SNOW COVER AND SNOWMELT RUNOFF MODEL IN THE FOREST ZONE

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MOTOVILOV, J. \& VEHVILÄINEN, B. 1989. Snow cover and snowmelt runoff model in the forest zone. Publications of the Water and Environment Research Institute, National Board of Waters and the Environment, Finland, No. 3

Conditions of snowmelt runoff formation are similar in the watersheds of Finland and in the north-western part of the USSR forest zone. In operative hydrological practice of the Finnish National Board of Waters and the Environment a modification of the conceptual model HBV-3 is used for calculations and forecasts of spring-flood hydrograph. Within the framework of Soviet-Finnish scientific co-operation investigations are carried out to improve this version, so that it could later be used for the evaluation of the influence of man-induced factors on the runoff in forest watersheds. At the first stage model blocks, describing snow cover formation and snow melting are improved. The paper presents the main model algorithms and the results of its testing in one of Finnish watersheds.

Index words: snowmelt, snow, runoff, hydrological model

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1 GENERAL STRUCTURE OF THE MODEL

The HBV-model (Bergström 1976) contains a simplified description of the following processes: snow cover formation and snow melting, infiltration and accumulation of meltwater in the soil in the aeration zone, formation of surface, subsurface and groundwater runoff (Fig. 1).

1.1 Precipitation

Phase composition of precipitation was determined by the daily average air temperature (T). It was assumed, that at \( T < T_{MIN} \) only snow falls, at \( T > T_{MAX} \) — only rain and within the interval \( T_{MAX} > T > T_{MIN} \) both snow and rain fall, and their correlation being determined by linear function on air temperature. \( T_{MIN} \) and \( T_{MAX} \) values
\[ \frac{d \text{MVS}(t)}{dt} = \text{YIELD}(t) - \text{ES}(t) - \text{INF}(t) \]  (1)

where

\[ \text{ES}(t) = \frac{\text{EP}(t) \cdot \text{MVS}(t)}{\text{LP}}; \]
\[ \text{INF}(t) = \frac{\text{YIELD}(t) \cdot \text{MVS}(t)^x}{\text{FC}}; \]

\text{YIELD} is the intensity of inflow of meltwater on the watershed surface; \text{ES} is evaporation intensity; \text{INF} is the intensity of water inflow into the upper soil zone (this zone may be regarded as the volume of non-capillary macro-pores in aeration zone, where subsurface flow can be generated);

\text{EP} is potential evaporation; \text{LP} is soil moisture content after which evaporation achieves its maximum; \text{FC} is the maximum soil moisture storage. The physical sense of this value is close to that of soil moisture storage under field capacity; \(x\) is a parameter; \(t\) is time.

1.4 Transformation model for runoff

1.4.1 Upper zone

Water storage in the upper zone (SUZ) (Fig. 2) is calculated according to:

\[ \frac{d \text{SUZ}(t)}{dt} = \text{INF}(t) - Q_0(t) - Q_1(t) - \text{PERC} \]  (2)

where \(Q_1(t) = K_1 \cdot \text{SUZ}(t)\);
\(Q_0(t) = K_0 \cdot \text{UZ}(t)\);
\(\text{UZ}(t) = \begin{cases} \text{SUZ}(t) - \text{LUZ} & \text{if SUZ}(t) > \text{LUZ}, \\ 0 & \text{if SUZ}(t) < \text{LUZ} \end{cases} \)

\text{PERC} is the intensity of water inflow to the lower zone; \(K_1, K_0\) are parameters.

Physical sense of \(Q_1\) value is close to that of subsurface runoff. \text{LUZ} can be regarded as the maximum volume of non-capillary macropores in aeration zone, after these macropores are filled begins the fast flow \(Q_0\).

1.4.2 Lower zone

The balance equation for the lower storage is as follows (Fig. 3):

\[ \frac{d \text{SLZ}(t)}{dt} = \text{PERC} - Q_2(t) \]  (3)

where \(Q_2 = K_2 \cdot \text{SLZ}(t)\) is groundwater runoff.

1.2 Snow cover

In the HBV-3 version used in Finland snow water equivalent was calculated by summing the amounts of snow precipitation. A method of temperature index is used to calculate snow melting. The model block, describing snow cover formation and snow melting was changed in order to improve it and will be presented in detail below.

1.3 Soil moisture storage (MVS)

Soil moisture storage in the aeration zone is determined from balance equation

\[ \text{Figure 1. General structure of the runoff model.} \]
2 SNOW COVER DURING ITS FORMATION AND SNOW MELTING

Snow cover characteristics undergo temporal variations, caused by precipitation, snow melting, freezing of meltwater in snow and snow compaction.

2.1 Account of precipitation

If snowfall occurs, the depth of snow cover \( H \) increases by \( H_p \) for time interval \( t \), and

\[
H_p = \frac{PSN \cdot RW}{RN}
\]

where \( PSN \) is layer of snow, fallen during \( t \) period (in water equivalent); \( RN \) is density of new snow; \( RW \) is density of water.

The average snow density (DSN) in this case is calculated using the condition of mass conservation.

\[
DSN = \frac{DSN' \cdot H' + H \cdot RN}{H' + H_p}
\]

Here and below a touch indicates snow characteristics at the moment of time \( t - t \), disregard of the process under consideration (in this case \( H' \) is depth of snow before snowfall at the moment \( t - t \)).

If rainfall occurs, the part of rain (PLIQ) can be retained in snow under the influence of capillary-sorption forces of snow. We assumed, that if the precipitated rain cannot fill the whole snow layer, so that its maximum water-holding capacity is achieved, all the rain is retained by snow.

Else surplus rain water flows into the soil surface (water yield of snow is observed). This assumption can be expressed in a mathematical form:

\[
W \begin{cases} 
(W' \cdot H + PLIQ)/H & \text{if } PLIQ < (WC - W')H \\
WC & \text{if } PLIQ \geq (WC - W')H 
\end{cases}
\]

\[
YIELD \begin{cases} 
0 & \text{if } PLIQ \leq (WC - W')H \\
PLIQ - (WC - W')H & \text{if } PLIQ > (WC - W')H 
\end{cases}
\]

\[
DSN = DSN' + (W - W') \cdot RW
\]

where \( W \) is snow moisture content (volumetric content of liquid water per unity volume of snow), \( WC \) is maximum water-holding capacity of snow (also in volumetric units).

1.4.3 River runoff

Water discharge in the outlet point in the basin \( Q \) is calculated as

\[
Q(t) = KR \cdot Q_3 + (1-KR) \cdot Q(t-t) - Q_{min} + Q_{min}(t)
\]

where \( Q_3 = Q_0 + Q_1 + Q_2 \); \( Q_{min} \) is the minimum water discharge; \( KR \) is a parameter.
2.2 Snow melting

Changes in snow characteristics during its melting are calculated by the following formulae:

\[ H_T = \frac{\text{MELT RW}}{DSN - W^*RW}; \]

\[ WM = \frac{W^*H^* + \text{MELT}}{H^* - H_T}; \]

\[ W = \begin{cases} WM, & \text{if } WM < WC; \\ WC, & \text{if } WM \geq WC; \end{cases} \]

\[ DSN = DSN' - W^*RW + W^*RW \]

\[ \text{YIELD} = \begin{cases} Q, & \text{if } WM < WC; \\ \frac{WM - WC)(H^* - H_T)}{W^*H^*}, & \text{if } WM \geq WC; \end{cases} \]

Here MELT is a layer of snow, melted at the \( \Delta t \) interval (expressed as water equivalent).

2.3 Freezing of meltwater in snow

The maximum possible amount of water, that can freeze for the \( \Delta t \) period (FMAX) is calculated for the heat exchange conditions at the snow-atmosphere interface. The actual layer of frozen water FROST and residual moisture content in snow is calculated by

\[ \text{FROST} = \begin{cases} \text{FRMAX, if } \text{FRMAX} < W^*H^*; \\ W^*H^*, & \text{if } \text{FRMAX} \geq W^*H^*; \\ 0, & \text{if } W^* = 0; \end{cases} \]

\[ W = W^* - \text{FROST}/H \]

2.4 Evaporation of snow

The change of snow depth during its evaporation is calculated as

\[ H_E = \frac{\text{ESN RW}}{DSN} \]

where ESN is a layer of snow, evaporated during \( t \) period (in water equivalent).

2.5 Compaction of snow

For the description of snow compaction and settling under the influence of wind loading and gravitational forces the following empirical equation is used:

\[ DSN = C (DSN)^2 (H^* \exp (C_2 T_s - C_3)) DSN' \Delta t + DSN' \]

as well as the balance equations

\[ H_c = \frac{DSN'H^*}{DSN}; \]

\[ W = \frac{W^*(H^* - H^*_c)}{H} + W^*_c; \]

where \( T_s \) is the temperature of the snow, \( C \) a parameter, \( C_2 \) and \( C_3 \) are empirical constants.

2.6 Depth of snow

The resulting of snow cover depth at the \( t \) moment of time is determined as

\[ H = H^* + H_p - H_T - H_E - H_c. \]

3 PHASE TRANSFORMATIONS OF WATER IN SNOW COVER

Heat flux at the snow surface is calculated using the method of energy balance, suggested by Kuzmin (1961):

\[ RTOT = \text{RSN} + \text{RLN} + \text{RSEN} + \text{RLAT} + \text{RP}, \]

where RTOT is a total flux of heat to the snow surface; RSN is a flux of short-wave solar radiation penetrating into the snow; RLN is the effective long-wave radiation; RSEN is sensible turbulent heat flux; RLAT is a latent heat flux; RD is a heat flux with liquid precipitation.

3.1 Short-wave solar radiation

The short wave radiation (wave length less than 3 \( \mu \)) comes originally from the sun, but a part comes as a diffused component after multiply dispersed from air molecules and reflected from clouds and terrain. Heavy cloud cover can reduce the short
wave radiation to one quarter of the clear-sky value.

A large amount of incoming short wave radiation is reflected by snow cover. This portion of incoming short wave radiation, called albedo, is dependent on the wave length, the geometry of the radiation and especially on the snow structure. New snow reflects over 80 per cent whereas coarse-grained old snow reflects only 30—40 per cent of the incoming short wave radiation.

Short-wave solar radiation is calculated by equations:

\[
\text{RSN} = \text{RS (1-A) CF,} \\
\text{CF} = 1 - F_1 (1 - (1 - F)^2)^{1/2}
\]

where RS is measured incoming short-wave radiation; A is albedo; CF is a coefficient of penetrating of short-wave radiation through tree crowns; \( F_1 \) is the parameter, introduced to take into account the type of forest; \( F \) is the forest cover density.

Snow cover albedo is calculated as (Kuchment et al. 1983):

\[
A = CA - DSN
\]

where CA is a parameter.

### 3.3 Sensible heat transfer

Sensible heat exchange is the product of turbulent exchange processes of heat in the layer of the first two three meters above snow surface. This turbulent heat flux can be measured directly by eddy correlation techniques, which requires sophisticated instrumentation.

A more common method is to evaluate sensible heat exchange on the basis of wind and temperature profiles above the snow surface (Prandtl, 1932):

\[
\text{RSEN} = -C_a D_a K_h \frac{dT_{pot}}{dz} \quad (18)
\]

where \( C_a \) = specific heat of air, kJ kg \(^\circ\)C\(^{-1}\) 
\( D_a \) = air density, kJ kg\(^{-1}\) m\(^3\) 
\( K_h \) = eddy diffusivity for convective energy transfer, m\(^2\) s\(^{-1}\) 
\( T_{pot} \) = potential temperature i.e. temperature at the surface pressure

In practice the eddy diffusivit is calculated from wind profile and assuming that the eddy diffusivity for convective heat transfer \( K_h \) equals eddy diffusivity for momentum \( K_m \) which is usually valid during snowmelt (Anderson, 1976).

This method of profiles is valid only for open areas, where well-defined wind and temperature profiles can exist. In forested area the vegetation prevents the formation of well-defined wind and temperature profiles 2—3 meters above snow. Therefore it is reasonable to use simplified equations for sensible heat exchange in the forested basins. These equations are usually the form:

\[
\text{RSEN} = CS U (T - T_{snow}) \quad (19)
\]

where \( CS \) = bulk transfer coefficient for sensible heat exchange kJ (m\(^{-3}\) \(^\circ\)C\(^{-1}\)) the value of which varies from 0.001 to 0.015 (Male et al. 1981) 
\( U \) = wind speed (m s\(^{-1}\))

The value of bulk transfer coefficients are usually averages over relative long periods and equation 84 should not be applied over intervals of less than 24 h at least with the presented coefficient values.

### 3.4 Latent heat transfer

Turbulent mixing of the air layers is the main process causing heat and moisture transfer above snow surface. Because the origin of these two exchange processes is the same they also occur simultaneously. Sensible and latent heat exchange needs the

where \( T_s \) is snow surface temperature, \( e_a \) is the pressure of water vapour of air, \( E \) is emissivity in the longwave portion of the energy spectrum \( E = 0.99 \), \( \delta \) is the Stefan-Boltzman constant, \( \text{TKEL} = 273 \circ\)K, \( a \) and \( b \) are constants.
gradient of temperature and moisture on one hand and wind on the other hand to exist. Moisture and temperature gradients are developed due to the energy supply of short wave and long wave radiati-
on. Wind tends to eliminate gradients increasing simultaneously turbulent heat absorption/intake and moisture evaporation/condensation. In the absence of wind heat and moisture transfer ceases although gradient exists there is no turbulent transport process. On the other hand wind can eliminate weak gradients easily and this ceases again the transfer of heat and moisture.

The same method of profiles or gradients, that is used for sensible heat exchange is also valid for latent heat exchange (Prandtl 1932):

\[
\text{RLAT} = -HV D_a K_e (e - e_a) / dz \quad (20)
\]

where
- \(HV\) = heat of vaporization, 335 kJ kg\(^{-1}\)
- \(K_e\) = eddy diffusivity of latent energy transfer m\(^2\) s\(^{-1}\)
- \(e_a\) = air moisture, mb

The eddy diffusivity of momentum \(K_m\) is calculated first from wind profile. With assumption that \(K_e/K_m\) is unity during snowmelt latent heat exchange can be evaluated based on measurements on two levels.

The same reasons, absence of well-defined wind and moisture profiles near surface, which led to the use of simplified equations of sensible heat in forested areas are valid also for latent heat exchange. For the simulation of latent heat exchange in forested basins equation is used as for sensible heat:

\[
\text{RLAT} = CL U (e - e_s) \quad (21)
\]

where
- \(CL\) = bulk transfer coefficient for latent heat exchange, kJ m\(^{-3}\) mb\(^{-1}\)
- \(e_s\) = vapour pressure on the snow surface, mb
- \(e_a\) = vapour pressure in air over snow, mb

The values of \(CL\) vary from 0.002 to 0.025 kJ m\(^{-3}\) mb according to Male et al. (1981).

3.5 Heat flux by rain

With precipitation heat it is considered the heat or energy, that is delivered to a snowpack, when water from precipitation reaches the temperature of snowpack and, if the temperature is under 0 °C, freezes. In the first case without freezing the energy supply to the snowpack, \(RP\), is:

\[
RP = D_w C_w (T_p - T) / 1000 \quad (22)
\]

where
- \(D_w\) = density of water, kg m\(^{-3}\)
- \(C_w\) = specific heat of water, kJ kg\(^{-1}\) °C\(^{-1}\)
- \(T_p\) = temperature of rain, °C
- \(P\) = precipitation, mm d\(^{-1}\)

The energy supply by this process is quite small compared to other components of energy balance equation.

In the case all liquid precipitation is freeze in snowpack the energy supply from liquid precipitation becomes important compared to other energy balance terms. The latent heat of the fusion of water is 335 kJ kg\(^{-1}\). This is possible only in the beginning of snowmelt, when the temperature of snowpack is below 0 °C.

The energy supply from 1 mm of rain at 5 °C air temperature when temperature of snowpack is −10 °C is about 6 J cm\(^{-2}\) after equation 22 and when the water is freezeed the additional energy supply to the snowpack is 34 J cm\(^{-2}\).

With abundant rainfall on snow the heat from liquid precipitation is distributed rapidly into the whole snowpack. Rainfall may change the structure of snow very quickly compared to normal melting process.

3.6 Snow melting and freezing of meltwater in snow

Snowpack during melting period is characterized by the following values: TS = 0 °C; ES = 6.11 mb. Substituting these values into formulae (17) — (20) we can determine RTOT. The amount of snow melted or water frozen in snow is determined from the relations:

\[
\text{MELT} = \begin{cases} 
\text{RTOT/CM,} & \text{if RTOT > 0;} \\
0, & \text{if RTOT ≤ 0;} 
\end{cases}
\]

\[
\text{FRMAX} = \begin{cases} 
0, & \text{if RTOT > 0;} \\
-\text{RTOT/CM,} & \text{if RTOT ≤ 0;} 
\end{cases}
\]

where CM is the heat of fusion of ice.
4 TESTING OF THE MODEL

Calibration of parameters and testing of the model were carried out in the basin of the Tuujuja river (Finland).

Tuujuja is a small experimental basin near western coast of Finland 64°N, 25°E. The area is 20.6 km² of which 82% is forest (canopy density 30%), 12% field, 4% bog and 2% urban area. There are no lakes in the area and the mean slope is 2.3%.

4.1 Calibration of parameters

The model parameters were calibrated on the basis of observed data of water discharges and the characteristics of snow cover during 1977—1981. The Rosenbrock optimization procedure was used for the calibration of the parameters with a quality criteria:

\[
R^2 = \frac{(Q_c - \bar{Q}_r)^2 - (Q_c - Q_t)^2}{(Q_c - \bar{Q}_r)^2}
\]

(24)

where \(Q_c\) is the calculated value, \(Q_r\) is the observed value, \(Q_t\) is the average from observed values.

At the first stage parameters of the submodel, describing snow cover, were calibrated using measured snow cover characteristics (depth, density and water equivalent of snow). Calculation showed, that the best agreement between calculated and observed values is achieved, when parameters are calibrated over snow depth. Calibration over the values of snow density or its water equivalent has considerably worsen results.

This is assumed to be connected mainly with low accuracy of measuring snow density.

At the second stage model parameters describing soil characteristics and storage zone were calibrated by measuring water discharge in the outlet point.

At the third stage parameters of blocks describing energy balance and snow cover formation were adjusted against measured water discharge.

4.2 Verification of the model

Model parameters, received as a result of calibration were used in test calculations for the Tuujuja river basin in 1970—1976. Table 1 contains the values of quality criterion for calibration and verification of the model.

Table 1. Quality criterion for calibration and verification of snowmelt runoff model.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>(R^2) Calibration</th>
<th>(R^2) Verification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth of snow</td>
<td>0.850</td>
<td>0.618</td>
</tr>
<tr>
<td>Density of snow</td>
<td>0.525</td>
<td>0.201</td>
</tr>
<tr>
<td>Water equivalent</td>
<td>0.807</td>
<td>0.620</td>
</tr>
<tr>
<td>Runoff</td>
<td>0.850</td>
<td>0.710</td>
</tr>
</tbody>
</table>

Figures 4 and 5 present some results of model calibration and verification.

SUMMARY

The aim of this study was to develop a physically based snowcover model for simulation of snow accumulation and snow melt, which can be used for operational purposes in large river basins with minimum input data on daily basis. The physically based model should also be better than the common degree-day snow models used normally in operational catchment hydrology. This criterion is not yet achieved with this model to be presented or with any other physical snowcover model. Anyway physically based snowcover model gives a lot of information of the different processes included in snow accumulation and snowmelt phenomenon, which can be usable in developing simpler snow models. Also physically based snow models can simulate quite accurately different snow characteristics as snow density and depth, which can be used in some other studies as soil frost simulation or in calculation of snow albedo.

REFERENCES


Figure 4. Simulation of water equivalent and runoff in the Tujuoja basin in the winters 1975—1977. Observed water equivalent is marked by (o) and observed runoff is full line.
Figure 5. Simulation of the depth and density of snow in Tujuoja basin in the winters 1975—1977. Observed values are marked by (o).
LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSN</td>
<td>snow density</td>
<td>kg m⁻³</td>
</tr>
<tr>
<td>EP</td>
<td>potential evaporation</td>
<td>cm d⁻¹</td>
</tr>
<tr>
<td>ES</td>
<td>actual evaporation</td>
<td>cm d⁻¹</td>
</tr>
<tr>
<td>ESN</td>
<td>snow evaporation</td>
<td>cm d⁻¹</td>
</tr>
<tr>
<td>FC</td>
<td>maximum value of soil moisture storage</td>
<td>mm</td>
</tr>
<tr>
<td>FROST</td>
<td>backfreezing in snowpack</td>
<td>cm d⁻¹</td>
</tr>
<tr>
<td>H</td>
<td>snow depth</td>
<td>cm</td>
</tr>
<tr>
<td>INF</td>
<td>water yield from soil moisture storage</td>
<td>mm d⁻¹</td>
</tr>
<tr>
<td>K₀</td>
<td>runoff coefficient</td>
<td>1 d⁻¹</td>
</tr>
<tr>
<td>K₁</td>
<td>runoff coefficient</td>
<td>1 d⁻¹</td>
</tr>
<tr>
<td>K₂</td>
<td>runoff coefficient</td>
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</tr>
<tr>
<td>KR</td>
<td>recession coefficient</td>
<td>0—1</td>
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<td>LP</td>
<td>soil moisture storage value after which the actual evaporation equals the potential evaporation</td>
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<td>MELT</td>
<td>snow melt</td>
<td>cm d⁻¹</td>
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<tr>
<td>MVS</td>
<td>soil moisture storage</td>
<td>mm</td>
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<td>PERC</td>
<td>water inflow to the lower zone</td>
<td>mm d⁻¹</td>
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<tr>
<td>PLIQ</td>
<td>liquid precipitation</td>
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<tr>
<td>PSN</td>
<td>snow precipitation</td>
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<tr>
<td>RN</td>
<td>density of new snow</td>
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<tr>
<td>RW</td>
<td>density of water</td>
<td>kg cm⁻³</td>
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<td>SLZ</td>
<td>water storage in the lower zone</td>
<td>mm</td>
</tr>
<tr>
<td>SUZ</td>
<td>water storage in the upper zone</td>
<td>mm</td>
</tr>
<tr>
<td>T</td>
<td>daily mean temperature</td>
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</tr>
<tr>
<td>Tₛ</td>
<td>temperature of snow</td>
<td>°C</td>
</tr>
<tr>
<td>TMAX</td>
<td>daily temperature over which all precipitation is liquid</td>
<td>°C</td>
</tr>
<tr>
<td>TMIN</td>
<td>daily temperature below which all precipitation is solid</td>
<td>°C</td>
</tr>
<tr>
<td>W</td>
<td>volumetric content of liquid water in snow</td>
<td>m³ m⁻³</td>
</tr>
<tr>
<td>WC</td>
<td>maximum water-holding capacity of snow in volumetric units</td>
<td>m³ m⁻³</td>
</tr>
<tr>
<td>YIELD</td>
<td>water yield from liquid precipitation and snow melt</td>
<td>mm d⁻¹</td>
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