Environmental Tax in a General Oligopoly Equilibrium Model

Minna Mäkelä

University of Helsinki

Faculty of Social Sciences

Department of Economics

Master's Thesis

May 2019

HELSINGIN YLIOPISTO
HELSINGFORS UNIVERSITET
UNIVERSITY OF HELSINKI

Tiedekunta/Osasto – Fakultet/Sektion – Faculty	Laitos – Institution – Department
Faculty of Social Sciences	Department of Economics

Tekijä – Författare – Author
Minna Mäkelä

Työn nimi – Arbetets titel – Title
An Environmental Tax in a General Oligopoly Equilibrium Model

Oppiaine – Läroämne – Subject
Economics

Työn laji – Arbetets art – Level	Aika – Datum – Month and year	Sivumäärä – Sidoantal – Number of pages
Master’s Thesis	May 2019	29

Tiivistelmä – Referat – Abstract

The aim of this paper is to show that increasing trade and having strict environmental policy are not mutually exclusive. I examine how the increase of trade under general oligopolistic equilibrium model affects environmental taxation. In particular, I show that with common assumptions, growth in trade increases environmental taxes.

In Neary’s (2015) general oligopolistic equilibrium model, there is a continuum of sectors in which the firms compete in Cournot manner. The firms in each sector are identical, having market power in their own sector, but not at the level of the whole economy. The sectors are otherwise identical, but open sectors trade with a foreign country, while the rest of the sectors are closed. There are two identical countries which I call Home and Foreign.

An environmental policy maker has perfect knowledge of the markets and the firms’ behavior. Because there is no abatement technology in the model, the policy maker sets a tax at the first and firms react to it at the second stage.

The main result is that with general assumptions, increasing trade leads to increasing environmental tax. When the transaction costs of trade decrease, environmental taxes increase in the closed sectors, but decrease in the open sectors, the less, the larger the proportion of trading sectors in the economy is. When new sectors are opened to trade, then, provided that the share of domestic pollutant and the marginal environmental damage are sufficiently high, environmental taxes increase both in open and closed sectors.

The main sources of the study are Neary (2015), which presents the general oligopolistic equilibrium model used in the analysis, and Richter (2015), which constructs strategic environmental policy in that setup.

Avainsanat – Nyckelord – Keywords
Trade and environment, general oligopolistic equilibrium, environmental policy

CONTENTS

Symbols used in the text

z	a sector
U	utility
n	the number of firms
x	output
I	income of a representative household
x(z)	a good produced in sector z
$\mathrm{p(z)}$	a price of a product $\mathrm{x(z)}$
λ	the marginal utility of income
D(E)	an environmental damage function
E(z)	damage caused to the environment by sector z
$e_i(z)$	emissions by a firm *i* in sector z
$x_i(z)$	a product produced and sold by firm *i* in domestic market
$y_i(z)$	a product produced domestically by a firm *i* but sold in a foreign market
φ	perceived environmental damage
γ	the percentage of pollution produced outside Home country's borders
t	environmental tax
$\pi_i(z)$	individual firm's profit function in sector z
w	wage
C	costs
η	an elasticity of demand
d	trade costs
L	labor supply

1. INTRODUCTION

The quality of the environment raises concerns, not only among scientists and activists, but also with the general public. Still, increasing trade and providing jobs appears to be enough reason to compromise environmental quality. This acceptance of lowered environmental standards stems from a belief that there must be a trade-off between high economic activities and environmental policies. The aim of this paper is to show that under a general oligopolistic equilibrium model and with certain assumptions, increasing trade increases environmental tax, too.

The principal idea in the general oligopolistic equilibrium model by Neary (2015) is that there is a continuum of sectors, and companies have market power in their own sector, but not at a national level. This means that firms can act strategically only in their own sector but must take wages and factor prices as given.

Of course, there are numerous excellent studies on oligopolistic markets and oligopolistic firms' behavior, but as discussed in Neary (2015), in some cases the problem might be that the firms are considered oligopolistic in an economy-wide level, the case in which the firms would have influence over national income and this would influence their decision making. As Richter (2014) shows, Neary's general oligopolistic equilibrium model might be better suited for studying environmental tax in oligopolistic competition than partial equilibrium models, because changes in one sector often affect the market equilibrium in others.

There exists a body of work that is based on general oligopolistic equilibrium model. Bastos and Kreickemeier (2009), for example, have used the model to study the effects of globalization on wages in sectors of which some have unionized and some non-unionized labor force. Similarly, there are excellent studies about trade and environment, like that of Kreickemeier and Richter (2014), in which they study the relationship between trade liberalization and the quality of the environment. But, perhaps surprisingly, there aren't many studies that use the general oligopolistic equilibrium model while concentrating on environmental issues. In this regard Richter's work is influential in its field. Another excellent study about environmental policies using a general oligopolistic equilibrium model, is by Colacicco (2017), but he focuses on the national economy, while Richter's work concentrates on sectors.

Palokangas (2015) has studied the interaction of policy-makers in the context of the wage determining process, but other than that, the policy making under general oligopolistic equilibrium has been scarcely researched. Therefore, the two studies, Richter (2014) and Colacicco (2017), are important, not only because of their environmental taxing insights, but because of studying public policy in Neary's model as well.

In this study I follow Richter's (2014) idea, in which he introduces the tax setting and the decision-making processes as a two-stage game where a policy-maker moves first. It is assumed that the policy-maker has perfect information about the firms' decision making and the markets. The tax is the only available instrument, because there is no abatement technology. At the second stage the firms react to the tax, and because they operate in Cournot oligopoly, they make their production decisions simultaneously.

I start by introducing the general equilibrium model and the basics of an optimal Pigouvian tax in oligopolistic markets. Next, I form the partial equilibriums in both open and closed sectors, and combine them into a general oligopolistic equilibrium. In chapter seven I analyze the effects the decreasing of the transaction costs of trade and opening new sectors to trade have on an optimal tax. The eighth chapter concludes.

I follow Richter's work closely because he provides excellent framework for my topic, but any mistakes, especially in derivations, are all mine.

2. THE MODEL

Following Neary (2015) there are only two countries, Home and Foreign, and they are considered identical. As in Richter (2014), any symbols representing the Foreign country are marked with (*) and an individual firm with subscript *j*. Subscript for a domestic individual firm is *i*.

It is assumed that there is a continuum of sectors marked as *z.* In each sector there are a few competing but identical firms that produce a homogeneous good x (Neary, 2015). As in Kreickemeier and Meland (2013), an α percentage of these sectors participate in trade with a Foreign country and are called open. The rest of the sectors produce only for the domestic market and are referred as closed sectors. In open sectors all firms trade, and in closed ones none do. The firms in both sector types, open and closed, treat variables such as wage level and labor supply that are determined at the national level, parametrically.

The firms in each sector compete in Cournot competition, so they make the production decisions simultaneously and are therefore unaware of the other firms' production amounts.

2.1 Demand

Another assumption in the general oligopolistic equilibrium model is that the preferences are additively separable. This allows the separation between the national economy level and sector level, and therefore an individual firm is small in economy but has some market power in its own sector (Neary 2015, 5). The representative household's utility function becomes a continuum of goods consumed:

$$U[x(z)] = \int_0^1 u[x(z)]dz, \tag{1}$$

where $u(z) = aX(z) - \frac{1}{2}\, b\big(X(z)\big)^2, \quad U' > 0, \quad U'' < 0$ and $X = \sum_{i=1}^{n} x_i$.

Because all firms in each sector are identical, we can write

$$X = nx,$$

where *n* represents the number of firms and *x* an output of one firm. The budget constraint is the income, *I*, of a representative household,

$$p(z)X(z) = I,$$

where $p(z)$ is a price of a product $x(z)$.

The utility maximizing problem now becomes

$$U\left[aX(z) - \frac{1}{2}\, b\big(X(z)\big)^2\right] - \lambda \int_0^1 \,[p(z)X(z) - I]\, dz,$$

where λ is the marginal utility of income I and serves as a Lagrange multiplier. Solving for the first order conditions we get

$$X(z) = \frac{1}{b}[a - \lambda p(z)] \tag{2}$$

as a demand function, and

$$p(z) = \frac{1}{\lambda}[a - bX(z)] \tag{3}$$

as an inverse demand function for good x produced by sector *z.* Because individual firms don't have market power over national income level I, the firms take the marginal utility of income, λ, as given. The marginal utility of income can be solved by inserting the X(z) to the budget constraint

$$\int_0^1 [p(z)X(z) - I]\, dz, = \int_0^1 \left[p(z)\frac{a - \lambda p(z)}{b} - I\right] dz.$$

We can write

$$p(z)\frac{a - \lambda p(z)}{b} = I,$$

And solve for λ:

$$\frac{ap(z) - \lambda p(z)^2}{b} = I, \qquad ap(z) - \lambda p(z)^2 - I = b, \qquad \lambda = \frac{ap(z) - bI}{p(z)^2},$$

$$\text{where} \qquad p(z) = \int_0^1 p(z)dz \qquad \text{and} \qquad p(z)^2 = \int_0^1 p(z)^2\ dz\,,$$

the first and the second moments of the price distribution. (Kreickemeier and Meland 2013, 777).

Since the firms treat the marginal utility of income as a parameter, an inverse demand function becomes linear. This linearity is a fundamental part of the general oligopolistic equilibrium model and makes it suitable for studying oligopolistic behavior. (Kreickemeier and Meland 2013, 777). Since *a* and *b* are parameters, we see that because of additive separability the inverse demand depends only on its own quantity and the marginal utility of income.

Because there is no money in the model, any nominal magnitude can be set equal to one. By choosing the marginal utility of income, λ, equal to one, the functions (3) become linear and independent of the income effect:

$$p(z) = a - bX(z).$$

3. ENVIRONMENTAL TAX

Because of the assumption that there is no abatement technology, the tax is the only tool available for a policy maker.

3.1. Damage to the Environment

A damage function, D(E), describes the negative effects of pollutants to environment (Ollikainen, 2017a, 4). Emissions, $e_i(z)$, can be written as

$$e_i(z) = x_i(z) + y_i(z),$$

where $x_i(z)$ is a product produced and sold by firm *i* in domestic market, and $y_i(z)$ is a product produced domestically by a firm *i* but sold in a foreign market. An environmental damage function, D(E), is defined as

$$D(z) = \varphi E(z), \quad \text{where} \quad E(z) = \sum_{i=1}^{n} e_i(z) \; + \gamma \sum_{j=1}^{n} e_j(z). \tag{4}$$

The letter φ symbols the perceived environmental damage, or a preference for clean environment. The bigger its value is, the higher are the society's preferences for clean environment. E(z) is a damage caused to the environment by sector z. The letter $\gamma \in (0,1)$ is a percentage of pollution that is produced outside of Home country's borders. Should its value be unity, all the pollution comes from Foreign country, and should it be zero, all the pollutants are domestic. An environmental tax *t* is set for each unit of emissions. The tax is set separately for every sector z, and all sectors have only one environmental tax. (Richter 2014, 9).

3.2. Pigouvian Tax in a Competitive Market

A policy maker seeks to tax this externality, the damage to environment, D(E), at the level that maximizes a benefit to whole society. The policy maker knows that firms react to the tax by maximizing they own welfare. After tax an individual firm's profit function $\pi_i(z)$ is

$$\pi_i(z) = p(z)x_i(z) - (c + t)x_i(z),$$

where $p(z)$ is a price of one product and $x_i(z)$ a production of the firm *i*. *C* is costs, and *t* is an environmental tax. After derivation we have:

$$\frac{\partial \pi_i(z)}{\partial x_i} = p - c_i - t = 0.$$

In competitive markets firms optimize they production amount at the point where marginal profit equals marginal cost:

$$p = c_i + t.$$

The social planner constructs a social welfare function, SW, by defining the consumer surplus, CS, as

$$CS = \int_0^X p(v)dv - pX, \qquad \text{where} \qquad X = \sum_{i=1}^{n} x_i.$$

As already stated in previous chapter, we can write

$$X = nx.$$

The producer surplus, PS, is defined as

$$PS = pX - ncx,$$

where c stands for private costs of an individual firm. The social costs, i.e. an environmental damage function (4) is already defined, so the social welfare function, SW, can be written as

$$SW = PS + CS - D(E) = \int_0^x p(v)dv - ncx - D(E).$$

Since the lower limit of the integral is zero, derivation becomes shorter:

$$\frac{\partial SW}{\partial x_i} \int_0^x p(X) \frac{\partial X}{\partial x_i} - nc - D'(E) = p(X)n - nc - D'(E) = 0.$$

We can divide this by the number of firms, n, and have

$$p(X) - c - D'(E) = 0.$$

Since we are constructing the Pigouvian tax first at the competitive markets, the firms must take the price as a given, so p(X) can be written as an average of p, and we can write

$$\frac{\partial SW}{\partial x_i} = p - c - D'(E) = 0.$$

At the optimum the private optimal benefit must equal to social optimal benefit, which they do when both derivatives equal zero:

$$\frac{\partial \pi_i(z)}{\partial x_i} = p - c - t = \frac{\partial SW}{\partial x_i} = p - c - D'(E) = 0.$$

This leads to a result

$$t = D'(E).$$

It can be concluded that in competitive markets an optimal Pigouvian tax equals the marginal environmental damage. (Ollikainen 2017a).

3.3. Tax in Cournot Oligopoly

Unlike in the case of competitive markets, in Cournot oligopoly firms seek to make some profit π. An individual firm's profit function is the same as before:

$$\pi_i(z) = p(z)x_i(z) - cx_i(z) - tx_i(z),$$

But when in competitive markets the firms take a price p as given, in oligopoly they can affect the price via production amounts. Therefore, when in the previous chapter $p(X)$ was an average of p, it is now a function. This means that the derivative is now

$$\frac{\partial \pi_i}{\partial x_i} = p(X)\frac{\partial X}{\partial x_i} + \frac{\partial P(X)}{\partial x_i}x_i - c - t = 0 = p(X) + p'x_i - c - t = 0.$$

The social welfare function, and hence its derivative, are also the same as in perfectly competitive markets:

$$\frac{\partial SW}{\partial x_i}\int_0^x p(X)\frac{\partial X}{\partial x_i} - nc - D'(E) = p(X)n - nc - D'(E) = 0,$$

and after dividing it with n we have

$$p(X) - c - D'(E) = 0.$$

Next, we substitute the oligopolist's optimal condition

$$p(X) = -p'x_i + c + t$$

to the derivative of a social welfare function and get

$$SW' = -p'x_i + c + t - c - D'(E) = -p'x_i + t - \varphi = 0.$$

This gives an optimal tax for an oligopoly as

$$t = \varphi + p'x. \tag{5}$$

The second term can be written as

$$p'x = \frac{p'X}{n},$$

After which we multiply both numerator and denominator by *p* and get

$$\frac{pp'X}{pn} = \frac{p}{\eta n},$$

where η is an elasticity of demand. Because an elasticity of demand of a normal good is negative, the whole second term is negative. This means that an optimal tax for oligopoly is lower than in a perfectly competitive market. The second term absorbs the damage of oligopoly market not being efficient, and the lower the number of firms in an oligopoly is, the larger the second term is, which in turn lowers the tax further. Also, the lower the elasticity of demand is, the lower is the tax. We can conclude that more distortion there is to the market, whether by the lack of firms operating in it or because of the inelasticity of demand, the higher is the cost of an inefficiency of the market to the society. (Ollikainen 2017b).

4. CLOSED SECTORS

We can now derive the partial equilibriums for closed and open sectors. The firms take a wage level as given. As already stated, the firms make some profit in Cournot oligopoly, and their profit functions is

$$\pi_i(z) = p(z)x_i(z) - cx_i(z).$$

By substituting for $p(z) = a - bX$ (λ has a value of one) from equation (3) we have:

$$\pi_i(z) = ax_i - bx_i \sum_{i=1}^{n} x_i - cx_i = ax_i - bx_i(x_1 + x_2 \ldots x_n) - cx_i.$$

Taking a derivative, we get

$$\frac{\partial \pi_i(z)}{\partial x_i} = a - b(x_1 + x_2 \ldots x_{n-1} + 2x_i) - cx_i,$$

and since all firms are identical, we can set

$$\sum_{i=1}^{n} x_i = nx_i.$$

The firms maximize their profit accordingly:

$$\frac{\partial \pi_i(z)}{\partial x_i} = a - b(n+1)x_i - cx_i = 0.$$

The firms' costs consist of wage, *w,* and the tax, t. An optimal production can now be solved, and the result is a familiar Cournot competition optimum:

$$x_i(z) = \frac{a - c}{b(n+1)} = \frac{a - w - t(z)}{b(n+1)}.$$

This optimal value of x_i and the tax in Cournot oligopoly already solved in a previous chapter can now be combined:

$$t^{closed}(z) = D'(E) + p'x = D'(E) - bx_i = D'(E) - b\frac{(a - w - t)}{b(n+1)}.$$

Solving for tax

$$(n + 1 - 1)t^{closed}(z) = (n+1)D'(E) - a + w$$

$$t^{closed}(z) = \frac{(n+1)D'(E) - (a - w)}{n}.$$

The tax includes one negative term, *-a/n.* As in previous chapter, the higher the number of firms, the closer to the value of the tax in fully competitive market this tax becomes. We can conclude that the lower the number of firms, n, is, the lower is the tax and the bigger the cost of market imperfection to the society. (Richter 2014, 11).

These two equations, $x_i(z)$ and tax, can now be combined by inserting tax into the equation of an output

$$x_i(z) = \frac{(a - w - (\frac{(n+1)D'(E) - (a-w)}{n})}{b(n+1)} \; .$$

Multiplying both sides by n we get

$$nx_i(z) = \frac{na - nw + (a-w) - (n+1)D'(E)}{b(n+1)},$$

which gives the optimal value of

$$x_i(z) = \frac{a - w - D'(E)}{bn}.$$

Inserting this to the price function (3) we get the price for products in closed sectors:

$$p(z) = a - bn\frac{a - w - D'(E)}{bn} = \; w + D'(E).$$

We can conclude that the price consists of the wage and the marginal damage to the environment, meaning that it includes both, the private and social costs. (Richter 2014, 12).

5. OPEN SECTORS

The firms in these sectors trade with the Foreign country. A good produced domestically but sold in the Foreign country is marked as y_i, and the trading costs are marked with *d.* As in closed sectors, the firms' costs include wages and an environmental tax, but now there are also the trade costs, so the cost function in open sectors becomes

$$c^{open}(z) = (w+t)x_i(z) + (w+t+d)y_i(z).$$

Therefore, the profit function of an individual firm is

$$\pi_i(z) = p(z)x_i(z) + p^*(z)y_i(z) - (w+t)x_i(z) - (w+t+d)y_i(z).$$

Since the two countries are identical, the number of firms operating in each open sector is now n+n* = 2n. Because of this identicality, it can also be assumed that domestic price equals the foreign price, i.e. $p(z) \ = p^*(z)$. Following Richter (2014) we separate the taxes by writing the tax of the product sold in a foreign country as t*, so the profit function is now

$$\pi_i(z) = p(z)(x_i(z) + y_i(z)) - (w+t)x_i(z) - (w+t*+d)y_i(z).$$

Substituting the function of price, equation (3), we get

$$\pi_i(z) = a(x_i + y_i) - bx_i(\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i) - c(x_i + y_i)$$

$$= a(x_i + y_i) - bx_i(x_1 + x_2 \ ... x_n) - bx_i(y_1 + y_2 \ ... y_n) - c(x_i + y_i).$$

Taking a partial derivative regarding the x_i we have

$$\frac{\partial \pi_i(z)}{\partial x_i} = a - b(x_1 + x_2 \ldots x_{n-1} + 2x_i) - b(y_1 + y_2 \ldots y_n) - w - t \ = 0.$$

Again, assuming the identicality of the firms, the derivative becomes

$$\frac{\partial \pi_i(z)}{\partial x_i} = a - b(n+1)x_i - bny_i - w - t \ = \ 0.$$

Partial derivative regarding to y_i can be solved in similar fashion

$$\frac{\partial \pi_i(z)}{\partial y_i} = a - b(n+1)y_i - bnx_i - w - t*-d = 0.$$

Now the optimal y_i can be solved:

$$y_i = \frac{a - bnx_i - w - t*-d}{b(n+1)}.$$

This result can now be substituted to derivative regarding to x_i:

$$\frac{\partial \pi_i(z)}{\partial x_i} = a - b(n+1)x_i - bn(\frac{a - bnx_i - w - t*-d}{b(n+1)}) - w - t \ = \ 0.$$

Rearranging we get

$$[(b(n+1))^2 \ - (bn)^2]x_i = -bn(a - w - t*-d) + b(n+1)(a - w - t)$$

$$[b^2(n^2+2n\ +1)-(bn)^2]x_i\ \ =-bn(a-w-t*-d)+b(n+1)(a-w-t).$$

The same can be done to $y_i(z)$, and we get the values

$$x_i(z)=\frac{a-w-(n+1)t(z)+nt^*(z)+nd}{(2n+1)b}$$

$$y_i(z)=\frac{a-w-(n+1)t(z)+nt^*(z)-(n+1)d}{(2n+1)b}.$$

As in closed sectors, the social welfare function of open sectors (Richter 2014, 13) includes the private costs *C* and the marginal damage function $D(E)$. The surpluses, however, are now calculated by adding the net trade surplus, S, in sector z, and the utility of domestic consumption, U together:

$$SW=U(z)+\ S(z)-D(E)-C(z). \tag{6}$$

We can solve this function by one term at the time, beginning from the U(z). This utility function (1) we already know, but because the firms now trade, the domestic consumption includes imported good y_j. Due to the symmetry of the countries, we can take an advantage of the value of y_i from above and write the function of

$$y_j(z)=\frac{a-w-(n+1)t^*(z)+nt(z)-(n+1)d}{(2n+1)b}.$$

The utility

$$u(z)=aX(z)-\frac{1}{2}\,b\big(X(z)\big)^2,$$

$$\text{where}\quad X=\sum_{i=1}^{n}x_i\ +\sum_{i=1}^{n}y_j\ =nx_i+ny_j\quad \text{so}\quad u(z)=a\big(nx_i+ny_j\big)-\frac{1}{2}\,b\big(nx_i+ny_j\big)^2.$$

Inserting the values of production, we get

$$u(z)=an\left(\frac{a-w-(n+1)t(z)+nt^*(z)+nd}{(2n+1)b}+\frac{a-w-(n+1)t^*(z)+nt(z)-(n+1)d}{(2n+1)b}\right)$$

$$-\frac{1}{2}\,bn\left(\frac{a-w-(n+1)t(z)+nt^*(z)+nd}{(2n+1)b}+\frac{a-w-(n+1)t^*(z)+nt(z)-(n+1)d}{(2n+1)b}\right)^2.$$

We can write down all the terms that include the tax t:

$$an\frac{(2n+1)}{b(2n+1)^2}(-t) - \frac{n^2\,(-4at+4wt+t^2+2tt^*+2dt}{2b(2n+1)^2} = \frac{n\,(-at-2nwt-\frac{n}{2}t^2-ntt^*-ndt}{b(2n+1)^2}.$$

After taking a derivative regarding to *t* we have:

$$\frac{\partial U}{\partial t} = \frac{n\,(-a-2nw-nt-nt^*-nd}{b(2n+1)^2}. \tag{7}$$

The net trade surplus, S, (Richter 2014, 13) consists of a domestic product sold in Foreign market, y_i, minus the transportation costs and the income that Foreign country earned by importing the product y_j into Home country:

$$S = (p^*(z)-d)y_i(z) - p(z)y_j(z).$$

Since the foreign price p^* is identical to the domestic one, we can set them equal in solving the equation. We multiply the first term, the function of $p*\,y_i(z)$, first, and again, write down only the terms that include the tax t:

$$\frac{-an(n+1)t(2n+1)}{b(2n+1)^2}\;\frac{-n^2[-2(n+1)at+2(n+1)wt+(n+1)^2t^2\;-2n(n+1)tt^*\;-2(n+1)dt]}{b(2n+1)^2}.$$

Next are the transportation costs, $dy_i(z)$. Again, to keep the functions more readable only the terms that include the tax t are written down:

$$\frac{nd(n+1)t(2n+1)}{b(2n+1)^2}.$$

Finally, the terms that include the tax t in the function of $p(z)y_j(z)$ are

$$\frac{an(n)t(2n+1)}{b(2n+1)^2}\;\frac{-n^2[2nat-2nwt+n^2t^2\;-2n(n+1)tt^*\;+2n(n+1)dt]}{b(2n+1)^2}.$$

Now we combine all these terms to write all the terms that include tax t in the function of net trade surplus, S:

$$\frac{n\,[-(2n+1)at-2n(2n+1)wt-n(2n+1)t^2+(2n+1)dt]}{b(2n+1)^2}.$$

And after derivation we get

$$\frac{\partial S}{\partial t} = \frac{n\,[-(2n+1)a-2n(2n+1)w-2n(2n+1)t+(2n+1)d]}{b(2n+1)^2}. \tag{8}$$

Private costs consist of wages and trade costs, but since trade costs are already included in the previous sectoral net trade surplus function, the costs are

$$C = -wn(x_i + y_i)$$
$$= -wn \frac{a - w - (n+1)t(z) + nt^*(z) + nd}{(2n+1)b} + \frac{a - w - (n+1)t(z) + nt^*(z) - (n+1)d}{(2n+1)b}$$
$$= \frac{-wn(2a - 2w - 2(n+1)t + 2nt^* - d)}{b(2n+1)}.$$

After derivation we have

$$\frac{\partial C}{\partial t} = \frac{n\,w(2(n+1)(2n+1)}{b(2n+1)^2}. \qquad (9)$$

The last term in the social welfare function to derivate is the marginal damage to the environment function (4):

$$E(z) = \sum_{i=1}^{n}[x_i(z) + y_i(z)] + \gamma \sum_{j=1}^{n}[x_j(z) + y_j(z)].$$

After inserting all four functions of domestic and foreign goods, $x_i(z)$, $y_i(z), x_j(z) and\ y_j(z)$, and performing the simple calculations, an environmental damage function can be written as

$$E(z) = n\frac{(2a - 2w - 2(n+1)t + 2nt^* - d)\varphi}{b(2n+1)} + \gamma n\frac{(2a - 2w - 2(n+1)t^* + 2nt - d)\varphi}{b(2n+1)},$$

where φ is a perceived marginal environmental damage. After the multiplications the terms including the tax are

$$\frac{n(-2(n+1)t + 2nt^*)\varphi}{b(2n+1)} + \gamma\frac{n(-2(n+1)t^* + 2nt)\varphi}{b(2n+1)}$$
$$= \frac{n(-2nt - 2t + 2nt^* - \gamma 2nt^* - \gamma 2t^* + \gamma 2nt)\varphi}{b(2n+1)}$$

$$= \frac{n(-2(n+1-n\gamma)t - 2(\gamma - n + \gamma n)t^*)\varphi}{b(2n+1)}.$$

Now we can take a derivative regarding to t and have

$$\frac{\partial D(E)}{\partial t} = \frac{-n\,(2n+1)2(n+1-n\gamma)\varphi}{b(2n+1)^2}\,. \tag{10}$$

These partial derivatives of the social welfare function, (7), (8), (9) and (10), can now be combined to form a derivative of the social welfare function of open sectors:

$$\frac{\partial SW}{\partial t} = \frac{\partial U}{\partial t} + \frac{\partial S}{\partial t} - \frac{\partial C}{\partial t} - \frac{\partial D(E)}{\partial t}$$

$$= \frac{n\,[-2(n+1)(a-w) - n(4n+3)t - nt^* + (n+1)d + 2(2n+1)(n+1-n\gamma)\varphi]}{b(2n+1)^2} = 0.$$

The optimal value of t becomes

$$t(z) = \frac{-2(n+1)(a-w) - nt^* + (n+1)d + 2(2n+1)(n+1-n\gamma)\varphi}{n(4n+3)}.$$

Still following Richter (2014), because the two trading countries are identical, the tax in both countries can be assumed to be set similarly. Using this we can write t = t^*:

$$(n(4n+3)+n)t = -2(n+1)(a-w) + (n+1)d + 2(2n+1)(n+1-n\gamma)\varphi,$$

and solve for an optimal tax:

$$t^{open}(z) = \frac{-2(n+1)(a-w) + (n+1)d + 2(2n+1)(n+1-n\gamma)\varphi}{4n(n+1)}. \tag{11}$$

As in closed sectors, the higher costs and domestically caused environmental damages increase the tax. (Richter 2014).

Now that the tax is set, we can insert it to the functions of x and y and solve for optimal production amounts:

$$x_i^{open}(z) = \frac{2(n+1)(a-w) + (n+1)(2n-1)\mathrm{d} - 2(n+1-n\gamma)\varphi}{4n(n+1)b}$$

$$y_i{}^{open}(z) = \frac{2(n+1)(a-w) - (n+1)(2n+1)d - 2(n+1-n\gamma)\varphi}{4n(n+1)b}.$$

Again, like in close sectors, the higher the costs, the lower the output. We see that the bigger the trade cost d is, the more is produced to domestic market. When trade costs decrease, a bigger share of domestic production is sold in a Foreign market. (Richter 2014).

6. COMPARISONS BETWEEN CLOSED AND OPEN SECTORS

Due to all sectors having the same number of domestically operating firms, n, and only the participation in trade setting them apart, we can determine which sector type has a higher environmental by calculating the difference between the taxes as

$$t^{open}(z) - t^{closed}(z) =$$

$$\frac{-2(n+1)(a-w) + (n+1)d + 2(2n+1)(n+1-n\gamma)\varphi}{4n(n+1)} - \frac{(4n+4)((n+1)\varphi - (a-w))}{4n(n+1)}$$

$$= \frac{-2(n+1)(a-w) + (n+1)d + (4n+2)(n+1-n\gamma)\varphi}{4n(n+1)} - \frac{(4n+4)(-a+w+(n+1)\varphi)}{4n(n+1)}$$

$$= \frac{2(n+1)(a-w) + (n+1)d - 2\big((n+1) + n(2n+1)\gamma\big)\varphi}{4n(n+1)}.$$

This result is unclear because several variables influence the tax. (Richter 2014). However, we can conclude that

the more likely the tax in open sectors is higher when trade costs are high, the wage level and perceived marginal environmental damage are low, or when a pollutant is mostly foreign.

To see the effects the different variables have on taxes, we can look at comparative statics starting with the wage level:

$$\frac{\partial t^{open}(z)}{\partial w} = \frac{2(n+1)}{n4(n+1)} = \frac{1}{2n} \quad < \quad \frac{\partial t^{closed}(z)}{\partial w} = \frac{(4n+1)}{n4(n+1)} = \frac{1}{n}.$$

An increase in the wage level increases the tax in both sector types, but because there are more firms producing to the open sectors' markets, the effect is smaller in open sectors. Therefore, it can be concluded that the more firms there are, the less the increase in wage level increases the tax.

A change in the perceived marginal environmental damage, φ, in both sector types:

$$\frac{\partial t^{open}(z)}{\partial \varphi} = \frac{(4n+2)(n+1-n\gamma)}{4n(n+1)} = \frac{(2n+1)(n+1-n\gamma)}{2n(n+1)} = \frac{(2n^2+3n+1-2n^2\gamma-n\gamma)}{2n(n+1)}$$
$$< \quad \frac{\partial t^{closed}(z)}{\partial \varphi} = \frac{(2n+2)(n+1)}{2n(n+1)} = \frac{(2n^2+4n+2)}{2n(n+1)}.$$

The tax in closed sectors increases more should there be an increase in perceived marginal environmental damage. An increase is the same in closed sectors regardless of pollutant being domestic or foreign.

Now, if all the pollutant is domestic, meaning γ = 0, and the perceived marginal environmental damage increases:

$$\frac{\partial t^{open}(z)}{\partial \varphi} = \frac{(4n+2)(n+1)}{4n(n+1)} = \frac{2n+1}{2n} \quad < \quad \frac{\partial t^{closed}(z)}{\partial \varphi} = \frac{n+1}{n}.$$

The difference between the sector types is much smaller.

Finally, if all the pollutant is foreign, meaning γ =1:

$$\frac{\partial t^{open}(z)}{\partial \varphi} = \frac{(4n+2)(n+1-n\gamma)}{4n(n+1)} = \frac{2n+1}{2n(n+1)} \quad < \quad \frac{\partial t^{closed}(z)}{\partial \varphi} = \frac{(2n+2)(n+1)}{2n(n+1)}.$$

The firms in closed sectors have a higher increase in tax.

While it can't be determined whether an environmental tax is higher in trading or non-trading sectors, the tax in closed sector is more vulnerable to changes in costs or pollution amounts. This is often due to fewer firms operating in closed markets. Richter (2014, 14) calculates the difference in sectoral taxes in a situation where both sector types, open and closed, have the same number of firms. This may first seem just a convenient way to achieve a more desirable result, but it can also be a more accurate way to compare taxes, because in open sectors there are twice as many firms operating in each sector, but only half of them, amount n, have production and emit pollution in the Home country. It can be assumed that the Home country can set a tax to domestic firms only. Should the foreign pollutant be taxed, it may be done via import taxes and be included in transport costs. So, setting the number of firms as n in all sectors, we can calculate the difference between the taxes again:

$$t^{open}(z) - t^{closed}(z)|_{n=n^{open}} =$$

$$\frac{-2(n+1)(a-w)+(n+1)d+2(n+1)(n+1-n\gamma)\varphi}{2n(n+1)} - \frac{(2n+2)((n+1)\varphi-(a-w))}{2n(n+1)}$$

$$= \frac{(n+1)d+(2n+2)(n+1-n\gamma)\varphi}{2n(n+1)} - \frac{(2n+2)(n+1)\varphi}{2n(n+1)}$$

$$= \frac{(n+1)d-2n(n+1)\gamma\varphi}{2n(n+1)} = \frac{d-2n\gamma\varphi}{2n}.$$

This result means that the tax in open sectors is higher only when the term $2n\gamma\varphi$ is smaller than transportation costs. This is more likely to happen if a perceived marginal damage to the environment is low, if the number of firms, n, is low, or if pollutant is mostly domestic (low γ). Should all emissions be results of a domestic production, meaning γ=0, the tax is higher in open sectors.

It can also be concluded that when the number of firms is set the same in all sectors, the changes in wage level and in marginal environmental damage caused by domestic pollutant have equally big effects on both taxes.

Since the tax in turn influences the firms' production decisions, let's compare the derivatives of domestic output, $\mathrm{x}^{\mathrm{open}}(\mathrm{z})$ and $\mathrm{x}^{\mathrm{closed}}(\mathrm{z})$ regarding to tax to see which sector is higher influenced should there be an increase in tax:

$$\frac{\partial x^{closed}(z)}{\partial t} = \frac{-1}{b(n+1)} \quad \text{and} \quad \frac{\partial x^{open}(z)}{\partial t} = \frac{\partial y^{open}(z)}{\partial t} = \frac{-1}{b(2n+1)}.$$

Because a tax is a cost to firms, the derivatives regarding to tax are negative. An increase in tax lowers the production amounts in both sector types, but the effect is smaller in open sectors. Again, this is due to a bigger number of firms. Should the number of firms be the same in all sectors, the effect would be the same too.

We can conclude that the more firms there are operating in the same market, the lesser is the effect of the change in tax on production.

7. GENERAL OLIGOPOLISTIC EQUILIBRIUM

The partial equilibriums can be combined at the national economy level to form a general oligopolistic equilibrium. The supply of labor, L, is inelastic and the full employment is assumed. As was with the wage and other variables determined at the national level, the firms must take the supply of labor as given. The general oligopolistic equilibrium is formed by adding all domestic production together, and supply of labor acting as a constraint:

$$L = L^{open} + L^{closed} = n\int_0^{\alpha} (x^{open}(z) + y^{open}(z))\ dz + n\int_{\alpha}^{1} (x^{closed}(z))\ dz$$

$$= \frac{\alpha n[2(n+1)(a-w) + (n+1)(2n-1)d - 2(n+1-n\gamma)\varphi}{4n(n+1)b}$$

$$+ \frac{2(n+1)(a-w) - (n+1)(2n+1)d - 2(n+1-n\gamma)\varphi]}{4n(n+1)b} + (1-\alpha)\, n\, \frac{a-w-\varphi}{bn}$$

$$= \frac{\alpha n[4(n+1)(a-w) - 2(n+1)d - 4(n+1-n\gamma)\varphi]}{4n(n+1)b} + (1-\alpha)\, n\, \frac{4(n+1)(a-w-\varphi)}{bn4(n+1)}$$

$$= \frac{\alpha[2(n+1)(a-w) - (n+1)d - 2(n+1-n\gamma)\varphi]}{2(n+1)b} + (1-\alpha)\, \frac{2(n+1)(a-w-\varphi)}{b2(n+1)}$$

$$= \frac{\mathrm{a} - w - \frac{\alpha}{2}d - \left(1 - \frac{\alpha n\gamma}{n+1}\right)\varphi}{b}.$$

Now all the sectors are linked. (Richter 2014, 15).

Next, we examine increasing international trade via two different avenues, decreasing the transaction costs of trade and therefore increasing the volume of trade, and opening new sectors to trade. We start with the lowered trade costs.

7.1. Changes in Trade Costs

In partial equilibriums the wage was a part of costs, so a decrease in wage level decreased an environmental tax and increased output in both sector types. From the full employment equation of general oligopolistic equilibrium, the wage can be solved as

$$w = \mathrm{a} - bL - \frac{\alpha d}{2} - (1 - \alpha \frac{n\gamma}{n+1})\varphi. \tag{12}$$

An increase of trade cost *d* decreases the wage *w*"

$$\frac{\partial w}{\partial d} = -\frac{\alpha}{2} < 0.$$

This is because a decrease in trade costs decreases total costs, which in turn increases production amounts. This leads to increasing demand of labor. Because the labor supply is inelastic, the wage must increase so that the demand of labor returns to a level of full employment. We see that the bigger the share of open sectors, α, and therefore the bigger the total share of firms directly affected by trade cost decrease, the bigger is also the effect of decreased trade costs on wage. (Richter 2014).

Because the wage effects the tax, the effect of lowered trade costs on wage effects environmental taxes via wage. This is an indirect effect the trade cost has on tax. It has already been shown that in open sectors lowering the trade cost directly decreases an environmental tax, and this is the direct effect the decreased trade cost has on environmental tax. The total effect can be calculated by combining these two effects, either by adding them together as in Richter (2014, 17) or by inserting the wage function (12) into the tax equation (11). Using the latter way, the environmental tax in open sectors can be written as

$$t^{open}(z) = \frac{-2(n+1)\left(a - a + bL + \frac{\alpha d}{2} + (1 - \alpha \frac{n\gamma}{n+1})\varphi\right) + (n+1)d + 2(2n+1)(n+1-n\gamma)\varphi}{4n(n+1)}.$$

Taking a derivative regarding trade cost by considering both, a direct and indirect effect we get

$$\frac{\partial t^{open}(z)}{\partial d} = \frac{-2(n+1)\left(\frac{\alpha d}{2}\right) + (n+1)d}{4n(n+1)}$$

$$\frac{\partial t^{open}(z)}{\partial d} = \frac{1-\alpha}{4n}.$$

An indirect effect is negative, but the direct effect is positive. The direct effect is bigger because 1-α > 0, and therefore the total derivative is positive. This means that a decrease in trade costs lowers an environmental tax in open sectors, but less so the bigger the indirect effect is. It can be concluded that more sectors there are already involved in trade, (bigger the α), the less environmental tax decreases when trade costs decrease. (Richter 2014).

The same analysis can be done with closed sectors. There trade costs have no direct effect on tax, but an indirect effect exists, so we insert a wage (12) into tax equation and get

$$t^{closed}(z) = \frac{(n+1)\varphi - a + a - bL - \frac{\alpha d}{2} - (1 - \alpha\frac{n\gamma}{n+1})\varphi}{n}.$$

And take a derivative regarding trade costs:

$$\frac{\partial t^{closed}(z)}{\partial d} = \frac{-\alpha}{2n}.$$

As in open sectors, the derivative of indirect effect is negative. This means that an environmental tax increases as a result of lowered trade costs. The more sectors there are involved in trade, (bigger the α), the bigger is the increase of tax in closed sectors. (Richter 2014).

Proposition 1. When the trade costs are lowered, an indirect effect raises the environmental tax in all sectors. However, in open sectors, the direct effect dominates, so the tax increases only in closed sectors.

7.2. Changes in The Share of Open Sectors

An increase in the number of open sectors, i.e. opening new sectors to trade, has no direct effect on environmental tax because α is not a component of tax in either sector type. However, as in the case of the lowered trade costs, there is an indirect effect via the wage level. As already shown, an increase in wage increases environmental taxes, and we can take a derivative regarding the share of open sectors to find out the indirect effect:

$$\frac{\partial w}{\partial \alpha} = -\frac{d}{2} + \frac{n\gamma\varphi}{n+1}.$$

If the perceived marginal environmental damage

$$\varphi > \frac{(n+1)d}{n\gamma 2},$$

an increase in the number of open sectors increases the wage level. The lower the share of domestically produced negative externality is (high γ), the more likely an increase in the number of open sectors will increase the wage level. The same effect happens if the perceived marginal environmental damage itself is high. The lower the trade costs, the more likely an increase in the percentage of open sectors will increase the wage level. (Richter 2014).

Proposition 2. An increase in the number of open sectors will more likely indirectly raise environmental tax in both open and closed sectors, if trade costs are low, if pollutant is mainly foreign, or if the perceived marginal environmental damage is sufficiently high.

7.3. Effects in General Equilibrium

Naturally, the change in wage causes the demand of labor to change to opposite direction. Should the supply of labor be treated as a variable, the wage level would be affected at the rate of -b. As already shown, decreased trade costs increase wage. In open sectors trade costs are included in production functions, and we get their total effect on labor market by inserting the wage function (12) in to the open sectors' demand of labor function:

$$L^{open} = \frac{\alpha n\left[4(n+1)\left(\mathrm{a} - (\mathrm{a} - bL - \frac{\alpha d}{2} - (1 - \alpha\frac{n\gamma}{n+1})\varphi)\right) - 2(n+1)d - 4(n+1-n\gamma)\varphi\right]}{4n(n+1)b}.$$

We derivate regarding the trade costs:

$$\frac{\partial L^{open}}{\partial d} = -\frac{\alpha}{2b} - \frac{\alpha}{b}\frac{(-\alpha\,)}{2} = \frac{-\alpha\, + \alpha^2}{2b} < 0.$$

The first term, the direct effect, is negative, and because it is larger than the effect via wage, the demand of labor moves to opposite direction from trade costs. This means that the demand of labor in open sectors increases, and as a result the labor moves from closed sectors to open sectors. The closed sectors lose labor exactly the amount that moves to open sectors. We can insert the wage in the demand of labor in closed sectors as we just did with open ones and get

$$L^{closed} = (1-\alpha)\,\frac{2(n+1)(\mathrm{a} - (\mathrm{a} - bL - \frac{\alpha d}{2} - (1 - \alpha\frac{n\gamma}{n+1})\varphi) - \varphi)}{b2(n+1)}.$$

Taking a derivative:

$$\frac{\partial L^{closed}}{\partial d} = \frac{(1-\alpha)(\alpha)}{2b} = \frac{\alpha - \alpha^2}{2b} > 0$$

Derivative is positive, meaning that lower trade costs lower the demand of labor in closed sectors freeing labor to move to open sectors. (Richter 2014).

8. CONCLUSION

The general oligopolistic equilibrium model is proper for studying oligopolistic behavior, because it enables the separation between the national economy and the sector level. For this reason, oligopolistic firms can be treated as having some market power inside their sectors, but not in the whole economy. For its flexibility the model provides a suitable approach for oligopolistic competition even when the companies differ from one another, or when there are multiple countries.

In chapter six I prove that international trade does not always lower environmental standards. In Neary's general equilibrium model, where two countries trade, an environmental tax can even be higher in open sectors, than in closed sectors, and more likely the higher the trade costs, the lower the wage level, the lower the perceived marginal environmental damage, and/or the lower the share of pollutant being foreign.

I also show that when the transaction costs of trade are lowered, an environmental tax does lower at open sectors, but less so when the share of trading sectors is bigger. On the closed sectors an environmental tax increases.

An increase in the number of open sectors will more likely raise environmental tax in both open and closed sectors, if trade costs are low, if pollutant is mainly foreign, or if the perceived marginal environmental damage is sufficiently high.

These results are encouraging now that the quality of environment is a cause of great concern, but hopefully there will be more studies of the subject. The more information there is, the more likely politicians and the general public alike can be convinced that environmental standards do not have to be lowered in order to increase trade.

9. REFERENCES

Bastos Paulo and Udo Kreickemeier. (2009). Unions, Competition and International Trade in General Equilibrium. *Journal of International Economics,* 79: 238-247.

Colacicco, Rudy. (2017). Environment, Imperfect Competition, and Trade: Insights for Optimal Policy in General Equilibrium. Department of Economics, Finance and Accounting, Maynooth University, February 3, 2017. Earlier version published as Working Paper N273-16.

Kreickemeier, Udo and Frode Meland. (2013). Non-traded Goods, Globalization and Union Influence. *Economica*, 80: 774-792.

Kreickemeier, Udo and Philipp M.Richter. (2014). Trade and the Environment: The Role of Firm Heterogeneity. *Review of International Economics,* 22(2): 209-225.

Neary P. (2015). International Trade in General Oligopolistic Equilibrium. CESifo Working Paper No 5671.

Ollikainen, Markku. (2017a). Environmental economics I: Basic Theory. The lecture notes, Chapter 1. Socially Optimal Production and Emissions, University of Helsinki, Fall 2017.

Ollikainen, Markku. (2017b). Environmental economics I: Basic Theory. The lecture notes, Chapter 2. Environmental Policy under Certainty, University of Helsinki, Fall 2017.

Palokangas, Tapio. (2015). The Welfare Effects of Globalization with Labor Market Regulation. IZA Discussion Paper No. 9142.

Richter, Philipp M. (2014). Strategic Environmental Policy in General Equilibrium. DIW Berlin August 8, 2014, Draft for the ETSG Annual Conference in 2014, Munich.