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Dynamo action and magnetic buoyancy in convection simulations with vertical shear

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ABSTRACT

Context. A hypothesis for sunspot formation is the buoyant emergence of magnetic flux tubes created by the strong radial shear at the tachocline. In this scenario, the magnetic field has to exceed a threshold value before it becomes buoyant and emerges through the whole convection zone.

Aims. We follow the evolution of a random seed magnetic field with the aim of study under what conditions it is possible to excite the dynamo instability and whether the dynamo generated magnetic field becomes buoyantly unstable and emerges to the surface as expected in the flux-tube context.

Methods. We perform numerical simulations of compressible turbulent convection that include a vertical shear layer. Like the solar tachocline, the shear is located at the interface between convective and stable layers.

Results. We find that shear and convection are able to amplify the initial magnetic field and form large-scale elongated magnetic structures. The magnetic field strength depends on several parameters such as the shear amplitude, the thickness and location of the shear layer, and the magnetic Reynolds number (Rm). Models with deeper and thicker tachoclines allow longer storage and are more favorable for generating a mean magnetic field. Models with higher Rm grow faster but saturate at slightly lower levels. Whenever the toroidal magnetic field reaches amplitudes greater a threshold value which is close to the equipartition value, it becomes buoyant and rises into the convection zone where it expands and forms mushroom shape structures. Some events of emergence, i.e. those with the largest amplitudes of the initial field, are able to reach the very uppermost layers of the domain. These episodes are able to modify the convective pattern forming either broader convection cells or convective eddies elongated in the direction of the field.

Conclusions. The results indicate that buoyancy is a common phenomena when the magnetic field is amplified through dynamo action in a narrow layer. It is, however, very hard for the field to rise up to the surface without losing its initial coherence.

Key words. magnetohydrodynamics (MHD) – convection – turbulence – Sun: magnetic fields – stars: magnetic fields

1. Introduction

Sunspots appear at the solar surface following a 11-year cycle. They reveal the presence of strong magnetic fields in the solar interior, and suggest the existence of a dynamo process governing its evolution. However, the process by which sunspots are formed is still unknown. At the solar surface, sunspots are observed as bipolar patches of radial magnetic field. This intuitively suggest that they are formed by the emergence of horizontal concentrations of magnetic field lines, often called magnetic flux tubes. Since Parker’s original proposal of magnetic buoyancy (Parker 1955), the model has evolved through the thin flux tube approximation (Spruit 1981), to numerical simulations of the emergence of 3D magnetic flux tubes (Fan 2008). The buoyant rise of flux tubes from the tachocline up to the surface is currently the most widely accepted mechanism of sunspot formation. During the last three decades much work has been done in order to understand the buoyancy phenomena and to reconcile the results of flux tube models with phenomenological sunspots rules such as the Joy’s law or the topological difference between the two spots in a pair. In spite of the fact that these basic observations have been qualitatively reproduced by the flux tube models, a set of complications have questioned the feasibility of this scenario. These may be summarized as follows:

1. Magnetic flux tubes at the base of the convection zone should have a strength of around $10^5$ G in order to become unstable and then cross the entire convection zone up to the surface (Caligari et al. 1995). This has been considered a problem since the magnetic energy density corresponding to such field is one to two orders of magnitude larger than the kinetic energy of the turbulent motions (the equipartition energy). Note, however, that in the presence of shear this is not any longer an upper limit for the amplitude of the magnetic field (see e.g. Käpylä & Brandenburg 2009).

2. Sunspots are coherent magnetic structures which means that the flux tubes should preserve their integrity while they rise through the entire convection zone. However, this region is highly turbulent and stratified, spanning more than 20 pressure scale heights, so in addition to the $10^5$ G strength, the tubes must have a certain amount of twist in order to resist the turbulent diffusion (see e.g. Emonet & Moreno-Insertis 1998; Fan et al. 2003). The current results do not conclusively yield the amount of twist required for the tubes to rise coherently up to the surface. On one hand, 2D simulations require a larger twist in order to prevent the fragmentation of the flux tube in two tubes due to the...
vorticity generated by the buoyant rise (Schüssler 1979, Moreno-Insertis & Emonet 1996, Lonecope et al. 1996, Emonet & Moreno-Insertis 1998, Fan et al. 1998). On the other hand, rising 3D flux tubes require less twist thanks to the tension forces due to the longitudinal curvature of the tube (Fan 2001). Three dimensional simulations in spherical coordinates require fine tuning of the initial twist for the tubes to emerge to the surface with the observed tilt (Fan 2008).

3. MHD simulations of convection in Cartesian coordinates have been able to produce large-scale magnetic fields through $\alpha^2$ (Käpylä et al. 2009b) and $\alpha \Omega$ (Käpylä et al. 2008) dynamo action. However, the magnetic fields generated on these simulations are more homogeneously distributed in space rather than in the form of isolated magnetic structures. To the date, however, no direct simulations have been able to spontaneously form sunspot-like magnetic structures.

4. Furthermore, instead of showing signs of buoyant rise and emergence of the magnetic field, the 3D dynamo simulations above have shown that the magnetic field is pumped down by convective downflows and tends to remain in the stable layer (see also Tobias et al. 1998, 2001; Ossendrijver et al. 2002). Fan et al. (2003) studied the evolution of an isolated flux tube in a turbulent convection zone. They found that coherent rise is possible as far as the magnetic buoyant force overcomes the hydrodynamic force from convection, i.e., obeying the condition $B_0 > (H_P/a)^{1/2} B_{eq}$, where $B_0$ is the initial magnetic field strength, $H_P$ is the pressure scale height, and $a$ is the radius of the tube. In their case where $(H_P/a)^{1/2} \approx 3$, magnetic flux tubes with $B_0 \geq 3B_{eq}$ are able to rise coherently without being affected by convection. Similar results were found in a similar setup in spherical geometry (Jouve & Braun 2009). However, this does not address the issue with the pumping since, firstly, a strong magnetic flux tube is imposed on the convective layer, and secondly, magnetic pumping is an effect related with the gradient of turbulence intensity (Kichatinov & Rüdiger 1992), which is not present in these simulations.

In addition to the thin flux tube approximation and simulations of rising flux tubes on stratified atmospheres, other recent attempts have been made in order to simulate the formation of a magnetic layer through the interaction of an imposed shear in convectively stable (Vasil & Brummell 2008) and unstable (Silvers et al. 2009a) atmospheres, with an imposed vertical magnetic field. They found that, unlike in the cases of imposed toroidal magnetic layers, buoyancy instability is harder to excite when the magnetic field is generated by the shear. More recently, Silvers et al. (2009b), have found, with a similar setup, that the buoyancy may be favored by the presence of double-diffusive instabilities (these in turn depend on the ratio between the thermal and magnetic diffusivities, $\chi/\eta$, often known as the inverse Roberts number). Later independent study of Chatterjee et al. (2010) has confirmed this result. However, in most of the current models, the presence of stratified turbulence, self-consistent generation of the magnetic field, or both, are omitted.

In view of the above mentioned issues, other mechanisms have been proposed in order to explain sunspots. These are related to instabilities due to the presence of a diffuse large-scale magnetic field in a highly stratified turbulent medium (Kleeorin & Rogachevskii 1994, Rogachevskii & Kleeorin 2007, Brandenburg et al. 2010). Mean-field models using this mechanism are able to produce strong flux concentrations but this has not yet been achieved in direct numerical simulations.

Here we present numerical simulations of compressible turbulent convection with an imposed radial shear flow located in a sub-adiabatic layer beneath the convective region. For numerical reasons, explained below, we do not include rotation in our setup, i.e. the turbulence is not helical, and an $\alpha \Omega$ dynamo is not expected. Nevertheless, recent studies have shown that mean-field dynamo action is possible due to non-helical turbulence and shear (Brandenburg 2005a, Yousef et al. 2008b, Brandenburg et al. 2008). The nature of this dynamo is not yet entirely clear and may be attributed to the so called shear-current effect (Rogachevskii & Kleeorin 2003, 2004) or to the incoherent, stochastic, $\alpha$-effect (Vishniac & Brandenburg 1997). According to (Brandenburg et al. 2008) the latter explanation is consistent with the turbulent transport coefficients.

Based on these results, we expect the development of a mean field magnetic field, i.e. dynamo action, with a system that mimics, as far as possible, the conditions of the solar interior, specifically in the lower part of the convection zone and the tachocline. As the shear is localized in a very narrow layer, we also expect the formation of a magnetic layer and the subsequent buoyancy of the magnetic fields.

A similar setup was studied recently by Tobias et al. (2008). They reported the appearance of elongated stripes of magnetic field in the direction of the shear. However, since they considered the Boussinesq approximation in their simulations, no buoyancy was observed. They also do not report the presence of a large scale dynamo.

Two important features distinguish the simulations presented here from previous studies in the context of flux tube formation and emergence. Firstly, we consider a highly stratified domain with $\approx 8$ scale heights in pressure and $\approx 6$ scale heights in density. Secondly, we do not impose a background radial magnetic field but allow the self-consistent development of the field from an initial random seed.

Even though this is a complicated setup where it is difficult to analyze the different processes occurring independently, we believe that these simulations may give us some light on the current paradigm of sunspot formation. There are several important issues that we want to address with the following simulations. (1) What are requirements for dynamo action in the present setup? This includes the dependence of the dynamo excitation on several parameters such as the amplitude of the shear, thickness and location of the shear layer, and the aspect ratio of the box. (2) What is the resulting configuration of the magnetic field? In particular, whether the field is predominantly in small or large scales, and whether it is organized in the form of a magnetic layer or isolated magnetic flux tubes. (3) Is the buoyancy instability (Parker 1955) operating on these magnetic structures? If yes, (4) how it depends on the parameters listed above? (5) Is it possible to have magnetic structures strong enough to emerge from the shear layer to the surface without being affected by the turbulent convective motions? (6) Finally, it is important to study how these strong structures back-react on the fluid motions, including the shear profile as well as the convective pattern.

Another important issue that may be addressed in this context is the mechanism that triggers the dynamo instability. With the recent developments on the test-field method, it is possible to compute the dynamo transport coefficients and have a better understanding of the underlying mechanism.
2. The model

Our model setup is similar to that used by e.g. Brandenburg et al. (1996) and Käpylä et al. (2008). A rectangular portion of a star is modeled by a box whose dimensions are \((L_x, L_y, L_z) = (4, 4, 2)d\), where \(d\) is the depth of the convectively unstable layer, which is also used as the unit of length. The box is divided into three layers, an upper cooling layer, a convectively unstable layer, and a stable overshoot layer (see below). The following set of equations for compressible magnetohydrodynamics is being solved:

\[
\frac{\partial A}{\partial t} = U \times B - \mu_0 \eta J, \tag{1}
\]

\[
\frac{D \ln \rho}{Dt} = -\nabla \cdot U, \tag{2}
\]

\[
\frac{D U}{Dt} = -\frac{1}{\rho} \nabla p + g + \frac{1}{\rho} J \times B + \frac{1}{\rho} \nabla \cdot 2\nu \rho S - \frac{U - \overline{U}^{(0)}}{\tau_f}, \tag{3}
\]

\[
\frac{D S}{Dt} = \frac{1}{\rho} \nabla \cdot K \nabla T + 2\nu S^2 + \frac{\mu_0 \eta}{\rho} J^2 - \Gamma, \tag{4}
\]

where \(D/Dt = \partial/\partial t + U \cdot \nabla\) is the total time derivative, \(A\) is the magnetic vector potential, \(B = \nabla \times A\) is the magnetic field, and \(J = \nabla \times B/\mu_0\) is the current density. The fluid obeys an ideal gas law \(p = \rho c(T - 1)\), where \(p\) and \(c\) are the pressure and internal energy, respectively, and \(T\) is the temperature, \(\rho\) is the specific entropy, \(K\) is the heat conductivity, \(\eta\) and \(\nu\) are the magnetic diffusivity and kinematic viscosity, respectively. The fluid is assumed to be isothermal cooling layer at the top. More details of the shear profiles are given in Sect. (2.3).

The last term of Eq. (1) relaxes the horizontally averaged mean velocity \(\overline{U}\) towards a target profile \(\overline{U}^{(0)}\) where \(\tau_f = 2\sqrt{T(dg)}\) is a relaxation time scale.

In a non-shearing case this value corresponds to \(1/2\tau_{turn}\), where \(\tau_{turn} = (\overline{u_{rms}} x k_x)^{-1}\) gives an estimate of the convective turnover time of the turnover target. The target profile is chosen so that it mimics the radial shear in the solar tachocline. More details of the shear profiles are given in Sect. (2.3).

The last term of Eq. (4) describes cooling at the top of the domain according to

\[
\Gamma_{cool} = \Gamma_0 f(z) \left(\frac{T - T_k}{T_k}\right), \tag{6}
\]

where \(f(z)\) is a profile function equal to unity in \(z > z_3\) and smoothly connecting to zero below, and \(\Gamma_0\) is a cooling luminosity chosen so that the sound speed in the uppermost layer relaxes toward \(T_k = T(z = z_k)\).

The positions of the bottom of the box, bottom and top of the convectively unstable layer, and the top of the box, respectively, are given by \((z_1, z_2, z_3, z_4) = (-0.8, 0, 1, 1.2)d\). Initially the stratification is piecewise polytropic with polytropic indices \((m_1, m_2, m_3) = (6, 1, 1)\), which leads to a convectively unstable layer above a stable layer at the bottom of the domain and an isothermal cooling layer at the top.

Fig. 1. Vertical profiles of mean density, pressure, temperature, specific entropy and azimuthal velocity \(\overline{U}_y\) in the initial state (solid lines) and the in the thermally saturated state (dotted). The dotted vertical lines at \(z = 0\) and \(z = d\) denote the bottom and top of the convectively unstable layer, respectively. The last two panels show the turbulent rms-velocity and the energy of the toroidal mean magnetic field in the saturated phase.

2.1. Nondimensional units and parameters

Dimensionless quantities are obtained by setting

\[
d = g = \rho_0 = c_T = \mu_0 = 1, \tag{7}
\]
where \( \rho_0 \) is the initial density at \( z_2 \). The units of length, time, velocity, density, entropy, and magnetic field are
\[
[x] = d, \quad [t] = \sqrt{g/d}, \quad [U] = \sqrt{g}, \quad [\rho] = \rho_0, \\
[s] = \text{cp}, \quad [B] = \sqrt{gd} \rho_0 \mu_0. 
\]
We define the fluid and magnetic Prandtl numbers and the Rayleigh number as
\[
Pr = \frac{\nu}{\chi_0}, \quad Pm = \frac{\nu}{\eta}, \quad Ra = \frac{gd \ell_0}{\nu \chi_0} \left( -\frac{1}{\text{cp}} \frac{ds}{dz} \right)_0, 
\]
where \( \chi_0 = K/(\rho_0 \text{cp}) \) is the thermal diffusivity, and \( \rho_m \) is the density in the middle of the unstable layer. For the magnetic diffusivity we consider a \( z \)-dependent profile which gives an order of magnitude smaller value in the radiative layer than in the convection zone through a smooth transition. Thus the magnetic Prandtl number in the stable layer is \( Pm = 15 \) and in the convection zone \( Pm = 1.5 \).

The entropy gradient, measured in the middle of the convectively unstable layer in the initial non-conveoting hydrostatic state, is given by
\[
\left( -\frac{1}{\text{cp}} \frac{ds}{dz} \right)_0 = \frac{\nabla - \nabla_{\text{ad}}}{H_p}, 
\]
where \( \nabla - \nabla_{\text{ad}} \) is the superadiabatic temperature gradient with \( \nabla_{\text{ad}} = 1 - 1/\gamma, \nabla = (\partial \ln T/\partial \ln p)_{\text{rad}}, \) where \( z_m = \frac{1}{\gamma} (z_3 + z_2). \)

The amount of stratification is determined by the parameter \( \xi_0 = (\gamma - 1) c v T_3^2 / (gd) \), which is the pressure scale height at the top of the domain normalized by the depth of the unstable layer. We use in all cases \( \xi_0 = 0.12 \), which results in a density contrast of about 120. We define the fluid and magnetic Reynolds numbers via
\[
Re = \frac{u_{rms}}{\nu k_l}, \quad Rm = \frac{u_{rms}}{\eta k_l} = \text{Pm} \Re, 
\]
where \( u_{rms} \) is the rms value of the velocity fluctuations and \( k_l = 2\pi / d \) is assumed as a reasonable estimate for the wavenumber of the energy-carrying eddies. Our definitions of the Reynolds numbers are smaller than the usually adopted ones by a factor of \( 2\pi \). The amount of shear is quantified by
\[
Sh = \frac{U_0}{d_s} \frac{u_{rms} k_l}{u_{rms} k_l}, 
\]
where \( U_0 \) is the amplitude and \( d_s \) the width of the imposed shear profile (see below). The equipartition magnetic field is defined by
\[
B_{eq} \equiv \left( \langle \mu_0 \rho U^2 \rangle \right)_{z=ref}^{1/2}, 
\]
where the angular brackets denote horizontal average. For a better comparison between the magnetic and kinetic energies, we evaluate \( B_{eq} \) at the center of the shear layer, \( z = z_{ref} \).

The simulations were performed with the PENCIL CODE\(^1\) which uses sixth-order explicit finite differences in space and third order accurate time stepping method.

2.2. Boundary conditions
In the horizontal \( x \) and \( y \) directions we use periodic boundary conditions and at the vertical (\( z \)) boundaries we use stress-free boundary conditions for the velocity,
\[
U_{x,z} = U_{y,z} = U_z = 0. 
\]

For the magnetic field vertical field condition is used on the upper boundary whereas perfect conductor conditions are used at the lower boundary, i.e.,
\[
B_x = B_y = 0, \quad (z = z_4) 
\]
\[
B_{x,z} = B_{y,z} = B_z = 0, \quad (z = z_1) 
\]
respectively. The upper boundary thus allows magnetic helicity flux whereas at the lower boundary does not. This is likely to be representative of the situation in a real star where magnetic helicity can escape via the surface but does not penetrate into the core.

2.3. Shear at the base of the convection zone
In order to mimic the tachocline at the base of the solar convection zone we introduce a shear profile
\[
\bar{U}^{(0)} = \frac{1}{2} U_0 \tanh \left( \frac{z - z_{ref}}{d_s} \right) \hat{e}_y, 
\]
where \( z_{ref} \) is the reference position of the shear layer. Given the uncertainties of the radial position and width of the tachocline we perform parameter studies where \( z_{ref} \) and \( d_s \) are varied. Furthermore, it is of general interest to study how dynamo excitation is depends on varying \( U_0 \) and the ratio of \( U_0/d_s \).

3. Results

With the purpose of addressing the questions raised in the introduction we perform a series of simulations with the model described above where some properties of the shear layer are varied. We first study the hydrodynamic properties of the system and the conditions for dynamo excitation. The results of this parameter study are summarized in Table 1. Then we study the topological and buoyant properties of the magnetic fields generated in some characteristic runs and compare them with models with different aspect ratio and higher resolution (see Table 2). We finalize this section with the study of the magnetic feedback on the plasma motion and the computation of the turbulent coefficients that govern the evolution of large-scale magnetic fields in our simulations.

3.1. Hydrodynamic instabilities
In our simulation setup, two kinds of hydrodynamical instabilities may develop, namely the Kelvin-Helmholtz (KH) instability due to the imposed shear and stratification, and the convective instability due to the superadiabatic stratification in the middle layer. From hydrodynamical runs, we find that the convective instability develops early, at \( t_{rms} k_l \approx 40 \), whereas the KH-instability develops at \( t_{rms} k_l \approx 100 \) in runs with the strongest shear. After a few hundred time units the velocity reaches a statistically steady state (constant rms-velocity), and the toroidal velocity achieves the desired shear profile. A thermally relaxed state (constant thermal energy), however, is reached only after a few thousand time units. This time depends on the radiative conductivity (\( K \)) but also on the imposed shear, which produces viscous heating that modifies the thermal stratification of the system as it may be seen in the top panels of Fig. 1. The final velocity profile which includes shear, convection, and the KH instability, does not allow the possibility of including rotation in the model. If it is done, the system develops mean field motions in the horizontal direction which are undesirable in the present study.

\(^1\) http://pencil-code.googlecode.com/
shearing case. With the runs in Set S we find that the critical 
4 large-scale dynamo due to non-helical turbulence and shea 
6 ment. We have considered 
8 heuristics for the growth rate of the small-scale dynamo. 
10 potential magnetic field is evaluated at $z = z_{\text{ref}}$ (see Eq. 15) and is given in units of $\sqrt{d\rho d\alpha_{eq}}$. Finally $B_y/B_T = \sqrt{(B_y^2 + B_T^2)}$ corresponds to fraction of mean magnetic field compared with the total field. The numbers are for the saturated state of the dynamo. In all these simulations the resolution is $128^3$ grid points, $Pr = 20$ and $Ra \approx 8.3 \times 10^6$. 

**Table 2.** Summary of runs with different aspect ratio (upper two rows) and higher resolution (lower rows).

<table>
<thead>
<tr>
<th>Run</th>
<th>$(L_x, L_y, L_z)$</th>
<th>$U_0/(dg)^{1/2}$</th>
<th>$d_z$</th>
<th>$z_{\text{ref}}$</th>
<th>Pr</th>
<th>Sh</th>
<th>Re</th>
<th>$M_a$</th>
<th>$M_a$</th>
<th>$\lambda(10^{-2})$</th>
<th>$B_{eq}$</th>
<th>$B_{rms}$</th>
<th>$B_y$</th>
<th>$B_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR01</td>
<td>(8, 4, 2) $d$</td>
<td>0.15</td>
<td>0.11</td>
<td>$-0.05$</td>
<td>20</td>
<td>4.4</td>
<td>15.8</td>
<td>26.3</td>
<td>0.050</td>
<td>0.050</td>
<td>0.89</td>
<td>0.25</td>
<td>6.08</td>
<td>0.40</td>
</tr>
<tr>
<td>AR02</td>
<td>(4, 8, 2) $d$</td>
<td>0.15</td>
<td>0.11</td>
<td>$-0.05$</td>
<td>20</td>
<td>3.8</td>
<td>18.4</td>
<td>30.6</td>
<td>0.058</td>
<td>0.058</td>
<td>0.30</td>
<td>0.32</td>
<td>4.73</td>
<td>0.29</td>
</tr>
<tr>
<td>D03b</td>
<td>(4, 4, 2) $d$</td>
<td>0.15</td>
<td>0.05</td>
<td>$-0.10$</td>
<td>12</td>
<td>7.9</td>
<td>31.8</td>
<td>47.7</td>
<td>0.062</td>
<td>0.062</td>
<td>2.57</td>
<td>0.43</td>
<td>3.16</td>
<td>0.31</td>
</tr>
<tr>
<td>T04a</td>
<td>(4, 4, 2) $d$</td>
<td>0.15</td>
<td>0.1</td>
<td>$-0.05$</td>
<td>12</td>
<td>5.1</td>
<td>25.0</td>
<td>37.6</td>
<td>0.047</td>
<td>0.047</td>
<td>2.23</td>
<td>0.26</td>
<td>2.16</td>
<td>0.06</td>
</tr>
<tr>
<td>T04b</td>
<td>(4, 4, 2) $d$</td>
<td>0.15</td>
<td>0.1</td>
<td>$-0.05$</td>
<td>12</td>
<td>4.9</td>
<td>25.6</td>
<td>38.5</td>
<td>0.052</td>
<td>0.048</td>
<td>2.19</td>
<td>0.27</td>
<td>4.30</td>
<td>0.37</td>
</tr>
<tr>
<td>T04c</td>
<td>(4, 4, 2) $d$</td>
<td>0.15</td>
<td>0.1</td>
<td>$-0.05$</td>
<td>3</td>
<td>4.4</td>
<td>26.6</td>
<td>43.0</td>
<td>0.056</td>
<td>0.054</td>
<td>1.21</td>
<td>0.30</td>
<td>4.13</td>
<td>0.36</td>
</tr>
</tbody>
</table>

**Notes.** Most of the quantities here are defined in Table 1. We have added here the scales of the domain, the Prandtl number and the Mach number at the kinematic ($M_a$) and saturated ($M_a$) stages. Models with different aspect ratio have the same spatial resolution than models in Table 1. In Runs D03b and T04a-c the resolution is $256^3$ grid points.

### 3.2. Dynamo excitation

In order to study the dynamo excitation we follow the evolution of a small random seed magnetic field on the order of $10^{-4} B_{eq}$. Since there is no rotation and because the vertical component of the large-scale vorticity $\vec{\omega}_i = \varepsilon_{ij} \vec{U}_{ij,k} \approx 0$, the average kinematic helicity and mean-field $\alpha$-effect are expected to vanish (e.g. Krause & Rädler 1980). However, it is still possible to excite a large-scale dynamo due to non-helical turbulence and shear (Brandenburg 2005a; Yousaf et al. 2008; Kitchatinov et al. 2008). In this part of the study we vary three parameters: the differential rotation amplitude, $U_0$ (Runs S00–S04), the location of the shear layer in the convectively stable region, $z_{\text{ref}}$ (Runs D02–D04), and the thickness of the shear layer, $d_z$ (Runs T02–T04).

We note that no small-scale dynamo is excited in the non-shearing case. With the runs in Set S we find that the critical value of shear above which a dynamo is excited is $Sh \approx 7$ (see the top panel of Fig. 2). The value of $Sh$ is roughly three times larger than that computed with typical values of the rms-velocity and length scales in the solar tachocline.

In Table 2, we have considered $\Delta = 33\text{kHz}$, $r_{\text{tac}} = 0.7 R_{\odot}$, $d_{\text{tac}} = 0.05 R_{\odot}$ (e.g. Christensen-Dalsgaard & Thompson 2007), and references therein. $u_{\text{rms}} = 10^3 \text{ cm s}^{-1}$, and $k_t = 2\pi/l$, with $l = 10^{10}$ cm (see Table 1 of Brandenburg & Subramanian 2005, which result in $Sh_{\text{tac}} \approx 2$).

Note that Run S04 is the same as Runs D01 and T01.
this turbulent motions are probably due to the KH instability. This difference in $u_{\text{rms}}$ reflects in the value of the equipartition magnetic field (Eq. 13) which differs from model to model.

We find that the amplitude of the magnetic energy increases for deeper tachoclines. This is an expected result since below the convection zone may hint how buoyant the magnetic field is on each model. We expect that the larger the magnetic field (Run D04) the more magnetic flux becomes buoyant and rises into the upper layers.

The time evolution of the magnetic field of the runs in Set D are shown in the middle panel of Fig. 2. We find that the growth rate of the magnetic field, $\lambda = \frac{1}{\Delta t} \ln \frac{B_{\text{rms}}(t)}{B_{\text{rms}}(0)}$, decreases slightly as the shear layer is moved deeper. This is not very clear in Table 1 since the error of this quantity is of the same order as the difference between different runs. However, it is clear from both, the figure and the table, that the volume averaged magnetic field increases with the tachocline depth.

In the third group of simulations (Runs T01–T04), we increase the width of the shear layer gradually from $d_z = 0.05 d$ in Run T01 to $d_z = 0.11 d$ in Run T04. This change implies lower values of the shear parameter $Sh$ (Eq. 12), but also larger fraction of the tachocline in the turbulent and stable layers. The results, presented in Table 1 and depicted in Figs. 1 and 2 (see legends), indicate that smaller values of the shear, $Sh \approx 3$, are still able to excite the dynamo at the price of a lower growth rate. We notice that the growth rate depends on the magnetic Reynolds number (compare Runs T04 and T04b). The fact that a smaller shear generates a larger magnetic field is a counter-intuitive result and does not agree with previous mean-field studies on the thickness of the solar tachocline (Guerrero & de Gouveia Dal Pino 2007). However, as it will be explained below, the important fact is that this configuration seems to be favorable to a longer storage of magnetic field in the stably stratified layer, with the advantage that here the effects of the shear on the thermal properties of the fluid are less important than in the previous sets of simulations. With the settings of Run T04 we find that the critical magnetic Reynolds number is between 20.8 (Run T04b) and 26.3 (Run T04c).

Based on the results above, we perform another series of simulations whose parameters and results are summarized in Table 2. Runs AR01 and AR02 correspond to Run T04 but with aspect ratios $(L_x, L_y, L_z) = (8, 4, 2)d$ and $(4, 8, 2)d$, respectively. Runs D03b, and T04a to T04c have essentially the same configuration than Runs D03 and T04 with $256^3$ grid points resolution. The value of $\nu$ and $K$ have been modified in order to obtain different Reynolds and Prandtl numbers.

The model with larger extent perpendicular to the direction of the shear does not show differences with respect to the reference case. On the other hand, the model with larger extent in the direction of the shear results in a reduced growth rate and in smaller values of the volume averaged magnetic field ($B_{\text{rms}}$), the maximum amplitude of the toroidal magnetic field and of the large-scale field. The reason for these changes is the increase of the $u_{\text{rms}}$ velocity, which diminishes the effective shear. Yousef et al. (2008) have obtained that both the growth rate and the scale where the maximum magnetic energy is concentrated, converge to a value which is independent of the relevant length scale ($L_z$ in their case). Such convergence analysis is computationally very expensive to be performed for our system. The models with higher resolution (see Runs D03b and T04b in Table 2) and bottom panel of Fig. 2 show a larger growth rate when compared with their corresponding low resolution cases. These runs, however, saturate at slightly smaller amplitudes of $B_{\text{rms}}$. Similar weak dependence on $Rm$ has been reported from simulations with horizontal shear (Käpylä et al. 2010a).

We have also changed the $Rm$ by considering a different input heat flux (i.e. different $Pr$). The results of Runs T04a–T04c
show that $\lambda$ depends on $Pr$. There is no considerable difference between the rms magnetic field of these runs.

### 3.3. Morphology of the dynamo generated magnetic field

In the initial stages of evolution, the magnetic field is dominated by small scales and although it is possible to distinguish the stretching effects of the shear, the structures formed are small compared with the size of the box. In Fig. 3 we present snapshots of the Run T04c in the kinematic phase.

Mean magnetic fields take longer to develop in all simulations but in the saturated state they correspond to a considerable fraction of the total field. This can be seen in the upper panel of Fig. 4 where the ratio $b_{\text{rms}}/B_{\text{rms}}$ for Run S04 is shown as a function of time. It is important to notice that mean values refer here to averages over the $y$ direction, i.e. $B_{\text{rms}} = \sqrt{\langle B^2 \rangle_y}$, and $b_{\text{rms}} = \sqrt{\langle B^2 \rangle_y} - \langle B \rangle_y^2$. We do not perform average over the $x$ direction because the toroidal magnetic field varies also in $x$ (see below). The average over $x$ and time is performed after the mean and fluctuation values are computed.

In this figure, the dashed line corresponds to the average value of this ratio (1.9) in the saturated phase of the dynamo, indicating that the mean magnetic field is roughly a third of the total magnetic field. For the sake of clarity only a single run is shown in this panel, but similar results are obtained for all simulations in Sets D and T.

In the bottom panel of Fig. 4 the vertical profiles of the mean (solid lines) and fluctuating (dashed lines) magnetic fields for four representative runs are shown. We find that the mean magnetic field is mainly located in the shear region, whereas the turbulent field, on the other hand, is more spread out inside the convection zone. It is noteworthy that for a thick tachocline the mean field is almost comparable with the fluctuating one, covering a fraction of the stable layer where the fluctuations are weak (see continuous blue line). Simulations with higher resolution (see blue lines with diamond symbols in Fig. 4) exhibit fluctuating magnetic field with vertical distribution and amplitude almost identical to the lower resolution case. The vertical profile of the mean field is roughly the same as in the lower resolution case but of smaller amplitude.

The structure of the magnetic fields depends strongly on the structure of convection. In Fig. 5 we show snapshots of vertical velocity ($U_z$) and azimuthal magnetic field ($B_y$) for arbitrary times in the saturated state for Runs S04, D04, and T04 (from left to right, respectively). From the bottom panels of this figure, it is possible to distinguish that at the base of the convection zone, the toroidal field organizes in elongated structures of both polarities which span all across the azimuthal direction if the penetrative downflows are less intense. These stripes coexist with more random magnetic fields in regions located where jet-like overshooting is able to reach the stable layer. The size of
the magnetic structures is at least equal or larger than the scale of the convective eddies. This is more evident for the streamline direction where the field occupies the full extent of the domain. The strong toroidal fields are mainly confined in the shear region. This can be seen in the bottom panels of Fig. 1 and also in the bottom panel of Fig. 5 where it is clear that the curve corresponding to Run T04 (blue line) has a broader profile than the one corresponding to Run S04 (black line). The presence of the KH instability is evident in the $yz$ plane where wave-like structures are observed. If the tachocline is thicker, the effects of the KH-instability are weaker. Another advantage of thicker shear layers is that they do not affect the thermodynamical properties of the fluid as strongly as in Sets S and T. The disadvantage is that they produce broad and more diffuse magnetic fields which do not totally agree with the picture of a flux tube. This is discussed in more detail in the next section.

These results are in contradiction with those of Tobias et al. (2008), who used a similar setup in the Boussinesq approximation. We believe that the lack of a large-scale dynamo in their simulations may be due either to the wider shear profile that they used or to the smaller amplitude of the shear parameter. However, a visual inspection of their figure 11 suggests that a large-scale dynamo could indeed exist in the horizontal plane.

### 3.4. Buoyancy

According to linear theory (Spruit & van Ballegooijen 1982), thin magnetic flux tubes become buoyantly unstable when the condition $3\beta \delta > -1/\gamma$ is fulfilled. Here $\beta = 2\mu_0 B^2/P$ is the plasma beta, $\delta = \nabla - \nabla_{ad}$ is the superadiabaticity (see Eq. 10), and $\gamma$ is the ratio of specific heats. This condition is satisfied in the convection zone where $\delta > 0$. From this relation it follows that below the convection zone, where $\delta < 0$, thin magnetic flux tubes require much higher field strength to become unstable. For a magnetic layer, the necessary and sufficient condition for the development of two-dimensional interchange modes, in which we are interested here, is given by (Newcomb 1961):

$$\frac{d\rho}{dz} > \frac{-\rho^2 g}{\gamma P + B^2/\mu_0}.$$  

(18)

We find that for the cases with thinnest shear layer, the instability region is above the center of the shear layer where the toroidal magnetic field reaches its maximum value (see Fig. 6). This implies that a large fraction of the magnetic field there becomes buoyantly unstable very quickly. For the cases with a thick shear layer (Set T), the magnetic field is unstable only within the convection zone. The magnetic field distribution of the Run T04 in Fig. 4 (blue line), indicates that a large fraction of the magnetic field is buoyantly stable. This configuration may explain why in these cases larger mean magnetic fields develop with smaller shear parameters than in sets S and D are used.

Note, however, that this conclusion is based on averaged quantities. The evolution of local magnetic structures is rather complex and possibly also affected by the KH instability. The radial velocity gradient in Run T04 is smaller, so it is expected to have a less efficient KH instability and this should allow a longer stay of the magnetic field in the stable layer. This is hard to demonstrate since it is difficult to disentangle these effects in the simulations. Nevertheless, it seems that the longer the magnetic field stays at the stable region the greater the final mean magnetic field strength. Hence, according to Fan (2001), the number of scale heights the magnetic flux concentrations may rise, and its final structure depends only on how strong the field is in comparison to the turbulent convective motions.

In the simulations it is observed that when the magnetic field rises it expands as a consequence of decreasing density. The morphology of the magnetic fields in the $xz$ plane (see bottom panels of Fig. 5) is reminiscent of the mushroom-shape that has been obtained in several 2D (Süssler 1979; Moreno-Insertis & Emonet 1996; Longcope et al. 1996; Fan et al. 1998; Emonet & Moreno-Insertis 1998) and 3D (Fan et al. 2001; Fan et al. 2003; Fan et al. 2008; Jouve & Brun 2009) simulations of flux tube emergence. Such expansion may result in the splitting and braking of the tube. We do not impose any twist on the tubes since the dynamo generates the magnetic field self-consistently and the possibilities span from events that do not rise at all to events where buoyant magnetic fields rise up to the very surface. In the horizontal $yz$ plane we observe that the magnetic field lines remain horizontal in the stable layer where they form. When the magnetic field rises, the field lines bend in the convection zone, with their rising part in the middle of upward flows and other parts withheld to the downward flows.

In Tables 1 and 2 we list the maximum values of $B_y$ in the simulations. In most of the cases we find $\max(B_y) \approx 5B_{eq}$, and only in the models with a thicker (Runs T04 and AR01) or a deeper shear layer (Run D04), $\max(B_y)$ can be somewhat greater than $6B_{eq}$, Fan et al. (2003) argue that if $B_0 > (H_B/a)^{1/2}B_{eq}$, $B_0$ being the field strength in the shear layer and $H_B$ the local pressure scale height, a buoyant flux-tube of radius $a$ may rise without experiencing the convective drag force. Here, although the shape of the magnetic field is not a tube, we compute the same condition using $a = dz$. We obtain values of $B_0$ going from $3.4B_{eq}$ in Run D04 to $2.2B_{eq}$ in Run T04 (2.3$B_{eq}$ in higher resolution runs). This indicates that the dynamo generated magnetic field is insufficient to reach the surface without being modified by the convection. There are, however, a few cases in Runs D04, T04 and AR01, where strongest magnetic fields are able to reach the surface. When this occurs the magnetic field reacts back on the flow and modifies the convective pattern. Broad convection cells elongated in the $y$ direction are formed. In Fig. 7 we present one of these events for Run AR01. In the upper layers, the field lines show a turbulent pattern except in the center and the right edge of the box where the field lines at the surface cross the entire domain in the $y$ direction. In the higher resolution cases, we do not observe events where the field lines, in the uppermost layers, remain horizontal across the whole toroidal direction. Nevertheless, the expanding magnetic field may reach the surface forming broad convection cells (at least two times larger than the regular cells). One of these cases, corresponding to Run T04c is shown in the bottom panels of Fig. 7. In this event two stripes of magnetic field of opposite polarity are reaching the surface simultaneously, the magnetic field lines of these ropes reconnect forming a large scale loop oriented in the $x$-direction.

In order to compute the rise speed of the magnetic field in these cases we have used the horizontal averages of $B_y$ and constructed the time-depth “butterfly” diagram shown in Fig. 8. The toroidal field is amplified in the shear layer, below $z = 0$ (see dotted line). When it becomes buoyant it travels through the convection zone. The tilt observed in the contours of $B_y$ may give a rough estimate of the vertical velocity. In the same figure we have drawn white dashed lines to guide the eye. For the emergence events in Run D04 (AR01), the estimated rise speed is $u_r \approx 0.034(d_4)^{1/2}$ ($\approx 0.030(d_4)^{1/2}$). These values are small in comparison to the $u_{rms}$ of the models (see Tables 1 and 2) and does not agree with the rise of a flux tube in a stratified atmosphere. For instance, Fan et al. (2003) obtain a final rise velo-
Fig. 5. Same as Fig. 3 but for Runs S04, D04 and T04 (from left to right) in the saturated phase. Movies corresponding to these runs may be found at: www.nordita.org/~guerrero/movies.

Fig. 6. Buoyancy instability condition, Eq. (18) evaluated for Runs S04, D04 and T04. Continuous (dashed) lines correspond to the left (right) hand side of the equation, respectively. Dotted lines correspond to the interfaces between convectively stable and unstable layers, and to the center of the tachocline in each simulation.

ity \approx 5 \text{ times larger than the rms-velocity. We should point out that in the simulations here, the strong shear increases } u_{\text{rms}} \text{ by a factor of two or even more. If we compare the rise velocity of the magnetic field with the rms-velocity of a non-shearing case (Run S00), we find that } w_0 \text{ is slightly larger. Magnetic pumping effects, which are evaluated below, may also play a role in braking the magnetic flux and in defining its final velocity.}

The results mentioned above do not fully agree with those found by Vasil & Brummell (2008), where a toroidal magnetic layer is generated from a purely vertical field through an imposed radial shear layer. Although their stratification is smaller than in our case, the buoyancy instability is hard to excite in their simulations and the buoyancy events are slowed down rather quickly. The lack of a dynamo mechanism, able to sustain the magnetic field, is maybe the reason of its rapid diffusion.

3.5. Back reaction

Temporal analysis of helioseismic data has revealed that the solar angular velocity varies by around 5 per cent with respect to its mean value. This fluctuation pattern, like the sunspots, follows an 11-year cycle, which suggests that it corresponds to the back reaction of the magnetic field on the plasma motion (e.g. Basu & Antia 2003, Howe et al. 2009). It is also known that the amplitude of the meridional circulation varies with the amplitude of the magnetic field, being smaller when the cycle reaches its maximum (see e.g Basu & Antia 2003). Apart from large scale effects, the magnetic field may also affect the local properties of the plasma. This is possible the case in the downflows found by Hindman et al. (2009) nearby the bipolar active regions.

In the simulations presented here, due to the different nature of the dynamo and to the numerical setup, there are neither periodic oscillations of the magnetic field nor large-scale circulation. We are able, however, to distinguish the differences between the plasma properties in the kinematic and the saturated stages. We also follow the deviations of the averaged flow occurring during peaks of the magnetic field amplitude.

We find that also the turbulent rms-velocity, as well as the imposed large-scale shear flow, are affected by the magnetic field. In the top panel of Fig. 9 we show the vertical profile of the horizontal and time averaged } u_{\text{rms}} \text{ at the kinematic (dotted line) and saturated (continuous line) regimes of the dynamo. A decrease in the amplitude is observed, especially in the region where the magnetic field is more concentrated. The volume averaged } u_{\text{rms}} \text{ also changes accordingly by } \sim 5 \text{ per cent, as indi-}
The dashed line in the top panel of Fig. 9 shows that it further decreases in the shear region.

In the middle and bottom panels of Fig. 9, butterfly diagrams of the toroidal velocity fluctuations, \( \delta U_y = U_y - \bar{U}_y \), in the \( z \)-time plane are shown for the Runs AR01 and T04b, respectively. The dashed white line indicates the center of the shear layer. Positive values of this quantity below (above) the dashed line indicate faster (slower) toroidal velocities with respect to its mean value. Accordingly, negative values below (above) the dashed line indicate deficiency (excess) of the local flow. The continuous white line corresponds to the time evolution of \( B_{\text{rms}} \) (for clarity normalized to its maximum value). We show this in order to demonstrate that the changes in \( U_y \) depend on the amplitude of the magnetic field. The overall result of the contour plots is that during the maximum of the magnetic field the shear is slightly stronger. In these diagrams, as well as in all of the simulations listed in Tables 1 and 2 (except Run T04\( \tau \)), the deviation around the averaged value of \( U_Y \) is found to be \( \sim 3 \) per cent.

The deviation of the mean shear profile depends on the forcing time scale \( \tau_f \) in Eq. 3. In Run T04\( \tau \), we consider the same settings that in Run T04 but varying \( \tau_f \) from 2(\( dg \))\(^{1/2} \) to 10(\( dg \))\(^{1/2} \) (\( \approx 1 \) to \( \approx 5 \) turnover times, respectively). The results show fluctuations of \( \approx 10 \) per cent of the shear velocity, and spread out now to a wider fraction of the domain (see bottom panel of Fig. 9). Since magnetic buoyancy is present also at the time of the maximum, the toroidal velocity in the middle of the convection zone is also affected. This change is not so strong as the one in the shear layer but is also a general characteristic of the simulations.

The perturbations shown in Fig. 9 are reminiscent of the equatorial branches of the solar torsional oscillations which have a leading excess in the zonal flow, followed by a slightly less pronounced deficit of it. Similar results have also been found in global numerical simulations of oscillating dynamos by Brown et al. (2011). They have found that the branches of zonal flows follow the evolution of a poleward dynamo wave.

### 3.6. Turbulent transport coefficients

In order to study the mechanism that generates a mean-field dynamo in our simulations, we compute the turbulent transport coefficients from a representative simulation (Run T04). In order to do this, we use the test-field method, introduced in the geodynamo context by Schrinner et al. (2005, 2007) and currently available in the PENCIL CODE (e.g. Brandenburg et al. 2008).

In mean-field theory, the electromotive force \( \mathbf{E} = u \times \mathbf{B} \) governs the evolution of the large-scale magnetic field (Krause & Rädler 1980). Under the assumption that the mean field varies smoothly in space and time, and that there is no small scale dynamo action, the electromotive force may be written in...
terms of the large-scale magnetic field:

$$ \mathbf{E} = \alpha_{ij} \mathbf{B}_j - \eta_{ijk} \mathbf{J}_k. \quad (19) $$

Considering large-scale fields that depend only on z we need four independent test fields in order to compute the 4+4 components of $\alpha_{ij}$ and $\eta_{ijk}$ (see a detailed description of the method in Brandenburg et al. 2008). The novel feature of the test field method is that the test fields do not act back on the flow and that the turbulent diffusivity can also be computed, thus avoiding many of the problems that plague other methods (cf. Kapylä et al. 2010b).

We discuss our results in terms of the following quantities:

$$ \alpha_{xx} \approx \alpha_{yy} = \alpha_{22}, \quad (20) $$
$$ \eta_\gamma = \frac{1}{2}(\eta_{11} + \eta_{22}), \quad (21) $$
$$ \gamma = \frac{1}{2}(\eta_{21} - \alpha_{12}, \quad (22) $$

which represent the inductive ($\alpha$), diffusive ($\eta_\gamma$), and pumping ($\gamma$) effects of turbulence, respectively. In Fig. 10 we show the vertical profiles of the kinetic helicity ($\mathbf{u} \times \omega$), where $\mathbf{u}$ is the vorticity, and the turbulent transport coefficients of Eqs. (20)–(22), normalized with the first order smoothing approximation (FOSA) quantities:

$$ \alpha_0 = \frac{1}{B_{\text{rms}}} \alpha_{\text{rms}}, \quad \eta_0 = \frac{1}{B_{\text{rms}}} \eta_{\text{rms}} k_f^{-1}. \quad (23) $$

For computing the coefficients in Fig. 10 we use Run T04 in the purely hydrodynamic state. The magnetic diffusivity considered for the test fields is one order of magnitude larger that that used for the magnetic field in the original run. However, the magnetic Reynolds number ($Rm \approx 4$) is still sufficiently large to yield reasonable results (Kapylä et al. 2009a).

We find that certain amount of helicity is generated in the system, but that it is still statistically consistent with zero. The coefficient $\alpha_{xy}$, despite large fluctuations, has a negative sign. This component of $\alpha$ contributes to amplifying the $y$ component of the magnetic field, but its contribution is likely negligible when compared with that of the shear in the present models.

Fig. 8. Butterfly diagram in the z-time plane for the toroidal magnetic field, $B_y$. The dotted lines corresponds to the top ($z = d$) and base ($z = 0$) of the convection zone. The tilted dashed lines show approximate trajectories of buoyant magnetic fields. A rough estimate of rise speed is computed with these lines as indicated. The event at $t = 90$ is also shown in Fig. 7.

Fig. 9. Top: time averaged vertical profile of $u_{\text{rms}}$ in the kinematic (dotted line) and dynamical (continuous lined) stages for the Runs T04b. The dashed lines show the deviation from the average at a peak in the magnetic field amplitude. The three lower panels are butterfly diagrams of the angular velocity deviation, $\delta U_y = U_y - \bar{U}_y$, for the Runs HR01,T04b and T04$\tau$. The continuous white line in these panels indicate the time evolution of $B_{\text{rms}}$ normalized to its maximum value in the time interval.

The component $\alpha_{xy}$ shows a zero mean value, but its variance has a large amplitude, especially in the middle of the convection zone. According to Vishniac & Brandenburg (1997), an incoherent $\alpha$ effect, zero in average but with finite variance, may generate enough inductive effects to sustain the dynamo. This suggests that the incoherent $\alpha$ effect may be the mechanism sustaining the dynamo.

The shear-current effect, arising from the inhomogeneity of the turbulence and the mean shear flow, may also result in the generation of a mean field (Rogachevskii & Kleerorin 2003, 2004). The existence of this effect has been studied with models of forced turbulence and latitudinal shear by (Brandenburg et al. 2008; Mitra et al. 2009), but no conclusive evidence has been found. Similar results have also been found from convection with horizontal shear (Kapylä et al. 2009b). In the case presented here, with vertical shear, the relevant coefficient for this effect would be $\eta_{13}$, which is not captured by the current version of
4. Conclusions

We perform numerical simulations of turbulent convection with a thin vertical shear layer with the aim of mimicking the conditions at the solar convection zone and the tachocline. This layer is located below the interface between a convectively stable and unstable layers. We consider the velocity gradient, the depth and the thickness of this layer as free parameters of the model and concentrated on two phenomena relevant for the solar cycle: dynamo process and magnetic buoyancy.

We find that it is possible to excite a large scale dynamo by the combined action of turbulent convection and a local radial shear. The efficiency of the dynamo (i.e., the growth rate), and the maximum amplitude of the magnetic field depend on the shear amplitude, the thickness of the shear layer, the aspect ratio and the magnetic Reynolds number of the system (see Fig. 2 and Tables 1 and 2).

The magnetic field is organized in elongated structures in the direction of the shear in the shear layer below the base of the convection zone. The large-scale magnetic field can have either polarity and may be up to the 40 per cent of the total field (solid lines in Fig. 4). It coexists with turbulent magnetic field which is distributed all across the convection zone. In the stable layer the magnetic field exists in the regions of strong downflows. For fixed depth and thickness (Set S) a smaller growth rate and larger $B_{\text{rms}}$ is obtained for a larger shear parameter. A critical value of $\text{Sh} \approx 7$ is required to excite the dynamo.

If the tachocline is located deeper (simulations in set D), the magnetic field develops mainly in the stable layer which allows a longer field storage. In these cases the dynamo grows faster and with a larger fraction of mean field magnetic field. This configuration, however, is not optimal since with the numerical resolution used here the thermodynamical properties of the fluid are affected by viscous heating in the shear region. If the depth of the shear layer remains constant but the thickness increases (Set T), the magnetic field grows slowly but it contains a larger fraction of mean magnetic field. This may be a consequence of the smoother shear profile which renders the magnetic field buoyantly unstable only in the superadiabatic part of the domain (see right panel of Fig. 5). A lower critical shear number is found to be able of excite the dynamo instability in these cases ($\text{Sh} \approx 3.5$).

Larger magnetic Reynolds numbers are achieved in two ways, first by doubling the initial resolution and second by changing the input flux (i.e., different $\text{Pr}$ number). On one side, for a fixed $\text{Pr}$ and larger $\text{Rm}$, the dynamo exhibits a much faster growth. On the other hand, for a fixed resolution and different input flux, the growth rate changes proportionally to $\text{Pr}$. In these higher resolution Runs the final value of $B_{\text{rms}}$ is slightly below that in the corresponding cases with lower resolution. This may correspond to the dependence of the saturation value of the magnetic field with the magnetic Reynolds number or may also be the result of insufficient statistics.

Since the system is not rotating, there is no kinematic helicity and hence no $\alpha$-effect. The test-field analysis suggests that the probable mechanism triggering the amplification of the magnetic

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Fig. 10. Normalized profiles of the kinetic helicity, and the turbulent transport coefficients $\alpha_{xx}, \alpha_{yy}, \gamma$ and $\eta_t$ (from top to bottom) computed with the test-field method.

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...the test-field method with only $z$-dependent test fields. Thus, we leave the study of the shear-current effect in turbulent convection with vertical shear for a forthcoming work.

The turbulent diffusivity, $\eta_t$, is roughly four times larger than the FOSA estimate. When compared with the molecular diffusivity, it is also four times larger (i.e. of the same order as $\text{Rm} \sim \eta_t/\eta$). This result agrees with previously computed turbulent diffusivity for convection [Käpylä et al. (2009)]. The vertical profile of $\eta_t$ differs somewhat from previous results, indicating the action of the radial shear on the turbulent diffusion. Another interesting point is that $\epsilon_t$ is not close to zero, as in [Käpylä et al. (2009)]. This is likely explained by the stronger shear used here. This is also an indication of the tensorial character of $\eta_t$ and suggests that the distinct components of the magnetic field may diffuse differently.

Finally, the vertical profile of the turbulent pumping ($\gamma$) in Fig. 10 shows that there is downwards transport of the mean magnetic field at the bottom of the convection zone and upward transport close to the top. The pumping velocity is comparable with $u_{\text{rms}}$ but it is not enough to retain the magnetic field within the stable layer, which indicates that buoyancy is a more efficient mechanism than magnetic pumping in transporting magnetic fields. Note, however, that the amplitude of this effect depends on the gradient of the level of turbulence, $\gamma = -\nabla \eta_T$, where $\eta_T = \eta + \eta_t$ (Kichatinov & Rüdiger 1992), which is not very large in the current simulations.
field could be incoherent $\alpha$-shear dynamo. However, other possibilities like the shear-current dynamo can not be ruled out for the time being.

Magnetic buoyancy is observed in all of the simulations. Based on horizontal averages we analyze the instability condition for 2D interchange modes (Eq. 18 and Fig. 6). We find that in models with a deeper tachocline the buoyancy instability develops even in the stable layer whereas in models with thicker shear layers the magnetic field is unstable in the convection zone only. This, however, does not mean that there are not emergence events. As far as the toroidal magnetic field is strong enough it rises throw the convection zone forming mushrooms shape structures.

When the buoyant magnetic field is weak it is strongly modified by local convective flows (see Fig. 5). On the other hand, magnetic fields of strong amplitude formed at the shear layer are able to rise up to the surface and modify the convective pattern (Fig. 7). They may form either large convection cells or convective rolls may occupy all of the y-extent. These clearest events are observed when the magnetic field in the shear layer exceeds the equipartition field by a factor $\geq 6$. Such strong magnetic field has been observed only in simulations with $128^3$ grid points resolution. The rise velocity of the buoyant field in the bulk of the convection zone, estimated from $\pm$-time “butterfly” diagrams, has been found that $u_\pm \approx 0.6 u_{\text{rms}}$. From the test-field method results, the maximal downwards pumping velocity of magnetic field is found to be $\gamma \approx 0.4 u_{\text{rms}}$. This indicates that buoyancy may indeed exist from the base of the convection zone, however, the magnetic field expands very quickly during the rise and magnetic structures observed close to the upper boundary are not so well organized.

Besides the emergence of the magnetic field that affects the flow pattern locally, other changes are observed due to the back-reaction of the magnetic force on the plasma. In the saturated phase, the rms-velocity varies between its kinematic value and a lower amplitude when the magnetic field is strong. In addition, during the peaks and valleys of the field amplitude, the imposed shear profile presents systematic variations with respect to its averaged profile. For most of the models this change is around 3 per cent of the shear velocity and is reminiscent of the fluctuations observed in the solar rotation profile or “torsional oscillations”. The variation signal is mainly observed in the places where $B$ is strong, with much weaker changes within the convection zone. The results encourage the study of the torsional oscillations in detail through direct numerical simulations.

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