

The thesis deals with Bergman spaces, Bergman projections and Toeplitz operators on the Bergman spaces. The theory of these integral operators is a combination of complex analysis, operator theory and functional analysis. Although Bergman and Toeplitz operators have been widely studied for decades, some fundamental questions remain still open.

The Bergman space A^2 over a bounded planar domain $\Omega \subset \mathbb{C}$ consists of all complex-valued, analytic functions which are defined on Ω and which are square-integrable with respect to the Lebesgue area measure. The Bergman projection P_Ω is the orthogonal projection from the Hilbert space $L^2(\Omega)$ onto $A^2(\Omega)$. In the Bergman space setting, Toeplitz operator T_a with a symbol a is defined by $T_a(f) := P_\Omega(af)$, where f is an analytic function on Ω and P_Ω is the Bergman projection.

In this thesis, we study the boundedness of (generalized) Bergman and Toeplitz operators on quite general bounded simply connected planar domains. The thesis consists of an introductory part and three published articles. In the introductory part we give a general overview of the theory of Bergman projections and Toeplitz operators and of the boundedness results concerning them. In the first article we give sufficient and necessary conditions for the boundedness of Bergman type projections on the growth space $L_v^\infty(\Omega)$, where the domain Ω is so called regulated domain and the weight v is a power of the boundary distance. We use the Riemann conformal mapping for transferring the consideration from the regulated domain to the unit disk.

In the second article we study generalized Toeplitz operators, with quite general symbols, on the Bergman spaces $A^p(\Omega)$, $1 < p < \infty$. More specifically, the symbols are locally integrable and the domain Ω is a bounded simply connected domain with a sufficiently smooth boundary. We give sufficient conditions for the boundedness and compactness of T_a in terms of “averages” of the symbol over certain Cartesian squares. The main tool in the proof is the Whitney decomposition of the domain Ω .

In the third article we continue the study of the previous one. Instead of smoothly bounded domains, we consider polygonal domains with corners. In this setting, we give sufficient conditions for the boundedness of generalized Toeplitz operators with locally integrable symbols on the Bergman spaces $A^p(\Omega)$.