Pricing and Market Structure

Mats Godenhielm  
University of Helsinki and HECER

and

Klaus Kultti  
University of Helsinki and HECER

Discussion Paper No. 338  
September 2011

ISSN 1795-0562
Pricing and Market Structure*

Abstract

We derive the equilibrium pricing strategies under three often observed market structures in a model with one large firm and a competitive fringe of small capacity constrained firms under uncertain demand. The pricing strategies reflect the varying levels of frictions and within-location competition induced by the market structures. An implication of the complexity of the pricing strategies is that a sample of posted prices and a simple index based on these is not enough for comparing the market structures in terms of expected prices paid. Knowledge of the market structure and expected demand is needed as well.

JEL Classification: D43, L10, L13

Keywords: firm location, market structure, firm size.

Mats Godenhielm
Department of Political and Economic Studies
University of Helsinki
P.O. Box 17 (Arkadiankatu 7)
FI-00014
FINLAND

e-mail: mats.godenhielm@helsinki.fi

Klaus Kultti
Department of Political and Economic Studies
University of Helsinki
P.O. Box 17 (Arkadiankatu 7)
FI-00014
FINLAND

e-mail: klaus.kultti@helsinki.fi

* Mats Godenhielm wishes to thank Matti Liski and Tanja Saxell for usefull comments. Financial support from the Academy of Finland and the Yrjö Jahnsson Foundation is greatly acknowledged.
1 Introduction

It is often observed that sellers of similar goods, say outdoor equipment, locate close to each other and that several smaller retailers are found near a larger one. Another frequently observed market structure is one with several small sellers in the city centre and large retailers in the outskirts of the city. We analyze the effect that different market structures have on expected prices and expected utilities and profits. This can be seen as investigating the effects of price competition between locations versus price competition within a location.

In our model aggregate supply is much larger than aggregate demand. If there were only one firm it could charge the monopoly price, whereas if there were two firms (still with enough capacity to satisfy the whole market) they would engage in Bertrand competition and drive the price down to zero. We model a market with one large firm (without capacity restrictions) and a competitive fringe of small capacity constrained firms and analyze the effect these firms have on the prices. Key assumptions are capacity constraints of the small firms and uncertain demand,\(^1\) as they induce the small sellers to use a mixed pricing strategy whenever they are together at a location.\(^2\)

We define market structure as the locational distribution of firms and analyze how market structure affects average posted prices, expected utilities, profits as well as the prices actually paid. The settings that we consider are

(A) All firms are in the same location, this setting can be interpreted as describing a city centre.

(B) The large firm is in one location and all the small firms are in a second location. This setting can be seen as corresponding to a city center with small firms and a large retailer at the outskirts of the city.

(C) All firms are at separate locations.

The ordering of the different market structures by average price and by expected price paid is often very different. There are several reasons for this. Firstly, when the small sellers are together in a location (as in market structures (A) and (B)) they use mixed pricing strategies.\(^3\) The cheapest goods are then bought first, leading to differences in the average and paid prices when demand is less than the small firms’ capacity. The large firm’s expected price is higher than that of the small firms in both market structures (A) and (B).\(^4\) Thus, the average prices and the expected prices paid are different

---

\(^1\)Without these assumptions the analysis would be uninteresting as firms would engage in Bertrand competition and drive prices to zero whenever demand is smaller than supply at a location. (The mirror case is just as uninteresting; when demand is at least as large as supply all firms would sell, thus all firms would charge the highest possible price).

\(^2\)This was first demonstrated by Prescott (1975).

\(^3\)The large firm might price using a mixed strategy as well when it is in the same location as the small firms.

\(^4\)In market structure (A) no small firm with a price higher than the large firm’s price sells ever sells. In market structure
also for high demand realizations. Secondly, when there are several locations as in market structures (B) and (C) the locations compete for customers affecting the prices. These market structures induce extra frictions as now some buyers visiting the small sellers are left without the good and not all small sellers are able to sell even when demand is relatively high. The small firms’ prices reflect the need to compensate the buyers for the possibility of being left without the good. It is clear that these frictions affect market structures (B) and (C) differently as the number of locations and goods per location are different. For the reasons above the effect of market structure on prices is highly nontrivial.

The different market structures lead to different pricing strategies for both the large and the small firms. An implication is that a sample of posted prices and a simple index based on these is not enough for comparing the market structures in terms of expected prices paid. Knowledge of the market structure and potential demand, or alternatively expected demand, is needed as well. The good news is that it is possible to construct indices that generate the prices paid as well as utilities and firm profits from a good sample of posted prices and knowledge of the market structure and expected demand. This can be useful given that data on prices paid can be hard to come by.

Even when the market structure is known the distribution of the demand can have large and surprising effects on the pricing behavior of firms. As an example of this consider market structure (A), where all sellers are together in a single location. In this setting the large seller has a pure pricing strategy when demand is exponentially distributed whereas it has a mixed pricing strategy with an atom at the highest price the buyers are willing to pay when demand is uniform.

The rest of the paper is structured the following way. In section two we describe the model and derive the pricing strategies of the firms under the three market structures. In section 3 we compare the average prices, the expected prices paid as well as utilities under the different settings when demand is uniformly distributed. In section 4 we show that the ranking of the market structures under the exponential distribution are different still. Section 5 concludes.

1.1 Related models

The study of firms’ choice of location has a long tradition in economics going back at least to Chamberlin (1933). More recently firms’ location choice has been analyzed by e.g. Kultti (2008). He considers the location choice of small capacity constrained firms that have the option of locating close together or separately. The paper derives the equilibrium prices in both markets and shows that both markets cannot coexist and that when sellers are allowed to choose markets they choose the clustered market.
Whenever the small firms are together in a location as in market structures (A) and (B) uncertain demand and capacity constraints induce them to use a mixed pricing strategy. This was first demonstrated by Prescott (1975) in his example of hotel competition. Later the effect of demand uncertainty on pricing has been modelled e.g. by Eden (1990) and Dana (1999). For a dynamic model of price posting with random demand see Deneckere and Peck (2010).

Whenever there are several locations as in market structures (B) and (C) we model the search behavior by the buyers similarly as in the directed search literature (see e.g. Moen (1997), Shimer (1996), Burdet Shi and Wright (2001), Watanabe (2010), or Godenhielm and Kultti (2011), where the last three papers allow for different capacities of sellers).

2 The model

There is a unit interval of small sellers who all have one good for sale. In addition there is a large seller with enough capacity to serve the whole market. There is a continuum $m > 1$ of potential buyers. The sellers value the good at zero. The number of actual buyers $\theta$ in the market is stochastic and follows distribution $H$. We assume that the support of $H(\theta)$ is $[0, m]$. The buyers value the good at unity. The sellers post prices and based on these, as well as on the quantities on offer at the different locations the buyers decide which location to visit.

Next we analyze the three market structures in detail.

2.1 Market structure (A); All firms in the same location

We assume that all sellers’ are in the same location. A buyer visiting the location will then choose to buy the cheapest good (as long as the price is at most unity). If a small firm charges the same price as the large firm we assume that the buyer prefers the small firm. To find the equilibrium prices we first assume that the large firm uses a pure strategy when all small firms are in the same location with it. The large firm asks price $q$. Now in a prospective equilibrium $q$ has to be the highest price. If the large firm quotes price $q = 1$ it will trade only when there are more buyers than small firms. As there is a unit interval of small firms this means that the large firm will trade only when realized demand $\theta > 1$.

We next determine whether there is a profitable deviation for the large firm to price $1 - H(1)$ from the prospective equilibrium where the large firm asks price 1. In the candidate equilibrium the large
firm earns
\[ \int_1^m (\theta-1)h(\theta)d\theta = \int_1^m \theta h(\theta)d\theta - [1 - H(1)] = m - H(1) - \int_1^m H(\theta)d\theta - [1 - H(1)] = m - 1 - \int_1^m H(\theta)d\theta. \]

where we have partially integrated to get the second equality. It is easy to show that the derivative of equation (1) is positive with respect to the price. Thus no small deviations exist. Next we look for larger deviations. A natural place to start is to look at deviations to the lower bound of the support of the prices of the small firms.

If the large firm quotes price \(1 - H(1)\) it can expect the following profit
\[ [1 - H(1)] \int_0^m \theta h(\theta)d\theta = m [1 - H(1)] - \int_0^m H(\theta)d\theta [1 - H(1)]. \]

where we have, again partially integrated.

The second is greater than the first if
\[ -mH(1) - \int_0^1 H(\theta)d\theta [1 - H(1)] - \int_1^m H(\theta)d\theta [1 - H(1)] > -1 + \int_1^m H(\theta)d\theta \]

which is equivalent to which is equivalent to
\[ 1 + H(1) > mH(1) + \int_0^1 H(\theta)d\theta. \]

**Claim 1** Whenever \(1 > \int_0^1 H(\theta)d\theta + mH(1) - H(1)\) the large firm has a profitable deviation from price unity.

**Proof.** The proof is sketched above. ■

An example of a demand distribution \((H)\) for which expression (4) does not hold is the exponential distribution. We show (in the appendix) that in this case the large firm prices at unity. An example of a distribution of \(H\) for which the expression holds is the uniform distribution. Next we derive the mixed strategy equilibrium in price when (4) holds.

Let us next derive the mixed strategies of the firms. Denote the small firms’ mixed strategy on \([a, A]\) by \(F\) and the large firm’s mixed strategy on \([b, B]\) by \(G\). Note first that as long as there are no atoms \(B = 1\) since otherwise there would be a profitable deviation upwards from \(B\). Notice that \(A = 1\) since otherwise there would be a gap between \(A\) and \(B\); in this case the large firm could deviate by choosing a mass point at \(B = 1\) and choosing prices between \(A\) and unity with probability zero. Then, again the small firms could profitably deviate upwards from \(A\).\footnote{Of course \(B\) cannot be less than \(A\) since small firms choosing a price above \(B\) would never sell.} A small firm would trade with probability zero
if it chose price $A = 1$ unless the large firm had a mass point at $B = 1$. This is the equilibrium we construct denoting the mass at unity by $\gamma$ Finally, let us note that it is quite possible that $b > a$.

Consider a small firm that chooses price $p \in [a, 1]$. Its expected profit is given by

$$p [1 - G(p)] \int_{F(p)}^{m} h(\theta)d\theta = p [1 - G(p)] [1 - H(F(p))] \quad (5)$$

when $p > b$, and by

$$p \int_{F(p)}^{m} h(\theta)d\theta = p [1 - H(F(p))] \quad (6)$$

when $p \leq b$.

Consider next a large firm that chooses price $q \in [b, 1]$. Its expected profit is given by

$$q \int_{F(q)}^{m} (\theta - F(q)) h(\theta)d\theta = q \left \{ m - F(q) - \int_{F(q)}^{m} H(\theta)d\theta \right \} \quad (7)$$

Price 1 yields a small firm profit $[1 - H(1)] \gamma$, and to the large firm it yields $\int_{1}^{m} (\theta - 1) h(\theta)d\theta$. We immediately see that $a = [1 - H(1)] \gamma$. Equating the small firms’ profit with $[1 - H(1)] \gamma$ allows to solve for

$$G(p) = \frac{p [1 - H(F(p))] - [1 - H(1)] \gamma}{p [1 - H(F(p))]} \quad (8)$$

Now $b$ is determined by $G(b) = 0$ which is equivalent to

$$b [1 - H(F(b))] - [1 - H(1)] \gamma = 0 \quad (9)$$

From this we immediately see that $b = a$ or equivalently $b = [1 - H(1)]$ when $H(F(b)) = 0$.

The small firms’ strategy is determined by the equality of profits for the large firm

$$q \left \{ m - F(q) - \int_{F(q)}^{m} H(\theta)d\theta \right \} = \int_{1}^{m} (\theta - 1) h(\theta)d\theta \quad (10)$$

One would like to show that in (12) the LHS becomes zero at some value $b > [1 - H(1)] \gamma$, and to solve $F$ from (13). This is, however, not possible unless one considers an explicit distribution $H$. To that end we focus on a uniform distribution$^6$. Now (12) becomes

$$b \left [1 - \frac{F(b)}{m} \right ] - [1 - H(1)] \gamma = 0 \quad (11)$$

and (13) becomes

$$q \left \{ m - F(q) - \frac{1}{2} m^2 - F(q)^2 \right \} = \frac{1}{2} m^2 - \frac{m - 1}{m} \quad (12)$$

This is equivalent to

$$qF(q)^2 - 2mqF(q) + m^2 q - (m - 1)^2 = 0 \quad (13)$$

$^6$We derive the explicit pricing strategies under the exponential distribution in the appendix.
From this one solves
\[ F(q) = \frac{mq - (m-1)\sqrt{q}}{q} \]  
(14)

**Claim 2** The unique mixed strategy of the small firms is 
\[ F(q) = \frac{mq - (m-1)\sqrt{q}}{q} \]
and the support is \( \left[ (\frac{m-1}{m})^2, 1 \right] \)

**Proof.** The proof is by construction above. ■

Inserting the condition \( F\left([1 - H(1)] \gamma \right) = F\left(\frac{m-1}{m} \gamma \right) = 0 \) and solving yields \( \gamma = \frac{m-1}{m} \).

Now we can solve for
\[ G(p) = 1 - \frac{m-1}{m\sqrt{p}} \]  
(15)

**Claim 3** The large firm uses a unique mixed strategy \( G(p) = 1 - \frac{m-1}{m\sqrt{p}} \) with probability \( 1 - \gamma \), with probability \( \gamma \) the large firm uses prices at unity.

**Proof.** The proof is by construction above. ■

Thus, in equilibrium the small firms price using mixed strategy \( F \) with support \([a, 1] \), and earn expected profits of \( [1 - H(1)] \gamma = (\frac{m-1}{m})^2 \). The large firm uses mixed strategy \( G(p) \) with support \([a, 1] \), it has an atom at price unity. The probability that the large firm has price one is profit is \( \gamma = \frac{m-1}{m} \).

The expected profit of the large firm is \( m - 1 - \int_1^m H(\theta)d\theta \).

### 2.2 Market structure (B): Large firm and small firms in two separate locations

We next consider price and expected utilities when the small firms are located together in one location but separately from the large firm. Kultti (2008) considered a model with small sellers and showed that they prefer to locate close together to locating separately. We proceed to find equilibrium prices and expected utilities in this case.

Assume that fraction \( z \) of buyers go to small firms and fraction \( 1 - z \) go to the large firm. Then the small firms set their price using a mixed strategy with support \( \gamma = (\frac{m-1}{m})^2 \). The large firm uses mixed strategy \( G(p) \) with support \([a, 1] \), it has an atom at price unity. The probability that the large firm has price one is profit is \( \gamma = \frac{m-1}{m} \).

The expected profit of the large firm is \( m - 1 - \int_1^m H(\theta)d\theta \).

\(^7\)It is clear that \( A=1 \) as a small firm pricing \( A \) would otherwise have a deviation to 1.
\[ F(\rho) = z H^{-1} \left( \frac{\rho - \left(1 - H \left( \frac{1}{z} \right) \right)}{\rho} \right) \]  

(17)

A large firm can expect the following profit

\[ P_B \int_0^m (1 - z) \theta h(\theta) d\theta = \Pi(P_B) \]  

(18)

To continue we first look at the expected utilities of buyers that go to the small firms. The small firms set their prices using a mixed strategy. To make calculations easier we follow Kultti (2008) and assume that all sellers charge the virtual price \( z \) described below. We denote the expected utility of buyers visiting the small firms by \( u(z, F) \). A buyer going to a large seller knows that he can expect to get \( 1 - P_B \).

In equilibrium the following must hold:

\[ u(z, F) = 1 - P_B \]  

(19)

and

\[ \Pi'(P_B) = 0. \]  

(20)

To solve this set of equations we begin by looking at the small firms’ pricing decision. To simplify we let every small firm asks the same price \( r \), where \( r \) can be thought of as the virtual price that gives the sellers the same expected profit as the sellers would get using the mixed strategy \( F \) derived above.

We get

\[ r \int_0^{1 \frac{1}{z}} \frac{\theta z}{1} h(\theta) d\theta + \left(1 - H \left( \frac{1}{z} \right) \right) r = 1 - H \left( \frac{1}{z} \right). \]  

(21)

In the first term on the LHS we integrate over levels of demand when there are fewer buyers than small sellers, the second term corresponds to levels of demand is higher than the number of sellers at the small firms’ location. Forcing the LHS to equal the expected profit from the mixed strategy we solve the small seller’s virtual price

\[ r = \frac{(1 - H \left( \frac{1}{z} \right))}{1 - H \left( \frac{1}{z} \right) + \int_0^{1 \frac{1}{z}} \frac{\theta z}{1} h(\theta) d\theta}. \]  

(22)

As the total number of trades is the same when using the virtual price \( r \) as under the sellers mixed strategy \( F \) so is the expected utility to the buyers visiting the small firms’ location.

This allows us to rewrite the buyers’ indifference condition as
\begin{equation}
(1 - r) \frac{1}{E(\theta)} \left[ \int_0^{\frac{1}{z}} \theta h(\theta) d\theta + \int_{\frac{1}{z}}^{m} \frac{1}{z} h(\theta) d\theta \right] = 1 - P_B.
\end{equation}

The LHS is the buyers’ expected utility from visiting the small firms’ location, the RHS is the buyers expected utility from visiting the large firm. It is clear that \( P_B \leq 1 \) as no buyer would otherwise visit the large firm.

The large firm maximizes
\[
\max_{P_B} \int_0^m (1 - z) \theta h(\theta) d\theta \cdot P_B
\]
. The first order conditions are:
\begin{equation}
\int_0^m (1 - z) \theta h(\theta) d\theta - \int_0^m \frac{dz}{dP_B} \theta h(\theta) d\theta \cdot P_B = 0
\end{equation}

We get \( \frac{dz}{dP_B} \) by totally differentiating the buyers indifference condition. We then solve for the large firm’s price which after some simple algebra simplifies to
\begin{equation}
P_B = \left(1 - z\right) \frac{\left[ E(\theta) - \int_0^{\frac{1}{z}} \theta h(\theta) d\theta + h(\frac{1}{z}) \frac{1}{z^2} \right]}{E(\theta)}.
\end{equation}

as \( \int_0^m \theta h(\theta) d\theta = E(\theta) \). The expected profit \( E(\pi_L) \) of the large firm is
\begin{equation}
E(\pi_L) = (1 - z) E(\theta) \cdot P_B = (1 - z)^2 \left[ E(\theta) - \int_0^{\frac{1}{z}} \theta h(\theta) d\theta + h(\frac{1}{z}) \frac{1}{z^2} \right].
\end{equation}

Claim 4 When the large firm is in one location and all small firms are in another location the large firm has a unique price \( P_B \) which is a function of \( z \) and \( m \) defined in (24). The small firms have a unique mixed strategy \( F(\rho) \) which is a function of \( z \) and \( m \) and is defined in (21).

Proof. The proof is above by construction. ■

2.3 Market structure (C); All firms in different locations

Now assume that all firms are in different locations. Assume that proportion \( z \) of buyers visit the small firms and proportion \( 1 - z \) visit the large seller. In equilibrium the buyers are indifferent between visiting the large firm or mixing over the small firms.

In equilibrium the price of the small sellers is
\begin{equation}
q = \frac{\int_0^m \left(1 - e^{-z\theta} - z e^{-z\theta} \right) h(\theta) d\theta}{\int_0^m \left(1 - e^{-z\theta} \right) h(\theta) d\theta}
\end{equation}
The only difference to the price derived in a standard directed search model with just capacity one firms is \( z \), the proportion of buyers going to the small sellers, which we will later derive. Next we go on by deriving the price of the large firm.

By going to the small firms a buyer thus expects to get

\[
(1 - q) \int_0^m \frac{(1 - ze^{-z\theta})}{z\theta} g(\theta)d\theta
\]

(28)

where \( \int_0^m \frac{(1 - ze^{-z\theta})}{z\theta} g(\theta)d\theta \) is the probability of getting the good by when the buyer visits a small firm. Again as \( g(\theta) = \frac{\delta h(\theta)}{E(\theta)} \) we can rewrite the above as

\[
\frac{1}{E(\theta)} \int_0^m \theta e^{-z\theta} h(\theta)d\theta.
\]

Thus we again proceed by writing the buyers indifferee condition between visiting a small seller or the large seller.

\[
\frac{1}{E(\theta)} \int_0^m \theta e^{-z\theta} h(\theta)d\theta = 1 - P_C
\]

(29)

The large firm’s price is found by maximizing the large firm’s expected profit with respect to \( P \).

The large firm maximizes

\[
\max_{P_C} \left( P_C \cdot (1 - z) \int_0^m \theta h(\theta)d\theta \right)
\]

(30)

The FOC is

\[
(1 - z) \int_0^m \theta h(\theta)d\theta - \int_0^m \frac{dz}{dP_C} \theta h(\theta)d\theta \cdot P_C = 0,
\]

(31)

where we find \( \frac{dz}{dP_C} \) by totally differentiating the buyers’ indifferee condition. We can now solve for the large firm’s price.

\[
P_C = \frac{(1 - z) \int_0^m \theta^2 e^{-z\theta} h(\theta)d\theta}{E(\theta)}
\]

(32)

To see that \( P_C \) is unique we show that there is a unique \( z \in [0, 1] \) that solves the buyers’ indifferee condition \( \frac{1}{E(\theta)} \int_0^m \theta e^{-z\theta} h(\theta)d\theta = 1 - P_C \).

We get

\[
E(\theta) - \int_0^m \theta e^{-z\theta} h(\theta)d\theta = (1 - z) \int_0^m \theta^2 e^{-z\theta} h(\theta)d\theta
\]

(33)
We begin by renaming the LHS and RHS of (33) as $f(z)$ and $g(z)$ respectively. As $f(0) = 0$ and $g(0) = E(\theta^2)$; $f(1) = E(\theta) - \int_{0}^{m} \theta e^{-\theta} h(\theta) d\theta > 0$ and $g(1) = 0$; and $f'(z) = \int_{0}^{m} \theta^2 e^{-\theta} h(\theta) d\theta > 0$ and $g'(z) = \int_{0}^{m} \theta^2 e^{-\theta} h(\theta) d\theta - (1 - z) \int_{0}^{m} \theta^3 e^{-\theta} h(\theta) d\theta < 0$; the result is immediate.

Hence $P_C$ is unique.

**Claim 5** When all sellers are separate the small sellers have a unique price $q$ and the large seller has a unique price $P_C$ defined as in (26) and (32)

**Proof.** The proof can be found above.

### 3 Market structure, utility and pricing

In this section we derive the expected prices actually paid in the market as well as the expected utilities of the buyers in the three market structures under consideration. After this we compare the different market structures.

#### 3.1 Expected price paid under market structure A (EPPA)

We now have the pricing functions for the firms under three different market structures. In order to answer questions regarding utilities and pricing under the different regimes we need to look at specific distributional forms of $H$. To this end we will assume that $H$ follows the uniform distribution. Thus when all firms are in the same location we know from section 2.1 that the small firms price using mixed prices.

The expected price in the market is

$$
\frac{1}{m + 1} \int_{a}^{1} qf(q) dq + \frac{m}{m + 1} \left( \int_{a}^{1} pg(p) dp + \frac{m - 1}{m} \cdot 1 \right)
$$

$$= \frac{1}{m(m + 1)} (m^2 + m - 2)$$

(34)

Next we find the expected price paid in the market. We begin by assuming that the large firm asks price $\bar{\theta}$. Then as long as $\theta \leq F(\bar{\theta})$ only the small firms sell and when $\theta > F(\bar{\theta})$ both types of firms sell. The expected price paid given that the large firm quotes price $\bar{\theta}$ is thus

$$
\Omega(\bar{\theta}) = \int_{0}^{F(\bar{\theta})} \int_{a}^{F^{-1}(\theta)} qf(q) dq h(\theta) d\theta + \int_{F(\bar{\theta})}^{m} \frac{\bar{\theta}}{\theta} (\theta - F(\bar{\theta})) h(\theta) d\theta + \int_{F(\bar{\theta})}^{m} \int_{a}^{\bar{\theta}} \frac{1}{\theta} f(q) dq h(\theta) d\theta. \quad (35)
$$

where the first term is the expected price of the small firms when demand is $\theta \leq F(\bar{\theta})$ multiplied by the probability that demand is at this level. When demand is higher than $F(\bar{\theta})$, the amount $F(\bar{\theta})$ of the
buyers buy the good from the small sellers and the rest buy the good from the large seller. The second and third terms capture this. The second term is the price of the large firm multiplied by the probability that a buyer buys from the large firm times the probability that demand is higher than $F(q)$. The third term is the expected price when buying from a small firm when demand is high $\theta > F(q)$ multiplied by the probability that a buyer gets to acquire the good from a small firm times the probability that demand is high.

To get the expected price paid under market structure (A) (EPPA) we need to integrate over the possible prices of the large firm. The following expression captures the idea:

$$EPPA = \int_{\bar{q}}^{1} \Omega(\bar{q}) \, g(\bar{q}) \, d\bar{q}$$

$$+ \gamma \cdot \int_{0}^{1} \int_{a}^{\theta^{-1}} q \, f(q) \, dq \, h(\theta) \, d\theta + \int_{1}^{m} \frac{1}{\theta} \cdot 1 \, h(\theta) \, d\theta + \int_{1}^{m} \int_{a}^{1} \frac{2}{\theta} \, f(q) \, dq \, h(\theta) \, d\theta .$$

$$\tag{36}$$

The first term in $EPPA$ is just $\Omega(\bar{q})$ integrated over the prices in the support of the large firm. The large firm has an atom at price unity. The second term is the probability $\gamma$ that the large firm prices at unity times the expected price paid when it does so. The terms in the parenthesis can be interpreted in a similar fashion as the terms in $\Omega(\bar{q})$ with $\bar{q}$ replaced by 1.

In section 4 we depict the expected price paid as a function of potential demand $m$.

### 3.2 Expected price paid under market structure B (EPPB)

We begin by looking at the situation for the large firm. First we show that the large firm has a unique pricing strategy given the proportion $z$ of buyers going to the small firms. To solve for $z$ we equate (22) with (24), and let $H$ follow the uniform distribution, thus we find that the proportion of buyers going to the small firms is

$$z = \frac{1}{3m^2 \sqrt[3]{\frac{1}{m^2} + \frac{1}{27m^2} + \frac{1}{m^2}}} + \frac{3}{\sqrt[3]{\frac{1}{m^2} + \frac{1}{27m^2} + \frac{1}{m^2}}}$$

$$\tag{37}$$

This expression is convex and decreasing as well as between 1 and 0 when $m > 1$.

\[8\] We derive the expression for the expected price paid under market structure A (EPPA) more explicitly in the appendix.
With the expression for \( z \) at hand we can now solve for the price of the large firm as a function of potential demand \( m \). The price of the large firm is

\[
P_B = \frac{(1 - z) \left[ E(\theta) - \int_0^{1/\pi} \frac{2}{\pi} \theta h(\theta) d\theta + h(\frac{1}{\pi}) \frac{1}{2\pi} \right]}{E(\theta)} = \frac{(1 - z)}{m^2 z^2} + 1 \frac{(1 - z)}{m^2 z^2}.
\]

where the second equality is a result of the assumption of an uniform \( H \).

Now we are ready to derive the expression for the expected price paid. It is

\[
(1 - z) \cdot P_B + z \cdot \left( \int_0^{1/\pi} \int_a^{F^{-1}(z\theta)} qf(q)dq d\theta + \int_{1/\pi}^m \int_a^{1} qf(q) \frac{1}{q} dq d\theta \right).
\]

The lower bound of the support of the mixed strategy of the small firms is

\[
a = 1 - H\left(\frac{1}{z}\right) = \frac{zm - 1}{zm}.
\]

When \( H \) is uniform the small firms mixed strategy is

\[
F(q) = zm \left( \frac{q - 1 + \frac{1}{zm}}{q} \right) = \frac{1}{q} (mzq - mz + 1)
\]

Thus

\[
f(q) = \frac{1}{q^2} (mz - 1)
\]

and

\[
F^{-1}(z\theta) = \frac{mz - 1}{z (m - \theta)},
\]

when \( \theta \in \left[0, \frac{1}{z}\right] \).

Thus the expected price paid when the large firm is in a different location than the small firms is

\[
EPPB = (1 - z) \cdot P_B + z \cdot \left( \int_0^{1/\pi} \int_a^{mz - 1} \frac{1}{q^2} \frac{(mz - 1)}{m} dq d\theta + \int_{1/\pi}^m \int_a^{mz - 1} \frac{1}{q^2} \frac{(mz - 1)}{m} dq d\theta \right)
\]

The first term of EPPB is just the price of the large firm \( C \) times the probability \( (1 - z) \) that a random buyer visits this firm. The second term consists of \( z \) multiplied by a parenthesis of which

\[
EPP2 \text{ simplifies to}
\]

\[
(1 - z) \cdot \frac{m^2 z^2 + 1}{m^2 z^2} \frac{(1 - z)}{m^2 z^2} + z \left( \frac{1}{m} (mz - 1) + (mz - 1) \ln \left( \frac{m - 1}{m} \right) + \frac{1}{m} (mz - 1) \ln \left( \frac{m - 1}{m} \right) + \left( \ln \frac{1}{z} m \right) \left( \ln \frac{1}{z} (mz - 1) - \ln m \right) \frac{mz - 1}{m} \right)
\]
the first term is the expected price of the small firms when demand is low \( (< \frac{1}{2}) \) multiplied by the probability that this happens. The second term in the parenthesis is the expected price of the small firms when they face high demand \( (> \frac{1}{2}) \) multiplied by the probability that demand is high multiplied by \( \frac{1}{2} \), the measure of goods divided by the measure of small firms (i.e. the probability that a buyer gets the good). (Note that when demand is low the buyer always manages to acquire the good even by visiting the small firms as there are then less buyers than there are small firms.)

Claim 6 When the large firm and the small firms are in two different locations the expected price paid is given by EPPB.

Proof. The proof is by construction and can be found above. 

3.3 Expected prices paid under market structure C (EPPC)

Now assume that all firms are in different locations. Assume that proportion \( z \) of buyers visit the small firms and proportion \( 1 - z \) visit the large seller. In equilibrium the buyers are indifferent between visiting the large firm or mixing over the small firms.

As derived earlier the equilibrium price of the small sellers is

\[
 q = \frac{\int_0^m (1 - e^{-z\theta} - z\theta e^{-z\theta}) \, d\theta}{\int_0^m (1 - e^{-z\theta}) \, d\theta} = \frac{2e^{-mz} + mz + mz e^{-mz} - 2}{e^{-mz} + mz - 1},
\]  

(40)

where the second equality results from imposing the uniform distribution on \( H \).

The expected utility of a buyer going to a small firm is \((1 - q)\) times the probability of ending up with the good. This is

\[
(1 - q) \int_0^m \frac{1 - e^{-z\theta}}{z\theta} g(\theta) \, d\theta = -2\frac{e^{-mz} + mz e^{-mz} - 1}{m^2z^2}.
\]

As \( g(\theta) = \frac{\theta h(\theta)}{E(\theta)} \) and \( H \) is uniform.

In equilibrium a buyer has to be indifferent between visiting a small seller or the large seller. Thus

\[
-2\frac{e^{-mz} + mz e^{-mz} - 1}{m^2z^2} = 1 - P_C,
\]

(41)

allowing us to solve for the price of the large firm

\[
P_C = 1 + 2\frac{e^{-mz} + mz e^{-mz} - 1}{m^2z^2} = \frac{2e^{-mz} + m^2z^2 + 2mz e^{-mz} - 2}{m^2z^2}.
\]

(42)

The proportion \( z \) of buyers visiting the small firms is found by maximizing the large firm’s expected profit with respect to \( z \).
The large firm maximizes
\[
\max_z \left( \frac{2e^{-mz} + m^2z^2 + 2mze^{-mz} - 2}{m^2z^2} \cdot (1 - z) \int_0^m \theta h(\theta) d\theta \right) \tag{43}
\]

The FOC is
\[
- \frac{1}{2mz^3} \left( 2z + 4e^{-mz} - 2ze^{-mz} + m^2z^3 + 2m^2z^2e^{-mz} - 2m^2z^3e^{-mz} + 4mze^{-mz} - 2m^2e^{-mz} - 4 \right) = 0, \tag{44}
\]

This expression allows us to solve for \( z \) as a function of \( m \). We are, however, not able to do so analytically but instead solve it numerically for specific values of \( m \).

The expected price paid in the market is\(^{10}\)
\[
EPPC = (1 - z) \cdot P + z \cdot q \cdot \frac{2}{m^2} \int_0^m \theta e^{-\theta} d\theta \tag{45}
\]

The expected price paid when all sellers are at separate locations (EPPC) is just the probability that a buyer goes to the large seller \((1 - z)\) times the large sellers price \( P \) plus the probability that a buyer goes to the small sellers \( q \) times the small sellers price \( q \) times the probability by which a buyer gets the good by visiting a small seller.

**Claim 7** *When the large firm and the small firms are in two different locations the expected price paid is given by EPPC.*

**Proof.** Above by construction. □

### 3.4 Comparing the expected prices and profits

In this section we compare the three market structures when demand \( H \) follows the uniform distribution. In the picture below we show the expected prices as a function of potential demand \( m \). We begin by describing the market structure (A) where all sellers are in the same location (denoted by red in the picture). For large values of \( m \) \((\gtrsim 3)\) the large firm prices at unity with a relatively high probability \((> \frac{2}{3})\). In addition the lower bound of the supports of the mixed strategies of both the large firm and the small firms is \((\frac{m-1}{m})^2\) which is increasing and approaches 1 in the limit as \( m \) approaches infinity. For

\[^{10}\text{EPPC simplifies to}\]
\[
(1 - z) \frac{2e^{-mz} + m^2z^2 + 2mze^{-mz} - 2}{m^2z^2} + \frac{2e^{-mz} + mz + mze^{-mz} - 2}{e^{-mz} + mz - 1} \left( - \frac{1}{m^2z^2} (2e^{-mz} + 2mze^{-mz} - 2) \right)
\]
small values of demand, e.g. when \( m \) approaches unity (from above) the probability \( \gamma = \frac{m-1}{m} \) that the large firm prices at unity approaches zero. Also the lower bound of the support of the pricing strategies of both the large firm and the small firms tends towards zero when \( m \) tends to unity. At values of \( m \) at unity or below there will be Bertrand competition and all firms will offer prices of zero.

In market structure (B), with two locations, values of \( m \) at or below unity lead to Bertrand competition and zero price just as in the one location case. For values of potential demand \( m \) above unity the average price is, however, always lower in market structure (B) than in market structure (A).

In market structure (C), where all sellers are separate, the sellers enjoy a locational monopoly and thus there is no competition within a location that would drive prices to zero even if \( m \) is below unity.

The expected price actually paid is lower than the average price in both market structures (A) and (B). This is clear as the buyers gobble up the cheapest goods upon arrival at a location. When all firms are separate the average price of the small firms is the same as the expected price paid when visiting the small firms. Likewise the price of the large firm is the same. The expected price paid is, however, lower than the average price. The reason is simply that the large firm’s share of all trades is smaller than its share of all goods and the large firm’s price is higher than the small firms’ price. The expected price actually paid is thus lower than the average price for all market structures.

More interesting is that the differences in the expected prices paid (EPPA, EPPB and EPPC) compared to the average prices in the three market structures are large enough to change the order of "expensiveness" for even quite large potential demands (values of \( m \) up to around four). This means
that a sample of posted prices a simple index is not enough for comparing the market structures in terms of expected prices paid or welfare. One needs to know the market structure and potential demand as well as the form of the demand distribution to be able to do so.

3.4.1 Expected profits in the three market structures

In this subsection we analyze how the large seller and small sellers fare in the different market structures. We find that the large firm always prefers market structure (A) to market structure (B). When potential demand is low \( m < 1.9 \) the large firm is even better off in market structure (C) where all firms are separate. The reason is that both in market structure (A) and in market structure (B) competition within the small sellers’ location drives the prices towards down zero when \( m \) approaches one. In both of these market structures the large firm then has to respond by lowering it’s expected price as well. When all sellers are separate there is no within location competition to drive the prices to zero when \( m \) approaches one as discussed in connection to the average prices. Market structure (C) becomes relatively worse for the large seller compared to the other market structures when \( m \) becomes larger. The reason for this is that the capacity constraints of the small firms’ locations forces them to quote relatively low prices even for high values of \( m \). This in turn means that the large firm must quote a low price as well (A direct consequence is e.g. that the large firm never quotes price unity) in order to entice any buyers to visit its location.
Proposition 8  The large firm is better-off in market structure (A) than in market structure (B)

Proof. The proof is by simple calculation of the expected profits of the large firm. ■

Proposition 9  The small firms are better off in market structure (B) than in market structure (A) when potential demand is low\(^{11}\), otherwise they are better of together with the large firm.

The result is obtained by comparing the expected profits of the small firms under the different market structures. First note that the small firms want to be in a market structure where they are all separate only when \(m\) is very close to one \((m<1.065)\) for reasons discussed above. When all firms are located together a small firm expects to get \((1 - H(1)) \gamma\), with \(H\) uniform this is equal to \(\left(\frac{m-1}{m}\right)^2\). When the large firm is located separately from the small firms the small firms expect to receive \(1 - H\left(\frac{1}{z}\right)\) with \(H\) uniform this is \(1 - \frac{1}{zm}\). Substituting \(z\) for \(z\) it is easy to verify that the small firms are better-off in market structure (B) than in market structure (A) when \(m \lesssim 2.27\).

\(^{11}\)When \(H\) is uniform low means \(m \lesssim 2.27\)
3.4.2 Expected utility of the buyers

The expected utility of the buyers in the three market structures is straightforward to calculate. When all firms are together it is just

$$1 - EPPA,$$

as all buyers are always served. When the large firm and the small firms are at two different locations a buyer must be indifferent visiting the two locations. As the large firm has enough capacity to satisfy the whole market the expected utility of a buyer is simply one minus the large firm’s price i.e.

$$1 - P_B(z(m), m).$$

The expected utility when all sellers are separate is analogous to the two locations case. The expected utility of a buyer when all sellers are separate is thus simply\(^\text{12}\)

$$1 - P_C(z(m), m).$$

Picture () shows the expected utilities as functions of potential demand \(m\). We immediately see that also the ordering of the curves depends on potential demand.

For relatively low \(m\) (\(\leq 5\)) the buyers are best off in market structure (A) (the red curve). In this setting there are no frictions and competition within the location keeps the prices low. As \(m\) grows the

\(^{12}\text{Not that the } z\text{'s are different for the two relevant market structures}\)
probability that the large firm prices at unity increases. So does the lower bound of the support of the mixed strategies of both the large and the small firms. These effects are large enough to overcome the costs of the frictions (some of the buyers visiting the small firms are left without a good) inherent in the other two market structures so that for a high $m (\geq 15.5)$ market structure (A) actually becomes the worst for the buyers.

Market structure (B) (the blue curve) is worse for the buyers than market structure (A) for values of $m$ lower than 15.5 for the reasons just described. The between locations competition, however, means that the large firm never prices at unity in the two-locations setting. In fact, with high potential demand the prices will still be low enough to compensate for the loss of the cost from frictions compared to market structure (A). Market structure (B) is thus better then market structure (A) for high values of potential demand.

When all firms are separate (the green curve) the cost of frictions are at their highest. There is competition between locations but no within location competition to drive prices to zero when potential demand is low. For small values of potential demand ($m \lesssim 2$) the last effect dominates and market structure (C) is actually the best for buyers.

![Picture 5: Expected utilities](image)

Expected utilities; A (red), B (blue), C (green)

### 3.5 Exponential distribution

In this section we compare the market structures when demand follows the exponential distribution. The reason we have chosen the exponential distribution is that it leads to quite different pricing strategies
compared to when demand is uniform. This is true especially for market structure A, where the large firm has a pure strategy price of unity when demand is exponential. As the pricing equilibria in all three market structures are unique also when demand is exponential the comparison of the three market structures is informative.

Realized demand $\theta$ follows the exponential distribution $\tilde{H}(\theta) = 1 - e^{\lambda \theta}$ and the support of $\tilde{H}$ is $[0, \infty]$. There is thus no upper bound for potential demand. In order to compare the outcomes to those in the uniform demand case we use the fact that the expected demand is $\frac{1}{\lambda}$ when $\tilde{H}$ is exponential and $\frac{m}{2}$ when $H$ is uniform. Thus we let

$$\lambda = \frac{2}{m}.$$  

The fact that the support of $\tilde{H}$ under the exponential demand is unbounded from above has the immediate effect that within-location competition will not drive prices to zero when all firms are together even for values of $m$ at or below unity. When $\tilde{H}$ is exponential the large firm will interestingly have a pure pricing strategy at unity when all sellers are in the same location (market structure (A)). The expected price of the large firm is thus higher than under the uniform distribution. The expected prices paid are driven down toward zero by competition between the small firms for low values of $m$, just as when demand is uniform. (The effect of the large firm’s price unity is that the expected prices under market structure (A) remain above zero even for very small levels of demand.) Under market structure (C) all firms have a locational monopoly thus protecting them from within-location competition that would drive the expected prices paid to zero for low levels of expected demand. (see picture 7)

The three pictures below compare the market structures under the exponential distribution. Given $m$ the relative ordering of the expected price, expected price paid and the expected utilities differ quite a lot compared to when $H$ is uniform as can be seen by comparing pictures 6-9 with pictures 1,2 and 5. Except for the effect of the changes in the large firm’s pricing strategy when all firms are together the intuitions from the uniform $H$ cases are valid.

---

13We derive the pricing strategies of the firms under exponential demand in the appendix.
Market structures A (red), B (blue), C (green)

EPPA (red), EPPB (blue), EPPC (green)
4 Conclusion

We derive the equilibrium pricing strategies under three often observed market structures in a model with one large firm and a competitive fringe of small firms. These pricing strategies are nontrivial and interesting in themselves as they reflect the varying levels of frictions and within-location competition induced by the market structures at different levels of expected demand. An implication of the complexity of the pricing strategies is that a sample of posted prices and a simple index based on these is not enough for comparing the market structures in terms of expected prices paid. Knowledge of the market structure and potential demand, or alternatively expected demand, is needed as well. The good news is that it is possible to construct indices that generate the prices paid as well as utilities and firm profits from a good sample of posted prices and knowledge of the market structure and expected demand. This can be useful given that data on prices paid can be hard to come by.

In addition to knowledge of the market structure also the specific distribution of realized demand is needed to describe the sellers’ pricing strategies. This becomes most apparent in market structure (A), where all sellers are in the same location; we show that the large firm has a unique pure price at unity when demand is exponential and a mixed pricing strategy with an atom at unity when demand is uniformly distributed.

The logical next step would be to endogenize the market structure. This, however, leads to surprising technical difficulties even with the simple demand distributions considered in this article and will be
left for future work.

References


5 Appendix

5.1 Appedix 1 : Deriving the pricing strategies in market structure (B)

In section (2.2) we saw that the in equilibrium the following must hold:
\[ u(z, F) = 1 - P_B \]  

and

\[ \Pi'(P_B) = 0. \] (47)

Next we look at the small firms' pricing decision. We pretend that every small firm asks the same price \( r \), where \( r \) can be thought of as the expected price when sellers use mixed strategies. We get

\[ r \int_0^{\frac{1}{z}} \frac{1}{1} h(\theta)d\theta + \left( 1 - H \left( \frac{1}{z} \right) \right) r = \left( 1 - H \left( \frac{1}{z} \right) \right) A. \] (48)

From this we solve the small seller's expected price

\[ r = \frac{(1 - H \left( \frac{1}{z} \right)) A}{1 - H \left( \frac{1}{z} \right) + \int_0^{\frac{1}{z}} \frac{1}{z} h(\theta)d\theta} \] (49)

Then the buyers' expected utility conditional on being alive is

\[ (1 - r) \left[ \int_0^{m} g(\theta) d\theta + \int_{\frac{1}{z}}^{m} \frac{1}{z} g(\theta) d\theta \right], \] (50)

where \( g(\theta) = \frac{\theta h(\theta)}{E(\theta)} \).

The buyer must be indifferent between visiting the large firm and going to the small firms.

\[ (1 - r) \left[ \int_0^{\frac{1}{z}} g(\theta) d\theta + \int_{\frac{1}{z}}^{m} \frac{1}{z} g(\theta) d\theta \right] = 1 - P_B \] (51)

it is clear that \( P_B \leq 1 \).

Using (25) we write \( 1 - r \) as

\[ 1 - r = \frac{1 - H \left( \frac{1}{z} \right) + \int_0^{\frac{1}{z}} \theta h(\theta)d\theta - (1 - H \left( \frac{1}{z} \right)) A}{1 - H \left( \frac{1}{z} \right) + \int_0^{\frac{1}{z}} \theta h(\theta)d\theta} = 1 - \frac{A}{1 - H \left( \frac{1}{z} \right) + \int_0^{\frac{1}{z}} \theta h(\theta)d\theta} \] (52)

Now we look at the rest of the LHS of (27). Remembering that \( g(\theta) = \frac{\theta h(\theta)}{E(\theta)} \) we get

\[ \int_0^{\frac{1}{z}} g(\theta) d\theta + \int_{\frac{1}{z}}^{m} \frac{1}{z} g(\theta) d\theta = \int_0^{\frac{1}{z}} \frac{\theta h(\theta)}{E(\theta)} d\theta + \int_{\frac{1}{z}}^{m} \frac{1}{z} \frac{\theta h(\theta)}{E(\theta)} d\theta = \frac{1}{E(\theta)} \left[ \int_0^{\frac{1}{z}} \theta h(\theta)d\theta + \frac{1}{z} \int_{\frac{1}{z}}^{m} h(\theta)d\theta \right] \] (53)
Partially integrating the first term in the last equation the expression becomes

\[
\frac{1}{E(\theta)} \left\{ \left[ \theta H(\theta) \right]_{\frac{z}{2}}^{1} - \int_{0}^{\frac{z}{2}} H(\theta) d\theta + \frac{1}{z} \left( 1 - H\left( \frac{1}{z} \right) \right) \right\},
\]

which simplifies to

\[
\frac{1}{E(\theta)} \left\{ \frac{1}{z} - \int_{0}^{\frac{z}{2}} H(\theta) d\theta \right\}
\]

We can now rewrite the buyers indifference condition (22) as

\[
1 - P_B = \frac{\int_{0}^{\frac{z}{2}} \theta z h(\theta) d\theta + \left( 1 - H\left( \frac{1}{z} \right) \right) \left( 1 - A \right)}{z E(\theta)}.
\]

Then

\[
P_B = \frac{z E(\theta) - \int_{0}^{\frac{z}{2}} \theta z h(\theta) d\theta - \left( 1 - H\left( \frac{1}{z} \right) \right) \left( 1 - A \right)}{z E(\theta)}.
\]

The large firm maximizes \( \int_{0}^{m} (1 - z) \theta h(\theta) d\theta \cdot P_B \). The first order conditions are:

\[
\int_{0}^{m} (1 - z) \theta h(\theta) d\theta - \int_{0}^{m} \frac{dz}{dP_B} \theta h(\theta) d\theta \cdot P_B = 0
\]

We get \( \frac{dz}{dP_B} \) by totally differentiating the buyers indifference condition. Rewriting the buyers indifference condition (33) we get

\[
z E(\theta)(1 - P_B) - \int_{0}^{\frac{z}{2}} \theta z h(\theta) d\theta - \left( 1 - H\left( \frac{1}{z} \right) \right) \left( 1 - A \right) = 0
\]

Totally differentiating the expression we get

\[
dP_B \{ -z E(\theta) \} +
\]

\[
dz \left\{ E(\theta)(1 - P_B) - \int_{0}^{\frac{z}{2}} \theta h(\theta) d\theta + \frac{1}{z^2} h\left( \frac{1}{z} \right) - \frac{1}{z^2} h\left( \frac{1}{z} \right) + Ah\left( \frac{1}{z} \right) \left( 1 \right) \right\}
\]

\[
= 0
\]

From which we solve

\[
\frac{dz}{dP_B} = \frac{z E(\theta)}{E(\theta)(1 - P_B) - \int_{0}^{\frac{z}{2}} \theta h(\theta) d\theta + Ah\left( \frac{1}{z} \right) \left( 1 \right)}
\]

Therefore the first order condition becomes

\[
\int_{0}^{m} (1 - z) \theta h(\theta) d\theta - \frac{z E(\theta)}{E(\theta)(1 - P_B) - \int_{0}^{\frac{z}{2}} \theta h(\theta) d\theta + Ah\left( \frac{1}{z} \right) \left( 1 \right)} \int_{0}^{m} \theta h(\theta) d\theta \cdot P_B = 0
\]
We get

\[
E(\theta) \int_0^m (1 - z) \theta h(\theta) d\theta - E(\theta) P_B \int_0^m (1 - z) \theta h(\theta) d\theta - \int_0^m (1 - z) \theta h(\theta) d\theta \int_0^1 \theta h(\theta) d\theta \ 
\]

\[
+ \int_0^m (1 - z) \theta h(\theta) d\theta Ah \left( \frac{1}{z} \right) \left( \frac{1}{z^2} \right) - z E(\theta) \int_0^m \theta h(\theta) d\theta \cdot P_B 
\]

\[
= 0 
\]

From which we solve the large firm’s price

\[
P_B^* = \frac{E(\theta) \int_0^m (1 - z) \theta h(\theta) d\theta - \int_0^m (1 - z) \theta h(\theta) d\theta \cdot \int_0^1 \theta h(\theta) d\theta + \int_0^m (1 - z) \theta h(\theta) d\theta \cdot Ah \left( \frac{1}{z} \right) \left( \frac{1}{z^2} \right)}{E(\theta) \int_0^m \theta h(\theta) d\theta} 
\]

Which simplifies to

\[
P_B^* = \frac{(1 - z) \left[ E(\theta) - \int_0^1 \theta h(\theta) d\theta + Ah \left( \frac{1}{z} \right) \frac{1}{z^2} \right]}{E(\theta)} 
\]

\[(65)\]

as \( \int_0^m \theta h(\theta) d\theta = E(\theta) \). The expected profit \( E(\pi_L) \) of the large firm is

\[
E(\pi_L) = (1 - z) E(\theta) \cdot P_B 
\]

\[
= (1 - z)^2 \left[ E(\theta) - \int_0^1 \theta h(\theta) d\theta + Ah \left( \frac{1}{z} \right) \frac{1}{z^2} \right], 
\]

\[(67)\]

where we have used the fact that \( A \), the upper bound of the support of the small firms’ pricing strategies, must be equal to 1.

### 5.2 Appendix 2: Deriving the price paid in market structure (A)

We begin by looking at the expected price paid when the large firm quotes price 1.

\[
\gamma \cdot 1 \left( \int_0^1 \int_a^F(q)^{-1} q f(q) dq h(\theta) d\theta + \int_1^m \frac{1(\theta - 1)}{\theta} \cdot 1 h(\theta) d\theta + \int_1^m \int_a^1 \frac{1}{\theta} f(q) dq h(\theta) d\theta \right) 
\]

\[
= \left( \frac{m - 1}{m} \right)^2 \left( \frac{m - 1}{m} \right) \left( \ln \left( \frac{m}{m - 1} \right) - \frac{1}{m} \right) + 1 - \ln m - \frac{1}{m} + \frac{1}{m^2} (\ln m) (m - 1) 
\]

Next we look at \( \int_\alpha^1 \Omega(\bar{\eta}) g(\bar{\eta}) d\bar{\eta} \). The first term in the integrand can be written as

\[
\frac{(m - 1)^2}{m} \left( \ln \left( \frac{m}{m - F(\bar{\eta})} \right) - \frac{F(\bar{\eta})}{m} \right) 
\]
The second term in the integrand simplifies to
\[
\int_{F(\bar{q})}^{\infty} \frac{q}{\theta} (\theta - F(\bar{q})) \frac{1}{m} d\theta \\
= \left[ -\frac{1}{m} \sqrt{\bar{q}} (\ln \theta - m \ln \theta - \sqrt{\bar{q}} \theta + m \sqrt{\bar{q}} \ln \theta) \right]_{F(\bar{q})}^{\infty} \\
= \frac{1}{m} \sqrt{\bar{q}} \left( m + \ln \frac{1}{\sqrt{\bar{q}}} (m \sqrt{\bar{q}} - m + 1) - \ln m - m \ln \frac{1}{\sqrt{\bar{q}}} (m \sqrt{\bar{q}} - m + 1) \\
+ m \ln m + m \sqrt{\bar{q}} \ln \frac{1}{\sqrt{\bar{q}}} (m \sqrt{\bar{q}} - m + 1) - m \sqrt{\bar{q}} \ln m - 1 \right).
\]

The third term can be written as
\[
\int_{F(\bar{q})}^{\infty} \frac{q}{\theta} \sigma(q) d\theta F(\bar{q}) = F(\bar{q}) \int_{F(\bar{q})}^{\infty} \frac{1}{\theta} \left( \sqrt{\bar{q}} (m - 1) - \frac{(m - 1)^2}{m} \right) h(\theta) d\theta \\
= F(\bar{q}) \frac{m - 1}{m^2} \left( \ln \frac{m}{F(\bar{q})} \right) \left( m \sqrt{\bar{q}} - m + 1 \right) \\
= \frac{1}{m^2} \sqrt{\bar{q}} \left( \ln \frac{mq}{m - 1} \sqrt{\bar{q}} \right) (m - 1) (m \sqrt{\bar{q}} - m + 1)^2.
\]

Now
\[
\int_{a}^{1} \Omega(\bar{q}) g(\bar{q}) d\bar{q} \\
= \int_{a}^{1} \Omega(\bar{q}) \left( \frac{m - 1}{2m} \right) \bar{q}^{-\frac{3}{2}} d\bar{q}
\]

The expected price paid when the large firm asks price 1 multiplied by the probability that the large firm asks price 1 is
\[
\left( \frac{m - 1}{m} \right)^2 \left( \frac{(m - 1)^2}{m} \left( \ln \left( \frac{m}{m - 1} \right) - \frac{1}{m} \right) + 1 - \ln m - 1 + \frac{1}{m} (\ln m) (m - 1) \right).
\]

The expected price paid assuming that the large firm mixes multiplied by the probability that this happens is
\[
\left( \frac{1}{2m} \right) (m - 1) \left( m + m \left( \ln \frac{1}{m} (m - 1)^2 \left( \frac{m-1}{m} \right) - \frac{1}{2m^2} (m - 1)^3 \left( m + \ln \frac{m^2}{(m-1)^2} - 1 \right) \right) \right) \\
+ \left( \frac{m - 1}{2m} \right) \frac{1}{m} \int_{(\frac{m-1}{m})^2}^{1} \left( m + \ln \frac{1}{\sqrt{\bar{q}}} (m \sqrt{\bar{q}} - m + 1) - \ln m - m \ln \frac{1}{\sqrt{\bar{q}}} (m \sqrt{\bar{q}} - m + 1) \\
+ m \ln m + m \sqrt{\bar{q}} \ln \frac{1}{\sqrt{\bar{q}}} (m \sqrt{\bar{q}} - m + 1) - m \sqrt{\bar{q}} \ln m - 1 \right) q^{-1} dq \\
+ \left( \frac{1}{(\frac{m-1}{m})^2} \right) \frac{1}{m^2} \left( \ln \frac{mq}{m - 1} \sqrt{\bar{q}} \right) (m - 1) (m \sqrt{\bar{q}} - m + 1)^2 \left( \frac{m - 1}{2m} \right) q^{-2} dq.
\]

Below we plot the expected price paid as a function of $m$. 

27
5.3 Appendix 3: Deriving the expected prices when $\hat{H}$ is exponential

5.3.1 Market structure (A); all firms in the same location

We assume that all sellers’ are in the same location. A buyer visiting the location will then choose to buy the cheapest good (as long as the price is at most 1). If a small firm charges the same price as the large firm we assume that the buyer prefers the small firm. To find the equilibrium prices we first assume that the large operator uses pure strategy when all small operators are in the same location with it. The large operator asks price $q$. Now in a prospective equilibrium $q$ has to be the highest price. The large operator is assumed to have unlimited capacity. If the large firm quotes price $q = 1$ it will trade only assuming that there are more buyers than small firms.

Assume $\hat{H}$ follows exponential distribution with support $[0, \infty)$

We next determine whether there is a profitable deviation for the large firm to price $1 - \hat{H}(1)$ from the prospective equilibrium where the large firm asks price 1. In the candidate equilibrium the large firm earns

$$\int_1^\infty (\theta - 1)\hat{h}d\theta = \int_1^\infty \theta\hat{h}d\theta - \left[1 - \hat{H}(1)\right] = \int_1^\infty \theta \lambda e^{-\lambda \theta} d\theta - e^{-\lambda} = \frac{1}{\lambda} e^{-\lambda}$$

(68)

A small player with price 1 makes

$$\int_1^\infty \lambda e^{-\lambda \theta} d\theta = e^{-\lambda}$$

28
It is, again, easy to see that the small players use mixed strategy $F$ with support $[e^{-\lambda}, 1]$. Consider a small firm that chooses price $p \in [e^{-\lambda}, 1]$. Its expected profit is given by

$$ p \int_{F(p)}^{\infty} \hat{h} \lambda e^{-\lambda \theta} d\theta = pe^{-F(p) \lambda} = e^{-\lambda} $$

(69)

as $\ln p - \lambda F(p) = -\lambda$ as $F(p) = \frac{\ln p + \lambda}{\lambda}$.

The large firm doesn’t have a profitable deviation as asking price $y \in (e^{-\lambda}, 1)$ yields

$$ y \int_{F(y)}^{\infty} \lambda e^{-\lambda \theta} (\theta - F(y)) d\theta $$

$$ = y \left( - \left[ \theta e^{-\lambda \theta}\right]_{F(y)}^{\infty} \right) + \int_{F(y)}^{\infty} e^{-\lambda \theta} d\theta + F(y) \left[ \theta e^{-\lambda \theta}\right]_{F(y)}^{\infty} $$

$$ = y \lambda e^{-\lambda F(y)} = \frac{y}{\lambda} e^{-\lambda - \ln y} = \frac{1}{\lambda} e^{-\lambda}. $$

(70)

The large firm asks price 1, the small firms use mixed strategy

$$ F(p) = \frac{\ln p + \lambda}{\lambda} $$

(71)

with support $p \in [e^{-\lambda}, 1]$

$$ f(p) = \frac{\partial}{\partial p} \left( \frac{\lambda + \ln p}{\lambda} \right) = \frac{1}{p\lambda} = \frac{1}{p\lambda} $$

(72)

Note that the equilibrium where the large firm asks price unity and the small firms mix using $F(p) = \frac{\ln p + \lambda}{\lambda}$ is unique. Below we sketch the proof. Assume first that both firms have mixed strategies. Denote the small firms’ mixed strategy on $[\bar{a}, \bar{A}]$ by $\bar{F}$ and the large firm’s mixed strategy on $[\bar{b}, \bar{B}]$ by $\bar{G}$. Note first that as long as there are no atoms $\bar{B} = 1$ since otherwise there would be a profitable deviation upwards from $\bar{B}$ for the large firm. Notice that $\bar{A} = 1$ since otherwise there would be a gap between $\bar{A}$ and $\bar{B}$; in this case the large firm could deviate by choosing a mass point at $\bar{B} = 1$ and choosing prices between $\bar{A}$ and unity with probability zero.$^{14}$ A small firm would trade with probability zero if it chose price $\bar{A} = 1$ unless the large firm had a mass point at $\bar{B} = 1$. Next we consider this equilibrium denoting the mass at unity by $\gamma$.

Now a small firm pricing at unity makes $\tilde{\gamma} \int_{1}^{\infty} \lambda e^{-\lambda \theta} d\theta = \tilde{\gamma} e^{-\lambda}$. It is clear that $\tilde{b}$ cannot be smaller than $\tilde{a}$ or the large firm would have a profitable deviation from price $\tilde{b}$ to price $\tilde{a}$. Thus $\tilde{b} \geq \tilde{a}$ and $\tilde{a} = \tilde{\gamma} e^{-\lambda}$.as it is in the support of the small firms. Now it is clear that $\tilde{b} > \tilde{a}$ as otherwise the large

$^{14}$Of course $B$ cannot be less than $A$ since small firms choosing a price above $B$ would never sell.
firm would have a deviation to unity where it makes \( \frac{1}{\lambda} e^{-\lambda} \) (at \( \bar{b} = \bar{a} \) it would get the whole demand but still make only \( \int_1^{\infty} \theta \hat{h}(\theta) d\theta = \frac{\lambda}{\lambda - 1} \)). In the case where \( \bar{b} > \bar{a} \) it must be that the small firms mixed strategy \( \bar{F}(p) = F(p) = \frac{\ln \gamma + \lambda}{\lambda} \) as for \( \bar{F}(y) > F(y) \) the large firm would again have a deviation to unity (as can be seen from eqv (70)). But as \( \bar{b} > \bar{a} \), \( \bar{F}(p) = F(p) \) at \( \bar{b} \) is impossible. Thus there cannot exist a mixed strategy for the large firm.

To scale assume that \( \lambda = \frac{2}{m} \)

The expected price is

\[
\frac{m}{m+1} + \frac{1}{m+1} \int_0^1 \left( 1 - \frac{e^{-\lambda \theta}}{\lambda} \right) \lambda e^{-\lambda \theta} d\theta = \frac{m}{m+1} - \int_0^1 e^{-\frac{2}{m} \theta} d\theta \frac{e^{-\frac{2}{m} \theta} - 1}{m+1} \quad (73)
\]

The expected price actually paid is

\[
\int_0^1 \int_1^{F^{-1}(\theta)} pf(p)dp \hat{h}(\theta)d\theta + \int_1^{\infty} \frac{1}{\theta} \cdot 1 \hat{h}(\theta)d\theta + \int_1^{\infty} \int_a^1 \frac{p}{\theta} f(p)dp \hat{h}(\theta)d\theta
\]

The first term is

\[
\int_0^1 \int_{\frac{1}{\lambda}}^{F^{-1}(\theta)} pf(p)dp \hat{h}(\theta)d\theta = \int_0^1 \int_{\frac{1}{\lambda}}^{F^{-1}(\theta)} \frac{1}{\lambda} dp \hat{h}(\theta)d\theta = \int_0^1 \left( \frac{F^{-1}(\theta)}{\lambda} - \frac{e^{-\lambda}}{\lambda} \right) \lambda e^{-\lambda \theta} d\theta = \int_0^1 \left( \frac{e^{-\lambda \theta} + \lambda \theta}{\lambda} - \frac{e^{-\lambda}}{\lambda} \right) \lambda e^{-\lambda \theta} d\theta
\]

\[
= \frac{1}{\lambda} e^{-\lambda} (\lambda + e^{-\lambda} - 1)
\]

The second term is

\[
\int_0^{\infty} \frac{(\theta - 1)}{\theta} \lambda e^{-\lambda \theta} d\theta = \lambda \int_0^{\infty} \frac{1}{\theta} e^{-\theta \lambda} (\theta - 1) d\theta
\]

\[
= \lambda \text{Ei}(\infty \lambda) - \lambda \text{Ei}(\lambda) + e^{-\lambda}
\]

The third term is

\[
\int_1^{\infty} \int_a^1 \frac{p}{\theta} f(p)dp \hat{h}(\theta)d\theta = \int_1^{\infty} \int_a^1 \frac{1}{\theta \lambda} dp \lambda e^{-\lambda \theta} d\theta = \int_1^{\infty} \left( \frac{1}{\theta \lambda} - \frac{e^{-\lambda}}{\theta \lambda} \right) \lambda e^{-\lambda \theta} d\theta = [\text{Ei}(\theta \lambda) (e^{-\lambda} - 1)]_1^{\infty}
\]

\[
= \text{Ei}(\lambda \infty) (e^{-\lambda} - 1) - \text{Ei}(\lambda) (e^{-\lambda} - 1)
\]

Plotting the expected price paid as a function of \( \lambda = \frac{2}{m} \) we get the following
Equivalently we can plot the expected price paid as a function of $m$.

Expected price (red) and expected actually paid price (green) as functions of $m$

### 5.3.2 Market structure (B); Large and small firms are at two separate locations

We next consider price and expected utilities when the small firms are located together in one location but separately from the large firm. Kultti (2008) considered a model with small sellers and showed that they prefer to locate close together to locating separately. We proceed to find equilibrium prices and expected utilities in this case.
Assume that fraction \( z \) of buyers go to small firms and fraction \( 1 - z \) go to the large firm. The small firms will in this case price using a mixed strategy with support \([ a, 1] \), and the large firm quotes price \( P_B \). As before, it is clear that \( F(1) = 1 \).

A firm quoting price \( A \) can then expect to get \( A \cdot (1 - \hat{H}(\frac{1}{z})) \). As the expected profit must be the same over the support we easily find that

\[
a = 1 - \hat{H}(\frac{1}{z}) = 1 - (1 - e^{-\frac{\lambda}{z}}) = e^{-\frac{\lambda}{z}}
\]

A small firm asking price \( \rho \in \left[ e^{-\frac{\lambda}{z}}, 1 \right] \) can thus expect

\[
\rho \left( 1 - \hat{H}\left( \frac{F(\rho)}{z} \right) \right) = e^{-\frac{\lambda}{z}}
\]

From this we solve

\[
1 - e^{-\frac{\lambda}{\rho}} = 1 - e^{-\frac{\lambda F(\rho)}{z}}
\]

\[
\Leftrightarrow -\frac{\lambda F(\rho)}{z} = -\frac{\lambda}{z} - \log(\rho)
\]

Thus

\[
F(\rho) = \frac{z}{\lambda} \left( \frac{\lambda}{z} + \log(\rho) \right) = 1 + \frac{z}{\lambda} \log(\rho)
\]

A large firm can expect the following profit

\[
P_B \int_{0}^{\infty} (1 - z) \theta \lambda e^{-\lambda \theta} d\theta = \Pi(P_B)
\]

To continue we first look at the expected utilities of buyers that go to the small firms. The small firms price using mixed strategies. To make calculations easier we follow Kultti (2008) and assume that all sellers charge the expected price. We denote the expected utility of buyers visiting the small firms by \( u(z, F) \). A buyer going to a large seller knows that he can expect to get \( 1 - P_B \).

In equilibrium the following must hold:

\[
u(z, F) = 1 - P_B
\]

and

\[
\Pi'(P_B) = 0.
\]

Next we look at the small firms’ pricing decision. We pretend that every small firm asks the same price \( r \). We get

\[
r \int_{0}^{\frac{r}{1}} \frac{\theta z}{1} \lambda e^{-\lambda \theta} d\theta + \left( e^{-\frac{\lambda}{z}} \right) r = e^{-\frac{\lambda}{z}}.
\]
From this we solve the small seller’s expected price
\[ r = \frac{e^{-\frac{z}{\lambda}}}{\left(e^{-\frac{z}{\lambda}} + \int_{0}^{\frac{z}{\lambda}} \frac{\theta z}{1} \lambda e^{-\lambda \theta} d\theta\right)} \] (80)

Then the buyers’ expected utility conditional on being alive is
\[ (1 - r) \left[ \int_{0}^{\frac{z}{\lambda}} g(\theta) d\theta + \int_{\frac{z}{\lambda}}^{\infty} \frac{1}{z\theta} g(\theta) d\theta \right], \] (81)

where \( g(\theta) = \frac{\theta h(\theta)}{E(\theta)} \).

The buyer must be indifferent between visiting the large firm and going to the small firms.
\[ (1 - r) \left[ \int_{0}^{\frac{z}{\lambda}} g(\theta) d\theta + \int_{\frac{z}{\lambda}}^{\infty} \frac{1}{z\theta} g(\theta) d\theta \right] = 1 - P_B \] (82)

it is clear that \( P_B \leq 1 \).

Using (80) we write \( 1 - r \) as
\[ 1 - r = \frac{\int_{0}^{\frac{z}{\lambda}} \frac{\theta z}{1} \lambda e^{-\lambda \theta} d\theta}{e^{-\frac{z}{\lambda}} + \int_{0}^{\frac{z}{\lambda}} \frac{\theta z}{1} \lambda e^{-\lambda \theta} d\theta} = \frac{\frac{z}{\lambda} - \frac{z}{\lambda} e^{-\frac{z}{\lambda} \lambda} \left(\frac{1}{2} \lambda + 1\right)}{e^{-\frac{z}{\lambda}} + \frac{z}{\lambda} - \frac{z}{\lambda} e^{-\frac{z}{\lambda} \lambda} \left(\frac{1}{2} \lambda + 1\right)} = -\frac{1}{z - ze^{-\frac{z}{\lambda} \lambda}} \left(ze^{-\frac{z}{\lambda} \lambda} - z + \lambda e^{-\frac{z}{\lambda} \lambda}\right) \] (83)

Now we look at the rest of the LHS of (82). Remembering that \( g(\theta) = \frac{\theta h(\theta)}{E(\theta)} \) we get
\[ \int_{0}^{\frac{z}{\lambda}} g(\theta) d\theta + \int_{\frac{z}{\lambda}}^{\infty} \frac{1}{z\theta} g(\theta) d\theta \]
\[ = \int_{0}^{\frac{z}{\lambda}} \frac{\theta \tilde{h}(\theta)}{E(\theta)} d\theta + \int_{\frac{z}{\lambda}}^{\infty} \frac{1}{z\theta} \frac{\theta \tilde{h}(\theta)}{E(\theta)} d\theta \] (85)
\[ = \lambda \left( \int_{0}^{\frac{z}{\lambda}} \theta \lambda e^{-\lambda \theta} d\theta + \frac{1}{z} \int_{\frac{z}{\lambda}}^{\infty} \lambda e^{-\lambda \theta} d\theta \right) \] (86)
\[ = -e^{-\frac{z}{\lambda} \lambda} \left(\frac{1}{2} \lambda + 1\right) + 1 + \lambda \frac{1}{z} e^{-\frac{z}{\lambda} \lambda} \] (87)
\[ = 1 - e^{-\frac{z}{\lambda} \lambda} \] (88)

We can now rewrite the buyers’ indifference condition (77) as
\[ 1 - P_B = -\frac{1}{z - ze^{-\frac{z}{\lambda} \lambda}} \left(ze^{-\frac{z}{\lambda} \lambda} - z + \lambda e^{-\frac{z}{\lambda} \lambda}\right), \left(1 - e^{-\frac{z}{\lambda} \lambda}\right) \] (89)
\[ = -\frac{1}{z} \left(ze^{-\frac{z}{\lambda} \lambda} - z + \lambda e^{-\frac{z}{\lambda} \lambda}\right) \] (90)
Thus
\[ P_B = 1 + \frac{1}{z} \left( z e^{-\frac{1}{2} \lambda} - z + \lambda e^{-\frac{1}{2} \lambda} \right). \] (91)

The large firm maximizes
\[ \max_{P_B} \int_0^\infty (1 - z) \theta \lambda e^{-\lambda \theta} d\theta \cdot P_B \]

The first order conditions are:
\[ \int_0^\infty (1 - z) \theta \lambda e^{-\lambda \theta} d\theta - \int_0^\infty \frac{dz}{dP_B} \theta \lambda e^{-\lambda \theta} d\theta \cdot P_B = 0 \] (92)

We get \( \frac{dz}{dP_B} \) by totally differentiating the buyers indifference condition. Rewriting the buyers indifference condition (89) we get
\[ 1 - P_B + \frac{1}{z} \left( z e^{-\frac{1}{2} \lambda} - z + \lambda e^{-\frac{1}{2} \lambda} \right) = 0 \] (93)

Totally differentiating the expression we get
\[ \frac{\partial}{\partial z} \left( \frac{1}{z} \left( z e^{-\frac{1}{2} \lambda} - z + \lambda e^{-\frac{1}{2} \lambda} \right) \right) = \frac{1}{z^2} \lambda^2 e^{-\frac{1}{2} \lambda}\]
\[ dP_B \{ -1 \} + dz \left\{ \frac{1}{z^3} \lambda^2 e^{-\frac{1}{2} \lambda} \right\} = 0 \]

From which we solve
\[ \frac{dz}{dP_B} = \frac{1}{\frac{1}{z^3} \lambda^2 e^{-\frac{1}{2} \lambda}} = \frac{z^3}{\lambda^2 e^{-\frac{1}{2} \lambda}} \] (94)

Therefore the first order condition becomes
\[ \int_0^\infty (1 - z) \theta \lambda e^{-\lambda \theta} d\theta - \int_0^\infty z^3 \lambda e^{-\frac{1}{2} \lambda} \theta \lambda e^{-\lambda \theta} d\theta \cdot P_B = 0 \] (95)

Solving for \( P_B \) we get
\[ P_B = \frac{(1 - z) \left( \lambda^2 e^{-\frac{1}{2} \lambda} \right)}{z^3} \] (96)

We solve for \( z \) by equating (91) and (96). The only real root between zero and one is
\[ z = \sqrt[3]{\frac{1}{2} \lambda^2 + \frac{7}{54} \lambda^3 + \sqrt{\frac{1}{36} \lambda^6 + \frac{7}{54} \lambda^5 + \frac{1}{4} \lambda^4 - \frac{2}{9} \sqrt{\frac{1}{2} \lambda^2 + \frac{z^2}{54} \lambda^3 + \sqrt{\frac{1}{36} \lambda^6 + \frac{7}{54} \lambda^5 + \frac{1}{4} \lambda^4}}} - \frac{1}{3} \lambda} \]
The expected price paid in the market is

\[(1 - z) \cdot P_B + z \cdot \left( \int_0^{\frac{1}{z}} \int_a^{F^{-1}(z\theta)} qf(q)dq \, d\theta + \int_{\frac{1}{z}}^{\infty} \int_a^{\frac{1}{z}} qf(q) \frac{1}{\theta} dq \, d\theta \right) \quad (97)\]

where \(F^{-1}(z\theta) = \frac{q}{z^\lambda(z\theta - 1)}\) and \(a = e^{\frac{1}{z}}\) and \(z\) is as defined in (\).

The expected price paid simplifies to

\[(1 - z) \cdot \left( 1 + \frac{1}{z} \left( ze^{-\frac{1}{z}\lambda} - z + \lambda e^{-\frac{1}{z}\lambda} \right) \right) + z \cdot \left( \frac{1}{\lambda} e^{-\frac{1}{z}\lambda} \left( \lambda - z + ze^{-\frac{1}{z}\lambda} \right) + z \left( e^{-\frac{1}{z}\lambda} - 1 \right) \left( \text{Ei}(\infty) - \text{Ei} \left( \frac{1}{z} \lambda \right) \right) \right)\]
When $\lambda = \frac{1}{2}$ the expected price paid is 0.45455 and when $\lambda = 0.05$ the expected price paid is 0.83923.

5.3.3 Market structure (C); all firms in different locations

Now assume that all firms are in different locations. Assume that proportion $z$ of buyers visit the small firms and proportion $1-z$ visit the large seller. In equilibrium the buyers are indifferent between visiting the large firm or mixing over the small firms.

In equilibrium the price of the small sellers is\footnote{It is well known that without a large firm the small firms have a unique symmetric price in this kind of a directed search model. The inclusion of a large firm changes this price only through $z$. As $z$ is unique given expected demand so is $q$.}

$$q = \frac{\int_0^\infty \left(1 - e^{-z \theta} - z \theta e^{-z \theta}\right) \lambda e^{-\lambda \theta} d\theta}{\int_0^\infty \left(1 - e^{-z \theta}\right) \lambda e^{-\lambda \theta} d\theta} = \frac{z}{z + \lambda} \quad (98)$$

By going to the small firms a buyer thus expects to get

$$(1 - q) \int_0^\infty \frac{(1 - e^{-z \theta})}{z \theta} g(\theta) d\theta = \frac{1}{E(\theta)} \int_0^\infty \theta e^{-z \theta} h(\theta) d\theta = \lambda^2 \int_0^\infty \theta e^{-(z + \lambda) \theta} d\theta = \frac{\lambda^2}{(z + \lambda)^2} \quad (99)$$

where the first equality follows from $g(\theta) = \frac{\partial h(\theta)}{\partial \theta} E(\theta)$.

In equilibrium a buyer has to be indifferent between visiting a small seller or the large seller. Thus

$$1 - P_C = \frac{\lambda^2}{(z + \lambda)^2}$$

Thus the large firms price is

$$P_C = 1 - \frac{\lambda^2}{(z + \lambda)^2}$$

The large firm’s price is found by maximizing the large firm’s expected profit with respect to $z$.

The large firm maximizes

$$\max_z \left( P_C \cdot (1 - z) \frac{1}{\lambda} \right) = \max_z \left( 1 - \frac{\lambda^2}{(z + \lambda)^2} \right) \frac{(1 - z) \lambda}{\lambda} \quad (101)$$
Solving for $z$ we get

$$z = \sqrt[3]{\frac{\lambda^2 + \lambda^3}{27}} + \sqrt[3]{\frac{28 \lambda^6 + 2 \lambda^5 + \lambda^4}{27}} - \lambda - \frac{1}{3} \sqrt[3]{\frac{\lambda^2}{\lambda^2 + \lambda^3 + \sqrt[3]{\frac{28 \lambda^6 + 2 \lambda^5 + \lambda^4}}}}$$

By charging $P$ the large firm expects to get

$$\frac{1}{\lambda} \left( \frac{\lambda^2}{(z + \lambda)^2} - 1 \right) (z - 1)$$

By deviating to $q$ the large firm would get

$$\frac{z}{z + \lambda} \left( \frac{\lambda^2}{(z + \lambda)^2} - 1 \right) (z - 1)$$

Thus a deviation is not profitable as

$$\left( \frac{\lambda^2}{(z + \lambda)^2} - 1 \right) (z - 1) - \frac{z}{z + \lambda} > 0.$$  

The expected price in this market is thus

$$\frac{z}{m+1} + \left( \frac{\lambda^2}{(z + \lambda)^2} - 1 \right) (z - 1) \frac{m}{m+1}$$

$$\frac{z}{m+1} + \left( \frac{\left( \frac{\lambda}{z} \right)^2}{(z + \frac{\lambda}{z})^2} - 1 \right) (z - 1) \frac{m}{m+1}$$

$$= m \frac{z}{(mz + 2)^2 (m+1)} (-m^2 z^2 + m^2 z - 3mz + 4m + 2)$$

When $m=3$ this is 0.36301.

The expected price actually paid takes into account that by going to a small firm a buyer gets the good with probability

$$\frac{\lambda^2}{z} \int_0^\infty (1 - e^{-z\theta}) e^{-\lambda\theta} d\theta = \frac{\lambda}{z + \lambda}.$$

Thus the expected price actually paid is
\[ \frac{\lambda z}{(z + \lambda)^2} z + (1 - z) \left( 1 - \frac{\lambda^2}{(z + \lambda)^2} \right). \]