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ASPECTS OF INFLATIONARY MODELS AT LOW ENERGY SCALES

SAMI NURMI

Division of Elementary Particle Physics
Department of Physics
Faculty of Science
University of Helsinki
Helsinki, Finland

ACADEMIC DISSERTATION
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Äh! Ei riviäkään. Miten pelkkä ajatustyö voi käydä näin paljon voimille?

Aku Ankka, 15/2005.

Contents

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Abstract

Cosmological inflation is the dominant paradigm in explaining the origin of structure in the universe. According to the inflationary scenario, there has been a period of nearly exponential expansion in the very early universe, long before the nucleosynthesis. Inflation is commonly considered as a consequence of some scalar field or fields whose energy density starts to dominate the universe. The inflationary expansion converts the quantum fluctuations of the fields into classical perturbations on superhorizon scales and these primordial perturbations are the seeds of the structure in the universe. Moreover, inflation also naturally explains the high degree of homogeneity and spatial flatness of the early universe. The real challenge of the inflationary cosmology lies in trying to establish a connection between the fields driving inflation and theories of particle physics.

In this thesis we concentrate on inflationary models at scales well below the Planck scale. The low scale allows us to seek for candidates for the inflationary matter within extensions of the Standard Model but typically also implies fine-tuning problems. We discuss a low scale model where inflation is driven by a flat direction of the Minimally Supersymmetric Standard Model. The relation between the potential along the flat direction and the underlying supergravity model is studied. The low inflationary scale requires an extremely flat potential but we find that in this particular model the associated fine-tuning problems can be solved in a rather natural fashion in a class of supergravity models. For this class of models, the flatness is a consequence of the structure of the supergravity model and is insensitive to the vacuum expectation values of the fields that break supersymmetry.

Another low scale model considered in the thesis is the curvaton scenario where the primordial perturbations originate from quantum fluctuations of a curvaton field, which is different from the fields driving inflation. The curvaton gives a negligible contribution to the total energy density during inflation but its perturbations become significant in the post-inflationary epoch. The separation between the fields driving inflation and the fields giving rise to primordial perturbations opens up new possibilities to lower the inflationary scale without introducing fine-tuning problems. The curvaton model typically gives rise to relatively large level of non-gaussian features in the statistics of primordial perturbations. We find that the level of non-gaussian effects is heavily dependent on the form of the curvaton potential. Future observations that provide more accurate information of the non-gaussian statistics can therefore place constraining bounds on the curvaton interactions.

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Sami Nurmi

Helsinki, April 2008

List of included papers

The three articles included in this thesis are:

1. K. Enqvist and S. Nurmi, "Non-gaussianity in curvaton models with nearly quadratic potential," JCAP **0510** (2005) 013 [arXiv:astro-ph/0508573].
2. K. Enqvist, L. Mether and S. Nurmi, "Supergravity origin of the MSSM inflation," JCAP **0711** (2007) 014 [arXiv:0706.2355 [hep-th]].
3. S. Nurmi, "Kahler potentials for the MSSM inflation and the spectral index," JCAP **0801** (2008) 016 [arXiv:0710.1613 [hep-th]].

The contribution of the present author to the joint publications

In the first publication, the idea to study the connection between the curvaton potential and the level of non-gaussianities was mainly due to K. Enqvist. The present author is responsible for the calculations and for writing most of the paper, except parts of the introduction and conclusions. K. Enqvist provided supervision and discussions.

The initial idea for the subject of study in the second publication also came from K. Enqvist and was thereafter elaborated by all the authors. The calculations are performed jointly by the present author and L. Mether. The first draft was written mainly by the present author.

Chapter 1

Introduction

The early stages in the history of the universe provide a unique opportunity to gain information about physics beyond the Standard Model. Direct terrestrial experiments are unlikely to ever reach many orders of magnitude beyond the electroweak scale 100 GeV whereas the history of the universe can probe physics all the way up to the Planck scale $M_{\rm P} \sim 10^{18}$GeV or even beyond. This early epoch determines the initial conditions for the subsequent evolution of the universe which at least from the nucleosynthesis ($T \sim 0.1$ MeV) onwards can be successfully described using the hot big bang model, for reviews see e.g. [1, 2]. The cosmic microwave background (CMB) is a direct probe of these initial conditions. The extremely small temperature anisotropies in the CMB show that the early universe was not completely homogeneous and isotropic but there have been small primordial perturbations. These perturbations are the origin of the structure in the universe and they must have been created in some process taking place before the era described by the hot big bang cosmology.

Cosmological inflation has become the dominant paradigm in attempting to explain the origin of primordial perturbations [1, 2, 3]. It is nowadays widely believed that in the very early universe, long before the nucleosynthesis, there has been an inflationary phase during which the universe has been expanding almost exponentially. This could be caused by the potential energy of some scalar field or fields which start to dominate the energy density of the universe. The inflationary expansion results into particle production for light scalar fields. Quantum fluctuations of the fields approach to a constant value outside the horizon and become essentially classical, stochastical perturbations around the homogeneous background. This provides a natural explanation for the origin of structure based on first principles of quantum field theory in curved space. Moreover, inflation also naturally solves the infamous horizon and flatness problems of the hot big bang model as well as dilutes away densities of undesired primordial relics, like topological defects.

Inflation is extremely successful as a phenomenological description of the very early universe but establishing a connection between the scalar field matter driving inflation and theories of particle physics has proven challenging. Details of inflationary physics can be indirectly probed by observing the properties of primordial perturbations. Present observations are however consistent with almost scale-invariant adiabatic and gaussian perturbations [4] which arise in a large class of different inflationary models, see e.g. [2, 3, 5]. In most of the models, the inflationary energy scale needs to be quite high to produce large enough amplitude for density perturbations. The Hubble parameter is typically only few orders of magnitude below the Planck scale, $H \lesssim 10^{-5} M_{\rm P}$, which is way above the scales covered by the Standard Model of particle physics and its extensions. This makes it difficult to seek for candidates for the inflationary matter and leaves freedom to construct a wide range of models with different physical motivations.

The inflationary scale is however not directly determined by the amplitude of primordial density perturbations and it is also possible to construct successful models with a significantly lower scale $H \ll 10^{-5} M_{\rm P}$. The low scale makes it easier to find theoretically motivated models but producing large enough perturbations poses difficulties. If inflation is driven by a single scalar field, its potential needs to be extremely flat which seems to imply unnatural fine-tuning [5]. Hybrid models [6] with several fields fare better although some fine-tuning is still required. Hybrid models however tend to yield too large spectral index [2, 4]. In this thesis we discuss a single field low-scale model [7, 8] in which inflation is driven by one of the flat directions [9] of the Minimally Supersymmetric Standard Model (MSSM). The great advantage of the model is that the gauge couplings of the inflationary matter are known, which allows to explain the generation of Standard Model fields after the inflation in a self-consistent manner. In the enclosed research papers [10, 11], it has been shown that the fine-tuning problems associated to the flatness of the potential along the flat direction can be solved quite naturally at the classical level in a class of supergravity models. The results are encouraging but more work is still needed to check the stability of the flatness against radiative corrections.

Another attractive possibility to lower the inflationary scale is the curvaton scenario [12, 13]. The primordial perturbations in this scenario are produced after the inflation by the decay of a curvaton field which is a scalar field different from the fields driving inflation. This considerably relaxes the constraints on the inflationary energy scale [14] and makes it easier to find particle physics candidates for the scalar fields. For example, flat directions of the MSSM have been shown to provide successful candidates for the curvaton [15]. The curvaton scenario can give rise to significant non-gaussian perturbations [16, 17, 18] which could be detected by the Planck satellite [19, 20, 21]. The dependence of the non-gaussianities on the form of the curvaton potential is examined in the enclosed research paper [18]. The non-gaussian statistics of primordial perturbations can act as an efficient discriminator between different inflationary models. Conventional single field models for example yield undetectable non-gaussianities [22, 23] and would therefore be ruled out by a detection of non-gaussianity.

The thesis is organized as follows. In Chapter 2 we review the basic features of inflationary cosmology. Chapter 3 concentrates on the MSSM inflation and introduces some relevant properties of supersymmetric theories. In particular, the form of the potential along flat directions is discussed. The supergravity embedding of the MSSM inflation as a possible solution to the fine-tuning problems of the model is reviewed following the work done in [10, 11]. Chapter 4 provides a brief outlook on the curvaton scenario and presents the results found in [18] on the relation between the curvaton potential and the level of non-gaussian perturbations. Finally, in Chapter 5 we summarize the discussion.

1.1 Notation

The natural units $c \equiv 1, \hbar \equiv 1, k_B \equiv 1$ are used throughout the thesis. The Planck mass is defined as $M_{\rm P} = (8\pi G)^{-1/2}$ where G is Newton's constant. The signature of the metric is chosen as $(-,+,+,+)$. Summation over repeated indices is understood. Greek letters $(\mu, \nu, \ldots)$ run over all the spacetime coordinates and Latin letters $(i, j, \ldots)$ over the spatial coordinates. The covariant derivative is denoted by ∇_μ and it is defined using the standard Christoffel connection. The notation used for the Kähler metric and the associated indices in supergravity is explained in Chapter 3. For the Fourier transformation we use the normalization

$$f(\mathbf{x}) = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\; f(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} \; . \tag{1.1}$$

Other notations are explained in the text. The notations used in the enclosed research papers differ partially from the ones used in the introductory part of the thesis.

Chapter 2

Inflation in the early universe

The dynamics of the universe at large is governed by gravity. Also the nearly exponential expansion during the inflationary stage is a gravitational effect which is caused by the domination of a matter content with large enough negative pressure. The inflationary matter can at least effectively be described by scalar fields [1, 2, 3] with a suitable potential. In this Chapter, we first review basic features of general relativity needed to understand the inflationary dynamics and then move on to discuss the properties of scalar fields. We outline the quantization of the scalar fields in the inflationary universe and discuss the generation of primordial perturbations. At the end of the Chapter, we discuss the relation between the properties of the primordial perturbations and the details of the inflationary models.

2.1 General relativity and cosmology

The structure in the universe is expected to originate from small primordial perturbations created in the very early universe and thereafter amplified by gravitational interactions [1, 2]. The observed anisotropies in the CMB radiation imply that deviations from homogeneity and isotropy have been extremely small in the early universe. At the time of photon decoupling, the amplitude of density fluctuations on superhorizon scales was $\delta\rho/\rho \sim 10^{-5}$ [4] [1]. Therefore it is an excellent leading order approximation to model the early universe as homogeneous and isotropic. The small density fluctuations can be described as perturbations around this symmetric solution [1, 2, 24].

The most general metric describing a homogeneous and isotropic spacetime is the Robertson-Walker (RW) metric for which the line element $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu$ reads [25, 26]

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\Big(\frac{\mathrm{d}r^2}{1-Kr^2} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2\Big)\ . \tag{2.1}$$

Here t is the time measured by a comoving observer, r, θ and ϕ are comoving polar coordinates and the dimensionless scale factor $a(t)$ describes the volume expansion of spatial hypersurfaces. The dimensional[2] parameter K describes the curvature of spatial hypersurfaces; $K > 0$ corresponds to positive and $K < 0$ to negative spatial curvature. If $K = 0$ the spacetime is spatially flat.

[1]This holds for the perturbations of photons and baryons. The cold dark matter has decoupled from photons already before the decoupling of baryonic matter and its fractional perturbations have thus grown larger [1].

[2]Another commonly used choice is to make K dimensionless and normalize it to $\{-1, 0, 1\}$. With this choice the scale factor is dimensional and the radial coordinate r dimensionless.

Observations seem to favour the spatially flat case [4] which is also consistent with simplest models of inflation.

According to general relativity, gravitation manifests itself as a geometry of the spacetime. Up to boundary terms [27], the action is written as

$$S = S_g + S_m = \frac{M_{\rm P}^2}{2}\int {\rm d}^4x\sqrt{-g}R + \int {\rm d}^4x\sqrt{-g}\mathcal{L}_m \ , \tag{2.2}$$

where the Ricci scalar R describes the curvature of the spacetime encoding gravitational degrees of freedom and $\mathcal{L}_m$ is the Lagrangian for the matter fields. The determinant of the metric is denoted by $g \equiv \det(g_{\mu\nu})$. Upon variation with respect to the metric one arrives at the equations of motion for the gravitational field [25, 26]

$$G_{\mu\nu} = M_{\rm P}^{-2}T_{\mu\nu} \ . \tag{2.3}$$

The Einstein tensor

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \tag{2.4}$$

describes local properties of the gravitational field. There is also a non-local part in the gravitational field which is described by the Weyl tensor [25, 26]. Gravitational waves are an example of non-local gravitational fields. The energy momentum tensor $T_{\mu\nu}$ is defined as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{\mu\nu}} \ , \tag{2.5}$$

and it describes the properties of matter fields. According to equation (2.3), the energy momentum tensor determines the coupling between the matter fields and the gravity.

The homogeneity and isotropy of the RW-metric (2.1) implies that the matter contents of a homogeneous and isotropic Friedmann-Robertson-Walker -universe (FRW) must consist of an ideal fluid or a mixture of non-interacting ideal fluids [28]. The energy momentum tensor of an ideal fluid with four velocity u^μ reads

$$T^{\mu\nu} = \rho\, u^\mu u^\nu + p\,(g^{\mu\nu} + u^\mu u^\nu) \ , \tag{2.6}$$

where $\rho = T_{\mu\nu}u^\mu u^\nu$ is the energy density and $p = 1/3\,(g^{\mu\nu} + u^\mu u^\nu)T_{\mu\nu}$ is the pressure of the fluid. In the comoving coordinates used in equation (2.1), the fluid is at rest $u^\mu = (1,0,0,0)$ and the energy momentum tensor (2.6) becomes diagonal $T^\mu_{\ \nu} = {\rm diag}(-\rho,p,p,p)$. The energy momentum tensor in general obeys the conservation equation $\nabla_\mu T^{\mu\nu} = 0$ which follows from the invariance of the matter action under general coordinate transformations. By contracting $\nabla_\mu T^\mu_{\ \nu} = 0$ with the fluid four velocity u^ν one obtains the continuity equation $u^\nu\nabla_\mu T^\mu_{\ \nu} = 0$. For ideal fluids the continuity equation in comoving coordinates reads

$$\dot\rho + 3H(\rho + p) = 0 \ , \tag{2.7}$$

where the Hubble parameter describes the volume expansion of the fluid, $H \equiv \nabla_\mu u^\mu/3 = \dot a/a$, and the dot denotes a derivative with respect to the comoving time coordinate.

The physical properties of the ideal fluid are determined by its equation of state $p = p\,(\rho)$ which is commonly written in the form

$$p\,(t) = w(t)\rho(t) \ . \tag{2.8}$$

For a constant equation of state parameter w, the continuity equation (2.7) can be solved for the energy density to yield

$$\rho(t) \propto a(t)^{-3(1+w)} \ . \tag{2.9}$$

The behaviour of the scale factor $a(t)$ is determined by the Einstein equations (2.3). In the comoving coordinates, the $0-0$ component of equation (2.3) gives the Friedmann equation

$$H^2 = \frac{\rho}{3M_{\rm P}^2} - \frac{K}{a^2} \ , \tag{2.10}$$

and the $i-i$ components give

$$2\frac{\ddot{a}}{a} + H^2 = -\frac{p}{M_{\rm P}^2} - \frac{K}{a^2} \ . \tag{2.11}$$

By subtracting equations (2.10) and (2.11), one obtains the so called acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{\rho + 3p}{6M_{\rm P}^2} \ . \tag{2.12}$$

The $i-i$ components of Einstein equations (2.11) can also be derived using (2.7) and (2.10) and provide no additional information above these two. The FRW universe thus contains three dynamical degrees of freedom, the scale factor $a(t)$, the density $\rho(t)$ and the equation of state parameter $w(t)$, and two independent equations governing their evolution. By fixing e.g. the matter contents described by $w(t)$, the functions $a(t)$ and $\rho(t)$ determining the evolution of the universe can be solved from equations (2.7) and (2.10).

2.1.1 Problems in the simplest model

In the hot big bang model [1, 2] the evolution of the early universe universe is described using essentially two kinds of matter contents: relativistic matter or radiation with $w = 1/3$ and non-relativistic matter with $w = 0$. In addition, vacuum energy or cosmological constant with $w = -1$ can be included at late times. According to equation (2.9) the energy densities behave as $\rho_r \propto a^{-4}$, $\rho_m \propto a^{-3}$ and $\rho_\Lambda =$ const. for radiation, matter and vacuum energy, respectively. In a spatially flat universe with constant w, equations (2.7) and (2.10) yield $a \propto t^{2/(3+3w)}$ for $w \neq -1$ and $a \propto e^{Ht}$ for $w = -1$.

The hot big bang model describes the history of the universe starting from some epoch before nucleosynthesis $T \sim 0.1$ MeV. Most of the matter is relativistic at this time and the universe is dominated by radiation. Since $\rho_r/\rho_m \propto a^{-1}$, the energy density of radiation eventually falls below the energy density of matter and the universe becomes matter-dominated. This happens at $T \sim 1$ eV and the formation of the CMB takes place at the photon decoupling temperature $T \sim 0.1$ eV.

At late times the universe might also become dominated by vacuum energy, or cosmological constant, since $\rho_m/\rho_\Lambda \propto a^{-3}$. If the FRW approximation is used to describe the late universe, the combined cosmological observations from different redshifts [29, 4]. i.e. from different distance scales, imply that the universe is undergoing an accelerated expansion and would thus be dominated by vacuum energy or something close to it. This dominating component is called dark energy [30] and explaining its physical origin and the required magnitude of its energy density is commonly phrased as the biggest problem in modern cosmology. It should also be kept in mind, however, that the homogeneity and isotropy is not a perfect approximation in the late universe which is filled with clusters of galaxies and large voids between them. Inhomogeneities in general can mimic the distance-redshift relation of an FRW universe dominated by the vacuum energy and the apparent acceleration could therefore be at least partially caused by the inhomogeneities, see [31, 32, 33] and references therein. This would also naturally explain the coincidence problem [34] between the beginning of the apparent acceleration and the structure formation since the inhomogeneities become important when the structure starts to form.

The hot big bang model is extremely successful [1, 2] in explaining the nucleosynthesis and the evolution of the early universe from nucleosynthesis onwards. However, it appears to require fine-tuning of initial conditions – known as the infamous horizon and flatness problems. The primordial perturbations also need to be put in by hand. The horizon problem is the problem [1, 2] in explaining the nearly perfect thermal equilibrium of the observed CMB radiation. The spectrum of the CMB photons is the most accurate black body spectrum ever measured, but it seems to originate from several causally disconnected regions. The comoving distance travelled by freely moving photons between some early time event $t \sim 0$ and the photon decoupling time $t_{\rm dec}$, when the CMB radiation was formed, is given by

$$r_{\rm dec} = \int_0^{t_{\rm dec}} \frac{{\rm d}t}{a(t)} = \frac{1}{a(t_{\rm dec})H(t_{\rm dec})} \ , \tag{2.13}$$

where we have assumed that the universe has been radiation dominated all the time before $t_{\rm dec}$. This gives an estimate of the size of the causally connected patch, or the particle horizon, at the decoupling time measured in coordinates fixed with the expansion. After the photon decoupling, the universe can be assumed to be dominated by matter and the comoving distance travelled by photons from $t_{\rm dec}$ up to the present time t_0 becomes

$$r_{\rm hor} = \int_{t_{\rm dec}}^{t_0} \frac{{\rm d}t}{a(t)} = \frac{2}{a(t_0)H(t_0)} - \frac{2}{a(t_{\rm dec})H(t_{\rm dec})} \approx \frac{2}{a(t_0)H(t_0)} \ . \tag{2.14}$$

In the last step we have employed the relation between the values of the scale factor in the present universe and at the decoupling time, which derives from the redshift of the CMB photons. The redshift z is defined as the ratio of the observed and emitted wavelengths, $1 + z \equiv \lambda_0/\lambda = a_0/a$, and the observed value for the CMB photons is $z(t_{\rm dec}) \approx 1100$ [1, 2]. According to equations (2.13) and (2.14), the present particle horizon is much larger than the horizon at the last scattering surface, $r_{\rm hor}/r_{\rm dec} \approx 2\sqrt{1+z} \gg 1$, indicating that the observed part of the CMB consists of a large number of causally disconnected regions. The thermal equilibrium can therefore not be explained by causal processes taking place during the hot big bang epoch but needs to be imposed by initial conditions.

The flatness problem can be understood by rewriting the Friedmann equation (2.10) in terms of the density parameter $\Omega \equiv \rho/\rho_c \equiv \rho/3M_{\rm P}^2H^2$ as

$$\Omega - 1 = \frac{K}{(aH)^2} \ , \tag{2.15}$$

where $\Omega = 1$ corresponds to a spatially flat universe with the critical density $\rho_c = 3M_{\rm P}^2H^2$. Assuming that the universe is not completely flat to begin with, the deviation from flatness grows as $(\Omega - 1) \propto t$ in a radiation-dominated universe and as $(\Omega - 1) \propto t^{2/3}$ in a matter-dominated universe. The observational bound for the deviation from flatness at the present time is $|\Omega - 1|_0 \lesssim 0.02$ [4], assuming a constant equation of state for the dark energy which effectively dominates the late universe [35]. Due to the age of the universe $t_0 \sim 13.7 \times 10^9$ yrs, this implies extreme fine-tuning of the initial conditions. At the time of the nucleosynthesis, one needs to require $|\Omega - 1|_{\rm nucl} \lesssim 10^{-16}$ and deviations from the flatness must be even smaller at earlier times [1, 2]. While the fine-tuning could of course just be accepted, it would certainly be more desirable to understand the origin of the initial conditions.

Unwanted relics constitute a third classical problem of the hot big bang model [1, 2]. Historically the main concern was the overproduction of heavy magnetic monopoles that necessarily arise in grand unified theories, assuming the symmetries are restored in the very early universe. Also other kind of topological defects that could contradict observations can be created in phase transitions of the early universe. Topological defects are not the only dangerous relics but the success of the hot big bang model can also be destroyed by other non-relativistic weakly interacting and stable particle species created in the very early universe. If their abundances are too high, the radiation dominated era can become too short and the predictions of nucleosynthesis can be altered.

2.2 Dynamics of inflation

Cosmic inflation was first introduced as a solution to the horizon and flatness problems. The beginning for the study of inflation is generally considered to be Guth's article in 1981 [36] but parts of the idea were presented already earlier by Starobinsky [37]. Very soon it was realized [38, 39] that besides explaining the homogeneity and flatness, inflation also provides a simple explanation for the origin of primordial perturbations based on the behaviour of quantum scalar fields in curved space. This remarkable feature is nowadays considered as the main motivation for the inflationary paradigm.

2.2.1 Inflationary expansion

Inflation is defined simply as a period of accelerating expansion in the early universe [1, 2, 3]

$$\ddot{a} > 0 \ . \tag{2.16}$$

In most of the inflationary models the expansion is almost exponential, $a(t) \sim e^{Ht}$. The acceleration condition (2.16) can equivalently be written as a condition for the comoving Hubble length $(aH)^{-1}$ characterizing the coordinate size of a causal patch,

$$\frac{d}{dt}(aH)^{-1} < 0 \ . \tag{2.17}$$

Viewed in comoving coordinates fixed with the expansion, the locally observable universe shrinks during inflation. All the large scale structure gradually drifts outside the horizon and the universe becomes locally empty and cold. Consequently, in a non-inflationary universe the horizon grows and the size of the observable patch increases with time.

The amount of inflationary expansion is commonly characterized by the number of e-foldings,

$$N(t) = \ln\frac{a(t_{\rm end})}{a(t)} \ , \tag{2.18}$$

which measures the growth of the scale factor from some time t during the inflation up to the end of inflation at $t_{\rm end}$. For an exponentially expanding universe $a(t) \propto e^{Ht}$ with a constant Hubble parameter, this becomes

$$N(t) = H(t_{\rm end} - t) \ . \tag{2.19}$$

The horizon size at the end of inflation is given by

$$r_{\rm end} = \int_t^{t_{\rm end}} \frac{{\rm d}t}{a(t)} \approx \frac{e^{N(t)}}{a(t_{\rm end})H(t_{\rm end})} \ , \tag{2.20}$$

which easily becomes larger than the current horizon $r_{\rm hor}$, given by equation (2.14), if inflation lasts long enough. The detailed evolution of the horizon scale depends on the post-inflationary physics and in particular on the temperature that the universe acquires in reheating after the end of inflation. In typical models the scales corresponding to the current horizon have exited the horizon about 60 e-foldings before the end of inflation [1, 2, 3, 40] and the total number of e-foldings is much greater. The horizon problem is therefore solved in a natural manner since the whole observable universe has been well within a single causally connected region during the early stages of inflation.

From equations (2.15) and (2.17), it is immediately seen that that the inflationary expansion dynamically drives the universe towards spatial flatness, $|\Omega-1| \to 0$. This happens exponentially fast in physical time if $a \propto e^{Ht}$ and the initial conditions with $|\Omega - 1|$ extremely close to zero thus arise naturally if inflation lasts long enough. The 60 inflationary e-foldings, or so, required to solve the horizon problem are also enough to solve the flatness problem.

The accelerating expansion of the universe implies according to equation (2.12) that the matter content needs to satisfy

$$p < -\frac{1}{3}\rho \ . \tag{2.21}$$

Expressed in terms of the equation of state parameter, the inflationary matter needs to have $w < -1/3$. A sufficiently long period of inflation thus also dilutes away relics of ordinary matter since, according to equation (2.9), the fractional energy density of radiation decays as $\rho_r/\rho \propto a^{(3w-1)}$ and matter as $\rho_m/\rho \propto a^{3w}$, where ρ and w refer to the inflationary matter.

2.2.2 Inflationary matter: scalar fields

As mentioned above, the inflationary expansion requires matter with negative pressure (2.21) and can thus not be caused by ordinary relativistic or non-relativistic particles. However, the condition (2.21) can easily be satisfied for scalar fields with a suitable potential [1, 2, 3, 5]. The scalar fields driving inflation could be fundamental scalars in theories of high energy physics or composite objects like fermion or gauge field condensates. They could also arise as effective fields describing for example the motion of branes in string theory motivated models or they could parameterize modifications of the general relativity. The field or fields driving the inflation are commonly called inflaton fields.

The action for a single scalar field in a spacetime with metric $g_{\mu\nu}$ is written as [1, 2, 3]

$$S_\phi = \int {\rm d}^4x\sqrt{-g}\mathcal{L}_\phi = \int {\rm d}^4x\sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right) \ , \tag{2.22}$$

where $\nabla_\mu\phi = \partial_\mu\phi$ and $V(\phi)$ is the potential of the field. The energy momentum tensor (2.5) for the scalar field matter becomes

$$T_{\mu\nu} = \nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}g^{\rho\sigma}\nabla_\rho\phi\nabla_\sigma\phi + V(\phi)\right) \ . \tag{2.23}$$

Using the RW metric (2.1), and setting the spatial curvature to zero $K = 0$, one finds the expressions for the energy density,

$$\rho = u^\mu u^\nu T_{\mu\nu} = \frac{1}{2}\dot\phi^2 + \frac{1}{2}\frac{(\nabla\phi)^2}{a^2} + V(\phi) \ , \tag{2.24}$$

and for the pressure

$$p = \frac{1}{3}(g^{\mu\nu} + u^\mu u^\nu)T_{\mu\nu} = \frac{1}{2}\dot{\phi}^2 - \frac{1}{6}\frac{(\nabla\phi)^2}{a^2} - V(\phi) \ , \tag{2.25}$$

which are both expressed in comoving coordinates. Here and elsewhere in the text, we use ∇ without indices to denote the spatial gradient in the Euclidean space, e.g. $(\nabla\phi)^2 \equiv \sum_i (\partial_i\phi)^2$. If the universe is exactly homogeneous, the scalar field ϕ needs to be homogeneous as well and the gradients in (2.24) and (2.25) vanish, $\nabla\phi = 0$. The inflationary condition (2.21) can be realized for homogeneous scalar field matter with $\dot{\phi}^2 < V(\phi)$. Typically this requires that the initial conditions for the field evolution are such that $\dot{\phi}^2_{\rm in} \ll V(\phi_{\rm in})$ and the potential is flat enough to keep $\dot{\phi}$ small for a sufficiently long period to obtain enough e-foldings.

The equation of motion for the scalar field is obtained by varying the action (2.22) with respect to ϕ and it reads,

$$\nabla^\mu\nabla_\mu\phi - V'(\phi) = 0 \ . \tag{2.26}$$

For a test field in a spatially flat FRW universe, this becomes

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2\phi}{a^2} + V'(\phi) = 0 \ , \tag{2.27}$$

and if the field is homogeneous, $\phi = \phi(t)$, the gradients vanish yielding,

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \ . \tag{2.28}$$

The Hubble parameter is given by the Friedmann equation (2.10) which in the spatially flat universe filled with homogeneous scalar field matter reads

$$H^2 = \frac{1}{3M_{\rm P}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \ . \tag{2.29}$$

2.2.3 Slow roll approximation

The coupled set of equations (2.28) and (2.29) determines the evolution of the FRW universe dominated by a single homogeneous scalar field. Instead of solving the exact equations, the inflationary dynamics is usually analyzed using the so called slow roll approximation [1, 2, 3]

$$\ddot{\phi} \ll 3H\dot{\phi} \ , \quad \dot{\phi}^2 \ll V(\phi) \ . \tag{2.30}$$

The Friedmann equation (2.29) then becomes

$$H^2 \simeq \frac{V(\phi)}{3M_{\rm P}^2} \ , \tag{2.31}$$

and the scalar field equation of motion (2.28) reads

$$3H\dot{\phi} \simeq -V'(\phi) \ . \tag{2.32}$$

While the slow roll approximation is strictly speaking not a necessary condition for the inflation to occur, the amount of e-foldings taking place when the slow roll conditions (2.30) are violated is generically small.

The exact set of equations of motion (2.28), (2.29) is of second order in time derivatives while the slow roll approximation results to first order equations (2.31), (2.32) and thus contains one degree of freedom less. In general there is no reason to expect that the reduced set of equations would provide a good approximation of the exact second order equations for arbitrary choices of the initial value $\dot{\phi}_{\rm in}$. The exact equations (2.28), (2.29) however exhibit an attractor behaviour in the region where $\dot{\phi}(t)$ is a monotonous function [41, 42]. When an exact solution satisfies the inflationary condition $\dot{\phi}^2 < V(\phi)$, deviations from the inflationary solution die out exponentially fast in ϕ. The slow roll equations (2.31) and (2.32) yield a good approximation to the attractor solution and can therefore be used instead of the full set of equations to analyze the inflationary dynamics.

In applying the slow roll approximation, it is convenient to introduce two dimensionless parameters

$$\epsilon = \frac{M_{\rm P}^2}{2}\left(\frac{V'}{V}\right)^2 , \tag{2.33}$$

and

$$\eta = M_{\rm P}^2 \frac{V''}{V} , \tag{2.34}$$

which describe the properties of the inflaton potential. The slow roll approximation (2.30) implies that [1, 2, 3]

$$\epsilon \ll 1 \ , \quad |\eta| \ll 1 \ , \tag{2.35}$$

which are necessary conditions for the approximation to be valid. In addition, one needs to choose the initial conditions such that $\dot{\phi}_{\rm in}^2 \ll V(\phi_{\rm in})$ to justify the use of the slow roll equations (2.31) and (2.32). For practical purposes the slow roll parameters (2.33) and (2.34) are very useful since they directly give the necessary conditions for the inflaton potential in slow roll inflation. Moreover, the properties of primordial perturbations generated in a given model can be straightforwardly related to ϵ and η at leading order.

The end of the inflationary stage is usually taken to be determined by the violation of the slow roll conditions (2.35); i.e. inflation ends when either $\epsilon \sim 1$ or $|\eta| \sim 1$. While this again is not a necessary nor a sufficient condition, it gives a good enough approximation in conventional single field models. Using the slow roll equations of motion (2.31) and (2.32), the number of e-foldings (2.18) can be written as

$$N(t) = \int_t^{t_{\rm end}} H {\rm d}t = \int_\phi^{\phi_{\rm end}} \frac{H}{\dot{\phi}}\, {\rm d}\phi \simeq -\frac{1}{M_{\rm P}^2}\int_\phi^{\phi_{\rm end}} \frac{V}{V'}\, {\rm d}\phi \ , \tag{2.36}$$

where $\phi_{\rm end} = \phi(t_{\rm end})$ denotes the field value at the end of inflation.

2.3 Generating the primordial perturbations

The almost exponential time evolution of the inflationary universe results into particle creation for light scalar fields. The quantum fluctuations of the fields become essentially classical perturbations outside the horizon and generate the primordial perturbations seen in the CMB. In this section we discuss the quantization of scalar fields and the relation between the scalar field perturbations and the metric perturbations.

2.3.1 Quantization of free scalar fields in de Sitter space

If the Hubble parameter $H = \dot{a}/a$ is constant so that the universe is expanding exponentially $a = e^{Ht}$, the spacetime is called de Sitter. According to the Friedmann equation (2.10), the de

Sitter space corresponds to a universe with constant energy density. This is the case with pure vacuum energy domination but slowly rolling scalar fields with $V(\phi) \simeq V(\phi_{\rm in})$ also yield an approximative de Sitter stage according to equation (2.31). Here we outline the quantization of light scalar fields in de Sitter space, for a thorough treatment see e.g. [43]. The scalar fields are treated as test fields that have no effect on the evolution of the spacetime. While this is not the case during inflation when the inflaton or inflatons dominate the universe, the results obtained here will prove useful when discussing inflationary perturbations.

Instead of using the proper time t of a comoving observer, it is convenient to switch temporarily to the conformal time η defined as

$$d\eta = \frac{dt}{a} \ . \tag{2.37}$$

The flat RW-metric in the conformal time is simply the Minkowski metric multiplied with a time dependent scale factor

$$g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu} = a^2(\eta)\mathrm{diag}(-1,1,1,1) \ . \tag{2.38}$$

In de Sitter space, equation (2.37) yields $\eta = -(aH)^{-1}$. For a free scalar field, $V = m^2\phi^2/2$, the action (2.22) becomes

$$S_\phi = \frac{1}{2}\int d\eta d^3x \ a^2(\phi'^2 - (\nabla\phi)^2 - a^2m^2\phi^2) \ , \tag{2.39}$$

where the prime denotes a derivative with respect to the conformal time $\phi' \equiv \partial\phi/\partial\eta$. The equation of motion for ϕ becomes

$$\phi'' + 2\mathcal{H}\phi' - \nabla^2\phi + a^2m^2\phi = 0 \ , \tag{2.40}$$

where $\mathcal{H} \equiv a'/a = aH$, and for Fourier modes this yields

$$\phi_k'' + 2\mathcal{H}\phi_k' + (k^2 + a^2m^2)\phi_k = 0 \ . \tag{2.41}$$

The general solution of (2.41) is given by [39]

$$\phi_k(\eta) = (-\eta)^{3/2}H\sqrt{\frac{\pi}{2}}\left(c_1(k)H_\nu^{(1)}(-k\eta) + c_2(k)H_\nu^{(2)}(-k\eta)\right) \ , \tag{2.42}$$

where $c_{1,2}(k)$ are arbitrary functions, $H_\nu^{(1,2)}$ are the Hankel functions of first and second kind and the parameter ν labeling the degree of the Hankel functions is defined as

$$\nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}} \ . \tag{2.43}$$

The result (2.42) holds for real values of ν and is thus valid for light fields $m \leqslant 3H/2$. For modes that are well inside the horizon $|k\eta| \gg 1$, the effect of curvature is negligible and the field equation (2.40) reduces to the flat space equation. The subhorizon limit $|k\eta| \gg 1$ of the solution (2.42) reads

$$\phi_k(\eta) \sim -\frac{\eta H}{\sqrt{k}}\left(c_1(k)e^{-i\left(k\eta + \frac{\pi}{2}(\nu+\frac{1}{2})\right)} + c_2(k)e^{i\left(k\eta + \frac{\pi}{2}(\nu+\frac{1}{2})\right)}\right) \ , \tag{2.44}$$

which coincides with the plane waves in Minkowski space up to the coefficients $c_{1,2}(k)$.

The canonical quantization of the scalar field ϕ in curved space proceeds in close analogue to the quantization in flat space. The classical field is promoted to the field operator [43]

$$\hat{\phi}(\eta, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^3}\left(e^{i\mathbf{k}\cdot\mathbf{x}}\phi_k(\eta)\hat{a}_\mathbf{k} + e^{-i\mathbf{k}\cdot\mathbf{x}}\phi_k^*(\eta)\hat{a}_\mathbf{k}^\dagger\right) , \tag{2.45}$$

where the creation $\hat{a}^\dagger_{\mathbf{k}}$ and annihilation operators $\hat{a}_{\mathbf{k}}$ satisfy the commutation relations

$$[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}] = 0 \ , [\hat{a}^\dagger_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}'}] = 0 \ , [\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}'}] = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') \ . \tag{2.46}$$

The mode functions $\phi_k(\eta)$ need to form a complete orthonormal set of solutions of (2.40). The orthonormality implies the condition [43, 24]

$$\phi_k(\eta)\phi^{*\prime}_k(\eta) - \phi^*_k(\eta)\phi'_k(\eta) = ia^{-2} \ , \tag{2.47}$$

which guarantees that the commutators (2.46) are equivalent to the canonical equal time commutation relations for the field $\hat{\phi}$ and its conjugate momentum $\hat{\pi} = a^2\hat{\phi}'$,

$$[\hat{\phi}(\eta, \mathbf{x}), \hat{\phi}(\eta, \mathbf{x}')] = 0 \ , \ [\hat{\pi}(\eta, \mathbf{x}), \hat{\pi}(\eta, \mathbf{x}')] = 0 \ , \ [\hat{\phi}(\eta, \mathbf{x}), \hat{\pi}(\eta, \mathbf{x}')] = i\delta(\mathbf{x} - \mathbf{x}') \ . \tag{2.48}$$

The commutation relations (2.46) and (2.48) are both expressed in terms of the flat space delta function, $\int d^3x\, \delta(\mathbf{x} - \mathbf{x}') = 1$.

In Minkowski space, there is a natural way to choose the basis of creation and annihilation operators, or the set of mode functions, since the spacetime is symmetric under both temporal and spatial translations. The mode functions can be chosen as eigenfunctions of the timelike Killing vector and they are uniquely divided into positive and negative frequency modes according to the sign of the eigenvalues. The positive frequency modes are then associated to annihilation operators $\hat{a}_{\mathbf{k}}$ and the negative frequency modes to creation operators $\hat{a}^\dagger_{\mathbf{k}}$. In curved space there are in general no such symmetries. Therefore there is no unique and time invariant way to define the positive and negative frequency modes. It is possible to separate the solutions into positive and negative frequency modes at a given time event η_0 but the time evolution will mix the modes [43]. The particle concept in curved space is therefore manifestly dependent on the choice of coordinates. Using a given set of creation and annihilation operators, one may define the vacuum state empty of particles at η_0, but this state will in general not be annihilated by the operators associated to the positive frequency modes of an observer at some later time event η because of the mixing of the mode functions. The mixing gives rise to an effective particle production since the observer at η will detect particles in the initially "empty" state. This is the basis for the inflationary particle production and it can be seen as a consequence of the nontrivial time evolution of the background fields in curved space.

Since the effects of curvature in de Sitter space are negligible for subhorizon modes $|k\eta| \gg 1$, the quantization should be identical to the Minkowski space in this limit. This can be achieved by choosing the coefficients $c_{1,2}(k)$ of the mode functions (2.42) such that $c_1(k) \to \exp(i\pi(\nu + 1/2)/2)/\sqrt{2}$ and $c_2(k) \to 0$ as $k \to \infty$. If inflation lasts long enough, all physical results will fortunately be independent of the specific choices satisfying these conditions [3, 24]. In particular, one may choose $c_1(k) = \exp(i\pi(\nu + 1/2)/2)/\sqrt{2}$ and $c_2(k) = 0$, which yields

$$\phi_k(\eta) = \frac{H\sqrt{\pi}}{2} e^{i\frac{\pi}{2}(\nu + \frac{1}{2})} (-\eta)^{3/2} H^{(1)}_\nu(-k\eta) \ . \tag{2.49}$$

It is straightforward to check that this satisfies the inner product condition (2.47). For massless fields, $\nu = 3/2$, the solution (2.49) reads [39]

$$\phi_k(\eta) = \frac{iH}{\sqrt{2k^3}} \left(1 + ik\eta\right) e^{-ik\eta} \ , \tag{2.50}$$

which in Fourier space gives the two point correlator

$$\langle \hat{\phi}_{\mathbf{k}}(\eta) \hat{\phi}_{\mathbf{k}'}(\eta) \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') |\phi_k(\eta)|^2 = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') \frac{H^2}{2k^3} (1 + k^2\eta^2) \ . \tag{2.51}$$

The brackets here denote the expectation value in the vacuum state annihilated by the operators $\hat{a}_{\mathbf{k}}$ associated to the mode functions (2.50).

On superhorizon scales $|k\eta| \ll 1$, the massless two-point function (2.51) freezes to a constant value [39]

$$\langle \hat{\phi}_{\mathbf{k}}(\eta)\hat{\phi}_{\mathbf{k}'}(\eta)\rangle \simeq (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\frac{H^2}{2k^3} \ . \tag{2.52}$$

For massless fields, the correlator $[\phi, \dot{\phi}]$ also vanishes exponentially fast in physical time t outside the horizon, which implies that quanta of the field ϕ become essentially classical stochastical quantities [3, 24]. From the viewpoint of the Minkowski vacuum, the quantum fluctuations of massless test scalar fields in de Sitter space thus appear as classical perturbations outside the horizon, with the statistical properties given by equation (2.52). For light fields, $m \ll H$, the results remain qualitatively the same but the time evolution outside the horizon is stronger [39, 24]. Note that this is not directly an example of particle production in curved space since the vacuum state defined by equation (2.50) remains invariant under the symmetries of de Sitter space. Being maximally symmetric, the de Sitter space admits a timelike Killing vector and the time-evolution thus does not lead to creation of particles with this choice of vacuum. A comoving observer will however detect a spectrum of particles corresponding to a thermal distribution with the temperature given by the Hubble parameter [39]; this is a manifestation of the curvature of the spacetime. The superhorizon limit of the two-point function (2.52) in turn is an unobservable quantity in pure de Sitter space but it will become observable if the early universe has been only approximatively in a de Sitter stage. This is the case in the real universe since the inflation has ended at some stage. From this point of view, equation (2.52) can be interpreted as the particle production arising from the period of the inflationary expansion of the spacetime.

2.3.2 The curvature perturbation

So far we have discussed the quantum fluctuations of generic light scalar fields in a fixed de Sitter background. In discussing inflationary physics, one also needs to take into account the coupling of the scalar field fluctuations to gravity since the scalar fields dominate the energy density of the universe. Moreover, it is technically convenient to express the perturbations generated during inflation in terms of perturbations of the metric. The inflaton or inflatons eventually need to decay into ordinary matter but the metric perturbations remain tractable all the way from the end of inflation to the formation of the CMB and thus provide a direct probe of the inflationary physics [1, 2, 3].

Slight deviations from the FRW background can be examined by considering small perturbations around the homogeneous and isotropic solution [24, 44, 45]. A technical complication in cosmological perturbation theory is that the background solution and the perturbed solution describe two different spacetimes and there is no unique way to compare them. The ambiguity in the choice of the mapping gives rise to non-physical gauge degrees of freedom. Without going into details, we quote here some results of perturbation theory.

As a mathematical construction, the cosmological perturbation theory can be conveniently formulated (see e.g. [46]) by considering a 5-dimensional manifold $\mathcal{N} = \mathcal{M}\times\mathbb{R}$. Each submanifold $\mathcal{M}_\lambda$, together with the tensor fields T_λ living on it, describes a spacetime model which interpolates between an ideal FRW background, at $\lambda = 0$, and the real inhomogeneous universe, at $\lambda = 1$. By defining a vector field X^A orthogonal to the slices $\mathcal{M}_\lambda$, one can construct a flow $X_\lambda : \mathcal{M}_0 \to \mathcal{M}_\lambda$ which defines a mapping between the background and the perturbed universe. The flow in turn defines a pull-back map, $X^*_\lambda : T^*_{X_\lambda(p)}\mathcal{M}_\lambda \to T^*_p\mathcal{M}_0$, between the cotangent spaces and it allows one to compare tensor fields on $\mathcal{M}_\lambda$ to tensor fields on $\mathcal{M}_0$. The perturbation of any tensor field T can be defined simply as the difference between the pull-back field and the background value,

$$(\Delta_X T)_\lambda \equiv X^*_\lambda\, T|_0 - T|_0 = \sum_{n=1}^{\infty} \frac{\lambda^n}{n!}\, \mathcal{L}^n_X T|_0 \ . \tag{2.53}$$

Here in the last step the pull-back $X^*_\lambda T$ has been written as a Taylor expansion in the parameter λ which acts as a coordinate along the direction of the flow. The derivatives $\mathcal{L}_X$ are Lie derivatives with respect to the vector field X^A. All the quantities in (2.53) are evaluated in the background space $\mathcal{M}_0$. The n-th order term in the series (2.53) is called the n-th order perturbation of T in the gauge defined by the flow of X^A and we denote it by

$$\delta^n_X T \equiv \mathcal{L}^n_X\, T|_0 \ . \tag{2.54}$$

Using some other vector field Y^A to define the flow $\mathcal{M}_0 \to \mathcal{M}_\lambda$, one obtains a different mapping between the background and the perturbed space and in general the perturbations in the two different gauges Y^A and X^A do not coincide, $(\Delta_X T)_\lambda \neq (\Delta_Y T)_\lambda$. The change of first order perturbations between the two gauges can be expressed as

$$\delta_Y T - \delta_X T = \mathcal{L}_\xi T_0 \ , \tag{2.55}$$

where $\xi^A = Y^A - X^A$ is a vector field tangential to the slices $\mathcal{M}_\lambda$ and T_0 is the background value of the tensor field T. This expression gives the gauge transformation rules for all the first order perturbations. The perturbations can be divided into scalar vector and tensor modes [24] according to their transformations properties under coordinate transformations in the background space $\mathcal{M}_0$. At first order the different modes evolve independently of each other.

The metric describing arbitrary first order scalar perturbations around a spatially flat FRW background can be expressed in component form as [2, 24]

$$g_{00} = -a^2(\eta)(1 + 2A(\eta, \mathbf{x})) \tag{2.56}$$

$$g_{0i} = a^2(\eta)\partial_i B(\eta, \mathbf{x}) \tag{2.57}$$

$$g_{ij} = a^2(\eta)((1 - 2\psi(\eta, \mathbf{x}))\delta_{ij} + 2\partial_i\partial_j E(\eta, \mathbf{x})) \ , \tag{2.58}$$

where $a(\eta)$ is the background scale factor in conformal time and the functions A, B, ψ and E describe first order perturbations. Here we concentrate on perturbations of the spatial metric g_{ij} which will play an important role in discussing inflationary cosmology. The quantity ψ in equation (2.58) is called curvature perturbation since it determines the perturbations of the scalar curvature ${}^{(3)}R$ on constant time hypersurfaces,

$${}^{(3)}R = \frac{4}{a^2}\nabla^2\psi \ . \tag{2.59}$$

The function E does not appear here since it can be removed from the metric by a suitable spatial gauge transformation whereas ${}^{(3)}R$ remains invariant under such transformations [2, 24]. From equation (2.55) it is readily seen that under temporal gauge transformations[3], $\xi^\mu = (-\delta\eta, 0) = (-\delta t/a, 0)$, the curvature perturbation transforms as

$$\tilde{\psi} - \psi = \mathcal{H}\delta\eta \qquad (= H\delta t) \ , \tag{2.60}$$

[3]The sign of the temporal shift $\delta\eta$ is chosen such that the gauge-transformation generated by ξ^μ can be seen as a transformation of coordinates while the point identification on the manifold is kept fixed. This is the so called passive point of view of gauge transformations [46].

where, instead of the subscript Y used in (2.55), the new gauge has been denoted by a tilde. A convenient quantity characterizing the metric perturbations is the curvature perturbation evaluated in uniform energy density gauge $\zeta \equiv -\psi|_{\delta\rho=0}$. The perturbations in energy density transform according to (2.55) as $\widetilde{\delta\rho} - \delta\rho = -\rho_0'\delta\eta$ and the transformation from an arbitrary gauge, say $\delta\rho$, into a uniform energy density gauge, $\widetilde{\delta\rho} = 0$, thus corresponds to a time shift $\delta\eta = \delta\rho/\rho_0'$. According to equation (2.60) the curvature perturbation in uniform energy density gauge can thus be written as [2, 24, 44, 47]

$$-\zeta \equiv \psi|_{\delta\rho=0} = \psi + \frac{\mathcal{H}}{\rho_0'}\delta\rho \qquad \left(= \psi + \frac{H}{\dot{\rho}_0}\delta\rho \right) \ , \tag{2.61}$$

where the perturbations on the right hand side of the equality sign are given in arbitrary gauge. This expression is by construction manifestly invariant under temporal gauge transformations.

On large scales $|k\eta| \ll 1$, where spatial gradients can be neglected, the curvature perturbation obeys a simple evolution equation [2, 24, 44, 47]

$$\zeta' = -\frac{\mathcal{H}}{\rho_0 + p_0}\delta p_{\rm nad} \qquad \left(\dot{\zeta} = -\frac{H}{\rho_0 + p_0}\delta p_{\rm nad} \right) \ , \tag{2.62}$$

where $\delta p_{\rm nad}$ denotes non-adiabatic pressure perturbations. For adiabatic perturbations, the curvature perturbation ζ thus remains constant outside the horizon. This makes ζ a useful quantity in discussing inflationary perturbations. After the horizon crossing the evolution of a given mode in single field models is essentially adiabatic due to the slow roll approximation, $\rho \simeq V \simeq -p$, and ζ remains constant. The particle production at the horizon crossing in turn can be seen as a non-adiabatic process during which the curvature perturbation ζ is created; we will consider this more carefully below. The conservation of ζ is of paramount importance since it allows one to establish a connection between the perturbations created during inflation and the perturbations entering the horizon at the time of photon decoupling and the formation of the CMB sky, without having a detailed knowledge of the history of the universe between these two epochs. Moreover, since ζ describes perturbations of the metric, it is straightforward to relate it to the temperature anisotropies observed in the CMB photons [1, 2, 24].

The conservation equation (2.62) for ζ can be derived without any reference to Einstein equations by simply using the continuity equation (2.7) for ideal fluid [47]. This can easily be understood intuitively since on large scales the curvature perturbation (2.61) describes the spatial variation of the volume expansion of the fluid on constant energy density hypersurfaces. If the fluid is in thermal equilibrium, the variation needs to remain unchanged in time since the specific volume is a unique function of the energy density. Consequently, time dependence of ζ implies deviations from thermal equilibrium and is thus sourced by non adiabatic perturbations, which is the physical contents in equation (2.62).

Another gauge invariant quantity commonly used in discussing the inflationary perturbations is the comoving curvature perturbation $\mathcal{R}$ [2, 24, 44, 47]. For a universe filled with a single scalar field ϕ, the gauge invariant expression for $\mathcal{R}$ reads

$$\mathcal{R} \equiv \psi|_{\delta\phi=0} = \psi + \frac{\mathcal{H}}{\phi_0'}\delta\phi \qquad \left(= \psi + \frac{H}{\dot{\phi}_0}\delta\phi \right) \ . \tag{2.63}$$

On large scales the quantities ζ and $\mathcal{R}$ coincide[4] $\zeta \simeq -\mathcal{R}$ since they differ only by gradients [2, 24, 44, 47]. For adiabatic fluids, the correspondence is exact $\zeta = -\mathcal{R}$ and holds on all scales. Thus $\mathcal{R}$ is conserved under the same conditions as ζ and outside the horizon one can trivially switch between them.

[4] Note that different sign conventions are also regularly used in the literature.

2.3.3 Generating the curvature perturbation during inflation

In this section we discuss the generation of the curvature perturbation ζ via quantum fluctuations of the inflaton field in single field models. Instead of ζ, it is technically more convenient to quantize the comoving curvature perturbation $\mathcal{R}$ since it is directly related to the perturbations of the inflaton field. The expression for ζ outside the horizon is obtained by setting $\mathcal{R} \simeq -\zeta$ at the end.

By expanding the action (2.2) around a background solution satisfying the slow roll equations of motion (2.31) and (2.32), the quadratic part of the action for $\mathcal{R}$ becomes [24]

$$\delta_2 S_{\mathcal{R}} = \frac{1}{2}\int \mathrm{d}\eta \mathrm{d}^3x \, \frac{a^2 \phi_0'^2}{\mathcal{H}^2}\left(\mathcal{R}'^2 - (\nabla\mathcal{R})^2\right) = \frac{1}{2}\int \mathrm{d}\eta \mathrm{d}^3x \, a^2 (2M_{\mathrm{P}}^2 \epsilon)\left(\mathcal{R}'^2 - (\nabla\mathcal{R})^2\right) \ , \qquad (2.64)$$

where several total derivatives have been dropped out. In exact de Sitter space, $\epsilon = 0$, the quadratic action vanishes and $\mathcal{R}$ is simply a non-propagating gauge mode. Since inflation needs to end at some point however, the real spacetime can not be exactly de Sitter, $\epsilon \neq 0$, and consequently the curvature perturbation becomes a physical propagating field describing the metric fluctuations around an FRW background.

When the slow roll parameters are small, the overall coefficient ϵ in the action (2.64) can be regarded as a constant during the horizon crossing of a given mode $k\eta \sim -1$. We denote this k - dependent constant by $\epsilon|_{k\eta=-1} \equiv \epsilon_*$. By rescaling $\mathcal{R}$ in (2.64) as $\mathcal{R} \to (\sqrt{2\epsilon_*}M_{\mathrm{P}})^{-1}\mathcal{R}$, the action (2.64) becomes the same as the action for a massless scalar field in de Sitter space (2.39). Therefore we can immediately read the expression for the two-point function of the curvature perturbation at the horizon crossing from equation (2.52) [24]

$$\langle \widehat{\mathcal{R}}_{\mathbf{k}}(\eta)\widehat{\mathcal{R}}_{\mathbf{k}'}(\eta)\rangle \simeq (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\frac{1}{2k^3}\frac{H_*^2}{2\epsilon_* M_{\mathrm{P}}^2} = (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\frac{H_*^2}{2k^3}\left(\frac{H_*}{\dot{\phi}_*}\right)^2 \ , \qquad (2.65)$$

where quantities with the star are evaluated at the horizon crossing $k\eta = -1$. Since the curvature perturbation is conserved outside the horizon, equation (2.65) is valid not only for $k\eta \simeq -1$ but also for scales outside the horizon $k\eta < -1$. The infrared limit of the two point function for the curvature perturbation on uniform energy density slices ζ is also given by (2.65)

$$\langle \widehat{\zeta}_{\mathbf{k}}(\eta)\widehat{\zeta}_{\mathbf{k}'}(\eta)\rangle \simeq (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\frac{1}{2k^3}\frac{H_*^2}{2\epsilon_* M_{\mathrm{P}}^2} = (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\frac{H_*^2}{2k^3}\left(\frac{H_*}{\dot{\phi}_*}\right)^2 \ , \qquad (2.66)$$

since on superhorizon scales one has the correspondence $\zeta \simeq -\mathcal{R}$. As for the massless fields in de Sitter, the commutator $[\zeta, \dot{\zeta}]$ vanishes exponentially fast in physical time outside the horizon. Single field slow roll inflation thus produces effectively classical curvature perturbations on superhorizon scales.

2.3.4 Gravitational waves

In addition to scalar perturbations, also tensor perturbations are generated during inflation. The tensor modes are called gravitational waves γ_{ij} and the quadratic part of their action becomes [24]

$$\delta_2 S_\gamma = \frac{M_{\mathrm{P}}^2}{8}\int \mathrm{d}\eta \mathrm{d}^3x \, a^2 \sum_{ij}\left(\gamma_{ij}'^2 - (\nabla\gamma_{ij})^2\right) \ . \qquad (2.67)$$

The gravitational waves are massless spin-2 fields and γ_{ij} can be expanded in plane waves with definite polarizations

$$\gamma_{ij}(\eta, \mathbf{x}) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \sum_{s=\pm} \epsilon^s_{ij}(k) \gamma^s_{\mathbf{k}}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} \ . \tag{2.68}$$

The physical modes are transverse and traceless, $\sum_i k_i \epsilon_{ij} = \sum_i e_{ii} = 0$, and normalized as $\sum_{ij} \epsilon^s_{ij} \epsilon^{s'}_{ij} = 2\delta_{ss'}$. The action for both of the physical polarization states is of the same form as for massless scalar fields in de Sitter space and the superhorizon limit of the two point function is again directly obtained from (2.51)

$$\langle \widehat{\gamma}^s_{\mathbf{k}}(\eta) \widehat{\gamma}^{s'}_{\mathbf{k}'}(\eta) \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') \frac{1}{k^3} \frac{H_*^2}{M_{\mathrm{P}}^2} \delta_{ss'} \ . \tag{2.69}$$

In analogue to the scalar perturbations, also gravitational waves can be regarded as classical fields outside the horizon.

2.3.5 The ΔN formalism

The CMB observations suggest that the amplitude of primordial perturbations generated in inflation is extremely small $\zeta \sim 10^{-5}$. First order perturbation theory is therefore more than an excellent approximation for most purposes. However, when discussing for example non-gaussian features of perturbations, one needs to go beyond the first order calculations. While it is in principle straightforward to consider higher orders in perturbation theory [46], the approach becomes technically complicated already at second order (see for example [21, 48]).

Instead of using the standard cosmological perturbation theory, the evolution of inhomogeneities and anisotropies can also be examined using non-perturbative approaches. One possibility is to construct a covariant and nonlinear generalization of the perturbation theory [49, 50, 51]. This approach is formulated entirely in terms of tensorial quantities in the real inhomogeneous universe and is formally free from gauge subtleties. Another extremely useful and widely used approach is the so called ΔN formalism, or the separate universe approach, which is based on gradient expansion [42, 52, 53, 54, 55, 56]. Since the spatial gradients are small on superhorizon scales, this is a well motivated procedure in discussing the infrared behaviour of perturbations. It leads to non-perturbative expressions in the sense that they capture all the orders of the standard perturbation theory.

Concentrating only on scalar perturbations, the spatial part of the metric can be written non-perturbatively as [42, 56]

$$g_{ij} = a^2(t) e^{-2\psi(t,\mathbf{x})} \delta_{ij} \ . \tag{2.70}$$

In first order gradient expansion, the evolution equations for each spatial point are the same as in a homogeneous FRW universe [42]. The local value of the scale factor at a given point is $a(t)e^{-\psi(t,\mathbf{x})}$ and this may be used to define the local Hubble parameter. Because the field equations are point by point the same as in FRW, the integrated expansion along a worldline of a comoving observer from an initial time t' to a final time t reads

$$N(t, t'; \mathbf{x}) = \ln \frac{a(t) e^{-\psi(t,\mathbf{x})}}{a(t') e^{-\psi(t',\mathbf{x})}} = \psi(t', \mathbf{x}) - \psi(t, \mathbf{x}) + \ln \frac{a(t)}{a(t')} \ . \tag{2.71}$$

By choosing the time coordinates such that the initial hypersurface is spatially flat, $\psi(t', \mathbf{x}) = 0$, and the final slice has uniform energy density $\rho = \rho(t)$, equation (2.71) yields [42, 56]

$$\zeta(t, \mathbf{x}) \equiv -\left.\psi(t, \mathbf{x})\right|_{\rho=\rho(t)} = N(t, t'; \mathbf{x}) - \ln \frac{a(t)}{a(t')} \equiv \Delta N(t, t'; \mathbf{x}) \ , \tag{2.72}$$

where ΔN measures the shift in the integrated expansion between the flat and uniform density hypersurfaces. For a universe filled with an ideal fluid with a fixed equation of state $p = p(\rho)$, equation (2.72) may be written as

$$\zeta(t, \mathbf{x}) = \frac{1}{3} \int_{\rho(t')}^{\rho(t', \mathbf{x})} \frac{\mathrm{d}\rho}{\rho + p} \ , \tag{2.73}$$

where $\rho(t')$ is the background value of the energy density and $\rho(t', \mathbf{x})$ is the perturbed value on the spatially flat slices at the initial time t'. The time $t > t'$ does not appear explicitly in (2.73) and ζ is therefore conserved for adiabatic perturbations to the precision of first order gradient expansion [42, 56]. On large scales, equation (2.72) can be regarded as a non-linear generalization of the first order curvature perturbation on uniform energy density hypersurfaces. The first order expression (2.61) is recovered by linearizing (2.72). An analogous conserved quantity can also be derived using the covariant approach [50, 51].

In single field inflation the number of e-foldings is within the slow roll approximation completely determined by the inflaton field, $N = N(\phi)$. Using equation (2.73), the curvature perturbation can therefore be written as [56, 57]

$$\zeta(t, \mathbf{x}) = \int_{\phi+\delta\phi}^{\phi} \frac{H}{\dot{\phi}} \mathrm{d}\phi = N'(\phi)\delta\phi(\mathbf{x}) + \frac{1}{2} N''(\phi)\delta\phi(\mathbf{x})^2 + \dots \ , \tag{2.74}$$

where the derivatives are taken with respect to the lower limit of the integral and $\delta\phi(\mathbf{x})$ stands for the field perturbations on spatially flat hypersurfaces. The background value of the field is denoted by ϕ. Since ζ remains constant for adiabatic perturbations, equation (2.74) can be evaluated at the time t_* when the smallest mode of interest exits the horizon during inflation. This gives an adequate approximation to the value of $\zeta(t > t_*)$, although the modes still inside the horizon at t_* would cause small time evolution when properly taken into account. For practical purposes, one may thus evaluate (2.74) at the time when the observable universe exits the horizon and neglect the subsequent time evolution during single field inflation. To leading order, the field perturbations $\delta\phi(\mathbf{x})$ in equation (2.74) are given by equation (2.51).

2.4 Testing inflationary models

The primordial perturbations generated during inflation act as a source for the temperature fluctuations seen in the CMB [1, 2, 21]. The CMB observations therefore provide precise information about the primordial perturbations and place rather strong constraints on the inflationary models. Here we discuss some of the observational constraints and their implications on the inflationary cosmology.

2.4.1 Amplitude and spectrum of primordial perturbations

In the previous chapter, we derived the expression (2.66) for the two point correlator of the curvature perturbation ζ produced in single field inflation. This expectation value can be interpreted as a statistical ensemble average since the perturbations become essentially classical stochastic quantities outside the horizon. The ensemble average is not directly observable since we can only measure fluctuations in a given realization of the ensemble; i.e. in our universe. However, for random fields, the spatial averages and the ensemble averages approach each other given enough measurements. This ergodic property makes it possible to establish a connection between theoretical predictions for the ensemble averages and the observed spatial averages. In the following we will interpret the brackets $\langle\rangle$ to represent ensemble averages in this sense.

Assuming statistical homogeneity and isotropy, the two point correlator of the curvature perturbation can be written as [1, 2]

$$\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') \frac{2\pi^2}{k^3} \mathcal{P}_\zeta(k) \ , \tag{2.75}$$

where $\mathcal{P}_\zeta(k)$ is called the spectrum[5]. The scale dependence of the spectrum is characterized by the spectral index n_s

$$n_s(k) - 1 \equiv \frac{\mathrm{dln}\mathcal{P}_\zeta}{\mathrm{dln}k} \ . \tag{2.76}$$

In single field inflation, the prediction for the spectrum of the curvature perturbation is according to equations (2.66) and (2.75) given by

$$\mathcal{P}_\zeta(k) \simeq \left(\frac{H_*}{2\pi}\right)^2 \left(\frac{H_*}{\dot{\phi}_*}\right)^2 \simeq \frac{1}{24\pi^2 M_{\mathrm{P}}^4} \frac{V_*}{\epsilon_*} \ , \tag{2.77}$$

where we have used the slow roll equations (2.31) and (2.32) in the last step. As before, the star denotes evaluation at the horizon crossing $k = aH = -\eta^{-1}$. To first order in the slow roll parameters, the spectral index (2.76) reads

$$n_s(k) - 1 = 2\eta_* - 6\epsilon_* \ . \tag{2.78}$$

The scale dependence of the spectral index, $\mathrm{d}n_s/\mathrm{dln}k$, is called running and it is of second order in slow roll parameters [58].

The spectrum and spectral index for gravitational waves can be defined in an analogous manner. Using equation (2.69) one obtains the spectrum

$$\mathcal{P}_\gamma(k) = \frac{2}{M_{\mathrm{P}}^2} \left(\frac{H_*}{2\pi}\right)^2 \tag{2.79}$$

and the spectral index is commonly defined as

$$n_\gamma(k) \equiv \frac{\mathrm{dln}\mathcal{P}_\gamma}{\mathrm{dln}k} = -2\epsilon_* \ . \tag{2.80}$$

Both the scalar and tensor perturbations, i.e. the curvature perturbation and the gravitational waves, contribute to the CMB anisotropies. In single field inflation the amplitudes of gravitational waves and the curvature perturbations are related by [2, 59]

$$r = \frac{4\mathcal{P}_\gamma}{\mathcal{P}_\zeta} = 16\epsilon \ . \tag{2.81}$$

The relation arises from equations (2.77) and (2.79). Using equation (2.80), it can also be expressed as a consistency relation for the tilt of the tensor spectrum

$$r = -8n_\gamma \ . \tag{2.82}$$

[5] Another commonly used definition for the spectrum is $P(k) = \frac{2\pi^2}{k^3}\mathcal{P}(k)$. The prefactor $(2\pi)^3$ in (2.75) corresponds to the normalization of the Fourier transformation (1.1) used in this thesis. Other normalizations are also frequently used in the literature.

Note that all the results presented here hold only to the precision of first order perturbation theory and they are expected to be slightly modified by both higher order corrections in classical perturbation theory and also by loop corrections in quantum field theory, see e.g. [60].

The amplitude of the primordial curvature perturbations at the horizon crossing of the observable universe is [4]

$$\mathcal{P}_{\zeta}^{1/2} \simeq 4.8 \times 10^{-5} \ . \tag{2.83}$$

Assuming negligible running of the scalar spectral index, the amplitude of gravitational waves is constrained by $r \lesssim 0.02$ [4]. In this case the best fit value for the spectral index is $n_s \simeq 0.96$ [4]. The bounds on gravitational waves and the spectral index are relaxed if the running is significant. An important feature in the recent WMAP data sets is that they strongly disfavour featureless primordial perturbations with exactly scale invariant spectrum. Although this is not a smoking gun evidence for the inflationary scenario, it implies that the primordial perturbations are generated in a non-trivial physical process. Plain vacuum energy domination which would yield the featureless spectrum is still not conclusively ruled out but the observations clearly favour models with more complicated dynamics.

2.4.2 Constraints on inflationary models

The amplitude of the CMB temperature fluctuations provides an important constraint on the inflationary energy scale in single field models. According to equations (2.77) and (2.83), the inflationary scale at the horizon crossing of the observable universe is given by

$$\left(\frac{V}{\epsilon}\right)^{1/4} \sim 0.03 M_{\rm P} \ , \tag{2.84}$$

or, expressed in terms of the Hubble parameter,

$$\frac{H}{\sqrt{\epsilon}} \sim 4 \times 10^{-4} M_{\rm P} \ . \tag{2.85}$$

The energy scale $V^{1/4}$ and the Hubble parameter H in single field inflation therefore necessarily need to be few orders of magnitude below the Planck scale since $\epsilon \lesssim 1$. To avoid excessive production of gravitational waves, one further needs to require $\epsilon \ll 1$. On purely phenomenologically grounds it appears that models with extremely small slow roll parameter values $\epsilon \lll 1$ imply fine-tuning. The "natural" scale of the single field inflation would thus not be much lower than $H \sim 10^{-5} M_{\rm P}$. This is presumably way beyond the scales covered by the Standard Model or its (supersymmetric) extensions and makes it difficult to construct theoretically motivated models for inflation.

Single field models can roughly be divided into two classes according to the form of the inflaton potential [2, 3]. Models with monomial potentials $V \propto \phi^n$ are called chaotic inflation and they require field values much larger than the Planck mass $\phi \gg M_{\rm P}$ to satisfy the slow roll conditions (2.35). Sub-Planckian vevs can be achieved in models with polynomial potentials. In chaotic models the initial state is highly inhomogeneous, chaotic, since the effects of quantum gravity play an essential role above the Planck scale. Without knowing the details of the high energy physics, it can be assumed that large enough regions in the chaotic state eventually become dominated by some scalar field and start to inflate. The advantage of the scenario is that it provides an explanation for the initial conditions [3]. However, our ignorance of the high energy physics makes it difficult to establish a connection between the fields driving the inflation and the Standard Model fields observed in the present universe.

Finding theoretically motivated models of inflation is easier at sub-Planckian scales since non-renormalizable extensions of the Standard Model can be trusted at least to some extent. Lowering the inflationary energy scale however implies tighter constraints on the flatness of the potential, as can be seen from (2.84), and a priori implies more fine tuning [2, 5]. In this thesis, we discuss a scenario [7, 8] where inflation is realized at an exceptionally low scale $H \sim 10^{-18} M_{\rm P}$ and is driven by a flat direction of the Minimally Supersymmetric Standard Model. The great virtue of the model is that the gauge couplings between the inflaton and the Standard Model fields are completely determined and in principle measurable in future collider experiments. The inflationary period and the generation of the Standard Model fields after the inflation are explained in a self-consistent manner by the physics of the flat directions. The apparent drawback of the MSSM inflation is that the inflaton potential is generically not flat enough but needs to be fine-tuned to produce the observed amplitude for perturbations [8] with the exceptionally low inflationary scale. However, the results obtained in the enclosed papers [10, 11] suggest that it is possible to construct reasonable supergravity models in which the flat inflaton potential arises automatically when supersymmetry is broken. This significantly alleviates the fine-tuning problems although some more work is needed to make the analysis of [10, 11] complete. We discuss these issues more closely in Chapter 3 below.

Another possibility to realize inflation at low scales is to consider models with several fields. Hybrid inflation [6] is a well studied example with two inflaton fields having sub-Planckian vevs. It tends to yield too large spectral index, though, and is therefore disfavoured by observations, unless the running of the spectral index is significant. An attractive class of inflationary models with several fields is comprised by the curvaton scenario [12, 13] which is the subject of Chapter 4. In this scenario, the curvature perturbations are mainly generated after the inflation during oscillations of a light late decaying scalar field, the curvaton, that has acquired a spectrum of perturbations during inflation. The curvaton does not need to have anything to do with the field or fields driving inflation which considerably deliberates the constraints on the inflationary scale [14]. The curvaton scenario typically predicts a significant level of non-gaussian perturbations and, as all the low scale models, a negligible amplitude for gravitational waves [16, 17]. The non-gaussian effects are however highly dependent on the form of the curvaton potential as discussed in the enclosed research paper [18].

2.4.3 Non-gaussian perturbations

Present observations are consistent with gaussian primordial perturbations [4]. However, even if the inflaton field would have no non-gravitational interactions at all, the non-linearities in the coupling to gravity necessarily give rise to non-gaussian features in perturbations, although these generically are very small. Non-gaussian effects directly probe non-trivial interactions and can therefore act as an efficient discriminator between different models of inflation. The possibility to detect non-gaussianities in the CMB anisotropies in future experiments, like the Planck satellite, has stimulated a large interest on the subject.

The statistical properties of the curvature perturbation ζ are determined by its correlators

$$\langle \zeta_{\mathbf{k_1}} \dots \zeta_{\mathbf{k_n}} \rangle = (2\pi)^3 \delta(\mathbf{k_1} + \ \dots \ + \mathbf{k_n}) P_n(\mathbf{k_1}, \ \dots \ , \mathbf{k_n}) \ , \tag{2.86}$$

where P_n is called the $n-1$ spectrum. For gaussian perturbations, the connected parts of the n-point functions vanish for $n > 2$. A non-vanishing connected three point function is thus the lowest order signal of non-gaussian statistics. Its spectrum, the bispectrum, is denoted by $B(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3}) \equiv P_3(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3})$. The magnitude of the bispectrum is commonly characterized by the non-linearity parameter $f_{\rm NL}$ [20, 21, 22, 57]

$$B(\mathbf{k_1},\mathbf{k_2},\mathbf{k_3}) = \frac{6}{5} f_{\rm NL}(\mathbf{k_1},\mathbf{k_2},\mathbf{k_3}) \left(\frac{4\pi^4}{k_1^3 k_2^3} \mathcal{P}(k_1)\mathcal{P}(k_2) + {\rm cycl.} \right) \ . \tag{2.87}$$

In general, the non-linearity parameter can have a non-trivial scale dependence but here we consider only the scale independent part of $f_{\rm NL}$, which is also called the local part. According to the definition (2.87), the constancy of $f_{\rm NL}$ implies that the curvature perturbation in coordinate space takes the form

$$\zeta(t,\mathbf{x}) = \zeta_g(t,\mathbf{x}) + \frac{3}{5} f_{\rm NL}\ \zeta_g(t,\mathbf{x})^2 + \dots \ , \tag{2.88}$$

where ζ_g is gaussian with the spectrum given by $\mathcal{P}(k)$. The current limits for the local contribution to $f_{\rm NL}$ based on the WMAP 5 year data are

$$-9 < f_{\rm NL} < 111 \tag{2.89}$$

at 95% confidence level [4]. The bounds on the negative value have become considerably tighter than in the earlier data sets and the current observations seem to favour a positive value for the $f_{\rm NL}$.

Obtaining a reliable theoretical estimate for the non-linearity parameter $f_{\rm NL}$ requires going beyond the first order perturbation theory. The ΔN approach [52, 53, 54, 55, 56, 57] described above provides a simple way to estimate the magnitude of non-gaussian effects. For single field inflation, the curvature perturbation can be written according to equation (2.74) as

$$\begin{aligned} \zeta(t,\mathbf{x}) &= N'(\phi)\delta\phi(\mathbf{x}) + \frac{N''(\phi)}{2N'(\phi)^2}\ (N'(\phi)\delta\phi(\mathbf{x}))^2 + \dots \\ &\equiv \zeta_g(\mathbf{x}) + \frac{N''(\phi)}{2N'(\phi)^2}\ \zeta_g(\mathbf{x})^2 + \dots \ . \end{aligned} \tag{2.90}$$

Assuming the field perturbations $\delta\phi$ to be gaussian, equations (2.88) and (2.90) yield [57]

$$f_{\rm NL} = \frac{5}{6}\frac{N''(\phi)}{N'(\phi)^2} + \dots \ , \tag{2.91}$$

where the ellipses denote contributions from higher order terms in (2.90). Using equation (2.91) and the slow-roll equations of motion (2.31), (2.32), the non-linearity parameter becomes $f_{\rm NL} \simeq -5(2\eta - 4\epsilon)/12$. This is of the same order of magnitude as the correct result for the local part of $f_{\rm NL}$,

$$f_{\rm NL} \simeq -\frac{5}{12}(2\eta - 6\epsilon) \ , \tag{2.92}$$

calculated by Maldacena [22] using quantum field theory at tree level (see also [23]) but the results do not exactly coincide because the non-gaussian parts of the field perturbations in (2.90) have been neglected. This yields an error proportional to the slow roll parameters. In this thesis we will apply the ΔN formalism only to the curvaton model where the non-gaussianities generated after the horizon exit are hierarchically larger than the non-gaussianities at the horizon crossing. For our purposes, it therefore suffices to treat the fields perturbations $\delta\phi$ in (2.90) as gaussian variables at the time of horizon crossing.

It is expected that the Planck satellite will either detect non-gassianity or reduce the bound to $|f_{\rm NL}| \lesssim 5$ [20, 21] and that foreseeable future observations can reach a level $|f_{NL}| \lesssim 3$ [20, 21]. The level of non-gaussianity produced in single field slow roll inflation, $|f_{\rm NL}| \ll 1$, is therefore too low to be observed and the simplest single field models would be ruled out by a detection of primordial non-gaussianities. More complicated single field constructions with for example non-canonical kinetic terms [61] or sharp features in the potential [62] are however capable of producing observable non-gaussian effects. Inflationary models with multiple scalar fields can also give rise to large non-gaussianities. In hybrid models, significant non-gaussianities can be generated in preheating after the inflation [63]. Preheating is a possible source for non-gaussian perturbations also in other models where light scalar fields are present at the end of inflation [64]. The curvaton scenario is yet another example of multiple field models producing detectable non-gaussianities [17, 18] and it will be discussed more closely in Chapter 4.

Chapter 3

Supersymmetry and MSSM inflation

Supersymmetry is an extension of the Poincaré spacetime symmetries, see e.g. [65, 66, 67]. The generators of supersymmetry, $Q_\alpha, \bar{Q}_{\dot{\alpha}}$, are anticommuting operators transforming as Weyl spinors under Lorentz transformations. The supersymmetry algebra is given by $\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^\mu{}_{\alpha\dot{\beta}} P_\mu$, where P_μ is the generator of spacetime translations. Being fermionic operators, the generators of supersymmetry transform fermionic states into bosonic states and vice versa. Each fermionic field in a supersymmetric theory is therefore accompanied by a bosonic superpartner.

Supersymmetry is motivated by both phenomenological and theoretical arguments [65, 66, 67]. It removes the problematic quadratic divergences in the Higgs mass that otherwise would occur in the Standard Model and makes unification of gauge couplings easier. Supersymmetry also appears to be the only possible physically compelling extension of the Poincaré group. If supersymmetry is realized in nature, it almost certainly plays a crucial role in the early universe. Supersymmetric theories are particularly interesting from the point of view of inflationary models since they naturally contain a large number of fundamental scalar fields.

In the space of scalar fields, there exist flat directions along which the potential vanishes if the supersymmetry is not broken [68, 69, 70]. The flat directions can have important cosmological consequences, for a review see [71]. They can seed the baryogenesis [72, 73], generate supersymmetric dark matter [74] or act as a source for isocurvature density perturbations [75]. In addition, they provide natural candidates for the curvaton field [15]. Flat directions as inflaton candidates have also been discussed in the literature, see e.g. [5]. In this Chapter we concentrate on the single field inflationary model presented in [7, 8] where the inflaton is a flat direction of the MSSM and the inflationary scale is very low. We start by reviewing some basic properties of supersymmetric models and flat directions.

3.1 Global supersymmetry

Supersymmetry can be realized either as a global or local symmetry. We begin by discussing properties of globally supersymmetric theories.

A chiral superfield [65, 66, 67] is an irreducible representation of the supersymmetry algebra that contains a chiral fermion ψ, a complex scalar field ϕ and an auxiliary scalar F. The auxiliary field encodes off-shell degrees of freedom alone. Gauge fields belong to vector superfields [65, 66, 67] that in addition to the gauge field contain the supersymmetric partner gaugino and an auxiliary field D. The tree level scalar potential in a globally supersymmetric theory with chiral superfields and gauge symmetries is written as [65, 66, 67]

$$V_{\text{global}}(\phi_M, \phi^*_M) = \sum_M \Big|\frac{\partial W}{\partial \phi_M}\Big|^2 + \frac{g^2}{2}\sum_a \Big(\sum_{M,N} \phi^*_M T^a_{MN}\phi_N\Big)^2 \equiv \sum_M |F_M|^2 + \frac{1}{2}\sum_a D^a D^a \ , \quad (3.1)$$

where ϕ_M are the scalar components of different chiral superfields. The superpotential $W(\phi_M)$ is an analytic function of the scalar fields and it determines the so called F-term

$$F^*_M = -\frac{\partial W}{\partial \phi_M} \equiv -W_M \ . \quad (3.2)$$

The gauge interactions enter in the D-term

$$D^a = -g \sum_{M,N} \phi^*_M T^a_{MN} \phi_N \ , \quad (3.3)$$

where T^a_{MN} are the generators of the gauge group in the representation relevant for the chiral fields and g is the associated gauge-coupling. Equations (3.2) and (3.3) are the equations of motion for the auxiliary fields contained by chiral and vector superfields.

Supersymmetric partners of the Standard Model fields have not been detected experimentally so far. If nature still is described by supersymmetric theories, supersymmetry needs to be broken at the currently observed energy scales. Global supersymmetry is spontaneously broken if either the F-terms or D-terms acquire non-vanishing vacuum expectation values [65, 66, 67]. This makes the vacuum state non-invariant under superymmetry transformations but preserves the Lorentz invariance. The scalar potential (3.1) vanishes in supersymmetry preserving vacua while it is strictly positive in vacua with broken susy. In the low-energy limit, it is also common to consider explicit supersymmetry breaking by adding effective supersymmetry violating terms[1] in the Lagrangian [65, 66, 67]. Such terms arise for example from supergravity with spontaneously broken susy after eliminating the gravitational field, which roughly corresponds to taking the limit $M_{\text{P}} \to \infty$.

3.2 Local supersymmetry

Local supersymmetry is called supergravity since locally supersymmetric theories include the gravitational field as a dynamical degree of freedom. This is because the algebra contains the generator of local spacetime translations P_μ. While globally supersymmetric theories can be renormalizable or non-renormalizable depending on the interactions, supergravity generally is non-renormalizable due to the inclusion of gravity[2].

The supergravity scalar potential is a generalization of the potential in global supersymmetry (3.1) and can be written in the form [65, 66, 67]

$$V(\phi_M, \phi^*_M) = M^4_{\text{P}} e^G (K^{M\bar{N}} G_M G_{\bar{N}} - 3) + \frac{M^4_{\text{P}}}{2}(\text{Re} f_{ab})^{-1} D^a D^b \ , \quad (3.4)$$

The lower indices $_M$ and $_{\bar{M}}$ refer to derivatives with respect to ϕ_M and ϕ^*_M. The indices are raised with $K^{M\bar{N}}$ which is the inverse of the Kähler metric $K_{\bar{N}M}$ and repeated indices are

[1] These terms should be "soft" in order not to reintroduce undesired divergences, like quadratic divergences in the Higgs mass. The allowed terms are either mass terms for scalars and gauginos or trilinear couplings.

[2] The statement of non-renormalizability is mainly based on power counting arguments and it might turn out to be too conservative in some cases. For example, there are studies suggesting that models with several supersymmetry generators could be finite in the ultraviolet limit [76].

summed over. The scalar potential (3.4) is completely determined by three functions; the analytic superpotential $W(\phi_M)$ and the non-analytic Kähler potential $K(\phi_M, \phi_M^*)$ and gauge kinetic function $f_{ab}(\phi_M, \phi_M^*)$. The function $G(\phi_M, \phi_M^*)$ in (3.4) is defined as

$$G(\phi_M, \phi_M^*) = K(\phi_M, \phi_M^*) + \ln\Big(\frac{|W(\phi_M)|^2}{M_{\rm P}^6}\Big) \ , \tag{3.5}$$

and G^M (or more precisely $M_{\rm P} e^{G/2} G^M$) is the local generalization of the F-term (3.2) in global susy,

$$G^M = K^{M\bar{N}} G_{\bar{N}} = K^{M\bar{N}} \left(K_{\bar{N}} + \frac{W_{\bar{N}}^*}{W^*} \right) \ . \tag{3.6}$$

Similarly, the term D^a in (3.4) is the generalization of the global D-term (3.3) and is given by

$$D^a = -g \sum_{M,N} G_M T^a_{MN} \phi_N \ , \tag{3.7}$$

where T^a_{MN} are the generators of the gauge group in the appropriate representation and g is the coupling constant. The kinetic term for the scalar fields is generically non-canonical

$$\mathcal{L}_{\rm kin} = -\frac{M_{\rm P}^2}{2} \sum_{M,\bar{N}} K_{M\bar{N}} g^{\mu\nu} \partial_\mu \phi_M \partial_\nu \phi^*_{\bar{N}} \ , \tag{3.8}$$

since the Kähler metric $K_{M\bar{N}}$ can have a non-trivial form.

In analogy to global susy, nonzero vevs of F-terms $\langle G^M \rangle \neq 0$ or D-terms $\langle D^a \rangle \neq 0$ signal spontaneous breaking of supersymmetry. The details of the supersymmetry breaking mechanism depend on the supergravity model, i.e. on the form of the functions K, W and f_{ab}. Given a fundamental theory that yields the non-renormalizable supergravity as a low energy limit, these functions can in principle be derived by integrating out the massive modes. Otherwise one simply needs to treat them on phenomenological grounds.

In this thesis we consider supersymmetry breaking taking place in some hidden sector which interacts only gravitationally with the visible sector containing the Standard Model fields. The supersymmetry breaking effects are then mediated to the visible sector by gravitational effects. The physical nature of the hidden sector fields depends on the underlying fundamental physics. In string theories, for example, the hidden sector can be formed by moduli fields that parameterize the size and properties of the compactified dimensions [77]. We assume the hidden sector fields to be gauge singlets although this is not compulsory. The supersymmetry is then broken by non-vanishing F-terms in the hidden sector.

3.2.1 F-term supersymmetry breaking

In F-term supersymmetry breaking the D-terms vanish and the relevant part of the supergravity scalar potential (3.4) reads

$$V(\phi_M, \phi_M^*) = M_{\rm P}^4 e^G (K^{M\bar{N}} G_M G_{\bar{N}} - 3) = M_{\rm P}^{-2} e^K |W|^2 (K^{M\bar{N}} G_M G_{\bar{N}} - 3) \ . \tag{3.9}$$

Supersymmetry is spontaneously broken if the F-terms of some hidden sector fields h_m acquire non-zero vacuum expectation values $\langle G^m \rangle \neq 0$. We adopt here the notation where the capital indices M refer to both the visible and the hidden sector fields and the lower case indices m refer to the hidden sector fields alone. The potential for the visible sector fields can then be directly read off from equation (3.9) once the Kähler potential and the superpotential are determined. The potential for visible sector fields contains supersymmetry violating terms with an overall scale given by the prefactor in (3.9),

$$m_{3/2} \sim \left\langle e^{K/2} \frac{|W|}{M_{\rm P}^2} \right\rangle , \tag{3.10}$$

which sets the scale of the graviton mass [65, 66, 67]. It is crucial to notice that unlike in globally supersymmetric theories, the supergravity scalar potential can vanish also in a supersymmetry breaking phase due to the term $-3M_{\rm P}^4 e^G$ in the potential (3.9). This is an appealing feature in light of cosmological observations that require an extremely small value for the vacuum energy density.

To make the discussion of F-term supersymmetry breaking more explicit, let us consider a simple example with only one visible sector field ϕ. We assume the superpotential to be of the form

$$W = \hat{W}(h_m) + I(\phi, h_m) , \tag{3.11}$$

where the part denoted by a hat depends only on hidden sector fields h_m. The scalar potential (3.9) can then be written as

$$V(\phi) = |\hat{W}|^2 V_{WW} + (\hat{W}^* I \ V_{WI} + {\rm h.c}) + |I|^2 V_{II} , \tag{3.12}$$

where

$$V_{WW} = M_{\rm P}^{-2} e^K \Big(K^{M\bar{N}} \Big(K_M K_{\bar{N}} + \frac{\hat{W}_M \hat{W}^*_{\bar{N}}}{|\hat{W}|^2} + \frac{K_M \hat{W}^*_{\bar{N}}}{\hat{W}^*} + \frac{K_{\bar{M}} \hat{W}_N}{\hat{W}} \Big) - 3 \Big) , \tag{3.13}$$

$$V_{WI} = M_{\rm P}^{-2} e^K \Big(K^{M\bar{N}} \Big(K_M K_{\bar{N}} + \frac{I_M \hat{W}^*_{\bar{N}}}{I \hat{W}^*} + \frac{K_M \hat{W}^*_{\bar{N}}}{\hat{W}^*} + \frac{K_{\bar{M}} I_N}{I} \Big) - 3 \Big) , \tag{3.14}$$

$$V_{II} = M_{\rm P}^{-2} e^K \Big(K^{M\bar{N}} \Big(K_M K_{\bar{N}} + \frac{I_M I^*_{\bar{N}}}{|I|^2} + \frac{K_M I^*_{\bar{N}}}{I^*} + \frac{K_{\bar{M}} I_N}{I} \Big) - 3 \Big) , \tag{3.15}$$

and the indices run over both the hidden sector $M = m$ and the visible sector $M = \phi$. To find a more explicit expression for the potential (3.12), one needs to determine the Kähler potential which we take to be given by the expansion

$$K = \hat{K}(h_m, h_m^*) + \hat{Z}_2(h_m, h_m^*) \Big(\frac{|\phi|}{M_{\rm P}} \Big)^2 + \hat{Z}_4(h_m, h_m^*) \Big(\frac{|\phi|}{M_{\rm P}} \Big)^4 \dots . \tag{3.16}$$

Using (3.16), the coefficients V_{WW}, V_{WI} and V_{II} in equation (3.12) can be expanded in powers of ϕ.

There are three kinds of different contributions to the potential (3.12) for ϕ generated in the F-term susy breaking. The first term including the factor $|\hat{W}|^2$ arises from the Kähler potential and the hidden sector superpotential and does not include the visible sector superpotential at all. It gives rise to terms proportional to powers of $|\phi|^2/M_{\rm P}^2$, in particular the soft mass term for ϕ. The second term in (3.12) with one factor of $\hat{W}$ includes the visible sector superpotential multiplied by powers of $|\phi|^2/M_{\rm P}^2$ arising from equation (3.14). This term would contribute to the possible trilinear soft terms and it will be the source of the so called A-term lifting the flat direction. The last term in (3.12) with no factors of $\hat{W}$ can be seen as a supergravity generalization of the visible sector F-term contribution in global case. It includes the $|I_\phi|^2$ part present in the global potential (3.1) but contains also higher order corrections arising from (3.15).

3.2.2 The η − problem in inflation

In supergravity models it is in general difficult to create potentials flat enough to support inflation. Such generic problems do not arise in global supersymmetry but supergravity corrections typically tend to violate the slow roll conditions (2.35) by yielding $|\eta| \sim 1$ [5]. The source of the η − problem is traced back to the form of the F-term part of the supergravity scalar potential (3.9). The visible sector Kähler potential can always be represented as an expansion around the canonical form $K = \sum_\alpha |\phi_\alpha|^2/M_{\rm P}^2$ and, by substituting this into the expression for the scalar potential (3.9), one can schematically write the potential as

$$V(\phi) \sim V_0 + V_0 \sum_\alpha \frac{|\phi_\alpha|^2}{M_{\rm P}^2} + \dots \, . \tag{3.17}$$

Here ϕ_α denote the appropriately normalized visible sector fields and V_0 is the vacuum energy arising from hidden sector fields. If inflaton is one of the visible sector fields, say $\phi_{\rm inf}$, and the higher order terms in (3.17) give a negligible contribution to the inflaton potential, the magnitude of the slow roll parameter η is given by

$$\eta \sim \frac{2V_0}{V_0 + V_0 \sum_\alpha |\phi_\alpha|^2/M_{\rm P}^2 + \dots} \sim 1 \, . \tag{3.18}$$

In the MSSM inflation [7, 8] this problem does not appear since $V_0 = 0$. Moreover, the higher order terms in (3.17) are comparable to the quadratic part, which makes it possible to adjust the potential such that $|\eta| \ll 1$ with field values $\phi_{\rm inf} \ll M_{\rm P}$.

3.3 Flat directions

Globally supersymmetric theories with gauge interactions generically possess degenerate vacua at renormalizable level [68, 69, 70]. This degeneracy manifests itself as the existence of flat directions in the scalar field space along which the potential vanishes. The flat directions arise because supersymmetry and gauge symmetries together imply more strict constraints on the allowed terms in the Lagrangian than the gauge symmetries alone and this is reflected as the apparently accidental degeneracy in the structure of vacua.[3]

Flat directions are defined as field configurations for which the renormalizable parts of the F-terms (3.2) and D-terms (3.3) in global susy vanish,

$$F_M^* = -W_M = 0 \ , \quad D^a = -g \sum_{M,N} \phi_M^* T^a_{MN} \phi_N = 0 \ , \tag{3.19}$$

which is equivalent to requiring that the renormalizable part of the scalar potential (3.1) vanishes. The non-renormalization theorem of global supersymmetry [68, 69, 70, 78] guarantees that the flatness of the tree level potential (3.1) will not be lifted by perturbative radiative corrections. Solutions for the F- and D-flatness conditions, $F = 0$ and $D^a = 0$, can be expressed as gauge-invariant operators formed out of chiral superfields [68, 69, 70] and their scalar components can be seen as condensate fields parameterizing the motion along a given flat direction. In the following we will mainly use the term flat direction to refer to these scalar components.

[3]The Lagrangian of a superymmetric theory with a gauge group G is invariant also under the complexified group G^c [70]. This gives rise to an "accidental" degeneracy since vacuum configurations lying on the orbits of the group G^c are physically equivalent.

3.3.1 Lifting the flatness

Phenomenologically interesting globally supersymmetric theories like the MSSM are expected to be effective low energy descriptions of fundamental physics. The superpotential should therefore contain non-renormalizable terms suppressed by some cutoff scale which we here assume to be the Planck mass $M_{\rm P}$. The non-renormalizable terms formally arise when integrating out massive ($m \gtrsim M_{\rm P}$) degrees of freedom in the fundamental theory but without a knowledge of the details of the theory, one should include all the possible terms allowed by symmetries of the effective model. The non-renormalizable terms eventually induce a non-vanishing potential for a given flat direction by violating the F-flatness condition and the flat directions are said to be lifted. The flatness is lifted also by supersymmetry breaking which removes the vacuum degeneracy.

Flat directions can be characterized by the mass dimension n of the lowest order operator in the superpotential that lifts the flatness of a given direction in global susy. A dimension n flat direction $\phi = |\phi|e^{i\theta}$, for example, would be lifted by terms like

$$W \supset \left\{ \frac{\lambda}{nM_{\rm P}^{n-3}}\phi^n, \frac{\lambda}{M_{\rm P}^{n-3}}\psi\phi^{n-1} \right\} , \tag{3.20}$$

where λ is an effective coupling constant and ψ is a visible sector scalar field with vanishing vacuum expectation value. According to equation (3.1), both of these terms will give a contribution

$$V(\phi) \supset |\lambda|^2 \frac{|\phi|^{2n-2}}{M_P^{2n-6}} , \tag{3.21}$$

to the scalar potential in global susy and thus lift the flatness.

The effects of supersymmetry breaking in the context of global susy are usually taken into account by adding phenomenological supersymmetry breaking terms in the scalar potential. Here we assume that supersymmetry is broken spontaneously by the F-terms of hidden sector fields in supergravity. The form of the effective supersymmetry breaking terms is then determined by the structure of the supergravity model. For definiteness, we consider flat directions lifted by the first type of terms in (3.20) such that the relevant superpotential will be of the form

$$W = \hat{W}(h_m) + \hat{\lambda}(h_m)\frac{\phi^n}{nM_{\rm P}^{n-3}} . \tag{3.22}$$

The potential for the flat direction ϕ is given by equation (3.12) and to leading order it can be expressed as (see e.g. [71, 73])

$$V(\phi) = \frac{1}{2}m^2|\phi|^2 + |A\lambda|\cos(n\theta + \theta_A)\frac{|\phi|^n}{nM_{\rm P}^{n-3}} + |\lambda|^2\frac{|\phi|^{2n-2}}{M_P^{2n-6}} + \dots , \tag{3.23}$$

where the ellipses denote higher order terms and we have assumed that the vacuum energy vanishes. The expressions for the parameters m, $|A|$, $|\lambda|$ and θ_A are given by equations (3.13 – 3.15). With the generic form for the Kähler potential (3.16), they become [10]

$$m^2 = 2M_{\rm P}^2 e^{\hat{G}}\Big(\hat{Z}_2(\hat{G}^m\hat{G}_m - 2) + \hat{G}_m\hat{G}_{\bar{n}}(\hat{Z}_2^m\hat{Z}_2^{\bar{n}}\hat{Z}_2^{-1} - Z_2^{m\bar{n}})\Big) \tag{3.24}$$

$$|A| = 2M_{\rm P}e^{\hat{G}/2}\hat{Z}_2^{1/2}\Big|\hat{G}^m(\hat{K}_m + \hat{\lambda}^{-1}\hat{\lambda}_m - n\hat{Z}_2^{-1}\hat{Z}_{2m}) + n - 3\Big| \tag{3.25}$$

$$|\lambda| = e^{\hat{K}/2}|\hat{\lambda}|\hat{Z}_2^{-1/2} \tag{3.26}$$

$$\theta_A = \arg\Big(\hat{G}^m(\hat{K}_m + \hat{\lambda}^{-1}\hat{\lambda}_m - n\hat{Z}_2^{-1}\hat{Z}_{2m}) + n - 3\Big) + \arg(\hat{\lambda}) + \arg(\hat{W}^*) , \tag{3.27}$$

where arg(...) stands for the phase of a complex variable and $\hat{G} \equiv \hat{K} + \ln(|\hat{W}|^2/M_{\rm P}^6)$. We remind the reader that the lower indices $_{m,\bar{m}}$ denote derivatives with respect to the hidden sector fields h_m, h_m^*. In equations (3.24) – (3.27), the lower case indices are raised with the inverse of the hidden sector Kähler metric $\hat{K}^{m\bar{n}}$.

3.3.2 Flat directions in the MSSM

The Minimally Supersymmetric Standard Model is the simplest globally supersymmetric extension of the Standard Model of particle physics. The MSSM superpotential is given by [66, 67]

$$W_{\rm MSSM} = \lambda_u Q H_u \bar{u} + \lambda_d Q H_d \bar{d} + \lambda_e L H_d \bar{e} + \mu H_u H_d \ , \tag{3.28}$$

where $H_u, H_d, Q, L, \bar{u}, \bar{d}, \bar{e}$ are chiral superfields which include the Standard Model fields. Barred fields represent SU(2) singlets and $\lambda_u, \lambda_d, \lambda_e$ are dimensionless Yukawa couplings. All the gauge and family indices in (3.28) are suppressed. The flat directions of the MSSM can be found by solving the F- and D-flatness conditions (3.19). The flat directions have all been classified in [9] together with the lowest dimension non-renormalizable operators in the superpotential that will lift them. The classification has been done assuming that the non-renormalizable part of the superpotential is restricted only by the gauge symmetries of the Standard Model and an additional discrete symmetry called R-parity [66, 67]. This imposes multiplicative conservation of the quantum number $P_R = (-1)^{3(B-L)+2S}$ where B is the baryon number, L is the lepton number and S is the spin of the particle.

As an example of the flat directions of the MSSM, let us consider the gauge-invariant monomial $LL\bar{e}$ [9]. Restoring the family indices $i, j, \dots$ and the $SU(2)$ indices $\alpha, \beta, \dots$, this is written as $\epsilon_{\alpha\beta} L_i^\alpha L_j^\beta \bar{e}_k$, where L^α are $SU(2)$ doublets. The combination LL and the field $\bar{e}$ are both invariant under $SU(2)$ and their $U(1)$ charges add up to zero. None of the fields carries an $SU(3)$ charge and the combination $LL\bar{e}$ is thus invariant under the MSSM gauge group $SU(3) \times SU(2) \times U(1)$. The D-flatness condition (3.19) is satisfied along the directions

$$L_i = \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \quad L_j = \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad \bar{e}_k = \phi \ , \tag{3.29}$$

provided that $i \neq j$. The complex field ϕ parameterizes the motion along a given one-dimensional direction $L_i L_j \bar{e}_k$ and with a slight abuse of notation we will call ϕ the flat direction. The MSSM superpotential gives rise to an F-term $F_{H_d}^\alpha = \lambda_e^{ij} L_i^\alpha \bar{e}_j$ and one needs to require $F_{H_d}^\alpha = 0$ in order to satisfy the F-flatness condition (3.19). Out of the 9 different family combinations in $L_i L_j \bar{e}_k$ only 5 are linearly independent and the condition $F_{H_d}^\alpha = 0$ places two additional constraints. Thus there are altogether three flat directions of type $LL\bar{e}$ in the MSSM. The lowest order operator in the superpotential that lifts all the three directions is of the form $W \supset M_{\rm P}^{-3} LL\bar{e}LL\bar{e}$ and the dimension of the flat direction $LL\bar{e}$ is therefore $n = 6$.

3.4 MSSM inflation

Recently it has been proposed that some of the flat directions of the MSSM could successfully act as the inflaton field [7, 8]. The field expectation values in the MSSM inflation are well below the Planck scale and the Hubble scale is exceptionally low, $H \sim 1$ GeV, which justifies the use of the MSSM. The great virtue of the model is that the gauge couplings of the inflaton are completely determined, which is in sharp contrast with conventional models based on ad hoc gauge singlets acting as the inflaton. In the MSSM inflation, both the inflationary period and the subsequent reheating is explained in a self consistent manner using the properties of the flat directions. Unlike in the gauge singlet models, there is no need to introduce additional couplings after inflation to explain the reheating and the creation of the Standard Model degrees of freedom. Moreover, since the model is based on the MSSM, it can be tested not only by cosmological observations but partially also by future collider experiments [7, 79].

3.4.1 Inflationary predictions

Suitable candidates for the inflaton field in the MSSM inflation turn out to be the dimension six flat directions $LL\bar{e}$ and $\bar{u}\bar{d}\bar{d}$ [7, 8]. Both of these directions are lifted by superpotentials of the form (3.22) where $n = 6$. In hidden sector F-term supersymmetry breaking, the leading order potential along these directions is thus given by (3.23). We assume that only one of the possible flat directions contained in $LL\bar{e}$ or $\bar{u}\bar{d}\bar{d}$ is excited and all the other visible sector fields are sitting at the origin. The hidden sector fields in the underlying supergravity model should also be stabilized before the beginning of inflation. Under these assumptions, the flat direction driving the inflation can be parameterized in terms of one complex field $\phi = |\phi|e^{i\theta}$. By choosing the phase such that the potential (3.23) is minimized along the angular direction, $\cos(6\theta + \theta_A) = -1$, the potential for the radial direction becomes

$$V(|\phi|) = \frac{1}{2}m^2|\phi|^2 - |A\lambda|\frac{|\phi|^6}{6M_{\rm P}^3} + |\lambda|^2\frac{|\phi|^{10}}{M_P^6} + \dots \ . \tag{3.30}$$

The potential (3.30) is generically not flat enough to yield a successful period of inflation with sub-Planckian field vevs $|\phi| \ll M_{\rm P}$ and a low inflationary scale. However, at the level of the MSSM, the supersymmetry breaking terms m and A in (3.30) are freely adjustable parameters and one can choose them to satisfy the condition [7, 8]

$$A^2 = 40m^2 \ . \tag{3.31}$$

Provided that the relation (3.31) holds, the potential (3.30) has a saddle point at

$$|\phi_0| = \left(\frac{mM_{\rm P}^3}{\sqrt{10}|\lambda|}\right)^{1/4} \ . \tag{3.32}$$

In this case the potential is flat enough to support inflation in the vicinity of the saddle point and can be expanded as

$$V(|\phi|) \simeq V(|\phi_0|) + \frac{1}{6}V'''(|\phi_0|)(|\phi| - |\phi_0|)^3 = \frac{4}{15}m^2|\phi_0|^2 + \frac{16}{3}\frac{m^2}{|\phi_0|}(|\phi| - |\phi_0|)^3 \ . \tag{3.33}$$

If the initial conditions are such that $|\phi| \simeq |\phi_0|$, and the energy density of the flat direction dominates the universe, there follows a period of slow roll inflation with the Hubble parameter given by

$$H^2 \simeq \frac{4}{45M_{\rm P}^2}m^2|\phi_0|^2 = \frac{4}{45(10)^{1/4}}\frac{m^2}{|\lambda|^{1/2}}\left(\frac{m}{M_{\rm P}}\right)^{1/2} \ , \tag{3.34}$$

where we have used equation (3.32) in the last step. Assuming that the effective coupling constant is of the order of unity $|\lambda| \sim 1$ and assigning phenomenologically plausible values for the soft mass, $m \sim 1$ TeV, the magnitude of the Hubble parameter becomes [7, 8]

$$H \sim 1 \text{ GeV} \ . \tag{3.35}$$

This is considerably less than the Hubble scale encountered in typical slow roll models $H \lesssim 10^{13}$GeV and the exceptionally low scale will have important consequences. The field value at the saddle point (3.32) becomes

$$|\phi_0| \sim 10^{14} \text{ GeV} , \tag{3.36}$$

which is well below the Planck scale $M_{\rm P} \simeq 2.4 \times 10^{18}$ GeV as required by consistency.

Using the potential (3.33), the slow roll parameters (2.33), (2.34) become

$$\epsilon = 1800 \frac{M_{\rm P}^2}{|\phi_0|^2} \left(\frac{|\phi| - |\phi_0|}{|\phi_0|} \right)^4 \tag{3.37}$$

and

$$\eta = 120 \frac{M_{\rm P}^2}{|\phi_0|^2} \left(\frac{|\phi| - |\phi_0|}{|\phi_0|} \right) . \tag{3.38}$$

The slow roll approximation breaks down when $|\eta| \sim 1$ which determines the field value at the end of inflation

$$|\phi_{\rm end}| \sim |\phi_0| \left(1 - \frac{1}{120} \frac{|\phi_0|^2}{M_{\rm P}^2} \right) . \tag{3.39}$$

Assuming that the saddle point condition (3.31) is exactly satisfied, there exists a region in the vicinity of $|\phi_0|$ where the quantum effects overcome the classical force. The quantum fluctuations turning into classical perturbations after the horizon crossing effectively amount to production of field perturbations with amplitude H in one Hubble time H^{-1}. This contribution acts as a stochastical noise and dominates over the classical motion if $H^2 \gtrsim |\dot{\phi}|$, where $\dot{\phi}$ is determined by the equation of motion (2.32). In our case, the classical force starts to dominate, i.e. $H^2 \lesssim |\dot{\phi}|$, for field values

$$|\phi_0| - |\phi_{\rm in}| \gtrsim \left(\frac{m}{M_{\rm P}} \right)^{1/2} \frac{|\phi_0|^2}{M_{\rm P}} , \tag{3.40}$$

where a numerical factor of order 10^{-2} has been omitted. This roughly determines the magnitude of the field value $|\phi_{\rm in}|$ at the beginning of the slow roll period, assuming that the inflaton field initially finds itself in the quantum regime.

The number of e-foldings (2.18) evaluated from some field value $|\phi|$ up to the end of inflation $|\phi_{\rm end}|$ becomes

$$N(|\phi|) = \int_{|\phi|}^{|\phi_{\rm end}|} H {\rm d}t \simeq \frac{|\phi_0|^3}{60 M_{\rm P}^2 (|\phi_0| - |\phi|)} . \tag{3.41}$$

Using equation (3.40), the total amount of e-foldings is approximately given by $N(|\phi_{\rm in}|) \sim (|\phi_0|^2/(m M_{\rm P}))^{1/2}$ where the factor 60 in equation (3.41) has been dropped for consistency with the accuracy of equation (3.40). For the values $|\phi_0| \sim 10^{14}$GeV, $m \sim 1$ TeV this yields $N_{\rm TOT} \gtrsim 10^2$ which is enough to solve the horizon and flatness problems and to dilute away possible relic densities. Note that due to the low scale of the MSSM inflation, $H \sim 1$ GeV, the number of e-foldings taking place after the visible scales exit the horizon is approximatively $N_* \sim 50$ [7, 40] which is less than the value $N_* \sim 60$ encountered in typical models [1, 2, 3]. The corresponding field value $|\phi_*|$ is obtained using equation (3.41), which allows to express the inflationary predictions in terms of N_*.

The amplitude of the curvature perturbations (2.77) at the horizon crossing of the observable universe is given by

$$\mathcal{P}_\zeta^{1/2}(|\phi_*|) \simeq \frac{1}{\sqrt{8\pi^2} M_{\rm P}} \frac{H}{\sqrt{\epsilon}} \simeq \frac{4\sqrt{5}}{\pi} N_*^2 \frac{m M_{\rm P}}{|\phi_0|^2} . \tag{3.42}$$

For the values $|\phi_0| \sim 10^{14}$ GeV, $m \sim 1$ TeV and $N_* \sim 50$, the amplitude becomes $\mathcal{P}_\zeta^{1/2} \sim 10^{-5}$ which is in perfect agreement with observations. Note that this is a non-trivial result because the reasonable soft mass scale m is dictated by phenomenology and the field value $|\phi_0|$ is related to this mass scale by equation (3.32). Thus there is a limited number of adjustable parameters and a priori no guarantee of obtaining the correct amplitude of perturbations. Indeed, as discussed in [7, 8], flat directions of the MSSM with dimension different from $n = 6$ fail to produce the correct amplitude. Together with the required form for the inflaton potential, i.e. the presence of the A-term in (3.30), this singles out the directions $LL\bar{e}$ and $\bar{u}\bar{d}\bar{d}$ as unique inflaton candidates.

The spectral index (2.78) in the MSSM inflation becomes

$$n_s(|\phi_*|) = 1 + 2\eta(|\phi_*|) - 6\epsilon(|\phi_*|) \simeq 1 - \frac{4}{N_*} \simeq 0.92 \ , \tag{3.43}$$

which is about 3σ off the observationally favoured value $n_s \simeq 0.96$ [4]. If the saddle point condition (3.31) is not exactly satisfied, the spectral index can be made larger [80, 81] by slightly adjusting the parameters m and A but the deviations from the saddle point need to be extremely small in order to preserve the success of the model. This fine-tuning issue is addressed below in discussing the supergravity embedding of the model. The amplitude of gravitational waves produced in the MSSM inflation is completely negligible due to the low scale.

3.4.2 End of the MSSM inflation

After the end of the MSSM inflation, the inflaton rolls down towards its global minimum $\phi = 0$ and the quadratic part in the potential (3.30) eventually starts to dominate, $V \sim m^2|\phi|^2/2$ [8]. Since the Hubble scale is much smaller than the inflaton mass, $H \sim 10^{-3}m$, the field starts to oscillate and can excite the light MSSM fields coupled to it. The inflaton candidates $\bar{u}\bar{d}\bar{d}$ and $LL\bar{e}$ are coupled to gauge/gaugino fields and to Higgses/Higgsinos. In addition, $\bar{u}\bar{d}\bar{d}$ can decay to (s)quarks and $LL\bar{e}$ to (s)leptons.

There can be various phases of particle creation in the model, but instant preheating is the most efficient one [8]. The gauge fields have largest couplings to the inflaton and they acquire a mass proportional to the inflaton vev, $m_{\rm gauge} \sim g\langle\phi\rangle$ where g is the gauge coupling. When the oscillating inflaton field crosses the origin, the gauge fields are effectively massless and the inflaton can efficiently decay into gauge field quanta. The particle production takes place in a short time interval and results to an exponential increase in the momentum space occupation numbers of the gauge fields. As the inflaton has passed the bottom of the potential, the gauge fields again become massive and can further decay into other MSSM fields. Immediately after the end of inflation, when the inflaton is still close to the saddle point, there can also be a period of tachyonic preheating since $V'' < 0$, but due to the large vev of the field $|\phi_0| \gg m$, the amount of particle production via this mechanism is suppressed.

After preheating the universe is in general far from equilibrium state and needs to thermalize in order to enter the hot big bang era. An upper limit for the thermalization temperature is given by

$$T_{\rm max} < (V(|\phi_{\rm end}|))^{1/4} \sim 10^9 {\rm GeV} \ , \tag{3.44}$$

corresponding to instantaneous thermalization after the end of inflation. The actual thermalization temperature can however be considerably lower than $T_{\rm max}$, which dilutes away the problems related to thermal overproduction of gravitinos. For further discussion of the preheating and thermalization in the MSSM inflation, see [8].

3.4.3 Fine-tuning problems

It is perhaps somewhat surprising that the flat directions of the MSSM provide successful candidates for the inflaton field. However, it should be kept in mind that the apparent success of the model discussed here strongly relies on the saddle point condition (3.31) that has been imposed by hand. Moreover, the condition needs to be satisfied to the precision 10^{-18} in order to produce large enough primordial perturbations [8, 80, 81, 10, 11]. The extreme fine-tuning arises because of the exceptionally low scale of the MSSM inflation. According to equation (2.77), the amplitude of the curvature perturbation is given by $\mathcal{P}_\zeta^{1/2} \sim H/(M_{\rm P}\sqrt{\epsilon})$. In typical models with the inflationary scale only few orders of magnitude below the Planck scale, the observed amplitude $\mathcal{P}^{1/2} \sim 10^{-5}$ implies $\epsilon \lesssim 1$, which is not a very restrictive constraint. The situation is completely different in the MSSM inflation, where the amplitude of the perturbations places an extremely tight constraint on the flatness of the potential, $\sqrt{\epsilon} \lesssim 10^{-13}$, due to the low scale $H \lesssim 10^{-17} M_{\rm P}$.

To make the argumentation more precise, consider a linear correction to the potential (3.33),

$$V(|\phi|) \approx \frac{4}{15} m^2 |\phi_0|^2 + \frac{16}{3}\frac{m^2}{|\phi_0|}(|\phi| - |\phi_0|)^3 + (\delta M_{\rm P})^3 (|\phi| - |\phi_0|) \ , \tag{3.45}$$

where δ is a dimensionless parameter. Corrections of this type arise if the saddle point condition (3.31) is not exactly satisfied [8, 80, 81] and also if one takes into account higher order supergravity corrections to the potential (3.23) [10, 11]. In order to obtain large enough primordial perturbations and to have a sufficiently long period of inflation, the corrections need to be very small [8, 80, 81]

$$\delta \lesssim \left(10^{-5} \left(\frac{m}{M_{\rm P}}\right)^2 \left(\frac{|\phi_0|}{M_{\rm P}}\right)^5 \right)^{1/3} \sim 10^{-18} \ . \tag{3.46}$$

This seems to imply that the MSSM inflationary scenario requires unnatural fine-tuning of the supersymmetry breaking parameters in the inflaton potential. However, since these parameters are determined by the supersymmetry breaking mechanism, it is reasonable to ask if the required flat potential (3.45), (3.46) could arise naturally in some supergravity models. This issue has been investigated in the enclosed papers [10, 11] where it is shown that the apparent fine tuning problem can indeed be solved, or at least considerably alleviated, in a class of supergravity models.

Initial conditions comprise another source of problems in the MSSM inflation. The slow roll conditions (2.35) are satisfied only in the immediate vicinity of the saddle point $|\phi_0|$ and to obtain a sufficiently long period of inflation, one needs to set the initial value of the field very close to the saddle point $|\phi - \phi_0| \lesssim (|\phi_0|^2 m^{1/2}/M_{\rm P}^{3/2}) \sim 10^{-12}|\phi_0|$. The required fine-tuning can again be traced back to the exceptionally low inflationary scale. Explaining the initial conditions requires knowledge of the physics taking place before the inflationary period driven by the flat direction. For example, the initial conditions might be connected to the details of the supersymmetry breaking mechanism. The problem has also been discussed in the context of string theory landscape in [82] by assuming that the MSSM inflation would occur as a last stage of a chain of inflationary periods driven by energy densities of several false vacua. At the time of writing, there is however no fully satisfactory explanation for the origin of the initial conditions and the issue remains an interesting open problem.

3.4.4 Supergravity embedding of the MSSM inflation

The supergravity embedding of the MSSM inflation has been considered in the two enclosed papers [10, 11]. In F-term supersymmetry breaking taking place in the hidden sector, the parameters $m, |A|$ and $|\lambda|$ in the inflaton potential (3.30) are given by equations (3.24) – (3.26) where $n = 6$. The saddle point condition (3.31) can then be written using equations (3.24) and (3.25) as [10]

$$\Big|\hat{G}^m(\hat{K}_m + \hat{\lambda}^{-1}\hat{\lambda}_m - 6\hat{Z}_2^{-1}\hat{Z}_{2m}) + 3\Big|^2 \;=\; 20\Big(\hat{G}^m\hat{G}_m - 2 + \hat{G}_m\hat{G}_{\bar{n}}(\hat{Z}_2^m\hat{Z}_2^{\bar{n}}\hat{Z}_2^{-2} - \hat{Z}_2^{-1}Z_2^{m\bar{n}})\Big) \;, \tag{3.47}$$

where the indices denote derivatives with respect to the hidden sector fields as before. This is a partial differential equation for the functions $\hat{K}, \hat{Z}_2$ and $\hat{W}, \hat{\lambda}$, appearing respectively in the Kähler potential (3.16) and in the superpotential (3.22). Since $\hat{K}$ and $\hat{Z}_2$ by definition are non-analytic functions while $\hat{W}$ and $\hat{\lambda}$ are analytic, equation (3.47) does not in general have non-trivial solutions valid for all hidden sector vevs h_m, h_m^*. However, if one neglects the hidden sector dependence of the superpotential by setting $\hat{W}_m = \hat{\lambda}_m = 0$, equation (3.47) is identically solved for Kähler potentials of the form [10]

$$K = \hat{K} + \hat{Z}_2\frac{|\phi|^2}{M_{\rm P}^2} + \ldots = \sum_m \beta_m \ln\left(\frac{h_m + h_m^*}{M_{\rm P}}\right) + \kappa\prod_m\left(\frac{h_m + h_m^*}{M_{\rm P}}\right)^{\alpha_m}\frac{|\phi|^2}{M_{\rm P}^2} + \ldots \;, \tag{3.48}$$

provided that the parameters $\alpha = \sum_m \alpha_m$ and $\beta = \sum_m \beta_m$ satisfy

$$\alpha(36\alpha + 16 - 12\beta) + (\beta + 7)^2 = 0 \;. \tag{3.49}$$

The constant κ can be chosen arbitrarily. For this class of Kähler potentials, the saddle point condition (3.31) crucial to the MSSM inflation is an identity that holds irrespectively of the hidden sector vevs.

It is interesting to note that the form of the Kähler potential (3.48) is quite common in various string theory compactifications. For example, large radius limits of Calabi-Yau compactifications [77, 83], Abelian orbifold compactifications of heterotic strings [84] and intersecting D-brane models [85] yield Kähler potentials of this form, although equation (3.49) represents a non-trivial constraint for the parameters. In the string context, the parameter $-\beta = -\sum_m \beta_m$ typically measures the number of hidden sector fields and from this point of view one would expect an integer value for β. Equation (3.49) admits integer solutions for example for the parameter values listed in Table (3.1).

In discussing the supergravity embedding of the MSSM inflation, it should be kept in mind that the potential (3.30) represents only the leading order part of the supergravity scalar potential. The supergravity corrections to the form (3.30) are suppressed by powers of $(|\phi_0|/M_{\rm P})^2$ but they still play an important role as pointed out in [10, 11]. The corrections are generically orders of magnitude larger than the deviations from the leading order form allowed by equations (3.45) and (3.46). In [10] it has been shown that one needs to consider next to leading and next to next to leading order supergravity corrections to guarantee the flatness of the inflaton potential. Therefore it is not enough to determine the Kähler potential only up to second order as in equation (3.48) but one also needs to discuss higher order terms. Put in another way, when the supergravity corrections are properly taken into account, the saddle point condition (3.31) alone does not ensure the flatness of the potential. The full supergravity scalar potential with all the

Table 3.1: Parameter values of the Kähler potential (3.48) for which the saddle point condition (3.31) is satisfied identically and β is an integer .

$\beta = \sum_m \beta_m$	$\alpha = \sum_m \alpha_m$
-7	0
-7	$-\frac{25}{9}$
-11	$-\frac{1}{9}$
-11	-4

relevant terms retained has been analyzed in [10, 11]. The outcome is that the sufficiently flat inflaton potential of the form (3.45) is obtained identically for Kähler potentials

$$\begin{aligned} K \;=\; & \sum_m \beta_m \ln\left(\frac{h_m+h_m^*}{M_{\rm P}}\right) + \kappa\prod_m\left(\frac{h_m+h_m^*}{M_{\rm P}}\right)^{\alpha_m}\frac{|\phi|^2}{M_{\rm P}^2} + \mu\left(\kappa\prod_m\left(\frac{h_m+h_m^*}{M_{\rm P}}\right)^{\alpha_m}\right)^2\frac{|\phi|^4}{M_{\rm P}^4} + \\ & \nu\left(\kappa\prod_m\left(\frac{h_m+h_m^*}{M_{\rm P}}\right)^{\alpha_m}\right)^3\frac{|\phi|^6}{M_{\rm P}^6} + \dots\,, \end{aligned} \tag{3.50}$$

where the parameters need to satisfy equation (3.49) and a set of additional constraints. For the values of α and β listed in Table (3.1), the additional constraints are shown in Table (3.2). Note

Table 3.2: The constraints on the parameters of the Kähler potential (3.50) implied by the flatness of the inflaton potential.

$\beta = \sum \beta_m$	$\alpha = \sum \alpha_m$	$\gamma = \sum \frac{\alpha_m^2}{\beta_m}$	$\delta = \sum \frac{\alpha_m^3}{\beta_m^2}$
-7	0	$\frac{1}{4} - 3\mu$	δ
-7	$-\frac{25}{9}$	$-\frac{46}{81} - \frac{22}{9}\mu$	$-\frac{2414}{16767} - \frac{628}{1863}\mu - \frac{2804}{207}\mu^2 + \frac{162}{23}\nu$
-11	$-\frac{1}{9}$	$\frac{28}{81} - \frac{26}{9}\mu$	$\frac{6556}{69255} - \frac{3736}{7695}\mu - \frac{12596}{855}\mu^2 + \frac{162}{19}\nu$
-11	-4	$-\frac{7}{8} - \frac{5}{2}\mu$	$-\frac{339}{1600} - \frac{73}{200}\mu - \frac{1371}{100}\mu^2 + \frac{36}{5}\nu$

that the constrains do not completely fix the Kähler potential but the coefficients μ and ν, for example, appear as adjustable parameters. It is also noteworthy that by choosing the parameters in (3.50) appropriately [11], it is possible to bring the slightly too small spectral index of the MSSM inflation (3.43) closer to the observationally favoured value $n_s \simeq 0.96$.

The results of [10, 11] suggest that the extremely flat MSSM inflaton potential could arise naturally as a consequence of the structure of the underlying supergravity model. This clearly alleviates the fine-tuning problems but the analysis of [10, 11] is still not complete in the sense that the hidden sector dependence of the superpotential has been neglected [86]. This does not allow for a proper discussion of the stabilization of the moduli fields but in the string theory context this is to large extent an open problem anyway. It would also be important to see if the radiative corrections will spoil the flatness of the potential in the supergravity embeddings. One-loop corrections to the leading order inflaton potential (3.30) have been studied in [8]. The outcome is that the corrections do not remove the existence of the saddle point but the running of the parameters m and $|A|$ shift its location. However, as we have pointed out above, the supergravity corrections to the leading order potential can not be neglected and one should also consider the running of these terms. In supergravity models with the class of Kähler potentials given by equation (3.50), the flatness of the tree level inflaton potential follows form the constraints in Table (3.2) and might be destroyed by radiative corrections. There is however a continuous trajectory in the parameter space (α_m, β_m) along which the inflaton potential remains flat [10, 11], although we have only shown solutions with integer values for β in Table (3.2). If this trajectory would have some specific geometrical interpretation, for example, it might turn out to be a renormalization group fixed point. In this case, the flatness of the inflaton potential would be protected from radiative corrections, which would be an extremely interesting result.

Chapter 4

The curvaton model

In conventional models of inflation, the accelerated expansion of the universe and the primordial perturbations are generated by the same scalar field or fields [1, 2, 3, 5]. This imposes strict constraints and makes it difficult to establish a connection between known theories of particle physics and inflation. The observed amplitude of primordial perturbations requires an extremely flat inflaton potential if inflation is to take place at low scales where extensions of the Standard Model are applicable. In the MSSM inflation discussed above, the flatness can be motivated without unnatural fine-tuning but in general it is difficult to produce large enough perturbations with theoretically plausible low scale models [5].

On generic grounds, there is however no reason to require that primordial perturbations would be generated by the same fields that drive inflation. A novel mechanism for generating the perturbations independently on the details of inflationary physics is the curvaton scenario [12, 13]. In this scenario, the adiabatic primordial perturbations are generated after inflation by a conversion of initially isocurvature perturbations of a curvaton field into adiabatic curvature perturbations. The curvaton is some late decaying scalar field which during inflation remains effectively massless and gives a negligible contribution to the total energy density. Like any other light scalar field, it acquires a spectrum of perturbations but since the curvaton contributes very little to the total energy density, the curvaton perturbations correspond to non-adiabatic isocurvature perturbations. After the end of inflation, the fractional contribution of the curvaton starts to increase since the dominating radiation component redshifts faster than the initially subdominant curvaton. The enhancement of the curvaton component can be a highly non-adiabatic process and can thus give rise to a significant production of curvature perturbations, sourced by the perturbations of the curvaton field. Once the curvaton eventually decays and thermalizes, the universe will enter the standard hot big bang era and evolve adiabatically. The universe may also become adiabatic already at an earlier stage if the curvaton comes to dominate the total energy density before decaying. If the amplitude of curvature perturbations generated during inflation is negligible, the primordial perturbations can be generated solely by the curvaton field.

There are many variations of the simplest curvaton scenario [87, 88], but the idea of converting the initially non-adiabatic perturbations of a light scalar field into adiabatic curvature perturbations after inflation is quite generic. The curvaton model leads to a significant relaxation of the constraints on the inflationary energy scale. In conventional single field models one typically finds $H \sim 10^{13}$GeV, or slightly less, whereas the curvaton scenario easily brings the scale down to $H \sim 10^{7}$GeV and even lower in some models [14]. This opens up new possibilities for seeking candidates for the curvaton and the inflaton within the extensions of the Standard Model.

In this section we briefly review the simplest curvaton scenario. A typical feature of the model is that the primordial perturbations can contain a significant non-gaussian component [16, 17]. In the enclosed research paper [18], it is shown however that the level of the non-gaussian effects is highly dependent on the form of the curvaton potential. We outline the results of [18] and discuss how deviations from the quadratic potential affect the observational bounds on the curvaton model.

4.1 The simplest curvaton model

The simplest possible curvaton model features two scalar fields, the inflaton ϕ and a light curvaton σ which is not coupled to the inflaton. Here we consider models with quadratic curvaton potential and the scalar potential during the inflation is then written as [12, 16]

$$V(\phi,\sigma) = V(\phi) + \frac{1}{2}m_\sigma^2\sigma^2 \ . \tag{4.1}$$

We assume that the curvaton component is completely subdominant $m_\sigma^2\sigma^2 \ll V(\phi)$ during inflation. A detailed discussion of the curvaton dynamics in more complicated models can be found in [87]. In principle there are no other restrictions on the inflaton potential than producing a sufficiently long period of inflation and not producing too large primordial perturbations. The observational upper bound for the amplitude of gravitational waves [4] requires that the scale of slow roll inflation can not be much larger than $H \sim 10^{13}$GeV. In addition, we assume for simplicity that the amplitude of curvature perturbations generated during inflation is completely negligible, which for slow roll inflation implies $H/\sqrt{\epsilon} \ll 10^{-4}\ M_\mathrm{P}$ according to equation (2.85). Models with mixed curvaton and inflaton perturbations have been discussed in [88, 89].

The curvaton field is supposed to be very light during inflation $m_\sigma \ll H$ and will therefore acquire a spectrum of perturbations given by equation (2.52)

$$\mathcal{P}_\sigma = \left(\frac{H_*}{2\pi}\right)^2 \ . \tag{4.2}$$

Since the curvaton energy density vanishes at $\sigma = 0$, the quantum effects start to dominate over the classical force before the curvaton reaches the minimum of its potential. This happens for field values $\sigma \lesssim H(H/m_\sigma)^2$. If inflation lasts long enough, the curvaton enters the quantum regime and the value of the background field σ_* at the end of inflation becomes a stochastical quantity. A typical value would then be given by $\sigma_* \sim H_*(H_*/m_\sigma)^2 \gg H_*$ which makes perturbations gaussian to leading order $\delta\sigma \ll \sigma_*$, as required by observations. If the curvaton stays in the classical regime, the value σ_* is determined by the initial conditions and the duration of the inflationary period [87].

After the end of inflation and thermalization of the decay products of the inflaton field, the universe is dominated by radiation and it is practically homogeneous $\rho_r \sim \rho_r(t)$ since the density perturbations generated during inflation are assumed to be negligible. The Hubble parameter evolves as $H = (2t)^{-1}$ and the equation of motion (2.27) for the curvaton field on superhorizon scales reads

$$\ddot{\sigma} + \frac{3}{2t}\dot{\sigma} + m_\sigma^2\sigma = 0 \ . \tag{4.3}$$

At some stage the curvaton becomes effectively massive $H_\mathrm{mass} \sim m_\sigma$ and starts to oscillate around the minimum $\sigma = 0$. We assume that the curvaton component is still subdominant at this stage, which implies $\sigma_\mathrm{mass} < M_\mathrm{P}$. If this was not the case, a period of inflation driven by the curvaton field might follow. The oscillating curvaton behaves as non-relativistic matter and its fractional energy density grows proportional to the scale factor $\rho_\sigma/\rho_r \propto a$. The equation of sate of the universe is therefore changing and the perturbations of the curvaton field generate curvature perturbations according to equation (2.62). Instead of directly solving the evolution equation for the curvature perturbation, we apply here the sudden decay approximation where the curvaton component and the radiation component do not interact until the instantaneous decay of the curvaton field into radiation. The sudden decay approximation gives a good (typically within 10% when compared to numerical computations) estimate for the order of magnitude of perturbations generated in the curvaton mechanism [89, 90].

The total curvature perturbation on uniform energy density hypersurfaces (2.61) can be written as a weighted sum of the radiation and the curvaton parts [12, 16]

$$\zeta = -\psi - \frac{H}{\dot{\rho}}\delta\rho = \left(1 - \frac{3\rho_\sigma}{4\rho_r + 3\rho_\sigma}\right)\zeta_r + \frac{3\rho_\sigma}{4\rho_r + 3\rho_\sigma}\,\zeta_\sigma \ , \tag{4.4}$$

where the curvature perturbations for the individual fluid components ζ_i are defined as

$$\zeta_i = -\psi - \frac{H}{\dot{\rho}_i}\delta\rho_i \ . \tag{4.5}$$

In a mixture of ideal fluids with the same four velocity, each component obeys its own continuity equation (2.7). Consequently, each ζ_i (4.5) obey its own conservation equation (2.62) as long as the different fluids do not interact [47, 50, 51]. Assuming that the radiation and the oscillating curvaton field do not interact non-gravitationally until the decay of the curvaton, the perturbations ζ_r and ζ_σ remain constant up to this point. Since the radiation is almost homogeneous, we can further set $\zeta_r \simeq 0$ and the curvature perturbation (4.4) becomes

$$\zeta \simeq \frac{3\rho_\sigma}{4\rho_r + 3\rho_\sigma}\,\zeta_\sigma \ . \tag{4.6}$$

All the time evolution of the total curvature perturbation is due to the evolution of background energy densities since ζ_σ remains constant within the approximations made here. The total curvature perturbation (4.6) generated in the model can be expressed as

$$\zeta \simeq \frac{3r}{4}\zeta_\sigma \ , \tag{4.7}$$

where the parameter

$$r = \left.\frac{4\rho_\sigma}{4\rho_r + 3\rho_\sigma}\right|_{\mathrm{dec}} \tag{4.8}$$

measures the fractional energy density of the curvaton component at the decay time. The case $r \simeq 1$ corresponds to the curvaton decaying after its energy density has become to dominate the universe. In the opposite case $r \ll 1$, the curvaton is still subdominant at the decay time and

$$r \sim \left.\frac{\rho_\sigma}{\rho_r}\right|_{\mathrm{dec}} \ . \tag{4.9}$$

We assume that the curvaton decays completely into radiation and thermalizes immediately after the decay such that from the decay onwards the universe is radiation dominated and evolves adiabatically. The curvaton model can also give rise to residual isocurvature perturbations if the curvaton does not dominate the energy density at the decay time and decays at least partially into particle species that cannot thermalize with the existing radiation [12, 16, 89, 91]. The isocurvature perturbations created in the curvaton scenario are fully correlated with the adiabatic perturbations. Present CMB observations place strict constraints on the allowed fraction ($\lesssim 5\%$) of the primordial perturbations comprised by a correlated isocurvature component [4, 92]. This rules out curvaton models where $r \ll 1$ and the cold dark matter is created by the curvaton decay [16].

For the quadratic curvaton potential, the first order perturbation of the energy density is given by

$$\frac{\delta\rho_\sigma}{\rho_\sigma} = 2\frac{\delta\sigma}{\sigma} = 2\frac{\delta\sigma_*}{\sigma_*} \ , \tag{4.10}$$

where the last equality holds since for a free field the background and perturbations obey the same equation of motion (4.3) outside the horizon and the fraction $\delta\sigma/\sigma$ thus remains constant. The energy density of the oscillation curvaton field behaves as non-relativistic matter $\rho_\sigma \propto a^{-3}$. Using a spatially flat time slicing, equation (4.5) then yields $\zeta_\sigma = 2/3\delta\sigma_*/\sigma_*$ and the total curvature perturbation (4.7) becomes

$$\zeta \simeq \frac{r}{2}\frac{\delta\sigma_*}{\sigma_*} \ . \tag{4.11}$$

The spectrum of the curvaton perturbations on spatially flat slices is given by equation (4.2) and the spectrum of the curvature perturbations thus reads [12, 16]

$$\mathcal{P}_\zeta(k) \simeq \frac{r^2}{4\sigma_*^2}\left(\frac{H_*}{2\pi}\right)^2 \ . \tag{4.12}$$

Significant amount of non-gaussian perturbations can also be generated in the curvaton model since the second order contribution to the perturbations of the energy density (4.10) and further to the curvature perturbation is generically not negligible. We will consider the non-gaussianities more closely below.

4.1.1 Inflationary scale

In the simplest curvaton model with the quadratic potential, the amplitude of the curvature perturbations (4.12) is determined by the inflationary scale H_* , the value of the curvaton field σ_* and the parameter r describing the curvaton energy density at the decay time. If the curvaton is subdominant when it decays $r \ll 1$, one obtains an estimate [12, 14, 16]

$$r \simeq \left.\frac{\rho_\sigma}{\rho_r}\right|_{\rm dec} \sim \frac{\sigma_*^2}{M_{\rm P}^2}\sqrt{\frac{m_\sigma}{\Gamma_\sigma}} \ , \tag{4.13}$$

where Γ_σ is the decay rate. The magnitude of the density parameter r therefore depends on the life-time of the curvaton field. Observational and theoretical constraints on the parameters of the simplest curvaton model have been analyzed in [93]. Combining equations (4.12) and (4.13), and taking into account that $r < 1$, one finds a bound on the inflationary scale, $H_*^2 \gtrsim \mathcal{P}_\zeta M_{\rm P}^2\sqrt{\Gamma_\sigma/m_\sigma}$. The curvaton necessarily needs to decay before nucleosynthesis, which implies $T_{\rm dec} \sim \sqrt{\Gamma_\sigma M_{\rm P}} \gtrsim 1$ MeV. Together with the observed amplitude of the primordial perturbations $\mathcal{P}_\zeta^{1/2} \sim 10^{-5}$ and the masslessness of the curvaton $m_\sigma \ll H_*$, this yields a lower limit for the inflationary scale [14]

$$H_* \gtrsim 10^7 {\rm GeV} \ . \tag{4.14}$$

This is less than the typical scale of the slow roll single field inflation $H \sim 10^{13}$ GeV, but still quite high from the point of view of supersymmetric extensions of the Standard Model for example. The inflationary scale can be further lowered if the curvaton mass increases after inflation [14]. This can be achieved e.g. if the curvaton is coupled to fields that develop a large vev in post-inflationary epoch.

The low inflationary scale implies that the amplitude of gravitational waves is negligible in the curvaton model. Combined with the possible non-gaussianities, this provides a characteristic prediction of the model.

4.1.2 Candidates for the curvaton

Several particle physics candidates for the curvaton field have been proposed in the literature. Flat directions of supersymmetric models provide a natural curvaton candidate but inflation needs to take place at some hidden sector to keep the curvaton field light enough [15]. In addition, one needs to take care that thermal effects do not evaporate the curvaton field before it starts to contribute to the total energy density. While the success of flat directions as the curvaton fields is not automatic, it is still possible to find viable candidates within the MSSM. Pseudo-Nambu-Goldstone bosons [94, 95, 14] related to some global symmetry have also been suggested as curvaton candidates. The curvaton mass in these models is proportional to the amount of symmetry breaking and it is kept small if the breaking is small. Brane-inspired scenarios have been coupled to the curvaton idea in [96].

4.2 Non-gaussian perturbations

During the radiation dominated era after inflation, the perturbations of the subdominant curvaton field act as isocurvature perturbations which in turn give rise to creation of adiabatic curvature perturbations. The properties of the primordial perturbations generated in the curvaton model are therefore dependent on the post-inflationary physics. Unlike in the slow roll inflation, there are no strict constraints on the curvaton potential except that the curvaton needs to be effectively massless during inflation. In particular, the post-inflationary evolution of curvaton perturbations can be highly non-linear and it can give rise to significant non-gaussinities [16, 17]. The magnitude of the non-gaussian effects is strongly dependent on the form of the curvaton potential, as discussed in [18].

In the curvaton model, the particle production takes place during inflation and the subsequent generation of curvature perturbations is due to changes in the equation of state of the background universe. Using the ΔN expression (2.74), the curvature perturbation in the post inflationary epoch is given by [57]

$$\zeta(t, \mathbf{x}) = N'(\sigma_*)\delta\sigma_*(\mathbf{x}) + \frac{1}{2}N''(\sigma_*)\delta\sigma_*(\mathbf{x})^2 + \dots \ , \tag{4.15}$$

where the star denotes evaluation at the end of inflation, or at the horizon crossing of the smallest scale of interest. The amount of expansion of the universe from the end of inflation t_* until the decay of the curvaton field $t_{\rm dec}$ can be written in terms of the energy density of the radiation component, $\rho_r \propto a^{-4}$, as

$$N(t_{\rm dec}, t_*) = \frac{1}{4}\ln\frac{\rho_r(t_*)}{\rho_r(t_{\rm dec})} = \frac{1}{4}\ln\frac{\rho_r(t_*)}{\rho(t_{\rm dec}) - \rho_\sigma(t_{\rm dec})} \ . \tag{4.16}$$

Assuming the curvaton potential to be quadratic during the oscillations of the field, the curvaton energy density at the decay time $\rho_\sigma(t_{\rm dec})$ can be solved from

$$\rho_\sigma(t_{\rm dec}) \simeq \frac{1}{2} m_\sigma^2 \sigma_{\rm osc}^2 \left(\frac{\rho(t_{\rm dec}) - \rho_\sigma(t_{\rm dec})}{\rho(t_{\rm osc})} \right)^{3/4} . \tag{4.17}$$

Here $\sigma_{\rm osc}$ is the field value at the beginning of oscillations and it depends on the curvaton potential and the curvaton value at the end of inflation σ_*. The derivatives of the number of e-foldings in (4.15) can now be evaluated using (4.16) and (4.17). Since the curvaton component is negligible at the end of inflation, $\partial_{\sigma_*} \rho_r(t_*) \simeq 0$, we find

$$\begin{aligned} N'(\sigma_*) &= \frac{r}{2\sigma_{\rm osc}}\ \sigma'_{\rm osc}(\sigma_*) \qquad (4.18)\\ N''(\sigma_*) &= \frac{r}{2\sigma_{\rm osc}}\ \sigma''_{\rm osc}(\sigma_*) + \frac{1}{2\sigma_{\rm osc}^2} \left(r - r^2 - \frac{3}{8} r^3 \right) (\sigma'_{\rm osc}(\sigma_*))^2 . \end{aligned}$$

The leading order expression (2.91) for the non-linearity parameter $f_{\rm NL}$ is then given by [57]

$$f_{\rm NL} \simeq \frac{5}{6} \frac{N''(\sigma_*)}{N'(\sigma_*)^2} = -\frac{5}{3} - \frac{5}{8} r + \frac{5}{3r} \Big(1 + \frac{\sigma''_{\rm osc}(\sigma_*)\sigma_{\rm osc}(\sigma_*)}{(\sigma'_{\rm osc}(\sigma_*))^2} \Big) . \tag{4.19}$$

Note that the sign convention chosen for $f_{\rm NL}$ here is the same as in [4] and differs from the one used e.g. in [57] and in the enclosed paper [18].

For a quadratic curvaton potential $\sigma''_{\rm osc}\sigma_{\rm osc}/{\sigma'_{\rm osc}}^2 = 0$ and $f_{\rm NL}$ is completely determined by the density parameter r. If the curvaton decays after it is has became to dominate the energy density $r \sim 1$, the non-gaussianities are small $|f_{\rm NL}| \sim 1$. In the opposite case where the curvaton is still subdominant at the decay time $r \ll 1$, the quadratic model predicts significant level of non-gaussianities $|f_{\rm NL}| \gg 1$. The present observational upper limit $|f_{\rm NL}| \lesssim 100$ [4] is strict enough to rule out quadratic curvaton potentials with very small density parameters $r \lesssim 0.02$ [16, 17].

The situation changes considerably if the curvaton potential deviates even slightly from the quadratic form. Although the curvaton needs to be light during inflation, it is unrealistic to assume that the potential would be exactly quadratic. Small self interactions of the form

$$V(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \lambda m_\sigma^{4-n} \sigma^n , \tag{4.20}$$

with $\lambda \ll 1$ have been studied in detail in [18]. If the correction to the quadratic potential is small at the end of inflation, the value of the curvaton field at the onset of oscillations can be approximated as [18]

$$\sigma_{\rm osc}(\sigma_*) \sim 0.8\sigma_* - 0.1\lambda n m_\sigma^{2-n} \sigma_*^{n-1} , \tag{4.21}$$

where we have dropped subdominant corrections presented in [18] to keep the discussion transparent. The first term in (4.21) is the quadratic result and the second term represents corrections arising from the interactions. This contribution in general yields a nonvanishing value for the term $\sigma''_{\rm osc}\sigma_{\rm osc}/{\sigma'_{\rm osc}}^2$ in the expression for the non-linearity parameter (4.19) and thus causes deviations from the quadratic results. In the limit $r \sim 1$, the deviations are not significant and the non-gaussianities remain small $|f_{\rm NL}| \sim 1$ as in the quadratic case. However, if $r \ll 1$, the last term in (4.19) dominates and the interaction term in the potential (4.20) becomes important. In particular, the solution (4.21) can yield $\sigma''_{\rm osc}\sigma_{\rm osc}/{\sigma'_{\rm osc}}^2 \sim -1$, which effectively cancels the quadratic contribution in (4.19). Therefore it is possible to obtain small non-gaussianities $|f_{\rm NL}| \sim 1$ even in the limit $r \ll 1$. This suppression generically takes place when the fraction of

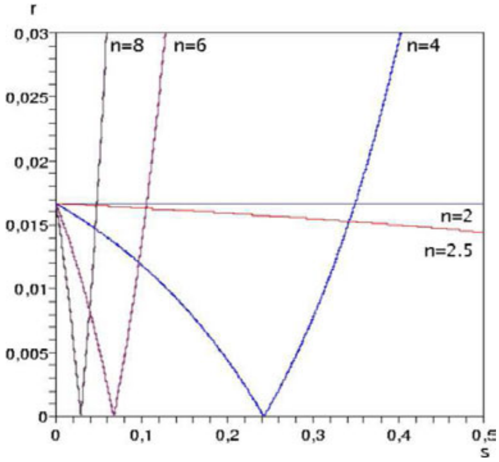

Figure 4.1: Allowed regions in the parameter space (s, r) where s describes the relative magnitude of the curvaton self-interactions compared to the quadratic part and r is the density parameter. The labels n denote the power of the self-interaction term and they are placed outside the region where the level of non-gaussianities is consistent with current observations. The line $n = 2$ corresponds to quadratic curvaton potential.

the interaction term in (4.20) over the quadratic part, denoted by $s \equiv 2\lambda(\sigma_*/m_\sigma)^{n-2}$, satisfies $s \lesssim 2/n$ [18]. The gaussian part of the curvature perturbation is not significantly affected by the small interactions. When the interactions become larger, $s \gtrsim 2/n$, the magnitude of the non-linearity parameter starts to grow but it changes sign when compared to the quadratic case. Given a detection of non-gaussianity, this information could be used to rule out certain forms of the curvaton potential. Observations seem to favour a positive value for $f_{\rm NL}$ [4] which suggests that the curvaton interactions need to be small $s \lesssim 2/n$ in the limit $r \ll 1$.

Since the predictions for non-gaussianities in the limit $r \ll 1$ are strongly modified even by small corrections to the quadratic potential, the WMAP upper limit $|f_{\rm NL}| \lesssim 100$ implies no strict lower bound for r unless the potential is exactly quadratic. In Figure 1, we have illustrated the bounds found in [18] for the density parameter. As compared to the quadratic case, $n = 2$ in Figure 1, the lower bounds on r are modified in particular for quartic and higher order interactions and the bounds are dependent on the magnitude s of the interactions.

Chapter 5

Summary

Cosmological inflation has become the dominant paradigm in explaining the origin of structure in the universe. The primordial perturbations are expected to arise from quantum fluctuations of some scalar field or fields that dominate the energy density of the universe during inflation and cause the almost exponential expansion. It is strikingly simple to construct phenomenological models that produce primordial perturbations which are in excellent agreement with the CMB observations but finding particle physics candidates for the inflationary matter has proven less straightforward [1, 2, 3, 5]. One of the major difficulties in constructing theoretically motivated models is the inflationary scale, which in the simplest models is only few orders of magnitude below the Planck scale $H \lesssim 10^{-5}M_{\rm P}$. Models with a considerably lower scale $H \ll 10^{-5}M_{\rm P}$ seem to require fine-tuning to produce large enough perturbations. On the other hand, the low scale would allow to describe the inflationary physics using plausible particle physics models like supersymmetric extensions of the Standard Model. Inflationary models based on the extensions of the Standard Model would be testable not only by their cosmological implications but in principle also by future collider experiments. This interplay between cosmology and high energy physics would open up new possibilities to gain insight about physics beyond the Standard Model.

In this thesis we have discussed two different scenarios in which the inflationary scale is well below the Planck scale. The MSSM inflation [7, 8] is a single field model where inflation is driven by one of the flat directions of the MSSM. Among all the flat directions, there are two candidates $LL\bar{e}$ and $\bar{u}\bar{d}\bar{d}$ for the inflaton field that yield the correct amplitude for primordial perturbations. The great advantage of the model is that the gauge couplings of the inflaton are known and the reheating of the universe after inflation along with the generation of the Standard Model fields can be explained in a self-consistent manner [7, 8]. The inflationary scale is very low $H \sim 1$ GeV which justifies the use of the MSSM but requires an extremely flat inflaton potential to produce large enough perturbations. The potential along the flat directions depends on the supersymmetry breaking mechanism. On purely phenomenological grounds, the flatness implies fine-tuning of the supersymmetry breaking parameters, which is an apparent drawback of the model. In the thesis work [10, 11], we have shown that the required flat inflaton potential can be obtained quite naturally without excessive fine-tuning in a class of reasonable supergravity models. The flatness is a consequence of the form of the Kähler potential and it arises irrespectively of the vacuum expectation values of the supersymmetry breaking fields. The Kähler potential needs to be chosen in a specific manner, but its general form is motivated by low energy limits of various string theory compactifications. The results are encouraging and alleviate the fine-tuning problems of the MSSM inflation but more work is still needed to check for example the stability of the flatness of the inflaton potential against radiative corrections.

The curvaton scenario [13] is another attractive low-scale inflationary model discussed in the thesis. In this scenario, the curvature perturbations are generated after inflation by oscillations of the curvaton field which is a light scalar field other than the inflaton. This leads to relaxation of the constraints on the inflationary scale but also makes the properties of the perturbations dependent on post-inflationary dynamics. The research work [18] consists of analyzing the connection between the level of non-gaussian perturbations and the form of the curvaton potential. Curvaton models with quadratic potentials typically produce large non-gaussianities [16, 17] but even small self interactions can significantly alter the result [18]. A detection of primordial non-gaussian statistics could therefore place strong limits on the curvaton interactions. This would be an interesting piece of information from the point of view of model building.

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