Bitcoin in Utility Function: The Demand for Bitcoin

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Tiivistelmä/Referat – Abstract

Bitcoin and other cryptocurrencies have been frequently on media lately. As these cryptocurrencies are relatively new, there are not much economic theory explaining their behavior and price developments. Due to these reasons, the goal of this thesis is to find an economic theory to study the demand for Bitcoin.

In this thesis, I will write about Bitcoin applying it to Walsh's Money in Utility function (MIU function). I will modify Walsh's original model by incorporating Bitcoin to it. In this model, Bitcoin is used as payment method and as a store of value. Both Bitcoin and money can be used to buy any goods, but there are certain goods that are easier to buy using bitcoin. Hence, Bitcoin has transaction benefit and the households will always need some bitcoin holdings in their portfolio. Using Walsh's MIU function, I will derive a demand function for Bitcoin.

In addition to this, I will go through the working paper "Bitcoin Pricing, Adoption, and Usage: Theory and Evidence" written by Athey et Al. (August 2016). In this paper, Bitcoin is used both as a payment method and a store of value. From the findings by Athey et Al., Bitcoin seems to be mainly used as a store of value. I will present an overview of the paper including the results and then concentrate on their aggregate analysis on Bitcoin exchange rate.

Based on the Bitcoin exchange rate equation presented by Athey et Al., I will study whether Bitcoin demand function derived from MIU model is able to explain the changes in Bitcoin's aggregate demand in real market. As expected, due to the assumptions and restrictions of the model, Bitcoin demand function derived in this thesis is not able to fully explain the changes in demand for Bitcoin in real world. Nonetheless, subject to the assumptions and restrictions of the model, Bitcoin demand function can be used to study the relationship between bitcoin demand, domestic nominal interest rate and consumption. Finally, I will present an alternative approach to further study Bitcoin's demand.

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Content

1. Symbols

U_t ≡ household's utility function

c_t ≡ per capita consumption at time t

z_t ≡ number of goods from money holdings at time t

M_t ≡ money holdings at time t

$1/P_t$ ≡ price of goods at time t

N_t ≡ population at time t

m_t ≡ real per capita money holding at time t

J_t ≡ number of goods from selling one's Bitcoin holdings at time t

j_t ≡ interest rate on Bitcoin holdings measured in domestic currency at time t

$\bar{O}_t$ ≡ value of Bitcoin holdings in domestic currency at time t

σ_t ≡ real per capita Bitcoin holding measured in domestic currency at time t

W ≡ the representative household's total utility

β ≡ subjective rate of discount

Y_t ≡ aggregate output at time t

K_{t-1} ≡ aggregate stock of physical capital hold by households at the beginning of period t

$\tau_t N_t$ ≡ aggregate real value of taxes or lump-sum transfers at time t

δ ≡ depreciation of physical capital

B_t ≡ bond holdings at time t

i_t ≡ nominal interest rate at time t

k ≡ per capita capital stock

n ≡ rate of population growth

y_t ≡ output per capita at time t

w_t ≡ level of resources = per capita wealth at time t

π ≡ inflation rate of domestic currency

b ≡ per capita bond holdings

$$u_c \equiv \frac{\partial V(w_t)}{\partial c}$$

$$u_b \equiv \frac{\partial V(w_t)}{\partial b}$$

$u_\sigma \equiv \frac{\partial V(w_t)}{\partial \sigma}$

$u_m \equiv \frac{\partial V(w_t)}{\partial m}$

$x = k, b, \sigma, m$

λ_t = marginal utility of consumption

θ_1 ≡ share of consumption in CES utility function

θ_2 ≡ share of money holding in CES utility function

s ≡ elasticity of substitution (constant) in CES utility function

$\eta \equiv \frac{s}{s-1}$

$Q_t(\sigma_t)$ ≡ aggregate demand of Bitcoins measured in domestic currency at time t

ψ ≡ aggregate supply of Bitcoins

2. Introduction

Bitcoin and other cryptocurrencies have been frequently on the news lately, especially regarding the wide fluctuations in their prices. As per the website blockchain.info, Bitcoin's current value at the time of writing is worth 5,406.71 US dollars, when back in 2010 it was under 1 US dollar worthy. What factors have made Bitcoin as worthy as it is today in only 9 years? I believe one of the reasons being the fact that Bitcoin can be used as an alternative to money and in such many ways.

There are many definitions for Bitcoin in the market. It can be seen as a currency, a protocol, a payment system, a store of value or even a technology platform, as mentioned in Bitcoin Pricing, Adoption, and Usage by Athey et al. (August 2016). As these cryptocurrencies are relatively new, there are not much economic theory explaining their behavior and large price fluctuations. Due to these reasons, the goal of this thesis is to find an economic theory to help explaining the pricing of Bitcoin and to give Bitcoin a demand function based on a theoretical model, even when having to significantly narrow down the use cases of Bitcoin.

In chapter 3, I will write about Bitcoin applying it to Walsh's Money in Utility Function (Monetary Theory and Policy, 2010). I will modify Walsh's original model by adding Bitcoin as one of the variables to the utility model. Assuming banks are not working well and households are lacking trust towards the banks, it is rational for the households to have at least a small marginal of Bitcoins in their portfolio. Additionally, assume households are living in a country where all goods can be bought using domestic money or Bitcoin holdings. However, there are certain goods that are more accessible by purchasing them using Bitcoin than using domestic money. Hence Bitcoin holdings have transaction benefits. In this thesis, I will be analyzing Bitcoin by incorporating it to Walsh's Money in Utility Function model and from there I will derive a simplified demand function for Bitcoin.

In chapter 4, I will shortly go through the main ideas of working paper "Bitcoin Pricing, Adoption, and Usage: Theory and Evidence" which is written by Susan Athey, Ivo Parashkevov, Vishnu Sarukkai and Jing Xia (August 2016). In my thesis, I will exclude the theoretical model of Bitcoin created by Athey et al. and concentrate mainly on the aggregate analysis on the exchange rate of Bitcoin which I will utilize in chapter 5 to further study the applicability of the demand function derived from Money in Utility model. In the working paper, Bitcoin is assumed to be used both as a payment method and a store of value. From the empirical findings by Athey et Al., Bitcoin seems to be mainly used as a store of value.

In chapter 5, following the same velocity equation presented in chapter 4, I will study whether the relationship between nominal interest rate, consumption and Bitcoin's demand presented in chapter 3 is valid. I will study whether the Bitcoin demand function derived from Money in Utility model can help to understand the cause to the main changes in the evolution of Bitcoin's price and demand from 2012 to 2015. As Money in Utility model is theoretical with certain assumptions and restrictions, it is usually used to study the relationship between money and prices and also to study certain subjects around inflation rate. Hence, I believe the Bitcoin demand function derived from Money in Utility model can only be used to get better understanding on the relationship between Bitcoin's demand, consumption and interest rate. Considering all the assumptions and restrictions of the model, it is probably not the most suitable to comprehensively analyze Bitcoin in real world market.

Finally, in chapter 6, I will provide conclusions of the findings and my opinions regarding the Bitcoin's demand model derived from Walsh's Money in Utility function. Additionally, I will propose an idea on how Bitcoin can be further studied.

2.1 Money in Utility Function (Walsh)

In the book of Monetary Theory and Policy (2010) written by Walsh, money demand is derived using household's utility function. There are some assumptions that need to take place in order for money to be positively valued. Walsh confronts this issue using three general approaches:

1. By including money balances into households' utility function, one can assume that money generates utility directly. This is based on Sidrauski's article "Rational Choice and Pattern of Growth in a Monetary Economy (1967)", where households' saving behaviour is seen as a wealth accumulation process aiming to maximize the intertemporal utility function.
2. Assume that in order to obtain consumption goods, money and time has to be combined to enable transaction services. (Brock 1974; McCallum and Goodfriend 1987; Croushore 1993)
3. Money is seen as an asset that enable moving resources across periods. (Samuelson 1958)

Using the mentioned approaches households, whose utility is dependent on good consumptions and money holdings, is introduced by Walsh to the basic neoclassical model. According to Walsh, with certain assumptions and restrictions on the utility function, money holding in equilibrium will be positive. Hence, the requirement that money is positively valued is valid.

In order to develop the basic Money in Utility function (MIU function), Walsh disregards uncertainty and all the labor-leisure choices. Also, households are assumed to be rational in every period hence he will maximize his utility in each period. In this model, goods are defined by domestic currency holdings that money yields. Additionally, as per Money in Utility Function, money holdings yield utility independently from the fact whether it is used to buy consumption or not. This means that the marginal utility of money has to be positive. Following the positive marginal utility of money, if one holds the path of real consumption constant for all periods, an increase in money holdings will increase the utility of the household. Furthermore, only household's real money holding at the end of the period, after purchases has been done, will yield utility.

Using the assumptions above, it is possible to incorporate money holdings to a utility function and derive money demand function based on it. Walsh presents some monetary issues where Money in Utility Function model can be used to examine the cases such as: it can be used to explain the relationship between money and prices, the impact of changes in inflation on equilibrium and also, to find the optimal inflation rate. In this thesis, however, Money in Utility Function is modified within the assumptions to analyse the demand for Bitcoin.

Using these assumptions, in chapter 3, I will go through Money in Utility function originally presented by Walsh in his book, but with Bitcoin incorporated to the model and further study the model in order to derive Bitcoin demand function to analyse what factors can affect the demand for Bitcoin.

2.2 Bitcoin Pricing, Adoption, and Usage: Theory and Evidence (Athey et al.)

The goal of the paper is to explore Bitcoin in both theoretical and empirical way concentrating on Bitcoin market, determination of the price and also Bitcoin usage. Athey et al. develop a theory of Bitcoin adoption and pricing and they use the theory to do an empirical analysis.

According to Athey et al., the core definition of Bitcoin is that it is an open source software that makes a public ledger of transactions possible. There are services, protocols and security related software attached to Bitcoin. These add-ons are run by independent companies and software developers. In order for an agent to be an owner of Bitcoin, there has to be a ledger entry moving Bitcoin to an address of the agent.

According to the writers, another important role in the analyses on the utility of Bitcoin is the exchange rate of fiat currency[1] to Bitcoin. On the internet, there are self-service currency exchange places to trade Bitcoin for fiat money. These web-based places enable agents to transfer fiat money to and from exchanges and also, send or receive Bitcoin in exchange. The exchange rate floats and it is determined by demand and supply. The supply of Bitcoin is exogenous: it is created by mining at a fixed rate over time up to a maximum.

As commonly known, Bitcoin has no fundamental value and its value is not guaranteed by government nor any authority nor company. Hence, no authorization from any company nor government is required when transferring Bitcoin from an address to another across the world, and also, no trust relationship is needed, except for the trust in the software itself. This makes determining the exchange rates more challenging.

Using the theoretical model on the paper, Athey et al. aim to show that there are two market fundamentals capable of determining the Bitcoin exchange rates: Bitcoin's transaction volume used for payments in the steady state, and the expectation that Bitcoin survives. According to the writers, steady state exchange rates are derived from the ratio of transaction volume to the supply of Bitcoins. If the demand for Bitcoin mainly consists of Bitcoin users, the exchange rate will also depend on the rate of adoption and the demand level. This is the case where the participation rate of investors is very low. In the model created by the writers, the increase of exchange

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[1] Fiat currency= is a money that is itself worthless. It can be used as money only. In addition, it is incovertible in a sense that it cannot be converted into any good with a value. Cash money is an example of fiat currency. (Money and Monetary course 2017, Jouko Vilmunen)

rates is continuous up until a steady state is reached. In a steady state, whenever the users' needs fit Bitcoin's use cases, they all will use Bitcoin to make payment transfers.

After developing a theoretical model for Bitcoin, the writers examine two types of empirical work: a work on relationship between transaction volume and exchange rates over time and a work on agent's behaviour towards Bitcoin. The long-run relationship *(between 2011 and 2015)* between exchange rates and transaction volume shows that, apart from few periods of price spikes, the exchange rates obey a pattern which is consistent with fundamentals. According to the writers, the few periods of price spikes are not significant as they are mostly due to speculative activities. As a side note to this, at the time of writing the working paper, the fluctuations of Bitcoin's exchange rate were relatively moderate. Based on these findings, Athey et al. conclude that the forces of supply and demand seems to be working at least directionally. The reason for examining an agent's behaviour on Bitcoin is to see, whether Bitcoin is being used for investment or for payments.

Athey et al. use grouped data to analyse adoption and behaviour of Bitcoin users. The finding shows a highly concentrated ownership of Bitcoin. Additionally, only a small portion of Bitcoin users are long-term active users and most of the users merely perform a small number of transactions. The data also shows that many users tend to buy and hold Bitcoin. Based on the findings the writers concluded that most of the current transactions seems to be done by the investors. As far as the theory is concerned, this means that the exchange rates are probably more exposed to expectations about the future of Bitcoin than to its current transaction volume.

Based on the results above, Athey et al. conclude investing being the most significant use case for Bitcoin in the time of writing. This means that Bitcoin seems to be used mainly as a store of value. Being a store of value, it is more challenging to link exchange rates to the fundamentals while expectations about the future of Bitcoin becomes more significant. And last but not least, the writers emphasized that all of their conclusions are subject to the fact that specific heuristics were implemented when transforming the data into entities.

In the working paper, Athey et al. concentrate on creating a theoretical model for Bitcoin demand and exchange rate with several assumptions. They go through interesting aspects of Bitcoin and do empirical studies based on their model and by utilizing the data from website blockchain.info. In this thesis, however, I will leave out the theoretical model created by Athey et al. and only present and utilize their aggregate analysis on Bitcoin exchange rate. The aggregate analysis by Athey et al. will be presented in chapter 4.

3. Money in Utility Function featuring Bitcoin

The Bitcoin demand model presented in this chapter is based on Walsh's Money in Utility Function (Monetary Theory and Policy, 2010). In this model, Bitcoin can be used both as a store of value and as an alternative payment method to money. Assume all the goods in the market can be bought using either Bitcoin or money, but there are certain goods that are more accessible using Bitcoins. Due to this reason, it is required that the households will have to have a portion of Bitcoin holdings in their portfolio all the time. As Bitcoin is used for payments, its holdings yield transaction benefits. Additionally, to simplify the utility model, it is required to assume that there are no goods that can be bought by Bitcoins only.

As Bitcoin holdings will yield return which, in this model, equals its expected deflation with respect to domestic currency, Bitcoin is also treated as a store of value. Unlike normal money which is regulated by central bank, Bitcoin's value is only affected by its demand and supply. The inflation rate of domestic currency equals the relative rise of domestic price level. Bitcoin has a different inflation rate compared to the inflation rate of domestic currency as the exchange rate between these two varies all the time. In case of an increase in inflation rate, the more valuable Bitcoin becomes as the household will choose Bitcoin holding over money holding as money loses its purchasing power.

Let's assume there are no transaction costs in selling or buying Bitcoins. Additionally, assuming all households have the same preferences and the same utility function, we can have one representative household in the model without changing the results. Let U_t be the utility function of the representative household:

$$U_t = u(c_t, z_t, J_t)$$

z_t is the flow of goods from money holdings, J_t is flow of goods from Bitcoin holdings measured in domestic currency and c_t is per capita consumption at time t. Let's assume the utility function to be increasing, strictly concave and continuously differentiable. Assume $lim_{z\to 0} u(c, z, J_t) = \infty$ for all c, where $u_z = \partial u(c, z, J_t)/\partial z$ and $u_J = \partial u(c, z, J_t)/\partial J$.

As the economic agents are rational, z should represent number of goods money holdings can get. Here we set z equal to real per capita money holdings.

$$z_t = \frac{M_t}{P_t N_t} \equiv m_t,$$

where M is money holdings, $1/P$ is the price of goods and $N_t$is the population.

Similarly, J_t represents number of goods Bitcoin holdings can get:

$$J_t = (1 + j_t)\frac{\bar{O}_t}{P_t N_t} \equiv (1 + j_t)\sigma_t,$$

where $\bar{\mathcal{O}}_t$ represents the value of Bitcoins in domestic currency, $1/P_t$ is the price of goods and N_t represents the population. j_t is the return on Bitcoin holdings measured in domestic currency.

For the monetary equilibrium to exists, we have to assume that for all per capita consumption c, there exists a finite real per capita money holding, $\bar{m} > 0$, and real per capita Bitcoin holding measured in domestic currency, $\bar{\sigma}$ so that $u_m(c, m, \sigma) \leq 0$ for all $m > \bar{m}$ and $u_\sigma(c, m, \sigma) \leq 0$ for all $\sigma > \bar{\sigma}$.

Now, in order to derive the Money in demand function, one will have to look into the representative household's maximization problem first. This requires knowing the household's preferences and budget constraints. The representative household's total utility is given by:

$$W = \sum_{t=0}^{\infty} \beta^t u(c_t, m_t, \sigma_t),$$

where $0 < \beta < 1$ denotes a subjective rate of discount.

To derive the household's budget constraint, let's first look into the aggregate economywide budget constraint of the whole household sector:

$$Y_t + \tau_t N_t + (1-\delta)K_{t-1} + \frac{(1+i_{t-1})B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} + \frac{(1+j_{t-1})\mathcal{O}_{t-1}}{P_t}$$

$$= C_t + K_t + \frac{M_t}{P_t} + \frac{\mathcal{O}_t}{P_t} + \frac{B_t}{P_t},$$

where the right-hand side of the equation represent the sources of funds and the left-hand side of the equation represent the use of funds. Y_t is aggregate output, K_{t-1} is the aggregate stock of capital hold by the households at the beginning of period t, δ is the depreciation of the physical capital and the aggregate real value of any lump-sum transfers or taxes are represented by $\tau_t N_t$. Also, assume in addition to money M, Bitcoins O and physical capital K, the households can hold bonds B that pay a nominal interest rate i_t. Hence, the economic wide budget constraint imply that household sector allocates its sources of funds between consumption, gross investment in capital and gross accumulation of real money holding, gross accumulation of bitcoin holding in domestic currency and gross accumulation of bonds. Money in Utility model assumes that only at the end of the period, household's real money holding and real Bitcoin holding, $\frac{M_t}{P_t}$ and $\frac{O_t}{P_t}$, yield utility. This means that utility arises only after purchasing consumption goods.

The relationship between output Y_t, available capital stock K_{t-1} and employment (population) N_t can be described by the aggregate production function $Y_t = F(K_{t-1}, N_t)$. Having assumptions of a linear homogenous production function with constant returns to scale, one can write output per capita y_t as a function of the per capita capital stock k_{t-1}:

$$y_t = f\left(\frac{k_{t-1}}{1+n}\right),$$

where n is the rate of population growth. In this thesis, we assume there is no population growth. Hence, $n = 0$. Output per capita y_t function becomes:

$$y_t = f(k_{t-1}).$$

The production function is by assumptions continuously differentiable and it satisfies the following conditions called as Inada conditions:

- $f_k \geq 0$
- $f_{kk} \leq 0$
- $\lim_{k \to 0} f_k(k) = \infty$
- $\lim_{k \to \infty} f_k(k) = 0.$

Household's per capita wealth at time t is derived by dividing both sides of the aggregate economywide household sector's budget constraint by population, N_t:

$$w_t \equiv f(k_{t-1}) + \tau_t + (1-\delta)k_{t-1} + \frac{(1+i_{t-1})b_{t-1} + m_{t-1} + (1+j_{t-1})\sigma_{t-1}}{(1+\pi_t)}$$

$$= c_t + k_t + m_t + \sigma_t + b_t,$$

where $\pi_t \equiv P_t/P_{t-1}$ is the inflation rate of domestic currency, $b_t = B_t/P_tN_t$, $\sigma_t = \mathcal{O}_t/P_tN_t$, and $m_t = M_t/P_tN_t$.

The household's maximizing problem is to choose c_t, k_t, b_t, σ_t, and m_t so that the total utility W subject to wealth w_t is maximized. This is a dynamic optimization problem, where we can formulate the problem as a value function. The dynamic optimization problem assumes the household to be rational in the future, hence the utility is optimally maximized in the future. Given the assumptions about the future, one can go backwards in time and assume that the household behaves rationally in current period as well. Hence, the utility will also be maximized at current period.

Given the current state of the utility, the value function produces the maximized discounted utility value which is achieved by choosing the optimal consumption c_t, capital holdings k_t, bond holdings b_t, Bitcoin holdings σ_t, and money balances m_t. The household's initial level of resources, w_t, is the state variable of the problem. Hence, the value function of the utility becomes a discreet model of:

$$V(w_t) = \max_{c_t,\, k_t, b_t, \sigma_t, m_t} \{u(c_t, m_t, \sigma_t,) + \beta V(w_{t+1})\},$$

subject to the household's per capita wealth at time t which denotes that wealth is being divided between assets:

$$w_t \equiv c_t + k_t + m_t + \sigma_t + b_t$$

and the following function:

$$w_{t+1} = f(k_t) + \tau_{t+1} + (1-\delta)k_t + \frac{(1+i_t)b_t + m_t + (1+j_t)\sigma_t}{(1+\pi_{t+1})}.$$

Now, noticing that $k_t = w_t - c_t - m_t - \sigma_t - b_t$, we can re-write the value function as:

$$V(w_t) = \max_{c_t, b_t, \sigma_t, m_t} \Bigg\{ u(c_t, m_t, \sigma_t) + \beta V \Bigg(f(w_t - c_t - m_t - \sigma_t - b_t) + \tau_{t+1} + (1-\delta)(w_t - c_t - m_t - \sigma_t - b_t) + \frac{(1+i_t)b_t + m_t + (1+j_t)\sigma_t}{(1+\pi_{t+1})} \Bigg) \Bigg\},$$

Hence, the maximization problem is constrained over c_t, b_t, σ_t and m_t.

By following derivation rule $D[f(g(x))] = f(g(x))g'(x)$, the first order conditions below are derived:

FOC of consumption:

$$\frac{\partial V(w_t)}{\partial c_t} = u_c(c_t, m_t, \sigma_t) - \beta V(w_{t+1})(f_k(k_t) + 1 - \delta)$$

$$= u_c(c_t, m_t, \sigma_t) - \beta[f_k(k_t) + 1 - \delta]V_w(w_{t+1})$$

$$= 0,$$

hence optimal level of consumption is

$$u_c(c_t, m_t, \sigma_t) = \beta[f_k(k_t) + 1 - \delta]V_w(w_{t+1}). \tag{1}$$

FOC of bonds:

$$\frac{\partial V(w_t)}{\partial b_t} = \beta V(w_{t+1})\left(-f_k(k_t) - (1-\delta) + \frac{1+i_t}{(1+\pi_{t+1})}\right)$$

$$= 0 \text{ (optimal level of bonds)}$$

As $\beta V(w_{t+1}) \neq 0$, the FOC of bonds implies:

$$-f_k(k_t) - (1-\delta) + \frac{1+i_t}{(1+\pi_{t+1})} = 0,$$

equivalently:

$$\frac{1+i_t}{(1+\pi_{t+1})} = (f_k(k_t) + 1 - \delta). \tag{2}$$

FOC of Bitcoins:

$$\frac{\partial V(w_t)}{\partial \sigma_t} = u_\sigma(c_t, m_t, \sigma_t) + \beta V(w_{t+1})\left(-f_k(k_t) - (1-\delta) + \frac{(1+j_t)}{(1+\pi_{t+1})}\right)$$

$$= u_\sigma(c_t, m_t, \sigma_t) - \beta[f_k(k_t) + 1 - \delta]V(w_{t+1}) + \frac{\beta(1+j_t)V(w_{t+1})}{(1+\pi_{t+1})}$$

$$= 0$$

Hence, the optimal level of Bitcoin holding, which is the transaction benefit from having Bitcoin, is:

$$u_\sigma(c_t, m_t, \sigma_t) = \beta V(w_{t+1})\left\{f_k(k_t) + 1 - \delta - \frac{(1+j_t)}{(1+\pi_{t+1})}\right\}. \tag{3}$$

FOC of money holdings:

$$\frac{\partial V(w_t)}{\partial m_t} = u_m(c_t, m_t, \sigma_t) + \ \beta V(w_{t+1})\left(-f_k(k_t) - (1-\delta) + \frac{1}{(1+\pi_{t+1})}\right)$$

$$= u_m(c_t, m_t, \sigma_t) - \ \beta[f_k(k_t) + 1 - \delta]V(w_{t+1}) + \frac{\beta V(w_{t+1})}{(1+\pi_{t+1})}$$

$= 0$ (optimal level of money holding)

Hence, the optimal money holding equals:

$$u_m(c_t, m_t, \sigma_t) = \ \beta V(w_{t+1})\left\{f_k(k_t) + 1 - \delta - \frac{1}{(1+\pi_{t+1})}\right\}. \tag{4}$$

Utilizing the equation (2), the optimal transaction benefit from Bitcoin (3) equals:

$$u_\sigma(c_t, m_t, \sigma_t) = \ \beta V(w_{t+1})\left\{\frac{1+i_t}{(1+\pi_{t+1})} - \frac{(1+j_t)}{(1+\pi_{t+1})}\right\}$$

$$= \ \beta V(w_{t+1})\left\{\frac{i_t - j_t}{(1+\pi_{t+1})}\right\}, \tag{5}$$

which implies that Bitcoin yield positive transaction benefit, if domestic nominal interest rate is higher than the interest rate of Bitcoin, $i_t > j_t$.

Similarly, by utilizing the equation (2), the optimal level of consumption (1) equals:

$$u_c(c_t, m_t, \sigma_t) = \beta[f_k(k_t) + 1 - \delta]V_w(w_{t+1})$$

$$= \beta V_w(w_{t+1})\left[\frac{1+i_t}{(1+\pi_{t+1})}\right] \tag{6}$$

Rearranging equation (6) we get:

$$\frac{u_c(c_t, m_t, \sigma_t)(1+\pi_{t+1})}{1+i_t} = \beta V_w(w_{t+1}) \tag{7}$$

And similarly, the optimal level of money holdings (4) equals:

$$u_m(c_t, m_t, \sigma_t) = \beta V(w_{t+1})\left\{f_k(k_t) + 1 - \delta - \frac{1}{(1+\pi_{t+1})}\right\}$$

$$= \beta V(w_{t+1})\left\{\frac{1+i_t}{(1+\pi_{t+1})} - \frac{1}{(1+\pi_{t+1})}\right\}$$

$$= \beta V(w_{t+1})\left\{\frac{i_t}{(1+\pi_{t+1})}\right\} \tag{8}$$

Now, by utilizing the equation (7), the optimal transaction benefit of Bitcoin (5) become:

$$u_\sigma(c_t, m_t, \sigma_t) = \frac{u_c(c_t, m_t, \sigma_t)(1+\pi_{t+1})}{1+i_t}\left\{\frac{i_t - j_t}{(1+\pi_{t+1})}\right\}$$

$$= \frac{i_t - j_t}{1+i_t} u_c(c_t, m_t, \sigma_t) \tag{9}$$

As we can see from the equation (9), the size of Bitcoin's transaction benefit depends on the optimal level of consumption, the nominal interest rate level and Bitcoin's interest rate. The interest rate of Bitcoin j_t equals the deflation expectations of Bitcoin. This holds as long as there are no bonds which are valued in Bitcoin and which interest are paid in Bitcoin. According to equation (9), as long as $i_t > j_t$, Bitcoin will have transaction benefit of $u_c(c_t, m_t, \sigma_t) > 0$. Equally, bitcoin holdings will yield utility in this case. Now, if Bitcoin doesn't have transaction benefit, meaning $u_c(c_t, m_t, \sigma_t) = 0$, in equilibrium the nominal interest rate should equal the interest rate of Bitcoin measured in domestic currency, $i_t = j_t$.

Again, utilizing the equation (7), the optimal level of money holdings (4) become:

$$u_m(c_t, m_t, \sigma_t) = \frac{u_c(c_t, m_t, \sigma_t)(1+\pi_{t+1})}{1+i_t}\left\{\frac{i_t}{(1+\pi_{t+1})}\right\}$$

$$= \frac{i_t}{1+i_t} u_c(c_t, m_t, \sigma_t) \tag{10}$$

The equation (10) implies that in order for money holdings to yield utility, $u_m(c_t, m_t, \sigma_t) > 0$, the nominal interest rate has to be $i_t > 0$.

As per the transversality conditions, at the very end, everything that the household owns should go to zero which means it is rational to consume everything and leave nothing behind when the household die. In case of infinite lifetime, the discounted value of capital k, bonds b, Bitcoin σ and money holdings m should go to zero, when time t goes to infinity:

$$\lim_{t\to\infty} \beta^t \lambda_t x_t = 0 \qquad \text{for } x = k, b, \sigma, m,$$

Where λ_t is the marginal utility of consumption in period t, c_t.

Now, according to the envelope theorem, where x is assumed to be fixed at the optimal level x^* defined by first order conditions of k, b, σ, m, change in the optimal value of function $V(w_t)$ with respect to w_t can be found by partially differentiating $V(w_t)$ and holding x at its optimal value x^*:

Hence, the envelope theorem states:

$$V^* = V(w_t) = \max_{c_t, b_t, \sigma_t, m_t} \Bigg\{ u(c_t{}^*(w_t), m_t{}^*(w_t), \sigma_t{}^*(w_t)) + \beta V\Bigg(f(k_t{}^*(w_t)) + \tau_{t+1} + (1-\delta)(w_t - c_t{}^*(w_t) - m_t{}^*(w_t) - \sigma_t{}^*(w_t) - b_t{}^*(w_t)) + \frac{(1+i_t)b_t{}^*(w_t) + m_t{}^*(w_t) + (1+j_t)\sigma_t{}^*(w_t)}{(1+\pi_{t+1})}\Bigg)\Bigg\},$$

$$\frac{dV^*}{dw_t} = \frac{\partial V(w_t)}{\partial w_t}\{x = x^*(w_t)\}$$

$$= V_w(w_t)$$

$$= \beta V_w(w_{t+1})[f_k(k_t) + 1 - \delta] > 0,$$

as $f_k(k_t) > 0$ and $1-\delta > 0$. This implies that an increase of wealth w_t improves total welfare.

Now, Let the utility function $u(c_t, m_t, \sigma_t)$ be a CES utility function:

$$u(c_t, m_t, \sigma_t) = \{\theta_1 c_t{}^\eta + \theta_2 m_t{}^\eta + (1-\theta_1-\theta_2)\sigma_t{}^\eta\}^{\frac{1}{\eta}},$$

Where $0 < \eta < 1$, $c_t = \frac{C_t}{P_t}, m_t = \frac{M_t}{P_t}, \sigma_t = \frac{O_t}{P_t},\ \theta_1 > 0.\ \theta_1$ represents the share of consumption c_t, θ_2 is the share of money holding m_t and $(1-\theta_1-\theta_2)$ is the share of Bitcoin holding σ_t. $\eta = \frac{s}{s-1}$, where $s \neq 1$. s is the elasticity of substitution which is constant in CES utility model. The utility function $u(c_t, m_t, \sigma_t)$ imply that both demand for Bitcoin and demand for money will increase as consumption increases: in CES utility model they increase in the same proportion.

The FOCs of $u(c_t, m_t, \sigma_t)$ equals:

$$\frac{\partial u(c_t, m_t, \sigma_t)}{\partial \sigma_t} = u_\sigma(c_t, m_t, \sigma_t) = \frac{1}{\eta} u(c_t, m_t, \sigma_t)^{\frac{1}{\eta}-1}(1-\theta_1-\theta_2)\eta {\sigma_t}^{\eta-1}$$

$$= u(c_t, m_t, \sigma_t)^{1-\eta}(1-\theta_1-\theta_2){\sigma_t}^{\eta-1}$$

$$= \frac{u(c_t, m_t, \sigma_t)}{\sigma_t} \times \frac{{\sigma_t}^{\eta}}{u(c_t, m_t, \sigma_t)^{\eta}}(1-\theta_1-\theta_2)$$

$$= \frac{\frac{u(c_t, m_t, \sigma_t)}{\sigma_t}}{\frac{u(c_t, m_t, \sigma_t)^{\eta}}{{\sigma_t}^{\eta}}}(1-\theta_1-\theta_2) = 0,$$

Equivalently:

$$u_\sigma(c_t, m_t, \sigma_t) = (1-\theta_1-\theta_2)\left(\frac{u(c_t, m_t, \sigma_t)}{\sigma_t}\right)^{1-\eta} \quad (11)$$

$$\frac{\partial u(c_t, m_t, \sigma_t)}{\partial c_t} = u_c(c_t, m_t, \sigma_t) = \frac{1}{\eta} u(c_t, m_t, \sigma_t)^{\frac{1}{\eta}-1}\eta\theta_1 {c_t}^{\eta-1}$$

$$= u(c_t, m_t, \sigma_t)^{1-\eta}\theta_1 {c_t}^{\eta-1} = 0,$$

Equivalently:

$$u_c(c_t, m_t, \sigma_t) = \theta_1\left(\frac{u(c_t, m_t, \sigma_t)}{c_t}\right)^{1-\eta} \quad (12)$$

$$\frac{\partial u(c_t, m_t, \sigma_t)}{\partial m_t} = u_m(c_t, m_t, \sigma_t) = \frac{1}{\eta} u(c_t, m_t, \sigma_t)^{\frac{1}{\eta}-1} \eta \theta_2 {m_t}^{\eta-1}$$

$$= u(c_t, m_t, \sigma_t)^{1-\eta} \theta_2 {m_t}^{\eta-1} = 0$$

Equivalently:

$$u_m(c_t, m_t, \sigma_t) = \theta_2 \left(\frac{u(c_t, m_t, \sigma_t)}{m_t} \right)^{1-\eta} \tag{13}$$

Now, rearranging equation (10) and using equations (12) and (13) we get:

$$\frac{u_m(c_t, m_t, \sigma_t)}{u_c(c_t, m_t, \sigma_t)} = \frac{i_t}{1+i_t} = \frac{\theta_2 \left(\frac{u(c_t, m_t, \sigma_t)}{m_t} \right)^{1-\eta}}{\theta_1 \left(\frac{u(c_t, m_t, \sigma_t)}{c_t} \right)^{1-\eta}}$$

$$= \frac{\theta_2}{\theta_1} \left(\frac{c_t}{m_t} \right)^{1-\eta}$$

The equation above equals:

$$\left(\frac{m_t}{c_t} \right)^{1-\eta} = \frac{\theta_2}{\theta_1} \frac{(1+i_t)}{i_t}$$

$$= \frac{\theta_2}{\theta_1} \left(\frac{1}{i_t} + 1 \right)$$

Raising both sides of the equation to an exponent of $\frac{1}{1-\eta}$, we get:

$$\frac{m_t}{c_t} = \left(\frac{\theta_2}{\theta_1} \right)^{\frac{1}{1-\eta}} \left(\frac{1}{i_t} + 1 \right)^{\frac{1}{1-\eta}},$$

where θ_1, θ_2 and η are constant. The equation implies that an increase in nominal interest i_t will decrease the money holdings m_t with respect to consumption c_t. Hence, the demand for money holding is:

$$m_t = \left(\frac{\theta_2}{\theta_1}\right)^{\frac{1}{1-\eta}} \left(\frac{1}{i_t} + 1\right)^{\frac{1}{1-\eta}} c_t \tag{14}$$

Continuing the same approach, by rearranging equation (9) and using equations (11) and (12) we get:

$$\frac{u_\sigma(c_t, m_t, \sigma_t)}{u_c(c_t, m_t, \sigma_t)} = \frac{i_t - j_t}{1 + i_t} = \frac{(1 - \theta_1 - \theta_2)\left(\frac{u(c_t, m_t, \sigma_t)}{\sigma_t}\right)^{1-\eta}}{\theta_1 \left(\frac{u(c_t, m_t, \sigma_t)}{c_t}\right)^{1-\eta}}$$

$$= \frac{(1 - \theta_1 - \theta_2)}{\theta_1}\left(\frac{c_t}{\sigma_t}\right)^{1-\eta},$$

which equals:

$$\left(\frac{\sigma_t}{c_t}\right)^{1-\eta} = \frac{(1 - \theta_1 - \theta_2)}{\theta_1}\frac{(1 + i_t)}{(i_t - j_t)}$$

Raising both sides of the equation to an exponent of $\frac{1}{1-\eta}$, we get:

$$\frac{\sigma_t}{c_t} = \left(\frac{1 + i_t}{i_t - j_t}\right)^{\frac{1}{1-\eta}} \left(\frac{1 - \theta_1 - \theta_2}{\theta_1}\right)^{\frac{1}{1-\eta}},$$

where, again, for the equation to be valid, $i_t \neq j_t$ and as mentioned previously, for bitcoin to have a transaction benefit, nominal interest rate should be larger than the interest rate of Bitcoin. Hence, $i_t > j_t$. This equation implies that an increase in

Bitcoin's interest rate increases bitcoin holdings σ_t with respect to consumption c_t. Dividing both sides of the equation by c_t, we get a demand function for bitcoin:

$$\sigma_t = \left(\frac{1+i_t}{i_t - j_t}\right)^{\frac{1}{1-\eta}} \left(\frac{1-\theta_1-\theta_2}{\theta_1}\right)^{\frac{1}{1-\eta}} c_t \tag{15}$$

From the equation (15) we can see that the demand for Bitcoin depends on consumption, nominal interest rate and the interest rate of Bitcoin. The relationship between Bitcoin demand, nominal interest rate and the interest rate of Bitcoin can be clarified by logarithmizing the equation above (15):

$$log\frac{\bar{O}_t}{P_t N_t} = log\sigma_t = log\left[\left(\frac{1+i_t}{i_t - j_t}\right)^{\frac{1}{1-\eta}} \left(\frac{1-\theta_1-\theta_2}{\theta_1}\right)^{\frac{1}{1-\eta}} c_t\right]$$

$$= logc_t + \frac{1}{1-\eta}log(1+i_t) - \frac{1}{1-\eta}log(i_t - j_t) + \frac{1}{1-\eta}log\left(\frac{1-\theta_1-\theta_2}{\theta_1}\right) \tag{16}$$

This function presents the real demand of Bitcoin measured in domestic currency as proportional to consumption. Now, by differentiating the function $log\sigma_t$ (16) with respect to i_t, we can study the relationship between nominal interest rate and the demand for Bitcoin:

$$\frac{\partial}{\partial i_t}[log(\sigma_t)] = \frac{1}{1-\eta}\left(\frac{1}{1+i_t} - \frac{1}{i_t - j_t}\right)$$

$$= \frac{1}{1-\eta}\left(\frac{i_t - j_t - 1 - i_t)}{(1+i_t)(i_t - j_t)}\right)$$

$$= \frac{1}{1-\eta} \times \left(-\frac{j_t + 1}{(1+i_t)(i_t - j_t)}\right) < 0.$$

The above result is negative due to the assumptions of $i_t > j_t$ and $0 < \eta < 1$. The results above imply that an increase in nominal interest rate i_t will decrease the demand for Bitcoins with respect to consumption.

By differentiating the function $log\sigma_t$ (16) with respect to j_t, we get:

$$\frac{\partial}{\partial j_t}[log(\sigma_t)] = -\frac{1}{1-\eta} \times \left(\frac{1}{i_t - j_t} \times (-1)\right)$$

$$= \frac{1}{1-\eta} \times \frac{1}{i_t - j_t} > 0.$$

Under the assumptions $i_t > j_t$ and $0 < \eta < 1$, this imply that an increase in Bitcoin's interest rate will increase Bitcoins demand with respect to consumption. As previously mentioned, Bitcoin's interest rate equals the return on Bitcoin holdings.

3.1 Aggregate Demand for Bitcoin

Using the above findings, aggregate demand for Bitcoins at time t measured in domestic currency equals:

$$Q_t(\sigma_t) = N\left(\frac{1+i_t}{i_t - j_t}\right)^{\frac{1}{1-\eta}} \left(\frac{1-\theta_1-\theta_2}{\theta_1}\right)^{\frac{1}{1-\eta}} c_t$$

Where $i_t > j_t$.

The above aggregate demand function for Bitcoins is based on assumption that all households have the same utility function and the same preferences.

Now, let ψ be the aggregate supply of Bitcoin. For simplification purposes, the supply ψ is assumed to be constant. This imply that all the Bitcoins are mined and available in the market. As the aggregate supply of Bitcoin is fixed, the price of Bitcoin at time t will be defined by the aggregate demand for Bitcoin at time t. In equilibrium, the aggregate demand for Bitcoins should equal the aggregate supply of Bitcoins. If the demand and supply equilibrium doesn’t hold, the price of Bitcoin will be adjusted by the market to find a new equilibrium, where aggregate demand of Bitcoin equals aggregate supply of Bitcoin.

4. Aggregate Analysis on Bitcoin Exchange Rates by Athey et Al.

This chapter presents the findings of Athey et Al. regarding Bitcoins exchange rate in their working paper Bitcoin Pricing, Adoption, and Usage: Theory and Evidence. As per the writers, Bitcoin can be seen as a private money[2], except that it has some unique characteristics such as not being affected by bank runs. In another hand, it faces other risks such as technological risks and cyber risks

According to Athey et al., there exists a remarkable risk regarding Bitcoin’s future value as it is not linked to any underlying asset and instead, its exchange rates fluctuate fully. As per the writers, if Bitcoin is being used as a currency or a way to transfer money, then its value in the future may be linked to the future volume of that use. Bitcoin also has some similar features as risky assets in a sense that expectations about the future

[2] Private money is a private party issued money such as notes issued by private banks.

value of the asset develops over time as information is uncovered. This implies that there can be speculative bubbles in the price of Bitcoin and investors may habe conflicting beliefs.

Athey et al. use the data provided by Blockchain.info to show the movement between exchange rates, transaction volume and the effective supply of Bitcoin. Blochain.info is a website providing estimates on transaction volume, but at the same time it excludes major Bitcoin exchanges. The website also provides charts and statistics about Bitcoin supply, block size, market price, Bitcoin circulation etc. According to Athey et al, Blockchain provides a complete and accurate data enough for the writers to use for the paper.

As we already know, in general, velocity is a ratio of transaction volume to money supply. Based on this, the writers define velocity of Bitcoin as:

$$Velocity = \frac{Transaction\,Volume}{Exchange\,rate\ \times\ Supply\ of\ Bitcoins}$$

By rearranging the equation above to solve for the exchange rate, we get:

$$Exchange\,rate = \frac{Transaction\,Volume}{Velocity\ \times\ Supply\ of\ Bitcoins} \tag{17}$$

Velocity times supply of Bitcoins equals the effective supply of Bitcoin. Hence, as transaction volume represents the demand, we can see from the equation (17), that exchange rate is the ratio of the demand and the effective supply of Bitcoin. Since this is only rearranging the definition of velocity, no assumptions are required for the equation (17) to hold. In the model by Athey et al., the supply is assumed to be exogenous and constant. According to Athey et al., in a case where future supply of Bitcoin is known, we can make estimates on the prices by forecasting the ratio of transaction volume to velocity.

There are many factors that can affect the velocity of Bitcoin, as an example Athey et al. listed the following: share of active Bitcoin users, share of lost or confiscated Bitcoins, opportunities to use Bitcoin in commercial activities, and the availability of Bitcoin related mobile applications. Also, possible transaction costs from converting Bitcoin to other currencies or Bitcoin spending opportunities can affect the velocity of Bitcoin. For example, in case of no transaction costs or in case of an increase in the ability to use Bitcoin, the velocity of Bitcoin would be higher as Bitcoin will be used more often and hence, this will increase the transaction volume. For these reasons, Athey et al. state that velocity and transaction volume appear to be moving hand in hand in the future.

According to the findings from the paper, agents will spend Bitcoins more often, if the possibility to use Bitcoin increases. Following this the transaction volume increases as well. New adoption of Bitcoin can also increase the transaction volume. In this case, increasing in transaction volume can get more substantial than the increase in velocity.

In order to understand better the effects of the forces to velocity and transaction volume, Athey et al. analyses transaction-level data. Velocity for M1 money[3] equals to the ratio of nominal GDP and the supply of M1 Money. In the case of Bitcoin, the writers are yet to find a measure of true economic activity that can be compared to GDP. The writers emphasize that the measurement issue may exaggerate non-exchange transaction in times when the amount of speculative activity is higher. This is due to the fact that their measure includes transaction volume.

Below are two figures: figure 1 displays the development of Bitcoin's market capitalization which is a product of the exchange rate and Bitcoin's supply (denominator of equation (17)). Figure 1 also shows two type of annual volume

[3] M1 Money=highly liquid money: currency notes and coins, demand deposits (transactions accounts, checking accounts) (Money and Monetary course 2017, Jouko Vilmunen)

measurements in US dollars (the numerator of (17)). From the figure 1 we can see, that market capitalization evolves approximately the same way as the volume transactions.

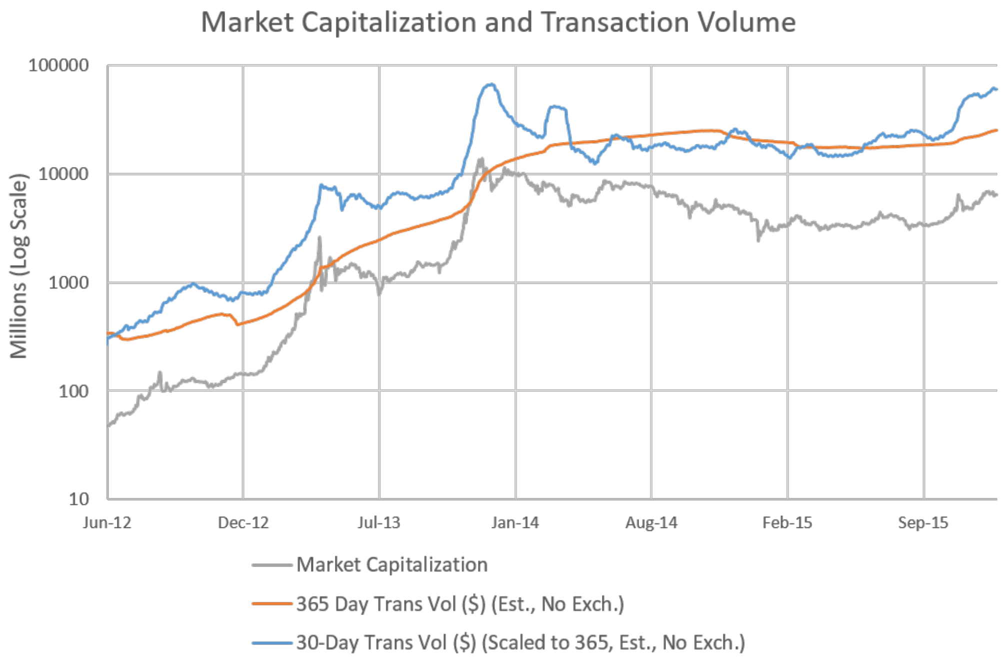

Figure 1: Market Capitalization as a product of exchange rate and Bitcoin supply. (Blockchain.info)

Figure 2 shows two versions of velocity, one regarding 30 days of transaction volume and the other one is regarding 365 days of transaction volume. The chart denotes that apart from some periods of high volume, velocity has been quite stable. As per Athey et al., high transaction volume may refer to high speculative activity due to high Bitcoin prices. Hence, in both of the figures, by applying several heuristics, transaction volume excludes change and volume related to the largest exchanges.

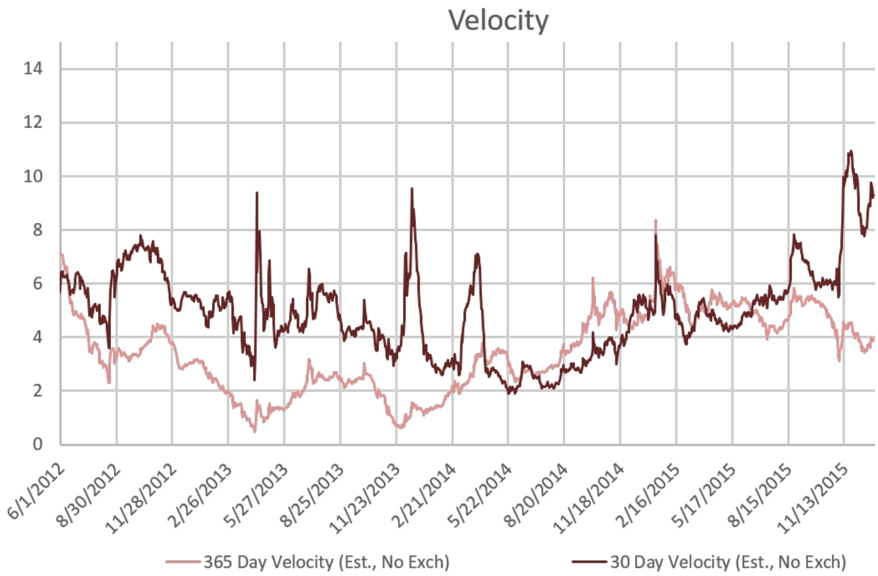

Figure 2. Velocity equals to the ratio between the transaction volume and supply. (Blockhain.info)

Athey et al. also compare the actual Bitcoin prices with the estimated prices in the case where Bitcoin velocity is held constant. This can be seen in figures 3 and 4. The figures shows that apart from the price spikes, estimated prices evolve fairly in a same way as the actual prices. This implies the theory of prices being determined by transaction volume and a quite stable velocity is consistent with the aggregate data.

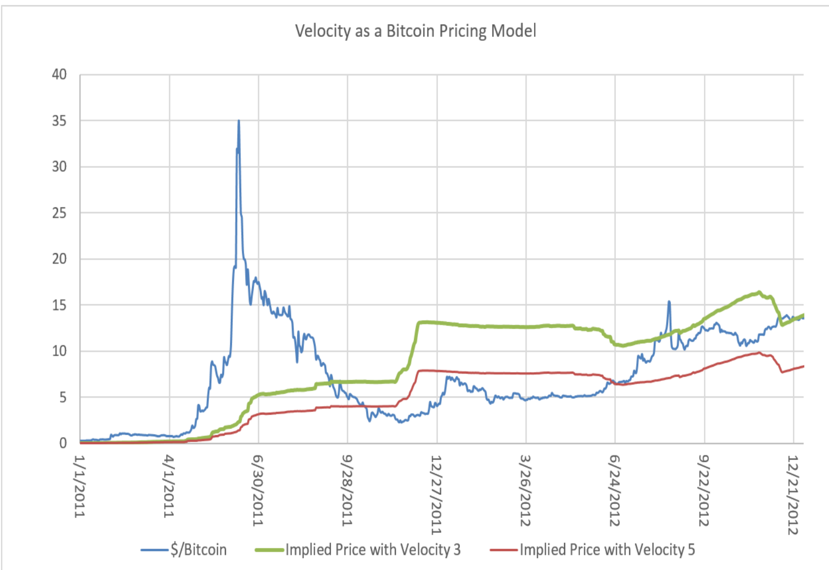

Figure 3. Estimated price from pricing model versus actual prices 2011-2012.

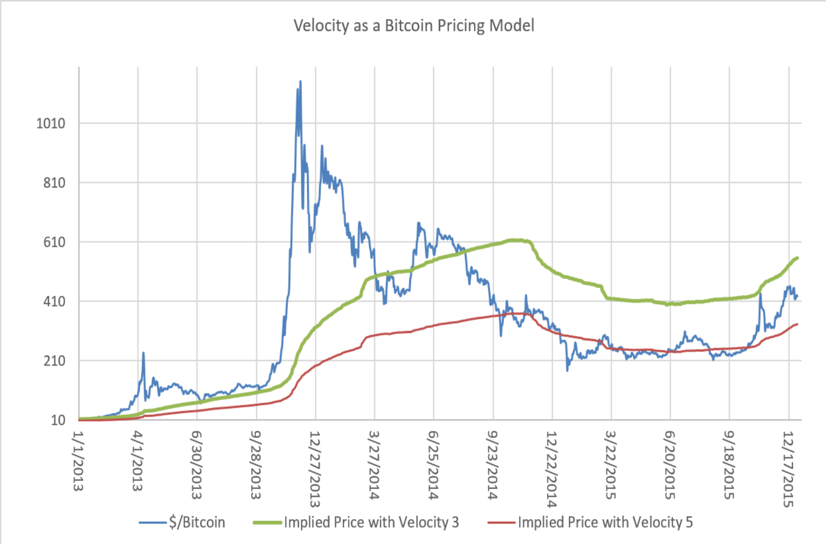

Figure 4. Estimated price from pricing model versus actual prices 2012-2015.

5. Bitcoin in Utility Model and the Aggregate Analysis on Demand for Bitcoin

As presented previously, according to Athey et Al., the exchange rate of Bitcoins equals:

$$Exchange\ rate = \frac{Transaction\ Volume}{Velocity \times Supply\ of\ Bitcoins},$$

where, as per Athey et Al., transaction volume represents the aggregate demand for Bitcoins in domestic currency and velocity times supply of Bitcoins is described as the effective supply of Bitcoins.

Now, let the aggregate demand be the function derived in chapter 3 based on Money in Utility Function. This implies changes in nominal interest rate, in consumption or in the interest rate of Bitcoin will have an effect on the demand of Bitcoin. Keeping the effective supply of Bitcoins constant, an increase in consumption or in Bitcoin's interest rate will cause the exchange rate of Bitcoin to increase as the Bitcoin's demand increases. In another hand, an increase in the domestic nominal interest rate will decrease the exchange rate of Bitcoin via the decrease in the demand for Bitcoin. Analyses and results presented below are subject to the assumptions and restrictions of the Bitcoin in Utility function model and the data provided by Athey et al., blockchain.info and tradingeconomics.com.

According to the findings by Athey et al. in chapter 4, Bitcoin prices are determined by transaction volume and a fairly stable velocity. Based on Athey et al., the velocity from June 2012 to November 2015 has been on average quite stable. This imply that changes in Bitcoin prices during the period are due to changes in transaction volume. As transaction volume represents the demand for Bitcoin, by comparing the evolvement of transaction volume with the nominal interest rate level and private consumption, we can attempt to find if there is a similar pattern in the evolvement of Bitcoin's demand and US private consumption and a reverse pattern between the evolvement of US nominal interest rate and Bitcoin's demand as denoted in chapter 3.

Figure 1 in chapter 4 shows us the evolution of transaction volume from June 2012 to September 2015. Both measures of transaction volume denote a fairly sharp increase from June 2012 to late 2013. From the end of 2013 to September 2015, after a decrease in volume in January 2014, transaction volume has been fairly stable. The figure shows that transaction volume started to increase again from September 2015. Now, as per the properties of the demand function for Bitcoin in chapter 3 and as per the above definition about transaction volume representing the demand for Bitcoin, there must have been changes in either private consumption, nominal interest rate or the interest rate of Bitcoin during these periods of time. Due to insufficient data of Bitcoin analyses in other currencies, we are only looking at the US interest rate, the US private consumption as these are in US dollars as the Bitcoin related charts and analyses from Blockchain.info are measured in US dollars.

Now, in order to study if there exists a connection between US nominal interest rate, consumption level and the transaction volume of Bitcoin as described in chapter 3, let's look into the evolvement of these factors during period of 2012-2015.

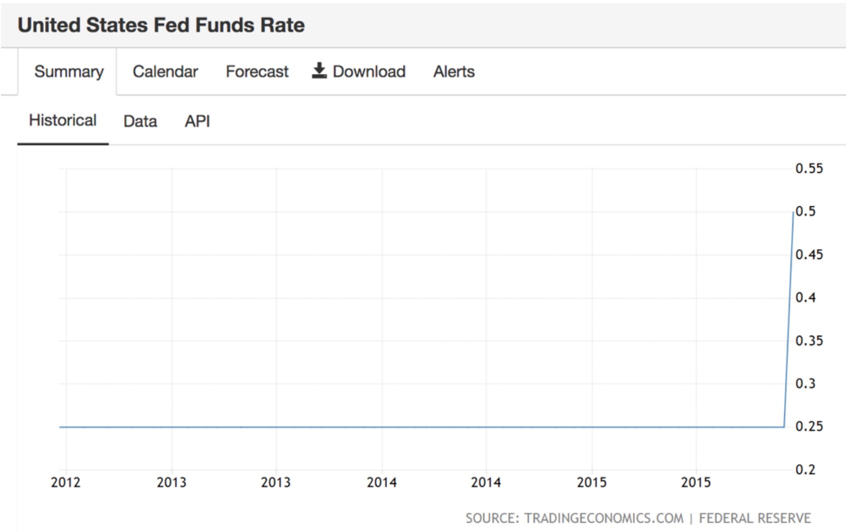

Figure 5. Fed interest rate 2012-2015

Figure 5 presents the evolution of US Fed Funds Rate from 2012 to late 2015. The Fed Funds rate act as a guiding rate for other interest rates in the US, we use evolvement of Fed Funds rate as nominal rate to do analysing. As per the assumptions and findings in chapter 3, the aggregate demand of Bitcoin is a negative function of the nominal interest rate. Hence, keeping all other factors constant, a decrease in nominal interest rate should increase the Bitcoin's transaction volume as Bitcoin's demand increases. Figure 5 shows that the development of US Fed Funds rate has been quite stable around 0,25% which is low in comparison to at what level the interest rate used to be few years back, whereas from figure 1 we can see that Bitcoin has been experiencing a remarkable increase in demand since 2012. This finding seems to be in line with the properties of Bitcoin's demand function in chapter 3.

Finally, Figure 6 presents the evolvement of the real private consumption C_t in US during May 2012 - September 2015. According to the figure 6, private consumption has been increasing fairly linearly from approximately 11,000.00 billion USD in 2012 to slightly over 12,000 billion USD in 2015. Comparing the increasing consumption to the transaction volume of Bitcoin during the same period of time (figure 1), we can see the growth rate in both have been positive. Although, Bitcoin's transaction volume has been growing in a larger scale. This finding affirms the positive relationship between Bitcoin's demand and consumption defined in chapter 3. However, the properties of Bitcoin's demand function states Bitcoin's demand as proportional to consumption which does not seem to apply in real life cases. This is not surprising given the fact that the utility function used in the demand model for Bitcoin is a CES utility function. CES utility function, as simple as it is, only considers certain factors, whereas a realistic utility function should include more variables.

Figure 6. US Real Personal Consumption 2012-2015 (https://fred.stlouisfed.org)

6. Conclusions

As Bitcoin and other cryptocurrencies such as Ethereum have been a lot on media and they have become a preferred investment option for a many people, I wanted to further study Bitcoin in my thesis. Bitcoins are being used in many ways and even inventions such as CryptoPay has been invented to mitigate the use of Bitcoin as a payment method. CryptoPay provides a digital wallet which enable the users to buy, send, receive and sell Bitcoins with no additional costs. In the digital wallet environment users can use Bitcoin holdings to buy certain currencies conveniently even via a SEPA bank transfer.

Due to the reasons above, I wanted to build up a demand function for Bitcoin using a theoretical economic model that is within my knowledge. However, it turns out that there were actually not much academic papers around this subject to start with, not to mention finding a paper studying Bitcoin within a context of a theoretical model. To my surprise, finding a theoretical model, which I could apply Bitcoin to, was fairly challenging. After a decent time used for searching, I found Walsh's Money in Utility function to be suitable for this study.

Incorporating Bitcoin to Money in Utility model as one additional variable having transaction benefits and with certain simplification assumptions, I was able to derive a demand function for Bitcoin. As shown in chapter 3, under certain assumptions, the demand function for Bitcoin is proportional to consumption. In addition, it is a negative function of the nominal interest rate and a positive function of the return on Bitcoin.

Now, as the demand function of Bitcoin is derived, I had to find a way to link it with the exchange rate of Bitcoin. In the working paper "Bitcoin Pricing, Adoption, and Usage: Theory and Evidence" by Athey et al., the writers studied Bitcoin's exchange rate using the fact that velocity is a ratio of transaction volume to money supply. Athey et al. define the velocity of Bitcoin as transaction volume divided by exchange rate times supply of Bitcoins. Hence, rearranging the equation, we can see that the exchange rate of Bitcoin equals transaction volume divided by the effective supply of Bitcoins which is velocity times supply of Bitcoins. Transaction volume represents the demand for Bitcoins. Using the data from blockchain.info, Athey et al. were able to show that the theory of prices being determined by transaction volume and a quite stable velocity is consistent with the aggregate data.

Using the findings by Athey et al. I wanted to study how well can the Bitcoin in Utility Function model explain the changes in Bitcoin demand in real world. Based on the data gathered, it seems that during period of 2012-2015, the US nominal interest was at a low level compared to how it used to be, while the demand for Bitcoin measured as transaction volume was increasing remarkably. At the same time, real consumption was increasing fairly linearly which seems to denote that there exists a positive relationship between private consumption and Bitcoin's demand. However, the increase in demand for bitcoin was not in proportion to the increase in private consumption. This finding was not surprising as the Bitcoin demand derived from Money in Utility model is not detailed enough to be able to explain all the changes in the demand for Bitcoin in real market where uncertainty is more a norm than an exception. Again, all my analyses and findings in this thesis are subject to the assumptions and restrictions of the model. Unfortunately, the theoretical model in this thesis is not able to provide further understanding around this topic due to the nature of Money in Utility Model, where any kind uncertainty is left out of scope, and due to the simplicity of the CES utility function. Also, certain assumptions are valid only in the model, when in real life Bitcoin demand is based on so many other factors such as expectations about the future of Bitcoin.

As Bitcoin is relatively new and its usage is versatile, there are probably many different approaches to study this topic. Closest being analysing Bitcoin under a scenario where certain goods can only be purchased using Bitcoin. This could also be done using Walsh's Money in Utility function, but instead of one consumption type, we would have to incorporate Bitcoin specific consumption type to the model and treat Bitcoin as a foreign currency which exchange rate is decided by the market. This approach would have made the model more complicated.

Finally, as the Bitcoin demand function derived in this thesis is limited to restrictions and assumptions related to Money in Utility model and the nature of CES utility function in general, the Bitcoin demand function presented can be used to help understanding the demand for Bitcoin in a simplified economy, where money holdings and bitcoin holdings have transaction benefit.

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