-1},
\end{align*}$$

respectively. The time scales $\Delta t_{F,I} = 1/\Delta E_{F,I}$ are proportional to $\gamma$ but their ratio scales as $\Delta t_I \sim \alpha \Delta t_F$ in all frames. Thus the internal time scales are Lorentz dilated, but transverse photon exchange is a rare event in all frames\(^1\): the probability of observing a transverse photon in the atom is $\propto \alpha$. Moreover, it is possible to check that all the other Fock states with, e.g., two transverse photons are suppressed by additional powers of $\alpha$. For the “Z diagrams” that have an extra fermion pair, i.e., when one of the fermion lines of Fig. 3.3 forms a Z, the lifetime is $\Delta t_Z \simeq \gamma/2m \sim \alpha^2 \Delta t_F$ due to the large energy $\sim 2m/\gamma$ needed to create the pair. Hence pair production is expected to be absent even at next-to-leading order in $\alpha$.

Retaining only the leading term in $\alpha$, which involves single Coulomb and transverse photon exchange, the time ordered BSE in a general frame works out to

$$\begin{align*}
\left[ \Delta M - \frac{1}{m} \left( q_\perp^2 + \gamma^{-2} q_\parallel^2 \right) \right] \varphi_P(q) &= -\frac{4\pi\alpha}{\gamma} \int \frac{d^3 k}{(2\pi)^3} \frac{\varphi_P(q-k)}{k_\perp^2 + \gamma^{-2} k_\parallel^2} \\
\text{(3.17)}
\end{align*}$$

where $\Delta M = M - 2m$, $\gamma = E/M = \sqrt{1 + \vec{P}^2/M^2}$, $\vec{k}$ is the momentum of the exchanged photon, and the equal-time wave function $\varphi_P$ is understood to be a function of the relative momentum $\vec{q} = \vec{p} - \vec{P}/2$ rather than a function of $\vec{p}$. The solutions of the equation (3.17) are Lorentz contracted (or rather Lorentz expanded in momentum space) w.r.t. the CM solution (2.14):

$$\begin{align*}
\varphi_P(q)_{\alpha\beta} &= \frac{1}{\sqrt{\gamma}} \sum_{s_1, s_2} u_\alpha(P/2, s_1) \bar{v}_\beta(P/2, s_2) \chi_{s_1, s_2} \phi_{CM}(q_\perp, q_\parallel/\gamma) \\
\text{(3.18)}
\end{align*}$$

where $\chi_{s_1, s_2}$ is a constant spin wave function, $\phi_{CM}$ is the ordinary hydrogen atom wave function in the CM frame, and the Dirac structure involves only the forward-moving components. The Lorentz contraction is manifest in the scaling of the longitudinal momentum component $q_\parallel$ by $\gamma$. The wave function is normalized\(^2\) as

$$1 = \int \frac{d^3 q}{(2\pi)^3} \text{Tr} \left\{ \varphi_P^\dagger(q) \varphi_P(q) \right\}$$

(3.19)

given that

$$1 = \sum_{s_1, s_2} |\chi_{s_1, s_2}|^2 = \int \frac{d^3 q}{(2\pi)^3} |\phi_{CM}(q)|^2. \quad (3.20)$$

\(^1\)In the CM frame transverse photons decouple from the equation at leading order in $\alpha$.

\(^2\)In the normalization (3.19) $\varphi_P$ is scaled by a factor $1/\sqrt{\gamma}$ w.r.t. the definition (2.3).
The wave function of the next-to-leading $e^-e^+\gamma$ Fock state which includes the photon distribution is also given in [II]. The probability distribution of the $e^-e^+\gamma$ Fock state is given by

$$d^6\mathcal{P}(q, k) = \frac{\alpha}{4\pi^2} \frac{\beta^2k_\perp^2}{|k|^3 (|k| - \beta k_\parallel)^2} \frac{|\phi_P(q) - \phi_P(q - k)|^2}{(2\pi)^3} \tag{3.21}$$

where $\beta = |P|/E$ and $\phi_P$ is the contracted scalar wave function

$$\phi_P(q) = \frac{1}{\sqrt{|q|}} \phi_{CM}(q_\perp, q_\parallel/\gamma). \tag{3.22}$$

Note that the distribution (3.21) vanishes in the CM frame ($\beta = 0$) where transverse photons are absent to leading order in $\alpha$.

As an illustration I derive the photon distribution for the ground state of the hydrogen atom. Let us define the Lorentz contracted electron and photon momenta as

$$\hat{q} = (q_\perp, q_\parallel/\gamma) ; \quad \hat{k} = (k_\perp, k_\parallel/\gamma). \tag{3.23}$$

Integrating (3.21) over the electron momentum $q$ and the remaining azimuthal angle one obtains the factorized form

$$\frac{d^2\mathcal{P}}{dk\cos \theta} = \frac{\alpha}{2\pi} f(\cos \theta) g(\hat{k}) \tag{3.24}$$

where $\hat{k} = |\hat{k}|$ and $\theta$ is the angle between $k$ and $P$. The functions $f$ and $g$ are given by

$$f(\cos \theta) = \frac{\gamma \beta^2 (1 - \cos^2 \theta)}{(1 + \beta^2 \gamma^2 \cos^2 \theta)^{3/2}} \left( \sqrt{1 + \beta^2 \gamma^2 \cos^2 \theta} - \beta \gamma \cos \theta \right)^2 \tag{3.25}$$

and

$$g(\hat{k}) = \frac{1}{\hat{k}} \int \frac{d^3\hat{q}}{(2\pi)^3} \left| \phi^{(0)}_{CM}(\hat{q} + \hat{k}/2) - \phi^{(0)}_{CM}(\hat{q} - \hat{k}/2) \right|^2 \tag{3.26}$$

where $\phi^{(0)}_{CM}$ is the ground state wave function of the hydrogen atom

$$\phi^{(0)}_{CM}(\hat{q}) = \sqrt{\frac{512\pi}{\alpha^3 m^3}} \frac{1}{\left[ 1 + 4\hat{q}^2 / (\alpha m)^2 \right]^2}. \tag{3.27}$$

While the $\hat{k}$-dependent function (3.26) contracts just as the $e^-e^+$ component, the angular dependence (3.25) is frame dependent and thus deviates from the classical Lorentz contraction. The evolution of $f$ in boosts is shown in Fig. 3.4. For small $P$ the distribution is rotationally symmetric, but for large $P$ backward-moving photons disappear. For $P \to \infty$ the distribution approaches the one found in light front quantization [60], where all photons are in the forward-moving hemisphere.
Figure 3.4: The angular dependence of the contracted and integrated photon distribution (3.24) in the positronium ground state [II]. The lines show the angular distribution \( f(\cos \theta) / (\gamma^2 \beta) \) [defined in (3.25)] for \( \beta = 0.001, 0.5, 0.9 \) and 0.999. For \( \beta = 0.001 \) (solid line) the distribution is almost symmetric. For \( \beta = 0.999 \) (dotted line) the photon is most likely moving forward \( (\cos \theta > 0) \).

To conclude this chapter I briefly comment on the weak-coupling solution for the covariant wave function \( \psi \) of (2.1) at arbitrary relative time \( t = x_0^1 - x_0^2 \). The leading dependence of \( \psi \) on the relative energy is given by (2.11). In terms of the relative momentum \( q = p - \frac{P}{2} = (p_1 - p_2)/2 \) we have in the CM frame

\[
\psi_{P=0}(q) = \frac{i e^2}{(q^0 + M/2 - E_q + i \varepsilon)(q^0 - M/2 + E_q - i \varepsilon)} \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2} \varphi(q - k)
\]

where \( q^0 \) is the relative energy. A Fourier transform to \((t, p)\) space gives

\[
\psi_{P=0}(t, q) = \exp \left[ i \left( \Delta M - q^2 / m \right) \right] \varphi(q)
\]

so that the Bethe-Salpeter wave function is independent of the relative time for \( |t| \ll 1/\alpha^2 m \) in the CM frame. This result may be understood in terms of the hydrogen atom dynamics in the time ordered picture (Fig. 3.3, see [I] for a complete time ordered proof in 1 + 1 dimensions). For \( |t| \ll 1/\alpha^2 m = \Delta t_F \) the probability of photon exchange is small. Then the evolution is that of free particles, giving phases \( \sim \exp \left[ i \Delta M t \right] \) which are negligible for \( |t| \ll 1/\alpha^2 m \). In particular, as the size of the wave function \( \sim 1/\alpha m \) is much less than \( \Delta t_F \), it follows that the equal-time and equal-light-front-time \((x^+ = x^0 + x^3)\) wave functions are the same in the
weak-coupling limit. Moreover, the dependence on relative time of the CM wave function can be neglected to leading order in $\alpha$ in transformation formulae like (3.3). Thus the approximate boosting formula suggested in [8,9] (and discussed above in Sec. 3.1) is in fact valid in all frames for the leading wave function in $\alpha$.

For $P \neq 0$ the result (3.29) becomes

$$\psi_P(t, q) = \exp \left[ -i \beta q \parallel t \right] \exp \left\{ i \left[ \Delta M - \frac{1}{m} \left( q_\perp^2 + \gamma^{-2} q_\parallel^2 \right) \right] \frac{|t|}{2\gamma} \right\} \varphi_P(q) \ (3.30)$$

where the first, larger phase factor accounts for the relativistic movement of the constituents with velocity $\beta$.

---

3The formula (3.30) is a better approximation for large $t$ than the coordinate space version represented in [II,61].
Chapter 4

QCD factorization and deep inelastic scattering on positronium

In this chapter I present the main ideas of QCD factorization using Deep Inelastic Scattering (DIS) as an example. I evaluate the electron distribution function using the wave function of moving positronium discussed in the preceding chapter.

The running coupling constant of QCD decreases with energy and becomes small $\alpha_s \ll 1$ in high-energy scattering. Asymptotic freedom ensures that in high-energy scattering the constituents of hadrons, the partons (quarks and gluons), interact weakly and perturbation theory can be applied. However, experimentally measured cross sections involve hadrons rather than partons, and the perturbative approach is unable to describe these cross sections directly. This fact shows up in a naive calculation of the hard parton QCD interactions as long distance, infrared divergences that spoil the convergence of the perturbation series.

The standard framework for handling the long distance problems is QCD factorization (see [62] for a review). It allows to separate the long distance physics from the hard subprocess in a systematic way. The soft physics is contained in universal parton distribution and fragmentation functions which can be understood as probability distributions of partons in the scattering hadrons. The separation into soft ($Q \sim \Lambda_{\text{QCD}} \sim 300 \text{ MeV}$) and hard ($Q \gg \Lambda_{\text{QCD}}$) physics is done at some factorization scale $\mu$, which should be high enough for perturbation theory to apply. The dependence on the factorization scale formally vanishes when all orders in perturbation theory are taken into account. In the standard picture a parton has a momentum that is collinear with the parent hadron momentum. Then the parton distributions are functions of the fraction $x$ of the hadron momentum carried by the parton. The physical cross section is obtained as a convolution of the hard parton interaction with the parton distribution and fragmentation functions.

Deep inelastic scattering ($e^- p \to e^- + X$) is a good example of QCD factorization. It also provides the most accurate determination of the parton distributions of hadrons or $e^- e^+ \to jets$.

\[\text{There are certain infrared safe cases where the QCD parton calculations can be applied directly such as } e^- e^+ \to \text{hadrons or } e^- e^+ \to jets.\]
the proton. In DIS the electron scatters on the proton via a virtual photon exchange (with momentum transfer $q$). The hard scale $Q$ of the interaction is given by the photon virtuality $Q^2 = -q^2$. The parton distributions are probed in the Bjorken limit: the limit of high $Q$ with fixed Bjorken variable $x_B = Q^2/(2P \cdot q)$ where $P$ is the target proton momentum. Factorization is valid at leading twist, i.e., to leading order in $1/Q$, and allows to express the cross section in the form

$$\sigma(e^- p \to e^- + X) = \int_0^1 dx \sum_f f_f(x) \tilde{\sigma}(e^- q_f \to e^- + \text{partons})$$

(4.1)

where the sum goes over different flavors $f$ (including antiquarks and gluons). The parton distributions $f_f(x, \mu^2)$ and the subprocess cross section $\tilde{\sigma}$ depend on the factorization scale $\mu^2$.

In order to evaluate DIS on a positronium target I review some basics of standard DIS at the parton model level. The $e^- \to e^- \gamma^*$ vertex can be treated in perturbative QED. Using the optical theorem one finds that the cross section for $\gamma^* p \to X$ is proportional to the imaginary part of the forward elastic $\gamma^* p \to \gamma^* p$ scattering amplitude.

Figure 4.1: The lowest order handbag diagram for the imaginary part of the forward photon-proton elastic scattering amplitude.

In the naive parton model the QCD (gluon) interactions are completely neglected in the hard scattering. The corresponding lowest order “handbag” diagram for $\gamma^* p \to \gamma^* p$ is shown in Fig. 4.1. To leading order in $1/Q$ the hard and soft terms of the diagram in Fig. 4.1 are separated. The hard part includes the intermediate quark propagator (of momentum $p + q$) and the photon vertices, while the lower part of the diagram is soft. It also includes the quark propagators with momentum $p$ that are almost on mass shell. The soft part is described by the correlator function $\Phi_P(p)_{\alpha\beta}$ which can be expressed as a soft matrix element (see, e.g., [16])

$$\Phi_P(p)_{\alpha\beta} = \int d^4y e^{ip \cdot y} \langle \psi, P | \bar{q} (0)_{\beta} q(y)_\alpha | \psi, P \rangle$$

(4.2)

where $|\psi, P\rangle$ is the proton state and $p$ is the momentum of the quark. Rescattering between the struck quark and the projectile system (which is absent in parton
model) would add to (4.2) Wilson lines making the definition gauge invariant. See Fig. 4.2 for a diagrammatic representation of $\Phi_P(p)_{\alpha\beta}$ for positronium. The unpolarized quark distribution is given by

$$f(x) = \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \text{Tr} \left[ \gamma^+ \Phi_P(p) \right] \delta \left( p^+ - xP^+ \right) \delta \left( p^- + xP^- \right)$$

$$= \frac{1}{8\pi} \int dy^- e^{ixP^+} \langle \psi, P | \bar{q}(0) \gamma^+ q(y^-) | \psi, P \rangle$$

(4.3)

where the trace goes over the Dirac indices $(\alpha, \beta)$.

### 4.1 Electron distribution in positronium

I present now a model calculation of the parton distribution using the wave function of a moving positronium atom discussed in the previous chapter. I only calculate the unpolarized electron distribution $f(x)$, which is a sharply peaked function at $x = 1/2$ for a nonrelativistic target. While the calculation leads to a well known result, it serves as a consistency check since the frame dependence vanishes in a nontrivial manner. It also demonstrates how the Bethe-Salpeter wave functions are used in scattering calculations. The model could be further developed to include subleading corrections in $\alpha$ and, e.g., a study of the rescattering effects. DIS on the hydrogen atom was considered in [63] using the Dirac wave function of hydrogen.

The cross section for DIS on positronium is given by (4.1), where one sums over electron and positron (and photon) contributions. I only evaluate the electron $f(x)$ in the weak-coupling limit. I use the wave function (3.18) of positronium in a general frame. All negative energy antiparticle propagators are suppressed by $\alpha$ (see Sec. 3.3) and one can thus restrict to the electron-positron Fock state in the weak-coupling limit.

![Figure 4.2: The electron correlator $\Phi_P(p)_{\alpha\beta}$ for positronium. $(\alpha, \beta)$ are Dirac indices. The blobs represent the Bethe-Salpeter wave functions of positronium.](image)

The electron correlator $\Phi_P(p)_{\alpha\beta}$ of (4.2) for positronium ($Ps$) is shown in Fig. 4.2. The blobs are the $Ps \rightarrow e^- e^+$ vertices given by the Bethe-Salpeter wave function.
function (2.1). The function (3.30) becomes in momentum space

\[ \psi_P(p) = -\frac{i (E - E_p - E_{P-p})}{(p^0 - E_p + i\varepsilon)(p^0 - E + E_{P-p} - i\varepsilon)} \varphi_P(p) \]  

(4.4)

up to subleading corrections in \(\alpha\). Here \(\varphi_P\) is the equal-time wave function of (3.18). The electron momentum \(p\) was used instead of the relative one \(q = p - P/2\) for notational simplicity. The Bethe-Salpeter wave function (4.4) is nonamputated, i.e., it includes the electron and positron propagators in addition to the \(Ps \to e^-e^+\) vertex. In Fig. 4.1 the wave functions thus share a common positron propagator which should be amputated from one of the wave functions. Because the backward-moving electron and positron components of \(\psi_P(p)\) are suppressed by \(\alpha^2\) one finds, e.g.,

\[ (\gamma^0 E_p - \gamma \cdot p - m) \psi_P(p) \simeq 0 \]  

(4.5)

as the operator \((\gamma^0 E_p - \gamma \cdot p - m)\) projects to the backward-moving components of \(\psi_p\). Thus the leading term of the amputated Bethe-Salpeter wave function \(\hat{\psi}_P\) (the plain \(Ps \to e^-e^+\) vertex) is given by

\[
\hat{\psi}_P(p) = \left(\begin{array}{c}
\gamma^0LIST\quad E_p - \gamma \cdot p - m \end{array}\right) \psi_P(p) \gamma^0
\]

(4.6)

where (4.5) and a similar identity for the positron were used.

The electron correlator (Fig. 4.2) may then be calculated in a straightforward manner. Using the amputated Bethe-Salpeter wave function (and its Dirac conjugate) one finds

\[ \Phi_P(p) = 2\pi \delta \left(E - p^0 - E_{P-p}\right) \varphi_P(p) \gamma^0 \overline{\varphi}_P(p) \]  

(4.7)

where the wave functions are understood as matrices in Dirac space. Here

\[ \overline{\varphi}_P(p) = \gamma^0 \varphi^\dagger_P(p) \gamma^0 \]  

(4.8)

is the Dirac conjugate of \(\varphi_P\) of (3.18). The electron distribution is thus given by

\[ f(x) = \frac{M}{2\pi} \int \frac{d^4 p}{(2\pi)^4} \text{Tr} \left[\gamma^+ \Phi_P(p)\right] \delta (p^+ - xP^+) \]  

(4.9)

where (4.3) was multiplied by \(2E\) to account for our nonrelativistic normalization of positronium wave function (3.19).

An explicit expression for \(f(x)\) may be obtained by inserting (3.18) and (4.7) into (4.9). Shifting to the relative momentum \(q = p - P/2\) one finds the final result

\[ f(x) = \frac{M}{2\pi} \int \frac{d^2 q_\perp}{(2\pi)^2} |\phi_{CM}(q_\perp, (x - 1/2)M)|^2 \]  

(4.10)
which is frame independent. Because $q_\parallel/\gamma = (x - 1/2)M \sim \alpha M$ the function is peaked around $x = 1/2$ as the natural result for a nonrelativistic system. Note that

$$\int_0^1 dx \, f(x) \simeq \int \frac{d^3q}{(2\pi)^3} |\phi_{CM}(q)|^2 = 1.$$  \hspace{1cm} (4.11)

Other (spin-dependent) electron distributions and, e.g., rescattering corrections can be analyzed similarly.

Naturally, the above calculation could also be done using the light front wave function of the atom. The fact that DIS probes the positions of the constituents at equal light front time is seen from the expression for $f(x)$ (4.3): the quark momentum $p^+$ is fixed but its light front energy $p^-$ is integrated, fixing the light front time difference between the constituents to zero. Consequently, on the light front an expression like (4.10), which relates $f(x)$ to the absolute value squared of the wave function, is obtained without using the weak-coupling limit (see [6, 7]) for two-particle Fock states when rescattering effects are neglected [64]. Hence the result (4.10) also reflects the fact that for $\alpha \to 0$ the bound state constituents move slowly in the CM frame, so that the light front and instant form treatments coincide.
Chapter 5

Single spin asymmetries

In this chapter I review the basics of single spin asymmetries. I present the main experimental results and previous theoretical work on the asymmetries.

Spin dependence of QCD offers many observables that can be used to test theoretical predictions (see [16, 65, 66] for reviews). In particular, one can study spin-dependent parton distribution functions. In the standard leading twist collinear factorization there are two spin-dependent distributions in addition to the unpolarized one $f(x)$, namely the helicity distribution $\Delta f(x)$ and the transversity distribution $\Delta_T f(x)$ (see [16] for a thorough discussion). The probabilistic interpretation of the functions is (ignoring rescattering effects [64])

$$
\begin{align*}
    f(x) &= \mathcal{P}_{q/N}(x) \\
    \Delta f(x) &= \mathcal{P}_{q-/-N}(x) - \mathcal{P}_{q-/N}(x) \\
    \Delta_T f(x) &= \mathcal{P}_{q_1/N\uparrow}(x) - \mathcal{P}_{q_\downarrow/N\uparrow}(x).
\end{align*}
$$

The helicity distribution $\Delta f(x)$ gives the probability of finding a longitudinally polarized quark inside a longitudinally polarized proton, and the transversity distribution $\Delta_T f(x)$ correspondingly gives the probability of finding a transversely polarized quark inside a transversely polarized proton. Similarly, (spin-dependent) fragmentation is described in terms of three distinct fragmentation functions.

The unpolarized parton distributions have been measured in DIS to a good precision (see [67] for a recent review). Measurements range over several orders of magnitude in the energy scale $Q$ and in the momentum fraction $x$. The helicity distributions of the light quarks are also known, in particular the combination $\Delta q + \Delta \bar{q}$ which is rather directly measured in polarized DIS. Progress on measuring the gluon helicity distribution has also been made. However, the transversity distributions are poorly known, because they are not directly accessible in DIS.

While longitudinal spin (such as the quark helicity distributions) received much attention during the first decades of QCD spin physics, there are also interesting phenomena linked to transverse spin. The simplest example is the (transverse) Single Spin Asymmetry (SSA). It is a dependence of the scattering cross section on a single measured spin. Usually the polarized particle is a spin 1/2 fermion. The
SSA is measured by the analyzing power

\[ A_N = \frac{\sigma(\uparrow) - \sigma(\downarrow)}{2\sigma} = \frac{\sigma(\uparrow) - \sigma(\downarrow)}{\sigma(\uparrow) + \sigma(\downarrow)} \]  

(5.2)

where the ↑ arrows indicate the transverse polarization. The denominator normalizes the analyzing power such that \(|A_N| \leq 1\). It is useful to express \(A_N\) in terms of helicity amplitudes \(\mathcal{M}_{\leftrightarrow,\{\sigma}\}}\)

\[ A_N = \frac{\sum_{\{\sigma\}} \left[ |\mathcal{M}_{\uparrow,\{\sigma}\}}^\ast|^2 - |\mathcal{M}_{\downarrow,\{\sigma}\}}|^2 \right]}{\sum_{\{\sigma\}} \left[ |\mathcal{M}_{\uparrow,\{\sigma}\}}|^2 + |\mathcal{M}_{\downarrow,\{\sigma}\}}|^2 \right]} = \frac{2 \sum_{\{\sigma\}} \text{Im} \left[ \mathcal{M}_{\uparrow,\{\sigma}\}}^\ast \mathcal{M}_{\downarrow,\{\sigma}\}} \right]}{\sum_{\{\sigma\}} \left[ |\mathcal{M}_{\uparrow,\{\sigma}\}}|^2 + |\mathcal{M}_{\downarrow,\{\sigma}\}}|^2 \right]} \]  

(5.3)

where ↔ denotes the helicity of the polarized particle and \(\{\sigma\}\} are the helicities of all other particles. It is understood that the numerator and denominator of (5.3) are integrated separately over the momenta of unobserved particles in the final state.

The numerator of (5.3) is given by an interference between amplitudes with opposite helicities of the polarized particle. Hence one of the amplitudes necessarily includes a helicity flip. Such a flip cannot occur in the standard distribution or fragmentation functions where the parton is collinear with its parent hadron due to conservation of angular momentum. Moreover, quark helicity flip in a hard process is proportional to the small current quark mass (see, e.g., [66]). Consequently, SSA’s are suppressed in the standard (collinear) factorization picture of QCD [15]. The asymmetry (5.3) also requires a (dynamical) phase difference between the helicity amplitudes. As the Born amplitudes are real, loop diagrams with imaginary (absorptive) parts are needed. However, loop corrections of the hard subprocess are suppressed by powers of \(\alpha_s\), further suppressing \(A_N\) in the standard factorization scheme.

Spin flip in the distribution functions is allowed if one generalizes factorization. Two alternatives are available: one can include the transverse momentum dependence of the partons, or consider subleading (twist-three) corrections.

Transverse-Momentum-Dependent (TMD) factorization was first suggested in [68] and applied to SSA’s almost a decade later [69]. In this approach the dependence on (typically small \(\lesssim \Lambda_{QCD}\)) parton transverse momentum is added to the parton distribution and fragmentation functions (see [16] for a review). In general, adding transverse momentum dependence leads to eight distinct parton distributions: in addition to the TMD generalization of the three distributions discussed above, one finds five additional ones which vanish when integrated over the transverse momentum [70, 71]. Similarly one finds eight different types of fragmentation functions.

The subleading, twist-three formalism was pioneered by Efremov and Teryaev [72, 73] in the early 80’s and later developed by Qiu and Sterman [74, 75]. The twist-three matrix elements involve three (rather than two as in Fig. 4.1) partons of the interacting hadrons. Remarkably, it has been shown recently that the two apparently different pictures, TMD and twist-three factorization, describe the same
physics in the kinematic regions where they are both applicable [76]. In fact, one can relate the Sivers function to certain twist-three quark-gluon correlation functions [77]. Thus the two pictures are consistent with each other.

5.1 SSA in semi-inclusive DIS

The TMD approach is natural for studying SSA’s in Semi-Inclusive Deep Inelastic Scattering (SIDIS, $\ell p^\uparrow \rightarrow \ell' + \pi + X$). A natural choice of frame is shown in Fig. 5.1: the large virtual photon momentum defines the $z$ direction and the lepton momenta lie in the $xy$ plane. The hard scale of DIS is given by the virtuality $Q$ of the exchanged photon, whence the transverse momentum of the pion $P_{h\perp}$ can be kept small. Consequently, the TMD distributions with small ($\lesssim \Lambda_{\text{QCD}}$) intrinsic transverse momenta are able to produce asymmetries at leading twist, i.e., at leading order in $1/Q$. The TMD factorization for SIDIS was recently shown to hold in QCD [78, 79].

Within the TMD formalism there are several helicity-flip parton distribution or fragmentation functions that can give rise to asymmetries. The most studied mechanisms are the Sivers [69] and Collins [25] effects. The Sivers distribution function $f_{1T}^\perp$ describes the angular distribution of an unpolarized quark inside a transversely polarized proton,

$$P_{q/N^T}(x, k_{\perp}) = f(x, k_{\perp}^2) - \frac{k_{\perp}}{M} \sin(\phi_k - \phi_S) f_{1T}^\perp(x, k_{\perp}^2), \quad (5.4)$$

where $\phi_k$ is the azimuthal angle of the quark momentum $k_{\perp}$, $\phi_S$ is defined in Fig. 5.1, and $M$ is the nucleon ($N$) mass. Hence the transverse momentum distribution of unpolarized quarks inside the proton is allowed to be asymmetric in $\phi_k$. The asymmetric quark distribution with $k_{\perp} \sim \Lambda_{\text{QCD}}$ then generates an asymmetry via a “trigger bias”: the correlation between $\phi_k$ and the azimuthal angle $\phi_h$ of the final state hadron in the hard scattering part of

$$A_N \sim f_{1T}^\perp \otimes \hat{\sigma} \otimes D \quad (5.5)$$

gives a $\phi_h$-dependent asymmetry. Here $D$ is the usual (unpolarized) fragmentation function. The Sivers function was believed to be subleading twist in SIDIS before the model calculation of Brodsky, Hwang, and Schmidt [20]. I will discuss the dynamics of the Sivers effect and the BHS calculation in more detail in the next chapter.

In the Collins [25] mechanism the asymmetry is due to an asymmetric fragmentation function, i.e., it is a final state effect. The Collins function $H_{1T}^\perp$ describes the fragmentation of a transversely polarized quark into an unpolarized hadron

$$\mathcal{N}_{h/q^T}(z, \kappa_{\perp}) = D(z, \kappa_{\perp}^2) + \frac{\kappa_{\perp}}{z M_h} \sin(\phi_\kappa - \phi_S) H_{1T}^\perp(z, \kappa_{\perp}^2) \quad (5.6)$$

where $z$ is the momentum fraction of the fragmenting quark carried by the formed hadron, $\kappa_{\perp}$ is the transverse momentum of the hadron w.r.t. the quark ($\kappa_T$ of
and $\phi_\kappa$ is the corresponding azimuthal angle. When the fragmenting quark transverse momentum is zero $\kappa_\perp$ equals $P_{h\perp}$ of Fig. 5.1. The Collins effect links the function $H_1^\perp$ to the transversity $\Delta_T f(x)$

$$A_N \sim \Delta_T f \otimes \Delta \hat{\sigma} \otimes H_1^\perp$$

(5.7)

where $\Delta \hat{\sigma}$ is the difference of the parton cross sections with opposite transverse polarizations of the fragmenting quark. Thus in the Collins picture the hadron polarization is inherited by the struck quark and the asymmetry is then generated in the fragmentation of the (transversely polarized) quark.

Figure 5.1: Definition of the azimuthal angles for SIDIS $[\ell(l) P^\uparrow \rightarrow \ell'(l') + \pi(P_h) + X]$ in the target rest frame. Here $S_\perp$ is the component of the target spin that is perpendicular to the virtual photon momentum. Figure from [80].

The Sivers and Collins effects can be separated in SIDIS due to their different dependence on the azimuthal angles,

$$\frac{\sigma(\uparrow) - \sigma(\downarrow)}{\sigma(\uparrow) + \sigma(\downarrow)} = A_{UT}^{\text{sin}(\phi_h - \phi_S)} \sin(\phi_h - \phi_S) + A_{UT}^{\text{sin}(\phi_h + \phi_S)} \sin(\phi_h + \phi_S)$$

(5.8)

where the first term arises from the Sivers effect and the second term from the Collins effect, and the angles are defined in Fig. 5.1. The subscript $UT$ indicates that an Unpolarized electron beam hits a Transversely polarized proton target.

Nonzero Sivers and Collins moments $A_{UT}^{\text{sin}(\phi_h \pm \phi_S)}$ of (5.8) have been observed in SIDIS on a polarized proton target by the HERMES collaboration [18]. The asymmetries are typically smaller than 10%. Similar measurements at higher energy on a polarized deuteron target by COMPASS [19] show asymmetries which are compatible with zero.
HERMES and COMPASS data have been used to extract the Sivers [81–84] and Collins [84–86] functions from the data and to make rough estimates of the transversity distributions $\Delta_T f$ [86]. Predictions for other processes such as the Drell-Yan process have also been made. The vanishing of the asymmetries at COMPASS [19] can be caused by a cancellation of the proton and neutron contributions. Predictions for $A_N$ at a large $P_{h\perp}$ of the pion have been made using the twist-three formalism [87, 88].

5.2 SSA in proton-proton collisions

The largest SSA’s have been observed in inclusive pion and $\Lambda$ production in proton-proton collisions ($p^\uparrow p \rightarrow \pi X$ and $pp \rightarrow \Lambda^\uparrow X$). The dynamics of these reactions is quite different from SIDIS because the hard scale of the process is given by the $k_\perp$ of the outgoing hadron. The TMD approach, which ascribes the symmetry to the small intrinsic transverse momenta of partons, predicts $A_N \propto \Lambda_{QCD}/k_\perp$. This contrasts with the large size and $k_\perp$ dependence of the data, which I review below.

In $p^\uparrow p \rightarrow \pi X$ and $pp \rightarrow \Lambda^\uparrow X$ parity requires the polarization to be transverse w.r.t. to the beam direction:

$$A_N \propto S \cdot k \times p \propto \epsilon_{\mu \nu \rho \sigma} p^\mu \tilde{p}^\nu k^\rho S^\sigma$$

(5.9)

where $p$, $\tilde{p}$ and $k$ are the beam, target, and pion (or $\Lambda$) momenta, respectively. Fixing $p$ in the $z$ direction and $S$ in the $y$ direction one finds

$$S \cdot k \times p \propto \cos(\phi)$$

(5.10)

where $\phi$ is the azimuthal angle of the produced particle. For inclusive pion (or hyperon) production it is thus convenient to define instead of (5.2)

$$A_N(x_F, k_\perp) \cos \phi = \frac{\sigma(\uparrow) - \sigma(\downarrow)}{\sigma(\uparrow) + \sigma(\downarrow)}$$

(5.11)

where $k = (k_\perp, x_F p_\parallel)$. Hence $A_N(x_F, k_\perp)$ is the asymmetry which is observed for pions in the $xz$ plane ($\phi = 0$). In $pp \rightarrow \Lambda^\uparrow X$ the asymmetry (5.11) is usually denoted as $P_\Lambda(x_F, k_\perp)$ and called “$\Lambda$ polarization”.

The polarization of $\Lambda$ (and other hyperons) can be measured through the asymmetric momentum distribution in the parity violating weak decay $\Lambda \rightarrow p\pi$. Hence it is possible to measure the SSA in $p$ nucleus $\rightarrow \Lambda^\uparrow(x_F, k_\perp)+X$ without a polarized beam. The first large high-energy SSA’s were observed in $\Lambda$ production already in the 1970s [30, 31]. Fig. 5.2 shows the $k_\perp$ dependence of the polarization $P_\Lambda$ for different $x_F$ bins in $p Be \rightarrow \Lambda^\uparrow X$ at 400 GeV [33]. The asymmetry does not decrease with $k_\perp$ up to the highest measured $k_\perp \simeq 3.5$ GeV. The asymmetry also increases with $x_F$ for all measured $k_\perp$ up to $x_F \simeq 0.8$ where $P_\Lambda \simeq -0.3$. Similar effects have been observed also in the production of other hyperons, e.g., $\Xi^-$ [32].
Figure 5.2: The transverse momentum dependence of Λ polarization in $pBe \rightarrow \Lambda\uparrow X$ for a 400 GeV proton beam on a fixed target with $x_F$ restricted to the ranges shown in (a)-(d). The lines show a parametrized fit to the data. Figure from [33].

The first high-energy measurements of $A_N$ in $p\uparrow p \rightarrow \pi X$ were done in the Fermilab E704 and E581 experiments [26–28] using polarized 200 GeV proton and antiproton beams and a fixed proton target. The results for the proton beam are shown in Fig. 5.3. As in polarized Λ production, the asymmetry increases with $x_F$ up to $x_F \simeq 0.8$ where the asymmetry reaches $|A_N| \simeq 40 \%$ for charged pions. The asymmetries of $\pi^\pm$ seem to be almost mirror symmetric. Recently, the asymmetry in neutral pion production was shown to persist to $E_{CM} = 200$ GeV at the Relativistic Heavy Ion Collider in Brookhaven [29]. The asymmetry again increases with $x_F$, and preliminary results [89] show that it tends to increase with $k_\perp$ up to $\simeq 2.5$ GeV for all available $x_F = 0.25 \ldots 0.56$.

The TMD approach has been used to fit the data on $p\uparrow p \rightarrow \pi X$ [90–94]. The $x_F$ dependence of the E704 data [26,27] can be reproduced by parametrizing the Sivers distribution [93] while the Collins mechanism is suppressed due to a cancellation
Figure 5.3: $A_N$ of (5.11) in $p^1p \rightarrow \pi X$ as a function of $x_F$ for a 200 GeV polarized proton beam on a fixed proton target, integrated over $k_\perp$ from 0.5 to 2.0 GeV for $\pi^0$ data and from 0.7 to 2.0 GeV for $\pi^\pm$ data. Figure from [27].

of phases [94] and the Soffer bound [92, 95] on the transversity distribution. TMD distributions have also been applied to the polarized inclusive $\Lambda$ data [96]. The twist-three formalism has been used to fit the E704 $p^1p \rightarrow \pi X$ data [97] and recently also the high-energy STAR data [17].

While the $x_F$ dependence of the data can be reasonably well parametrized, the $k_\perp$ dependence seems to fail. The expected decreasing behavior $A_N \propto \Lambda_{QCD}/k_\perp$ has not been observed in any experiment, with the asymmetries instead increasing up to $2 - 3$ GeV. These transverse momenta are so much higher than the energy scale of soft QCD $\Lambda_{QCD} \simeq 300$ MeV, that one would expect perturbation theory to work. These shortcomings of the present approaches motivated our studies in articles III and IV.
Chapter 6

Model calculations of single spin asymmetries

In this chapter I discuss several model calculations of SSA’s. I start with the model of Brodsky, Hwang and Schmidt (BHS, [20]) of SIDIS and its application to spin-flip rescattering [II]. I then discuss nonperturbative effects in $p^1p \to \pi X$ [98].

As reviewed in the previous chapter, spin asymmetries have mainly been discussed in terms of transverse-momentum-dependent or twist-three parton distributions. These methods lead to rather complicated calculations where the dynamics of the soft processes is parametrized using parton distributions which are a priori unknown. Model calculations can shed light on the underlying QCD dynamics of the various effects present in the generalized factorization schemes. The BHS model is an example of such an approach.

Recall (5.3) that the asymmetry $A_N$ is expressed in terms of helicity amplitudes as

$$A_N = \frac{2 \sum_{\{\sigma\}} \text{Im} \left[ M^*_{-,-,\{\sigma\}} M_{-,-,\{\sigma\}} \right]}{\sum_{\{\sigma\}} \left[ |M_{-,-,\{\sigma\}}|^2 + |M_{-,-,\{\sigma\}}|^2 \right]},$$

(6.1)

which leads to two stringent requirements for a nonzero asymmetry: a helicity flip and a helicity-dependent dynamical phase. These conditions play a key role in model calculations, which must have both features in order to produce an asymmetry.

6.1 The BHS scalar spectator model of SIDIS

The BHS model [20] is a toy model of semi-inclusive deep inelastic scattering. The proton is modeled as a massive single quark that emits a scalar diquark before scattering with the virtual photon (see Fig. 6.1). The scalar diquark mimics the spectator quark system and QCD is modeled as an Abelian gauge theory. The original BHS calculation shows that a coherent rescattering (within the Ioffe coherence length $L_I \sim 1/x_B M$ [5]) between the active quark and the scalar spectator causes a leading twist asymmetry. The result came as a surprise since the rescattering
effects were thought to be suppressed by a power of the photon virtuality $Q$. Soon afterwards it was realized that this so-called “Sivers effect” \cite{Sivers} is indeed allowed at leading twist in QCD factorization \cite{QCDfactorization}.

![Figure 6.1: The two diagrams of the BHS model. The dashed propagator is the scalar diquark. The dashed vertical cut represents the imaginary part of the diagram.](image)

The diagrams of the BHS model are shown in Fig. 6.1. An asymmetry arises from the interference between the Born and one-loop diagrams. The loop integral provides the necessary imaginary part in the numerator of (6.1). The spin flip occurs at the soft proton-quark-scalar vertex. Consequently $A_N$ is proportional to the interference between light front wave functions having orbital angular momenta $L_z = 0$ and $L_z = \pm 1$. The different wave functions of the two amplitudes also allow the dynamical phase generated by the rescattering to be helicity dependent. The phases of the nonflip ($\psi_1$) and flip ($\psi_2$) amplitudes are

$$\arg \psi_i \sim \alpha_s \int_0^1 dy \frac{(r_\perp^2 + B) y^{i-1}}{y(1-y) r_\perp^2 + (1-y)B}. \quad (6.2)$$

Here $r_\perp$ is the transverse momentum of the outgoing quark and

$$B = x_B \left[ m_s^2 - (1 - x_B)M^2 \right] \quad (6.3)$$

where $m_s$ ($M$) is the scalar (proton) mass\footnote{I have set the quark and gluon masses of the BHS model to zero.}. The infrared divergence of the loop in Fig. 6.1 appears at $y \to 1$, giving a helicity independent infinite Coulomb phase. The infinite phase arises from the long distance region (distances much larger than the Ioffe length $L_I$) that is incoherent with the hard subprocess. It cancels in (6.1) which only depends on the phase difference $\arg \psi_1 - \arg \psi_2$. The resulting asymmetry reads

$$A_N(x_B, r_\perp) = -\alpha_s \frac{x_B M (r_\perp^2 + B)}{r_\perp (x_B^2 M^2 + r_\perp^2) \log \frac{r_\perp^2 + B}{B}}. \quad (6.4)$$

Rescattering diagrams like that of Fig. 6.1 form the (future-pointing) Wilson line contribution to the transverse-momentum-dependent distribution functions. Although the rescattering is coherent with the hard process, it is soft and universal
and thus included in the parton distributions. In particular, the imaginary part of the rescattering causes a leading twist, naively time reversal odd Sivers function [99]. T-odd parton distributions such as the Sivers function were thought to be suppressed in QCD before the BHS model calculation and they vanish in the absence of rescattering effects [25].

The calculation can be repeated for the Drell-Yan process \( p\bar{p} \rightarrow \ell^+\ell^- + X \), where initial state rescattering between the scalar and the active antiquark in the (anti)proton gives rise to an asymmetry [100]. The Drell-Yan asymmetry has an opposite sign w.r.t. that of SIDIS. This is due to the rescattering occurring in the initial state, which gives rise to a past-pointing Wilson line. The Sivers functions of SIDIS and Drell-Yan are thus expected to have the same \( x \) dependencies but opposite signs [99].

### 6.2 Rescattering helicity flip within SIDIS

Effects arising from the nonperturbative sector of QCD [101–103] can also give rise to an asymmetry in SIDIS. Spin-dependent soft, nonperturbative interactions can be coherent with and thus affect hard scattering cross sections. While such contributions have not been observed in unpolarized cross sections, spin observables like \( A_N \) of (6.1) might be more sensitive to them.

![Figure 6.2:](image)

**Figure 6.2:** Amplitudes which contribute to the asymmetry. The blob represents the Pauli coupling in (6.5). (a) Elastic scattering amplitude with Pauli coupling. (b) The Born amplitude. (c) The discontinuity of the loop amplitude.

In the third article of this thesis [III] the effect of coherent spin-flip rescattering is considered in SIDIS. We use the BHS model but insert a rescattering vertex that flips helicity (see Fig. 6.2). A nonperturbatively generated Pauli coupling (anomalous magnetic moment) is possible because the rescattering is soft. This effect is absent in perturbative QCD but might arise from nonperturbative (e.g., instanton) effects [21–24].

The key observation is that the forward elastic Pauli spin-flip amplitude of Fig. 6.2 (a) is not suppressed at high CM energies. Consequently the loop diagram of Fig. 6.2 (c) contributes at leading twist. Here the Pauli coupling (the blob
in the figure) is defined through the replacement

\[-ig_s\gamma^\mu \rightarrow -ig_s\gamma^\mu + a(p^2)\sigma^{\mu\nu}p_\nu\]  

(6.5)

where the effective coupling \(a(p^2)\) should vanish for large virtualities \(-p^2\) of the exchanged gluon. Taking

\[a(p^2) = a_0 \exp\left(-Ap_2^2\right)\]  

(6.6)

it is straightforward to evaluate the asymmetry. The helicity-dependent phases [corresponding to (6.2) in the BHS calculation] are finite due to the helicity flip. The result may be obtained in a closed form for large parameters \(A\) in (6.6), i.e., in the limit of very soft rescattering,

\[A_N \simeq \frac{e_2a_0}{2\pi A} \frac{1 - y}{1 + (1 - y)^2} \frac{1}{\{r_\perp + (Mx_B)^2\}} \left[1 + O\left(\frac{1}{A}\right)\right]\]  

(6.7)

where \(r_\perp\) is the transverse momentum of the outgoing quark and the azimuthal angles are defined in Fig. 5.1, except that here \(\phi_h\) refers to the outgoing quark or jet axis direction rather than to the observed hadron. The angular dependence of the second term is the same as in the Collins term of (5.8). The first term is also present in the transverse-momentum-dependent picture where it involves the convolution of the Collins fragmentation function with the \(h_{1T}\) distribution of the proton (see [104]).

While the angular dependence of the Pauli contribution (6.7) is similar to the Collins effect, the physics behind it is different. We consider a rescattering that is coherent with the hard subprocess, whereas the Collins fragmentation effect is incoherent with the hard scattering. Our model does not include fragmentation at all. The fact that in our definition the angle \(\phi_h\) refers to the jet axis direction rather than to the hadron reflects this substantial difference. Rescattering with helicity flip cannot be distinguished from the Collins effect in pion production \(ep \uparrow \rightarrow e + \pi + X\) but might be observed in \(ep \uparrow \rightarrow e + jet + X\) where the Collins fragmentation effect is absent.

Perturbative QCD also includes an anomalous magnetic moment that is proportional to the quark mass. To conclude this section I explain why it does not contribute to DIS at leading twist. An amplitude containing the perturbative anomalous magnetic moment in DIS is shown in Fig. 6.3(a). The Pauli contribution is isolated by requiring a helicity flip of the active quark as indicated by the \(\pm\) signs in Fig. 6.3(a). One may check that the spin flip occurs at either of the vertices where the transverse loop gluon (momentum \(\ell\)) attaches to the active quark. The longitudinal (Coulomb) gluon exchange does not flip spin at leading twist. The spin-flip contribution of the loop integral over \(\ell\) is finite despite the apparent logarithmic divergence and \(\ell_\perp \sim \Lambda_{\text{QCD}}\) for the dominant contribution. The diagram turns out to be subleading twist.
Figure 6.3: The Pauli coupling in perturbative DIS. (a) A contribution to DIS that includes the perturbative anomalous magnetic moment (Pauli coupling). The plus and minus signs indicate the helicities of the active quark. (b) DIS with soft transverse gluon emission. $A(p, k)$ is a soft amplitude.

One can illustrate the suppression of the diagram in Fig. 6.3(a) by considering soft transverse gluon emission as in the diagram of Fig. 6.3(b). Choosing the frame

$$q \approx (-p^+, q^-, 0_\perp)$$

with $p^+ = \mathcal{O}(Q^0)$ and $q^- = \mathcal{O}(Q^2)$, the amplitude is

$$\mathcal{M}({\text{Fig. 6.3(b)})} \propto \int dk^+ \frac{1}{(p - k + q)^2 + i\varepsilon} \frac{1}{(p - k + q - \ell)^2 + i\varepsilon} A(p, k)$$

$$\approx \frac{1}{(q^-)^2} \int dk^+ \left[ k^+ + \mathcal{O} \left( \frac{\Lambda^2_{\text{QCD}}}{q^-} \right) + i\varepsilon \right] \left[ k^+ + \mathcal{O} \left( \frac{\Lambda^2_{\text{QCD}}}{q^-} \right) + i\varepsilon \right].$$

Evaluating the proportionality factor one finds that a leading twist contribution requires (6.9) to be $\mathcal{O}(1/q^-)$. Naively, this seems to be achieved for $k^+ \sim \Lambda^2_{\text{QCD}}/q^-$. However, the soft amplitude $A(p, k)$ is independent of the large scale $q^-$ and hence a constant for $k^+ \sim \Lambda^2_{\text{QCD}}/q^-$. Then the integration over $k^+$ gives zero: the contributions from the two poles cancel. Thus the amplitude of Fig. 6.3(b) is subleading.

This result may be understood as follows. The length scale of the gluon-quark pair formation from the struck quark is $x^+ \sim 1/\Lambda_{\text{QCD}}$ in the quark rest frame. When the quark is boosted to a high momentum $\mathcal{O}(q^-)$ the length transforms to $x^+ \sim q^-/\Lambda^2_{\text{QCD}}$ which is much longer than the Ioffe length $L_I = 1/x_B M$. Hence the coherent production of a soft gluon in Fig. 6.3(b) is suppressed: the two residue contributions of (6.9) where the struck quark is on-shell before and after the gluon emission cancel. Such an argument may not apply to a Pauli coupling generated by the QCD vacuum, where gluons are “preformed”.

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6.3 Nonperturbative effects in $p\uparrow p \to \pi X$

I analyze now $p\uparrow p \to \pi X$ in a similar manner as SIDIS above [98]. An essential difference between SIDIS and $p\uparrow p \to \pi X$ is that in SIDIS the momentum scale of the hard subprocess is given by the virtuality $Q$ of the exchanged photon, but in $p\uparrow p \to \pi(x_F,k_\perp) + X$ the hard scale is the $k_\perp$ of the pion. Hence a perturbative analysis requires that the pion transverse momentum $k_\perp \gg \Lambda_{\text{QCD}}$. However, all perturbative mechanisms predict subleading twist behavior of $A_N$ in $p\uparrow p \to \pi X$,

$$A_N \propto \frac{\Lambda_{\text{QCD}}}{k_\perp} \quad (6.10)$$

This can intuitively be understood as follows. The asymmetric contribution to the cross section needs to be odd in $k_\perp$, as seen from (5.9): $A_N \propto S \times P \cdot k_\perp$. For the numerator of (6.1) to be odd in $k_\perp$ implies an interference between the leading amplitude in $1/k_\perp$ and a nonleading one.

In the case of the Sivers effect the suppression arises as follows. The Sivers mechanism involves soft interactions within the projectile system. In particular, the spin flip occurs at a soft vertex “inside” the polarized proton that corresponds to the proton-quark-scalar vertex of the BHS model in SIDIS. Consequently, the numerator of (6.1) is proportional to $\ell_\perp e^{i\psi}$, where $\ell_\perp = \ell_\perp(\cos \psi, \sin \psi)$ is the soft momentum. The large $k_\perp$ of the pion is obtained from the hard interaction with the target. The hard process is incoherent with the internal proton dynamics and thus independent of $\ell_\perp$ to leading order in $1/k_\perp$. Hence integrating over the (unobserved) $\ell_\perp$ gives

$$A_N \sim \int_0^{2\pi} d\psi \, e^{i\psi} = 0 \quad (6.11)$$

The Sivers effect only shows up as a subleading twist “trigger bias” effect $\propto 1/k_\perp$ when the dependence of the hard scattering on $\ell_\perp$ is taken into account.

However, as shown in Sec. 5.2, the $A_N$ in $p\uparrow p \to \pi X$ (and in $pp \to \Lambda\uparrow X$) is observed not to decrease with $k_\perp$ up to $2-3$ GeV. This motivates us to study a qualitatively different scenario, where the helicity-dependent phase arises from soft scattering on the unpolarized proton. The SSA data in $p\uparrow p$ are roughly independent of the center of mass energy, being comparable in the fixed target E704 data ($E_{\text{CM}} \simeq 20$ GeV) and the RHIC STAR data ($E_{\text{CM}} = 200$ GeV). This suggests that only Pomeron and Odderon exchanges contribute between the (polarized) projectile and (unpolarized) target regions – and these two exchanges have nearly maximal ($90^\circ$) phase difference. The Pomeron is a pseudoparticle that carries the quantum numbers of the vacuum. In particular, it is colorless, has $C = P = +1$ and is connected to two-gluon exchange in QCD. The Odderon is the $C = P = -1$ partner of the Pomeron. Previously, Pomeron-Odderon interference has been suggested to cause charge and single spin asymmetries in diffractive $c\bar{c}$ [105] and $\pi^+\pi^-$ [106–110] photoproduction.

One also needs a quark helicity flip in the pion emission. There is in fact experimental evidence that quark helicity flip can dominate even at high momentum
transfers in (semi) exclusive hadronic processes. The H1 experiment recently confirmed [111] an earlier observation by ZEUS [112] that $\rho^0$ mesons are transversely polarized in photoproduction, $\gamma p \rightarrow \rho^0 p$, for $|t| \lesssim 10$ GeV$^2$. Thus the helicity of the photon is carried by the $\rho$, whereas quark helicity conservation would require the $\rho$ to be longitudinally polarized [113]. Hence the possibility of quark helicity flip also in the $p^\uparrow \rightarrow \pi(x_F, p_\perp) + X$ process cannot be excluded.

I first study the above scenario using a simple perturbative model. The Pomeron is described by two (Abelian) gluon exchanges, which gives the required energy dependence and imaginary phase. Analogously the Odderon is modelled by single gluon exchange. In order to keep the model as simple as possible we describe the proton by a single quark line, and generate the transverse momentum by gluon emission. The pion is identified with the quark in the final state.

![Figure 6.4: The Abelian model](image)

The SSA mechanism in the model for $p^\uparrow p \rightarrow \pi(k_\perp) + X$ is then analogous to the one studied in [20], namely $e^\uparrow \mu \rightarrow \gamma + e(k_\perp) + \mu$ where only the momentum of the electron is measured in the final state (see Fig. 6.4). In QCD, the photon and the electron would be replaced by a gluon and a quark which fragments into a pion. The electron-photon vertex is the hard part which generates the large $k_\perp$ of the pion. Since the data indicate that $A_N$ is independent of the center of mass energy, we take the limit $E_{CM} \rightarrow \infty$ at a fixed (and large) $k_\perp$ of the photon emitted from the electron. At leading order in $E_{CM}$ only (Coulomb) photon exchange between the projectile (electron) and target (muon) systems contributes. The electrons and the photons are referred to as “quarks” and “gluons” in the following.

Before going to the actual Pomeron-Odderon calculation, I shall consider two general scenarios involving soft exchange with the target that fail to generate a SSA. This is useful in order to better understand the dynamics, and motivates the structure of the Pomeron-Odderon model.
Figure 6.5: Diagrams for the Coulomb rescattering amplitudes $B_{ss'\lambda}$ in $e\mu \rightarrow \gamma e\mu$. Only the imaginary (absorptive) parts of loop amplitudes (as indicated by the vertical dashed cuts) contribute to the SSA. The first two diagrams (a) cancel in the high-energy limit. The contribution thus arises from the diagrams (b) and (c).

6.3.1 Coulomb rescattering

I demonstrate first that because Coulomb gluons are independent of the spin of the particles to which they couple one finds $A_N = 0$ in the high-energy limit ($E_{CM} \gg k_\perp$), even though the quark is massive and flips its helicity. The rescattering diagrams with two Coulomb exchanges are shown in Fig. 6.5. Only the discontinuities (absorptive parts) of the loop amplitudes $B_{ss'\lambda}$ contribute to the asymmetry (6.1). They may be evaluated using Cutkosky’s rules. The discontinuity of the diagram in Fig. 6.5(a) vanishes as the contributions from the two possible cuts cancel at leading order in $1/E_{CM}$. This may be understood intuitively as a consequence of the formation time of the emitted photon being much longer than the target traversal time, due to Lorentz dilation. The one-loop result is then solely given by the diagrams in Fig. 6.5(b) and in Fig. 6.5(c) where the loop momentum $\ell$ only flows through the Coulomb vertices. As the vertices do not depend on helicity, the additional Coulomb exchange only adds a helicity independent, infinite...
Coulomb phase factor

\[ \text{Disc} B_{ss',\lambda} = -ie^2 q_{\perp}^2 A_{ss',\lambda} \int \frac{d^2 \ell_{\perp}}{(2\pi)^2} \frac{1}{\ell_{\perp}^2 (q_{\perp} - \ell_{\perp})^2} \]  

(6.12)

where $A_{ss',\lambda}$ are the Born amplitudes. The proportionality factor cancels in (6.1), resulting in $A_N = 0$. This result is quite general, as it depends only on the helicity independence of Coulomb scattering. It apparently holds for any number of Coulomb exchanges, and illustrates the essential difference between $p^1p \rightarrow \pi X$ and SIDIS.

Figure 6.6: Gluon rescattering model for an asymmetry in $pp \rightarrow \pi X$. The large $k_{\perp}$ is generated at the gluon emission vertex. The soft gluon rescattering becomes incoherent for $\ell_{\perp} \ll k_{\perp}$.

6.3.2 Rescattering within the projectile system

Rescattering within the projectile system may give rise to a nonzero asymmetry. The rescattering must be soft as a hard process cannot give rise to a large asymmetry: if the helicity flip required by (6.1) occurs in the hard process we get a suppression by $m/k_{\perp}$ where $m$ is a current quark mass. However, soft rescattering with transverse momentum $\ell_{\perp}$ much smaller than the hard scale $p_{\perp}$ is not coherent with the hard process which leads to a suppression by the factor of $\ell_{\perp}/k_{\perp}$. We illustrate the incoherence by adding soft gluon rescattering between the emitted gluon and the outgoing quark in the model described above (see Fig. 6.6).

To describe the three-gluon vertex we use QCD. To leading order in $\ell_{\perp}/k_{\perp}$, the rescattering conserves helicity

\[ \mathcal{M}_{\text{m}} \simeq \delta_{\lambda,\lambda'} \delta_{s',s} \frac{2e^2 \delta_{t}}{t} \]  

(6.13)

The model also includes the suppression $m/k_{\perp}$ as the helicity flip must occur at the hard vertex. Its purpose is to illustrate the suppression due to incoherence.
where we dropped the color factors. The discontinuity of the loop amplitude of Fig. 6.6

\begin{equation}
\text{Disc}\mathcal{M}(\text{Fig. 6.6})_{ss'\lambda} \simeq i \int \frac{d^4\ell}{(2\pi)^4} A_{ss'\lambda} 2\pi\delta((p' - \ell)^2)2\pi\delta((k + \ell)^2)\mathcal{M}_{rs}
\end{equation}

(6.14)
is given by the Cutkosky rules as a convolution of the rescattering amplitude (6.13) and the Born amplitudes \(A_{ss'\lambda}\). Softness of the rescattering can be guaranteed adding a constraint \(\ell_\perp < \ell_{\text{cut}}\) in the integration in (6.14). To leading order in \(\ell_\perp/k_\perp\) the dependence of the amplitudes \(A_{ss'\lambda}\) in (6.14) on \(\ell_\perp\) can be neglected: the soft rescattering is incoherent with the hard vertex. Then \(\text{Disc}\mathcal{M}(\text{Fig. 6.6})_{ss'\lambda}\) is proportional to the corresponding Born amplitude \(A_{ss'\lambda}\). As in the Coulomb rescattering, the factor cancels in (6.1) so the asymmetry vanishes at leading order in \(\ell_{\text{cut}}/k_\perp\). Taking into account the \(\ell_\perp\) dependence of the Born amplitudes \(A_{ss'\lambda}\) (coherence effect), we find a contribution which is suppressed by \(\ell_{\text{cut}}/k_\perp\).

### 6.3.3 Asymmetry from Pomeron-Odderon interference with a helicity flip

The effect of a nonperturbative Pauli spin-flip coupling at rescattering in \(pp \to \pi X\) may be studied \cite{98} similarly as in SIDIS \cite{III}. An asymmetry arises due to a Pauli vertex contribution at the Coulomb rescattering vertex that could be generated by QCD vacuum effects. The above calculation of Coulomb rescattering in \(e^+\mu \to \gamma + e(k_\perp) + \mu\) with the diagrams of Fig. 6.4 and Fig. 6.5 can be extended to the case where the gluon scatters with the projectile quark also through a Pauli coupling. Two-gluon exchange serves as a model of the Pomeron, while the (Abelian) single gluon exchange represents the Odderon. The Pauli vertex is defined as above in (6.5), (6.6). The small terms which result from spin flips at vertices other than the Pauli vertex may be dropped by taking the quark mass \(m\) to zero.

A straightforward but lengthy calculation gives the asymmetry, which for soft rescattering \(q_\perp^2 \sim 1/A \ll k_\perp^2\) and for small flip contribution \(a_0k_\perp \ll 1\) is

\begin{equation}
A_N \propto -\frac{e a_0}{1 + x_F^2} \frac{k_\perp}{x_F}.
\end{equation}

(6.15)

The flip amplitudes with Pauli coupling are \(\propto 1/k_\perp\), while the nonflip amplitudes behave as \(1/k_\perp^2\) for soft rescattering \(q_\perp \ll k_\perp\). Thus the asymmetry (6.15) is \(\propto k_\perp\) in the region where the nonflip amplitudes dominate \((a_0k_\perp \ll 1\)\). The \(x_F\) and \(k_\perp\) dependencies of (6.15) arise from the hard gluon emission vertex. The soft Coulomb exchange only affects the proportionality constant: it is given by a ratio of soft integrals and its magnitude depends of the size of the phase between the flip and nonflip amplitudes. The phase turns out to be small since the contributions from the (imaginary) double Coulomb exchange diagrams are smaller than the (real) Born contributions. Hence a rather small asymmetry \((A_N \lesssim 5\%)\) is generated.
The nonzero asymmetry obtained in the perturbative “Pauli” model motivates us to consider a Regge-like model for the Pomeron exchange. In this model the dynamical phase is not generated by rescattering but through an interference between Pomeron and Odderon exchange amplitudes. The Pomeron spin-flip amplitudes are observed to be small (see, e.g., [114]). However, the Odderon may have a sizeable spin-flip coupling. Then the different signature of the Pomeron and the Odderon guarantees a large phase difference between the helicity amplitudes whence a sizeable SSA is possible.

\[ A_N \approx - \frac{2a_0 x_F k_\perp}{1 + x_F^2 + 2a_0^2 k_\perp^2} \]  

(6.17)

The magnitude of \( A_N \) in the data requires an Odderon exchange amplitude which is about 30% of the Pomeron exchange one (then \( a_0 k_\perp \simeq 0.3 \)). The \( x_F \) and \( k_\perp \) dependence of (6.17) is fixed by the hard gluon emission vertex and is only weakly dependent on the model of the soft part. This explains the similarity to (6.15).

Our model with Pomeron-Odderon interference thus suggests that QCD vacuum effects give rise to large spin-flip amplitudes and thus sizeable single spin asymmetries in \( p^1 p \rightarrow \pi X \). Note that the model produces similar \( x_F \) and \( k_\perp \) dependencies as seen in the data. If the asymmetry is indeed generated by colorless exchange it should persist in events with large rapidity gaps. This prediction of the model can be tested experimentally.
Chapter 7

$p^\uparrow p \to \pi X$ at large $k_\perp$ and $x_F$

In the last article [IV] of this thesis a novel dynamical mechanism is suggested for the large SSA’s observed in $p^\uparrow p \to \pi(x_F, k_\perp) + X$. Recall from Sec. 5.2 that the E704 data ($E_{CM} \simeq 20$ GeV) [27, 28] increase with $x_F$ up to $x_F \simeq 0.8$ where the asymmetry reaches $|A_N| \simeq 40\%$. Similarly at the higher $E_{CM} = 200$ GeV in STAR [29, 89] the asymmetry in $\pi^0$ production increases with $x_F$. This indicates that the asymmetry is a large-$x_F$ coherence effect. The asymmetry being almost an order of magnitude larger than those observed in SIDIS further supports the idea that the mechanisms which create the asymmetries are different in the two reactions.

QCD factorization requires $k_\perp^2 (1 - x_F) \gg \Lambda_{QCD}^2$ and is thus expected to hold better in the central rapidity region with small $x_F$. In fact, it was shown in [115] that leading twist QCD fails to describe the unpolarized $p^\uparrow p \to \pi^0 X$ cross section for $x_F \gtrsim 0.5$ at $E_{CM} \approx 20 \ldots 50$ GeV. At $x_F \simeq 0.8$, where the observed $A_N$ is largest, the disagreement between the measured and calculated cross sections is about an order of magnitude. For STAR ($E_{CM} = 200$ GeV) the agreement is better, but the data do not extend to high values of $x_F \gtrsim 0.5^1$. When using the twist expansions to describe the E704 asymmetries at large $x_F$ one thus needs to assume that the large “K factor” observed in the unpolarized data are spin independent and cancels in the ratio of (6.1). Hence there is ample room for spin-dependent high-$x_F$ coherence effects. Since the SSA involves phases it might be more sensitive to new effects than the unpolarized cross sections.

In general, the increase of coherence effects at large $x_F$ can be understood as follows (see [116]). The lifetime $\tau$ of a Fock state inside a rapidly moving proton is the inverse of the (light-front) energy difference $\Delta E$ between the Fock state and the proton

$$P^+ \Delta E = M^2 - \sum_i \frac{k_{i\perp}^2 + m_i^2}{x_i}$$  \hspace{1cm} (7.1)$$

where $x_i > 0$, $k_{i\perp}$, and $m_i$ are the momentum fraction, the transverse momentum, and the mass of parton $i$, respectively, $P^+$ is the proton light front momentum, and

$^1$The transverse momentum scale is about the same (few GeV) in STAR and in E704.
\( M \) is the proton mass. From momentum conservation it follows that
\[
\sum_i x_i = 1 .
\] (7.2)

In a typical Fock state the momentum is shared equally among the participants: all \( x_i \) are of the same order which maximizes the lifetime \( \tau_{\text{soft}} \sim P^+ / \Lambda_{QCD}^2 \). If in such a configuration one parton scatters with an external particle obtaining a large transverse momentum \( k_\perp \), that parton will dominate the sum in (7.1). The hard scattering timescale \( \tau_{\text{hard}} \sim P^+ / k_\perp^2 \ll \tau_{\text{soft}} \) as required for factorization.

Let us now consider a Fock state where one quark carries a large \( x_i \sim x_F \rightarrow 1 \). Then due to (7.2) all other partons \( j \) with \( j \neq i \) must have \( x_j \sim 1 - x_F \rightarrow 0 \). Hence the energy fraction of (7.1) grows large \( \sim 1 / x_j \) and the lifetime of the state \( \tau \sim (1 - x_F)P^+ / \Lambda_{QCD}^2 \) becomes short. The incoherence of such state with a hard quark with transverse momentum \( k_\perp \) (and finite \( x \)) requires \( (1 - x_F)P^+ / \Lambda_{QCD}^2 \ll \tau_{\text{hard}} \) or
\[
k_\perp^2 (1 - x_F) \ll \Lambda_{QCD}^2 .
\] (7.3)

When \( x_F \) grows large enough, i.e., \( k_\perp^2 (1 - x_F) \sim \Lambda_{QCD}^2 \), the hard scattering becomes coherent with the soft physics and factorization is lost.

### 7.1 High \( x_F \) in unpolarized scattering

I start by reviewing previous work on high-\( x_F \) effects. Berger and Brodsky [117] consider high-\( x_F \) coherence effects in (unpolarized) Drell-Yan \( (\pi p \rightarrow \mu^- \mu^+ + X) \) where the dimuon pair carries a large fraction of the incoming pion momentum. The situation is described by the two diagrams of Fig. 7.1. Let us concentrate on diagram (a). The pion enters with the quark and the antiquark carrying similar shares of its momentum \( p_\pi^+ \). Almost all of the quark momentum is then transferred to the antiquark via gluon exchange, giving the antiquark a large fraction \( x_F \rightarrow 1 \) of \( p_\pi^+ \). The Fock states after the gluon emission are short lived \( \tau \sim (1 - x_F)p_\pi^+ / \Lambda_{QCD}^2 \). Thus the hard virtual photon interaction becomes coherent with the gluon exchange for large enough\(^2 x_F \). Indeed, taking
\[
p_\pi = (p_\pi^+, 0, 0_\perp)
\] (7.4)
and giving to the quark and the antiquark an equal share \( p_\pi / 2 \) of the pion momentum in the initial state, one may check that the gluon and intermediate antiquark virtualities for \( x_F \rightarrow 1 \) are
\[
2p_g^2 \simeq p_q^2 \simeq -\frac{p_{q_\perp}^2}{(1 - x_F)}
\] (7.5)

where I set all masses to 0.

\(^2\)In this analysis the coherence effects set in already at \( Q(1 - x_F) \sim \Lambda_{QCD} \). The factorizable contribution is suppressed by an extra \( (1 - x_F) \) due to the helicity mismatch between the pion and the virtual photon.
Figure 7.1: The two Drell-Yan diagrams ($\pi p \rightarrow \mu^- \mu^+ + X$) considered in [117]. The quark that annihilates with the antiquark of the pion comes from the proton (not shown). The virtual photon carries a large fraction of the incoming pion momentum which is transferred to the muon pair (not shown).

Since both pion constituents are coherent with the hard scattering the process must be described using the pion wave function rather than in terms of the antiquark distribution of the pion. The effective upper limit of the transverse loop momentum $p_{g\perp}$ in Fig. 7.1(a) is given by the virtuality of the gluon $\sim \Lambda^2_{\text{QCD}}/(1 - x_F)$. Hence the components of the pion wave functions which contribute have a small transverse size

$$r_{\perp} \sim 1/p_{g\perp} \sim \sqrt{1 - x_F}/\Lambda_{\text{QCD}}.$$  \hspace{1cm} (7.6)

The coherence effect can be experimentally observed via the polarization of the produced virtual photon. When $x_F$ is small usual factorization holds and the antiquark in the intermediate state is nearly on-shell. Then the virtual photon is transversely polarized giving the muon pair the angular distribution $d\sigma \propto 1 + \cos^2 \theta$ in the rest frame of the pair. When $x_F$ grows, the intermediate antiquark goes off-shell leading to an increased production longitudinally polarized photons for which the muon angular distribution is $d\sigma \propto \sin^2 \theta$. The change in the angular distribution of the muons was seen in the E615 experiment at Fermilab [118]. When higher-twist kinematics are taken into account [119] the data are in agreement with predictions.

High-$x_F$ coherence effects have also been studied in heavy quark pair production (such as $c\bar{c}$) at high $x_F$ from a hadron [116, 120, 121]. In the kinematic limit of high pair mass $\mathcal{M}$ with $\mathcal{M}^2(1 - x)$ fixed several quarks from the same hadron are seen to contribute coherently. Typical contributions for a pion projectile are shown in Fig. 7.2. The Fock states that contain slow quarks with momentum fractions $x \sim (1 - x_F)$ are short lived and compact, as above in Drell-Yan. The heavy quark pair can be produced coherently from such a state when $\mathcal{M}^2(1 - x) \sim \Lambda^2_{\text{QCD}}$. While the pion enters in a compact state with $r_{\perp} \sim \sqrt{1 - x_F}/\Lambda_{\text{QCD}} \sim 1/\mathcal{M}$, the time dilation of the slow quarks is reduced by $1 - x_F$ which allows them to expand to hadronic size $1/\Lambda_{\text{QCD}}$ during the hard timescale $\tau_{\text{hard}} \sim 1/\mathcal{M}$. The large-$x_F$ quark pair stays compact, and its interaction with the target is reduced due to the small dipole moment of the pair. Hence the target interacts dominantly with the slow quarks, which “frees” the heavy quarks due to the coherence. The high-$x_F$
Figure 7.2: Typical diagrams for $c\bar{c}$ production from a pion at large $x_F$ of the pair. The pair is produced in a compact, short lived Fock state. The interaction with the target is dominated by soft gluons ($\ell$) that scatter from the slow quarks carrying momentum fractions $x \sim (1-x_F)$. (a) An “extrinsic” contribution where the $c\bar{c}$ pair only interacts with a single quark. (b) An “intrinsic” contribution involving interactions with both quarks of the pion.

dynamics allows also coherent interactions between the produced pair and several quarks of the projectile [see Fig. 7.2(b)]. The presence of such diagrams at leading order signals the breaking of standard factorization.

7.2 SSA in $p^+p \rightarrow \pi X$ at high $x_F$

We suggest [IV] that the large asymmetries observed in $p^+p \rightarrow \pi X$ at E704 are high-$x_F$ coherence effects. The largest asymmetries were observed at $x_F \simeq 0.8$. Hence a very large fraction $x \simeq 0.9$ of the proton momentum must be transferred to the active quark, and the produced pion must carry a similarly large fraction $z \simeq 0.9$ of the outgoing quark momentum. Sizeable coherence effects were observed in Drell-Yan at E615 for $x_F \simeq 0.9$ and for much larger photon virtuality $Q \simeq 4-5$ GeV than the typical $k_\perp \sim 1$ GeV of the pion at E704. Hence coherence is expected to be significant in the kinematic region where the asymmetries are large.

We consider the $p^+p \rightarrow \pi(x_F,k_\perp) + X$ in the kinematic limit of large $k_\perp$ and $x_F$:

$$k_\perp \rightarrow \infty \quad \text{with} \quad (1-x_F)k_\perp^2 \sim \Lambda^2_{\text{QCD}} \quad \text{fixed.} \quad (7.7)$$

Our mechanism for the asymmetry is shown in Fig. 7.3. The proton first has a typical Fock state configuration where all the quarks carry fractions of $\mathcal{O}(1)$ of the total (plus) momentum. A short lived Fock state with one fast quark ($x \sim 1$) is then created via gluon exchange similarly as in the analysis of Berger and Brodsky. The fast quark scatters with the target obtaining a large transverse momentum

$^3$The hard $k_\perp$ cannot be obtained from scattering within the projectile system. This would lead to at least one of the slow quarks having a large $\sim k_\perp$ transverse momentum and thus to a double suppression $\tau \sim (1-x_F)P^+/k_\perp^2$ of the state.
The hard scattering is coherent with the gluon exchanges in the limit (7.7): the soft timescale \( \tau_{\text{soft}} \sim (1 - x_F)P^+ / \Lambda_{\text{QCD}}^2 \) coincides with the hard \( \tau_{\text{hard}} \sim P^+ / k_\perp^2 \) one. The fast quark then picks up a slow antiquark and the pion is formed through a gluon exchange which equalizes the momentum fractions.

The interactions within the slow quark system (indicated by the dashed circle in Fig. 7.3) are soft with momentum scale \( \sim \Lambda_{\text{QCD}} \). Thus they cannot be described using perturbation theory. However, since \( \tau_{\text{soft}} \sim \tau_{\text{hard}} \) all parts of the diagram in Fig. 7.3 stay fully coherent. As in the \( c \bar{c} \) production above, soft (re)interactions between the slow quarks and the target would also contribute at leading order.

For a nonzero asymmetry we need a helicity flip and a large, helicity-dependent phase. As helicity flip is suppressed in the hard interactions, the flip must occur in the soft subprocesses. A possible mechanism is shown in Fig. 7.3 where the flip vertex is indicated by a dot and \( \pm \) are the helicities of the quarks in the two interfering amplitudes. The soft interactions are modeled by a single soft gluon exchange. The interference between the flip and nonflip amplitudes is then \( \sim \ell_\perp e^{i\psi} \) where \( \ell_\perp \) is the antiquark momentum. Since we integrate over \( \ell_\perp \) in the end, a correlation between the antiquark \( \ell_\perp \) and the pion \( k_\perp \) transverse momenta is needed. While such a correlation is absent in the Sivers mechanism as explained in Sec. 6.3, it is possible here due to the coherence of the soft slow quark system with the pion formation. A dynamical phase is obtained from the hard subprocess as indicated by the vertical dashed cut in Fig. 7.3. As the soft part with the helicity flip is coherent with the hard subprocess, the phase will depend on helicity.

In [IV] we estimate the contribution of the diagram in Fig. 7.3 using Abelian interactions. We drop one quark from the proton, use constant hadron wave functions and leave out the gluon exchange inside the pion. In this model we show that a sizeable helicity-dependent phase indeed arises in the kinematic limit (7.7). The

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**Figure 7.3:** Our mechanism for a sizeable asymmetry at large \( x_F \) and \( k_\perp \) of the pion in \( p^1p \rightarrow \pi(x_F, k_\perp) + X \). See text for explanation.
phase is given by
\[ \tan \theta = \frac{\sqrt{AB}}{A + 2B} \] (7.8)
where \( B = k_\perp^2 \) and \( A \sim \Lambda_{QCD}^2/(1 - x_F) \). A sizeable phase requires \( A \sim B \). It is thus suppressed when the coherence between the hard and soft parts of the diagram is lost, i.e., when \( k_\perp^2(1 - x_F) \) is not kept fixed. To obtain a numerical estimate for the asymmetry further assumptions would have to be made. In particular, the pion dynamics containing the correlation of the soft antiquark transverse momentum \( \ell_\perp \) and the pion momentum \( k_\perp \) should be modeled. In any case, the size of the asymmetry requires the phase (7.8) and thus \( A_N \) is suppressed unless we take the limit (7.7).

Preliminary STAR data show that the asymmetry in \( \pi^0 \) production increases with \( k_\perp \) up to \( k_\perp \simeq 2.5 \) GeV. As stressed above, factorization based mechanisms predict \( A_N \propto \Lambda_{QCD}/k_\perp \). In STAR \( x_F \) is rather small \( \lesssim 0.5 \), which suggests that our high-\( x_F \) framework cannot be applied. In our model, the maximum of \( A_N \) is expected at \( k_\perp \sim \Lambda_{QCD}/\sqrt{1 - x_F} \) which is not large enough even at \( x_F \simeq 0.8 \). So even if our mechanism was applicable at STAR, it seems that the increasing behavior of \( A_N \) with \( k_\perp \) would not be explained. However, note that the energy difference (7.1)
\[ P^+ \Delta E = M^2 - \sum_i \frac{k_{\perp i}^2 + m_i^2}{x_i} \] (7.9)
contains only one fast and hard quark but many (say \( n \)) soft partons. Each of them carries a fraction \( \sim (1 - x_F)/n \) of the proton momentum. Thus our kinematic limit (7.7) becomes merely
\[ k_{\perp i}^2(1 - x_F) \sim n^2 \Lambda_{QCD}^2 \] (7.10)
so that the maximum of \( A_N \) is shifted to larger \( k_\perp \) and the region of applicability is shifted to smaller \( x_F \) if \( n \) is large.

The dependence of \( A_N \) on the species of the produced particle is particularly striking in \( p^1p \to \pi X \). With the proton spin up, the positively charged pions bend mostly to the left relative to the beam direction and the negatively charged pions to the right, with the two distributions being almost mirror symmetric (see Sec. 5.2). Neutral pions bend to the left, but the asymmetry is smaller than for \( \pi^+ \)'s. This supports the idea of the asymmetry being a valence quark effect and the naive picture from the nonrelativistic quark model: \( \pi^+ \) shares a \( u \) quark with the proton, while \( \pi^- \) shares a \( d \) quark. In the quark model the spin of the \( u \) quark is mostly parallel to the proton spin, while the spin of \( d \) points mostly in the opposite direction, which explains the sign difference of the asymmetries [122]. \( \pi^0 \) is a mixture of both \( u \) and \( d \) but because there are two \( u \)'s in the proton, its contribution wins and \( \pi^0 \)'s go to the left. Also polarized hyperon production data seem to support the SSA's being valence quark effects. Merging this simple picture with our high-\( x_F \) approach seems possible. The difference between the \( \pi^+ \) and \( \pi^- \) must then arise from helicity-dependent proton wave functions of the \( u \) and \( d \) quarks.
Chapter 8
Conclusions and outlook

The first topic of this thesis was the relativistically moving hydrogen atom or positronium. In [I, II] the wave function of the $e^-e^+$ Fock state of positronium in a general frame both in $1+1$ and in $3+1$ dimensional QED was studied. The leading component of the wave function was seen to Lorentz contract in the same way as a rod in classical special relativity. In the case of the physical $3+1$ dimensional theory the wave function of the next-to-leading $e^-e^+\gamma$ Fock state was studied and found not to obey classical contraction.

Surprisingly little attention has been paid to the frame dependence of bound state wave functions in the literature. Classical objects are known to contract in the direction of motion according to Lorentz transformation rules. However, no quantum level description of contraction exists, even though contracted hadrons and nuclei are often drawn in qualitative discussions. A general understanding of the boosts of equal-time wave functions is desirable. The frame dependence of the proton wave function could shed light on the relation between the nonrelativistic quark model of the CM frame and the parton picture of the infinite momentum frame. My nonrelativistic study serves as a first step in this direction.

In general, relativistic bound states involve Fock states with arbitrarily many particles. It would be surprising if all the Fock state wave functions Lorentz contracted in a classical manner, since the Fock state content of Feynman diagrams is frame dependent. We showed that the frame dependence of the $e^-e^+\gamma$ component of positronium indeed deviates from the classical expectation in the weak-coupling limit. Our work could be extended to include higher order relativistic effects that may violate the classical contraction of the two-body Fock state as well. The extension might be first tried in a simpler theory such as $\phi^3$ theory (with light scalar exchange) or scalar QED and possibly in $1+1$ dimensions.

The Single Spin Asymmetry (SSA) is a simple observable that cannot be described using the standard tool of perturbative QCD, collinear factorization. This has made the understanding of its dynamics a difficult task. For example, the leading twist asymmetry from rescattering in SIDIS found in the model calculation of Brodsky, Hwang and Schmidt only a few years ago came as a surprise. Their observation started a discussion which clarified the picture of QCD rescattering effects
in general. This example illustrates the potential benefits for the progress of QCD of understanding spin-dependent phenomena.

In [III] we modeled novel effects in polarized SIDIS that could arise due to the nontrivial vacuum of QCD. A spin flip in soft rescattering was seen to cause observable asymmetries also in $p^l p \rightarrow \pi X$. These effects may be enhanced in spin observables such as SSA which involve interference terms and are thus sensitive to the phases of the scattering amplitudes.

We argued that the contribution of spin-flip rescattering might be observed in $e p^l \rightarrow e + jet + X$ where it is not obscured by fragmentation effects. A further study should clarify whether this is feasible. A further elaboration of spin-flip rescattering in $p^l p \rightarrow \pi X$ also seems desirable, since the model with Pomeron-Odderon interference seems to naturally produce asymmetries that increase with $k_\perp$ in accord with the data. The asymmetries should show up also in diffractive events with large rapidity gaps if they are indeed generated by colorless exchange between the projectile and target systems. This prediction can be studied experimentally.

Finally in [IV] we presented a novel perturbative mechanism for $p^l p \rightarrow \pi (x_F, k_\perp) + X$ at large $k_\perp$ and $x_F$ in the limit where $(1-x_F)k_\perp^2 \sim \Lambda_{\text{QCD}}^2$ is held fixed. We demonstrated that sizeable flip and nonflip amplitudes may arise from diagrams involving more than one quark of the polarized proton. Such contributions are suppressed in the usual leading twist limit where $x_F$ is fixed. At large $x_F$ the soft and hard subprocesses are coherent, which naturally produces the helicity-dependent phase needed for sizeable asymmetries.

It is worthwhile to do a more quantitative analysis based on the ideas presented here. This would mean not only expanding our model to include features neglected here such as the pion structure, but also to express the soft amplitudes as specific matrix elements. We stressed that the standard factorization framework is inapplicable in the kinematic limit we have considered here. It would be important to understand if an analogous factorization can be established at fixed $k_\perp^2 (1-x_F)$. Such a framework would serve as a starting point for quantitative studies.


[38] C. Itzykson and J. B. Zuber, Quantum Field Theory (Mcgraw-hill, New York, 1980).


