MODEL OF COMBINED WATER USE AND WATER DERIVATION PLANNING

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KOCHARIAN, A., KHRANOVICH, I. 1989. Model of combined water use and water derivation planning. Publication of the Water and Environment Research Institute, National Board of Waters and the Environment, Finland, No. 3.

The paper presents a flow model describing the selection of optimum parameters of water resource system elements: sources of water and pollutants, water reservoirs, reaches of rivers and canals, water users, water protection facilities. The model takes into account both water use and water quality control.

Index words: water resource system (WRS), two-stage stochastic flow model, interacting flows, variants of element's development, characteristic function of development alternatives, multiextremal task.

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1 INTRODUCTION

A water resource system (WRS) involves an assembly of interacting sources of water, reservoirs, seas and water users, connected by river and canal reaches.

A WRS coordinates water demands with available water resources. According to this function of the system, its parameters are to be coordinated in a most effective way. In this particular paper under the term "optimum" such WRS parameters and operational regimes are meant, with which national economy expenses for the designing period are minimized. Costs involve capital investments and operation and maintenance costs. Effect of water use is also accounted in them.

Considering the selection of WRS parameters it
is supposed that parameters of elements of the existing WRS (volumes of reservoirs, canal capacities and efficiency, production capacities, i.e. capacities of enterprises — water-users and purification facilities, etc.) and probable regimes of water and pollutant entry into the system are known. Reconstruction elements and elements under construction with possible values of their parameters, received as a result of preliminary studies, are known too. Solution of this problem consists in selection of the optimum set of parameters of WRS elements among the probable ones.

It requires a great number of possible sets of elements’ parameters and relevant operation regimes of the system to be compared. To solve the problem an optimization model is used, where all permissible sets of alternatives are compared. Water distribution and water quality control are analysed jointly in the suggested model. Such an approach, unlike the traditional separate analysis of quantitative and qualitative characteristics, allows all water management measures to be solved in one complex.

The system is assumed to operate in discrete time. The model takes into account random processes of water and pollutants entry and utilization, which is presented by a finite set \( \Omega \) of characteristic realizations of conditions \( \omega \).

**2 MODELS OF WRS ELEMENTS**

A WRS is presented by a network \( \Gamma'(J,S) \), the configuration of which corresponds to a diagram WRS presentation. In the network \( \Gamma'(J,S) \) all the assumed WRS elements, existing or possible, are represented. Elements of this network have their own characteristics, their interaction is produced by flow movement, corresponding to water and admixture flows in the simulated system.

For any WRS element a finite set of possible development alternatives are singled out. Each of them is characterised by a set of elements’ parameters.

In the model every version of WRS elements’ development is presented as an operational element. Operation of the \( z \)-th element according to the \( z \)-th alternative of development is described by a characteristic function of the version \( \eta_{oz} = 1 \), if the \( z \)-th alternative is accepted; \( \eta_{oz} = 0 \), if the \( z \)-th version is not accepted. As a WRS element can operate according to one version of the finite sets \( a_z \), characteristic functions of elements’ development alternatives are linked by:

\[
\eta_{zo} \in \{0; 1\}, \sum_{z \epsilon \mathcal{A}} \eta_{oz} = 1
\]

River and canal reaches and water users, except those using water from reservoirs without its diversion and considered as a single whole together with reservoirs, are depicted by \( \Gamma'(J,S) \) arcs with gain and retardation.

The flow \( \dot{q}_{sot}^{\omega} \) at the beginning of the \( s_{o} \)-th arc corresponds to the \( s_{o} \)-th variant of the set \( a_{\omega} \) of possible development variants of the \( s \)-th user under \( \omega \) stochastic conditions, is connected with the flow \( q_{sot}^{\omega} \) at the end of this arc by the inequality:

\[
q_{sot}^{\omega} = K_{sot}^{\omega} q_{sot}^{\omega}, t - \theta
\]

without non-negative retardation \( \theta^{\omega} \) and an amplification factor, having the meaning of a ratio of water, returned by the user, to the amount of water, allocated to the user. Its values are within the range \( 1 \geq K_{sot}^{\omega} \geq 0 \).

In the model users’ requirements to water resources quantity and water consumption restrictions cause the appearance of sets of possible values of arc flows:

\[
\underline{q}_{sot}^{\omega} \leq q_{sot}^{\omega} \leq \overline{q}_{sot}^{\omega}
\]

**Water sources with uncontrolled consumption** (natural river runoff) in the model are presented by sources of prescribed intensities \( b_{it}^{\omega} \) located at \( i \in J \).

**Water sources with controlled consumption** (presented by systems of interregional water transfer) are described by fragments of the network \( \Gamma'(J,S) \), each having vertices, the number of which equals the number of versions of the source’s development. Every vertex is connected with the rest of the network by outgoing arcs \( s_{o1} \) and \( s_{o2} \) with the gain coefficients \( K_{sot1}^{\omega} = 1, K_{sot2}^{\omega} = 0 \).

**Reservoirs and seas** in the model are regarded as a single whole with users, located on them. They are expressed by storages in the graph’s vertices. Their supplies \( Q_{sot}^{\omega} \) equal the amount of water in the reservoirs and seas. These constraints cause sets of possible values of storage supplies:

\[
\underline{Q}_{sot}^{\omega} \leq Q_{sot}^{\omega} \leq \overline{Q}_{sot}^{\omega}
\]

In the model relationship between the intensity of water losses in reservoirs and seas due to seepage and evaporation are approximated by linear ones:

\[
\delta Q_{sot}^{\omega} = \gamma_{sot}^{\omega} Q_{sot}^{\omega}
\]
3 POLLUTANTS AND WATER QUALITY

In addition to "basic" flows \( q_{\text{s}} \) and supplies \( Q_{\text{o}} \), expressing water storage and discharge, "additional" flows \( y_{\text{salt}} \) and supplies \( Y_{\text{salt}} \), simulating pollutants (\( l \) is the type of pollutant), are included in the model. Interaction of these flows corresponds to representation of processes of pollutant proliferation and transformation by a system of linear differential equations of the Streeter-Phelps type, (Vavinil et al. 1977).

In the elements of the network flows and supplies of pollutants of each set \( L_i \), should meet the following requirements:

\[
\sum_{l \in L_i} d_{\text{salt}}^{0} Z_{\text{salt}}^{0} \leq \beta_{\text{salt}}^{0} \gamma_{\text{salt}}^{0}, z_{\alpha} \in R = \Omega \cup I. \tag{6}
\]

The set of types \( L \) of pollutants are divided into subsets \( L_{\text{s}} \), which have unempty crossing according to a limiting factor in such a way that \( L_{\text{s}} = L_{\text{s}} \), where \( d_{\text{salt}}^{0} \) is the value, inverse to the maximum permissible concentration \( C_{\text{salt}}^{0} \), of the \( l \)-th admixture in the \( z_{\alpha} \)-th element of \( \Gamma (J, S) \) under stochastic conditions \( \omega \); if \( z_{\alpha} \in S \), \( Z_{\text{salt}}^{0} \) denotes a pollutant flow of \( l \)-type in the \( z_{\alpha} \)-th arc \( y_{\text{salt}}^{0} \), where \( z_{\alpha} \in I \), \( Z_{\text{salt}}^{0} \) pollutant supply of \( l \)-type in the \( z_{\alpha} \)-th storage \( y_{\text{salt}}^{0} \); \( X_{\text{salt}}^{0} \) denotes a water flow \( q_{\text{salt}}^{0} \) and supply \( Q_{\text{salt}}^{0} \). \( \beta_{\text{salt}}^{0} \) is the coefficient of incomplete mixing, which is equal to the ratio of the average pollutant concentration to maximum pollutant concentration. If \( z_{\alpha} \in S \), the conditions (6) correspond to arcs' inlets and reflect users' requirements to the quality of water, allocated to them. Requirements to the quality of water, returned to the system, are prescribed by similar conditions, imposed on the pollutant composition at arcs' outlets:

\[
\sum_{l \in L_i} d_{\text{salt}}^{0} y_{\text{salt}}^{0} \geq \beta_{\text{salt}}^{0} \gamma_{\text{salt}}^{0}, s_{\alpha} \in S. \tag{7}
\]

Moreover, the values of pollutant flows and supplies have lower limits:

\[
Z_{\text{salt}}^{0} \geq Z_{\text{salt}}^{0}, y_{\text{salt}}^{0} \geq y_{\text{salt}}^{0}, z_{\alpha} \in R, s_{\alpha} \in S \tag{8}
\]

The constants \( d_{\text{salt}}^{0} \), \( d_{\text{salt}}^{0} \), \( Z_{\text{salt}}^{0} \), \( y_{\text{salt}}^{0} \) are non-negative. It should be noted, that conditions (6) and (7) impose stricter requirements on the pollutant composition than non-excess of pollutant concentration \( C_{\text{salt}}^{0} = Z_{\text{salt}}^{0} / X_{\text{salt}}^{0} \) and \( C_{\text{salt}}^{0} = y_{\text{salt}}^{0} / q_{\text{salt}}^{0} \) of all the pollutants, taken separately.

From the assumptions of complete momentary mixing and linearity of pollutant composition variations it follows that pollutant supplies in the storage satisfies the following:

\[
Y_{\text{salt}, t}^{0} + 1 = \sum_{\gamma \in L} A_{\gamma}^{\text{salt}} Y_{\text{salt}, \gamma t}^{0} + h_{\text{salt}}^{0} Y_{\text{salt}}, \tag{9}
\]

where \( A_{\gamma}^{\text{salt}} \), \( \gamma, l \in L \) is the non-singular square matrix of substances transformation in the storage.

Solving the system of linear differential Streeter-Phelps equations a relationship is obtained between the vectors of pollutants at the inlets and outlets of the arcs, forming the set \( S_{\uparrow} \subset \Gamma, S \subset \Gamma \) users without treatment facilities, river and canal reaches:

\[
y_{\text{salt}}^{0} = \sum_{\gamma \in L} A_{\gamma}^{\text{salt}} y_{\text{salt}, \gamma t}^{0}, \tag{10}
\]

A functional relationship of type (10) between pollutant flows at the arcs' inlets and outlets, representing users and sources with treatment facilities in the model and forming the set \( S_{\uparrow} = S \setminus S_{\downarrow} \), is absent, since, due to operational regimes of treatment facilities, for the same value \( y_{\text{salt}}^{0} \), different values \( y_{\text{salt}}^{0} \) can be received. Pollutant flows \( y_{\text{salt}}^{0} \) and \( y_{\text{salt}}^{0} \) are interconnected by operation and maintenance costs of elements with treatment facilities.

Inevitability of receiving water with pollutants brings about a requirement of coincidence of a heterogenous flow composition at the inlets of arcs, starting from one and the same vertex:

\[
y_{\text{salt}}^{0} = y_{\text{salt}}^{0} y_{\text{salt}}^{0} y_{\text{salt}}^{0}, s_{\alpha} \in S_{\downarrow}. \tag{11}
\]

4 CONTINUITY EQUATIONS

In the WRS water and pollutant distribution, subjected to the mass conservation law, in accordance with water and pollutant flows in \( \Gamma (J, S) \), meet the following system of continuity equations:

\[
\sum_{i \in a} (Q_{i}^{0} + 1 - Q_{i}^{0}) = h_{i} \cdot \sum_{i \in a} q_{\text{salt}}^{0} =
\]

\[
= h_{i} \cdot \sum_{i \in a} q_{\text{salt}}^{0} = h_{i} \cdot \sum_{i \in a} q_{\text{salt}}^{0} - \sum_{i \in a} q_{\text{salt}}^{0} \delta Q_{i}^{0} + 6_{i}^{0} \tag{12}
\]

\[
\sum_{i \in a} y_{\text{salt}}^{0} = \sum_{i \in a} y_{\text{salt}}^{0} - \sum_{i \in a} y_{\text{salt}}^{0} \delta y_{\text{salt}}^{0} + 6_{i}^{0}, \tag{13}
\]

where \( S_{\uparrow}^{0} \) is the set of arcs, coming into the \( i \)-th vertex; \( S_{\uparrow}^{0} \), the set of arcs, outgoing from the \( i \)-th vertex; \( 6_{i}^{0} \) is the pollutant flow of the \( i \)-th type, coming into the \( i \)-th vertex under the conditions \( \omega \).
5 RESOURCE CONSTRAINTS

The suggested model involves conditions, reflecting constraints of labour, financial and other resources and tasks on production targets, produced by water users; those conditions include constraints on the resources, connected with the WRS development and operation.

Resource constraints on development are represented by the system of inequalities

$$\sum_{z \in R} m^d_{za} \eta_{za} \leq M^d, \quad d \in D_I,$$

where $M^d$ is the total amount of the $d$-type resource, which can be allocated for the WRS development (capital investments, building materials, etc); $D_I$ is the set of resource types taken into account, used in the WRS elements' construction and reconstruction; $m^d_{za}$ is the amount of $d$-type resource, necessary for the introduction of the $a$-version of development of the $z$-th element of the WRS.

In operating WRS restricted stored and unstored resource are used. Stored resources involve utilized raw materials and reagents for water treatment; unstored resources are labour resources, electric power, resources of irrigation facilities, etc.

Employment conditions of stored resources cover the whole design period and are described by the system of inequalities:

$$\sum_{t \in \{T_0, T\}} \left[ \sum_{z \in R} \varphi^{dw}_{zat} (X^{u}_{zat}, U^{\text{wat}}_{zat}) + \sum_{s \in S_{II}} \varphi^{edw}_{sat} (X^{u}_{sat}, U^{\text{wat}}_{sat}) \right] \leq M^{dw}, \quad d \in D_{II}.$$  

(15)

Here $M^{dw}$ and $d$ have the same meaning as in (14); $D_{II}$ is the the set of stored resources, $\varphi^{dw}_{zat}$ and $\varphi^{edw}_{sat}$ relationships of the resource magnitude being utilized and water quality and quality in the element ($\varphi$) and returned into the system ($\varphi^e$); $U^{\text{wat}}_{zat} = \sum_{l \in L} \delta^{\text{wat}}_{zat} Y^{\text{wat}}_{zat}$ and $U^{\text{wat}}_{sat} = \sum_{l \in L} \delta^{\text{wat}}_{sat} Y^{\text{wat}}_{sat}$ pollutant complexes in the element and returned into the WRS; $\delta^{\text{wat}}_{zat}$ and $\delta^{\text{wat}}_{sat}$ are non-negative numbers, reflecting the significance of the l-th pollutant in a complex.

Functions $\varphi^{dw}_{zat}$ and $\varphi^{edw}_{sat}$ are convex in each of the variables $X^{u}_{zat}, U^{\text{wat}}_{zat}$ and $X^{u}_{sat}, U^{\text{wat}}_{sat}$, they are non-convex in their totality, (Priazhinskaya et al., 1978).

Unstored resources are used only in time intervals, when they are not singled out. Conditions of their utilization correspond to these time intervals and are as follows:

$$\sum_{z \in R} \varphi^{dw}_{zat} (X^{u}_{zat}, U^{\text{wat}}_{zat}) + \sum_{s \in S_{II}} \varphi^{edw}_{sat} (X^{u}_{sat}, U^{\text{wat}}_{sat}) \leq M^{dw}, \quad d \in D_{III},$$

(16)

where $D_{III}$ is the set of unstored resources. Functions of consumption of unstored resources, presented in (16), have the meaning and characteristics, similar to those, presented in (15).

Conditions of achieving production targets, depending on the possibility of accumulating during the desiging period, are introduced into the model in the form of (15) or (16). Besides, functions $\varphi^{dw}_{zat}$ have the meaning of the relationship of production targets amount of water resources quality and quantity.

6 TASK FUNCTION

The WRS development over the design period is estimated by the cost function, including expenses on construction and renovation, operation and maintenance, on output, delivery, and purification of water, as well as expenses, arising from the deviation of water quantity and quality from the optimum.

The functions describe the mathematical expectation of costs over the design period:

$$f_\epsilon(\eta_{zt}, x_{zt}, y_{zt}) = \sum_{z \in R} \left[ K_{zt} \eta_{zt} + \sum_{\omega \in \Omega} \rho_{\omega} \sum_{t \in \{T_0, T\}} \left[ f^{\omega}_{zt}(x^{\omega}_{zt}, u^{\omega}_{zt}) + f^{\omega}_{zt}(x^{\omega}_{zt}, u^{\omega}_{zt}) \right] \right],$$

(17)

where coefficients $K_{zt}$ reflect constant costs independent of operational regimes $x^{\omega}_{zt}$ and $u^{\omega}_{zt}$. Functions $f^{\omega}_{zt}$ and $f^{\omega}_{zt}$ estimate variable costs, depending on $x^{\omega}_{zt}$ and $u^{\omega}_{zt}$. Components $f^{\omega}_{zt}$ and $f^{\omega}_{zt}$ are not equal to zero, only if $z_{zt} \in S_{II}$. Functions $f^{\omega}_{zt}$ and $f^{\omega}_{zt}$ have characteristics similar to those of functions $\varphi^{dw}_{zat}$ and $\varphi^{edw}_{sat}$ i.e they are convex in each of the variables $x^{\omega}_{zt}$ and $u^{\omega}_{zt}$ and non-convex in their totality, (Priazhinskaya et al., 1978); $f^{\omega}_{zt}(0,0) = f^{\omega}_{zt}(0,0) = 0.$

7 DEVELOPMENT TASK

Determination of optimum WRS parameters and operation regimes is described by the task A of
The determination of the optimum parameters and sources of the network \( \Gamma(J, S) \). The task deals with the estimations of vectors \( \eta^0, x^0, y^0 \) with strategic components \( x^0 = [x^0_{zo}, x^0_{zo}] \), tactical components \( y^0 = [y^0_{zo}, y^0_{zo}] \), \( z_{zo} \in R, s_{zo} \in S_{II}, \omega \in \Omega, t \in [T_0, T] \), \( y^0 = [y^0_{zo}, y^0_{zo}] \), \( z_{zo} \in R, s_{zo} \in S_{II}, \omega \in \Omega, t \in [T_0, T] \) which minimize the mathematical expectation of the WRS costs:

\[
\bar{f}(\eta, x, y) = \sum_{z \in R} f_z(\eta_z, x_z, y_z) = \frac{\sum_{z \in R} K_{zo} \eta_{zo} + \sum_{\omega \in \Omega} \rho^\omega \sum_{t \in [T_0, T]} f^\omega_{zo} (x^\omega_{zo}) + \sum_{\omega \in \Omega} f^\omega_{zo} (x^\omega_{zo}, u^\omega_{zo})}{ \text{in the set } G, \text{ formed by constraints (1)—(16).} G \text{ is prescribed by the system of continuity equations of water and pollutant flows:}}
\]

\[
\sum_{\omega \in \Omega} \left[ Q^\omega_{zo} t + 1 - K^\omega_{zo} Q^\omega_{zo} \right] = h_t \left[ \sum_{s \in S} K^\omega_{so} s - \sum_{s \in S} \beta^\omega_{so} s - \sum_{s \in S} \gamma^\omega_{so} s + \delta^\omega_{so}, \right], t \in [T_0, T], \omega \in \Omega.
\]

by transforming equations of pollutants in arcs, depicting river and canal reaches, users and treatment facilities:

\[
y_{\text{sal}} = \sum_{\gamma \in \Gamma} A^\gamma_{\text{sal}} y^\gamma_{\text{sal}}, t \in [T_0, T], \omega \in \Omega.
\]

by equations of pollutant concentration in arcs, outgoing from one and the same vertex

\[
y^\omega_{\text{sal}} = y^\omega_{\text{sal}} q^\omega_{\text{sal}}, s, s \in S_{II}, \omega, t \in [To - \delta^\omega_{\text{sal}}, To - 1],
\]

\[
\eta_{\text{zo}} \leq x^\omega_{\text{zo}} \leq \eta_{\text{zo}}, \omega \in \Omega, t \in [To, T], \gamma \in \Omega.
\]

by upper and lower limits of flow magnitudes in arcs and storage supplies as well as amounts and compositions of impurities (pollutants) in arcs and storages:

\[
\eta_{\text{zo}} \leq Z^\omega_{\text{zo}} \leq \eta_{\text{zo}}, \omega \in \Omega, t \in [To, T], \gamma \in \Omega.
\]

8 SOLUTION OF THE PROBLEM

Task A is multiextremal, i.e. it has local minima, different from the global one. Multiextremity of
this task is caused by the non-convexity of the biseparable task function (18) and non-convexity of the permissible set $G$, assigned by bilinear constraints of the equation (22), biseparable non-convex constraints of the equation (24) and integer requirements (25) of strategic variables $\eta$. Specific features of the task A allow us to find methods of its solution.

This solution is reduced to the selection of optimum vectors of a finite sequence of estimating convex tasks on the graph $\Gamma(J, S)$. working out details of the diagram of branches and boundaries, (Lazebnik et al. 1981). As a result, the vector is obtained, corresponding to the value of the task function, different from the optimum one by not more than the prescribed value and exceeding the permissible set by not more than the prescribed error. This method is described in detail elsewhere, (Lazebnik et al. 1981).

REFERENCES


