

Generalized Forecast Error Variance Decomposition for Linear and Nonlinear Multivariate Models*

MARKKU LANNE[†] and HENRI NYBERG[‡]

[†]*Department of Political and Economic Studies, and HECER, University of Helsinki, Helsinki, Finland and CREATES, Aarhus University, Aarhus, Denmark (e-mail: markku.lanne@helsinki.fi)*

[‡]*Department of Mathematics and Statistics, University of Turku, Turku, Finland and Department of Political and Economic Studies, and HECER, University of Helsinki, Helsinki, Finland (e-mail: henri.nyberg@utu.fi)*

Abstract

We propose a new generalized forecast error variance decomposition with the attractive property that the proportions of the impact accounted for by innovations in each variable sum to unity. Our decomposition is based on the generalized impulse response function, and it can easily be obtained by simulation. The new decomposition is illustrated in an empirical application to U.S. output growth and interest rate spread data.

JEL classification numbers: C13, C32, C53.

Keywords: Forecast error variance decomposition, generalized impulse response function, output growth, term spread

* We would like to thank the Editor Anindya Banerjee, two anonymous referees, Pentti Saikkonen, Timo Teräsvirta and the participants in the 8th International Conference on Computational and Financial Econometrics in Pisa (2014) for useful comments. Financial support from the Academy of Finland is gratefully acknowledged. The first author also acknowledges financial support from CREATES (DNRF78) funded by the Danish National Research Foundation, while the second author is grateful for financial support from the OP-Pohjola Group Research Foundation and the Research Funds of the University of Helsinki.

I Introduction

Impulse response and forecast error variance decomposition analyses are the prominent tools in interpreting estimated linear and nonlinear multivariate time series models. These methods call for the identification of the structural shocks by imposing a sufficient number of identification restrictions on a reduced-form linear vector autoregressive (VAR) model. However, in many cases it is difficult to come up with adequate credible identification restrictions. In these cases as well as in nonlinear models, the generalized impulse response functions (GIRF) and generalized forecast error variance decompositions (GFEVD) offer alternative means of structural analysis.

The main difference between the impulse response function (IRF) and forecast error variance decomposition (FEVD) and their generalized counterparts is the interpretation of the shocks: in the former, they are uncorrelated and carry an economic meaning, while in the latter, each of them is just a shock to a given equation of the model. Moreover, because the latter shocks are not necessarily uncorrelated, the interpretation of the GFEVD as the proportions of the impact accounted for by innovations in each of the variables of the total impact of all innovations after h periods ($h = 0, 1, 2, \dots$) is somewhat nebulous, as these ‘proportions’ may not sum to unity.

Our contributions are twofold. First, we propose a simple modification of the GFEVD in linear multivariate models due to Pesaran and Shin (1998) that, by construction, forces the relative contributions to the h -period impact of the shocks to sum to unity, and hence, facilitates convenient interpretation. Second, we generalize this modification to obtain a GFEVD in nonlinear models that, to the best of our knowledge, has not been entertained in the previous literature. Overall, structural analysis in nonlinear models has not been frequently considered in the previous literature albeit it has recently awoken increasing interest (see, e.g., Karamé, 2012 and 2015, and Hubrich and Teräsvirta, 2013, 313–315, who discuss

GIRFs in Markov-switching and threshold and smooth transition vector autoregressive models, respectively).

The paper is organized as follows. The new GFEVD is introduced and its relation to the orthogonalized FEVD and GIRF is discussed in Section II. In Section III, we illustrate the GFEVD in an empirical application to U.S. output growth and the spread between long-term and short-term interest rates. Section IV concludes.

II A new generalized FEVD

Let us consider a K -dimensional VAR(p) model

$$\mathbf{y}_t = \sum_{i=1}^p \Phi_i \mathbf{y}_{t-i} + \boldsymbol{\varepsilon}_t, \quad (1)$$

where $\boldsymbol{\varepsilon}_t$ is an independent and identically distributed (iid) error term with zero mean and covariance matrix $\boldsymbol{\Sigma}$. Assuming weak stationarity, \mathbf{y}_t obtains the infinite-order moving-average representation

$$\mathbf{y}_t = \sum_{j=0}^{\infty} \mathbf{A}_j \boldsymbol{\varepsilon}_{t-j}, \quad (2)$$

and if suitable identification restrictions are available such that $\boldsymbol{\Sigma}$ can be written as the product of a $K \times K$ matrix \mathbf{P} and its transpose, the orthogonalized error $\boldsymbol{\xi}_t = \mathbf{P}^{-1} \boldsymbol{\varepsilon}_t$ has identity covariance matrix. In other words, the components of $\boldsymbol{\xi}_t$ are the uncorrelated structural shocks. The orthogonalized impulse response function on the i th component of \mathbf{y}_t , $y_{i,t+l}$, of the j th shock is then (see, e.g., Lütkepohl, 2005, Section 2.3)

$$IRF_{ij}(l) = \frac{\partial y_{i,t+l}}{\partial \xi_{jt}} = [\mathbf{A}_l \mathbf{P}]_{ij}, \quad l = 0, 1, 2, \dots, \quad (3)$$

and the corresponding FEVD component for horizon h equals

$$\gamma_{ij}(h) = \frac{\sum_{l=0}^h IRF_{ij}^2(l)}{\sum_{j=1}^K \sum_{l=0}^h IRF_{ij}^2(l)} = \frac{\sum_{l=0}^h IRF_{ij}^2(l)}{\sigma_i^2(h)}, \quad i, j = 1, \dots, K, \quad (4)$$

with $\sum_{j=1}^K \gamma_{ij}(h) = 1$ for a given i , $l = 0, \dots, h$, and $\sigma_i^2(h)$ denotes the h -step forecast error variance of the i th variable.

For the case where sufficient restrictions are not available to identify the structural shocks $\boldsymbol{\xi}_t$, Pesaran and Shin (1998) have proposed an approach building upon Koop et al. (1996). For generality, consider a K -dimensional nonlinear multivariate model (with the linear VAR(p) model in (1) or (2) as a special case),

$$\mathbf{y}_t = G(\mathbf{y}_{t-1}, \dots, \mathbf{y}_{t-p}; \boldsymbol{\theta}) + \boldsymbol{\varepsilon}_t, \quad (5)$$

where $G(\cdot)$ is a nonlinear function depending on the parameter vector $\boldsymbol{\theta}$ and $\boldsymbol{\varepsilon}_t$ is an iid error term. Following Pesaran and Shin (1998), we concentrate on shocks hitting only one equation at a time, and define the GIRF of \mathbf{y}_t to the shock δ_{jt} at horizon l as

$$GI(l, \delta_{jt}, \boldsymbol{\omega}_{t-1}) = E(\mathbf{y}_{t+l} | \varepsilon_{jt} = \delta_{jt}, \boldsymbol{\omega}_{t-1}) - E(\mathbf{y}_{t+l} | \boldsymbol{\omega}_{t-1}), \quad l = 0, 1, 2, \dots, \quad (6)$$

where $\boldsymbol{\omega}_{t-1}$ and δ_{jt} are the history and the shock to the j th equation that the expectations are conditioned on, respectively. The GIRF (6) can be interpreted as the time profile of the effect of the shock δ_{jt} hitting at time t , obtained as the difference between the expectations conditional on the shock and the history $\boldsymbol{\omega}_{t-1}$, and the expectations conditioned only on the history $\boldsymbol{\omega}_{t-1}$. Each history $\boldsymbol{\omega}_{t-1}$ consists of the matrix of initial values needed to compute the two conditional expectations (forecasts) in (6) which are typically obtained by averaging a large number of realizations from model (5) with and without the shock δ_{jt} , respectively (for details on the procedure and the specification of the shocks, see Koop et al.

(1996, Section 5).

In the linear VAR model (2), with no identification restrictions imposed, the GIRF (6) reduces to $GI(l, \boldsymbol{\delta}, \boldsymbol{\omega}_{t-1}) = \mathbf{A}_l \boldsymbol{\delta}$, which is independent of history $\boldsymbol{\omega}_{t-1}$, but depends on the hypothetical $K \times 1$ vector of shocks of size $\boldsymbol{\delta} = (\delta_1, \dots, \delta_K)'$, with only one of the elements nonzero. Assuming normality of the error term $\boldsymbol{\varepsilon}_t$ and setting a shock to the j th element of $\boldsymbol{\varepsilon}_t$, the unscaled GIRF of the shock δ_j is given by

$$GI(l, \delta_j, \boldsymbol{\omega}_{t-1}) = \mathbf{A}_l \boldsymbol{\Sigma} \mathbf{e}_j \sigma_{jj}^{-1} \delta_j, \quad (7)$$

where $\boldsymbol{\Sigma} = \{\sigma_{ij}, i, j = 1, \dots, K\}$ and \mathbf{e}_j is an $K \times 1$ selection vector with unity as its j th element and zeros elsewhere. By analogy to (4), scaling the GIRF (7) by setting $\delta_j = \sqrt{\sigma_{jj}}$, i.e., a positive one standard deviation shock, leads to the GFEVD of Pesaran and Shin (1998)

$$\theta_{ij}(h) = \frac{\sigma_{ii}^{-1} \sum_{l=0}^h (\mathbf{e}_i' \mathbf{A}_l \boldsymbol{\Sigma} \mathbf{e}_j)^2}{\sigma_i^2(h)}, \quad i, j = 1, \dots, K. \quad (8)$$

Because the shocks are not uncorrelated, in general $\sum_{j=1}^K \theta_{ij}(h) \neq 1$. Thus, the GFEVD (8) has the shortcoming that the contributions of the shocks to the forecast error variance of a given variable at horizon l do not sum to unity if the covariance matrix of the error $\boldsymbol{\varepsilon}_t$ is not a diagonal matrix. This makes their interpretation problematic.

Our new GFEVD is also defined analogously to (4), but in contrast to Pesaran and Shin (1998), it is not restricted to the linear VAR(p) model with normally distributed errors. It is obtained by replacing the IRF in (4) by the GIRF:

$$\lambda_{ij, \boldsymbol{\omega}_{t-1}}(h) = \frac{\sum_{l=0}^h GI(l, \delta_{jt}, \boldsymbol{\omega}_{t-1})_i^2}{\sum_{j=1}^K \sum_{l=0}^h GI(l, \delta_{jt}, \boldsymbol{\omega}_{t-1})_i^2}, \quad i, j = 1, \dots, K. \quad (9)$$

Here j and i refer to shock and variable, respectively, and h is the horizon, and $\boldsymbol{\omega}_{t-1}$ denotes the history. The denominator measures the aggregate cumulative effect of all the shocks, while the numerator is the cumulative effect of the j th shock.

By construction, as in (4), $\lambda_{ij,\omega_{t-1}}(h)$ lies between 0 and 1, measuring the relative contribution of a shock to the j th equation in relation to the total impact of all K shocks on the i th variable in \mathbf{y}_t after h periods, and these contributions sum to unity. Our GFEVD is thus easily interpretable and applicable in any linear (Gaussian or non-Gaussian) or nonlinear model for which the conditional expectations in (6) can be computed. It is worth noting that in the common case of the orthogonalized Gaussian structural VAR(p) model, the proposed GFEVD (9) reduces to the FEVD (4) (given the identification restrictions imposed on the matrix \mathbf{P} , i.e., the ordering of the variables).

In a nonlinear model, the GIRFs (6) and hence the GFEVDs, cannot typically be expressed in closed form, but the effects of a shock δ_{jt} typically depend on its size and sign as well as the history, and simulation methods are needed. Specifically, the GFEVD of interest can be obtained by averaging over the relevant shocks and histories in the same way as shown for the GIRF by Koop et al. (1996). These authors also discuss the efficiency of this approach, which is reached under certain monotonicity assumptions in Ripley (1987). They also point out that the approach is intuitively appealing even when these assumptions are not satisfied.

In practice, we recommend computing the GFEVD as the average of $\lambda_{ij,\omega_{t-1}}(h)$ over shocks obtained by bootstrapping from the residuals of the estimated model, and over all the histories. This should yield the GFEVD characteristic of the data at hand, and it naturally solves the problem of setting the size of shocks to each equation in a multivariate model. The following steps detail the computation of the GFEVD in any (potentially nonlinear) model (5):

1. Draw N vectors of shocks $(\delta_{1t}, \delta_{2t}, \dots, \delta_{Kt})'$ from the residuals of the estimated model. Given the form of $G(\cdot)$, the residuals $\widehat{\boldsymbol{\varepsilon}}_t$ of the model (5) are obtained in the usual way as

$$\widehat{\boldsymbol{\varepsilon}}_t = \mathbf{y}_t - G(\mathbf{y}_{t-1}, \dots, \mathbf{y}_{t-p}; \widehat{\boldsymbol{\theta}}),$$

where $\widehat{\boldsymbol{\theta}}$ denotes the vector of estimated parameters.

2. Pick a history $\boldsymbol{\omega}_{t-1}$ from among the set of all histories. The history $\boldsymbol{\omega}_{t-1}$ consists of the information used to compute the conditional expectations in (6), i.e., p lags of \mathbf{y}_t in the case of model (5).
3. Pick a shock vector, and compute $GI(\cdot)$ for each δ_{jt} ($j = 1, \dots, K$). In the nonlinear case, the conditional expectations in $GI(\cdot)$ in (6) can be obtained standard multistep forecasting methods, described, e.g., in Teräsvirta, Tjøstheim and Granger (2010, Section 14.2, pp. 345–351), including the bootstrap-based method employed in our empirical illustration in Section III.
4. Plug in the GIs computed in Step 3 into (9) to obtain $\lambda_{ij, \boldsymbol{\omega}_{t-1}}(h)$ ($h = 0, 1, 2, \dots$) for the particular history and shock.
5. Repeat Steps 3 and 4 for all N vectors of shocks.
6. Repeat Steps 2–5 for all the histories.
7. Finally, compute the average of $\lambda_{ij, \boldsymbol{\omega}_{t-1}}(h)$, ($h = 0, 1, 2, \dots$) over all the histories and shocks.

If the interest concentrates on only a subset of the histories, averaging can be restricted to the relevant histories, with shocks bootstrapped from among the residuals related to these histories only. For instance, we might be interested in finding the GFEVDs of positive and negative shocks to the j th equation separately, or we might want to compute the GFEVDs of all shocks only in periods satisfying certain conditions.

Finally, identification restrictions can be imposed on the parameters of nonlinear models in the same way as in structural linear VAR models prior to computing the conditional expectations in the GIRF (6). Such restrictions change the steps of the above procedure only in that the residuals in Step 1 and the conditional

expectations in Step 3 are computed from the restricted (identified) model. There are, however, very few examples of GIRF analyses based on a restricted nonlinear VAR model in the literature. For example, Weise (1999) and Balke (2000) impose recursive restrictions when examining asymmetric effects of monetary policy and the relationship between credit and economic activity, respectively, while Caggiano et al. (2015) apply this approach to a nonlinear VAR model of fiscal policy.

III Empirical illustration

We illustrate the different generalized FEVDs in the bivariate linear and nonlinear autoregressive leading indicator models of Anderson et al. (2007) for the U.S. GDP growth rate and term spread between the long-term (10-year) and short-term (3-month) interest rates. The term spread reflects the stance of monetary policy: It tends to decrease prior to recessions and increase during recessions, which suggests that it might be a useful leading indicator of output growth, and the previous empirical literature lends support to this conjecture. In particular, the term spread above has often been found the best leading indicator in the surveys of Stock and Watson (2003) and Wheelock and Wohar (2009), and Estrella (2005) for a rational expectations model that justifies the predictive power of the term spread by expectations of future monetary policy.¹

The estimates of the fifth-order (Gaussian) VAR and logistic smooth-transition vector autoregressive (LSTVAR) models on quarterly data from 1961Q3 to 1999Q4 are reported in Anderson et al. (2007, Appendix B). They first selected the lag length by the Akaike information criterion (AIC), and then eliminated redundant lags so that the residuals still remain uncorrelated. In the bivariate LVSTAR model,

¹It is worth pointing out that a closely related literature provides evidence of various credit and corporate bond spreads having predictive power for real output (see, e.g., Gilchrist, Yankov and Zakrajšek (2009), Gilchrist and Zakrajšek (2012) and Hubrich and Tetlow (2015), and the references therein). These interest rate spreads are generally indicators of changes in the supply of credit and the expectations of default, which are also partly reflected by the effective monetary policy set by central banks. Hubrich and Tetlow (2015) also find them essential ingredients in a U.S. financial stress index.

the transition variable for both equations is the lagged GDP growth rate (the second lag in the output growth equation and the first lag in the term spread equation).

To obtain the GFEVDs in the nonlinear LVSTAR model, the conditional expectations (forecasts) in (6), and subsequently in the GFEVD (9), are computed by the procedure described in Section II. In particular, we first compute the residuals of the estimated LSTVAR model (Step 1). Then we generate m sequences of forecasts $\mathbf{y}_t^{(m)}, \dots, \mathbf{y}_{t+h}^{(m)}$, $m = 1, \dots, M$, by replacing the unknown future error terms by independent draws with replacement from the set of residuals (see the case (iv) in Teräsvirta et al. (2010, p. 348)). Finally, the conditional expectations in (6) are obtained by averaging the simulated forecasts.

As an example of the GIRFs behind the GFEVDs, in Figure 1 we depict the GIRFs of a unit shock in the VAR and LSTVAR models. In the latter case, the GIRFs are presented for two histories (1991:Q1 and 1999:Q4) which exemplify low and high output growth histories, respectively. The two models seem to produce impulse response functions rather similar in shape but somewhat different in magnitude. The differences in magnitude are especially pronounced in the cases of the effect of the shock in output growth to the term spread and the shock to the term spread to output growth.

The GFEVDs of the VAR and LSTVAR models are reported in Tables 1 and 2, respectively. In accordance with the discussion in Section II, the (scaled) GFEVDs of Pesaran and Shin (1998) in the linear VAR model (see (8)) do not sum to unity at all horizons (see, especially, the decomposition for the term spread), whereas this problem does not arise with our GFEVD measure, which facilitates interpretation. For example, for the horizon of eight quarters ($h = 8$), according to the GFEVD of Pesaran and Shin (1998) the relative importance of the output shock to the output growth is 0.91, and the contributions of the two shocks only sum to 0.96. In contrast, according to our GFEVD, the relative contributions of the output growth shock is 0.82 and 0.80 in the VAR and LVSTAR models, respectively. For

the term spread, the differences are even larger and clearly the sum of the relative contributions of the shocks implied by the Pesaran-Shin GFEVD deviates even more from unity.

The GFEVDs based on the VAR and LSTVAR models appear somewhat different. Especially at short forecast horizons, the shock to the term spread has a larger relative contribution to the forecast error variance of output growth in the LSTVAR model than in the VAR model. This is in line with the importance of nonlinearity found by Galbraith and Tkacz (2000) and Anderson et al. (2007), among others, suggesting that the linear model is incapable of fully exploiting the predictive information of the term spread. The stability of the forecast performance of the term spread in the linear model has been questioned (see Giacomini and Rossi, 2006, and the references therein), and it may be another explanation to the differences. As to the term spread itself, it is dominated by its own shock in the VAR model while the contribution of the shock to output growth is far more important for it in the LSTVAR model.

We finally computed the GFEVDs of the LSTVAR model in the low growth regime consisting of histories with lagged output growth rate less than 0.32% (see Anderson et al. (2007)). In computing these, the shocks were bootstrapped from the residuals related to the low growth histories only. The results reported in the right panel of Table 2 suggest that in the low growth regime the relative importance the term spread shock is slightly emphasized compared to the GFEVDs from both regimes. This difference is more pronounced at shorter horizons. In other words, it seems that matching the business cycle-specific shocks with the relevant histories (in contrast to all the histories) results in somewhat different conclusions in the short run, with little effect in the long run.

IV Conclusions

We propose a new easily implementable generalized forecast error variance decomposition for multivariate linear and, in particular, nonlinear models. In the commonly used linear (Gaussian) VAR model with orthogonal shocks, the proposed GFEVD reduces to the usual FEVD, and it has a convenient interpretation also when the shocks are non-orthogonal. In nonlinear models this is not the case, and it is in these models that the proposed decomposition is likely to be particularly useful, as it allows for studying the effects of shocks in subsets of data of particular interest. An empirical application to U.S. output growth and term spread highlights the advantages of the new GFEVD in interpreting estimated linear and nonlinear multivariate models.

References

Anderson, H., Athanasopoulos, G. and Vahid, F. (2007). ‘Nonlinear autoregressive leading indicator models of output in G-7 countries’, *Journal of Applied Econometrics*, Vol. 22, pp. 63–87.

Balke, N.S. (2000). ‘Credit and economic activity: Credit regimes and nonlinear propagation of shocks’, *Review of Economics and Statistics*, Vol. 82, pp. 344–349.

Caggiano, G., Castelnuovo, E., Colombo, V. and Nodari, G. (2015). ‘Estimating fiscal multipliers: News from a nonlinear world’, *Economic Journal*, Vol. 125, pp. 746–776.

Estrella, A. (2005). ‘Why does the yield curve predict output and inflation?’, *Economic Journal*, Vol. 115, pp. 722–744.

Galbraith, J.W. and Tkacz, G. (2000). ‘Testing for asymmetry in the link between the yield spread and output in the G-7 countries’, *Journal of International Money and Finance*, Vol. 19, pp. 657–672.

Giacomini, R. and Rossi, B. (2006). ‘How stable is the forecasting performance

of the yield curve for output growth?', *Oxford Bulletin of Economics and Statistics*, Vol. 68, pp. 783–795.

Gilchrist, S., Yankov, V. and Zakrajšek, E. (2009). 'Credit market shocks and economic fluctuations: Evidence from corporate bond and stock markets', *Journal of Monetary Economics*, Vol. 56, pp. 471–493.

Gilchrist, S. and Zakrajšek, E. (2012). 'Credit spreads and business cycle fluctuations', *American Economic Review*, Vol. 102, pp. 1692–1720.

Hubrich, K. and Teräsvirta, T. (2013). 'Thresholds and smooth transitions in vector autoregressive models', *Advances in Econometrics*, Vol. 32, pp. 273–326.

Hubrich, K. and Tetlow, R.J. (2015). 'Financial stress and economic dynamics: The transmission of crises', *Journal of Monetary Economics*, Vol. 70, pp. 100–115.

Karamé, F. (2012). 'An algorithm for generalized impulse-response functions in Markov-switching structural VAR', *Economics Letters*, Vol. 117, pp. 230–234.

Karamé, F. (2015). 'Asymmetries and Markov-switching structural VAR', *Journal of Economic Dynamics and Control*, Vol. 53, pp. 85–102.

Koop, G., Pesaran, H.M. and Potter, S. (1996). 'Impulse response analysis in nonlinear multivariate models', *Journal of Econometrics*, Vol. 74, pp. 119–147.

Lütkepohl, H. (2005). *New Introduction to Multiple Time Series Analysis*. Springer, Berlin.

Pesaran, H.M. and Shin, Y. (1998). 'Generalized impulse response analysis in linear multivariate models', *Economics Letters*, Vol. 58, pp. 17–29.

Ripley, B.D. (1987). *Stochastic Simulation*. Wiley, New York.

Stock, J. H. and Watson, M.W. (2003). 'Forecasting output and inflation: The role of asset prices', *Journal of Economic Literature*, Vol. 41, pp. 788–829.

Teräsvirta, T., Tjøstheim, D. and Granger, C. (2010). *Modelling Nonlinear Economic Time Series*. Oxford University Press, Oxford.

Weise, C.L. (1999). 'The asymmetric effects of monetary policy: A nonlinear vector autoregression approach', *Journal of Money, Credit and Banking*, Vol. 31, pp. 85–108.

Wheelock, D.C. and Wohar, M. E. (2009). 'Can the term spread predict output growth and recessions? A survey of the literature', *Federal Reserve Bank of St. Louis Review*, Vol. 91, pp. 419–440.

Tables and Figures

TABLE 1

GFEVDs of the linear VAR model

Variable: Shock to: h	Pesaran and Shin (1998)				GFEVD (9)			
	Growth		Spread		Growth		Spread	
	Growth	Spread	Growth	Spread	Growth	Spread	Growth	Spread
1	1.00	0.01	0.04	1.00	1.00	0.00	0.00	1.00
2	0.98	0.01	0.10	0.99	0.96	0.04	0.01	0.99
3	0.96	0.02	0.16	0.97	0.92	0.08	0.01	0.99
4	0.94	0.03	0.20	0.96	0.88	0.12	0.02	0.98
8	0.91	0.05	0.59	0.80	0.82	0.18	0.11	0.89
16	0.91	0.05	0.66	0.77	0.82	0.18	0.13	0.87
20	0.91	0.05	0.67	0.77	0.82	0.18	0.13	0.87

Notes: The GFEVDs for the different forecast horizons (quarters) h are based on the Pesaran and Shin (1998) (scaled GIRFs) approach and the new formulation (9) in the left and right panels, respectively. The latter are given by expression (3) assuming $\mathbf{P} = \mathbf{I}_K$.

TABLE 2

GFEVDs (9) of the LSTVAR model

Variable: Shock to: h	LSTVAR				LSTVAR, Low Growth Regime			
	Growth		Spread		Growth		Spread	
	Growth	Spread	Growth	Spread	Growth	Spread	Growth	Spread
1	1.00	0.00	0.27	0.73	1.00	0.00	0.21	0.79
2	0.82	0.18	0.36	0.64	0.81	0.19	0.31	0.69
3	0.81	0.19	0.39	0.61	0.78	0.22	0.36	0.64
4	0.81	0.19	0.42	0.58	0.78	0.22	0.40	0.60
8	0.80	0.20	0.53	0.47	0.77	0.23	0.52	0.48
16	0.79	0.21	0.54	0.46	0.77	0.23	0.53	0.47
20	0.79	0.21	0.54	0.46	0.77	0.23	0.53	0.47

Notes: Following the step-by-step procedure described in Section II, the GFEVDs are based on 1,000 shocks bootstrapped from among the residuals of the estimated LSTVAR model. For each pair of shocks the GIRF is computed for each of the 154 histories (consisting of five consecutive observations), yielding, in total, 154,000 GIRFs, over which (9) is averaged. The conditional expectations in (6) are based on 1,000 simulated realizations. In the right panel, the low growth regime applies when the GDP growth rate is less than 0.32%

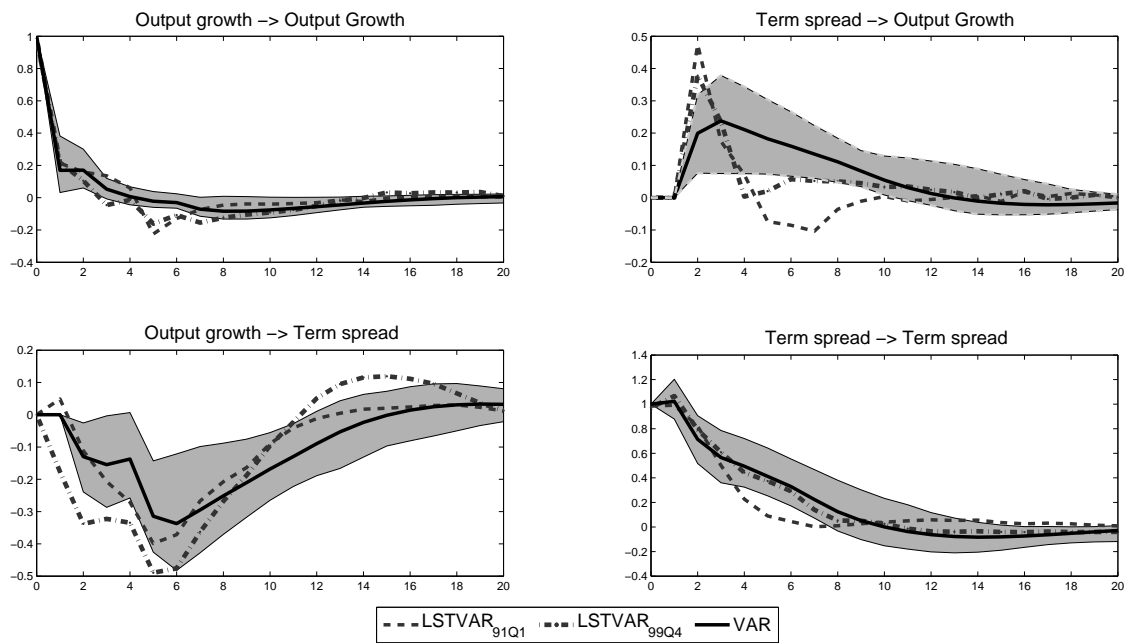


Figure 1: The GIRFs of positive unit shocks in the selected VAR (solid line) and LSTVAR models (histories 1991:Q1 (dashed line) and 1999:Q4 (dashed-dotted line)). The forecast horizon is up to 20 quarters. The shaded areas depict the bootstrapped (1000 replications) 95% confidence intervals of the GIRFs of the VAR model.