Tax Reforms, Collective Bargaining and

Imperfect Capital Markets

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This paper discusses possible consequences of tax reforms. The tax system consists of labor income, capital income and consumption taxes. I consider the effects on Gross Domestic Product, capital stock, the employment rate, the wage rate, the representative worker's wealth, private consumption and the welfare of the households. I assume that the government's budget is balanced.

I establish a macroeconomic model that consists of heterogeneous households, firms, labor market organizations and the government. There are two types of households – workers and firm owners. In the model, the firm owners and the firms are merged into one agent, entrepreneurs. The labor market organizations – the labor union representing the workers and the employers' federation representing the entrepreneurs – bargain over the wages and the level of employment. Hence, the labor market is neither perfect nor based on a monopoly labor union. I consider imperfect capital markets, where borrowing is subject to a risk premium, and an open economy with the possibility to import or export capital.

I show that decreasing the taxes on labor income, capital income and consumption separately, combined with decreased government expenditures, has mostly positive effects on the Gross Domestic Product, the capital stock, the employment, the consumption of workers and entrepreneurs as well as the utility of the entrepreneurs. However, the reforms’ effects on the workers' wealth are contradictory. The same concerns the workers’ utility.

A combined decrease of the taxes on labor income, capital income and consumption with a decrease of government expenditures generally impacts the economy positively. The exception is a decreased gross wage, which is, however, compensated by the lower labor income tax.

The contribution of this paper is that I examine taxation in an open economy with imperfect labor and capital markets. I show that a decreased tax burden with decreased government expenditures has a number of positive macroeconomic effects. Thus, the reforms examined in this paper indicate what kinds of policy action could be of benefit to the economy.
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List of Variables and Parameters

A  total factor productivity
B  borrowing of the representative entrepreneur
C_W consumption of the representative worker
C_E consumption of the representative entrepreneur
G  government expenditures
I  investments of the representative entrepreneur
K  capital stock of the representative entrepreneur
L  number of employed workers
N  number of atomistic workers
r  periodic risk-free interest rate
Q  stock of debt of the representative entrepreneur
S_W savings of the representative worker
T_C consumption tax rate
T_L labor income tax rate
T_{pi} capital income tax rate (including tax on interest income)
U_W utility of the representative worker
U_E utility of the representative entrepreneur
W  wealth of the representative worker
\bar{W} median wealth of the workers
w  wage rate
X  foreign capital
\( Y \) Gross Domestic Product

\( Z \) unemployment benefit

\( \alpha \) relative bargaining power of the labor union

\( \beta \) discount factor for the whole population

\( \gamma \) borrowing cost parameter

\( \delta \) depreciation rate of capital

\( \eta \) production function parameter determining the distribution of income between labor and capital

\( \theta \) distribution parameter between the representative worker’s consumption and the government expenditures in the representative worker’s utility function

\( \kappa \) investment cost parameter

\( \mu \) weight parameter for the amount of employed workers in the labor union’s function

\( \nu \) weight parameter for the difference between the wage rate and the unemployment benefit in the labor union’s function

\( \pi_E \) operative profit of the representative entrepreneur

\( \rho \) rate of time preference

\( \sigma \) production function parameter expressing the elasticity of factor substitution

\( \varphi \) weight parameter for status in the representative worker’s utility function

\( \chi \) elasticity of substitution between the representative worker’s consumption and the government expenditures in the representative worker’s utility function
1 Introduction

Finland is generally considered as an example of a society with high taxation. It is therefore interesting to investigate what the effects of a lower level of taxation would be. Additionally, it could be interesting to investigate the results of decreased public expenses combined with a lower general level of taxation. My research topic is how reforming the tax code, combined with possible changes in public expenditures, impacts macroeconomic variables, including the welfare of individuals.

In the USA, President Ronald Reagan’s tax reform, ”The Tax Reform Act of 1986”, simplified taxation for persons and companies.\textsuperscript{1} In contrast, President Bill Clinton’s reform, ”Taxpayer Relief Act”, complicated the system\textsuperscript{2}. President Donald Trump’s reform, ”Tax Cuts and Jobs Act”, temporarily decreased personal income tax rates and permanently decreased the corporate tax rate\textsuperscript{3}.

In Europe, there were substantial changes in the tax systems during the 1990s. The Baltic countries introduced various forms of flat tax systems\textsuperscript{4}. This is described in, for example, Hall and Rabushka (1983). The system can simply be a flat tax for all kinds of income, personal or corporate, or refined with various deductions. In Finland, the corporate tax was lowered from 24.5 to 20 percentage in 2014.\textsuperscript{5}.

The literature dealing with taxation is very extensive. In practice the differences between articles concern the wage setting in the labor market and whether an economy is open or closed. Altig et al. (2001) and Aronsson and Schöb (2017) assume a perfectly competitive labor market. Díaz-Giménez and Pijoan-Mas (2011) differ from these with the assumption that the labor supply is subject to an idiosyncratic stochastic process. Aronsson et al. (2002), Aronsson and Sjögren (2003) as well as Aronsson and Sjögren (2004a)

\textsuperscript{1}Chamberlain (2006).
\textsuperscript{2}Altig et al. (2001).
\textsuperscript{3}Tax Foundation (2017).
\textsuperscript{4}European Central Bank (2007).
\textsuperscript{5}Taxpayers Association of Finland (2018).

Aronsson and Sjögren (2003) and Aronsson and Sjögren (2004b) consider an open economy. The former paper assumes that goods are traded, while the latter assumes that firms can move their production abroad.

Altig et al. (2001) simulate the impact and efficiency of various tax regimes by a dynamic macroeconomic model. They consider households, firms and the government in an intragenerational setup, focusing on the consequences of a proportional income tax, a proportional consumption tax and flat taxes. In that setup, there are "winners" and "losers". The conclusion is that in some cases retired people lose when the situation of future generations is improved. In other cases middle- and upper-income citizens will be better off and current and future poor people will suffer. In line with the article, the big question is "are the gains to the winners worth the costs to the losers?".

Aronsson et al. (2002) use a general equilibrium model where the labor unions determine the wages. The agents are the households, labor unions, the firms and the government. The tax revenue is returned to the households as a lump sum and the size of the population is exogenous. Their paper examines the effects of the progressivity of taxation in the economy. The conclusion is that real wages are increased and other variables such as working time, employment, output and consumption are decreased.

Aronsson and Sjögren (2003) use a model of a small economy, where the world market determines the prices of products. The players are the consumers with two levels of productivity, deciding about the labor supply, the firms deciding about the labor demand, the labor unions deciding about the wages and the government choosing the tax rates and the public expenditures. They show that with low unemployment benefits, the level of provided public goods should be increased and the commodity taxes decreased. Furthermore, if the wage ratio between the two consumer types changes, the
labor input of the consumer types reacts similarly.

Aronsson and Sjögren (2004a) use a general equilibrium model of a unionized economy. The agents are three types of consumer - firm owners as well as employed and unemployed workers - identical firms, labor unions and the utilitarian government. Labor is the only input in production and the labor unions decide either the wage rates or the wage and the number of working hours. The paper shows that if the labor unions set only the wages and the income tax is unrestricted, the free market solution, including zero marginal income tax, can be implemented. On the other hand, if the income tax is restricted, there will be unemployment and a progressive tax on labor income.

In Aronsson and Sjögren (2004b), the wage is determined through negotiations between the labor unions and firms. The article considers different countries that coordinate their policy concerning taxation, the support of unemployed households and public production. If such a coordination increases leisure time for workers, possibly combined with decreased financial support for the unemployed, welfare will also increase.

Díaz-Giménez and Pijoan-Mas (2011) describe a modified neoclassical growth model where the households are heterogeneous and cannot insure themselves against idiosyncratic risks. The authors study the effects of various flat-tax reforms on the distribution of income and the welfare of individuals in a model economy. Díaz-Giménez and Pijoan-Mas assume stochastic aging and retirements to shape features connected to the life-cycle of individuals. The government taxes capital income, labor income, consumption and estates. The taxes are spent on the benefits given to retired households and on government consumption. The authors show that progression on the consumption-based flat tax increases the government income and supports the weakest people in the society.

Aronsson and Wikström (2011) examine the impact of the concentration of wage settlement on the progressivity of the labor income taxation. The utilitarian government applies an unrestricted tax of profits as well as a pro-
gressive tax on working income. The paper shows that with a decentralized wage setting, the free market solution is attainable, in contrast to the centralization, which leads to unemployment and a progressive taxation.

Hummel and Jacobs (2016) analyze an economy with workers, trade unions, owners of firms and the government. The labor market is unionized, workers face heterogeneous labor participation costs and the wage rate depends on the type of work. Workers decide if they want to take part in the labor market. Workers in different sectors belong to different labor unions. The owners of the firms possess a capital stock and hire different types of labor. The trade unions and the owners of the firms bargain over the wages. It is shown that as labor unions often raise the wages above the market level, the result is involuntary unemployment. Furthermore, with a higher level of unionization of the labor market, the optimal income tax is lower.

Aronsson and Schöb (2017) use a general equilibrium overlapping-generations model that contains consumers, firms and a government. A consumer lives for three periods and tends to make irrational decisions, which the government tries to correct by adjusting the marginal labor income tax rate as well as taxes on savings. These tools are generation-specific.

In this paper, I establish a macroeconomic model that consists of heterogeneous households, firms, labor market organizations and the government. There are two types of households - workers and firm owners. In the model, the firm owners and the firms are merged into one agent, entrepreneurs. The labor market organizations - the labor union representing the workers and the employers’ federation representing the entrepreneurs - bargain over the wages and the level of employment. Hence, the labor market is neither perfect nor based on a monopoly labor union.

Following Gertler and Karadi (2011), Yépez (2017) and Vachadze (2018), I assume an imperfect capital market where borrowing is subject to a risk premium. Furthermore, I assume an open economy with the possibility to import or export capital. My contribution to the literature is that I combine
imperfect labor and capital markets with an open economy.

The model is described in chapter 2 and in chapter 3, I present the values used for calibrating the model. In chapter 4, I collect the results and discuss them. Finally in chapter 5, I draw conclusions of this research.

2 Description of the Model

The model is solved as a Stackelberg game. I normalize the price of the single good at 1.

2.1 Workers

There is an exogenous large number N of atomistic workers, who are either employed or unemployed. Together they form a representative worker. An employed worker supplies one unit of labor during one period and earns a working income \( w \). The number of workers employed by the entrepreneurs in a period is \( L \). The unemployed worker receives an unemployment benefit \( Z \), paid by the government.

The representative worker accumulates wealth \( W \) and saves \( S_W \) in one period. Thus, the wealth accumulation equation is

\[
W_{t+1} = W + S_W. \tag{1}
\]

The representative worker receives the interest on the accumulated wealth at the end of each period. The periodic risk-free interest rate is \( r \in [0, \infty) \). Hence, the value of the interest income is \( rW \).

The consumption of the representative worker in one period is denoted by \( C_W \), which is taxed at the rate \( T_C \). The labor income is taxed at the rate \( T_L \). The unemployment benefit is not taxed. The representative worker pays a tax \( T_\pi \) on the received interest income. The tax rates are assumed to fulfill

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6I denote variables with the subscript \(+n\), \( n \) indicating the difference between the period concerned and the present one.
the condition $T_C, T_L, T_\pi \in (-\infty, 1)$.

Consequently the budget constraint of the representative worker is

$$(1 - T_\pi)rW + (1 - T_L)Lw + (N - L)Z = (1 + T_C)C_W + S_W. \tag{2}$$

Given equation (1), the budget constraint (2) can be rewritten as

$$(1 - T_\pi)rW + (1 - T_L)Lw + (N - L)Z = (1 + T_C)C_W + W_{+1} - W \tag{3}$$

or equivalently

$$C_W = \frac{W}{1 + T_C} - \frac{W_{+1}}{1 + T_C} + \frac{(1 - T_\pi)rW}{1 + T_C}$$
$$+ \frac{(1 - T_L)Lw + (N - L)Z}{1 + T_C}. \tag{4}$$

The representative worker derives utility from its own consumption $C_W$, the government expenditures $G$ and its status\footnote{Following Futagami and Shibata (1998), the status is here used in order to separate the interest rate $r$ from the rate of time preference $\rho$, guaranteeing the existence of a risk premium in borrowing.} in society, which for technical reasons is measured by the natural logarithm of the ratio between between its own average wealth $W/N$ and the workers’ median wealth $\overline{W}$, which a single worker takes as given. The representative worker’s periodic utility function in a period is

$$U_W(C_W, G, W, N, \overline{W}) = [\theta C_W^{\frac{\chi - 1}{\chi}} + (1 - \theta)G^{\frac{\chi - 1}{\chi}}]^{\frac{1}{\chi}} + \varphi \ln \frac{W}{N\overline{W}},$$

$$= C_W f(g) + \varphi(\ln W - \ln N - \ln \overline{W}), \tag{5}$$

where $\theta, \chi \in (0, 1)$, $\varphi > 0$ and $g = \frac{G}{C_W}$, which is taken as given\footnote{The definition of $g$ here describes the congestion of government expenditures relative to the workers’ consumption. The more an individual uses public services, the greater is its benefit from those services. A similar model has been used by Chatterjee and Ghosh (2011) and Gómez (2014).}. Parameter $\theta$ is the distribution parameter between the representative worker’s consumption and the government expenditures, while $\chi$ expresses the constant elasticity of substitution between the representative worker’s consumption and
the government expenditures\(^9\). The function \(f(g)\) is defined as

\[
f(g) = \left[ \theta + (1 - \theta)g^{\frac{\chi - 1}{\chi}} \right]^{\frac{\chi}{\chi - 1}}.
\]

The periodic utility (5) is discounted by the factor \(\beta = \frac{1}{1 + \rho}\), where \(\rho > 0\) is the constant rate of time preference.

In each period, the representative worker chooses the new level of wealth, \(W_{+1}\) in order to maximize its lifetime utility. The maximization is solved by dynamic programming. When choosing \(W_{+1}\), the variables \(T_L, T_\pi, T_C, r, Z, g, \overline{W}\) and \(W\) are taken as given. Based on (4) and (5), the maximization problem becomes

\[
\mathcal{V}_W = \mathcal{V}(T_L, T_\pi, T_C, r, Z, g, W, W) = \max_{W_{+1}} \left\{ C_W f(g) + \varphi(\ln W - \ln N - \ln \overline{W}) + \beta \mathcal{V}(T_{L,+1}, T_{\pi,+1}, T_{C,+1}, r_{+1}, Z_{+1}, g_{+1}, \overline{W}_{+1}, W_{+1}) \right\},
\]

where \(\mathcal{V}_W\) is the value function of the representative worker. The maximization is carried out in Appendix B.2.

The savings decision of the representative worker determines its consumption path, which is described by the Euler equation

\[
\frac{f(g)(1 + \rho)}{1 + T_C} = \frac{f(g_{+1})(1 + (1 - T_{\pi,+1})r_{+1})}{1 + T_{C,+1}} + \frac{\varphi}{W_{+1}}.
\]

### 2.2 Entrepreneurs

The entrepreneurs are atomistic. Together they form a representative entrepreneur. The representative entrepreneur possesses the capital stock \(K\). This is built up by the entrepreneur’s investment \(I\) in one period. The accumulation of capital by the entrepreneurs follows the equation

\[
K + I = K_{+1} + \delta K,
\]

\(^9\)Restricting \(\chi\) to the interval \((0, 1)\) guarantees that the representative worker’s consumption and the government expenditures are complements.
or equivalently
\[ I = K_{+1} - (1 - \delta)K, \] (10)
where \( \delta \) is the constant depreciation rate of capital, \( \delta \in [0, 1] \). Thus the investments equal the difference between the successive period’s capital stock, \( K_{+1} \), and what is left of the previous period’s stock after depreciation.

The cost of investment \( I \) is defined as a function \( D \) of the ratio between the investment and the capital stock in one period,
\[ D \left( \frac{I}{K} \right) = 1 + \frac{\kappa I}{2K}, \] (11)
where \( \kappa \geq 0 \) is a parameter.\(^{10}\)

The investments are financed by borrowing from the capital market and retained profits. In one period the representative entrepreneur borrows the amount \( B \) and the stock of debt is \( Q \). The representative entrepreneur’s accumulation of debt between two successive periods can be described by the following equation:
\[ Q_{+1} = Q + B, \] (12)
where the value of the new total debt is \( Q_{+1} \).

The representative entrepreneur faces a risk in its borrowing. Therefore, the interest rate related to the debt is defined as a function \( H \) of the risk-free interest rate \( r \) as well as of the ratio between the amount of debt and the capital stock in a period. The interest rate paid by the representative entrepreneur on debt \( Q \) is defined as
\[ H \left( r, \frac{Q}{K} \right) = r + \frac{\gamma Q}{2K}, \] (13)
where \( \gamma \geq 0 \) is a parameter. An increase of the ratio of the borrowed capital to the existing capital stock increases the borrowing interest rate\(^{11}\).

\(^{10}\)This follows from Nickell (1978) as well as from Heijdra and van der Ploeg (2002). The concavity of the adjustment costs of investments guarantees a unique and stable equilibrium.

\(^{11}\)This follows from Yépez (2017). The concavity of the risk premium when borrowing guarantees a unique and stable equilibrium.
The representative entrepreneur produces a homogeneous final good using the production technology \( F(A, K, L) \), which exhibits constant returns-to-scale with regard to the capital stock \( K \) and the labor demand \( L \). The production function \( F(A, K, L) \) is defined as
\[
F(A, K, L) = A[\eta K^{\frac{\sigma - 1}{\sigma}} + (1 - \eta)L^{\frac{\sigma - 1}{\sigma}}]^{\frac{\sigma}{\sigma - 1}},
\]
(14)
where \( A > 0 \) and \( \eta, \sigma \in (0, 1) \) are parameters. Parameter \( A \) expresses the total factor productivity, \( \eta \) the distribution parameter between labor and capital and \( \sigma \) is the constant elasticity of factor substitution.

The partial derivatives of the production function \( F(A, K, L) \), given by equation (14), with regard to \( K \) and \( L \) are denoted by the corresponding subscripts, e.g. \( F_K(A, K, L) = \frac{\partial F(A, K, L)}{\partial K} \). The same pattern concerns second as well as cross partial derivatives.\(^{12}\)

I assume the good to be the numeraire. Then, the representative entrepreneur’s revenue equals the output, i.e.
\[
Y = F(A, K, L),
\]
(15)
while the operative profit of the representative entrepreneur is defined as the difference between the output and the labor costs, i.e.
\[
\pi_E = F(A, K, L) - wL.
\]
(16)

The labor costs as well as the costs of investments and accumulated debt are deducted from the sum of the gross income and new loans. The remaining part of the gross income is taxed at the rate \( T_\pi \), where \( T_\pi \in (-\infty, 1) \). The consumption of the representative entrepreneur is denoted by \( C_E \) and is assumed to be taxed at the rate \( T_C \).

The budget constraint of the representative entrepreneur is
\[
(1 - T_\pi) \left[ F(A, K, L) + B - wL - \left( r + \frac{\gamma Q}{2K} \right) Q - \left( 1 + \frac{\kappa I}{2K} \right) I \right] = (1 + T_C)C_E.
\]
(17)
\(^{12}\)The partial derivatives are calculated in Appendix B.
Based on equations (10) and (12), equation (17) can be rewritten as

\[
(1 - T_\pi) \left[ F(A, K, L) + Q_{+1} - Q - wL - rQ - \frac{\gamma Q^2}{2K} 
- K_{+1} + (1 - \delta)K - \frac{\kappa (K_{+1} - (1 - \delta)K)^2}{2K} \right] = (1 + T_C)C_E
\]  

(18)

or equivalently

\[
C_E = \frac{(1 - T_\pi)}{(1 + T_C)} \left[ F(A, K, L) + Q_{+1} - Q - wL - rQ - \frac{\gamma Q^2}{2K} 
- K_{+1} + (1 - \delta)K - \frac{\kappa (K_{+1} - (1 - \delta)K)^2}{2K} \right].
\]  

(19)

The representative entrepreneur’s utility is a logarithmic function of its own consumption, i.e.

\[
U_E(C_E) = \ln C_E.
\]  

(20)

The periodic utility is discounted by the factor \( \beta = \frac{1}{1+\rho} \), where \( \rho > 0 \) is the rate of time preference.

In one period the representative entrepreneur, which maximizes its lifetime utility, chooses the next period’s capital stock \( K_{+1} \) and debt level \( Q_{+1} \). The maximization is solved by dynamic programming. When choosing \( K_{+1} \) and \( Q_{+1} \), the variables \( T_\pi, T_C, r, Q \) and \( K \) are taken as given. Based on (19) and (20), the maximization problem takes the following form:

\[
\mathcal{V}_E = \mathcal{V}(T_\pi, T_C, r, Q, K)
= \max_{K_{+1}, Q_{+1}} \left\{ \ln C_E + \beta \mathcal{V}(T_{\pi+1}, T_{C+1}, r_{+1}, Q_{+1}, K_{+1}) \right\},
\]  

(21)

where \( \mathcal{V}_E \) is the value function of the representative entrepreneur. The maximizations are carried out in Appendix B.2.

The decision of the representative entrepreneur regarding the capital stock,
$K_{+1}(K, Q, T_\pi, T_C, r)$, is described by the following equation:

\[
(1 - T_{\pi,+1})C_E(1 + T_C) \times \left[ Y_{K,+1} + \frac{\gamma Q_{+1}^2}{2K_{+1}^2} + 1 - \delta \right] - \left( Y_{L,+1} - w_{+1} \right) \left( \alpha \mu Y_{K,+1} + (1 - \alpha) Y_{LK,+1} L_{+1} \right) - \frac{\alpha \nu Y_{K,+1}}{\alpha \nu + 1 - \alpha} + \frac{\kappa (K_{+2} - (1 - \delta) K_{+1})(1 - \delta)}{K_{+1}} + \frac{\kappa (K_{+2} - (1 - \delta) K_{+1})^2}{2K_{+1}^2} = (1 - T_{\pi})(1 + \rho)C_{E,+1}(1 + T_{C,+1}) \left( 1 + \frac{\kappa (K_{+1} - (1 - \delta) K)}{K} \right),
\]

where the expressions for $Y_{K,+1} \doteq F_K(A_{+1}, K_{+1}, L_{+1})$, $Y_{L,+1} \doteq F_L(A_{+1}, K_{+1}, L_{+1})$, $Y_{LL,+1} \doteq F_{LL}(A_{+1}, K_{+1}, L_{+1})$ and $Y_{LK,+1} \doteq F_{LK}(A_{+1}, K_{+1}, L_{+1})$ are based on the CES production function, given by equation (14).

The representative entrepreneur’s decision regarding the stock of debt, $Q_{+1}(Q, K, T_\pi, T_C, r)$, is described by the following equation:

\[
(1 - T_\pi)(1 + \rho)C_{E,+1}(1 + T_{C,+1}) = (1 - T_{\pi,+1})C_E(1 + T_C) \left( 1 + r_{+1} + \frac{\gamma Q_{+1}}{K_{+1}} \right).
\]

2.3 Labor market

Decisions on the labor market are determined through collective bargaining between the labor union, representing the workers, and the employers’ federation, representing the entrepreneurs. The labor market organizations bargain over the wage and the number of employed workers.

The labor union observes the representative worker’s rent $M$ as a function of the wage $w$, the labor income tax $T_L$, the unemployment benefit $Z$ and the level of employment $L$. This rent is defined as the economic gain from working, i.e. how much more money the representative worker earns being
employed instead of unemployed, i.e.

\[
M(T_L, w, Z, L) = [(1 - T_L)w - Z]^\nu L^\mu, \quad (24)
\]

where \(\nu, \mu > 0\) are parameters.

The employers’ federation considers the representative entrepreneur’s operative profit \(\pi_E\), given by equation (16). The wage and the number of workers to be employed are simultaneously determined by an alternative-offers game between the labor union and the employers’ federation\(^\text{13}\). The solution of the game is obtained, following Osborne and Rubinstein (1990), by maximizing the geometric average of the parties’ utilities, i.e.

\[
\alpha \ln \left\{ (1 - T_L)w - Z \right\}^\nu L^\mu \right. + (1 - \alpha) \ln \pi_E \\
= \alpha \nu \ln [ (1 - T_L)w - Z ] + \alpha \mu \ln L + (1 - \alpha) \ln \left[ F(A, K, L) - wL \right], \quad (25)
\]

where the constant \(\alpha \in (0, 1)\) characterizes the relative bargaining power of the labor union. The maximizations are carried out in Appendix B.1.

The maximization with respect to \(w\) gives the optimal wage as a function of the representative entrepreneur’s capital stock \(K\) and the unemployment benefit \(Z\). The wage function \(w(K, Z)\) is defined by the following equation

\[
\alpha \nu (1 - T_L)(Y - wL) - (1 - \alpha)L[(1 - T_L)w - Z] = 0, \quad (26)
\]

where the expression for \(Y = F(A, K, L)\) is given by equation (14).

The maximization with respect to \(L\) gives the optimal number of employed workers as a function of the representative entrepreneur’s capital stock \(K\). The labor function \(L(K)\) is defined by the following equation

\[
\alpha \mu (Y - wL) + (1 - \alpha)(Y_L - w)L = 0, \quad (27)
\]

where the expressions for \(Y = F(A, K, L)\) and \(Y_L = F_L(A, K, L)\) are based on the CES production function, given by equation (14).

\(^{13}\)The data used in this paper are more suitable for calculations based on an efficient bargaining than on a right-to-manage model.
Substituting the optimality condition (27) into (26), gives the following combined optimality condition

\[ \nu(1 - T_L)(Y_L - w) + \mu[(1 - T_L)w - Z] = 0, \]  

(28)

where the expression for \( Y_L = F_L(A, K, L) \) is based on the CES production function, given by equation (14).

2.4 The government

The government collects taxes from the workers and the entrepreneurs. The taxes, paid by the representative worker, are

\[ T_\pi rW, \quad T_L Lw \quad \text{and} \quad T_C C_W. \]

The taxes, paid by the representative entrepreneur, are

\[ T_\pi \left[ F(A, K, L) + Q_{+1} - Q - wL - rQ - \frac{\gamma Q^2}{2K} - K_{+1} + (1 - \delta)K - \frac{\kappa(K_{+1} - (1 - \delta)K)^2}{2K} \right] \quad \text{and} \quad T_C C_E. \]

It is to be noted that the workers’ interest income and the entrepreneurs’ profit are taxed at the same rate \( T_\pi \).

The expenditures for the government are the public expenditures \( G \) and the unemployment benefits \( (N - L)Z \), paid to the unemployed workers. The tax rates \( (T_\pi, T_L, T_C) \) and the public expenditures \( G \) are exogenous. The government’s budget constraint is

\[ T_\pi rW + T_L Lw + T_C C_W + T_\pi \left[ F(A, K, L) + Q_{+1} - Q - wL - rQ - \frac{\gamma Q^2}{2K} - K_{+1} + (1 - \delta)K - \frac{\kappa(K_{+1} - (1 - \delta)K)^2}{2K} \right] + T_C C_E \]

(29)

\[ = G + (N - L)Z, \]

where the production function \( F(A, K, L) \) is given by (14).
2.5 Capital and goods markets

The stock of assets in the capital market is based on the representative worker’s current wealth $W > 0$, the current net amount of foreign capital $X$, and the representative entrepreneur’s current stock of debt $Q > 0$. Hence, the capital market is defined by the equation

$$W + X = Q. \quad (30)$$

The total income of the economy is based on the net value of foreign capital $X_{t+1} - X$ and the value of the production of goods $F(A, K, L)$. The total expenditures are based on the value of interest costs of foreign capital $rX$, the value of private consumption $C_W$ and $C_E$ as well as of government expenditures $G$, the entrepreneur’s extra borrowing cost $\frac{\gamma Q^2}{2K}$ and the entrepreneur’s investment costs $K_{t+1} - (1 - \delta)K + \frac{\kappa [K_{t+1} - (1 - \delta)K]^2}{2K}$. The equation of the single-good market in the open economy is

$$X_{t+1} - X + F(A, K, L) = rX + C_W + C_E + G + \frac{\gamma Q^2}{2K} + K_{t+1} - (1 - \delta)K + \frac{\kappa [K_{t+1} - (1 - \delta)K]^2}{2K}. \quad (31)$$

2.6 Description of the steady state

The model is examined in the steady state of (3), (5), (8), (18), (20), (22), (23), (28), (29), (30) and (31).

3 Calibration

The model is calibrated based on data received from the Federation of Unemployment Funds in Finland (2018), the Finnish Tax Administration (2016, 2018), Statistics Finland (2018) and the Taxpayers Association of Finland (2017). The values, mentioned in Table 2, for parameters $\delta$, $\eta$, $\sigma$ and $\rho$ are based on the paper of Alvarez-Cuadrado et al. (2014). In order to simplify the calculations, the labor income tax is assumed to be 30 % and the consumption tax rate to be 24 %, which is the general rate applied in Finland.
For simplicity I have fragmented the data on consumption and wealth with the assumption that the highest quintile represents the entrepreneurs. This assumption is reasonable as wealth and income level, following Headey and Wooden (2004), correlate when measuring the households’ standard of living.

The real values used in the calibration are shown in Table 1 below and the parameter values in Table 2 below. It should be noted that the euro-values are calibrated based on the Gross Domestic Product being normalized at 1.

### Table 1: Real values used for calibration

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y^*$</td>
<td>Gross Domestic Product</td>
<td>1</td>
</tr>
<tr>
<td>$K^*$</td>
<td>entrepreneurs’ capital stock</td>
<td>1.32519</td>
</tr>
<tr>
<td>$C_W^*$</td>
<td>workers’ consumption</td>
<td>0.31005</td>
</tr>
<tr>
<td>$C_E^*$</td>
<td>entrepreneurs’ consumption</td>
<td>0.14650</td>
</tr>
<tr>
<td>$W^*$</td>
<td>workers’ wealth</td>
<td>0.88494</td>
</tr>
<tr>
<td>$\bar{W}^*$</td>
<td>worker’s median wealth</td>
<td>$3.04189 \times 10^{-7}$</td>
</tr>
<tr>
<td>$Q^*$</td>
<td>entrepreneurs’ stock of debt</td>
<td>0.30342</td>
</tr>
<tr>
<td>$G^*$</td>
<td>public expenditures</td>
<td>0.24090</td>
</tr>
<tr>
<td>$w^*$</td>
<td>wage rate</td>
<td>$1.88962 \times 10^{-7}$</td>
</tr>
<tr>
<td>$Z^*$</td>
<td>unemployment benefits</td>
<td>$1.13377 \times 10^{-7}$</td>
</tr>
<tr>
<td>$L^*$</td>
<td>entrepreneurs’ labor demand</td>
<td>2452400 persons</td>
</tr>
</tbody>
</table>

### Table 2: Parameter values used for calibration

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_L^*$</td>
<td>labor income tax rate</td>
<td>30 %</td>
</tr>
<tr>
<td>$T^*_\pi$</td>
<td>capital income tax rate</td>
<td>20 %</td>
</tr>
<tr>
<td>$T^*_C$</td>
<td>consumption tax rate</td>
<td>24 %</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>wage negotiation power of workers</td>
<td>0.5</td>
</tr>
<tr>
<td>$\delta$</td>
<td>depreciation rate</td>
<td>0.05</td>
</tr>
<tr>
<td>$\eta$</td>
<td>income share of capital</td>
<td>0.423</td>
</tr>
<tr>
<td>$\theta$</td>
<td>ratio between $C^<em>_W$ and $C^</em>_W + G^*$</td>
<td>0.563</td>
</tr>
<tr>
<td>$\rho$</td>
<td>rate of time preference</td>
<td>0.06</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>elasticity of substitution between $K$ and $L$</td>
<td>0.6735</td>
</tr>
<tr>
<td>$\chi$</td>
<td>elasticity of substitution between $C^<em>_W$ and $G^</em>$</td>
<td>0.5</td>
</tr>
</tbody>
</table>
The values received by the calibration are shown in Table 3 below.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A^*$</td>
<td>total factor productivity</td>
<td>0.12825</td>
</tr>
<tr>
<td>$r^*$</td>
<td>real interest rate</td>
<td>0.78644 %</td>
</tr>
<tr>
<td>$N^*$</td>
<td>workers’ labor supply</td>
<td>2933120 persons</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>entrepreneur’s borrowing parameter</td>
<td>0.22770</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>entrepreneur’s investment parameter</td>
<td>140.63737</td>
</tr>
<tr>
<td>$\mu$</td>
<td>labor union weight parameter for labor</td>
<td>0.06219</td>
</tr>
<tr>
<td>$\nu$</td>
<td>labor union weight parameter for wage</td>
<td>0.00891</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>worker’s status parameter</td>
<td>0.03406</td>
</tr>
</tbody>
</table>

4 Results

Based on the introduced model, I simulate four alternative tax reforms, combined with decreased government expenditures. The original situation and the reforms are described in Table 4 below, noting that $G$ is based on the Gross Domestic Product being normalized at 1:

<table>
<thead>
<tr>
<th>Instrument</th>
<th>start</th>
<th>reform 1</th>
<th>reform 2</th>
<th>reform 3</th>
<th>reform 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_L$</td>
<td>30 %</td>
<td>25 %</td>
<td>30 %</td>
<td>30 %</td>
<td>25 %</td>
</tr>
<tr>
<td>$T_\pi^*$</td>
<td>20 %</td>
<td>20 %</td>
<td>19 %</td>
<td>20 %</td>
<td>19 %</td>
</tr>
<tr>
<td>$T_C^*$</td>
<td>24 %</td>
<td>24 %</td>
<td>24 %</td>
<td>22 %</td>
<td>22 %</td>
</tr>
<tr>
<td>$G^*$</td>
<td>0.24090</td>
<td>-0.33 %</td>
<td>-0.33 %</td>
<td>-0.33 %</td>
<td>-1 %</td>
</tr>
</tbody>
</table>

The model of this paper shows that there are a number of positive effects of reforms 1, 2 and 3. Reform 4 combines the positive effects. These are described below.

4.1 Wages

Reform 1 lowers the wage rate, while reforms 2 and 3 have an insignificant impact on the wage rate. The combined result, given by reform 4, represents
a wage decrease of about 6.7%.

The wage rate is only influenced by the labor income tax. This is because labor income taxation is taken into consideration during the wage negotiations on the labor market. Other taxes do not affect the decisions of the labor union or the employers’ federation. Despite the lower gross wage, the change of the after-tax wage is insignificant. These effects are shown in the chart below:

![Chart showing wage changes](chart.png)

### 4.2 Employment

Reforms 1-3 all increase the employment. The combined effect, given by reform 4, represents an increased employment rate of about 4.5 percentage points.

Reform 1 decreases the wage, thus increasing the employment. The decreased capital income tax in reform 2 and consumption tax in reform 3 increase entrepreneurs’ incentives to employ workers. Thus, the bargaining over employment leads to a slight improvement of the employment rate, despite the unchanged wage. All reforms 1-3 support each other. These effects are shown in the chart below:
4.3 Capital stock

Reforms 1-3 all increase the capital stock. The combined effect, given by reform 4, represents an increase of about 6.1%.

Because labor and capital are complements, employment and capital stock shift to the same direction following a tax decrease. These effects are shown in the chart below:
4.4 Gross Domestic Product

Reforms 1-3 all increase the Gross Domestic Product (GDP). The combined effect, given by reform 4, represents an increase of about 6.1%.

The tax reforms influence GDP through labor and capital. These effects are shown in the chart below:

![Gross domestic product chart](chart)

4.5 The representative worker’s wealth

Reforms 1 and 2 both increase the representative worker’s wealth, while reform 3 decreases it. The combined effect, given by reform 4, represents an increase of about 0.2%.

The higher employment rate, due to the lower labor income tax in reform 1, enables the representative worker to increase its wealth. The decrease of the capital income tax in reform 2 directly encourages workers to increase their wealth. Furthermore, because the lower capital income tax slightly increases employment, keeping the wage unchanged, it increases the labor income of the representative worker, encouraging the workers to increase their wealth. The lower consumption tax rate in reform 3 discourages workers from saving, thus decreasing their wealth. These effects are shown in the chart below:
Reforms 1-3 all increase the consumption of the representative worker and the representative entrepreneur. The combined effect, given by reform 4, is beneficial for all households. It increases the workers’ consumption by about 2.3 % and the entrepreneurs’ consumption by about 25.6 %.

Because reforms 1 and 2 increase the employment rate, they increase the representative worker’s income and therefore promote its consumption. However, the increase of income promotes saving more than consumption. The decrease of the consumption taxation in reform 3 creates a strong incentive for the workers to increase their consumption.

Reform 1 generates a higher GDP, encouraging the representative entrepreneur’s consumption. The decrease of the capital income tax in reform 2 and that of the consumption tax in reform 3 increases the representative entrepreneur’s consumption directly. Furthermore, also reforms 2 and 3 increase the GDP, positively affecting the representative entrepreneur’s consumption. These effects are shown in the charts below:
4.7 Welfare

Reforms 1 and 3 are good for both the workers’ and the entrepreneurs’ utility. Reform 2 is bad for the workers, but good for the entrepreneurs. The combined effect, given by reform 4, is positive for all households.

The utility of the representative worker is an increasing function of its consumption, its wealth and government expenditures. With reform 1, the increase of the consumption and wealth dominates over the effect of decreased government expenditures, thus increasing the utility level. With reform 2,
the increases of consumption and wealth do not dominate over the effect of decreased government expenditures. Reform 3 increases the utility level for workers as their consumption faces a significant increase.

The utility of the representative entrepreneur depends directly on its consumption. The changes are shown in the charts below, where the starting levels of the utilities are normalized to 1.
4.8 Discussion

In this paper, I examine the results of tax reforms combined with decreased government expenditures. This is done by a model with collective bargaining, an imperfect capital market with borrowing costs and adjustment costs of investment. Such structural features are uncommon in the literature dealing with taxation. Furthermore, papers considering tax reforms normally ignore the effects of changed government expenditures.

Aronsson et al. (2002) argue that higher progressivity in labor income taxation causes a decrease of the Gross Domestic Product, the capital stock and the private consumption. These changes resemble reform 1 in my paper. If a higher progressivity of the labor income tax is interpreted as an increase of the labor income tax rate, the results of Aronsson et al. (2002) correspond to those of this paper, although they do not separate workers and entrepreneurs.

The findings of Aronsson et al. (2002) are in line with the results of Díaz-Giménez and Pijoan-Mas (2011). These authors conclude that increasing the labor income taxation leads to a smaller Gross Domestic Product, capital stock as well as a lower consumption. These findings are similar to those of my paper, although I, in contrast to Díaz-Giménez and Pijoan-Mas (2011), distinguish between workers and entrepreneurs. A major difference between my paper and that of Díaz-Giménez and Pijoan-Mas (2011) is that their paper uses a growth model, while I do not consider economic growth at all.

Altig et al. (2001) attempt to simplify the tax system so that all types of income are taxed at the same rate. There are five different versions, mainly depending on the extent of possible deductions. Although I do not consider a situation where all types of income are taxed equally, the conclusions of Altig et al. (2001) about the effect on the Gross Domestic Product and the capital stock, are close to those in my paper. However, Altig et al. (2001) assume perfect labor and capital markets, while I assume them to be imperfect.
5 Conclusions

This paper discusses consequences of certain tax reforms. The tax system consists of labor income, capital income and consumption taxes. The effects are in terms of Gross Domestic Product, capital stock, the employment rate, the wage rate, the representative worker’s wealth, private consumption and the welfare of the households. This paper examines the effects of a reform, where the government’s budget is balanced and the taxes on labor income, capital income and consumption as well as government expenditures are decreased.

Decreasing the labor income tax, the capital income tax as well as the consumption tax separately, combined with decreased government expenditures, has positive effects on the Gross Domestic Product, the capital stock, the employment, the consumption of workers and entrepreneurs as well as the utility of the entrepreneurs. However, the reforms’ effects on the workers’ wealth are contradictory. The same concerns the workers’ utility.

Simultaneously decreasing the taxation of labor income, capital income and consumption, combined with decreased government expenditures, increases most economic indicators dealt with in this paper. The exception is the decreased gross wage. This negative effect is, however, compensated by the lower labor income tax.

In contrast to most of the literature on taxation, I combine imperfect labor and capital markets with an open economy. I show that a decreased tax burden, combined with decreased government expenditures, has a number of positive macroeconomic effects. Thus, the reforms examined in this paper indicate what kind of policy actions could be of benefit to the economy.
A Validity of Walras’ law

When checking if Walras’ law holds, one needs to consider the budget constraints of the representative worker, the representative entrepreneur and the government as well as the capital market equation.

The budget constraint of the workers, according to equation (3), is

\[(1 - T_w)rW + (1 - T_L)Lw + (N - L)Z = (1 + T_C)C_W + W_{t+1} - W. \tag{32}\]

The budget constraint of the entrepreneurs, according to equation (18), is

\[(1 - T_e) \left[ F(A, K, L) + \frac{Q_{t+1} - Q + wL - rQ - \frac{\gamma Q^2}{2K}}{2K} - K_{t+1} + (1 - \delta)K - \frac{\kappa(K_{t+1} - (1 - \delta)K)^2}{2K} \right] = (1 + T_c)C_E. \tag{33}\]

The budget constraint of the government, according to equation (29), is

\[T_{\pi} rW + T_L Lw + T_C C_W + T_{\pi} \left[ F(A, K, L) + Q_{t+1} - Q - wL - rQ - \frac{\gamma Q^2}{2K} - K_{t+1} + (1 - \delta)K - \frac{\kappa(K_{t+1} - (1 - \delta)K)^2}{2K} \right] + T_C C_E = G + (N - L)Z. \tag{34}\]

The capital market equation, according to equation (30), is

\[W + X = Q. \tag{35}\]

Adding equations (32), (33) and (34), taking into consideration equation
which describes the general goods market balance outside the steady state.

\[ (1 - T_\pi) rW + (1 - T_L) Lw + (N - L) Z + \]
\[ + (1 - T_\pi) \left[ F(A, K, L) + Q_{+1} - Q - wL - rQ - \frac{\gamma Q^2}{2K} \right] - K_{+1} + (1 - \delta) K - \frac{\kappa (K_{+1} - (1 - \delta) K)^2}{2K} + T_\pi rW + T_L Lw \]
\[ + T_C C_W + T_\pi \left[ F(A, K, L) + Q_{+1} - Q - wL - rQ - \frac{\gamma Q^2}{2K} \right] - K_{+1} + (1 - \delta) K - \frac{\kappa (K_{+1} - (1 - \delta) K)^2}{2K} + T_C C_E \]
\[ = (1 + T_C) C_W + W_{+1} - W + (1 + T_C) C_E + G + (N - L) Z \]

\[ \Leftrightarrow rW + Lw + F(A, K, L) + Q_{+1} - Q - wL - rQ - \frac{\gamma Q^2}{2K} - K_{+1} + (1 - \delta) K - \frac{\kappa (K_{+1} - (1 - \delta) K)^2}{2K} \]
\[ = C_W + W_{+1} - W + C_E + G \]

\[ \Leftrightarrow F(A, K, L) + Q_{+1} - W_{+1} + W - Q - \frac{\gamma Q^2}{2K} - K_{+1} + (1 - \delta) K - \frac{\kappa (K_{+1} - (1 - \delta) K)^2}{2K} \]
\[ = r (Q - W) + C_W + C_E + G \]
\[ \Leftrightarrow X_{+1} - X + F(A, K, L) = r X + C_W + C_E + G + \frac{\gamma Q^2}{2K} + K_{+1} - (1 - \delta) K + \frac{\kappa (K_{+1} - (1 - \delta) K)^2}{2K}, \]

which describes the general goods market balance outside the steady state.

\[ \text{B Solving the Model} \]

The model is below solved step-by-step. The production function \( F(A, K, L) \) is assumed to be approximated using a second degree Taylor approximation
polynomial:


This means that any third or higher degree derivative of the production function must equal zero.

**B.1 Labor market**

Based on equation (25), the maximization problem of the labor market negotiations is

\[
\max_{w, L} \left\{ \alpha \nu \ln (1 - T_L)w - Z + \alpha \mu \ln L + (1 - \alpha) \ln F(A, K, L) - wL \right\}. \tag{38}
\]

Maximization with respect to \(w\) leads to

\[
\frac{\alpha \nu (1 - T_L)}{(1 - T_L)w - Z} - \frac{(1 - \alpha) L}{F(A, K, L) - wL} = 0 \Leftrightarrow \alpha \nu (1 - T_L)(F(A, K, L) - wL) - (1 - \alpha) L((1 - T_L)w - Z) = 0, \tag{39}
\]

where the expression for \(F(A, K, L)\) is given by equation (14).

Equation (39) is the optimality condition for the negotiated wage rate. Totally differentiating equation (39) with respect to \(w, K\) and \(Z\) generates

\[
- \left( \alpha \nu + 1 - \alpha \right) (1 - T_L) L dw + \alpha \nu (1 - T_L) F_K(A, K, L) dK + (1 - \alpha) L dZ = 0. \tag{40}
\]

With \(dZ = 0\), equation (40) can be rewritten as

\[
\frac{\partial w}{\partial K} = \frac{\alpha \nu F_K(A, K, L)}{(\alpha \nu + 1 - \alpha) L}. \tag{41}
\]
With $dK = 0$, equation (40) can be rewritten as

$$\frac{\partial w}{\partial Z} = \frac{1 - \alpha}{(\alpha \nu + 1 - \alpha)(1 - T_L)}.$$  (42)

Based on equations (41) - (42), the wage rate can be expressed as

$$w(K, Z)$$  (43)

Based on the production function (14), the expression for $F_K(A, K, L)$ in equations (40) and (41) is received from the following calculation:

$$F_K(A, K, L) = A \frac{\sigma}{\sigma - 1}[\eta K^{\frac{\sigma - 1}{\sigma}} + (1 - \eta)L^{\frac{\sigma - 1}{\sigma}}]^{\frac{1}{\sigma - 1}} \frac{\sigma - 1}{\sigma} K^{-\frac{1}{\sigma}}$$

$$\Leftrightarrow F_K(A, K, L) = A \eta K^{\frac{\sigma - 1}{\sigma}} + (1 - \eta)L^{\frac{\sigma - 1}{\sigma}}]^{\frac{1}{\sigma - 1}} K^{-\frac{1}{\sigma}}.$$  (44)

which can, based on equation (14), be developed as per the calculation below

$$F_K(A, K, L) = A^{\frac{\sigma - 1}{\sigma}} A^\frac{1}{\sigma} \eta [\eta K^{\frac{\sigma - 1}{\sigma}} + (1 - \eta)L^{\frac{\sigma - 1}{\sigma}}]^{\frac{1}{\sigma - 1}} \sigma K^{-\frac{1}{\sigma}}$$

$$\Leftrightarrow F_K(A, K, L) = \eta A^{\frac{\sigma - 1}{\sigma}} \left( \frac{F(A, K, L)}{K} \right)^{\frac{1}{\sigma}}.$$  (45)

Maximization with respect to $L$ leads to

$$\frac{\alpha \mu}{L} + \frac{(1 - \alpha)(F_L(A, K, L) - w)}{F(A, K, L) - wL} = 0$$

$$\Leftrightarrow \alpha \mu (F(A, K, L) - wL) + (1 - \alpha)(F_L(A, K, L) - w)L = 0,$$  (46)

where the expression for $F(A, K, L)$ is given by equation (14).

Based on production function (14), the expression for $F_L(A, K, L)$ in equation (46) is received from the following calculation:

$$F_L(A, K, L) = A \frac{\sigma}{\sigma - 1}[\eta K^{\frac{\sigma - 1}{\sigma}} + (1 - \eta)L^{\frac{\sigma - 1}{\sigma}}]^{\frac{1}{\sigma - 1}} (1 - \eta) \frac{\sigma - 1}{\sigma} L^{-\frac{1}{\sigma}}$$

$$\Leftrightarrow F_L(A, K, L) = A(1 - \eta)[\eta K^{\frac{\sigma - 1}{\sigma}} + (1 - \eta)L^{\frac{\sigma - 1}{\sigma}}]^{\frac{1}{\sigma - 1}} L^{-\frac{1}{\sigma}}.$$  (47)
which can, based on equation (14), be developed as per the calculation below

\[
F_L(A, K, L) = A \frac{\sigma - 1}{\sigma} A^\frac{1}{2} (1 - \eta)[\eta K^{\frac{\sigma - 1}{\sigma}} + (1 - \eta)L^{\frac{\sigma - 1}{\sigma}}]^{\frac{1}{\sigma - 1}} L^{-\frac{1}{\sigma}} \\
= (1 - \eta)A^{\frac{\sigma - 1}{\sigma}}(F(A, K, L))^{\frac{1}{2}} L^{-\frac{1}{2}} \\
\Leftrightarrow F_L(A, K, L) = (1 - \eta)A^{\frac{\sigma - 1}{\sigma}} \left( \frac{F(A, K, L)}{L} \right)^{\frac{1}{2}}. 
\]

(48)

Equation (46) is the optimality condition for the negotiated amount of workers to be employed by the entrepreneurs. Totally differentiating equation (46) with respect to \( L \) and \( K \) generates

\[
[(\alpha\mu + 1 - \alpha)(F_L(A, K, L) - w) + (1 - \alpha)F_{LL}(A, K, L)L]dL \\
+ [\alpha\mu F_K(A, K, L) + (1 - \alpha)F_{LK}(A, K, L)L]dK = 0 \\
\Leftrightarrow \frac{\partial L}{\partial K} = -\frac{\alpha\mu F_K(A, K, L) + (1 - \alpha)F_{LK}(A, K, L)L}{(\alpha\mu + 1 - \alpha)(F_L(A, K, L) - w) + (1 - \alpha)F_{LL}(A, K, L)L}. 
\]

(49)

where the expression for \( F_K(A, K, L) \) is given by equation (44) and the expression for \( F_L(A, K, L) \) by equation (47). Based on equation (49), the amount of employed workers can be expressed as

\[
L(K). 
\]

(50)

Based on equation (47), the expression for \( F_{LL}(A, K, L) \) in equation (49) can be received from the following calculation:

\[
F_{LL}(A, K, L) = A(1 - \eta)\frac{1}{\sigma - 1}[\eta K^{\frac{\sigma - 1}{\sigma}} + (1 - \eta)L^{\frac{\sigma - 1}{\sigma}}]^{\frac{2}{\sigma - 1}}(1 - \eta) \\
\times \frac{\sigma - 1}{\sigma} L^{-\frac{1}{2}} L^{-\frac{1}{2}} + A(1 - \eta)[\eta K^{\frac{\sigma - 1}{\sigma}} + (1 - \eta)L^{\frac{\sigma - 1}{\sigma}}]^{\frac{1}{\sigma - 1}}(1 - \eta)L^{-\frac{\sigma - 1}{\sigma}} \\
\Leftrightarrow F_{LL}(A, K, L) = A\frac{(1 - \eta)^2}{\sigma}[\eta K^{\frac{\sigma - 1}{\sigma}} + (1 - \eta)L^{\frac{\sigma - 1}{\sigma}}]^{\frac{2}{\sigma - 1}} L^{-\frac{2}{\sigma}} \\
- A\frac{(1 - \eta)}{\sigma}[\eta K^{\frac{\sigma - 1}{\sigma}} + (1 - \eta)L^{\frac{\sigma - 1}{\sigma}}]^{\frac{1}{\sigma - 1}} L^{-\frac{\sigma - 1}{\sigma}}. 
\]

(51)

Furthermore, based on equation (47), the expression for \( F_{LK}(A, K, L) \) in equation (49) can be received from the following calculation:

\[
F_{LK}(A, K, L) = A(1 - \eta)\frac{1}{\sigma - 1}[\eta K^{\frac{\sigma - 1}{\sigma}} + (1 - \eta)L^{\frac{\sigma - 1}{\sigma}}]^{\frac{1}{\sigma - 1}} \frac{\sigma - 1}{\sigma} K^{-\frac{1}{2}} L^{-\frac{1}{2}} \\
\Leftrightarrow F_{LK}(A, K, L) = A\frac{(1 - \eta)^2}{\sigma}[\eta K^{\frac{\sigma - 1}{\sigma}} + (1 - \eta)L^{\frac{\sigma - 1}{\sigma}}]^{\frac{2}{\sigma - 1}} K^{-\frac{1}{2}} L^{-\frac{1}{2}}. 
\]

(52)
Taking the logarithm of equation (48) gives

\[
\log(F_L(A, K, L)) = \log(1 - \eta) + \frac{\sigma - 1}{\sigma} \log A + \frac{1}{\sigma} (\log(F(A, K, L)) - \log L). \tag{53}
\]

Differentiating equation (53) with regard to \(L\) gives the expression for \(F_{LL}(A, K, L)\) based on the following calculation:

\[
\frac{F_{LL}(A, K, L)}{F_L(A, K, L)} = \frac{1}{\sigma} \left( \frac{F_L(A, K, L)}{F(A, K, L)} - \frac{1}{L} \right) \Rightarrow F_{LL}(A, K, L) = \frac{F_L(A, K, L)}{\sigma} \left( \frac{F_L(A, K, L)}{F(A, K, L)} - \frac{1}{L} \right). \tag{54}
\]

Differentiating equation (53) with regard to \(K\) gives the expression for \(F_{LK}(A, K, L)\) based on the following calculation:

\[
\frac{F_{LK}(A, K, L)}{F_L(A, K, L)} = \frac{1}{\sigma} \frac{F_K(A, K, L)}{F(A, K, L)} \Rightarrow F_{LK}(A, K, L) = \frac{F_L(A, K, L)}{\sigma F(A, K, L)} F_K(A, K, L). \tag{55}
\]

Furthermore, the optimality conditions (39) and (46) can be combined as per the below calculation:

\[
- \nu (1 - T_L) (1 - \alpha) (F_L(A, K, L) - w) L \mu - (1 - \alpha) L (1 - T_L) w - Z = 0 \\
\Leftrightarrow \nu (1 - T_L) (F_L(A, K, L) - w) + \mu (1 - T_L) w - Z = 0. \tag{56}
\]

B.2 Entrepreneurs’ stock of capital and debt; workers’ wealth

When choosing the stock of capital \(K_{+1}\) and debt \(Q_{+1}\) the state variables of the representative entrepreneur are \(T_{\pi}, T_C, r, Q\) and \(K\). Based on (19) and (20), the maximization problem takes the following form:

\[
V_E = V(T_{\pi}, T_C, r, Q, K) = \max_{K_{+1}, Q_{+1}} \left\{ \ln C_E + \beta V(T_{\pi, +1}, T_{C, +1}, r_{+1}, Q_{+1}, K_{+1}) \right\}, \tag{57}
\]
where
\[
C_E = \frac{(1 - T_\pi)}{(1 + T_C)} \left[ F(A, K, L) + Q_{+1} - Q - wL - rQ - \frac{\gamma Q^2}{2K} \right.
\]
\[
- K_{+1} + (1 - \delta)K - \frac{\kappa(K_{+1} - (1 - \delta)K)^2}{2K} \right].
\]

The production function \( F(A, K, L) \) is expressed by formula (14), \( \beta = \frac{1}{1+\rho} \) is the discount factor and \( \rho \) is the rate of time preference.

Maximization of the value function (57) with respect to \( K_{+1} \), taking into consideration equation (19), leads to
\[
\frac{(1 - T_\pi)}{C_E(1 + T_C)} \left( -1 - \frac{\kappa(K_{+1} - (1 - \delta)K)}{K} \right) + \beta \frac{\partial V_{E,+1}}{\partial K_{+1}} = 0. \quad (58)
\]

The capital stock \( K_{+1} \) is dependent on the previous capital stock \( K \), i.e. \( K_{+1} \) can be expressed as the decision rule \( K_{+1}(K) \). Now the value function (57) is differentiated with respect to \( K \), keeping the decision rule \( K_{+1}(K) \) and equations (19), (41), (49) and (58) in mind.

\[
\frac{\partial V_E}{\partial K} = \frac{(1 - T_\pi)}{C_E(1 + T_C)} \left[ \frac{\partial F(A, K, L)}{\partial K} + \frac{\partial F(A, K, L)}{\partial L} \right] \frac{\partial L}{\partial K} - \frac{\partial w}{\partial KL} - w \frac{\partial L}{\partial K}
\]
\[
+ \frac{\gamma Q^2}{2K^2} - \frac{\partial K_{+1}}{\partial K} + 1 - \delta - \frac{\kappa(K_{+1} - (1 - \delta)K)(\partial K_{+1} - (1 - \delta))}{K}
\]
\[
+ \frac{\kappa(K_{+1} - (1 - \delta)K)^2}{2K^2} \right] + \beta \frac{\partial V_{E,+1}}{\partial K_{+1}} \frac{\partial K_{+1}}{\partial K}
\]
\[
= \frac{(1 - T_\pi)}{C_E(1 + T_C)} \left[ F_K(A, K, L) + (F_L(A, K, L) - w) \frac{\partial L}{\partial K} - \frac{\partial w}{\partial KL} + \frac{\gamma Q^2}{2K^2}
\]
\[
+ 1 - \delta + \frac{\kappa(K_{+1} - (1 - \delta)K)(1 - \delta)}{K} + \frac{\kappa(K_{+1} - (1 - \delta)K)^2}{2K^2}
\]
\[
+ \left[ \frac{(1 - T_\pi)}{C_E(1 + T_C)} \left( -1 - \frac{\kappa(K_{+1} - (1 - \delta)K)}{K} \right) + \beta \frac{\partial V_{E,+1}}{\partial K_{+1}} \right] \frac{\partial K_{+1}}{\partial K}.
\]
\[
\Rightarrow \frac{\partial V_E}{\partial K} = \frac{(1 - T_\pi)}{C_E(1 + T_C)} \left[ F_K(A, K, L) + (F_L(A, K, L) - w) \frac{\partial L}{\partial K} \right.
\]

\[
- \frac{\partial w}{\partial K} L + \frac{\gamma Q^2}{2K^2} + 1 - \delta + \frac{\kappa(K_{+1} - (1 - \delta)K)(1 - \delta)}{K}
\]

\[
+ \frac{\kappa(K_{+1} - (1 - \delta)K)^2}{2K^2} \right].
\] (59)

Forwarding equation (59) by one period generates

\[
\frac{\partial V_{E,+1}}{\partial K_{+1}} = \frac{(1 - T_{\pi,+1})}{C_{E,+1}(1 + T_{C,+1})} \left[ F_K(A_{+1}, K_{+1}, L_{+1}) - \frac{\partial w_{+1}}{\partial K_{+1}} L_{+1} + \frac{\gamma Q^2_{+1}}{2K^2_{+1}} \right.
\]

\[
+ (F_L(A_{+1}, K_{+1}, L_{+1}) - w_{+1}) \frac{\partial L_{+1}}{\partial K_{+1}} + 1 - \delta
\]

\[
+ \frac{\kappa(K_{+2} - (1 - \delta)K_{+1})(1 - \delta)}{K_{+1}} + \frac{\kappa(K_{+2} - (1 - \delta)K_{+1})^2}{2K^2_{+1}} \right].
\] (60)

and substituting the received equation into the first-order condition (58), taking into consideration the fact that \( \beta = \frac{1}{1 + \rho} \), leads to

\[
- \frac{(1 - T_\pi)}{C_E(1 + T_C)} \left( -1 - \frac{\kappa(K_{+1} - (1 - \delta)K)}{K} \right) + \frac{1}{1 + \rho} \times \frac{(1 - T_{\pi,+1})}{C_{E,+1}(1 + T_{C,+1})}
\]

\[
\times \left[ F_K(A_{+1}, K_{+1}, L_{+1}) + (F_L(A_{+1}, K_{+1}, L_{+1}) - w_{+1}) \frac{\partial L_{+1}}{\partial K_{+1}} \right.
\]

\[
- \frac{\partial w_{+1}}{\partial K_{+1}} L_{+1} + \frac{\gamma Q^2_{+1}}{2K^2_{+1}} + 1 - \delta + \frac{\kappa(K_{+2} - (1 - \delta)K_{+1})(1 - \delta)}{K_{+1}}
\]

\[
+ \frac{\kappa(K_{+2} - (1 - \delta)K_{+1})^2}{2K^2_{+1}} \right] = 0
\]

\[
\Rightarrow -(1 - T_\pi)(1 + \rho)C_{E,+1}(1 + T_{C,+1}) \left( 1 + \frac{\kappa(K_{+1} - (1 - \delta)K)}{K} \right)
\]

\[
+ (1 - T_{\pi,+1})C_E(1 + T_C) \times \left[ Y_{K,+1} + \frac{\gamma Q^2}{2K^2_{+1}} + 1 - \delta
\]

\[
- (Y_{L,+1} - w_{+1}) \left( \alpha \mu Y_{K,+1} + (1 - \alpha)Y_{L,K,+1}L_{+1} \right)
\]

\[
- (\alpha \mu + 1 - \alpha)(Y_{L,+1} - w_{+1}) + (1 - \alpha)Y_{L,L,+1}L_{+1}
\]

\[
- \frac{\alpha \mu Y_{K,+1}}{\alpha \mu + 1 - \alpha} + \frac{\kappa(K_{+2} - (1 - \delta)K_{+1})(1 - \delta)}{K_{+1}}
\]

\[
+ \frac{\kappa(K_{+2} - (1 - \delta)K_{+1})^2}{2K^2_{+1}} \right] = 0,
\]
where the expression for $Y_{K,+1} \equiv F_K(A_{+1}, K_{+1}, L_{+1})$ is based on equation (44), the expression for $Y_{L,+1} \equiv F_L(A_{+1}, K_{+1}, L_{+1})$ is based on equation (47), the expression for $Y_{LL,+1} \equiv F_{LL}(A_{+1}, K_{+1}, L_{+1})$ is based on equation (51) and the expression for $Y_{LK,+1} \equiv F_{LK}(A_{+1}, K_{+1}, L_{+1})$ is based on equation (52).

Equation (60) is the optimality condition for the representative entrepreneur’s capital stock. Based on equations (19) and (60), the capital stock can be expressed as

$$K_{+1}(K, Q, T, T_C, r) \quad (61)$$

Maximization of the value function (57) with respect to $Q_{+1}$, taking into consideration equation (19), leads to

$$\frac{(1 - T_π)}{C_E(1 + T_C)} + \beta \frac{\partial V_{E,+1}}{\partial Q_{+1}} = 0. \quad (62)$$

The stock of debt $Q_{+1}$ is dependent on the previous stock of debt $Q$, i.e. $Q_{+1}$ can be expressed as the decision rule $Q_{+1}(Q)$. Now the value function (57) is differentiated with respect to $Q$, keeping the decision rule $Q_{+1}(Q)$ and equations (19) and (62) in mind.

$$\frac{\partial V_{E}}{\partial Q} = \frac{(1 - T_π)}{C_E(1 + T_C)} \left[ \frac{\partial Q_{+1}}{\partial Q} - 1 - r - \gamma \frac{Q}{K} \right] + \beta \frac{\partial V_{E,+1}}{\partial Q_{+1}} \frac{\partial Q_{+1}}{\partial Q}$$

$$= -\frac{(1 - T_π)}{C_E(1 + T_C)} \left[ 1 + r + \gamma \frac{Q}{K} \right] + \frac{(1 - T_π)}{C_E(1 + T_C)} \left[ \frac{\partial V_{E,+1}}{\partial Q_{+1}} \frac{\partial Q_{+1}}{\partial Q} \right]$$

$$\Leftrightarrow \frac{\partial V_{E}}{\partial Q} = -\frac{(1 - T_π)}{C_E(1 + T_C)} \left[ 1 + r + \gamma \frac{Q}{K} \right]. \quad (63)$$

Forwarding equation (63) by one period generates

$$\frac{\partial V_{E,+1}}{\partial Q_{+1}} = -\frac{(1 - T_π,+1)}{C_{E,+1}(1 + T_{C,+1})} \left[ 1 + r_{+1} + \gamma \frac{Q_{+1}}{K_{+1}} \right]$$

and substituting the received equation into the first-order condition (62), taking into consideration the fact that $\beta = \frac{1}{1+\rho}$, leads to

$$\frac{(1 - T_π)}{C_E(1 + T_C)} - \frac{(1 - T_π,+1)}{(1 + \rho)C_{E,+1}(1 + T_{C,+1})} \left[ 1 + r_{+1} + \gamma \frac{Q_{+1}}{K_{+1}} \right] = 0$$

36
\[ (1 - T_\pi)(1 + \rho)C_{E,+1}(1 + T_{C,+1}) \]
\[ - (1 - T_{\pi,+1})C_E(1 + T_C) \left[ 1 + r_{+1} + \gamma \frac{Q_{+1}}{K_{+1}} \right] = 0. \] (64)

Equation (64) is the optimality condition for the representative entrepreneur’s decision concerning borrowing. Based on equations (19) and (64), the decision concerning the stock of debt of the representative entrepreneur can be expressed as

\[ Q_{+1}(Q, K, T_\pi, T_C, r) \] (65)

When choosing the wealth \( W_{+1} \) the state variables of the representative worker are \( T_L, T_\pi, T_C, r, Z, g, W \) and \( W \). Based on (4) and (5), the maximization problem becomes

\[ V_W = V(T_L, T_\pi, T_C, r, Z, g, W, W) \]
\[ = \max_{W_{+1}} \left\{ C_Wf(g) + \varphi(\ln W - \ln N - \ln \overline{W}) \right. \]
\[ \left. + \beta V(T_{L,+1}, T_{\pi,+1}, T_{C,+1}, r_{+1}, Z_{+1}, g_{+1}, \overline{W}_{+1}, W_{+1}) \right\}, \]

where

\[ C_W = \frac{W}{(1 + T_C)} - \frac{W_{+1}}{(1 + T_C)} + \frac{(1 - T_\pi)rW}{(1 + T_C)} \]
\[ + \frac{(1 - T_L)Lw + (N - L)Z}{(1 + T_C)} \]

and \( \beta = \frac{1}{1 + \rho} \) being the discount factor and \( \rho \) the rate of time preference.

Maximization of the value function (66) with respect to \( W_{+1} \) leads to

\[ - \frac{f(g)}{1 + T_C} + \beta \frac{\partial V_{W,+1}}{\partial W_{+1}} = 0. \] (67)

Based on the first-order condition (67), the wealth \( W_{+1} \) is dependent on \( W \), i.e. \( W_{+1} \) can be expressed as the decision rule \( W_{+1}(W) \). Now the value function (66) is differentiated with respect to \( W \), keeping the decision rule
\( W_{t+1}(W) \) and equation (67) in mind.

\[
\frac{\partial \mathcal{V}_W}{\partial W} = f(g) \left[ \frac{1}{1 + T_C} - \frac{\partial W_{t+1}}{\partial W} \right] + \frac{(1 - T_\pi)r}{1 + T_C} + \frac{\varphi}{W} + \beta \frac{\partial \mathcal{V}_{W_{t+1}}}{\partial W_{t+1}} \frac{\partial W_{t+1}}{\partial W} \\
= \frac{f(g)(1 + (1 - T_\pi)r)}{1 + T_C} + \frac{\varphi}{W} + \left[ - \frac{f(g)}{1 + T_C} + \beta \frac{\partial \mathcal{V}_{W_{t+1}}}{\partial W_{t+1}} \right] \frac{\partial W_{t+1}}{\partial W} \\
\iff \frac{\partial \mathcal{V}_W}{\partial W} = \frac{f(g)(1 + (1 - T_\pi)r)}{1 + T_C} + \frac{\varphi}{W}. \tag{68}
\]

Forwarding equation (68) by one period generates

\[
\frac{\partial \mathcal{V}_{W_{t+1}}}{\partial W_{t+1}} = \frac{f(g_{t+1})(1 + (1 - T_{\pi,t+1})r_{t+1})}{1 + T_{C,t+1}} + \frac{\varphi}{W_{t+1}}
\]

and substituting the received equation into the first-order condition (67), taking into consideration the fact that \( \beta = \frac{1}{1 + \rho} \), leads to

\[
- \frac{f(g)}{1 + T_C} + \frac{1}{1 + \rho} \left( \frac{f(g_{t+1})(1 + (1 - T_{\pi,t+1})r_{t+1})}{1 + T_{C,t+1}} + \frac{\varphi}{W_{t+1}} \right) = 0 \iff \frac{f(g)(1 + \rho)}{1 + T_C} = \frac{f(g_{t+1})(1 + (1 - T_{\pi,t+1})r_{t+1})}{1 + T_{C,t+1}} + \frac{\varphi}{W_{t+1}}. \tag{69}
\]

Equation (69) is the Euler equation for the representative worker.

### C Comparative statics

The comparative statics analysis will here be performed by first collecting the steady state equations and then utilizing a numerical mathematical calculation program. The following budget constraints, utility functions, optimality conditions and market equilibrium conditions will be transformed into their steady state counterparts: (3), (5), (6), (8), (18), (20), (22), (23), (28), (29), (30) and (31).

Based on equation (3), the steady state budget for the representative worker is expressed by

\[
(1 - T_\pi^*)r^*W^* + (1 - T_L^*)L^*w^* + (N^* - L^*)Z^* = (1 + T_C^*)C_W^* + W^* - W^*
\]
\[(1 - T^*_\pi) r^* W^* + (1 - T^*_L) L^* w^* + (N^* - L^*) Z^* - (1 + T^*_C) C^*_W = 0. \tag{70}\]

Based on equations (5), (6) and the fact that \( g = \frac{G}{C_W} \), the steady state utility of the representative worker can be expressed by

\[ U^*_W - C^*_W [\theta + (1 - \theta) \left( \frac{G^*}{C^*_W} \right) \frac{\lambda}{\lambda^*} - \varphi (\ln W^* - \ln N^* - \ln W^*)] = 0. \tag{71} \]

Applying steady state values to the Euler equation for the representative worker, i.e. equation (8) and taking into consideration equation (6), \( g = \frac{G}{C_W} \) and the fact that \( \beta = \frac{1}{1 + \rho} \), the following calculation is received

\[ f(g^*)(1 + \rho) = f(g^*)(1 + (1 - T^*_\pi)r^*) + \frac{\varphi}{W^*} \]

\[ \Leftrightarrow f(g^*) (1 + \rho) W^* = f(g^*) (1 + (1 - T^*_\pi)r^*) W^* + \varphi (1 + T^*_c) \]

\[ \Leftrightarrow f(g^*) \rho W^* = f(g^*) (1 - T^*_\pi)r^* W^* + \varphi (1 + T^*_c) \]

\[ \Leftrightarrow \left[ \theta + (1 - \theta) \left( \frac{G^*}{C^*_W} \right) \frac{\lambda}{\lambda^*} \right] \frac{\lambda^*_W}{\lambda} W^* (\rho - (1 - T^*_\pi)r^*) - \varphi (1 + T^*_c) = 0. \tag{72} \]

Based on equation (18), the steady state budget for the representative entrepreneur is received by the calculation below:

\[(1 - T^*_\pi) \left[ Y^* + Q^* - Q^* - w^* L^* - r^* Q^* - \frac{\gamma(Q^*)^2}{2K^*} \right. \]

\[ - K^* + (1 - \delta) K^* - \frac{\kappa(K^* - (1 - \delta)K^*)^2}{2K^*} \] \[= (1 + T^*_C) C^*_E \]

\[ \Leftrightarrow (1 - T^*_\pi) \left[ Y^* - w^* L^* - r^* Q^* - \frac{\gamma(Q^*)^2}{2K^*} \right. \]

\[- \delta K^* - \frac{\kappa\delta^2(K^*)^2}{2K^*} \] \[= (1 + T^*_C) C^*_E \]

\[ \Leftrightarrow (1 - T^*_\pi) \left[ Y^* - w^* L^* - r^* Q^* - \frac{\gamma(Q^*)^2}{2K^*} - \left( \delta + \frac{\kappa\delta^2}{2} \right) K^* \right. \]

\[- (1 + T^*_C) C^*_E = 0, \tag{73} \]

where the expression for \( F(A, K, L) \) is given by equation (14).
Based on equation (20), the steady state utility of the representative entrepreneur can be expressed by

\[ U^*_E - \ln C^*_E = 0. \]  

(74)

The entrepreneur’s optimality condition for the capital stock (22) in steady state is received by the calculation below:

\[
(1 - T^*_c)C^*_E(1 + T^*_c) \times \left[ Y^*_K + \frac{\gamma(Q^*)^2}{2(K^*)^2} + 1 - \delta \right]

- \left( Y^*_L - w^* \right) \left( \alpha \mu Y^*_K + (1 - \alpha)Y^*_{LK}L^* \right)

- \left( \alpha \mu + 1 - \alpha \right) Y^*_L - \left( 1 - \alpha \right) Y^*_{LL}L^*

- \frac{\alpha \nu Y^*_K}{\alpha \nu + 1 - \alpha} + \frac{\kappa K^*}{K^*} \left( 1 - \delta \right) \left( \kappa + \frac{\delta^2(K^*)^2}{2(K^*)^2} \right) = (1 + \rho)(1 + \kappa \delta K^*)

\[
\iff Y^*_K + \frac{\gamma(Q^*)^2}{2(K^*)^2} + 1 - \delta - \left( Y^*_L - w^* \right) \left( \alpha \mu Y^*_K + (1 - \alpha)Y^*_{LK}L^* \right)

- \frac{\alpha \nu Y^*_K}{\alpha \nu + 1 - \alpha} + \frac{\kappa \delta K^*}{K^*} \left( 1 - \delta \right) \left( \frac{\kappa \delta^2}{2(K^*)^2} \right) = (1 + \rho)(1 + \kappa \delta)

\[
\iff Y^*_K + \frac{\gamma(Q^*)^2}{2(K^*)^2} + 1 - \delta - \left( Y^*_L - w^* \right) \left( \alpha \mu Y^*_K + (1 - \alpha)Y^*_{LK}L^* \right)

- \frac{\alpha \nu Y^*_K}{\alpha \nu + 1 - \alpha} + \frac{\kappa \delta}{1 - \delta} \left( \frac{\kappa \delta^2}{2} \right) = 1 + \kappa \delta + \rho + \rho \kappa \delta

\]
\[ Y_K^* + \frac{\gamma(Q^*)^2}{2(K^*)^2} - \frac{(Y_L^* - w^*)}{(\alpha \mu + 1 - \alpha)(Y_L^* - w^*) + (1 - \alpha)Y_{LL}^*L^*} \]

\[ = \frac{\alpha \nu Y_K^*}{\alpha + 1 - \alpha} - \frac{\delta}{2} - \rho - \rho \kappa \delta = 0, \] 

where the expression for \( Y_K \equiv F_K(A, K, L) \) is given by equation (44), the expression for \( Y_L \equiv F_L(A, K, L) \) is given by equation (47), the expression for \( Y_{LL} \equiv F_{LL}(A, K, L) \) is given by equation (51) and the expression for \( Y_{LK} \equiv F_{LK}(A, K, L) \) is given by equation (52).

The entrepreneur’s optimality condition for the stock of debt (23) in steady state is received by the calculation below:

\[ (1 - T^*_\pi)(1 + \rho)C_E^*(1 + T^*_C) \]

\[ = (1 - T^*_\pi)C_E^*(1 + T^*_C) \left[ 1 + r^* + \gamma \frac{Q^*}{K^*} \right] \]

\[ \Leftrightarrow 1 + \rho = 1 + r^* + \gamma \frac{Q^*}{K^*} \]

\[ \Leftrightarrow (\rho - r^*)K^* - \gamma Q^* = 0. \] 

The combined optimality condition for the labor market (28) in steady state is

\[ \nu(1 - T^*_L)(Y_L^* - w^*) + \mu(1 - T^*_L)w^* - Z^* = 0, \] 

where the expression for \( Y_L \equiv F_L(A, K, L) \) is given by equation (47).

Based on equation (29), the steady state budget for the government is expressed by

\[ T^*_\pi r^* W^* + T^*_L w^* + T^*_C C_W^* + T^*_\pi \left[ Y^* + Q^* - Q^* - w^* L^* \right] \]

\[ - r^* Q^* - \frac{\gamma(Q^*)^2}{2K^*} - K^* + (1 - \delta)K^* - \frac{\kappa(K^* - (1 - \delta)K^*)^2}{2K^*} \]

\[ + T^*_C C_E^* = G^* + (N - L^*)Z^* \]
Based on equation (30), the steady state equation for the capital market is:

\[ W^* + X^* - Q^* = 0. \]  

(79)

The steady state version of the goods market equilibrium condition, based on equations (31) and (79), is

\[ X^* - X^* + Y^* = r^*X^* + C^*_W + C^*_E + G^* + \frac{\gamma (Q^*)^2}{2K^*} + K^* - (1 - \delta)K^* \]
\[ + \frac{\kappa (K^* - (1 - \delta)K^*)^2}{2K^*} \]
\[ \iff Y^* = r^*(Q^* - W^*) + C^*_W + C^*_E + G^* + \frac{\gamma (Q^*)^2}{2K^*} + \delta K^* + \frac{\kappa \delta^2 (K^*)^2}{2K^*} \]
\[ \iff Y^* + r^*W^* - r^*Q^* - C^*_W - C^*_E - G^* \]
\[ - \frac{\gamma (Q^*)^2}{2K^*} - (\delta + \frac{\kappa \delta^2}{2})K^* = 0, \]  

(80)

where the expression for \( Y = F(A, K, L) \) is given by equation (14).

Based on the equations (70) - (78), and (80), the following system of steady state equations is received:
\[
(1 - T^*_π)r^*W^* + (1 - T^*_L)L^*w^* + (N^* - L^*)Z^* - (1 + T^*_C)C^*_W = 0 \quad \Rightarrow \quad f^1
\]
\[
U^*_W - C^*_W[\theta + (1 - \theta)\left(\frac{G^*_W}{C^*_W}\right)^{\frac{\lambda - 1}{\rho}} - \varphi(\ln W^* - \ln N^* - \ln \bar{W})] = 0 \quad \Rightarrow \quad f^2
\]
\[
[\theta + (1 - \theta)\left(\frac{G^*_W}{C^*_W}\right)^{\frac{\lambda - 1}{\rho}}]^{\frac{1}{\gamma - 1}} W^*\left(\rho - (1 - T^*_π)r^*\right) - \varphi(1 + T^*_C) = 0 \quad \Rightarrow \quad f^3
\]
\[
(1 - T^*_π)\left[Y^* - w^*L^* - r^*Q^* - \frac{\gamma(Q^*)^2}{2K^*} - \left(\delta + \frac{\kappa^2}{2}\right)K^*\right] - (1 + T^*_C)C^*_E = 0 \quad \Rightarrow \quad f^4
\]
\[
U^*_E - \ln C^*_E = 0 \quad \Rightarrow \quad f^5
\]
\[
Y^*_K + \frac{\gamma(Q^*)^2}{2K^*} - \frac{(Y^*_L - w^*)}{\alpha (Y^*_K + (1 - \alpha)Y^*_L L^*)} - \frac{\alpha \nu Y^*_K}{\alpha v + 1 - \alpha} - \delta - \frac{\nu \delta^2}{2}
\]
\[
- \rho - \rho \kappa \delta = 0 \quad \Rightarrow \quad f^6
\]
\[
(\rho - r^*)K^* - \gamma Q^* = 0 \quad \Rightarrow \quad f^7
\]
\[
\nu(1 - T^*_L)(Y^*_L - w^*) + \mu((1 - T^*_L)w^* - Z^*) = 0 \quad \Rightarrow \quad f^8
\]
\[
T^*_π r^*W^* + T^*_π L^*w^* + T^*_C C^*_W + T^*_π \left[Y^* - w^*L^* - r^*Q^* - \frac{\gamma(Q^*)^2}{2K^*} - \left(\delta + \frac{\kappa^2}{2}\right)K^*\right]
\]
\[
+ T^*_C C^*_E - G^* - (N - L^*)Z^* = 0 \quad \Rightarrow \quad f^9
\]
\[
Y^* + r^*W^* - r^*Q^* - C^*_W - C^*_E - G^* - \frac{\gamma(Q^*)^2}{2K^*} - \left(\delta + \frac{\kappa^2}{2}\right)K^* = 0 \quad \Rightarrow \quad f^{10}
\]
where the formulas for \(Y^* \equiv F(A^*, K^*, L^*)\), \(Y^*_K \equiv F_K(A^*, K^*, L^*)\), \(Y^*_L \equiv F_L(A^*, K^*, L^*)\), \(Y^*_{LL} \equiv F_{LL}(A^*, K^*, L^*)\) and for \(Y^*_{LK} \equiv F_{LK}(A^*, K^*, L^*)\) are based on the CES production function from (14).
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