FROM DIFFUSION TO TRACTS

Viljami Sairanen

Department of Physics
Faculty of Science
University of Helsinki

HUS Medical Imaging Center
University of Helsinki and
Helsinki University Hospital

ACADEMIC DISSERTATION

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UNIVERSITY OF HELSINKI
FINLAND
Supervisors

Professor Sauli Savolainen, Ph.D.
Department of Physics
Faculty of Science
University of Helsinki
Helsinki, Finland

Professor Sampsa Vanhatalo, Ph.D.
Department of Clinical Neurophysiology
Faculty of Medicine
University of Helsinki
Helsinki, Finland

Preliminary examiners

Adjunct Professor Juha Nikkinen, Ph.D.
Department of Oncology and Radiotherapy
Oulu University Hospital
Oulu, Finland

Research Scientist Kerstin Pannek, Ph.D.
The Australian E-Health Research Centre
CSIRO
Brisbane, Australia

Opponent

Professor Miika Nieminan, Ph.D.
Research Unit of Medical Imaging
Physics and Technology
University of Oulu
Oulu, Finland

Cover art

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“The Guide is definite. Reality is frequently inaccurate.”

—The Hitchhiker’s Guide to the Galaxy by Douglas Adams
Abstract

Diffusion of water molecules within the brain tissue can be used to modulate the nuclear magnetic resonance signal that is used to form magnetic resonance images (MRI). As the signal itself can be noisy and its meaning challenging to interpret, mathematical models are generally fitted to these measurements to obtain the more accurate characterization of the brain microstructure. This, of course, requires that the mathematical model itself is sound in respect to the measurement setup. This dissertation focuses on the extensively used tensor models as they have been shown to unravel details of the physical diffusion phenomena along with various applications in the basic neuroscience, the clinical research, and even in the neurosurgery.

One of the greatest challenges in the diffusion weighted MRI measurements is subject motion during the image acquisition as that can cause a complete loss of the measurement which is especially highlighted in ill or uncooperative patients studies. Due to the used acquisition technique, this loss extends to multiple measurements simultaneously resulting in an enormous gap in the sampling. Such gaps can be problematic for any model fitting, even for the currently available robust means developed to exclude outlier measurements from affecting the estimate. Hence in this dissertation, a tool coined as SOLID was developed to detect these outliers and to robustly process them during the tensor based model estimation. SOLID was implemented as a part of the widely used ExploreDTI toolbox to allow the rapid international distribution of the tool.

Unfortunately, any reduction in the measurement sampling will lead to increasing error propagation during the model estimation. Mathematically this is detailed in terms of a condition number for the matrix inversion in the linear least squares fitting. Previously, the condition number has been used to optimize the diffusion weighted MRI acquisition gradient scheme but in this dissertation it was renovated into a novel quality control tool. The condition number of the matrix inversion that provides the model estimate can be calculated after the outliers are excluded to assess spatially and directionally varying error propagation to obviate any bias in subject or population studies.

To motivate the importance of the robust methods and diffusion weighted MRI at large, neurocognitive studies with neonates’ visual abilities and bilinguals’ acquisition age of the second language were conducted as a part of this thesis. The findings in these studies indicated that premature birth affects the white matter structures across the brain whereas the age of acquisition of the second language affects only the speech related brain structures.

Keywords: Magnetic resonance imaging, diffusion, tensor, subject motion, robust estimation, neurocognitive correlations.
Preface

The support from the doctoral school MATRENA of the University of Helsinki and the HUS Medical Imaging Center of the Helsinki University Hospital allowed me to accomplish this dissertation during the years from 2014 to 2018. Additional financial support for the project has been received from Sigrid Jusélius Foundation, the Academy of Finland, and Finnish Cultural Foundation.

I express my sincere gratitude to my supervisors. To Professor Sauli Savolainen for always supporting my ambitions in research and in practical Medical Physicist training during the past six years. To Professor Sampsa Vanhatalo for the equal support in research and arranging the financial support that allowed me to escape the previously mentioned practical training. The most influential of these flights was my three month visit to the University Medical Center Utrecht in 2016 where Professor Alexander Leemans warmly welcomed me to his PROVIDI Lab. It has been a privilege not to just work for these fine scientists but with them.

I thank Adjunct Professor Juha Nikkinen and Ph.D. Kerstin Pannek for their valuable and heartening feedback during the preliminary examination of this thesis. I am grateful to Professor Miika Nieminen for accepting the invitation to be the opponent in the public defense of this dissertation.

This thesis was not a one-man job. My enthusiasm towards diffusion weighted MRI was initially sparked by a curious lab assignment in 2013 from a colleague, a friend, and a co-author M.Sc. Linda Kuusela. Her support during this dissertation has been invaluable. Adjunct Professor Outi Sipilä deserves thanks for being an inspiring co-author and introducing the MRI to me in the first place. I would like to thank co-authors M.A. (Psych.) Susanna Stjernå and M.A. (Psych.) Sini Hämäläinen for the illuminating discussions on- and off-topic of this thesis. Finally, the kindest thanks go to a co-author, a mentor, and “the naive reviewer” Ph.D. Chantal Tax for her valuable feedback and never-ending ideas.

To my colleagues, family, and friends - as I am immensely fond of you all - I find no better words to end this preface and to describe my thoughts than a quote from a dear childhood hero: “I don’t know half of you half as well as I should like; and I like less than half of you half as well as you deserve.”1

1Bilbo Baggins, The Fellowship of the Ring by J.R.R. Tolkien
List of original articles

This thesis entails four original studies and their summary. Studies are referred with Roman numerals I-IV throughout the document. Studies II and IV are reprinted with the permission from Elsevier and Study III under the CC-BY 4.0 license.


Author’s contribution

For Study I, the author surveyed the previously published algorithms, developed the methods, performed all simulations and experiments with subject data and drafted the manuscript. For Study II, the author designed the study, revised the theory, performed all simulations and experiments with subject data and drafted the manuscript. For Study III, the author did diffusion MRI data processing and developed the statistical methods used in the analysis of the diffusion MRI data and test results for newborns’ visual abilities, participated in writing the manuscript and result visualization. For Study IV, the author designed the diffusion MRI acquisition scheme, processed the diffusion MRI data and participated in analyzing the results and writing the manuscript. The results of these studies have not been used in previous PhD studies.
# Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>AD</td>
<td>Axial Diffusivity</td>
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<td>ADC</td>
<td>Apparent Diffusion Coefficient</td>
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<td>AUC</td>
<td>Area Under Curve</td>
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<td>CN</td>
<td>Condition Number</td>
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<tr>
<td>CPU</td>
<td>Central Processing Unit</td>
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<td>DWI</td>
<td>Diffusion Weighted Image</td>
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<td>EK</td>
<td>Excess Kurtosis</td>
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<td>EP</td>
<td>Electrical Potential</td>
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<td>EPI</td>
<td>Echo Planar Imaging</td>
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<tr>
<td>FA</td>
<td>Fractional Anisotropy</td>
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<tr>
<td>FE</td>
<td>The direction of the First Eigenvector of diffusion tensor</td>
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<tr>
<td>GB</td>
<td>Gaze Behaviour</td>
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<td>GIB</td>
<td>Gradual Intensity Band</td>
</tr>
<tr>
<td>IFOF</td>
<td>Inferior Fronto-Occipital Fasciculus</td>
</tr>
<tr>
<td>IQR</td>
<td>Interquartile Range</td>
</tr>
<tr>
<td>IWLLS</td>
<td>Iteratively Re-Weighted Linear Least Squares</td>
</tr>
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<td>KA</td>
<td>Kurtosis Anisotropy</td>
</tr>
<tr>
<td>L2</td>
<td>Second Language</td>
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<tr>
<td>LLS</td>
<td>Linear Least Squares</td>
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<tr>
<td>MAD</td>
<td>Median Absolute Deviation</td>
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<td>MD</td>
<td>Mean Diffusivity</td>
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<tr>
<td>MK</td>
<td>Mean Kurtosis</td>
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<tr>
<td>MRI</td>
<td>Magnetic Resonance Imaging</td>
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<tr>
<td>NC</td>
<td>Normalized Cross-correlation</td>
</tr>
<tr>
<td>PRC</td>
<td>Precision-Recall Curve</td>
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<tr>
<td>RD</td>
<td>Radial Diffusivity</td>
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<tr>
<td>ROC</td>
<td>Receiver Operating Characteristic</td>
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<tr>
<td>ROI</td>
<td>Region Of Interest</td>
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<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
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<tr>
<td>TBSS</td>
<td>Tract-Based Spatial Statistics</td>
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<tr>
<td>V1</td>
<td>See FE</td>
</tr>
<tr>
<td>VF</td>
<td>Visual Fixation</td>
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1 Introduction

Diffusion weighting augments magnetic resonance imaging (MRI) with sensitivity toward the motion of water molecules which has been shown to be advantageous for the microstructural investigation of the living brain [1–5]. This fascinating ability to explore the molecular motion provides information beyond any normal structural MRI and numerous studies since the initial publication of the technique have utilized it to decipher the white matter connectivities [6–12], structural differences due to language learning [13–20] or premature birth [21–26] and aging of the brain [27–32]. In addition, diffusion weighted MRI has provided powerful clinical applications [33, 34] to detail brain injuries, diseases, and disorders [35–41]. However, to obtain such reliable and sound findings from diffusion weighted MRI data, several aspects of the MRI acquisition, physical diffusion phenomenon, and practical limitations must be first familiarized.

An image acquired using a MRI scanner consists of small three-dimensional volume elements called ‘voxels’. The position relative to the scanned subject and the size of this voxel are controlled with spatial gradient fields. To obtain the sensitivity toward the motion of water molecules, two additional carefully timed diffusion gradient fields are also required [42–45]. The first diffusion gradient dephases the spins of the target nuclei e.g. protons within hydrogen atoms. After a specific diffusion time, a reverse diffusion gradient is applied to rephase the spins [46–50]. If atoms in water molecules have remained stationary relative to the voxel during this process, only spin-to-spin relaxation effects are observed resulting in a small signal decrease [51]. However, if a notable motion in the direction of these diffusion gradient fields occurs, the signal will suffer much larger decay depending on the amplitude and duration of the gradient fields and diffusion time which can be summarized with a diffusion weight factor known as the \( b \)-value [52]. Information about pathways that water can effortlessly move along to or microstructures that restrict motion and thus the diffusion process can be studied in detail by applying these gradient fields in multiple directions [2–5, 53–59].

The diffusion itself is a random phenomenon occurring due to the thermal energy bound in the molecules [60, 61]. The expected displacements of molecules follow a Gaussian distribution in the case of free diffusion which is likely in the brain ventricles, for example. However, in tightly packed brain structures such as white matter, the diffusion can be highly restricted in many directions and the expected molecular displacements could become non-Gaussian [62, 63]. This explains why a more detailed characterization of the underlying diffusion phenomenon can be obtained by using multiple \( b \)-values; if the diffusion is highly restricted in some direction the signal in that direction will not decay due to diffusion even after a long diffusion time. It should be noted that also other effects such as perfusion or molecular motion due to concentration differences can confound these measurements [64, 65]. Regardless how interesting further theoretical underpinnings would be, they largely stand outside the main scope of this dissertation and as such are omitted. For a more complete and detailed descriptions of nuclear magnetic resonance, MRI, and diffusion weighting the reader is advised to study the
works of Callaghan, Liang, Slichter, Jones, or Hecke [51, 52, 66–68] and the work of Berg [69] for diffusion in biology. For the remainder of this thesis, it is enough to appreciate that the random motion of water molecules results in a measurable signal decay in diffusion weighted images (DWI). Thus, acquiring multiple DWIs with diffusion gradients in different orientations and $b$-values it is possible to uncover the underlying tissue microstructure [2–5, 57–59].

The DWI intensity, however, is affected by various errors [70–72] arising from technical sources such as noise [73–77], signal drift [78], bandwidth limitations [51, 79, 80], eddy current [81–84] and susceptibility distortions [85–88] and physiological sources e.g. subject motion [89–94] and cardiac pulsation [95]. Thus, unlike the commonly used T1 or T2 weighted images [51], deducing causalities directly from DWIs would range from challenging to altogether impossible and a prior knowledge of the tissue of interest is required to form a sound mathematical model that holds the explicable characteristics of the tissue such as the direction of the fiber population. The most used methods of modelling the brain microstructure are the voxelwise diffusion and kurtosis tensors [3, 62, 63, 96–106] but other models with a higher number of degrees of freedom have also been suggested in order to elucidate voxels affected by partial volume effects [107] such as crossing white matter fiber populations [108–117]. Modelling also adds yet another layer of uncertainty to the analysis, as noise or any other artefacts in the raw measurements will propagate to the model, depending on both the mathematical properties of the model and how the data was acquired [118, 119].

The two most undesirable errors that affect the diffusion MRI are scanner hardware related issues and the subject motion as they can potentially lead to geometrical or intensity errors in the measured DWIs [70, 72, 120]. Even with the recent technological advancements, such as the simultaneous acquisition of multiple slices [121–123], the total acquisition time of a diffusion MRI study can surpass ten minutes. For an unanaesthetised human, especially for uncooperative patients, this is a long time to stay completely still. Any drastic motion of the target nuclei can lead to the different spatial placement of measured signals between DWIs or, if occurring during the acquisition of a DWI, motion can affect the dephasing and rephasing cycle. The former means that the signal in the same voxel of two different DWIs might not originate from the same region or could be affected by a different degree of partial voluming [107] and thus the intensity of these voxels in different DWIs might not fully relate to each other and impede the model estimation. The latter is the reason for outlier measurements. While geometrical image distortions and misalignments between subsequent DWIs can be amended using image registration algorithms [84, 89, 90, 93, 94, 124, 125], a more tedious problem of inaccuracy in the signal intensity due to the imperfect spin manipulation with the diffusion gradients cannot be corrected and the outliers will remain in the data.

Erroneous diffusion MRI measurements will affect any model estimation and multiple attempts have been made to improve simple least squares model estimators [3, 97, 126] with robust means [127–132] or with the ability to detect and exclude outliers based on the model residual values [133–136]. Maximum likelihood estimators that consider the true Rician noise distribution of DWIs have also been suggested [73, 75, 76, 137–141], however, the widespread usage of maximum likelihood methods has been restricted by the long computational times. As DWIs are typically acquired with a fast slicewise echo planar imaging (EPI) techniques [87, 142–146], subject motion related artefacts obfuscate all voxels within the slice simultaneously and this
could be utilized in the outlier detection as well [147–154]. Regardless of these previous studies, in this dissertation a need for an improvement in slicewise outlier detection and robust model estimation was established, especially for neonatal studies as newborns tend to move more during the acquisition. The tool was coined as SOLID (Slicewise OutLIer Detection) and is distributed as a part of ExploreDTI toolbox [155].

The subject motion could also affect the direction in which the diffusion was measured i.e. diffusion gradient vectors. Since the early days of diffusion MRI, an immense effort has been put into optimizing the best acquisition directions i.e. the design matrix or b-matrix [156] for different applications and models [118, 157–168]. However, the fact that subject motion can potentially eradicate this optimization was largely neglected as it was assumed that it would be enough simply to apply the rotations from DWI corrections on the diffusion gradient vectors [93]. Such post-acquisition corrections, however, cannot solve the issue that if the original gradient vectors were optimized by uniformly sampling points on a spherical surface, when a subject rotate their head the measurements are no longer sampled uniformly. Especially, if motion results in intensity artefacts which are excluded as outlier measurements and all information about diffusion in that direction is lost resulting in a gap in the sampling. This could cause a serious bias between subjects in population studies or even spatial differences within a single subject, hence a quality control tool for assessing these biases was developed during this dissertation. The method is based on a mathematical property of the least squares problem design matrix called ‘condition number’ (CN) which defines the upper boundary for the error propagation during the model estimation. While the condition number was originally used to optimize the gradient directions [118, 161], it had not been utilized previously to data quality assessment after all corrections and robust estimation were made.

To highlight the enthusiasm towards the robust diffusion MRI data processing, an extensive set of experiments has been conducted ranging from the theoretical aspects of acquisition [81, 118, 157, 159–163, 169], to processing [82, 83, 85, 87, 89, 90, 125, 147, 152, 153, 155, 170–174], analyzing and modelling [91, 175–190], visualizing the measurements [22, 24, 25, 57, 101, 102, 191–195], and performing quality control [120, 147, 151, 152, 155, 196, 197] denoting that diffusion MRI, as a technique, is far from complete and opening the possibility to re-investigate previous studies with the new methods or to conduct the entirely new sets of practical experiments.

This thesis presents novel additions to three parts of the complex diffusion MRI analysis: firstly to slicewise outlier detection, secondly to the quality control of the final model estimate, and lastly to the statistical analysis. In Study I, SOLID framework was developed to disclose outliers occurring due to subject motion or hardware issues as the uncertainty of measurements to guide a robust model estimation. To further evaluate the reliability of the processed diffusion MRI measurements, a quality control method was constructed around the rotational variant condition number in Study II. Both methods were evaluated with comprehensive simulations and validated with newborn and adult human data. Finally, two practical Studies III and IV on neurocognitive correlations between the white matter structures and the visual fixation (VF) of the newborns and the age of acquisition of the second language (L2) of adults motivate how the methods from Studies I and II could practically push forward research in the wide field of neuroscience.
2 Aim of the study

The aim of this dissertation was to deploy advanced diffusion MRI acquisition protocols and improve their analyses methods including visualization in order to help clinical research at Helsinki University Hospital. However, during the work, the insufficient means or even a total absence of quality control tools for diffusion MRI data was noted and the aim of the thesis was shifted towards the development of robust preprocessing methods including tools for evaluating the comparability and quality of the final results. Thus, the studies were designed to:

1. Develop a fast and accurate method for disclosing artefactual data and a framework to handle outliers properly during model estimation;
2. Generate a quality control tool that can be used to investigate error propagation in a voxelwise manner in both individual and population experiments;
3. Apply the robust diffusion MRI analysis on newborn and adult human data along with the statistical analysis and visualizations of the results.
3 From Diffusion to Tracts

The chapter is divided into three sections, the first two summarizing Studies I and II and the third combining neurocognitive Studies III and IV. The sections begin with the theoretical background of the problem and continue through the improvements and investigations conducted on the theory and end in descriptions of the methodological aspects in each study. Experiments involving patients or healthy volunteers (57 neonates and 34 adults in total) were approved by the relevant ethics committees detailed in Studies I-IV respectively.

3.1 Slicewise OutLier Detection (SOLID)

As diffusion weighted MRI is typically acquired using EPI sequences [143, 145, 146], data is gathered in a slicewise manner. Consequently, any error occurring during the acquisition, whether due to a moving subject [89–94] or sub-optimal hardware functionality, will likely corrupt a whole slice. Furthermore, any errors in the measured signal intensities will propagate into errors in the estimated models [118] and they should be controlled to avoid unnecessary bias in the further analyses. However, this cannot be achieved by simply detecting outliers during the voxelwise modelling [133, 134, 136, 148] as the intensity errors could become interpolated with neighbouring data points during the subject motion correction which is used to ensure the geometrical comparability of acquired images [84, 89, 90, 125]. For example, an axially acquired slice with a signal intensity artefact that is rotated along anterior-posterior (nod) or left-right (tilt) will form a gradual intensity band (GIB) in the corrected image space, as shown in Study I, and the edge of GIB could be problematic for any voxelwise robust estimator as only a part of the signal is corrupted.

3.1.1 Theoretical aspects of outlier detection and handling

Statistical outlier detection is usually based on some assumption on the distribution of the observations or measurements [198]. If observations \( Y \) are drawn from a normal distribution \( \mathcal{N}(\mu, \sigma^2) \), where \( \mu \) is the mean and \( \sigma \) is the standard deviation, an observation several \( \sigma \) distances to the mean could be considered an outlier. This was translated into a practical slicewise outlier detection method for diffusion MRI in Study I by defining an observation as a slice \( k \) and DWI \( l \) specific intensity metric \( y_{k,l} \). For example, the variance of all brain voxel intensities in one slice of a DWI would give one observed \( y_{k,l} \) value. By assuming that the slicewise intensity metric across DWIs follows a normal distribution, it is possible to calculate the slicewise Z-score \( z_{k,l} \) (eq. 1.1) to determine if observation \( y_{k,l} \) is an outlier. As this normality assumption is likely violated due to outliers skewing the distribution and likely masking milder outliers, the implementation of the modified Z-score [198] \( \zeta_{k,l} \) (eq. 3.1) was chosen instead.
of the normal Z-score. In the modified Z-score, the sample mean $\bar{y}$ and the sample standard deviation $s$ are replaced with the sample median $\tilde{y}$ and the median absolute deviation (MAD) [130]:

$$\zeta_{k,l} = \frac{y_{k,l} - \tilde{y}_k}{\text{MAD}_k}.$$  \hspace{1cm} (3.1)

While the simplest approach would be to reject observations with large $\zeta_{k,l}$, this could lead rapidly to a missing data problem and thus interpreting $\zeta_{k,l}$ as an estimate for the measurement certainty was chosen instead. A schematic overview of this is shown in Figure 3.1. Intensity variance was chosen as the slicewise intensity metric $y_{k,l}$ for it is fast and easy to compute and provided a good capability to disclose outliers. As $\zeta_{k,l}$ are also slice and DWI specific measures, they can be used to fill a 3D volumes $\xi_l$ ($l = 1,...,N$ where $N$ is the number of measured DWIs) so that all voxels $x$ in each slice and DWI have the same value. After the subject motion, eddy current and other geometrical misalignments and distortions are corrected for [84, 89, 90, 93, 94, 125] and DWI volume specific transformations $T_l$ are obtained from the registrations, they are onward applied on these volumes $\xi_l$ to obtain the interpolated modified Z-scores in the corrected image space i.e. $\xi'_l(x') = \xi_l(T_l(x))$. Voxelwise $\zeta_l(x')$ values can now be linearly scaled to SOLID weights $S_l(x')$ between 0 (outlier) and 1 (normal data) using lower and upper thresholds $t_L$ and $t_U$ (eq. 3.2)

$$S_l(x') = \begin{cases} 0, & \text{if } \zeta_l(x') > t_U \\ \frac{\zeta_l(x') - t_U}{t_L - t_U}, & \text{if } t_L \leq \zeta_l(x') \leq t_U \\ 1, & \text{if } \zeta_l(x') < t_L \end{cases}.$$  \hspace{1cm} (3.2)

As DWIs are acquired in multiple orientations to capture the diffusion phenomenon within the tissue, it is presumable that intensity metric across DWIs with the same slice position varies accordingly even in a hypothetical situation of infinite signal to noise ratio (SNR). A lower threshold $t_L = 3.5$ can be used to account for this normal variation of the observations and an upper threshold of $t_U = 10$ to prevent extreme outliers from saturating the downweighting framework. A clear benefit in such slicewise outlier detection is that it does not depend on the estimated model as previously used M-estimators such as the Geman-McClure algorithms [129, 130, 173] do and potentially has more statistical power to disclose the outliers than voxelwise approaches [133, 134, 136]. The linear relation in eq. 3.2 could be replaced with a sigmoid function for a more natural change between normal to outlier data but that would also require adjusting the lower and upper thresholds differently.

### 3.1.2 Theory behind SOLID-informed tensor estimator

SOLID weights can inform a generally used estimation method that models the voxelwise diffusion weighted signal $s(x' : b, g) := s(b,g)$ based on the symmetric diffusion
Figure 3.1: Proposed framework for slicewise outlier detection and handling. A) Intensity histograms show examples of typical DWI slices: normal (top), signal decrease (middle), and signal increase (bottom). B) Slice \( k \) and DWI \( l \) specific metric \( y_{k,l} \) is calculated from all voxels \( x \). C) The modified Z-score \( \zeta_{k,l} \) is obtained with the help of the slicewise median \( \bar{y}_k \) and the median absolute deviation MAD\(_k\). Thresholding of the \( \zeta_{k,l} \) shows immediately that most of the slices are below normal variation threshold (3.5) whereas some are clearly outliers with \( \zeta_{k,l} > 10 \). D) The geometrical transformations \( T_l \) of DWIs are obtained during the correction of geometrical distortions. Due to the rotation of the artefactual DWI, the axial slice with signal decrease is distributed over multiple axial slices forming a gradual intensity band in the corrected image space. Slice orientation is indicated with the red dashed line for reference. E1) \( T_l \) are used to calculate the voxelwise interpolated modified Z-scores \( \xi_l'(x') \) which are E2) linearly scaled to SOLID weights \( S_l(x') \) and used as data certainty estimates in E3) the tensor model estimation.
\[ D = \{ D_{ij} : i, j = 1, \ldots, 3 \} \] and kurtosis \( K = \{ K_{ijkl} : i, j, k, l = 1, \ldots, 3 \} \) tensors [3, 62, 63, 96–106] as function of a \( q \)-space shell [66] i.e the gradient direction \( g = \{ g_i : i = 1, \ldots, 3 \} \) and its strength \( b \):

\[
s (b, g) = s (0, 0) \cdot \exp \left( -b \sum_{i,j=1}^{3} g_i g_j D_{ij} + \cdots \right)
\]

\[
\cdots + \frac{b^2}{6} \left( \sum_{i=1}^{3} D_{ii} \right)^2 \sum_{i,j,k,l=1}^{3} g_i g_j g_k g_l K_{ijkl} + \varepsilon
\]

(3.3)

where \( \varepsilon \) is the error term independent of the gradient direction or strength. The relation between diffusion weighted signal \( s (b, g) \) is thus represented with an approximately exponential decay of the non-diffusion weighted signal \( s (0, 0) \) i.e. \( b^0 \)-image which has been shown to correlate well with in vivo measured signal decay for \( b \)-values below \( 300 \text{mm}^2/\text{s} \) [68, p. 411]. This can be expressed more conveniently as

\[
s (b, g) = \exp (X \beta) + \varepsilon
\]

(3.4)

with \( X \) being the \( N \times 22 \) design or encoding matrix (eq. 3.5) [156, 162] whose rows \( X_{i=1,..,N} \) describe the relations for each diffusion gradient direction and its strength for each measurement

\[
X_i := \left( 1, \quad b_1 g_{11}^2, \quad 2 b_1 g_{11} g_{12}, \quad 2 b_1 g_{11} g_{13}, \quad b_1 g_{12}^2, \quad 2 b_1 g_{12} g_{13}, \quad b_1 g_{13}^2, \quad \cdots \right)
\]

\[
\cdots \frac{1}{54} b_i^2 g_{i1}^2, \quad \frac{1}{54} b_i^2 g_{i2}^2, \quad \frac{1}{54} b_i^2 g_{i3}^2, \quad \frac{4}{54} b_i^2 g_{i1} g_{i2}, \quad \frac{4}{54} b_i^2 g_{i1} g_{i3}, \quad \frac{4}{54} b_i^2 g_{i2} g_{i3}, \quad \cdots
\]

\[
\cdots \frac{4}{54} b_i^2 g_{i1} g_{i2}, \quad \frac{4}{54} b_i^2 g_{i1} g_{i3}, \quad \frac{4}{54} b_i^2 g_{i2} g_{i3}, \quad \frac{4}{54} b_i^2 g_{i1}^2, \quad \frac{6}{54} b_i^2 g_{i2} g_{i3}^2, \quad \frac{6}{54} b_i^2 g_{i1} g_{i3}^2, \quad \frac{6}{54} b_i^2 g_{i2}^2, \quad \cdots
\]

\[
\cdots \frac{6}{54} b_i^2 g_{i1}^2 g_{i2}, \quad \frac{12}{54} b_i^2 g_{i1} g_{i2}^2, \quad \frac{12}{54} b_i^2 g_{i1} g_{i2} g_{i3}, \quad \frac{12}{54} b_i^2 g_{i2} g_{i3}^2, \quad \frac{12}{54} b_i^2 g_{i1}^2 g_{i3}, \quad \frac{12}{54} b_i^2 g_{i2} g_{i3}^2, \quad \frac{12}{54} b_i^2 g_{i1}^2 g_{i3}^2, \quad \frac{12}{54} b_i^2 g_{i2} g_{i3}^2 g_{i1} g_{i2} \right)
\]

(3.5)

\( \beta \) is the unknown \( 22 \times 1 \) vector of the natural logarithm of noiseless non-diffusion weighted \( s (0, 0) \) signal and tensor elements to be estimated:

\[
\beta := \left( \ln (s (0, 0)), \quad D_{11}, \quad D_{12}, \quad D_{13}, \quad D_{22}, \quad D_{23}, \quad D_{33}, \quad \cdots \right)
\]

\[
\cdots c W_{1111}, \quad c W_{2222}, \quad c W_{3333}, \quad c W_{1112}, \quad c W_{1113}, \quad c W_{2221}, \quad \cdots
\]

\[
\cdots c W_{2223}, \quad c W_{3331}, \quad c W_{3332}, \quad c W_{1122}, \quad c W_{1133}, \quad \cdots
\]

\[
\cdots c W_{2233}, \quad c W_{1123}, \quad c W_{2213}, \quad c W_{3312} \right)^T
\]

(3.6)

and \( c \) being the square of the trace of the diffusion tensor \( c = \text{tr} (D)^2 = (\sum_{i=1}^{3} D_{ii})^2 \). Thus, a solution for the diffusion tensor from eq. 3.4 requires measuring at least six DWIs with
non-collinear directions and the kurtosis expansion additional 15 DWIs as both tensors are symmetric. The non-diffusion weighted signal can also be estimated giving the total of 22 parameters to solve. By taking the natural logarithm from eq. 3.4 the problem is transformed to a simple linear regression [52]

\[ z = X\beta + \epsilon (X, \beta) \]  

(3.7)

where \( N \times 1 \) vector \( z \) consists of natural logarithms of measured signals and \( N \times 1 \) vector \( \epsilon (X, \beta) \) is now an error term that depends on the gradient direction and its strength as shown in the next chapter. If SNR of DWIs is adequate, it has been shown that the best linear unbiased estimator [199] for tensors is a weighted linear least squares estimator:

\[ \hat{\beta} = \left( X^T \tilde{W} X \right)^{-1} X^T \tilde{W} z \]  

(3.8)

with the voxelwise weights \( \tilde{W} \) estimating the unknown noise-free measurements \( W = \text{diag} \left( s(b,g)^2 \right) \) [131]. The weight estimates are obtained from the ordinary linear least squares estimator and can be improved with an iteratively re-weighted linear least squares (IWLLS) approach [131] by updating the estimate on each iteration step \( m \):

\[ \tilde{W}_m = \text{diag} \left( \exp \left( 2X\hat{\beta}_{m-1} \right) \right) . \]  

(3.9)

SOLID information (eq. 3.2) is applied to these weights by concatenating \( S_l \left( x' \right) \) over all shells to a single voxelwise vector \( S \) and using that as an additional factor in eq. 3.9:

\[ \tilde{W}_m = \text{diag} \left( S \cdot \exp \left( 2X\hat{\beta}_{m-1} \right) \right) . \]  

(3.10)

In practice this ensures that any voxel \( x' \) with \( \xi_l \left( x' \right) < t_L \) will receive a SOLID weight of 1 and the IWLLS estimator handles it as normal data whereas voxels with \( \xi_l \left( x' \right) > t_U \) receive a weight of 0 and thus are excluded from the model estimation process and voxels in between i.e. \( t_L \leq \xi_l \left( x' \right) \leq t_U \) are downweighted depending how far off from the normal data they are.

### 3.1.3 Evaluation of SOLID

SOLID was evaluated with outlier simulations based on the MASSIVE database [190]. The ground truth consisted of 30 DWIs on two shells with \( b \)-values 1000\( s/mm^2 \) and 2000\( s/mm^2 \) and infinite SNR. Ten different simulations were arranged. In the first two, one slice in one DWI based on \( b \)-value 1000\( s/mm^2 \) was replaced with a -100% signal decrease outlier or a +50% signal increase outlier. In simulations three and four, five slices in eight DWIs based on \( b \)-value 1000\( s/mm^2 \) were replaced with signal decrease and increase outliers. In simulations five and six the number of outliers was kept the same but the \( b \)-value was increased to 2000\( s/mm^2 \). In the
last four simulations $b$-value $1000\text{s/mm}^2$ shell was again used with multiple signal decrease and increase outliers but now each artefactual DWI was either rotated by $5^\circ$ degrees around the left-right or anterior-posterior axes to simulate head nods and tilts.

These ten setups cover a range of typical clinical $b$-values, situations with a few outliers, and situations with plenty of outliers. Rotations were used to simulate slight nods and tilts that subjects might make during the acquisition that could potentially hinder the SOLID performance as anatomical regions between artefactual and normal DWIs would not be exactly the same. Random selection of artefactual DWIs and slice positions was repeated 1000 times for each arrangement before introducing Rician noise with three different SNR levels: 8, 16, and 32. Finally, the sensitivity, specificity, and positive predictive value of SOLID were evaluated with receiver operating characteristic (ROC) profiles and precision-recall curves (PRC) by applying hypothetical thresholds from 0 to 10 to the modified Z-score $\zeta_{k,l}$ and recording the number of true and false positive or negative findings. The threshold is referred as hypothetical as with our downweighting framework it does not directly reject outliers.

3.1.4 Comparison to previously published tools

A slightly different simulation was designed to compare SOLID with previously published tools DTIPrep [152] and DTI Studio [147]. DTIPrep and DTI Studio have a fundamentally different approach to outlier detection i.e. they search for outliers within DWIs whereas SOLID searches for them across DWIs. Thus, only a qualitative example was given with 30 DWIs based on $b$-value $1000\text{s/mm}^2$ of which one DWI had multiple signal decrease and increase outliers. For a more quantitative comparison with FSL EDDY [153], which also searches for outliers across DWIs, simulations of eight artefactual DWIs out of 30 based on $b$-value $1000\text{s/mm}^2$ each having five outlier slices with a combination of signal increase and decreases along with a $5^\circ$ rotation around the left-right axis were generated. SOLID performance was evaluated again with the hypothetical modified Z-score thresholds from 0 to 10 whereas FSL EDDY was evaluated by its standard deviation threshold from 0 to 10 to obtain ROC profiles and PRCs.

3.1.5 Evaluation of SOLID-informed tensor estimator

SOLID-informed model estimator (eq. 3.10) was investigated by selecting one voxel from the corpus callosum in the white matter and one voxel from the deep gray matter from the MASSIVE database [190]. The ground truth signals of these two voxels were used to estimate the ground truth diffusion and kurtosis tensors, five non-diffusion weighted signals, and diffusion weighted signals on two shells with 60 DWIs each based on $b$-values of $1000\text{s/mm}^2$ and $2000\text{s/mm}^2$. Different regions of a GIB were simulated by replacing 0, 1, 2, 4, 8, or 16 signals on both shells with outliers of varying magnitudes from a full signal decrease of -100% to a modest +50% signal increase before introducing Rician noise (SNR levels of 16, 32, and 64). Noise iterations were repeated 1000 times, each time selecting new random signals to be replaced with outliers, before fitting the diffusion kurtosis equation (eq. 3.3) [131, 155]. Finally, to simulate the ameliorating effect of the SOLID-information, outlier intensities were gradually decreased from one to zero with the additional SOLID weights $S(x^\prime)$. 

10
3.1.6 SOLID validation with human data

The validation of SOLID was done against ‘a gold standard’ i.e. manually detected outliers from the 54 neonatal subjects in Study III. In total, the gold standard consisted of 156 sets of 15 DWIs each, as the acquisition was repeated up to four times if the subject was co-operative. PRCs indicated that the initial SOLID quality control resulted in numerous false positive findings thus a revised manual analysis of SOLID output was done and false or true positives were updated accordingly. One neonatal subject with severe motion artefacts was picked for further visualization purposes to motivate the adjusting of the lower threshold $t_L$ in eq. 3.2 along with one additional adult subject from Study IV with no visible motion artefacts to be used as as an good quality data reference.

3.2 Error propagation in the linear least squares estimation

Rejecting outliers or even downweighting them properly can potentially lead to a missing data problem. As the optimal acquisition schemes for diffusion MRI are based on specific diffusion gradient orientations [118, 157–168], rotating these vectors to correct for the subject motion [84, 89, 90, 93, 94, 125] and rejecting [133, 134, 136] or weighting [127–132] the outlier measurements during the estimation can lead to a situation where the vectors are no longer distributed optimally. It is unlikely that with modern acquisition schemes with a moderate or even high angular resolution the inversion of the design matrix $X$ would be fully ill-conditioned resulting in a non-finite linear least squares (LLS) estimate. However, an increased subjectwise variation is plausible and should be considered especially when comparing healthy volunteers and patient groups who might move more during the acquisition.

3.2.1 Theoretical aspects of the error propagation in LLS estimation

A popular method to obtain the optimized gradient vectors for the acquisition can be explained with an analogy to the Thomson problem i.e. minimizing the electrical potential of a system of point charges on the surface of a unit sphere [158, 200]. If one or more of these point charges would be removed, this hypothetical electrical potential would change and as proposed by Chen et al. 2015 [186] this could be used as a quality assurance tool for diffusion MRI data after outlier rejections. The electrical potential, however, is rotationally invariant [118] meaning that the unit sphere with charges fixed on its surface could be rotated to any orientation without affecting the potential. As mentioned in the previous chapter, the error term $\epsilon$ in eq. 3.7 depends on both the gradient direction $g$ and its strength $b$ thus quality control with the electric potential clearly disregards theoretical aspects of the LLS estimation. Rewriting eq. 3.4 helps to understand why this directional dependent error occurs

$$s (b, g) = \exp (X\beta) + \epsilon = \exp (X\beta) \left(1 + \frac{\epsilon}{\exp (X\beta)}\right)$$

(3.11)
and when taking the natural logarithm from both sides with the Taylor series approximation
\[
\ln \left(1 + \frac{\epsilon}{\exp(X\beta)}\right) \approx \frac{\epsilon}{\exp(X\beta)} = \epsilon \, (X, \beta)
\]
the directional dependency of the error term is confirmed:

\[
\ln \left(s (b, g)\right) \approx X\beta + \epsilon \, (X, \beta).
\]

(3.12)

In general, it would be preferred to have a uniform error propagation in all directions to avoid directionally biased tensor estimates. To achieve this the design matrix \(X\) should be optimized so that its condition number \(\kappa (X) := \sqrt{\frac{\lambda_{\text{max}}(X^T X)}{\lambda_{\text{min}}(X^T X)}}\) that controls the relative boundaries of the error propagation in the LLS system [118]:

\[
\frac{\text{relative error in } \hat{\beta} (= X \setminus z)}{\text{relative error in measurements } z} \leq \kappa (X)
\]

(3.13)

remains as uniform as possible in all directions. In theory, the best value for the condition number is one as that would mean that error does not propagate during the model estimation and no estimator can decrease the measurement error. In practice, however, this is not possible and achieving a rotationally invariant condition number should be desirable instead as all directions would have equal error propagation and the model would not become biased. Unfortunately, such optimized gradient schemes are not possible either as the subjects always move during the acquisition and this must be corrected using image registration algorithms on DWIs [84, 89, 90, 93, 94, 125] and the design matrix \(X\) [93] to model the diffusion process correctly, thus likely rendering the scheme rotationally variant.

Intensity outliers makes the situation even more challenging. When such outliers are excluded, also the respective row in the design matrix \(X_l\) has to be removed causing potentially large deviations in the condition number. Whether the outliers are detected in slicewise [147, 152, 153] or voxelwise [130, 133, 134, 136] manner, this causes spatial differences in the error propagation within subject and could result in severe bias in studies where comparisons are done between healthy volunteers thus likely still subjects and patients that due to pain or discomfort tend to move more. Hence, investigating the condition number after the geometrical transformations are done and outliers rejected or downweighted should be preferred over the verification of the electrical potential.

3.2.2 Simulation experiments

Study II aimed to investigate changes in the condition number and, more interestingly the changes in the tensor derived measures fractional anisotropy (FA) [4] and direction of the first eigenvector (FE) [4] as a function of the incremental data rejections. The ground truth cylindrically symmetric diffusion tensors were simulated based on FA values from 0.05 to 0.95 with a step size of 0.10 and assuming mean diffusivity of \(2.1 \cdot 10^{-3} \text{mm}^2/\text{s}\). As the white matter
bundles can travel in any direction within the brain, tensors were rotated to 100 different orientations providing in total 1000 unique diffusion tensors. The initial diffusion gradient scheme based on 30 nearly uniformly distributed directions [158] and a $b$-value of $1000 \text{s/mm}^2$ were used to simulate DWIs from these tensors with 1000 Rician noise iterations with SNRs of 18 and 36.

Outlier rejections were simulated by calculating all possible sub-schemes of the initial 30 direction gradient scheme. The diffusion tensor has six degrees of freedom i.e. at least six DWIs are needed for an unambiguous estimation. As the sum of possible $h_{i=6...30}$ combinations from 30 DWIs is $\sum_{i=6}^{30} \binom{30}{i} = 1073567387$, it would not been feasible to use them all to estimate the tensors to calculate FA and FE values due to the immense computational time required. Instead, the electric potential (EP) was calculated and sub-schemes were rotated to 100 different orientations (Figure 3.2A) to evaluate the minimum and the maximum condition numbers (CN). Four groups of sub-schemes were created based on these minima and maxima values: the best CN and the best EP with the smallest maximum CN and the smallest EP values and the worst CN and the worst EP with the largest maximum CN and the largest EP values. Even these simple metrics required a computational time of approximately 5 ms for each sub-scheme resulting in a total computing time of two months with one CPU. Parallelization of the problem was achieved by enumerating the sub-schemes in the lexicographical order [201] followed by unranking [202] them into a blocks of $10^5$ sub-schemes for each CPU. Finally, these four groups of sub-schemes were used to select parts from the ground truth DWIs to be used in the LLS diffusion tensor estimations [155]. Derived 1000 FA and FE values were reported as differences from the ground truth FA (mean ± standard deviation) and FE (median ± interquartile range) for each ground truth FA and FE value separately.

### 3.2.3 Human data experiments

Six neonatal subjects (three from Study III) and three adult subject experiments were carried out to visualize these effects on real diffusion MRI data derived FA and FE values along with a streamline tractography [57, 155, 203] of the corticospinal tract [204]. Subject data was visually inspected to contain no slicewise outliers as they would have hindered this experiment. The preprocessing was done with FSL topup and EDDY [173] or eddy correct [87, 205] depending on the data acquisition detailed in the manuscript II. The adult subjects were scanned with the same 30 direction scheme used in the ground truth simulation part. As the data was corrected for subject motion, the sub-schemes had to be recalculated for each of them separately, highlighting the need for parallelization of the $h$-combinations problem.

Neonatal data consisted of two groups with three different subjects in both of them. Group A) was scanned with 32 non-collinear directions which were vendor specific [161] but not optimized for rotational variance [118]. The first two vectors along with x and y-axes were removed from the set to form a set of 30 DWIs with almost uniform angular sampling and group B) subjects from the previous Study III with vendor specific 15 direction scheme that was repeated at least twice to form a set of 30 DWIs. Group B also provided an option to investigate how the condition number would change in the case of repeated acquisitions and was further
Figure 3.2: Illustration of the spiral sampling on the spherical surface. A) Nearly uniformly sampled 100 points on a unit sphere showing the rotational variance of the condition number (CN) without any outlier rejections. B) An interpolated 2D map is used to visualize the whole spherical surface. C) Differences between the deterioration rates of electric potential (EP) and condition number as function of incremental data rejections. (Reprinted from II with permission of Elsevier).

divided into two sub-group for which B1) rejections were applied consecutively to both sets of 15 DWIs and B2) rejections were applied on the full 30 DWI set. Studying repeated acquisitions with the ground truth set of 30+30 DWIs was not possible due to enormous number ($1.15 \cdot 10^{18}$) of possible combinations which would correspond to a computational time of 182 megayears.

In summary, every subject experiment had an original dataset of one non-diffusion weighted image and 30 DWIs which were used as the baseline in comparisons with the sub-schemes.

3.3 Neurocognitive correlations of tensor derived measures

After the model has been successfully estimated it can be used to derive voxelwise eigenvalues or combinations of them to characterize the brain microstructure in a meaningful way and be used in statistical analyses.

3.3.1 Diffusion measures

The amount of diffusion can be described with the apparent diffusion coefficient (ADC) which is usually referred as the mean diffusivity (MD) defined in eq. 3.14 if it is calculated from the diffusion tensor elements $D = \{D_{ii} : i = 1, \ldots 3\}$ instead from the average of three
orthogonally acquired DWIs [4]:

$$\text{MD} = \frac{D_{11} + D_{22} + D_{33}}{3}. \quad (3.14)$$

In some applications, such as tractography [57], it is beneficial also to know the orientation of the diffusion in addition to its magnitude. This can be achieved through the eigendecomposition of the diffusion tensor that gives the eigenvectors $$e = \{ e_i : i = 1, \ldots, 3 \}$$ and corresponding eigenvalues $$\lambda_i : i = 1, \ldots, 3$$ [52]:

$$D = \begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{13} & D_{23} & D_{33} \end{pmatrix} = E \Lambda E^{-1}$$

$$= \begin{pmatrix} e_{11} & e_{21} & e_{31} \\ e_{12} & e_{22} & e_{32} \\ e_{13} & e_{23} & e_{33} \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \begin{pmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{pmatrix}. \quad (3.15)$$

Simple measures, such as the axial diffusivity (AD) or $$\lambda_1$$ and the radial diffusivity (RD) i.e. $$\frac{\lambda_2 + \lambda_3}{2}$$ that describe the magnitude of the diffusion along the first eigenvector $$e_1$$ or perpendicular to it are complemented usually with the dimensionless FA (eq. 3.16) that characterizes the anisotropy of the diffusion profile:

$$\text{FA} = \sqrt{3} \sqrt{\frac{(\lambda_1 - \text{MD})^2 + (\lambda_2 - \text{MD})^2 + (\lambda_3 - \text{MD})^2}{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}}. \quad (3.16)$$

FA is scaled between 0 and 1 meaning that areas with isotropic diffusion such as cerebrospinal fluid should have zero FA and high MD whereas white matter bundles should have FA closer to one and lower MD [4]. However, due to noise and partial volume effects, FA values below 0.2 are usually considered unreliable. While the noise in the images could be isotropic, the propagation of the error is not and this will amplify deviations in FA values as well.

Similar presentation for the mean kurtosis (MK, eq. 3.17) based on the kurtosis tensor [62, 104] is more complex to define as it does not have a similar physical analogy as MD or ADC:

$$\text{MK} = \frac{1}{4\pi} \int \int_{S^2} \text{EK}(g) \ ds_{g}^2$$

(3.17)

with $$\text{EK}$$ being the excess kurtosis in the direction defined on the surface of a unit sphere i.e. $$g \in S^2$$:

$$\text{EK}(g) = \left( \sum_{i=1}^{3} \frac{D_{ii}}{3} \right)^2 \sum_{i,j,k,l=1}^{3} g_ig_jg_kg_l K_{ijkl} \left( \sum_{i,j=1}^{3} g_ig_j D_{ij} \right)^2. \quad (3.18)$$
Poot et al. [104] also defined the kurtosis anisotropy (KA) as a metric that describes the variability of kurtosis without normalization from 0 to 1 which was used for the FA:

\[
KA = \sqrt[1/4]{\int \int \int_{S^2} (E_K(\mathbf{g}) - MK)^2 \, dS^2_g}.
\] (3.19)

### 3.3.2 Region of interest based analysis

Descriptive voxelwise values e.g. FA can be extracted from a region of interests (ROI) so they can be compared to other measurements such as cognitive or physiological test results. ROI can be defined with various methods such as a hand drawn shape or an atlas based area within an FA image or as a volume specified by a white matter fiber bundle modelled with the tractography. However, in studies such as III and IV with multiple subjects, drawing ROIs by hand is both time-consuming and is highly dependent on the drawer. One of the most commonly used automatic voxel-based analysis tools is Tract-Based Spatial Statistics (TBSS) [180, 185] implemented in FSL [205] was designed to help exactly with this kind of problem.

### 3.3.3 Voxel based analysis

TBSS registers FA images of all subjects to the same space using a nonlinear registration tool FNIRT [206, 207], it calculates an average for them and thresholds this to a mean FA skeleton that should contain the largest white matter structures common for the whole group. Registered FA images are projected onto the mean skeleton to form the subjectwise FA skeletons. The rationale behind skeletonization is to reduce multiple comparisons thus increasing the power of statistical tests. Projections should ensure that the comparisons are done on the same white matter structures. Finally, the subjectwise FA skeletons can be used in voxelwise permutation tests [182, 208, 209] to locate the differences between groups or correlations between FA values in specific brain structures and some other measurements such as cognitive test results. The mean FA skeleton could also be used for a projection target for other tensor derived values such as MD or RD. However, TBSS has been questioned as the nonlinear registrations rarely succeed well with the same settings for a large number of varying brains, especially if the subjects are young or brains are abnormal. In such case, the skeletonization could fail as wrong white matter structures could be projected onto the skeleton resulting in utterly false comparisons [185].

### 3.3.4 Tractography based analysis

A more arduous but accurate approach is to form actual white matter bundle models with the tractography for each of the subjects and calculate descriptive statistics from them. Tractography can be implemented in various ways with the most common being a deterministic streamline tractography [57] that is based on voxelwise tensors with each having the first eigenvector indicating the orientation of the fiber bundle. This also means that this simple
diffusion tensor based tractography is unable to distinguish crossing fiber populations as there is only one principal orientation per voxel \cite{25, 110, 194, 203, 210, 211}. Tractogram is formed by following these tensor orientations starting from a manually selected seed ROI and continuing from voxel to voxel until predefined boundaries such as low FA, the angle between two tensors is too large to be physically plausible, or a manually defined exclusion ROI is reached.

3.3.5 Visual abilities of neonatal subjects

Study III investigated if white matter structure characteristics of neonatal subjects would correlate with their performance in visuomotor or visual reasoning tasks \cite{212, 213}. Diffusion MRI data from sixty infants was visually inspected for slicewise outliers and six subjects were rejected due to lack of the non-corrupted data. From the remaining subjects, 45 had also test results for visual fixation (VF) and gaze behaviour (GB). A full diffusion MRI data analysis from corrections of geometric distortions with FSL’s eddycorrect algorithm \cite{84, 89, 90, 93, 94, 125, 205} to the voxelwise robust tensor estimation with non-linear RESTORE algorithm implemented in Camino \cite{133, 214} was performed to obtain FA images. The standard TBSS pipeline was used to study the relation between FA and the other metric (here VF or GB) and possible covariates such as gestational age as a linear relation \cite{209}. As there was no reason to assume such a simplistic relation, the Pearson correlation test used in the standard permutation test was replaced with the Spearman rank test \cite{215}. In practice this should increase the power of the statistical test with any non-linear but monotonic relation with a small cost in power for a truly linear correlations \cite{216}.

3.3.6 White matter changes in bilingual adults

In Study IV, two groups of fourteen healthy volunteers were imaged to investigate if any structural differences due to the age of acquisition of the second language (L2) could be detected with diffusion weighted MRI \cite{18, 217}. The first group had learned the second language early in life whereas the second group consisted of late bilinguals. A similar full diffusion MRI data analysis as used in Study III was conducted to obtain FA, MD, and RD images to be used in TBSS \cite{180, 185} with the updated FSL EDDY algorithm to correct for the off-resonance effects and subject motion \cite{173} and tensor estimates were calculated using ExploreDTI \cite{155}. Additionally, subjectwise tractograms of two speech related tracts: the arcuate fasciculus and the inferior fronto-occipital fasciculus (IFOF) \cite{41, 204, 218} were segmented to extract tractwise mean FA, MD, and RD values as well as to aid TBSS by restraining the full FA skeleton with the tractwise masks.
4 Results

4.1 SOLID

Simulations in Study I were based on full brain DWIs from the MASSIVE database [190] by setting hypothetical modified Z-score $\xi'(x')$ thresholds from 0 to 10 each time counting number of true and false positive or negative findings to form ROC curves and PRC profiles. The optimal threshold was calculated from each curve as the smallest Euclidean distance to the left top point with full sensitivity and specificity for comparison with the suggested lower threshold $t_L = 3.5$. Area-Under-Curve (AUC) values of ROC curves and PRCs summarized the overall effectiveness of SOLID.

4.1.1 Evaluation of SOLID

Six outlier setups were simulated to evaluate the SOLID performance. Figure 4.1 A-B) represent a typical clinical acquisition scheme with 30 DWIs based on a $b$-value of $1000\text{s/mm}^2$ with one artefactual DWI having one signal decrease or increase outlier. SOLID performed well for such a simple setup having an ROC AUC value of 0.98 even with the lowest SNR of 8. While the optimal modified Z-score threshold deviates from the recommended 3.5 in most of the cases, the visual inspection of ROC curves show that the recommended threshold is actually quite near the optimal region. In C-D) the number of artefactual DWIs and slicewise artefacts are increased to eight and five respectively with 30 DWIs based on the same $b$-value of $1000\text{s/mm}^2$ to simulate a typical acquisition of uncooperative subjects such as neonates with severe motion artefacts. The increase in the number of outliers reduced the smallest ROC AUC value to 0.97 and slightly increased the difference between the recommended threshold and optimal threshold. In E-F) the number of outliers was kept high but the $b$-value was increased to $2000\text{s/mm}^2$. The lowest SNR simulation had the ROC AUC value of 0.95 still indicating good performance even for such low SNR case with a small number of available DWIs. However, the differences between the optimal and the recommended thresholds become more apparent suggesting that for higher $b$-value experiments the lower threshold could be smaller.

Figure 4.2 shows the results for the last four simulations that included subject motion in the form of 5-degree rotations around the left-right (nod) or the anterior-posterior axis (tilt). Simulated DWIs were based on a $b$-value of $1000\text{s/mm}^2$ with eight of them having five signal decrease (left column) or increase (right column) outlier slices. Differences in ROC curves between simulations shown in Figure 4.1 C-D) are apparent only with the signal increase outliers that produced the lowest AUC value of 0.95 with SNR 8.

Further investigation of PRC profiles (Supplementary Figs. I.1 and I.4) confirmed that SOLID did not make excess false positive findings as the positive predictive value remained overall
Figure 4.1: SOLID sensitivity - specificity characteristics as ROC curves. Each simulation was based 30 DWIs with SNR values of 8, 16, 32, and infinite. Signal decrease outliers had a full -100% deviation from the ground truth (left column) thus consisting only of noise whereas signal increase outliers deviated by +50% from the original image intensity. A-B) One of the 30 $b = 1000$ s/mm$^2$ DWIs had one artefactual slice. C-D) Eight out of 30 $b = 1000$ s/mm$^2$ DWIs had five outlier slices. E-F) Eight out of 30 $b = 2000$ s/mm$^2$ DWIs had five outlier slices. Rician noise was reiterated with the randomized selection of artefactual DWIs and slice positions 1000 times for each simulation and true and false positive and negative identifications of slicewise outliers were counted.
high with the smallest PRC AUC for the simulation experiments being 0.8. Detection of lower intensity deviations with -10% and -50% decreases were explored in Supplementary Fig. I.2. The -50% decrease outliers were identified with nearly similar effectivity as the full -100% decrease outliers whereas the -10% deviation was too close to normal intensity variation between DWIs for SOLID to detect them reliably.

4.1.2 Comparison to previously published tools

Figure 4.3 A-D) illustrates the qualitative differences between SOLID and the previously published tools: DTIPrep [152] and DTI Studio [147]. In A) a set of 30 DWIs based on b-value 1000s/mm² with one artefactual volume containing multiple signal increase and decrease outlier slices was used to highlight the problems in the algorithms used in DTIPrep and DTI Studio. B) The DTIPrep algorithm identifies outliers within DWI based on normalized cross-correlation (NC) [197] between two adjacent slices. This resulted in multiple false positives as normal slices adjacent to outliers were also flagged as outliers. C) SOLID identified correctly all simulated outliers and gave them a modified Z-score value of at least 10. While SOLID did find false positives, they were located in the inferior part of the DWI volume with only a few brain voxels thus having relatively small impact on any further analysis. D) The DTI Studio algorithm is based on the morphological closing of the DWI volumes in a perpendicular direction to the slice acquisition plane. This approach failed to detect multiple adjacent i) signal increases or ii) decreases and iii) positioned interleaved signal increase outliers incorrectly.

Figure 4.3 E) summarizes a quantitative comparison with FSL EDDY [153]. Sensitivity - specificity characteristics of SOLID and FSL EDDY were compared with 1000 sets of 30 DWIs based on a b-value of 1000s/mm². Eight DWIs were replaced with artefactual volumes with a 5° rotation around the left-right axis. Each DWI had five outlier slices having randomly either -100% decrease or +50% increase in the signal and Rician noise with SNR 16. SOLID ROC curve was calculated using the hypothetical modified Z-scores thresholds between 0 and 10 whereas EDDY ROC curve was based on the standard deviation thresholds between 0 and 10. By default, EDDY uses a threshold of 4 standard deviations. SOLID had overall higher sensitivity than EDDY with 9% higher ROC AUC value. Due to the proposed SOLID downweighting framework, the actual differences in the model estimation could be even higher as EDDY either merely rejects the outlier or replaces them with the prediction from the Gaussian process model [116, 153].

4.1.3 Evaluation of SOLID-informed tensor estimator

Investigation of SOLID-informed IWLLS estimator based on the ground truth voxel from the corpus callosum (Fig. I.5) confirmed that the full downweighting of measurements that are only partially affected by outlier data, i.e. the edge regions of GIB artefact, is not needed to obtain correct tensor estimates. The deviation from the ground truth estimate increased as a function of outlier signal intensity and the number of outlier signal per shell. Even in the case of 16 outlier signals on both shells of 1000s/mm² and 2000s/mm² SOLID-information was able to ameliorate the iterative estimation process to produce nearly correct results. Interestingly, SOLID-information also kept the numerical problem well-conditioned. For example, in the case
Figure 4.2: SOLID sensitivity - specificity characteristics as ROC curves in the presence of subject motion. Each simulation was based on 30 DWIs based on $b$-value $1000\text{s/mm}^2$ with SNR values of 8, 16, 32, and infinite. Eight DWIs were replaced by artefactual volumes with five slicewise outliers each. A-B) Artefactual volumes with signal decrease of -100% were rotated by $5^\circ$ around the anterior-posterior axis and volumes with signal increase of +50% were rotated around the left-right axis. C-D) Artefactual volumes with signal decreases were rotated around the left-right axis whereas the volumes with signal increases were rotated around the anterior-posterior axis. Rician noise was reiterated with the randomized selection of artefactual DWIs and slice positions 1000 times for each simulation and true and false positive and negative identifications of slicewise outliers were counted.
Figure 4.3: Comparison between SOLID and previously published tools. A) An example of a problematic outlier setup with one out of 30 DWIs being artefactual with multiple slicewise outliers. B) Detection based on the normalized cross-correlation produces multiple false positives (indicated with the red arrows) whereas C) SOLID detects outliers correctly and has outliers only in the lower extreme of the DWI volume. D) Detection based on morphological closing fails to detect i-ii) multiple adjacent outliers and iii) interleaved signal increase outliers. E) Sensitivity - specificity comparison of SOLID and FSL EDDY based on 1000 samples of 30 DWIs.
of 16 outliers per shell, even though the condition number \([118, 161]\) of the weighted design matrix (eq. 3.8) was in the normal range, the noise started to propagate rigorously on each iteration step leading finally to the fully ill-conditioned problem with a non-finite solution and to failed tensor estimates. While applying SOLID weights will likely increase the condition number, downweighting the outlier intensity reduced the overall noise so the iterative process could be finished and produce a finite solution. Further comparison between SOLID-informed IWLLS and LLS, standard IWLLS, and voxelwise robust tensor model estimators (Fig. I.6) showed that while both the robust estimator and SOLID-informed IWLLS were able to reduce the bias due to outliers to nearly identical results, the voxelwise approach missed several outliers in the GIB edge.

4.1.4 SOLID validation with human data

The data / certainty principle is further visualized in Figure 4.4 with two subject datasets, showing SOLID validation results, and highlighting a situation were SOLID was able to detect outliers that the technician had missed. A1-2) A sagittal slice of a DWI before and after motion correction is shown for an adult with no visible intensity artefacts and a neonate with severe intensity artefacts. B) Final weights of a standard IWLLS (eq. 3.9) indicate that the model estimated from the neonatal data is affected by the outliers as their weight is not zero. C) SOLID-informed IWLLS (eq. 3.10) with lower threshold of \(t_L = 0\) correctly removes outlier weights from neonate but now the good quality adult data gets partially downweighted incorrectly. D) Increasing the lower threshold to the suggested \(t_L = 3.5\) corrects the problem with incorrect good quality data downweighting on the adult subject yet preserving the correct outlier removal on the neonatal subject. E) Manually labeled slicewise outliers from 54 neonatal subjects were used to calculate true and false positive and negative findings by SOLID with the modified Z-score threshold from 0 to 10. An ROC AUC value of 0.92 indicates a good agreement with both methods, however, F) a closer inspection of outliers detected by SOLID revealed that in some cases the technician had missed outliers that SOLID was able to detect. In this case, the outlier was due to sub-optimal hardware behaviour and while the clear signal deviations are local such hardware problems likely affect the whole slice due to the slicewise acquisition technique and should be excluded or downweighted accordingly.
Figure 4.4: In-vivo examples of SOLID downweighting framework. A1) Non-corrected and A2) motion corrected DWIs, the adult subject had no visible intensity artefacts but the neonate had both signal increase and decrease outliers. B) The final weights of the standard IWLLS estimator are non-zero for the neonatal outlier slices indicating that the final model is affected by the outliers. C) The SOLID-informed IWLLS estimator final weights with the lower threshold $t_L = 0$ result in undesirable downweighting of good quality adult data whereas the outliers in the neonatal data are now correctly downweighted. D) The increased lower threshold of $t_L = 3.5$ rectifies the incorrect good quality data downweighting yet still correctly removing outliers from the model estimation. The intensity scale in the weight images was normalized between 0 and 1 for visualization purposes. E) The ROC and F) the PRC profiles for SOLID validation with the initial and revised manual slicewise outlier labeling from 54 neonatal subjects. G) Both slices are from the same subject with nearly identical spatial positioning and the same diffusion gradient direction from to subsequent acquisitions. The upper slice shows no outliers indicating that the intensity errors in the lower are not due to a pathology but caused by hardware malfunction. The technician missed this artefact whereas SOLID gave it the modified Z-score of 8.9.
4.2 Error propagation in the least squares estimation

Study II emulated white matter bundles travelling in various directions with the set of cylindrically symmetric diffusion tensors to investigate data rejections and changes in diffusion gradient scheme EP and CN values on tensor derived FA and FE measures. As hypothesized, the EP metric was not as effective in disclosing error prone sub-schemes as the CN metric (Fig. II.3). While a general correlation between CN value and tensor derived metrics is difficult to define, we recommend investigating CN values across the compared subjects or within subject in longitudinal studies as that could reveal direction-wise biases in the data.

4.2.1 Simulation experiments

Figure 4.5 shows examples angular maps of A) FA and B) FE errors for selected three sub-schemes (rows of subfigures) with 30, 18 and 6 gradient directions and three ground truth FA values (columns of subfigures) 0.85 (high FA simulation), 0.55 (medium FA simulation), and 0.25 (low FA simulation). The results shown here were based on the worst CN as it was shown to produce the largest errors (Fig. II.3). The results for all cylindrically symmetric tensors based on the 10 different FA values after rejections using the best and the worst CN and EP sub-schemes are available as supplements to Study II¹. While in most of the cases the worst EP and CN sub-scheme were actually the same, there were multiple situations when the worst CN sub-scheme was notably more error prone resulting in larger deviations from the ground truth tensors.

The comparison of such complex angular maps (Figure 4.5) between all schemes is challenging, thus FA error maps were averaged with one standard deviation error bars and the median of FE error maps with an interquartile range (IQR) as error bars were used to form 2D scatter plots (Figure 4.6D and E and Fig. II.3C and D and Supplementary Figs. II.3-5). As the number of rejections increased, the simulation experiment and all subject experiments showed that the worst CN metric was more effective in identifying problematic sub-schemes.

4.2.2 Human data experiments

The full set of 30 DWIs in Figure 4.5 visualized a reference when no data rejections were applied and only the Rician noise with SNR 36 affects the FA and FE estimates. While both FA and FE show directional variances, their magnitudes for each tensor type remains relatively low. With the incremental data rejections, these directional variances become more distinct with all tensors for the both FA and FE estimates. In practice this means that fiber populations traveling in varying directions within the brain could have widely different errors. Especially, if the outliers are rejected in a voxelwise manner, as these changes would vary from voxel to voxel leading to both spatially and directionally shifting errors.

Interestingly, in the neonatal experiment B i.e. two acquisitions of 15 DWIs did not agree fully with the simulation or other subject experiment results. Visualizations for both rejection

¹Available online at http://dx.doi.org/10.1016/j.neuroimage.2016.11.061
schemes with rejections applied B1) consecutively on the two sets of DWIs e.g. the first set would have to be rejected completely before the first volume of the second set would be rejected and B2) on both DWI sets simultaneously thus maximizing the decline in angular sampling.

Figure 4.6 A) shows colour coded FA maps for the both neonatal rejection schemes with no, four, and eight rejections. B) FA errors and C) FE i.e. V1 errors were calculated as differences from the original data set without applied rejections as it was assumed to provide the most accurate estimates. B2 group with DWIs rejected from both sets was clearly more error prone as both FA and FE errors were higher. Finally, to demonstrate the higher sensitivity of the CN metric, D) and E) show FA (mean ± standard deviation) and FE (median ± IQR) results: While the EP criteria did show a gradual decline in both FA and FE, it did miss the large deviations seen in the worst CN sub-schemes.

**Figure 4.5:** Directional dependence of the tensor derived parameters FA and FE increases as function of DWI rejections. The colour scale is normalized according to minimum and maximum values for each row of subfigures and represents A) the mean FA deviation B) the median FE deviation from the ground truth values. (Modified from II with permission of Elsevier).
Figure 4.6: Spatially varying FA and FE errors for one of the neonatal subjects in group B in which two acquisitions of 15 DWIs acquired consecutively. A) Colour coded FA map with no, four, and eight rejections applied for both rejection schemes with the worst CN criteria. B) FA error and C) FE error maps visualize that white matter regions with overall higher FA value can tolerate a larger number of rejections as their deviations from the original dataset were lower. D) Mean FA error ± standard deviation and E) Median FE error ± interquartile range when rejections were applied on both sets of 15 DWIs simultaneously. (Reprinted from II with permission of Elsevier).
4.3 Neurocognitive correlations of tensor derived measures

Two studies were conducted to investigate the correlations between the tensor model characteristics and the visuomotor and visual reasoning task performance of neonatal subjects (III) or the age of L2 acquisition of adult subjects (IV).

4.3.1 Visual abilities of neonatal subjects

Study III presents the correlations in the white matter FA values and visual ability test for the infants. Nearly half of the white matter voxels, even after the correction for multiple comparisons [216], showed significant correlations between the FA and VF scores in the TBSS based voxelwise comparison tests (Figure 4.7). The differences were spread uniformly across the whole FA skeleton, indicating that the infants with a low VF score had overall lower white matter FA values. A ROI based analysis (Figure 4.7B) was in agreement with the TBSS results: the changes in FA values were global with no asymmetries over the hemispheres or other spatial preferences.

FA correlations with GB score were notably more localized (Figure 4.7A) as TBSS located significantly correlating voxels only near to the region that could include the optic radiation. Even with a higher significance threshold of $p < 0.10$, findings remained near the white matter pathways trailing from the visual cortex to the deep brain structures.

4.3.2 White matter changes in bilingual adults

The TBSS results for Study IV, however, were not as prominent as no differences between late and early bilingual groups were disclosed with FA, RD, or MD values. As the power of the statistical test could be low due to a large number of voxels [216], the TBSS skeleton was further restricted with tractwise masks. These masks were based on manually traced three segments of the arcuate fasciculus and the IFOF (Figure 4.8A). This post-hoc test restricted the number of multiple comparisons and was able to disclose significant differences in the MD values in the frontal parts of the IFOF. Average FA, RD, and MD values of the tractograms of the three arcuate fasciculus segments and the IFOF were used further in a mixed model analysis. The mean FA of the left long segment of the arcuate fasciculus was significantly higher for the early bilinguals, the mean MD was significantly higher in the right posterior segment of the arcuate fasciculus for the late bilinguals (Figure 4.8).
Figure 4.7: Voxelwise and ROI based analysis of the TBSS FA skeleton (green) shown on the mean FA image calculated over all infants after registration to the same space. A) Voxels with significant correlation with VF or GB scores are shown on top of the skeleton. B) The mean FA correlations with VF score calculated from the large blue ROIs drawn on the skeleton. (Modified from III under the CC-BY 4.0 license).

Figure 4.8: A) Diffusion tensor tractography results for the three segments long (red), anterior (green), and posterior (yellow) of the arcuate fasciculus and IFOF (purple). B) FA skeleton mask (light green) drawn on the standard MNI152 brain showing regions with significant differences (light blue) in MD values between late and early bilingual groups. Regions were dilated to improve the visualization. (Modified from IV with permission of Elsevier).
5 Discussion

5.1 SOLID

The unambiguous characterization of the true physical diffusion [60, 61] of water molecules within the brain can be hindered by the slicewise artefacts arising from subject motion [89–94] or hardware issues during the MRI acquisition. As it is necessary to correct the geometrical alignments of the DWIs before model estimation [84, 89, 90, 93, 94, 125], these slicewise outliers will likely be distributed into neighbouring slices forming GIB artefacts and obscuring the partial volume effects [107] even further. The manual labelling of slicewise outliers is challenging, error prone, and time consuming as the amount of data with modern acquisition schemes is calculated in thousands of DWI slices per subject. Previously published tools [147, 151–153, 197, 219] have addressed this issue but their methods have been either too conservative excluding whole DWIs at a time if they contained slicewise outliers [152, 197], are limited to certain types of outliers such as interleaved signal decreases [147], or require time consuming modelling and prediction of the DWIs which also sets tight restrictions on the acquisition protocols [151, 153]. To lift such restrictions, a versatile and fast algorithm, coined SOLID, was developed with a framework to interpret the detected outliers as increased uncertainty in the measurements so they could be downweighted accordingly during the model estimation [131] (eq. 3.10). SOLID was evaluated with comprehensive simulations with a range of clinical $b$-values and acquisition protocols, compared to the previously published tools, and validated with a unique set of 54 neonatal subjects.

As SOLID is not dependent on any model, it sets no restrictions on the acquisition protocol including $b$-values, gradient directions or measurement repetitions. Practically, of course, if over the half of the measurements within the same spatial location are artefactual, SOLID reaches its breakpoint and cannot identify what is normal and abnormal. The situation could be even more difficult with extremely high $b$-values, as the intensity in such images is near the noise floor [74] and the signal decrease artefacts could be challenging to identify reliably. The SOLID framework was implemented as a part of ExploreDTI toolbox [155] to allow a rapid distribution of the algorithm.

5.1.1 Evaluation of SOLID

To identify abnormal slices, SOLID assumes that the slicewise metrics follow a normal distribution. While this assumption could be avoided with non-parametric approaches [216], their speed would not be nearly sufficient for practical use as the amount of data in large population studies can be enormous. Non-parametric approaches could also have less power if the actual
distribution is normal [216]. And as our simulations revealed (Figs. I.2, I.3, and I.4), outlier detection was not hindered even if such a normality assumption was violated.

A more likely shortcoming of SOLID is related to eddy current and susceptibility artefacts [81–88] or subject motion artefacts [89–95] as all of these could cause a situation where the compared slices in different DWIs have either different geometrical distortions or the anatomical regions are different due to head rotations or translations. The simulated subject motion decreased the performance of SOLID (Figs I.2 and I.3) but not drastically. It should be noted that mild false positives, e.g. a modified Z-score of 5, that could be a result of these ill-posed comparisons would not be fully excluded due to the informed model estimation framework. Furthermore, slices with a full signal decrease or large increase would likely still produce a modified Z-score above 10 as such slices would surely be different than slices with any anatomical information resulting in a full and correct exclusion of the measurement.

In addition to the simple slicewise statistics with mean, variance, median, and MAD (Fig. I.8), a non-parametric Kolmogorov-Smirnov [216] and mutual information [124] based SOLID frameworks were tested. As these more complex methods did not provide increased efficiency in the preliminary tests but resulted in notably longer computing times, the simple variance based detection was chosen as the recommended slicewise metric.

5.1.2 Comparison to previously published tools

Three of most similar algorithms from previously published tools [147, 149, 151–154] were selected to conduct more thorough comparison: DTIPrep [152, 197], DTI Studio [219], and FSL EDDY [153]. Algorithms in DTIPrep and DTI Studio searched for outliers within DWI whereas FSL EDDY and SOLID searched for outliers across DWIs. While the within DWI approach would avoid the problem of geometrical misalignments between different DWIs due to subject motion during the acquisition, both algorithms produced multiple false positive findings (Figure 4.3B and D). DTIPrep rejects the whole DWI volume if a predefined number of outlier slices is found, thus these false positives could lead to a large scale unnecessary exclusion of good quality data.

The most similar detection algorithm, used in FSL EDDY, is based on the modelling of the diffusion signal as a Gaussian process [116] which is used to predict error-free DWIs. If a measurement deviates over four standard deviations from the corresponding prediction, it is deemed to be an outlier. However, the Gaussian process modelling sets tight restrictions on the data acquisition and especially on the used gradient scheme. This highly limits the usage of EDDY on older diffusion MRI datasets that were not acquired within these limits or even new studies with uncooperative subjects as high end protocols require longer acquisition times. EDDY provides an option to replace outliers with their predicted counterparts to avoid GIB artefacts but it has not been investigated how tensor estimates are affected in such cases when they are based on partially original DWIs and partially predictions from the Gaussian process. SOLID avoids such difficulties, as the output can be used in any model estimator as an additional weight to avoid excess noise propagation due to multiple successive model estimations.
5.1.3 Evaluation of SOLID-informed tensor estimator

To investigate the SOLID downweighting i.e. the data uncertainty framework would work, GIB artefacts were simulated using the ground truth tensors selected from the deep gray matter and corpus callosum. Artefacts were simulated by deviating randomly selected diffusion weighted signals from the ground truth by -100% to +50% before introducing Rician noise. Next, to simulate data uncertainty, the weight of these altered signals was decreased gradually from 1 to 0 (Fig. I.5 and Supplementary Figs. I.6-10). For both GM and CC tensors, the results were quite similar and expected: the increase in the outlier deviation from the ground truth and the number of outlier signals resulted in increasing deviations for all of the studied tensor derived metrics FA, MD, KA, and the model residual. As hypothesized, applying the data uncertainty principle as SOLID weights during the IWLLS model estimation (eq. 3.10) led to clearly smaller deviations from the ground truth values.

Interestingly, Study I linked directly to Study II at this point. Even though, the condition number of the original acquisition scheme was close to one, outliers with large intensity deviations led to rapid and cumulative noise propagation in the iterative estimation process. As the weights of each step were based on the estimated diffusion signals of the previous step this prevented the IWLLS estimator from converging to a finite solution and the algorithm would return the initial LLS estimate instead. By applying SOLID weights to the design matrix a slight increase in the condition number [118, 161] is likely but the decrease in noise was more prominent and the IWLLS estimation problem became well-conditioned again. An example of this is visualized in Fig. I.5 FA deviations with 16 outliers per shell as the outlier with -100% signal decrease had lower deviation from the ground truth than the outlier with -90% signal decrease.

Figure I.6 compares the SOLID-informed IWLLS algorithm to a simple LLS, IWLLS [131], and a voxelwise robust REKINDLE [136] algorithm. While REKINDLE and SOLID-informed IWLLS produced quite similar improvements when compared to the non-robust estimators, as hypothesized, the edge regions of GIB were problematic for the voxelwise approach and multiple voxels were not detected correctly as outliers. The REKINDLE outlier detection parameter $\kappa$ could be tuned to improve the detection but it could require multiple iterations for each subject to find the proper value and thus would be impractical in large population studies. As an optimal solution, REKINDLE could use voxels with zero SOLID weight as the initial outlier map to avoid this issue and potentially even decrease the computing time of REKINDLE.

5.1.4 SOLID validation with human data

Finally, SOLID was validated with real subject experiments (Fig. I.7) based on manually labeled outlier slices and there was a high agreement between the two methods as the AUC value of the ROC curve was 0.92. A more specific investigation of false positives found with SOLID showed that the technician had missed some of the milder outliers and local intensity changes due to technical problems during the acquisition. This means that SOLID did not only agree with the manually labelled outliers but was able to uncover artefacts that a human observer was not. A qualitative visualization of the IWLLS weights (eq. 3.10) with different lower thresholds $t_L$
was done to motivate the selection of $t_L = 3.5$ in eq. 3.2. It was shown that a standard IWLLS model estimate would be highly affected, especially by signal increase outliers. Furthermore, if no lower threshold was used i.e. $t_L = 0$, good quality adult data would be unnecessary downweighted as the normal variation in the data would be seen as increased uncertainty.

5.2 Error propagation in the least squares estimation

Study II investigated the expected loss in the accuracy and the precision of the diffusion tensor [3, 96–100, 102, 103] derived parameters FA and FE [4] due to incremental DWI rejections to extend the previous study by Chen et al. 2015 [186]. Chen et al. suggested that investigating the EP [158] of the gradient scheme after outlier rejections could be used as a quality control tool. Our hypothesis was that the EP, as it is a rotationally invariant measure, cannot disclose problematic sub-schemes in such detail as the rotationally variant CN could. Previously, CN had been used only to optimize the initial diffusion gradient scheme [118, 161] but here we showed that it could also serve as a post-estimation quality control tool for investigating voxel to voxel varying and angularly dependent error propagation within subject and ensure that population studies would not become biased due to the same reason.

5.2.1 Simulation experiments

While the worst CN and the worst EP criteria could produce identical sub-schemes, the worst CN metric revealed notably larger deviations in FA and FE for the non-identical schemes (Figure 3.2C). This leads to the conclusion that there is no reason to use EP based metrics for quality control purposes, at least based on the studied gradient scheme. As these results are gradient scheme specific, a general guideline for how many rejections can be safely made without compromising the final result cannot be made. Especially, as the subject motion correction generates a totally unique set of diffusion gradient vectors for each acquisition. Fortunately, CN can be rapidly assessed easily after any least squares estimation allowing different MRI sites to build their own knowledge of their acquisition schemes and typical patient populations.

The CN metric considered in this study is exact only for the LLS estimation of the diffusion tensor. Models based on multi-shell acquisitions or estimated with other algorithms could potentially have different error propagation. This limitation, however, is minor when compared to the limitations of the previously proposed EP metric [186] that can be seen in Figure 3.2C: for the studied gradient scheme EP values did not decrease monotonically with the incremental number of rejections. This means that if the EP value were used as a quality control tool, after rejecting exactly the half of the directions it would always be better to reject one more to achieve a lower EP value. Since rejecting good quality data hardly makes sense in an already missing data problem, the usage of EP is not advised. This non-monotonic behaviour of the EP was not observed with the vendor specific 30-direction gradient scheme [158] which suggests that different gradient schemes could indeed have highly varying tolerances to the error propagation.
5.2.2 Human data experiments

FA errors (Fig. II.4D) for the adult data were in a good agreement with the simulation experiments (Fig. II.3C) i.e. they started to deviate from the best possible estimate slowly as incremental data rejections were applied. FE errors (Fig. II.4E), however, deteriorated at a more rapid pace than their simulation counterparts (Fig. II.3D). This can be explained by the fact that the human white matter structures were modelled with a single tensor characterization [3] which is not sufficient to disclose crossing fibers [211, 220] or other partial volume effects [72, 107] in detail. Thus, even a few rejections could result in a widely different estimate than the best possible estimate i.e. situation with no rejections applied whereas the simulations were based on a known cylindrically symmetric tensor which simply models a single orientation fiber population. The rapid deterioration of FE also explains the swift decline of the streamline tractography [57, 203] results shown in Fig. II.5 with even a small number of rejections.

A clinically relevant question is, if a repeated acquisition tolerates the incremental data rejections better than a single acquisition with twice as many non-collinear diffusion gradients? As both would require the same acquisition time, but the latter could perhaps allow the usage of a more sophisticated models [110, 117, 211, 221] in addition to the simple tensor. However, in clinical settings where scanning times are a limited supply, it is also necessary to consider how likely the experiment is to succeed at the first attempt as many different MRI sequences are generally acquired sequentially and repeating the whole examination might not be feasible. This was investigated with the neonatal group B with two acquisitions of the same vendor specific 15 diffusion gradient directions. These results were, of course, affected by the subject motion corrections [89–94] resulting in small changes between the diffusion gradient vectors [93]. In the first test B1, rejections were applied to the two sets of 15 DWIs successively, meaning that the first set was fully rejected before the first volume of the second was excluded. In the second test B2, rejections were allowed on both acquisitions at the same time. Intuitively, as long as the other set was intact, the estimates stayed relatively stable whereas the second test produced progressively much worse results (Supplementary Fig II.2). Notably, this was not seen with the worst EP based sub-schemes, further motivating the usage of the CN metric.

5.3 Neurocognitive correlations of tensor derived measures

5.3.1 Visual abilities of neonatal subjects

An investigation into the visual abilities of the infants and their white matter characteristics found a significant correlation between FA values and the VF score. Previous studies have suggested that the infants’ visual functions would correlate with the white matter structures associated with the optic radiation [24, 26]. The findings presented in Study III, however, encompassed the whole white matter volume of the infant brain thus suggesting that the overall maturity of the brain could be lower with infants who had lower scores in visuomotoric or visual reasoning tasks [212, 213]. Although the used method, TBSS [180], has been criticised [185] due to its pitfalls in correctly projecting white matter structures onto the FA skeleton and affine multi-subject image registration problems [84, 207, 222], these findings were also
confirmed using ROI regions within the original subject image spaces. The previous studies [24, 26] also agreed well with the GB test. A significant correlation between white matter structures and GB score was recorded in a brain region which is likely to contain the optic radiation (Figure 4.7).

5.3.2 White matter changes in bilingual adults

As for the early and late bilingual adult experiments of Study IV i.e. investigation if the age of L2 acquisition [18, 217] results in significant differences within the white matter, the standard TBSS [180, 185] inspection did not find any differences between the groups. The tractwise restricted TBSS, however, was able to disclose differences in the anterior part of the IFOF and when the tractwise mean FA, MD, and RD values were compared in the mixed model analysis showing the mean FA was significantly higher in the left long segment of the arcuate fasciculus whereas the mean MD was significantly lower in the right posterior segment for the early bilinguals [41, 204, 218]. While all prudent measures were undertaken to exclude false discoveries, these findings need to be confirmed in independent datasets to better understand their neurobiological meaning [15, 17, 19, 20, 217].

5.4 Future aspects

SOLID in Study I could be improved by introducing the within-DWI information along with angular neighbourhood matching [174]. As these potentially could decrease the current error proneness toward subject motion between DWIs and allow the use of SOLID with extremely high b-value research studies as well [12]. The data / certainty principle driven framework for the robust tensor model estimations presented here could also be extended for other diffusion MRI models [109, 110, 211, 223] utilizing other model fitting algorithms [116, 138, 139, 141] including quality control with the condition number [118, 161].

The designed CN metric for post-estimation quality control of diffusion MRI estimates in Study II remains to be refined into a readily usable tool and, as discussed previously, the presented condition number is exact only for the LLS estimator. In Study I, preliminary tests were made with the IWLLS [131] condition number but the theoretical aspects were not investigated in detail. As similar error propagation measures can likely be defined for non-linear [133, 134] and maximum likelihood estimators [138, 141] as well, investigating the error propagation in such data processing pipelines will be one of the topics of future studies.

The neonatal study III could be re-iterated with these updated tools, not only to confirm FA findings but to complement them with MD, RD, and other such as Westin measures [192]. The latter could also be applied in the bilingual experiments as they can reveal details from the different symmetries of the diffusion tensor. While the neonatal experiment was based on sparsely sampled data, building tractograms [21, 24, 57] might be feasible at least with the subjects with multiple successful acquisitions to get tractwise statistics instead of error prone TBSS analysis [180, 185].
The tractogram analyses used in the bilingual study could also be improved with a more accurate diffusion model, such as spherical deconvolution [110, 117, 211, 221, 224] that could distinguish crossing fibers and instead of calculating the average metrics of these white matter models, an along tract analysis [79] could be performed. As both, the bilingual experiment and the neonatal, are currently rebooting in a form of new data acquisition with multiple $b$-values that allow the usage of the kurtosis tensor model [40, 62, 63, 104–106, 220, 225] re-iterating this same dataset might not be necessary as these new measurements will likely clarify the modelling issues faced in Studies III and IV.

Finally, the measurement error could be estimated using bootstrapping [179, 203, 216, 226] approaches to provide distributions for the estimated tensor derived values such as FA. While the mean FA of two different groups might be the same the underlying distribution could reveal that the variance of FAs for one of the groups might be significantly different. As these estimate errors are not commonly reported in diffusion weighted MRI studies, it could be that their theoretical readiness has not been sufficient and further investigations on how robustly estimated models should be bootstrapped are needed before they can be applied in more practical studies such as III or IV.
6 Conclusion

In this thesis, a fast and accurate method for disclosing artefactual diffusion MRI data, coined SOLID, was developed along with a framework to use the found outliers as estimates for data uncertainty during model estimation (I). This was further accompanied with a quality control method (II) that could reveal rotational variant errors in the diffusion tensor model emerging from the information loss due to necessary data augmentations to exclude subject motion from the measurements. Methods were investigated in theory, evaluated with simulations and validated with newborn and adult human data.

The SOLID-informed model estimation supports the convergence of the iterative estimators as it does not completely exclude individual measurements but downweights them according to their degree of uncertainty. Comprehensive simulations covering a range of clinically plausible situations showed that the sensitivity and the specificity of SOLID were comparable or better to previously published tools but with less complexity and evading the need for time-consuming model estimation and signal prediction. SOLID was validated with manually labelled outliers from a unique set of 54 neonatal subjects which also demonstrated its versatility in large population studies.

The problem with the data rejection or downweighting is that the original LLS problem could become ill-posed due to the increased condition number of the design matrix inversion (II). This, as well, was first investigated in theory with simulations and then applied to human MRI measurements. This does not mean that outliers should not be excluded or the subject motion corrected but that after they have been done, the condition number should be evaluated to ensure comparability within a subject and across all of the subjects in population studies.

Finally, to motivate the necessity of such tools, the power of robust diffusion MRI analysis was shown in two clinical studies that found novel correlations between the brain white matter structures and newborn’s visual abilities (III) or adults’ language skills (IV).
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