HELDA
HELSINGIN YLIOPISTO
HELSINGFORS UNIVERSITET
UNIVERSITY OF HELSINKI

UNIVERSITY OF HELSINKI

https://helda.helsinki.fi

þÿExtreme Apprenticeship Emphasising conceptu understanding in undergraduate mathematics

Rämö, Johanna

Charles University in Prague, Faculty of Education and ERME
2015

þÿRämö, J, Oinonen, L & Vikberg, T 2015, Extreme Apprenticeship conceptual understanding in undergraduate mathematics . in K Krainer & N Vondrová (eds) , CERME9 : Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education . Charles University in Prague, Faculty of Education and ERME , Prague , pp. 2242-2248 , Congress of the European Society for Research in Mathematics Education , Prague , Czech Republic , 04/02/2015 .

http://hdl.handle.net/10138/303099

acceptedVersion

Downloaded from Helda, University of Helsinki institutional repository.

This is an electronic reprint of the original article.

This reprint may differ from the original in pagination and typographic detail.

Please cite the original version.

HAL
archives-ouvertes.fr

Extreme Apprenticeship – Emphasising conceptual understanding in undergraduate mathematics

Johanna Rämö, Lotta Oinonen, Thomas Vikberg

To cite this version:

Johanna Rämö, Lotta Oinonen, Thomas Vikberg. Extreme Apprenticeship – Emphasising conceptual understanding in undergraduate mathematics. Konrad Krainer; Nad̆a Vondrová. CERME 9 - Ninth Congress of the European Society for Research in Mathematics Education, Feb 2015, Prague, Czech Republic. pp.2242-2248, Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education. <hal-01288624>

HAL Id: hal-01288624
https://hal.archives-ouvertes.fr/hal-01288624

Submitted on 15 Mar 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Extreme Apprenticeship – Emphasising conceptual understanding in undergraduate mathematics

Johanna Rämö, Lotta Oinonen and Thomas Vikberg

University of Helsinki, Helsinki, Finland, johanna.ramo@helsinki.fi

Extreme Apprenticeship (XA) is an educational method that has been used in teaching undergraduate mathematics in the University of Helsinki. In this paper, we analyse the course assignments and exam questions of a certain lecture course that has recently been reformed to an XA-based course. The results show that the XA method has made it possible to move the emphasis from rote learning towards understanding the concepts behind the procedures.

Keywords: University mathematics education, Extreme Apprenticeship, conceptual knowledge, tasks, large classes.

INTRODUCTION

The pedagogical decisions in introductory mathematics courses at university-level are of great importance. On these courses the students form their first perceptions of what university mathematics is about, and most importantly, what studying mathematics in university will be like. Traditionally, the first year courses have concentrated on memorising procedures and algorithms. The procedure-centred approach can lead to problems since procedures that lack connections with conceptual knowledge may deteriorate quickly and do not transfer easily to new situations (Hiebert & Lefevre, 1986).

Over the past years many student-centred approaches have been used in mathematics teaching for facilitating conceptual understanding (Abdulwahed, Jaworski, & Crawford, 2012). These approaches include Inquiry-based learning (IBL) and Problem-based learning (PBL), in which learning revolves around real life based problems and questions that require critical thinking (Chang, 2011; Mokhtar, Tarmizi, Ayub, & Tarmizi, 2010; Retsek, 2013). Both IBL and PBL emphasise collaborative work, presentation of conclusions and development of learning skills. Another innovative and widely used approach in mathematics teaching is Flipped classroom or Inverted classroom (Jungić, Kaur, Mulholland, & Xin, 2014; Talbert, 2014). In this approach students familiarize themselves with the new information through online video resources outside of class, and class time is reserved for discussions and cooperative problem solving. Peer instruction (Lucas, 2009) and Just-in-Time Teaching (Natarajan & Bennett, 2014) are further examples of interactive teaching methods that allow teacher to adjust teaching to the needs of students. These can be used separately or with the Flipped classroom approach.

A new student-centred method for teaching large introductory courses, Extreme Apprenticeship (XA), has been adopted in the Department of Mathematics and Statistics in the University of Helsinki (Hautala, Romu, Rämö, & Vikberg, 2012; Vihavainen, Paksula, & Luukkainen, 2011). The XA method has similarities with the approaches mentioned above, as it promotes active engagement of the students and preparation prior to the class. However, the main idea of the XA method is to support students in becoming experts in their field by making them participate in meaningful activities, which resemble those carried out by professionals. This means there are neither video lectures nor screencasts. Instead, students read the course material with the help of the teaching assistants. Another difference between XA and the other novel approaches is that in XA the main method of teaching is personal instruction: students have to do a lot of work outside of the classroom, but they are offered guidance in a drop-in basis several hours a day.

Previously it has been shown that students find the XA method satisfactory, and the passing rates do not drop even though the workload is significantly increased and the requirement level raised (Hautala et al., 2012). Also, the XA method has increased student engagement and effort (Rämö & Vikberg, 2014).

In this paper, we study whether the XA method has shifted the emphasis towards tasks that promote the development of conceptual knowledge and the linking of procedural and conceptual knowledge. This is done by comparing the assignments and exams used on an XA-based course to those used traditionally. The course under study is the first year course "Linear algebra and matrices I". While this paper focuses on the linear algebra course, the XA method has been used on many undergraduate mathematics courses, including algebra, logic and probability, and our experience leads us to believe that the conclusions drawn in this paper apply more generally than just in linear algebra.

We use the definitions of conceptual and procedural knowledge by Hiebert and Lefevre (1986): Conceptual knowledge is characterized as knowledge that is rich in relationships of many kinds, whereas procedural knowledge is made up of two distinct parts: the formal language of mathematics and the algorithms for completing mathematical tasks.

EXTREME APPRENTICESHIP

The Extreme Apprenticeship (XA) method is an educational model for organising instruction in an effective and scalable manner. Its theoretical background is in situated view on learning and Cognitive Apprenticeship (Brown, Collins, & Duguid, 1989). The method was originally invented as an instrument for teaching university-level computer programming (Vihavainen et al., 2011), and later employed in mathematics courses (Rämö & Vikberg, 2014).

In XA, the amount of tasks is substantially larger and the number of lectures smaller than traditionally. Students learn skills and gain knowledge by doing tasks that offer them small and approachable goals (Vihavainen et al., 2011). Passive activities, such as sitting in the lectures, are reduced to the minimum and active work done by the students is emphasised.

The main method of teaching in XA is personal instruction, which is based on the concepts of coaching and instructional scaffolding in Cognitive Apprenticeship (Collins, Brown, & Holum, 1991; Lave & Wenger, 1991). Instructional scaffolding refers to temporary support given to students (Wood, Bruner, & Ross, 1976), and is interlinked with the concept of zone of proximal development introduced by Vygotsky (1978). Coaching refers to the broader perspective of regulating pace and difficulty of assignments in the course. A necessity for coaching students' progress is bi-directional feedback between the students and the teaching team (Kurhila & Vihavainen, 2011). The teaching team receives feedback on the progress of the students by evaluating their solutions during the course, and also from the conversations with the students during personal instruction. Students receive feedback on how they are performing, but also encouragement and support in completing the assignments.

EDUCATIONAL SETTING

University studies in Finland resulting in a Master's degree are intended to last five years with three years of Bachelor's studies and two years of Master's studies. There are no tuition fees. The students are selected by their performance in the upper secondary school matriculation examination, an entrance exam, or both.

A traditional lecture course

On a traditional lecture course, there are 4–5 hours of lectures per week, in which the lecturer covers all the theory of the course. Every week the students are given problem sheets consisting of 6–7 tasks they have to solve. The solutions to the tasks are discussed under the guidance of a teaching assistant in a group session that lasts for two hours. In each group there are approximately 20–30 students, who usually take turns in explaining their solutions to the problems on a blackboard.

Extreme Apprenticeship based course

On an XA-based course, the amount of tasks is substantially larger than traditionally, approximately 15–20 problems per week. There are relatively easy problems on new topics, but also more challenging tasks regarding more familiar concepts studied in the previous weeks. The tasks are designed to support the development of conceptual knowledge, and to aid the students in building relationships between procedural and conceptual knowledge.

Students are offered guidance by the teaching team to complete the assignments in drop-in sessions, approximately 20 hours per week. This one-on-one or small group instruction forms the main part of teaching. The purpose of the instruction is to lead the student subtly towards the discovery of a solution through a process of questioning and listening, instead of simply giving away the answers.

One or two of the tasks are selected for inspection each week. Students receive written feedback on their reasoning and also on the readability and language of the solution, and they are encouraged to improve their solutions when necessary.

New kinds of learning spaces have been created to encourage student collaboration. The main corridor of the department has become a huge drop-in class where the tables are arranged into groups and act as whiteboards, and the walls are covered with blackboards for the students to share their thoughts with each other and with the instructors.

The amount of lectures is significantly smaller than on traditional lecture courses, only 2–3 hours per week. As the assignments force the students to investigate the topics by reading the course material prior to the lectures, it is not necessary to deliver content or go through details in the lectures. Instead, it is possible for example to discuss the meaning and consequences of definitions and to address student misconceptions through various small group activities.

Linear algebra and matrices I

The course investigated in this study is Linear algebra and matrices I. It is a first year course, and for most students it is the first mathematics course they take in the university. Approximately half of the students on the course have mathematics as a major, and the rest study mathematics as a minor subject. Among these students the most common majors are computer science, physics and economy. The amount of students taking the course has increased over the last few years. In 2008, there were 394 students enrolled for the course, whereas in 2013 the number was 484.

The content of the course has varied slightly from year to year, but the main topics have remained the same. They are systems of linear equations, matrices, spanning sets, linear independence, basis and coordinates. The course lasts for 6 weeks. The workload of the course is approximately one third of the total workload of the students.

In this paper, we investigate the years 2008–2013. Two different lecturers taught the course as a traditional lecture course during the years 2008–2010, one of them in 2008 and 2010 and the other in 2009. In 2011, the course was transformed to an XA-based course (Hautala et al., 2012). Improving the implementation of the XA method continued in 2012–2013 (Rämö & Vikberg, 2014). The teacher responsible for the XA-based courses was the first author of this paper.

METHOD

The aim of this study is to compare the assignments and exams used on an XA-based course to those used traditionally. This was done by classifying the tasks using the classification scheme by Pointon and Sangwin (2003), which was modified slightly to fit the purposes of this study.

The classification of Pointon and Sangwin consists of 8 categories shown in Table 1 (categories 1–8). When the tasks of this study were analysed, it became clear that one category, namely category 9, had to be added.

The classification was executed as in the paper of Pointon and Sangwin: Each question was evaluated individually, and given equal value. If a question had multiple parts, it was classified by estimating the proportion of each category. The evaluation was done by the second author.

1. Factual recall
2. Carry out a routine calculation or algorithm
3. Classify some mathematical object
4. Interpret situation or answer
5. Proof, show, justify (general argument)
6. Extend a concept
7. Construct example/instance
8. Criticize a fallacy
9. Information transfer

Table 1: The task classification scheme

The categories are described briefly here, and more detailed descriptions with examples can be found in the paper by Pointon and Sangwin (2003).

1) Factual recall: A question that requires only the recall of some factual knowledge, usually verbatim.

2) Carry out a routine calculation or algorithm: A question that requires routine use of algebra, calculus or matrix operations. Often such tasks may be performed by a computer algebra system.

3) Classify some mathematical object: Solving the task requires recalling a definition and providing justification to show that some specific object satisfies the definition.

4) Interpret situation or answer: The task requires modelling of a physical situation or interpretation of a mathematical model.

5) Proof, show, justify (general argument): A question that requires a general argument involving abstract or general objects rather than specific examples.

6) Extend a concept: Students are asked to evaluate previously acquired knowledge in a new situation.

7) Construct example/instance: Students are required to provide an object satisfying certain mathematical properties.

8) Criticize a fallacy: Students are asked to find mistakes in supposed proofs, or criticize reasoning.

9) Information transfer: A question that requires transformation of information from one form to another, as well as processing this information. This category was added by the authors of this paper. It is explained below in detail.

Category 9: Information transfer

Category 9 did not occur in the original classification scheme of Pointon and Sangwin that consists of categories 1–8. A category bearing the same name, information transfer, can be found in the MATH Taxonomy proposed by Smith and colleagues (1996). Our category resembles theirs, but is only a subset of it. Introducing a new category was necessary, as many of the tasks did not fit in any of the categories 1–8. Questions in category 9 require transformation of information from one form to another, as well as processing this information. Typically, in these questions students are asked to draw pictures, interpret diagrams, explain something in their own words or draw concept maps.

Examples of category 9:

— Denote $v_1 = (-3, 4)$, $v_2 = (1, 1)$ and $v_3 = (\frac{2}{3}, -2)$. Draw pictures of the subspaces $span(v_1)$, $span(v_1, v_2)$ and $span(v_1, v_3)$. You do not need to justify your answer.

— The vector space $\mathbb{R}^2$ has a basis $B = ((1, 1), (2, 3))$. Determine, by drawing a picture, a vector $u \in \mathbb{R}^2$ whose coordinates with respect B to are 3 and −2.

— Explain in your own words why an elementary matrix always has an inverse matrix.

RESULTS

Course assignments

The weekly course assignments were evaluated using the classification described in the previous section. Table 2 shows that in category 2 (routine calculation), the proportion of tasks has decreased. In 2008–2010, when the course was a traditional lecture course, 36–46% of the assignments were from category 2. In 2011, when the XA method was introduced, the proportion was still high (43%), but it dropped to 28% in 2013.

In category 9 (information transfer), the proportion of tasks has risen. In 2008–2010, 1–5% of the assignments were from this category, whereas in 2011–2013, the proportion was 15–18%.

Exam tasks

Also the exam tasks were analysed in order to find out how much weight each of the categories had in the final exam. The tasks were divided into the nine categories, and the maximum score of the tasks in each category was calculated. Table 3 shows the proportions of maximum scores in each category.

It can be seen that the weight of category 2 (routine calculation) decreased when the XA method was introduced: in 2008–2010, the proportion of points that could be obtained from category 2 tasks was 42–54%, whereas in 2011–2013 when the XA method was used, the corresponding percentage was 0–25%.

The weight of category 5 (proof) has not decreased when using the XA method. A new category (7, construct example) has appeared with the introduction of XA.

Categories, %	1. Factual recall	2. Routine calculation	3. Classify object	4. Interpret	5. Proof	6. Extend a concept	7. Construct an example	8. Criticize a fallacy	9. Information transfer	No. of tasks
2008		41	22	4	15	3	12		3	30
2009		46	13	8	28		3		1	30
2010		36	22	8	15	3	12		5	30
2011	2	43	14	2	13	2	7		18	133
2012		29	21	4	21		10		15	89
2013		28	27	7	11		10		18	89

Table 2: Proportions of course assignments in different categories. In 2011 the XA method was introduced

Categories, %	1. Factual recall	2. Routine calculation	3. Classify object	4. Interpret	5. Proof	6. Extend a concept	7. Construct an example	8. Criticize a fallacy	9. Information transfer	No. of exam points
2008	8	42	25		25					24
2009	8	54		13	25					24
2010	17	42	42							24
2011	8	25	25		25		17			48
2012	6	8	33	25	27					48
2013	13		50		25		13			48

Table 3: Proportions of exams points in each category. The points are divided into nine categories according to the category of the task they are awarded for. In 2011 the XA method was introduced

	2008	2009	2010	2011	2012	2013
Number of students	307	262	280	324	345	383
Mean (%)	69	63	72	69	75	65
Standard deviation (%)	24	19	23	19	19	21
Lower quartile (%)	50	54	58	58	65	52
Median (%)	71	67	75	73	79	67
Upper quartile (%)	92	75	92	81	90	81

Table 4: The number of students taking the exam and statistical parameters of the exam scores

Exam performance

Information on the performance of the students was obtained by studying the exam scores. From Table 4 it can be seen that the average scores were 63–72% in traditional teaching and 65–75% in XA. There seems to be no distinctive change in the student performance since the XA method was introduced.

CONCLUSIONS

The aim of the XA method is to educate skilled professionals. As professional mathematicians need to understand the concepts they are working with, this should be emphasised also when teaching future mathematicians, even first year students.

The results show that when the XA method was introduced, the weight of routine calculations in the course assignments decreased and the weight of category 9 (information transfer) increased. It can be concluded that there was a change of focus from rote learning of routine procedures towards tasks that require also conceptual understanding. However, there are still categories that are almost non-existent, such as "extend a concept" or "criticize a fallacy".

Also in the course exams the weight of category 2 decreased when the XA method was introduced. Instead of routine calculations, the tasks required constructing examples or interpreting situations or answers. At the same time, there was no drastic change in the performance of the students. In the light of these results, we can draw the conclusion that the students have developed conceptual understanding. However, when interpreting the exam results, one should note that the level of difficulty of the exams may have varied slightly over the years.

Conceptual understanding is not developed at the cost of routine skills: the actual amount of course assignments in category 2 has not dropped. Since the number of tasks in XA is greater than in traditional teaching, the amount of category 2 tasks has actually risen from 12 tasks in 2008 to 25 tasks in 2013 (Table 3). This means that also in the XA method the students get plenty of practice in routine calculations.

There are many features in the XA method that facilitate emphasising conceptual knowledge. The teaching revolves around the tasks, and there are plenty of them for the students to work on. Therefore, it is easier to give students a wide range of diverse tasks. Because of the one-on-one instruction, also the weaker students have a change to fully work on the problems, and they do not need to give up if a task seems too difficult for them. The lectures support the development of conceptual understanding by focusing on motivating the concepts and discussing how they are linked together. However, not all the students take advantage of the instruction: less than half of the students who submit course work speak with the teaching assistants, and many of the students do not attend the lectures. Our next goal is to find ways to encourage students to take part in the instruction.

In this study, the tasks were categorised by the second author. The reliability of the study would be improved if the tasks were given to an independent evaluator who does not know which tasks are from which year. Also, a detailed statistical analysis would give more information about the changes that have taken place.

The tasks given to the students should be versatile and varied because mathematical competence involves knowledge of both concepts and procedures, as well as understanding the relations between them (Hiebert & Lefevre, 1986). Also, when students are offered diverse problems, they learn problem-solving strategies of experts (Collins et al., 1991). In this light, our results indicate that the Extreme Apprenticeship method is a step in the right direction.

REFERENCES

Abdulwahed, M., Jaworski, B., & Crawford, A. (2012). Innovative approaches to teaching mathematics in higher education: a review and critique. *Nordic Studies in Mathematics Education, 17*(2), 49–68.

Brown, J. S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational researcher*, *18*(1), 32–42.

Chang, J. M. (2011). A practical approach to inquiry-based learning in linear algebra. *International Journal of Mathematical Education in Science and Technology*, *42*(2), 245–259.

Collins, A., Brown, J.S., & Holum, A. (1991). Cognitive apprenticeship: Making thinking visible. *American educator*, *15*(3), 6–46.

Hautala, T., Romu, T., Rämö, J., & Vikberg, T. (2012). Extreme apprenticeship method in teaching university-level mathematics. In *Proceedings of the 12th International Congress on Mathematical Education, ICME*. Seoul, Korea: ICMI. Available online http://www.icme12.org/upload/UpFile2/TSG/1801.pdf

Hiebert, J., & Lefevre, P. (1986). Conceptual and Procedural Knowledge in Mathematics: An Introductory Analysis. In J. Hiebert (Ed.), *Procedural and Conceptual Knowledge: The Case of Mathematics* (pp. 1–27). London, UK: Lawrence Erlbaum Associates.

Jungić, V., Kaur, H., Mulholland, J., & Xin, C. (2014). On flipping the classroom in large first year calculus courses. *International Journal of Mathematical Education in Science and Technology 46*(4), 508-520. doi:10.1080/0020739X.2014.990529

Kurhila, J., & Vihavainen, A. (2011). Management, structures and tools to scale up personal advising in large programming courses. In *Proceedings of the 2011 conference on*

Information technology education, SIGITE '11 (pp. 3–8). West Point, NY, USA: ACM.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge, UK: Cambridge University Press.

Lucas, A. (2009). Using peer instruction and I-clickers to enhance student participation in calculus. *Primus, 19*(3), 219–231.

Mokhtar, M., Tarmizi, R. A., Ayub, A. F. M., & Tarmizi, M. A. A. (2010). Enhancing calculus learning engineering students through problem-based learning. *WSEAS transactions on Advances in Engineering Education, 7*(8), 255–264.

Natarajan, R., & Bennett, A. (2014). Improving student learning of calculus topics via modified just-in-time teaching methods. *Primus*, *24*(2), 149–159.

Pointon, A., & Sangwin, C. J. (2003). An analysis of undergraduate core material in the light of hand-held computer algebra systems. *International Journal of Mathematical Education in Science and Technology*, *34*(5), 671–686.

Retsek, D. Q. (2013). Chop wood, carry water, use definitions: Survival lessons of an IBL rookie. *Primus, 23*(2), 173–192.

Rämö, J., & Vikberg, T. (2014). Extreme Apprenticeship – Engaging undergraduate students on a mathematics course [Special issue]. *European Journal of Science and Mathematics Education,* 26–33.

Smith, G., Wood, L., Coupland, M., Stephenson, B., Crawford, K., & Ball, G. (1996). Constructing mathematical examinations to assess a range of knowledge and skills. *International Journal of Mathematical Education in Science and Technology*, *27*(1), 65–77.

Talbert, R. (2014). Inverting the linear algebra classroom. *Primus, 24*(5), 361–374.

Vihavainen, A., Paksula, M., & Luukkainen, M. (2011). Extreme apprenticeship method in teaching programming for beginners. In *Proceedings of the 42nd ACM technical symposium on Computer science education* (pp. 93–98). Dallas, Texas, USA: ACM.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Wood, D., Bruner, J.S., & Ross, G. (1976). The role of tutoring in problem solving*. The Journal of Child Psychology and Psychiatry and Allied Disciplines*, *17*, 89–100.