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PhD dissertation

Nonlinear mixture autoregressive model: economic applications

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1 Introduction

1.1 Background

The aim of this dissertation is to study how the interest rate and stock market are related to inflation using nonliner time series models. According to traditional economic assumptions the nominal interest rate and stock returns move one-to-one with the inflation rate. Thus, the real interest rate and real stock returns do not correlate significantly with inflation. In the case of the interest rate this assumption was first proposed by Irving Fisher (1930). He postulated the so called Fisher hypothesis which states that the nominal interest rate can be expressed as the sum of expected constant real interest rate and expected inflation.

However, there are several empirical studies which contradict these aforementioned hypotheses. A conventional method to test the Fisher effect is to use a linear cointegration analysis between inflation and nominal interest rate. The Fisher effect implies that the real interest rate has no unit root. Rose (1988), Evans & Lewis (1995) and Koustas & Serletis (1999) have found evidence that the nominal interest rate and inflation do not move one-to-one and the real interest rate seems to be a nonstationar unit-root process which has no tendency to revert to its long-run mean. Researcher such as Rose (1988) is pointed out unit root in real interest rate is inconsistent with the standard consumption-based asset pricing model.

A plausible explanation for real interest rate persistence is that the relationship between the nominal interest rate and inflation is nonlinear. Garcia & Perron (1996) and Rapach & Wohar (2005) have during the postwar period found evidence that the US real interest rate and the inflation rate have undergone level shifts in their means and variancse and such level shifts cannot be described by a stationary linear model. The results of these authors have shown that timings of the breaks in the real interest rate often coincide with breaks in the inflation rate. Thus, increases (decreases) in the mean of the inflation rate are associated with decreases (increases) in the mean of the real interest rate.

Garcia & Perron (1996) have used a nonlinear regime switching model in order to study this kind of behavior whereas Christopoulos & Leon-Ledesma (2007) have found evidence of nonlinear cointegration between the nominal interes rate and inflation rate. According to their results, mean reversion of the real interest rate depends on the level of the inflation rate. Lanne (2006) has used a bi-variate regime-switching model to investigate the long-term relationship between U.S inflation and interest rate. He has found that the dynamics between these variables differs both high and low inflation regimes.

Garcia & Perron (1996), Rapach & Wohar (2005) and Christopoulos & Leon-Ledesma (2007) have suggested that nonlinearities and nonstationarities in real interest time series are a consequence of monetary policy. Another explanation for the observed behavior of the real interest rate is systematic bias in inflation expectations. Empirical evidence that survey inflation expectations are not perfectly rational have been provided by various researchs such as DeBondt and Bange (1992), Roberts (1998) and Thomas (1999). According to these studies survey participants usually underestimate (overestimate) the actual inflation rate during a period of rising (falling) inflation. There is also empirical evidence that the New-Keynesian Phillips curve fit the data much better when survey data are used instead of rational expectation (see Roberts (1995,1997)). These studies indicate that survey data can be useful to model the relationship between inflation and the interest rate.

An alternative model to rational expectations is the so-called sticky-information model of Mankiw & Reis (2002,2003). The sticky-information model assumes that economic agents form their expectations rationally, but do this only occasionally due to costs of acquiring and processing information. In each period, a fraction of economic agents update their expectations based on new information about the state of the economy whereas the rest of population continues to act according to old plans and outdated information. In this model inflation expectations are a weighted average of what they were in the preceding period and the rational expectation.

Mankiw & Reis (2002,2003) have claimed that their model can explain some stylized facts in macroeconomics, which are difficult to explain by models with fully rational expectations. These include the cost of disinflation (Ball (1994)), the delayed effect of monetary shocks (Christiano et al. (2005)) and a positive correlation between the change of inflation and economic activity (Abel & Bernanke (1998)).

There is also empirical evidence that the Fisher hypothesis does not hold in the stock market. Several studies (e.g. Fama and Schwert (1977), Modigliani & Cohn (1979), Ritter & Warr (2002) and Campbell & Vuolteenaho (2004)) show that common stock returns are negatively related to inflation.

A rational explanation for the negative correlation between stock returns and inflation is based the assumption that inflation is a proxy for an omitted macrofactor (Fama (1981)) or risk aversion (Brandt & Wang (2003)). A behavioral explanation is that stock market participants are subject to inflation illusion (Modigliani & Cohn (1979) and Ritter & Warr (2002)). According to this explanation stock market participants fail to understand the effect of inflation on nominal dividend growth rates.

The behavioral explanation is based on the assumption that the financial markets are not fully efficient and stock prices do not reflect rationally all known information. According to behavioral finance some agents are not fully rational in their decision making and these agents also influence the aggregate market due to limit of arbitrage (Barberis & Thaler (2003)). Several studies have shown that individuals are subject to cognitive biases and herding behavior in their decision making.

Behavioral finance provides explanations for stock market bubbles which refers to time periods when asset prices exceed their fundamental values (Brunnermeier (2008)). Shiller (2005) has written a popular describtion of stock market bubbles. He claims that bubbles are usually associated with so called "new era" thinking about the economy. An example of the stock market bubble is the recent dot-com bubble when stocks of internet-based companies were highly valued. There qre also rational models for stock market bubbles; for an overview see LeRoy (2004).

Behavioral explanations for stock market bubbles assume that bubbles arise in the interaction between rational and irrational noise traders. Many behavioral models for stock market valuation (e.g. Shleifer & Vishny (1997)) assume that there are two kind of investors: noise traders and rational arbitrageurs. Noise traders are irrational investors whose trading activities are based on mistaken beliefs or disinformation. Rational arbitrageurs are investors who try to exploit the noise traders' incorrect beliefs. Due to limited arbitrage they still cannot fully correct mispricing.

Arbitrage is limited due to risk aversion, short horizons, agency problems (Shleifer & Vishny (1997)) and synchronization risks (Abreu & Brunnermeier (2003)). A rational arbitrageur who tries to exploit mispricing faces the risk that noise traders' sentiments will become even more extreme in the near future. Secondly, he is uncertain when other arbitrageurs will trade against the misspricing.

In this thesis we study how bubbles are related to inflation. These studies are based on an assumption that noise traders' attitudes toward inflation are different than rational investors' attitudes. There is survey-based evidence on heterogenous attitudes towards inflation. Shiller (1997) has found that people dislike inflation much more than is rationally anticipated. Non-economists' attitudes towards inflation are more negative than economists' attitudes. Non-economists commonly associate a high inflation rate with economic disarray and lower purchasing power.

In our model, noise traders are subject to inflation illusion but rational investors are not. Piazzesi & Schneider (2008) have provided a similar model for housing booms. In their model illusionary investors mistake changes in nominal interest rates for changes in real rates, while smart investors understand the Fisher equation. This model can explain a nonmonotonic relationship between house price-rent ratios and inflation.

These studies imply that noise traders' participation in the stock market depends on inflation. According to this assumption low inflation is related to a period when stock prices are determined more by noisy information than news of fundamentals and valuation ratios are less mean reverting. If this assumption holds the relationship between stock returns and inflation is highly nonlinear.

1.2 Non-linear time series models

Many economic time series contain properties which cannot be described by linear and stationary time series models. For instance dramatic changes in the properties of time series can occurs due to economic crises. Regime switching time series models (e.g. Lange & Rahbek (2009)) have been widely used to describe this kind of behavior. These models allow the conditional mean and variance to vary with some finite-valued switching variable $s_t \in \{1, 2, ..., r\}$.

An example of regime switching models is the threshold autoregressive (TAR) model (Tong & Lim (1980) and Tong(1990)) where regimes are determined by an observable variable such as a lagged value of series. A change in regime occurs when this variable exceeds a certain threshold level. This model can be viewed as a piecewise linear approximation of a nonlinear autoregressive (AR) model. When regimes are determined by a lagged value of the process the model is called a self-exciting htreshold autoregressive (SETAR) model.

In the case of two regimes the SETAR model can be expressed as

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + ... + \alpha_p y_{t-p} + \sigma_1 \epsilon_t \text{ if } y_{t-k} < r$$
$$y_t = \beta_0 + \beta_1 y_{t-1} + ... + \beta_p y_{t-p} + \sigma_2 \epsilon_t \text{ if } y_{t-k} \geq r$$

where the innovation ϵ_t is usually assumed to be independently and identically distributed random variable.

In many economic applications it is counter-intuitive that the regime switch is abrupt such as assumed in the TAR model. An extension of the TAR model is the smooth transition autoregressive (STAR) model (e.g. Granger & Teräsvirta (1994)) where the thick regime indicator $I[y_{t-k} > r]$, which get only values 0 or 1, is replaced with a smooth function. This transition function is continuous and bounded between 0 and 1. In this model there is a smooth transition from one regime to the other.

The STAR -model can be expressed as

$$y_t = (\alpha_0 + \alpha_1 y_{t-1} + ... + \alpha_p y_{t-p} + \sigma_1 \epsilon_t)(1 - G(y_{t-k}, \gamma, r))$$
$$+(\beta_0 + \beta_1 y_{t-1} + ... + \beta_p y_{t-p} + \sigma_2 \epsilon_t) G(y_{t-k}, \gamma, r),$$

where $G(y_{t-k}, \gamma, r)$ is a transition function. The most popular functional forms of the transition function are the exponential function, the logistic function and the probit function. When the logistic function is used, the transition function is specified as
$G(y_{t-k}, \gamma, r) = \frac{1}{1+e^{-\gamma(y_{t-k}-r)}}$,

where the parameter r can be interpreted as the threshold and the parameter γ determines speed of transition. When the parameter goes to infinity the STAR model converge to the TAR model.

The TAR and STAR models assume that regimes are determined deterministically by recent and past information. There are also regime switching models where regimes are determined stochastically. An example of these models is the mixture autoregressive (MAR) model (Le et al.(1996) and Wong & Li (2000,2001)). The MAR -model consists of K linear autoregressive components. At any given point of time one of these autoregressive components is randomly selected to generate a new observation for the time series.

In the case two regimes the MAR model can be expressed as

$$y_t = (\alpha_0 + \alpha_1 y_{t-1} + ... + \alpha_p y_{t-p} + \sigma_1 \epsilon_t)(1 - z_t)$$
$$+(\beta_0 + \beta_1 y_{t-1} + ... + \beta_p y_{t-p} + \sigma_2 \epsilon_t) z_t,$$

where z_t is a non-observable Bernoulli distributed regime indicator variable.

The mixture probability can be constant over time (Wong & Li (2000)) or a direct function of a lagged value of the time series process or some exogenous variable ((Wong & Li (2001)). The latter model can be also called to the logistic mixture autoregressive (LMARX) model. The mixture probability is originally assumed to be a logistic function of switching variables. Naturally, the probit function can be also used as the mixture probability. This model can be representd as an extension of the threshold autoregressive (TAR) model adding a random error to the threshold parameters (Lanne & Saikkonen (2003)). In the case of a probit function this nonobservable random error is normally distributed.

The MAR model is related to the Markov regime switching (MS) model (Hamilton (1989)) where the regimes are determined by an unobserved discrete-state Markov chain. Thus, probabilities of regimes depend on the past through the value of the most recent regime. Diebold et al. (1994) have proposed an extension of the Markov switching model where the transion probabilities depend on observed variables. In this model probabilities of regimes depend on the past through the value of the most recent regime and observable variables.

Mathematically, the MAR model can be represented as a special case of the MS model when rows of the transition matrix are the same. Then transition probabilities do not depend on the most recent regime. The MAR model with constant mixing probabilities is a special case of the MS model with constant transition probabilities. The LMARX model is a special case of the MS model with time-varying transition probabilities. An advantage of MAR models is that probabilities of regime switches are directly observable.

Wong & Li (2000,2001) mention several reasons which make MAR models attractive for economic time series. An interesting property of MAR models is that a mixture of a nonstationary AR component and a stationary AR component can result in a stationary process. An example of this kind of model is given by Bec et al. (2008). Models of this kind can be appealing for many economic time series, such as interest rate and stock market valuation ratios, where conventional unit root tests fail to reject the null of an unit root but economic theory and common sense strongly support stationarity. The fact that standard unit root tests can have low power against some stationary nonlinear alternatives has been demonstrated by Kapetianos et al. (2003).

A problem with nonlinear regime switching models is that standard asymptotic theory is usually not valid when the null hypothesis of lineary is tested. The reason is that nuisance parameters are not identified under the null hypothesis. For example in the TAR model the threshold parameter r is not identified under the null of linearity when $\alpha_i = \beta_i$ for $i \in \{0, 1, ..p\}$ and $\sigma_1 = \sigma_2$. When linearity is tested against the STAR model the same identification problem concerns parameters r and γ.

In the case of the TAR model Tsay (1989) and Hansen (1997) have developed asymptotically valid tests for linearity. When linearity is tested against the STAR model Luukkonen et al. (1988) have proposed Lagrange multiplier type tests by using a Taylor series approximation for the transition function. Unfortunately, valid linearity tests are not available for MAR models. Diagnostic checks and information criteria are useful tools when a linear model is compared to a MAR model.

1.3 Overview of essays

In this dissertation the MAR model is used to model stock market bubbles and a relationship between inflation and the interest rate. This dissertation contains four essays. The first two essays contain the model for the interest rate and inflation. The interest rate is decomposed to inflation expectation and the ex-ante real interest rate. We use the Livingston survey as the proxy for inflation expectations. This survey contains forecasts of US economists and is available on the Philadelphia Fed's web page (http://www.phil.frb.org).

In the first essay, *A survey-based model for inflation and inflation expectation*, introduces a bi-variate nonlinear model for inflation and survey inflation expectations. We employ a non-linear MAR model for inflation with randomly occuring level shifts. The probability of a level shift is described as a probit function of past inflation.

The inflation expectations are not assumed to be fully rational. In the case of inflation expectations we have employed the sticky information model where economic agents adjust their expectations gradually. Finally, we arrive at a bi-variate nonlinear model where the inflation rate follows the MAR model and the expected inflation follows the smooth transition (STR) model. This model has two regimes: a low inflation regime and a high inflation regime. In the high inflation regime inflation behaves almost like an integrated process.

The second essay, *Fisher effect, survey data and time-varying volatility* is related to the first essay. In this essay we study a relationship between the interest rate and survey data on inflation expectations. Thus, we provide a model for the ex-ante real interest rate. In this essay we propose a simple AR(1) -model for the ex-ante real interest rate where the standard deviation of survey forecasts is used to correct for heteroskedasticity.

We find supportive evidence for the Fisher hypothesis that the nominal interest and expected inflation move one-for-one both for the short and the long run. Evidence for the Fisher hypothesis is much more favourable in this study than in previous studies which assume rational expectations. Our results also contradict previous findings which claim that the dynamics of the U.S ex-ante interest rate has undergone a structural shift in the turn of 1970 and 1980 due to the influence of monetary policy. This essay shows how useful survey data can be in statistical modelling. Using survey data we arrived at a very simple and adequate model for the interest rate.

The last two essays discuss stock market bubbles and their implications to investment management. In these essays a MAR model is used to model bubbles and crashes. In the third essay, *Stock market bubbles, inflation and investment risk*, we introduce a statistical model for bubbles and crashes whereas the fourth essay, *The long-term risk caused by the stock market bubble*, analyses the implications bubbles have on risk management.

The model introduced in the third essay has two regimes: the bubble regime and the error correction regime. In the error correction regime price depends on a fundamental factor, the price-dividend ratio, and in the bubble regime, price is independent of fundamentals. In this model a stock market crash is usually caused by a regime switch from a bubble regime to an error-correction regime. The probability of a bubble regime depends on exogenous inflation and the lagged change of the stock price.

In our model the bubble regime can be interpreted as one where the stock market is dominated by noise traders who do not rely on the relationship between dividend and stock price. The error-correction regime can be interpreted as being dominated by arbitrageurs, who try to exploit the noise traders' incorrect beliefs.

According to many previous studies the stock returns is negatively related to inflation. In our paper we have found evidence that this relationship can be nonlinear. Furthermore, our results indicate that market participants have heterogenous attitudes toward inflation. In our model only noise traders are subject to inflation illusion.

In the fourth essay a model similar to that in the third essay has been used to study how the existence of stock market bubbles influences long term investment risk. The results highlight the importance of higher moments in risk and investment management. An interesting property of the model is that also higher moments are time varying.

The findings of the fourth essay indicate that risk management, which only concentrates on the two lowest moments, can be misleading. We find support for old wisdom that stocks are a more attractive investment in the long run than in the short run. Thus, short term log real returns are negatively skewed and long term log real returns are positively skewed.

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