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Early Universe Higgs dynamics in the presence of the Higgs–inflaton and non–minimal Higgs–gravity couplings

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Abstract. Apparent metastability of the electroweak vacuum poses a number of cosmological questions. These concern evolution of the Higgs field to the current vacuum, and its stability during and after inflation. Higgs–inflaton and non–minimal Higgs–gravity interactions can make a crucial impact on these considerations potentially solving the problems. In this work, we allow for these couplings to be present simultaneously and study their interplay. We find that different combinations of the Higgs–inflaton and non–minimal Higgs–gravity couplings induce effective Higgs mass during and after inflation. This crucially affects the Higgs stability considerations during preheating. In particular, a wide range of the couplings leading to stable solutions becomes allowed.

Contents

1 Introduction

The currently favored values of the top quark and Higgs masses imply that our Universe is metastable [1–3], while its lifetime is much longer than the present age of the Universe. The extra minimum of the Higgs potential is much deeper than the electroweak vacuum. This raises pressing cosmological questions: how did the Universe evolve to the energetically disfavored state and why it remained there during inflation despite large field fluctuations. In the minimal framework that includes the Standard Model (SM) and the inflaton, the key to these puzzles may lie in the Higgs coupling to gravity [4] or inflaton [5]. On general grounds, one expects the presence of the following interactions at the renormalizable level

$$-\mathcal{L}_{hR} = \xi H^\dagger H \hat{R}\,, \tag{1.1}$$
$$-\mathcal{L}_{h\phi} = \lambda_{h\phi} H^\dagger H \phi^2 + \sigma H^\dagger H \phi\,, \tag{1.2}$$

where $\hat{R}$ is the Ricci scalar and ϕ is the inflaton. During inflation, these generate large effective mass for the Higgs field, which can drive it to small field values and stabilize it at the origin [5]. Studies of the Early Universe Higgs evolution without the extra interactions can be found in [6] and [7–12].

Even if one suppresses the above terms at tree level, both of them are generated by renormalization group (RG) running. Indeed, the RG equation for ξ can be found in [13–15], while the Higgs–inflaton coupling is generated since successful reheating requires some inflaton coupling to the SM fields, which at loop level induces a Higgs–inflaton coupling [16]. We also note that Higgs–inflaton interaction is generated when one eliminates $H^\dagger H \hat{R}$ from the action by going to the Einstein frame. In general, we expect $\mathcal{L}_{hR}$ and $\mathcal{L}_{h\phi}$ to be equally important.

The Higgs–gravity and Higgs–inflaton couplings can stabilize the Higgs field during inflation, yet the same couplings may have a destabilizing effect after inflation. In particular, in the preheating epoch, the inflaton oscillates around its central value thereby inducing an oscillating mass term for the Higgs field. This can lead to explosive Higgs production through parametric, tachyonic or mixed resonance [17–20]. The resulting Higgs field variance can exceed the barrier separating the two vacua thus leading to vacuum destabilization. The consequent constraints on the couplings were derived in [20–22]. If only the non–minimal Higgs–gravity coupling is present, one requires $\xi \lesssim 10$ [22]. This assumes the inflationary Hubble rate $H \sim 10^{14}$ GeV and the SM instability scale of order 10^{10} GeV, which we take as representative values for our analysis. On the other hand, if only the Higgs–inflaton couplings are present, the constraints are roughly $\lambda_{h\phi} \lesssim 10^{-8}$, $|\sigma| \lesssim 10^{8}$ GeV [20]. The bounds are quite tight which restricts the range of the couplings consistent with vacuum stability *both* during and after inflation to a small window.

As mentioned above, one generally expects $\mathcal{L}_{hR}$ and $\mathcal{L}_{h\phi}$ to be equally important. Thus, in this work, we allow for the Higgs–gravity and Higgs–inflaton couplings to be present simultaneously. We find that different combinations of the couplings are responsible for stabilizing the Higgs during inflation and after inflation. In this case, the resonances during preheating can be suppressed which allows for a wide range of ξ and $\lambda_{h\phi}$ consistent with vacuum stability in the Early Universe.

The paper is organized as follows. In Section 2, we derive the scalar equations of motion in the Einstein frame. Higgs potential stability during inflation is studied in Section 3. The centrepiece of our analysis is Section 4, where preheating in the presence of $\mathcal{L}_{hR}$ and $\mathcal{L}_{h\phi}$ is analyzed. We conclude with Section 5.

2 Equations of Motion in the Einstein Frame

We start with the Jordan frame action of the form

$$S = \int \mathrm{d}^4x\sqrt{-\hat{g}}\left[\frac{1}{2}\left(1-\xi h^2\right)\hat{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu h\partial^\mu h - \hat{V}\left(\phi, h\right)\right], \tag{2.1}$$

where the hats serve to distinguish the Jordan frame quantities from those in the Einstein frame. In this work, we set $\hbar = c = M_{\mathrm{Pl}} = (8\pi G)^{-1/2} = 1$. The potential $\hat{V}$ is given by[1]

$$\hat{V}\left(\phi, \chi\right) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}\lambda_{h\phi}\phi^2h^2 + \frac{1}{2}\sigma\phi h^2 + \frac{1}{4}\lambda h^4. \tag{2.2}$$

Here ϕ is the inflaton and h is the Higgs field in the unitary gauge, $H = (0, v+h)^T/\sqrt{2}$. In this work, we focus on the chaotic ϕ^2 inflation [23] for definiteness, although our results apply more generally. The above potential includes the most general Higgs–inflaton interaction terms at the renormalizable level. In what follows, we will use the step function approximation for the running Higgs quartic coupling,

$$\lambda(h) \simeq 0.01 \times \mathrm{sign}\left(h_c^{\mathrm{SM}} - \sqrt{\langle h^2\rangle}\right)\,, \tag{2.3}$$

where we take for definiteness $h_c^{\mathrm{SM}} \sim 10^{10}$ GeV as the critical scale at which λ flips its sign.

The non–minimal Higgs coupling to gravity [24] can be eliminated by a conformal transformation. That is, the action in the Einstein frame is obtained via the conformal transformation of the metric,

$$g^{\mu\nu} = \Omega^{-1}\hat{g}^{\mu\nu}, \tag{2.4}$$

[1]Note the difference in the definition of $\lambda_{h\phi}$ compared to that of [20].

where

$$\Omega\left(h\right)\equiv1-\xi h^{2}. \tag{2.5}$$

This leads to

$$S=\int \mathrm{d}^{4}x\sqrt{-g}\left[\frac{1}{2}R-\frac{1}{2\Omega}\partial_{\mu}\phi\partial^{\mu}\phi-\frac{1}{2}\,\frac{6\left(\xi h\right)^{2}+\Omega}{\Omega^{2}}\partial_{\mu}h\partial^{\mu}h-V\left(\phi,h\right)\right], \tag{2.6}$$

where $V\left(\phi,h\right)$ is the Einstein frame potential defined by

$$V\left(\phi,h\right)\equiv\frac{\hat{V}\left(\phi,h\right)}{\Omega^{2}}. \tag{2.7}$$

As a result, the gravity part of the action assumes the Einstein-Hilbert form but the scalar field kinetic terms become non–canonical. One can show that the field space in eq. (2.6) is curved which makes it impossible to canonically normalize both fields simultaneously. One can however introduce a canonically normalized Higgs field h_c via

$$\mathrm{d}h_{\mathrm{c}}\equiv\sqrt{\frac{6\xi^{2}h^{2}+\Omega}{\Omega^{2}}}\,\mathrm{d}h. \tag{2.8}$$

Although it is possible to integrate eq. (2.8) analytically, we have no use for such an expression. Instead, we are interested in the small h regime where the difference between the Jordan and Einstein frames is suppressed by h^2 (in Planck units), that is,

$$\left|\xi\right|h^{2}\;,\;\xi^{2}h^{2}\ll1. \tag{2.9}$$

In this approximation,

$$h_{\mathrm{c}}\simeq h\left[1+\left(\xi+\frac{1}{6}\right)\xi h^{2}\right]. \tag{2.10}$$

The Einstein frame potential then reads

$$V\left(\phi,h_{\mathrm{c}}\right)=\frac{1}{2}m^{2}\phi^{2}+\frac{1}{2}\left(\lambda_{h\phi}+2\xi m^{2}\right)\phi^{2}h_{\mathrm{c}}^{2}+\frac{1}{2}\sigma\phi h_{\mathrm{c}}^{2}+\frac{1}{4}\lambda h_{\mathrm{c}}^{4}+\ldots, \tag{2.11}$$

where the dots denote higher dimensional operators which are small and therefore neglected in the following analysis.

The equations of motion are found by varying the action with respect to ϕ and h_{c}. For our purposes, it suffices to keep the homogeneous parts of the inflaton and the metric, in which case we obtain

$$\begin{aligned}&\ddot{\phi}+\left(3H+2\xi h_{\mathrm{c}}\dot{h}_{\mathrm{c}}\right)\dot{\phi}+\left[m^{2}+\left(\lambda_{h\phi}+\xi m^{2}\right)h_{\mathrm{c}}^{2}\right]\phi+\frac{1}{2}\sigma h_{\mathrm{c}}^{2}\simeq0, \qquad (2.12)\\&\ddot{h}_{\mathrm{c}}+3H\dot{h}_{\mathrm{c}}-\partial_{i}\partial^{i}h_{\mathrm{c}}+\left[\left(\lambda_{h\phi}+2\xi m^{2}\right)\phi^{2}-\xi\dot{\phi}^{2}+\sigma\phi\right]h_{\mathrm{c}}+\lambda h_{\mathrm{c}}^{3}\simeq0.\end{aligned}$$

Here $\partial_i\partial^i=a^{-2}\partial_i\partial_i$ with a being the scale factor. The Hubble parameter H is given by

$$3H^{2}\simeq\frac{1}{2}\left(1+\xi h_{\mathrm{c}}^{2}\right)\dot{\phi}^{2}+\frac{1}{2}\dot{h}_{\mathrm{c}}^{2}+V\left(\phi,h_{\mathrm{c}}\right). \tag{2.13}$$

3 Vacuum Stabilization During Inflation

The Higgs couplings to the inflaton and to gravity modify the scalar potential during inflation. If these are sufficiently large, the Higgs field evolves to electroweak values. As a case study, we choose the inflaton potential $m^2\phi^2/2$ [23] with $m \sim 10^{-5}$ as required by the COBE normalization of the primordial perturbations. Although this model is on the edge of the Planck–allowed 2σ parameter region [25] (see however [26]), it captures main features of the large field inflation framework such that our conclusions apply more generally.

3.1 Constraints from the Higgs evolution

Since the main thrust of our study concerns postinflationary Higgs dynamics, we will make certain simplifying assumptions which facilitate our analysis during inflation. Let us assume that at the initial stage the scalar potential is dominated by the inflaton contribution $m^2\phi^2/2$ and ϕ undergoes a slow roll to its VEV. Furthermore, we require that the Higgs potential be convex in the relevant field range so that the Higgs evolves to smaller values. Thus,

$$\begin{aligned} m^2 &\gg \left(\lambda_{h\phi} + 2\xi m^2\right) h_{c0}^2 \, , \\ \left(\lambda_{h\phi} + 2\xi m^2\right) \phi_0^2 &\gg |\lambda| h_{c0}^2 \, , \end{aligned} \tag{3.1}$$

where h_{c0} and ϕ_0 are the initial values of the Higgs field and the inflaton, respectively. We neglect the $\sigma\phi$ term during inflation (but not after) so that our stabilization mechanism is independent of the sign of ϕ. The above conditions constrain the Higgs values to be small in Planck units,

$$|\xi|\, h_c^2 \ll 1 \, , \tag{3.2}$$

$$|\lambda_{h\phi}|\, h_c^2 \ll m^2 \, , \tag{3.3}$$

where the first condition was imposed in the previous subsection, while the second follows then from (3.1). We note that larger Higgs values almost up to the Planck scale can also be treated consistently [5].

The equations of motion (2.12) take the form

$$\ddot{\phi} + \left(3H + 2\xi h_c \dot{h}_c\right)\dot{\phi} + \tilde{m}_\phi^2 \phi \simeq 0, \tag{3.4}$$

$$\ddot{h}_c + 3H\dot{h}_c + \tilde{m}_h^2 h_c + \lambda h_c^3 \simeq 0, \tag{3.5}$$

where we have introduced the effective mass terms

$$\tilde{m}_\phi^2 \equiv \left(1 + \xi h_c^2\right) m^2 + \lambda_{h\phi} h_c^2, \tag{3.6}$$

$$\tilde{m}_h^2 \equiv -\xi\dot{\phi}^2 + \left(\lambda_{h\phi} + 2\xi m^2\right)\phi^2, \tag{3.7}$$

and neglected the Higgs spatial gradient terms which are unimportant during inflation.

We assume that initially the Hubble parameter in eq. (2.13) is dominated by the potential term, that is, $\dot{\phi}^2,\, \dot{h}_c^2 \ll V(\phi, h_c)$. Slow–roll inflation is achieved if the Hubble friction term in the inflaton equations of motion (3.4) dominates $2\xi h_c \dot{h}_c$ and the effective inflaton mass term $\tilde{m}_\phi^2$. At the same time, the Higgs evolves exponentially quickly to zero if the Higgs effective mass term $\tilde{m}_h^2$ exceeds the Hubble term and the (negative) Higgs self–interaction term. That is,

$$H^2 \gg \left(\xi h_c \dot{h}_c\right)^2, \ \tilde{m}_\phi^2 \quad , \tag{3.8}$$

$$\tilde{m}_h^2 \gg H^2, \ \lambda h_c^2. \tag{3.9}$$

In this case, the Higgs field evolution is well described by

$$h_c \simeq h_{c0} e^{-\frac{3}{2}Ht} \cos\left(\tilde{m}_h t\right). \tag{3.10}$$

As follows from (3.1), the above conditions are satisfied in our set–up as long as $\phi^2 \gg 1$ which is the usual large field inflation condition and

$$\tilde{m}_h \gtrsim H \ . \tag{3.11}$$

The Hubble rate and ϕ are almost constant in this regime, and

$$\tilde{m}_\phi^2 \simeq m^2, \tag{3.12}$$

$$\tilde{m}_h^2 \simeq \left(\lambda_{h\phi} + 2\xi m^2\right)\phi^2. \tag{3.13}$$

3.2 Constraint from the flatness of the inflaton potential

As seen from eq. (2.11), both the non–minimal coupling ξ and the Higgs portal coupling contribute to the $\phi^2 h_c^2$ interaction in the Einstein frame. Closing the Higgs field in the loop, this induces the following Coleman–Weinberg correction to the inflaton potential (see e.g. [5]),

$$\Delta V_{\rm infl} \simeq \frac{(\lambda_{h\phi} + 2\xi m^2)^2}{64\pi^2} \ \phi^4 \ \ln \frac{(\lambda_{h\phi} + 2\xi m^2)\phi^2}{m^2} \ . \tag{3.14}$$

During the last 60 e–folds of inflation, this contribution should not exceed $m^2\phi^2/2$. Thus, taking $\phi \sim 10$, one finds[2]

$$\lambda_{h\phi} + 2\xi m^2 \lesssim 10^{-6} \ . \tag{3.15}$$

We note that the inflaton kinetic terms are Higgs–dependent, so in principle there are further radiative corrections from the term $h^2 \partial_\mu \phi \partial^\mu \phi$. These are however much less important as they lead to higher derivative interactions as well as small corrections to the kinetic terms, which we neglect.

3.3 Summary of constraints

Fast evolution of the Higgs field to small values requires (cf. eq. (3.11)) $\lambda_{h\phi} + 2\xi m^2 \gtrsim m^2$. Combining this lower bound with the upper bound coming from the radiative corrections to the inflaton potential, we obtain the allowed range for the couplings,

$$10^{-10} \lesssim \lambda_{h\phi} + 2\xi m^2 \lesssim 10^{-6}. \tag{3.16}$$

This is the range we will focus on in our subsequent discussion.

[2]These considerations of course apply as well if only the non–minimal coupling is present. The direct Higgs–inflaton coupling in the Einstein frame induces a radiative correction to the inflaton potential which leads to the constraint $|\xi| \lesssim 10^4$. This point has not been discussed in the literature.

There are further constraints on the initial values of the Higgs and the inflaton. In particular, requiring that the Higgs potential be convex in the relevant field range (cf. eq. (3.1)), we get

$$\phi_0 \gtrsim \sqrt{\frac{|\lambda|}{\lambda_{h\phi} + 2\xi m^2}}\, h_{c0}. \tag{3.17}$$

Since $|\lambda| \sim 10^{-2}$, this implies that the initial inflaton value must be between 2 and 4 orders of magnitude greater than the initial Higgs value. ϕ_0 is also constrained by the condition that inflation last about 60 e-folds such that $\phi_0 > \phi_* \simeq 15$, where ϕ_* is the inflaton value when cosmological scales exit the horizon.

The approximations we have employed in our analysis restrict further the initial Higgs field values (cf. eqs. (2.9,3.3)),

$$\frac{|\lambda_{h\phi}|}{m^2} h_{c0}^2 \ , \ |\xi| h_{c0}^2 \ , \ \xi^2 h_{c0}^2 \ \ll \ 1 \ . \tag{3.18}$$

We note that although some of the above conditions, e.g. that the inflaton dominates the energy density, can be relaxed (see [5],[27]), these are not essential for our analysis since our goal is to understand an interplay of the inflationary and preheating constraints on the Higgs couplings.

4 Higgs Evolution after Inflation: Preheating Epoch

At the end of inflation, the slow–roll conditions are violated and the inflaton rolls rapidly to the origin where it oscillates with high frequency. These oscillations cause resonant particle production of all the fields which interact strongly enough with the inflaton, the process called "preheating" [17, 19]. In the case of the Higgs field, this could have disastrous consequences since the field can be excited beyond the stability scale. Thus, the same interactions which stabilize the Higgs during inflation could destabilize it afterwards. This phenomenon was studied in detail in refs. [21, 22] for the $\lambda_{h\phi} = 0$ case and in refs. [20, 22] for $\xi = 0$.[3] Here we consider a general situation with $\lambda_{h\phi} \neq 0$ and $\xi \neq 0$. We find that an interplay of the two couplings is important leading to different conclusions compared to the cases considered before. In what follows, we drop the scale dependence of ξ over the energy range relevant to inflation/preheating (see the RG equations in [14]) such that it should be understood approximately as $\xi(H)$, while the scale dependence of $\lambda_{h\phi}$ is completely negligible.

At the end of inflation, the Higgs field is anchored at the origin, making the contribution of Higgs–inflaton crossterms in eq. (2.12) negligible. The Hubble friction term is also small compared to the inflaton mass. Hence, eq. (2.12) reduces to the harmonic oscillator equation with a decaying amplitude,

$$\phi\,(t) \simeq \Phi\,(t) \cos\,(mt)\,. \tag{4.1}$$

Strictly speaking, the harmonic approximation above is valid somewhat after the end of inflation. For definiteness, we can take

$$\Phi\,(t_0) \simeq 1. \tag{4.2}$$

[3]Related analyses can be found in [28, 29].

Thereafter, the universe is dominated by inflaton oscillations, which effectively behave as pressureless dust. Hence the oscillation amplitude decays as

$$\Phi\left(t\right)\simeq\frac{\sqrt{8/3}}{mt}, \tag{4.3}$$

while the scale factor a grows as $a \propto t^{2/3}$. Plugging eq. (4.1) into the Higgs equation of motion (2.12), we get (cf. [20])

$$\frac{\mathrm{d}^2 X_k}{\mathrm{d}z^2}+\left[A_k\left(z\right)+2p\left(z\right)\cos 2z+2q\left(z\right)\cos 4z+\frac{\delta m^2(z)}{m^2}\right]X_k\simeq 0, \tag{4.4}$$

where $X_k \equiv a^{3/2}h_k$ is the rescaled Fourier k–mode of the Higgs field h_c and we have defined

$$z\equiv\frac{1}{2}mt, \tag{4.5}$$

$$A_k\left(z\right)\equiv\left(2\frac{k}{am}\right)^2+2\left(\lambda_{h\phi}+\xi m^2\right)\frac{\Phi\left(z\right)^2}{m^2}, \tag{4.6}$$

$$p\left(z\right)\equiv 2\frac{\sigma\Phi\left(z\right)}{m^2}, \tag{4.7}$$

$$q\left(z\right)\equiv\left(\lambda_{h\phi}+3\xi m^2\right)\frac{\Phi\left(z\right)^2}{m^2}, \tag{4.8}$$

$$\delta m^2(z)\equiv 3\lambda a^{-3}\langle X^2\rangle. \tag{4.9}$$

In this equation, we have resorted to the Hartree approximation $h^3 \to 3h\langle h^2\rangle$ such that the Higgs self–interaction is replaced by an effective mass term. In this case, the equations for different k–modes decouple. However, in our lattice simulations we do not employ this approximation.

We note that during preheating, the $\sigma\phi h_\mathrm{c}^2$ term plays an important role since it decreases slower than the quartic $\phi^2 h_\mathrm{c}^2$ interaction and at some stage becomes dominant [20]. However, the effect of this term can be isolated and it is instructive to start with the case $\sigma = 0$.

4.1 $\sigma = 0$ case: modified Mathieu equation

Let us set $\sigma = 0$ and, furthermore, neglect for the moment the Universe expansion and Higgs self–interaction, $\delta m^2 \to 0$. These are good approximations at the beginning of the preheating epoch. In this case, eq. (4.4) reduces to the familiar Mathieu equation [17],

$$\frac{\mathrm{d}^2 X_k}{\mathrm{d}z^2}+\left[A_k+2q\cos 4z\right]X_k\simeq 0, \tag{4.10}$$

In effect, the Higgs field receives an oscillating mass term which can cause resonance. Certain k–modes up to $k_*/a \sim \left(\lambda_{h\phi}+3\xi m^2\right)^{1/4}\sqrt{m\Phi}$ would undergo a resonant growth, which results in the increase of the Higgs variance $\langle h_c^2\rangle$,

$$\langle h_c^2\rangle \simeq \int\frac{d^3k}{(2\pi a)^3}\frac{n_k}{\omega_k}, \tag{4.11}$$

where the mode frequency is determined by $\omega_k^2 = \left(A_k + 2q\cos 4z\right)m^2/4$ and n_k is the corresponding occupation number (see [20] for further details). If the fluctuations become too large, the Higgs field crosses over the barrier into the catastrophic vacuum.

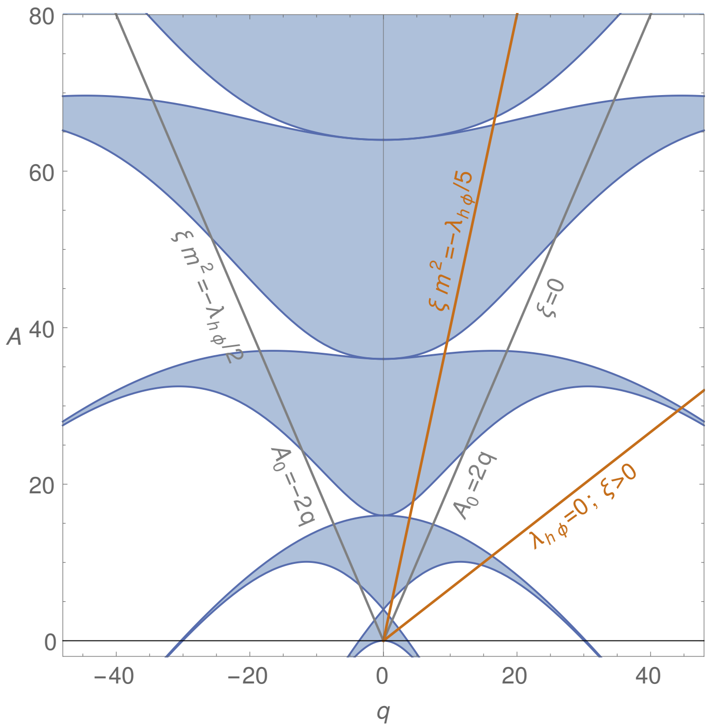

Figure 1: Stability chart of the Mathieu equation.

Behaviour of the Mathieu equation solutions is described by the stability chart of fig. 1. If the point (A_k, q) lies in the white regions, the corresponding solution grows exponentially in time. On the other hand, points in the shaded regions lead to stable solutions. In an expanding Universe, both A_k and q decrease in time crossing many instability regions. The resonance ends when the system enters the last stability region at $q \lesssim 4$. If the resulting $\langle h_c^2 \rangle$ is below its critical value, no destabilization occurs. The position of the barrier separating the two vacua is affected by the Higgs–inflaton couplings,

$$h_{\rm crit} \sim \sqrt{\frac{2(\lambda_{h\phi} + 2\xi m^2)}{|\lambda|}}|\phi| \ . \tag{4.12}$$

Therefore even when the Higgs fluctuations exceed 10^{10} GeV, the system may remain stable. One finds that the destabilization criterion amounts roughly to $\sqrt{\langle h_c^2 \rangle} > h_{\rm crit}$ with some average value of $|\phi|$. This is discussed in detail in [20] (for $A = 2q$).

An important feature of the stability chart in fig. 1 is that larger A/q lead to a weaker resonance since the instability regions become shorter along the corresponding evolution lines. In particular, $q \simeq 0$ or

$$\lambda_{h\phi} \simeq -3\xi m^2 \tag{4.13}$$

suppresses the resonance completely. It is in fact sufficient to have $A/|q| \sim$ few to avoid vacuum destabilization. Such a relation is not very unnatural in our framework: since $m^2 \sim 10^{-10}$, values of $|\xi|$ in the range $1...10^4$ correspond to $\lambda_{h\phi}$ between 10^{-10} and 10^{-6}. The latter is the range considered in [5].

In reality, the Universe expands and the Hartree approximation for the h^4 term is not necessarily reliable (see [20]), hence the Mathieu equation gives only an approximate description

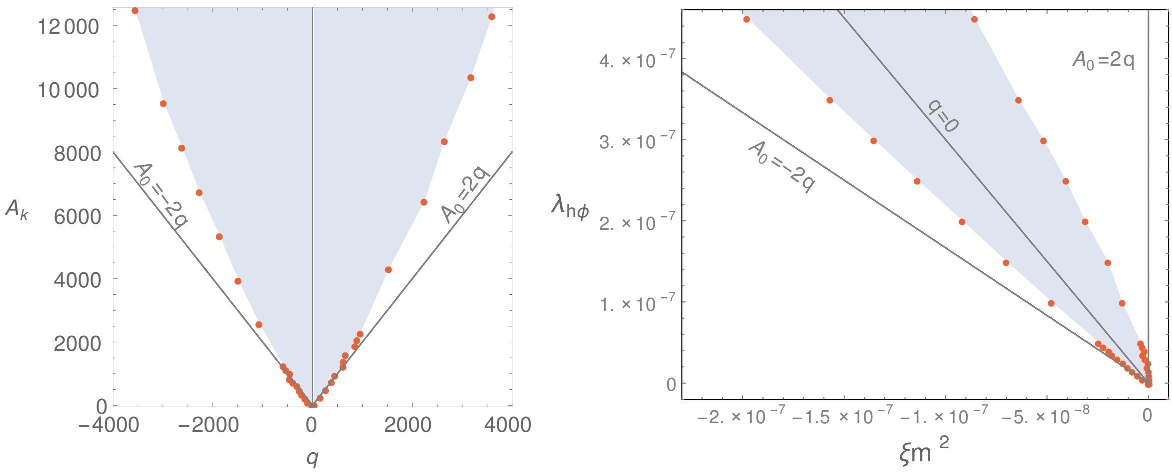

Figure 2: Vacuum stability regions (shaded) during preheating in the (A, q) plane (left) and $(\lambda_{h\phi}, \xi m^2)$ plane (right). Here $m^2 \simeq 10^{-10} M_{\rm Pl}^2$ and the SM instability scale is 10^{10} GeV. The red dots indicate the boundary of the stability region obtained with a modified version of `Latticeeasy`.

of the system behaviour. To determine the stability regions in our parameter space reliably, we have resorted to classical lattice simulations (for a recent overview see [30]). We have used two different implementations, in the Jordan and Einstein frames, with two different codes: a modified version of `Latticeeasy` [31] and our own lattice code. The results of our numerical simulations are presented in fig. 2. These show maximal allowed $|q|$ for a given A and the corresponding range of ξm^2 for a given $\lambda_{h\phi}$. The displayed points correspond to stable configurations in the sense that the Higgs variance does not blow up before the end of the resonance. The resonance ends when the occupation numbers in the comoving frame remain constant. We find that variations of these requirements lead to similar results. Our main result is that a wide range of positive $\lambda_{h\phi}$ and negative ξ is allowed. On the other hand, negative $\lambda_{h\phi}$ lead to larger q versus A and therefore are ruled out (apart from small values around the origin). Note that the line $A = 2|q|$ separates the parametric resonance from the tachyonic one.

The qualitative behavior of our bound can be understood as follows. We are mostly interested in region with a substantial ratio A/q. At $A/q > 2.3$ or so, the parametric resonance in the expanding Universe simplifies. Whether the system gets destabilized or not is mostly determined by its behavior in the last instability region, that is, the one closest to the origin. This is because, in the parameter range of interest, the system spends little time in other instability bands since they are shorter (fig. 1) and the inflaton evolves faster at earlier times. This tendency is clearly seen in fig. 3: at $A/q = 3$ only the last band contributes significantly, whereas in the "usual" case of $A/q = 2$ many bands are important. The increase of the Higgs amplitude for a given k, which can be taken $k = 0$ as a representative value, is determined approximately by the Floquet exponent

$$X_0 \propto e^{\mu_{\rm eff} \Delta m t}, \tag{4.14}$$

where $\mu_{\rm eff}$ is an average Floquet parameter μ along the relevant trajectory in the last instability band and Δmt is the time the system spends there. Since both A and q have the same

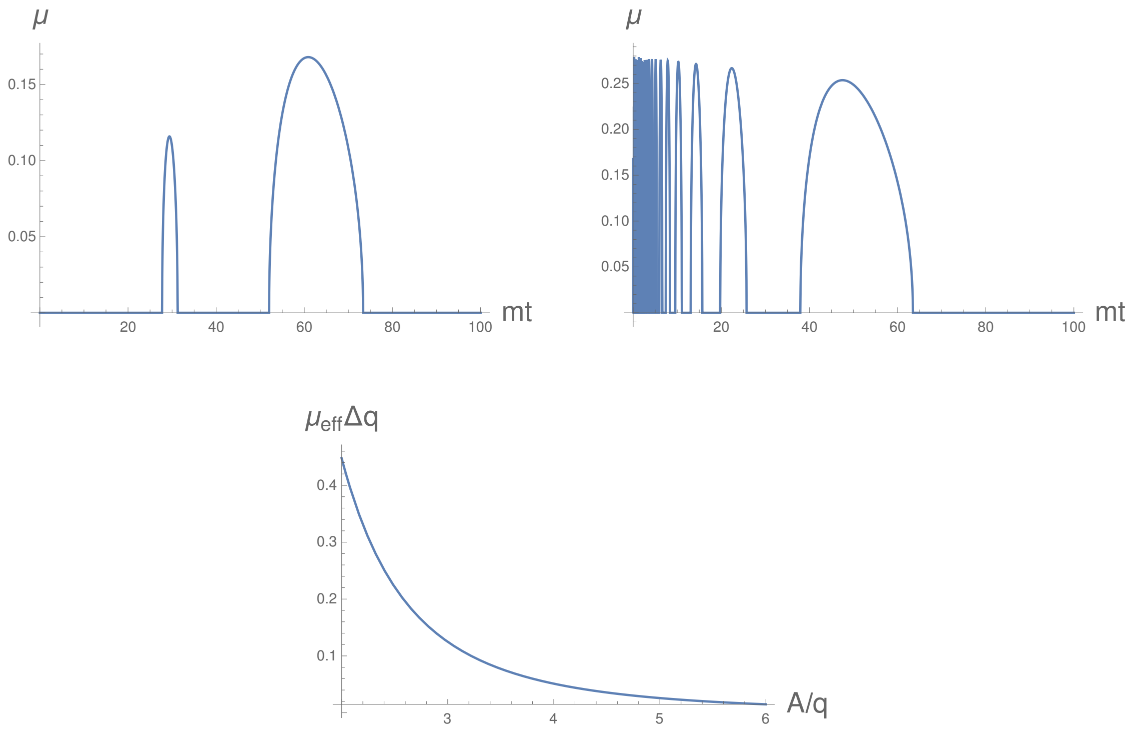

Figure 3: Top: the Floquet exponent μ (cf. eq. (4.14)) along the trajectory $A/q = 3$ (left) and $A/q = 2$ (right). Here the initial value of q is chosen to be 2000. Bottom: $\mu_{\rm eff}\Delta q$ vs A/q.

dependence on the inflaton amplitude, the system evolves along a straight line in the (A, q) plane.[4] Clearly, the resonance becomes inefficient for $\mu_{\rm eff}\Delta mt \lesssim \mathcal{O}(1)$. The quantity Δmt depends on two factors: (a) A/q determines the width Δq (as well as $\mu_{\rm eff}$) of the instability band with larger A leading to smaller Δq; (b) the rate of the inflaton change in the last band which is controlled by the duration of the resonance $mt_{\rm end}$. For reference, in fig. 3 (bottom) we display the scaling of $\mu_{\rm eff}\Delta q$ with A/q. This can be converted to $\mu_{\rm eff}\Delta mt$ using the approximate relations $\Delta q/q \simeq 2\Delta mt/mt$ and $mt_{\rm end} \simeq \text{const} \times \sqrt{\lambda_{h\phi} + 3\xi m^2}$. We thus get the following scaling

$$\mu_{\rm eff}\Delta mt \simeq \text{const} \times \sqrt{\lambda_{h\phi} + 3\xi m^2}\; \mu_{\rm eff}\Delta q \,. \tag{4.15}$$

This shows that the resonance can be suppressed at larger couplings $\lambda_{h\phi} + 3\xi m^2$, i.e. larger initial q, by increasing A/q. For instance, a tenfold increase in $\lambda_{h\phi} + 3\xi m^2$ can be compensated by increasing A/q from 2 to 3. This is roughly what we observe in fig. 2. While for larger A/q the above scaling works well, at $A/q \leq 2.3$ this approximation breaks down and many instability bands contribute to the resonance. In fact, the tachyonic resonance is also consistent with vacuum stability as long as the relevant couplings are small, i.e. around the origin in fig. 2.

The final results of our study are shown in fig. 4, which displays the parameter space consistent with vacuum stability during and after inflation. The region is finite due to the large couplings being cut off by the constraint of sufficiently flat inflaton potential. Negative

[4] We have verified that this also applies to the relevant range of $k \neq 0$.

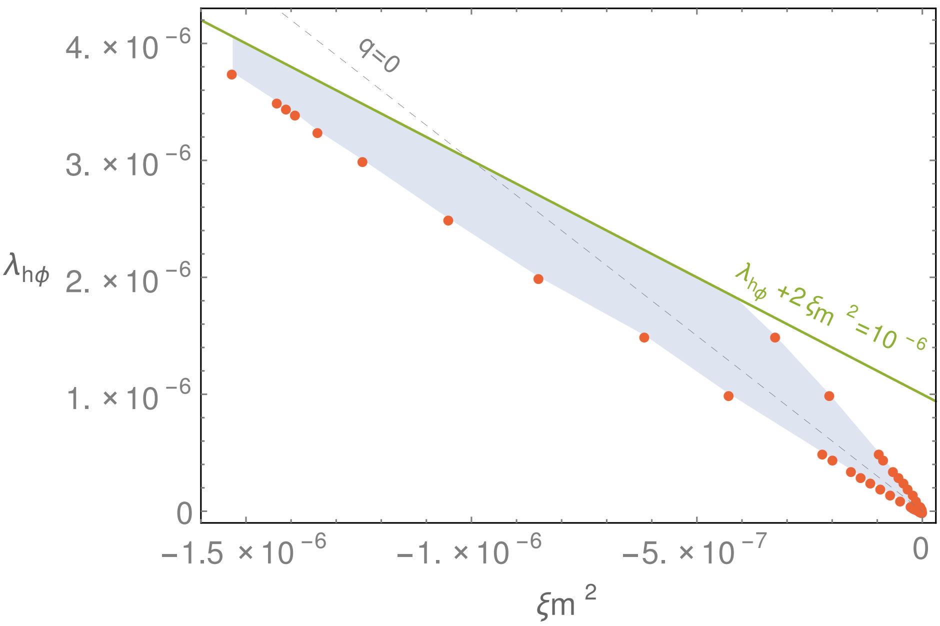

Figure 4: Parameter space consistent with vacuum stability during inflation and preheating (shaded). Here $m^2 \simeq 10^{-10} M_{\rm Pl}^2$ and the SM instability scale is 10^{10} GeV. The green line marks the upper bound on the couplings from the flatness of the inflaton potential. The red dots indicate the boundary of the stability region obtained with our lattice simulations.

values of $\lambda_{h\phi}$ (and positive ξ) lead to stronger resonance and thus are excluded except for points close to the origin (as in [22],[20]). In the plot, the red dots indicate the boundary of the stability region obtained with our lattice simulations. These center around the "no resonance" $q = 0$ line. The allowed parameter space extends to about $\lambda_{h\phi} \sim 6 \times 10^{-6}$, although the tip of this region is not shown in the plot. The inflationary constraint $\lambda_{h\phi} + 2\xi m^2 > 10^{-10}$ is satisfied automatically by points in the shaded region.

Let us emphasize again the difference between the analyses with and without the non–minimal Higgs coupling to gravity. The ξ–term brings in both the $h_c^2\phi^2$ and $h_c^2\dot{\phi}^2$ interactions during preheating, thereby modifying A_k and q in different ways. This changes radically the stability analysis compared to the pure Higgs–inflaton coupling case since A/q becomes a free variable. Hence, the resonance can be suppressed while still retaining the positive stabilizing effect of the couplings during inflation.

4.2 $\sigma \neq 0$ case: modified Whittaker–Hill equation

The trilinear interaction $h_c^2\phi$ can be neglected compared to $h_c^2\phi^2$ during inflation, however when the inflaton field decreases in the preheating epoch, it can become dominant. While the quantities q and A_k (for small k) in eq. (4.4) decrease as $1/t^2$, the coefficient p decays as $1/t$ which leads to a tachyonic resonance at late times. This leads to very efficient Higgs production and thus to a strong constraint on σ [20]. Neglecting the Universe expansion and the Higgs self–interaction, the Higgs dynamics are described by the Whittaker–Hill equation (cf. [32–35])

$$\frac{{\rm d}^2 X_k}{{\rm d}z^2} + \left[A_k + 2p\cos 2z + 2q\cos 4z\right] X_k \simeq 0. \tag{4.16}$$

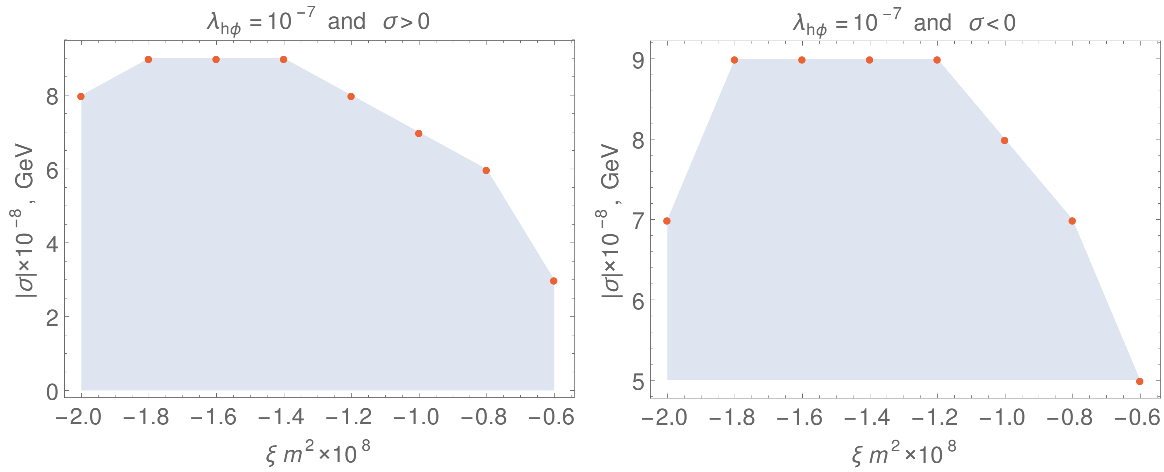

Figure 5: Vacuum stability regions (shaded) in the presence of the trilinear Higgs–inflaton coupling σ. Left: $\sigma > 0$, right: $\sigma < 0$. The red dots indicate the boundary of the stability region obtained with our lattice simulations.

This is a good approximation as long as A_k, p and q change adiabatically. The relevant stability charts for the Whittaker–Hill equation have been studied in [20]. The results of that study can be directly applied here since the presence of the non–minimal coupling to gravity does not affect the trilinear coupling.

Nevertheless, we have performed an additional lattice study to set limits on σ. It includes the effects of the Universe expansion and the full Higgs self–interaction h_c^4. Choosing $\lambda_{h\phi}$ and ξ values from the stability region in fig. 2, we increase $|\sigma|$ until the tachyonic resonance destabilizes the system. Thus we determine the maximal allowed $|\sigma|$ for given $\lambda_{h\phi}$, ξ shown in fig. 5. The resulting dependence is non–monotonic, yet the order of magnitude of the maximal $|\sigma|$ remains between 10^8 and 10^9 GeV, as in our previous study [20]. Note that positive and negative σ are inequivalent due to the Universe expansion.

Finally, let us comment on two effects which are not taken into account in our simulations: (a) perturbative Higgs decay and (b) possibility of late time Higgs destabilization (after preheating). The effect of the Higgs decay into top quark pairs was studied in [22],[20]. It was found that although it reduces $\langle h_c^2 \rangle$, it does not significantly affect the stability properties of a configuration with given couplings. In particular, unstable configurations typically remain unstable except the destabilization time moves to somewhat larger values. The decay has a stabilizing effect on the system, yet it is not significant enough for our purposes and we neglect it. Concerning the late time behavior, $\langle h_c^2 \rangle$ decreases slower in time than the position of the barrier between the two vacua [22],[20] which can lead to eventual destabilization. However, there are many factors that affect the system after the end of the resonance: thermalization, non–perturbative effects, etc. Therefore, this question requires a dedicated study. Let us note however that our mechanism allows one to suppress the resonance completely such that no significant $\langle h_c^2 \rangle$ would arise. In this case, there is no danger of late time vacuum destabilization.

5 Conclusions

The Higgs–inflaton and Higgs–gravity couplings are essential for understanding the Early Universe Higgs dynamics. These can stabilize the electroweak vacuum during inflation by inducing large effective Higgs mass. On the other hand, the same couplings can have a destabilizing effect after inflation due to explosive Higgs production via parametric or tachyonic resonance.

In this work, we have studied a combined effect of the Higgs–inflaton $\lambda_{h\phi}$ and non–minimal Higgs–gravity ξ couplings. We find that a non–trivial interplay of the two allows us to stabilize the Higgs field during inflation while avoiding excessive Higgs production after inflation. In particular, different combinations of $\lambda_{h\phi}$ and ξ enter the A and q parameters of the Mathieu equation which makes it possible to suppress the parametric and tachyonic resonance. This mechanism is effective even at "large" couplings up to $\lambda_{h\phi} \sim 6\times10^{-6}$ (fig. 4). It requires $\lambda_{h\phi} > 0$ and $\xi < 0$ (unless the couplings are small, e.g. $\xi < 10$). To delineate parameter space, we have resorted to classical lattice simulations based on `Latticeeasy` as well as an independent code.

We have focused on a chaotic m^2 inflation model as a representative example of large field inflation and also fixed the Standard Model instability scale to be around 10^{10} GeV. In this case, $\lambda_{h\phi}$ and ξm^2 (where m is the inflaton mass) must be of the same order of magnitude for our mechanism to work. Given that $m^2 \sim 10^{-10}$ in Planck units, this allows for a wide range of the couplings including $|\xi| \gg 1$, unlike the previous analyses.

We have also studied the effect of the mixed parametric–tachyonic resonance induced by the trilinear Higgs–inflaton coupling ϕh^2. The resulting bound on the trilinear parameter $|\sigma|$ is between 10^8 and 10^9 GeV, which is consistent with our previous study [20].

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A Vacuum Fluctuations and Lattice Simulations

In this appendix, we summarize the issues of vacuum fluctuations relevant to our simulations. In order to quantify particle production, we need to account for vacuum fluctuations appropriately. Indeed, since the Mathieu equation is homogeneous, no particle production occurs unless X_k or its derivative is non–zero initially. Such initial conditions are provided by the vacuum fluctuations. In `Latticeeasy`, one employs the probability distribution for the ground state of a scalar field in a Friedmann Universe [36],

$$P(X_k) \propto \exp\left(-2\omega_k^2|X_k|^2\right) \ , \tag{A.1}$$

where ω_k is the frequency of oscillations of the eigenmode at $t = 0$ and we set the scale factor $a = 1$ initially. The resulting distribution for $|X_k|$ is of Rayleigh type, while the phase of X_k has a random uniform distribution. This gives the mean–squared value

$$\langle|X_k|^2\rangle = \frac{1}{2\omega_k} \, , \tag{A.2}$$

which corresponds to the vacuum contribution to $\langle h_c^2 \rangle$. Formally, it diverges in the UV and must be regularized. For instance, one can impose a momentum cutoff, compute the Higgs variance as a function of time and subtract the vacuum contribution: $\langle h_c^2 \rangle_{\rm reg} = \langle h_c^2 \rangle - \langle h_c^2 \rangle_{\rm vac}$. This is known as adiabatic regularization (see [37] for a recent discussion).

Let us focus for definiteness on the "standard" parametric resonance $A = 2q$ (fig. 1). When the resonance is efficient ($q \gg 1$), X_k for comoving momenta $k < k_* \sim mq^{1/4}$ grow fast and the mode occupation numbers become large. Away from the inflaton zero crossings, these can be approximated by $n_k \simeq \omega_k |X_k|^2$. The modes much above k_* do not get amplified and remain in the "vacuum state". Clearly, they do not contribute to $\langle h_c^2 \rangle_{\rm reg}$.

In practice, if the momentum cut–off is of order k_*, the vacuum subtraction is unimportant since for this momentum range $\langle h_c^2 \rangle$ is dominated by modes with large occupation numbers. In our simulations, the cut–off is taken to be $35m$ which is enough to capture the UV behavior of the system. The boundary of the stability region in fig. 4 corresponds to substantial occupation numbers up to $\mathcal{O}(100)$ such that the system is in the semi–classical regime $n_k \gg 1$.

We emphasize that large Higgs variance $\langle h_c^2 \rangle_{\rm reg}$ leads to destabilization only if the system is semi–classical, i.e. the occupation numbers are large. For a purely quantum system, large $\langle h_c^2 \rangle_{\rm reg}$ cannot change the "vacuum state". Indeed, a single highly energetic particle (or a narrow wave packet) can lead to $\sqrt{\langle h_c^2 \rangle_{\rm reg}} > h_{\rm crit}$, but that does not lead to destabilization. For the same reason, extremely high energy cosmic rays cannot destabilize the electroweak vacuum [38] since a sufficiently large semi–classical patch of the field in a new phase must be created for the transition. This is relevant to the weak resonance regime $q \lesssim 1$, where particle creation is inefficient. Even if at late times $\sqrt{\langle h_c^2 \rangle_{\rm reg}} > h_{\rm crit}$, no transition occurs. We also note that when the resonance has faded away, the dynamics become rather complicated due to rescattering, thermalization and possible non–perturbative phenomena.

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