Top Quark Mass Measurement in the lepton+jets Channel
Using a Matrix Element Method and in situ Jet Energy Calibration

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Top Quark Mass Measurement in the Lepton + Jets Channel Using a Matrix Element Method and in situ Jet Energy Calibration

A precision measurement of the top quark mass $m_t$ is obtained using a sample of $t\bar{t}$ events from $p\bar{p}$ collisions at the Fermilab Tevatron with the CDF II detector. Selected events require an electron or muon, large missing transverse energy, and exactly four high-energy jets, at least one of which is tagged as coming from a $b$ quark. A likelihood is calculated using a matrix element method with quasi–Monte Carlo integration taking into account finite detector resolution and jet mass effects. The event likelihood is a function of $m_t$ and a parameter $\Delta_{\text{JES}}$ used to calibrate the jet energy scale in situ. Using a total of 1087 events in 5.6 fb$^{-1}$ of integrated luminosity, a value of $m_t = 173.0 \pm 1.2$ GeV/c$^2$ is measured.

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The top quark is the heaviest known fundamental particle in the standard model of particle physics. Since the 1995 discovery of the top quark at the Fermilab Tevatron [1], both the CDF and D0 experiments have been improving the measurement of its mass $m_t$, which is a fundamental parameter in the standard model [2]. Loop corrections in electroweak theory relate $m_t$ (along with the W boson mass $m_W$) to the mass of the predicted Higgs boson. Thus, precision measurements of $m_t$ help to constrain the value of the Higgs boson mass [3].

This Letter describes the single most-precise measurement to date of the top quark mass. It is performed on data collected by the CDF II detector [4] during Run II of the Fermilab Tevatron $p\bar{p}$ collider operating at $\sqrt{s} = 1.96$ TeV with a total integrated luminosity of 5.6 fb$^{-1}$. The measurement is performed on candidate $t\bar{t}$ events containing a lepton and four jets [5]. For each event selected in this analysis, we calculate the probability of observing that event by integrating the matrix element for $t\bar{t}$ production and decay over phase-space variables. We use a neural network discriminant to distinguish between signal and background events to correct for the contribution due to background, and employ a cut on the peak likelihood for a given event for additional rejection of background and poorly modeled signal events. This analysis offers a gain of nearly 20% in statistical precision over our previous measurement [5] given an equal number of events; there, in order to make the likelihood integration computationally tractable, we introduced kinematic assumptions to reduce the dimensionality of the integral. In the present analysis, we use a quasi–Monte Carlo integration technique [5], which converges more rapidly than the typical $O(N^{-1/2})$ convergence of standard Monte Carlo integration. This allows us to integrate over a total of 19 dimensions in a computationally practical time, resulting in a more accurate modeling of the event. Furthermore, in addition to the increased data sample available with more integrated luminosity delivered by the Tevatron, we have expanded our muon selection ability, which increases the size of our data sample by nearly 30%. In total, this measurement improves our statistical precision by a factor of two over our previous analysis with 1.9 fb$^{-1}$ [3].

In this measurement, the largest uncertainty is due to the uncertainty in the jet energy scale (JES) determination. To reduce this uncertainty, we calculate the likelihood as a two-dimensional function of $m_t$ and a second parameter, $\Delta_{\text{JES}}$, which corrects the jet energies by a factor of $1 + \Delta_{\text{JES}} \cdot \sigma_j$, where $\sigma_j$ is the fractional systematic uncertainty on the energy for a given jet [7] [8]. The known $W$ boson mass is used to constrain the $W \to q\bar{q}'$ decay, which yields information on the $\Delta_{\text{JES}}$ parameter. We can thus optimally combine events to reduce the total uncertainty on $m_t$ due to JES.

Within the standard model, the top quark decays almost exclusively into a $W$ boson and a $b$ quark. We define a “lepton + jets” event as an event where one of the $W$ bosons produced by the $t\bar{t}$ pair decays into a charged lepton (in this analysis, an electron or a muon) and a neutrino, and the other into a $q\bar{q}'$ pair. The two $b$ quarks and two quarks from the $W$ boson then produce jets in the detector [9]. We thus require candidate events to have an electron with $E_T > 20$ GeV or a muon with $p_T > 20$ GeV/c in the central detector ($|\eta| < 1$ [10]), or a muon with $p_T > 20$ GeV/c obtained with a trigger [3] on missing transverse energy, $E_T$ [10], instead of a central muon. As the neutrino energy is not detected, we require $E_T > 20$ GeV in the event. We also require exactly four jets with $E_T > 20$ GeV within the region $|\eta| < 2.0$, at least one of which must be tagged as a $b$ jet using a secondary vertex tagging algorithm [11]. To model $t\bar{t}$ events, we use Monte Carlo–simulated events generated with the PYTHIA [12] generator for 15 different $m_t$ values ranging from 162 to 184 GeV/c$^2$.

Background events contributing to the selected sample are: a) events in which a $W$ boson is produced in conjunction with heavy-flavor quarks ($b\bar{b}$, $c\bar{c}$, or $c\bar{c}$); b) events in which a $W$ boson is produced along with light quarks, at least one of which is mistagged as heavy flavor; c) QCD events that do not contain a true $W$ boson; d) diboson ($WW$, $WZ$, or $ZZ$) or $Z +$ jets events; e) single top events. We model the contribution from $W +$jets events using ALPGEN [13], single top events using MADGRAPH [14], and diboson events with PYTHIA. The $Z+$jets contributions are not modeled separately, but are included in the $W+$light flavor contribution. All Monte Carlo samples are processed with the CDF II detector response simulation package [15]. The non-$W$ QCD background is modeled using a sideband of data events selected to have a small contribution from heavy boson decay. The numbers of background events are estimated with the method used for the $t\bar{t}$ cross section measurement [15], and are shown in Table [1].

For each event, we construct a likelihood as a function of $m_t$ and $\Delta_{\text{JES}}$ using the following integral:

$$L(\vec{y} \mid m_t, \Delta_{\text{JES}}) = \frac{1}{N(m_t)} A(m_t, \Delta_{\text{JES}}) \times \frac{1}{\mathcal{F}(m_t, \Delta_{\text{JES}})} \times \int \mathcal{F}(\vec{x} \mid \Delta_{\text{JES}}) d\Phi(\vec{x}),$$

where $\vec{y}$ are the quantities measured in the detector (the momenta of the jets and charged lepton), $\vec{x}$ are the parton-level quantities that define the kinematics of the event, $N(m_t)$ is a global normalization factor, $A(m_t, \Delta_{\text{JES}})$ is the event acceptance as a function of
TABLE I: Expected sample composition for an integrated luminosity of 5.6 fb$^{-1}$. The $t\bar{t}$ contribution is estimated using a cross-section of 7.4 pb$^{-1}$ and $m_t = 172.5$ GeV/c$^2$.

<table>
<thead>
<tr>
<th>Event type</th>
<th>1 b tag</th>
<th>$\geq 2$ b tags</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W$+heavy flavor</td>
<td>129.5 ± 42.1</td>
<td>15.7 ± 5.5</td>
</tr>
<tr>
<td>non-$W$ QCD</td>
<td>50.1 ± 25.5</td>
<td>5.5 ± 3.8</td>
</tr>
<tr>
<td>$W$+light flavor mistag</td>
<td>48.5 ± 17.1</td>
<td>1.0 ± 0.4</td>
</tr>
<tr>
<td>diboson (WW, WZ, ZZ)</td>
<td>10.5 ± 1.1</td>
<td>1.0 ± 0.1</td>
</tr>
<tr>
<td>Single top</td>
<td>13.3 ± 0.9</td>
<td>4.0 ± 0.4</td>
</tr>
<tr>
<td>$Z \to t\bar{t}$ + jets</td>
<td>9.9 ± 1.2</td>
<td>0.8 ± 0.1</td>
</tr>
<tr>
<td>Total background</td>
<td>261.8 ± 60.6</td>
<td>28.0 ± 9.6</td>
</tr>
<tr>
<td>$t\bar{t}$ signal</td>
<td>767.3 ± 97.2</td>
<td>276.5 ± 43.0</td>
</tr>
<tr>
<td>Total expected</td>
<td>1029.1 ± 114.5</td>
<td>304.5 ± 44.1</td>
</tr>
<tr>
<td>Events observed</td>
<td>1016</td>
<td>247</td>
</tr>
</tbody>
</table>

$m_t$ and $\Delta_{\text{JES}}$, $f(z_1)$ and $f(z_2)$ are the parton distribution functions (PDFs) for incoming parton momentum fractions $z_1$ and $z_2$, $FF$ is the relativistic flux factor, $\text{TF}(\vec{y} \mid \vec{x}, \Delta_{\text{JES}})$ are the transfer functions that describe the measured jet-momentum distributions given the quark kinematics, $d\Phi(\vec{x})$ the phase space for the eight particles in the $t\bar{t}$ production and decay process, and $M(m_t, \vec{x})$ is the matrix element for the process. The integral is calculated for each of the 24 possible permutations of jet-parton assignment and then summed with weights $w_i$ determined by the probability that a 6 or light parton will result in a $b$-tagged or untagged jet.

We use the Kleiss-Stirling matrix element [18], which is a leading-order matrix element including both $qq \rightarrow t\bar{t}$ and $gg \rightarrow t\bar{t}$ production processes, as well as all spin correlations. For the PDFs, we use the CTEQ5L functions [19] for the incoming $q\bar{q}$ and gluons. The normalization factor $N(m_t)$ is obtained by integrating the Kleiss-Stirling matrix element with the PDFs and the flux factor over the phase space formed by the two initial and the six final-state particles. The acceptance $A(m_t, \Delta_{\text{JES}})$ is obtained from simulated events where the parton directions and momenta are smeared to simulate final-state jets. The transfer functions connect the measured jets to the partons. We construct the transfer functions by taking simulated $t\bar{t}$ → lepton + jets events in a wide range of masses and matching the simulated jets to their parent partons. The transfer functions are separated into momentum and angular terms; both are constructed with dependence on the true jet $p_T$ and mass from the Monte Carlo simulation. The transfer functions are constructed separately for $b$ and light quarks, as well as for each of four bins of jet $\eta$. There are 32 phase space integration variables in Eq. (1) (for the two initial partons and six final partons). Four of these are eliminated by energy and momentum conservation, and four more by taking the charged lepton, neutrino, and initial parton masses as known. In addition, we assume that the lepton momentum is perfectly measured, and we neglect the effects of the individual transverse momenta of the initial partons so that we model only the transverse momentum of the total $t\bar{t}$ system, for which we use a prior derived from Monte Carlo simulation. This leaves a total of 19 dimensions over which the integral must be evaluated, which we perform using a quasi–Monte Carlo technique.

Handling of background events is unchanged from our previous publication [5]. We identify events likely to be background using a JETNET 3.5 artificial neural network [20] with ten inputs. We construct distributions of the neural network output weight $u$ for signal, $S(u)$, and background, $B(u)$, events, normalized to their overall expected fractions, and calculate the expected background fraction for a given event as $f_{\log}(u) = B(u)/(B(u) + S(u))$.

We calculate the likelihood for all candidate events under the assumption that they are signal, but the combined likelihood contains contributions from both signal and background events. However, only the signal events contain information about $m_t$, so using Monte Carlo–simulated events we compute the average likelihood for background events and subtract it from the total likelihood:

$$\log L_{\text{adj}}(m_t, \Delta_{\text{JES}}) = \sum_{i \in \text{events}} \left[ \log L(\vec{y}_i|m_t, \Delta_{\text{JES}}) - f_{\log}(u_i) \log L_{\text{bg}}(m_t, \Delta_{\text{JES}}) \right]$$

where $L_{\text{adj}}$ is the adjusted total likelihood for a given set of events, $L(\vec{y}_i|m_t, \Delta_{\text{JES}})$ is the likelihood for an individual event from Eq. (1), $f_{\log}(u_i)$ is the background fraction for a given event with a neural network output $u_i$, and $L_{\text{bg}}(m_t, \Delta_{\text{JES}})$ is the average likelihood for a background event.

Besides background events, the sample includes events which contain a real $t\bar{t}$, but where one or more of the four jets and/or the lepton observed in the detector do not come directly from the $t\bar{t}$ decay, and are not well-modeled by the signal likelihood or handled by the background subtraction above. These events, which we refer to as “bad signal,” have a variety of sources (extra jets from gluon radiation, $t\bar{t}$ events where both $W$ bosons decay into leptons or hadrons, $W \rightarrow \tau\nu$ decay, etc.) and make up 36% of the simulated $t\bar{t}$ events for $m_t = 172.5$ GeV/c$^2$. We suppress these events by requiring that the peak log-likelihood value for an event be at least 10. This cut retains 96.3% of the signal, while rejecting 30.8% of the bad signal and 37.3% of the background.

We test and calibrate the method by constructing simulated experiments using the Monte Carlo samples of $t\bar{t}$ events and background described earlier. For a given input $m_t$ and $\Delta_{\text{JES}}$, we perform 2000 experiments using a Poisson distribution with mean of 1089 events (the num-
number of events expected to pass the likelihood cut), and use these to calibrate the measurement as a function of the input $m_t$ and $\Delta_{\text{JES}}$. Figure 1 shows the output mass before calibration and the calibrated expected uncertainty.

In the data we find a total of 1087 events which pass all of the selection requirements (including the likelihood peak cut), of which 854 have 1 $b$ tag and 233 have >1 $b$ tag. Figure 2 shows the resulting 2-D likelihood contours for 1 $\sigma$, 2 $\sigma$, and 3 $\sigma$ after all calibration.

To obtain a 1-D likelihood curve in $m_t$ only, we treat $\Delta_{\text{JES}}$ as a nuisance parameter and eliminate it using the profile likelihood method [21], where we take the maximum value of the likelihood along the $\Delta_{\text{JES}}$ axis for each $m_t$ value. The top quark mass value extracted from the profile likelihood is $m_t = 173.0 \pm 0.9$ GeV/$c^2$. We can separate this uncertainty into the statistical uncertainty on $m_t$ and the uncertainty due to $\Delta_{\text{JES}}$ by fixing the $\Delta_{\text{JES}}$ value to its maximum likelihood value. We find that the uncertainty from the resulting 1-D likelihood is 0.7 GeV/$c^2$, so we assign the remaining uncertainty of 0.6 GeV/$c^2$ to $\Delta_{\text{JES}}$ and conclude $m_t = 173.0 \pm 0.7$ (stat.) $\pm 0.6$ (JES) GeV/$c^2$.

To validate the likelihood cut procedure, we compare the peak values of the log-likelihood curves obtained with data to those obtained with Monte Carlo–simulated events at $m_t = 172.5$ GeV/$c^2$ (the nearest available mass value). The results are shown in Fig. 3.

The systematic uncertainties on $m_t$, given in Table II, are derived using the methods described in Ref. [3]. In brief, we include uncertainties coming from: the calibration method; signal Monte Carlo modeling, evaluated by comparing events simulated with the PYTHIA and HERWIG [22] generators; variations of the parameters used for initial state radiation (ISR) and final state radiation (FSR); a residual JES uncertainty because the JES uncertainty contains several components with different $p_T$ and $\eta$ dependence; additional uncertainties on the energy scale for $b$ jets; uncertainty on the lepton $p_T$ scale; multiple hadron interactions, to take into account uncertainty on the jet corrections as a function of the number of interactions in the event; uncertainties arising from the PDFs used in the integration; and the background modeling. This analysis includes a systematic uncertainty due to color reconnection effects, not considered in our previous analysis. We use PYTHIA version 6.4.20, which includes a color reconnection model [23], and measure the difference between two tunes, Tune A, which is the tune used in this analysis, and Tune ACR, which adds color reconnection effects to Tune A. The individual systematic uncertainties are added in quadrature to obtain the final total of 0.9 GeV/$c^2$. 

![FIG. 1: Simulated experiment results using Monte Carlo signal and background events. Top: output $m_t$ vs. input $m_t$, before calibration is applied. Bottom: expected uncertainty $\sigma_{m_t}$ vs. input $m_t$, with calibration applied. The lines are linear best fits.](image1)

![FIG. 2: Measured 2-D likelihood on the data events, with the contours corresponding to a 1-$\sigma$, 2-$\sigma$, and 3-$\sigma$ uncertainty in the final $m_t$ measurement from the profile method. The marker shows the point of maximum likelihood.](image2)

![FIG. 3: Comparison of the log-likelihood value of the peak of likelihood curves for data and Monte Carlo events. The vertical line at 10 indicates the likelihood cut used in this analysis. A Kolmogorov-Smirnov test gives a confidence level of 0.93, showing good agreement between the two.](image3)
We thank the Fermilab staff and the technical staffs of the participating institutions for their vital contributions. This work was supported by the U.S. Department of Energy and National Science Foundation; the Italian Istituto Nazionale di Fisica Nucleare; the Ministry of Education, Culture, Sports, Science and Technology of Japan; the Natural Sciences and Engineering Research Council of Canada; the National Science Council of the Republic of China; the Swiss National Science Foundation; the A.P. Sloan Foundation; the Bundesministerium für Bildung und Forschung, Germany; the World Class University Program, the National Research Foundation of Korea; the Science and Technology Facilities Council of the United Kingdom; the Foundation for Basic Research; the Ministry of Education, Culture, Sports, Science and Technology of Japan; the Natural Sciences and Engineering Research Council of Canada; and the Academy of Finland.

In conclusion, the measured top quark mass in a sample with 5.6 fb$^{-1}$ of integrated luminosity, with 1087 events passing all cuts, is $m_t = 173.0 \pm 0.7$ (stat.) $\pm 0.6$ (JES) $\pm 0.9$ (syst.) GeV/c$^2$, for a total uncertainty of 1.2 GeV/c$^2$. The improved integration techniques and increased data sample make this the best single measurement of the top quark mass to date, and it is comparable in precision to the most recent combination for the top quark mass at the Tevatron [2].

We thank the Fermilab staff and the technical staffs of the participating institutions for their vital contributions. This work was supported by the U.S. Department of Energy and National Science Foundation; the Italian Istituto Nazionale di Fisica Nucleare; the Ministry of Education, Culture, Sports, Science and Technology of Japan; the Natural Sciences and Engineering Research Council of Canada; the National Science Council of the Republic of China; the Swiss National Science Foundation; the A.P. Sloan Foundation; the Bundesministerium für Bildung und Forschung, Germany; the World Class University Program, the National Research Foundation of Korea; the Science and Technology Facilities Council and the Royal Society, UK; the Institut National de Physique Nucleaire et Physique des Particules/CNRS; the Russian Foundation for Basic Research; the Ministerio de Ciencia e Innovación, and Programa Consolider-Ingenio 2010, Spain; the Slovak R&D Agency; and the Academy of Finland.

TABLE II: List of systematic uncertainties on $m_t$. 

<table>
<thead>
<tr>
<th>Systematic source</th>
<th>Uncertainty (GeV/c$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration</td>
<td>0.10</td>
</tr>
<tr>
<td>MC generator</td>
<td>0.37</td>
</tr>
<tr>
<td>ISR and FSR</td>
<td>0.15</td>
</tr>
<tr>
<td>Residual JES</td>
<td>0.49</td>
</tr>
<tr>
<td>$b$-JES</td>
<td>0.26</td>
</tr>
<tr>
<td>Lepton $p_T$</td>
<td>0.14</td>
</tr>
<tr>
<td>Multiple hadron interactions</td>
<td>0.10</td>
</tr>
<tr>
<td>PDFs</td>
<td>0.14</td>
</tr>
<tr>
<td>Background modeling</td>
<td>0.33</td>
</tr>
<tr>
<td>Color reconnection</td>
<td>0.37</td>
</tr>
<tr>
<td>Total</td>
<td>0.88</td>
</tr>
</tbody>
</table>

[10] A particle’s transverse momentum $p_T$ and transverse energy $E_T$ are given by $|\vec{p}| \sin \theta$ and $E \sin \theta$ respectively, where $\theta$ is the polar angle with respect to the proton direction (z-axis). The pseudorapidity $\eta$ of a particle’s three-momentum is defined by $\eta = -\ln(\tan(\theta/2))$. The missing $E_T$, $E_T$, is defined by $E_T = -|\sum_i E_i \hat{n}_i|$, where $\hat{n}_i$ is the unit vector in the $x$-$y$ plane pointing from the primary vertex to a given calorimeter tower $i$, and $E_i$ is the $E_T$ measured in that tower.