Some search twice

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Abstract

We consider duopoly pricing and sequential consumer search with the twist that the firms can make the search time-consuming. We provide a characterization of equilibria in which buyers search twice and sellers deliberately raise the search time despite the costs. The firms with low and intermediate prices let the consumers search freely. The firms with intermediate and high prices stop continued search. The opportunity cost of obfuscation is non-monotonic. We construct an explicit example that features a continuous price distribution on a connected support. The solution technique might be useful in applications.

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1 Introduction

How would you buy, say, a cheap sofa? Would you just go to the nearest retailer, find a sofa, take a look at your watch, and purchase the sofa whatever the price? Have you ever continued searching for a better deal? The classic oligopolistic pricing model with sequential consumer search by Stahl [10] predicts that you have not — unless you really enjoy shopping. Casual observation and a recent extension by Ellison and Wolitzky [3], however, suggest that you may have. Search is costly but, typically, not so expensive as to stop continued search altogether. Besides, technical advances have made, especially, on-line search much faster. There are hopes that this would improve consumer welfare. Buyers would search longer and find better matching goods for lower prices. Sellers would compete more intensively.

How long do you usually search then? Probably not very long. Search is not that cheap either. As shown by a pair of complementary, seminal contributions by Ellison and Ellison [2] and Ellison and Wolitzky [3], technological improvement may be offset by deliberate debilitation. [2] document a myriad of ways how firms intentionally make on-line shopping confusing or complicated. They conjecture that at least part of the practices cannot be explicated by claiming that firms incur a cost of releasing clear and accurate information. On the contrary, it must be costly to obfuscate. To explain the findings, [3] go through a number of settings in which obfuscation occurs in equilibrium. The sellers may have an incentive to raise the buyer search time even if it costs. The buyers may be worse off.1

1[2] analyze competition between a large number of small, undifferentiated e-retailers selling computer parts (memory upgrades, central processing units, etc.) through Pricewatch.com, an Internet price search engine. They report various forms of obfuscation. For instance, when the price lists used to be sorted solely by item prices, it was commonplace to see a low item price (say, $1 for memory chips, observable on the price comparison lists) combined with high shipping costs (up to $40, observable only at checkout). After Pricewatch changed its policies to render the practice impossible, the firms started bundling low priced items with low priority for packing, otherwise unattractive contractual terms, or so low quality that, if ordered, the firms preferred to deliver a higher quality item instead to avoid the hassles of returns etc. It seemed to be a typical firm strategy to attract consumer attention by offering them low prices but, when they follow to the firm web-page, to flood them with offers to switch to higher qualities sold for higher prices. To place an order for the lowest prices, the consumers sometimes needed to unclick a series of suggested upgrades. Or, it was difficult to find the Pricewatch prices from the firm web-page. They were not be listed among the other prices, and there was no direct link to them. In some cases, the Pricewatch prices could only be ordered on the phone, not on the Internet.

In marketing literature, the effect of store or web-page atmosphere on shopping behavior, “atmospherics”, is a much explored topic. It is widely believed to be well-established that “experienced
We start from two observations

(i) some consumers search more than once,
(ii) some firms raise consumer search time,

and formalize them building on [3] and [4]. We describe “transparency-obfuscation equilibria”, in which both (i) and (ii) are satisfied, and provide an explicit example. The solution technique might be useful in applications.

In general, it is not easy to derive an equilibrium of this particular type analytically. [3] calculate an example equilibrium numerically, but they resort to a discrete cost function and they end up with a price distribution with several gaps. Though the illustrated patterns of prices, obfuscation and search are highly interesting, the black box nature of computation leaves open some questions and raises more. Under what conditions do equilibria with multiple searches arise? What do they look like? Are jumps in the primitives and gaps in the price distribution necessary?

To answer the questions, we consider duopoly pricing under sequential consumer search and characterize equilibria in which (i) holds. This requires that (ii) holds as well. Transparency and obfuscation coexist. “Transparency-obfuscation equilibria” are found to be quite special. Firms increase the consumer search time in the highest (and intermediate) parts of the price distributions but not in the lowest and intermediate parts. If there are gaps, they form when obfuscation would be appropriate but too expensive. In the simplest equilibrium with continued search, the opportunity cost of obfuscation is inverse-U-shaped and has two roots. As a result, the growth of obfuscation cost must be first sufficiently strong and then subdued enough relative to the growth of obfuscation revenue. Intuitively, for the outcome (search costs) to be non-

pleasantness of the in-store environment is a significant predictor of willingness to spend time in the store and intentions to spend more money than originally planned” ([1], and the references therein). Moreover, it is “suggested that the characteristics of products and web-sites ... can significantly influence the level of arousal and pleasure that consumers experience, and thereby can influence their later shopping behavior” [7]. A quick Internet search also reveals various tips for retailers on encouraging their customers to stay longer – and buy more. Coffee shops, sandwich stores and bakeries are, for example, advised to offer free wi-fi, putting out games, starting some trivia, initiating a deal hunt, or offering half-priced refills to delay the customers (SwipelyWorks.com for local retailers, article by Amanda MacArthur, July 19, 2011). It is also suggested that adding in-store demonstrations and videos, engaging customers’ all senses, stocking the store by surprising finds, improving customer service, catching their eyes while they are queuing, and giving them discount coupons with the receipts, to turn them around on their way out etc., makes the customers stay longer and spend more (MainStreet.com, article by Seth Fiegerman, June 2, 2011).
monotonic, the primitives (opportunity costs) have to be also non-monotonic (see Fig. 1 in Appendix B).

We construct a simple analytical example of a “transparency-obfuscation equilibrium” founded on continuous primitives and resulting in a continuous price distribution on a connected support (see Fig. 2 in Appendix B). The solution technique utilizes the inverse of the obfuscation time to essentially linearize the opportunity cost of obfuscation. This makes it possible to analyze the firm and consumer problems separately. In stead of solving a system of seven nonlinear equations to find an equilibrium, we can solve a smaller system of linear equations (the firm problem) and check that the remaining non-linear equations (the consumer problem) hold as well. This might be helpful in applied work since alternative analytical approaches are more contrived. The inverse of obfuscation time could also be transformed, if the linear pattern would not be appropriate.

We make a few remarks on welfare. Overall, the option to obfuscate makes firms better off and consumers worse off. As shown by [3, Corollary 2], prices are increased in the sense of first-order stochastic dominance. We decompose the profits to clarify the implications. Obfuscation benefits both individual firms and the industry. The expanded revenues offset the elevated obfuscation costs. Since the other firms set higher prices, it is easier for a firm to extract profits from the consumers who search twice. As the other firms also obfuscate, the consumers are more reluctant to search, and a firm can obfuscate less and incur lower obfuscation costs. Interestingly, “transparency-obfuscation equilibria” may generate higher welfare for both buyers and sellers than “obfuscation equilibria”. Less time is wasted and lower obfuscation costs are paid.

The structure of the note is the following. In Section 2, we introduce the model and, in Section 3, we define and characterize “transparency-obfuscation equilibria”. In Section 4, we analyze welfare. We discuss the construction of an example equilibrium in Section 5, and we conclude in Section 6.
2 Model

There are two firms and a unit mass of consumers. The firms set the prices $p$ and choose the added search time $t$ (obfuscation, the time it takes for the consumers to discover the product and the price at the store or at the web-page) knowing the basic search time $\tau$ (the time it takes for the consumers to get to a store or open a web-page). Obfuscation is expensive. It takes time and other resources. The obfuscation cost $c$ is increasing as a function of obfuscation time $t$. The firms are similar but the consumers differ. A mass $\mu$ of shoppers incur zero search costs. They like shopping. A mass $1 - \mu$ of searchers encounter increasing, convex search costs $g$ as a function of cumulative search time $t$. Consumers go to a firm and wait until the price is revealed (they cannot leave earlier). They can either stop the search and buy for the best price so far (recalled without cost) or continue the search. Both consumer types have identical (inverse) demands $D$ as a function of price $p$.

We consider Perfect Bayesian equilibria (PBE) of the following two stage game. In stage 1, the firms fix prices and obfuscation levels simultaneously. In stage 2, the consumers search the firms sequentially.

The sellers do not see each others’ prices or obfuscations. As they cannot coordinate, they have to resort to symmetric pricing and obfuscating strategies. A buyer has to search a seller to observe the price and obfuscation. The firms look the same at first. Therefore, the consumers use symmetric contact strategies. Both firms are approached by a mass $\frac{1-\mu}{2}$ of searchers who have not discovered the other firm’s price and obfuscation yet. They continue the search if expected gains exceed expected costs. The state vector of this search problem is a pair $(p, t)$, the best price and the time spent. The firm with the lowest price attracts all the shoppers $\mu$. The cost and demand functions $c$, $g$ and $D$ are assumed to be continuous and, unless indicated otherwise, continuously differentiable. The demand is such that the revenue $R$, $R(p) := D(p)p$, is concave and has a unique maximum at the monopoly price $p^m$. The cost functions $c$ and $g$ are zero at zero. The parameter space is $\{(\tau, \mu)|0 < \tau, 0 < \mu < 1\}$.

The equilibrium strategies are characterized as follows.
Lemma 1.
(a) Firms price in mixed strategies $F(p)$.
(b) The price distribution is continuous and does not have atoms in its support $\text{supp}F(p)$.
(c) Firms obfuscate in pure strategies $t(p)$.
(d) If the state is $(p_0, t_0)$, searchers continue if and only if the value of continuing exceeds the cost of continuing
\[
V(p_0) = \int_{\hat{p}}^{p_0} (CS(x) - CS(p_0))f(x)dx = \int_{\hat{p}}^{p_0} D(x)F(x)dx > E_t(g(t + t_0) - g(t_0)),
\]
where $V$ is the expected value of continuing the search, $CS$ is the consumer surplus, and
$E_t(g(t + t_0) - g(t_0))$ is the expected cost of continuing the search.

Proof. We repeat the basics to keep the exposition self-contained (see also [10] and [3]). (a) There is no symmetric pure strategy price equilibrium. To see this, consider a candidate pure strategy equilibrium $p$. If $p = 0$ and $\tau > 0$, there is a higher price $p(\varepsilon) > 0$ which stops the first-time searchers $\frac{1-\mu}{2}$ and yields positive profits $\frac{1-\mu}{2}p(\varepsilon)$. If $p > 0$, there is a lower price $p - \varepsilon$ that attracts all the shoppers and yields additional profits $\frac{1}{2}\mu(p - \varepsilon) - \frac{1}{2}\mu\varepsilon$. (b) If there is a jump at $p$ in the price distribution, the firms can increase the profits discretely by decreasing the price continuously as the probability of attracting the shoppers increases from $(1 - F(p))$ to $(1 - F(p - \varepsilon))$. Concerning obfuscation, they can keep on doing what they did at the jump price $p$ as the primitives $c$, $g$ and $R$ are continuous. If there were an atom, the same logic would apply. (c) Since obfuscation is costly and unobservable to the other firm, the firms choose either no obfuscation or the lowest level of obfuscation that stops continued search. For item (d), see [3, Lemma 1], and [11]. ■

3 Transparency-Obfuscation Equilibria

We define and characterize non-trivial equilibria with both continued search and obfuscation. An equilibrium is non-trivial if the searchers find it profitable to enter the market. There is potential for obfuscation if the search costs $\tau$ and $E_t(t)$ are not too high to deter continued search at the monopoly price $p^m$. We are interested in the search
costs across the price distribution. There is price dispersion as long as the search costs are neither so low (both the searchers and the shoppers find out both prices) nor so high (the searchers do not enter but the shoppers do) as to invoke Bertrand competition.

**Definition 1.** A *transparency-obfuscation equilibrium* is a PBE in which,

(i) for some prices, there is no obfuscation and the searchers go to both firms and,

(ii) for some other prices, there is obfuscation and the searchers go to only one firm.

We focus on transparency-obfuscation equilibria with continuous price distributions on connected supports. In particular, it turns out that transparency is necessarily accompanied by obfuscation. The search costs and the opportunity cost of obfuscation induce a partition of the support. In brief, the firms never obfuscate for the lowest prices and always obfuscate for the highest prices. For the prices in between, the opportunity cost of obfuscation determines whether it pays off to increase the search costs or to keep them at the minimum. The firms let the consumers search undisturbed if and only if the opportunity cost of obfuscation is non-negative.

**Proposition 1.** In a transparency-obfuscation equilibrium with connected support $\text{supp} F(p) = [\underline{p}, \overline{p}]$,

(a) there is *neither obfuscation nor continued search* for the lowest prices $[\underline{p}, p_0]$,

(b) there is *obfuscation* for the highest prices $[p_i, \overline{p}]$,

(c) there is *continued search* for prices $p \in [p_0, \overline{p}]$ such that the opportunity cost of obfuscation is non-negative, and

(d) there is *obfuscation* for prices $p \in [p_0, \overline{p}]$ such that the opportunity cost of obfuscation is negative.

The bound prices are given by

(i) $\underline{p} \leq p$ for all $p \in \text{supp} F(p)$ ($\underline{p}$ is the lowest price),

(ii) $p \leq \overline{p}$ for all $p \in \text{supp} F(p)$ ($\overline{p}$ is the highest price),

(iii) $V(p) = E_t(g(2\tau + t) - g(\tau))$ for $p = p_0$, and (continued search is possible above $p_0$)

(iv) $h(p) = 0$ for $p \in \{p_{j-1}, p_j\}$ for all even $j$ below some even $i$

(obfuscation stops above $p_{j-1}$ and starts above $p_j$).

The opportunity cost of obfuscation is given by $h : [p_0, \overline{p}] \to \mathbb{R}, h(p) := c(t(p)) - \frac{1}{2} \mu : F(p) R(p)$. 

Proof. (a) & (iii) The searchers who have discovered very good deals have no incentive to keep on looking for even better deals as search is costly. The firms need not obfuscate to stop continued search. There are low prices \( p \) for which the base-line cost of continuing the search \( E_t(g(2\tau + t) - g(\tau)) \) exceeds the value of continuing the search \( V(p) \). The function \( V \) is increasing and zero at the lowest price \( p \). It equals the cost \( E_t(g(2\tau + t) - g(\tau)) \) for \( p = p_0 \), and exceeds it for all \( p > p_0 \). (b) By the definition of a transparency-obfuscation equilibrium, the cost \( E_t(g(2\tau + t) - g(\tau)) \) is lower than the value \( V(p) \) for the highest prices \( p \) in the support \( \text{supp}F(p) \). The searchers who have discovered very bad deals have an incentive to keep looking for better deals. Without obfuscation, they leave and rarely return. The firms pricing near the high end of the distribution must obfuscate to cease continued search. Otherwise, they get negligible profits \( (1 - F(p))(\mu + 1 - \mu)R(p) \). That is impossible. All prices \( p \) in the support \( \text{supp}F(p) \) have to yield the same positive profits \( \pi \) to support mixed strategies. (c) & (d) & (iv) There is a profitable deviation in obfuscation if the opportunity cost is non-negative when there is obfuscation, and the other way around. The opportunity cost of obfuscation is the difference between the costs of obfuscation and the gains from obfuscation. By obfuscating, the firm loses the obfuscation cost \( c(t(p)) \) but gains the first-time searchers even when the other firm prices lower \( \frac{1 - \mu}{2}F(p)R(p) \). Since the opportunity cost of obfuscation is continuous and negative at \( p_0 \) (given (a) and given that \( c(t(p_0)) = 0 \)) and \( \bar{p} \) (given (b)) and positive for some prices in between, it clearly has an even number of roots in a transparency-obfuscation equilibrium (according to Bolzano’s theorem). For items (a) and (b), see also [3, Proposition 9]. ■

As shown by the example in Section 5, there exist transparency-obfuscation equilibria for appropriately chosen function forms and parameter values. The equilibrium price distribution is pinned down by profit equivalence. The equilibrium obfuscation is uniquely determined by the opportunity cost of obfuscation and the minimal level of obfuscation deterring continued search.

To elaborate, notice that all prices \( p \) in the support \( \text{supp}F(p) \) must yield the same profit \( \pi \) for the sellers to mix. The price distribution can be derived from the equivalence of profits. There are three cases. When there is neither obfuscation nor continued search,
the profit is given by
\[
\pi = \left( a(p) \left( \frac{1 - \mu}{2} + \frac{1 - \mu}{2} + (1 - F(p)) \mu \right) \right) R(p),
\]
where \(a(p)\) is the probability of selling to the searchers who initially approached the other firm. The firm sells to the first-time searchers \(\frac{1 - \mu}{2}\) for sure and to the shoppers \(\mu\) if the other firm has chosen a higher price. When there is continued search, the profit is given by
\[
\pi = \left( a(p) \left( \frac{1 - \mu}{2} + (1 - F(p)) \left( \frac{1 - \mu}{2} + \mu \right) \right) \right) R(p).
\]
The firm sells to the returning searchers \(\frac{1 - \mu}{2}\) and to the shoppers \(\mu\) if its price is lower than the other firm’s price. When there is obfuscation, the profit is given by
\[
\pi = \left( a(p) \left( \frac{1 - \mu}{2} + \frac{1 - \mu}{2} + (1 - F(p)) \mu \right) \right) R(p) - c(t(p)).
\]
By obfuscating, the firm wins the revenue from the first-time searchers when the other firm has a lower price \(\frac{1 - \mu}{2} R(p) F(p)\) but loses the cost \(c(t(p))\). Given the structure of profits, it is also easy to see that gaps can form only if obfuscation (a) increases the profits in comparison to continued search but (b) is still not profitable enough to generate the equilibrium profits \(\pi\) given the costs.

**Remark 1.** In a transparency-obfuscation equilibrium, the support is connected \(\text{supp} F(p) = [\underline{p}, \overline{p}]\) if the obfuscation cost \(c \circ t\) does not grow too fast in relation to the revenue \(R\) when the opportunity cost of obfuscation is negative.

**Proof.** Suppose there is a gap. As a consequence, \(F\) is constant within the gap. Consider a small deviation upwards to \(p'\) from the low end of the gap \(p\). The appropriate obfuscation is given by the opportunity cost. If obfuscation at \(p'\) generates lower profits than continued search, the deviation is always profitable. The probability of attracting the shoppers and the continued searchers \((1 - F(p))\) is constant but the revenue \(R(p)\) increases as seen from (1) and (2). Instead, if obfuscation at \(p'\) generates higher profits than continued search, the deviation is profitable only as long as the obfuscation cost \(c(t(p))\) does not grow too fast in relation to the revenue \(R(p)\) between \(p\) and \(p'\). When
that is not the case, there might be a gap in the price distribution $F$.

The underlying structure of transparency-obfuscation equilibria is illustrated in Fig. 1 in Appendix B. In the florid case (Fig. 1, above), the opportunity cost of obfuscation is M-shaped and has four roots – or $2n$ more for $n$ integer. That seems improbable. In the simple and, perhaps, more reasonable case (Fig. 1, below), the opportunity cost of obfuscation is inverse-U-shaped and has two roots. The basic search costs are enough to stop continued search for $[p, p_0]$. There is obfuscation for both intermediate $[p_0, p_1]$ and high prices $[p_2, p]$, but continued search for intermediate prices $[p_1, p_2]$. Transparency is based on obfuscation.

Obviously, the equilibria of this particular type are based on quite specific cost and revenue structures. The growth of the cost of obfuscation $c(t(p))$ must be first sufficiently strong and then subdued enough relative to the growth of the revenue from obfuscation $\frac{1-\mu}{2} F(p) R(p)$. Intuitively, for the added search costs to be non-monotonic, the opportunity costs of adding to the search costs must be non-monotonic.

The described pattern of obfuscation cost might arise in some familiar setups. For instance, consider a manager who can either obfuscate herself or hire sales personnel to do the obfuscation for her, for example, in the disguise of customer service. Suppose the marginal cost of managerial time is first lower and then higher than the marginal cost of personnel time – the fixed salary. Therefore, we conjecture that under appropriate circumstances there might be managerial obfuscation for the lower prices and sales personnel obfuscation for the higher prices, and two obfuscation cost regimes, a steep one and a (locally) flat one.

There are other examples. We return to the idea of two distinct obfuscation cost regimes with a more detailed setup in Chapter 5. To ease the analysis, we spell out the equilibrium pricing and obfuscation strategies. In simple transparency-obfuscation equilibria with connected supports, the strategies have a piece-wise form.
Corollary 1. In a simple transparency-obfuscation equilibrium with connected support \( \text{supp} F(p) = [\underline{p}, \overline{p}] \), the firm strategies are given by

(a) for all \( p \in [\underline{p}, p_0] \), \( t(p) = 0 \) and \( F(p) = \frac{\mu + (1+F(p_2)-F(p_1))\frac{1-\mu}{2}R(p)-\pi}{\mu R(p)} \),

(b) for all \( p \in (p_0, p_1] \), \( t(p) > 0 \) and \( F(p) = \frac{\mu + (1+F(p_2)-F(p_1))\frac{1-\mu}{2}R(p)-(\pi+c(p))}{\mu R(p)} \),

(c) for all \( p \in (p_1, p_2) \), \( t(p) = 0 \) and \( F(p) = \frac{\mu + (1+F(p_2))\frac{1-\mu}{2}R(p)-\pi}{\mu R(p)} \), and

(d) for all \( p \in [p_2, \overline{p}] \), \( t(p) > 0 \) and \( F(p) = \frac{\mu + (1+F(p_2))\frac{1-\mu}{2}R(p)-(\pi+c(p))}{\mu R(p)} \).

Whenever \( t(p) > 0 \), it is the unique solution of \( V(p) = E_t (g(2\tau + t + t(p)) - g(\tau + t(p))) \).

Proof. The price distribution is derived by direct calculation based on Proposition 1 and the equivalence of (1), (2) and (3). The probability of selling to the searchers who started from the other firm is \( a(p) = F(p_2) - F(p_1) \) for \( p \in [\underline{p}, p_1] \), \( a(p) = F(p_2) - F(p) \) for \( p \in (p_1, p_2] \), and \( a(p) = 0 \) for \( p \in [p_2, \overline{p}] \). Since obfuscation is costly, the firms choose either no obfuscation or the lowest level of obfuscation that stops continued search. The lowest level \( t(p) \) is uniquely given by \( V(p) = E_t (g(2\tau + t + t(p)) - g(\tau + t(p))) \).

4 Welfare

Obfuscation is inefficient. It wastes time and it costs. But how is the surplus divided? Are the firms better off if they have the option to obfuscate? It turns out that they are.

Corollary 2. The profit is higher if obfuscation is possible than if obfuscation is not possible when \( p_0 < p^m \).

Proof. The condition \( p_0 < p^m \) implies that \( p_0 \) is the highest price when obfuscation is not possible (see [3, Proposition 2]). Therefore, the profit is given by \( \pi_\tau = \frac{1-\mu}{2} R(p_0) \) because the firms with the high end price almost never attract shoppers or second-time searchers. In contrast, if obfuscation is possible, the profit is obtained from \( \pi_{\tau+t} = \frac{1-\mu}{2} R(p_0) + (F(p_2) - F(p_1))\frac{1-\mu}{2} R(p_0) + (1 - F(p_0))\mu R(p_0) \). That is at least as high as \( \pi_\tau = \frac{1-\mu}{2} R(p_0) \) since \( p_0 \) need no longer be the highest price. In addition to the revenue from the own first-time searchers, it may be possible to extract revenue from the second-time searchers and the shoppers. (See also [3, Corollary 2].) ■

The decomposition \( \pi = \frac{1-\mu}{2} R(p_0) + (F(p_2) - F(p_1))\frac{1-\mu}{2} R(p_0) + (1 - F(p_0))\mu R(p_0) \) proofs that the profits are inflated in all equilibria with obfuscation, but the combination
of transparency and obfuscation can raise them even more. Without obfuscation, there is no revenue from the shoppers \((1 - F(p_0)) \mu R(p_0)\) at \(p_0\). Without transparency, there is no revenue from the second-time searchers \((F(p_2) - F(p_1)) \frac{1 - \mu}{2} R(p_0)\) at \(p_0\). Notice, however, that obfuscation equilibria and transparency-obfuscation equilibria cannot be directly compared. First, the level of \(F(p_0)\) need not be the same and, second, the opportunity cost of obfuscation determines the equilibrium type.

All in all, obfuscation allows the firms to charge higher prices more often and raise the high end price. The effect on profits is positive even after accounting for the additional costs of obfuscation. Consumers are worse off. Still, when the equilibrium features transparency, the sellers and the buyers may both gain as there is less wasteful obfuscation.

Obfuscation not only advantages the firms who are engaged in it. The profits are increased for all firms. The firms who are not obfuscating benefit from the industry practice as well. Since the expected level of search costs and the expected price level are higher, the buyers are more reluctant to continue the search. The firms can, therefore, set higher prices and, if they decide to obfuscate, they can obfuscate less and incur lower obfuscation costs. Interestingly, the transparency-obfuscation equilibria are based on symmetric strategies and do not presuppose any coordination from either firm or consumer side.

5 Example

[3, Proposition 8] present a numerical example of a transparency-obfuscation equilibrium based on two feasible obfuscation levels resulting in a price distribution with several gaps. To demonstrate that neither discrete jumps nor gaps are necessary, we construct an analytical example of a transparency-obfuscation equilibrium with a continuous price distribution on a connected support founded on continuous primitives. This is not trivial. It is not easy to come up with an equilibrium generating an inverse-U-shaped opportunity cost of obfuscation with two roots. Many other function forms and parameter values would have created an equilibrium with neither continued search nor obfuscation, or with only obfuscation.
Modeling choices  We set the stage by describing a favorable environment. For the sake of a story, consider a bachelor sleeping on a mattress in a garret – a busy man thriving on the ascetics of his whereabouts. For all the obvious reasons and for no reason, suppose he decides to buy himself a futon one day. He has a unit demand for a futon of any kind up until a price $p$ normalized to unity, $D(p) = 1$ for $p \leq 1$. He values his leisure time to such an extent that his search costs are an exponential function of the search time $t$, $g(t) = \sigma (e^t - 1)$ for $t \geq 0$. Moreover, let us presume there is not just a singleton but a measure $1 - \mu$ of such bachelors sleeping on mattresses in garrets (searchers, hereafter), and a measure $\mu$ of bachelorettes (shoppers) also looking for a futon but enjoying shopping and incurring zero search costs, $g(t) = 0$ for $t \geq 0$. In addition, assume there are two furniture retailers who sell virtually identical futons, say, a multinational and its local competitor. They choose both the price of the futon $p$ and the time it takes for the consumers to find the futon $t$. The retailers face obfuscation cost, $c(t)$ for $t \geq 0$, since designing and putting into operation effective ways of delaying the customers binds resources. Getting to an outlet takes a fixed time $\tau$. By Lemma 1, the shoppers purchase for the lowest price and the searchers continue to the other outlet if and only if

$$\int_{p_0}^p F(p)dp > \sigma e^{\tau + t(p)}(e^{\tau + E_t(t)} - 1).$$

To make the construction of the equilibrium as easy as possible, we suppose that the obfuscation cost has a piece-wise representation as

$$c(t) = \begin{cases} \frac{3}{5} \left( t^{-1} \circ t - p_0 \right), & \text{for } p \leq p_0 + \frac{5}{32}, \\ \frac{3}{32}, & \text{for } p > p_0 + \frac{5}{32}. \end{cases}$$

The obfuscation cost increases first linearly and plateaus then to $\frac{3}{32}$ at $p_0 + \frac{5}{32}$. By Corollary 1, the equilibrium obfuscation is determined by

$$t(p) = \ln \left( \frac{V(p)}{\sigma e^{\tau}(e^{\tau + E_t(t)} - 1)} \right),$$

whenever $t(p) > 0.$
Observe that the obfuscation time \( t \) is a strictly increasing function of the price since the value of searching \( V \) is strictly increasing and the expected cost of searching \( \sigma e^{\tau E_t(t)}(e^{\tau} + E_t(t)) - 1 \) is constant. Therefore, the inverse \( t^{-1} \) exists. Since \( t \) is strictly increasing and continuous, \( t^{-1} \) is also strictly increasing and continuous. Moreover, the scale parameter \( \sigma \) can be chosen conveniently. It fine-tunes the search cost \( g \) so that the price at which obfuscation becomes necessary is exactly \( p_0 = \frac{5}{24} \approx 0.208 \) independent of \( \tau \) and \( E_t(t) \). The obfuscation cost \( c \) is defined to be continuous and increasing as a function of both time and price, first strictly increasing and then constant. This function form essentially linearizes the opportunity cost of obfuscation. The linear part of the cost \( c \) and the linear revenue \( R \) help to render obfuscation not profitable for intermediate prices. The constant part of the cost \( c \) in conjunction with the linear revenue \( R \) makes obfuscation worthwhile for higher prices. All these decisions are also essential for tractability. Otherwise, the equilibrium is characterized by a system of non-linear equations in seven unknowns, and numerical solution methods are usually needed. Observe, however, that the piece-wise linear obfuscation cost function \( c \) can be approximated arbitrarily well by smooth, strictly increasing functions.\(^2\)

We fix the following parameter values: \( \tau = \frac{1}{2} \) (if the time unit is hours, it takes half an hour to drive to a furniture outlet) and \( \mu = \frac{1}{2} \) (there are equally many bachelors and bachelorettes). A range of other values would do equally well. The construction of an equilibrium is documented in Appendix A.

\(^2\)The modeling choices underline the specific nature of transparency-obfuscation equilibria. It takes effort to find explicit function forms that produce a inverse-U-shaped opportunity cost of obfuscation \( h \) with two roots \( p_1 \) and \( p_2 \) and a continuous price distribution \( F \) on a connected support supp\( F(p) \) if it is required that (a) obfuscation cost \( c \) is continuous and increasing, (b) search cost \( g \) is continuous, strictly increasing and strictly convex, and (a) revenue \( R \) is continuous, strictly increasing and concave. The main obstacles are getting the opportunity cost of obfuscation \( h \) above zero fast enough and getting it below again before the highest price. The function forms and the parameter values have been selected keeping that in mind. It can be shown that a combination of two linear functions, \( c \circ t \) and \( R \), allows the fastest possible incline after \( p_0 \), and that of a linear \( R \) and a constant \( c \circ t \) permits the fastest possible decline before \( p_3 \).

Two other choices are worth noting. The use of the appropriate scale parameter \( \sigma \) and the application of the inverse \( t^{-1} \) make it feasible to solve the firm problem essentially separately from the consumer problem. Since \( c(t) \) is a function of \( t^{-1}(p) \), the firm problem becomes linear in prices and there is no need to solve the value of searching \( V \) simultaneously with the price distribution \( F \). That simplifies a lot. In general, obtaining \( V \) for the different price intervals \([p_0, p_1] \), \([p_1, p_2] \), and \([p_2, p_3] \) may require integration and solution of differential equations. In stead of asking indirectly what is the cost of obfuscating the time \( t \) needed to keep the customers at price \( p \), it is possible to ask directly what is the cost of keeping the customers at price \( p \). After all, it is impossible to say which one is the ultimate primitive of the obfuscation cost, time or price, since the two are in a 1-to-1 relationship.
**Constructed Equilibrium** The constructed equilibrium is displayed in Fig. 2 in Appendix B. It is evident that there are neither gaps nor jumps in the price distribution. About 20% of (the times) the firms set so low prices that they can retain the first-time searchers without obfuscation. In comparison, only 3% of the firms choose the somewhat higher prices which need to be supported by slight obfuscation. 40% of the firms quote intermediate prices and let the searchers continue, and 37% of the firms charge high prices and obfuscate extensively to prevent the searchers from continuing. Since there is only one competitor and the shoppers and the second-time searchers represent more than a half of the consumers, low prices are relatively profitable and, hence, chosen by a fifth of the firms. Instead, the prices slightly above are not very attractive as the costs of obfuscation rise abruptly. In fact, most firms do not obfuscate. It is costly and the chances of selling to the shoppers and the second-time searchers are high enough for low and intermediate prices. However, over a third of the firms set high prices and obfuscate considerably. The revenue is still ascending but the obfuscation costs have already plateaued for high prices.

Fig. 2 makes it clear that many neighboring function forms and parameter values would have generated transparency-obfuscation equilibria as well. Still, as the price distribution \( F \) grows strongly for the low prices \([p_0, p_0]\), small changes in \( \sigma \) and, therefore, in \( p_0 \) can generate large changes in the opportunity cost of obfuscation \( h \) for the lower intermediate prices \([p_0, p']\). If \( \sigma \) and \( p_0 \) are higher, \( h \) starts lower and is flatter to begin with. If it fails to have two roots, the equilibrium fails to feature both transparency and obfuscation. The exact function forms for the obfuscation cost \( c \) and the revenue \( R \) matter. If \( c \) rises faster with respect to \( R \) for the prices \([p_0, p_1]\), \( h \) attains zero earlier. But, if \( c \) rises too fast in relation to \( R \) for the lower intermediate prices \([p_0, p_1]\), \( F \) has a gap. Moreover, the opportunity cost of obfuscation \( h \) must have a second root \( p_2 \) above the first root \( p_1 \). If \( c \) increases more relative to \( R \) for the higher intermediate and high prices \([p_2, \overline{p}]\), the highest price \( \overline{p} \) is lower and there is less room for the two roots.\(^3\)

\(^3\)For differentiable cases, the highest price \( \overline{p} \) has to satisfy the first-order condition

\[-\mu F'(\overline{p})R(\overline{p}) + \left( \frac{1-\mu}{1} + (1 - F(\overline{p})) \mu \right) R'(\overline{p}) = c'(t(\overline{p}))t'(\overline{p}).\]

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6 Conclusion

We characterize an equilibrium in which some consumers search twice and some firms deliberately raise the search time despite the costs. It turns out that the opportunity cost of obfuscation must have an even number of roots. We construct an explicit analytical example with a continuous price distribution on a connected support. The solution technique might be useful in applied work.

In the model we use, some firms decide to inflate the consumer search costs for high prices but sustain them as low as possible for intermediate prices. That provides rational foundations for two frequently observed phenomena: some buyers search more than once, though they do not enjoy it, and some sellers fail to make the search for prices and products fast, though it appears to be very easy for them to avoid it.

It seems plausible to us, however, that obfuscation might sometimes be only a by-product of other costly marketing practices such as customer communication, product presentation or product placement. In that case, it might be challenging to separate obfuscation, information provision and merchandizing. For instance, the prospective buyers of expensive goods tend to receive more patient and personalized service. Some buyers may prefer that. Crowdedness can also make shopping time-consuming. Some sellers may have capacity constraints, either in-store on on-line, and that can create congestion.

We have built our model on [3], who define obfuscation as the time it takes for the consumers to discover the full price structure once they have entered the store or opened the web-page. There is endogenous information concerning the prices. However, [2] and [3] suggest that obfuscation could also be understood more generally. In certain contexts, the sellers appear to make it difficult for the buyers to obtain not only price information but also other relevant information affecting the willingness-to-pay. Taking that into account in a resembling setup seems like a nice avenue for future work.

The findings can be also juxtaposed with earlier results stating that the sellers will choose to reveal the buyers all information ([5], [6], [8], and [9]). Certainly, the settings are quite different. The obfuscation papers focus on search frictions whereas the adverse selection and the auction papers concentrate on information frictions. If prices are
observable but quality is not, sellers can signal high quality by costly warranties or information disclosure under the threat of harsh out-of-equilibrium-path beliefs ([6] and [8]). Alternatively, sellers can choose to release information that shifts and sharpens buyer valuation. Full revelation is found out to be optimal. Auction houses have an incentive to provide the bidders as much information as possible about the value of the auctioned good (the linkage principle, [9]). Especially, if the auctioneer controls information disclosure, he extracts its profits and has, therefore, an incentive to reveal the bidders all information [5]. The obfuscation papers abstract from that. In contrast, they postulate that goods are homogeneous and buyers have similar demands. This enables them to analyze oligopolistic competition in a market with search frictions – and reach interestingly different conclusions.

References

Appendix A: Construction of Equilibrium

We describe the construction of the equilibrium in detail. The above modeling choices and Corollary 1 jointly imply that the price distribution $F$ is given as

$$F(p) = \begin{cases} 
1 + \frac{1}{2}(1 + F(p_2) - F(p_1)) - 2\frac{\pi}{p}, & \text{for } p \in [p_2, \frac{5}{24}], \\
1 + \frac{1}{2}(1 + F(p_2) - F(p_1)) - \frac{6}{5} - 2\frac{\pi - \frac{1}{5}}{p}, & \text{for } p \in (\frac{5}{24}, p_1), \\
\frac{1}{2} + \frac{1}{3}(1 + F(p_2)) - \frac{\pi}{p}, & \text{for } p \in [p_1, p_2), \\
\frac{3}{2} - 2\frac{\pi + \frac{3}{32}}{p}, & \text{for } p \in [p_2, \bar{p}],
\end{cases}$$

and the opportunity cost of obfuscation $h$ as

$$h(p) = \begin{cases} 
\left( \frac{13}{20} - \frac{1}{5} \right) (1 + F(p_2) - F(p_1)) p + \frac{1}{4} \pi - \frac{3}{10}, & \text{for } p \in (\frac{5}{24}, p_1), \\
\left( \frac{19}{10} - \frac{1}{16} \right) (1 + F(p_2)) p + \frac{1}{4} \pi - \frac{1}{5}, & \text{for } p \in [p_1, \frac{35}{96}], \\
- \left( \frac{1}{8} + \frac{1}{16} \right) (1 + F(p_2)) p + \frac{1}{4} \pi + \frac{3}{32}, & \text{for } p \in (\frac{35}{96}, p_2), \\
- \frac{3}{8} p + \frac{1}{2} \pi + \frac{9}{64}, & \text{for } p \in [p_2, \bar{p}].
\end{cases}$$

$F$ is increasing as long as $\pi - \frac{1}{5} > 0$, and $h$ is clearly linear, increasing below $\frac{35}{96}$ and decreasing above that.

There are seven endogenous variables to be determined: the prices $p_2$, $p_0$, $p_1$, $p_2$, and $\bar{p}$, the profit $\pi$ and the expected level of obfuscation $E_t(t)$. First, we solve for the highest price $\bar{p}$ and the profit $\pi$ (Part 1). Then, we use the properties of $h$ to set the values of the other bound prices $p_2$, $p_1$, and $p_2$ (Part 2). Finally, we derive the value of continuing the search $V(p)$ for $p \in \{p_0\} \cup \{p_1, p_2\}$ and show how to obtain the expected level of obfuscation $E_t(t)$ (Part 3).

**Part 1: Support and Profit** Since the obfuscation cost is constant for high prices, it pays off to increase the highest price until it reaches the monopoly price one. As a result, the high price and the high end profit are given by

$$\bar{p} = 1 \text{ and } \pi = \frac{1}{4} p - c(\bar{p}) = \frac{1}{4} - \frac{3}{32} = \frac{5}{32}.$$ 

The lowest price $\underline{p}$ yields the same profit $\pi$ (given by (1), (2) and (3)) as the highest
price \( \bar{p} \) (and other prices)

\[
\left( \frac{3}{4} + \frac{1}{4} F(p_2) - \frac{1}{4} F(p_1) \right) p = \frac{5}{32},
\]

from which we notice that the larger the measure of second searchers \( F(p_2) - F(p_1) \) the smaller the lowest price \( p \), which varies within \( \left( \frac{5}{32}, \frac{5}{24} \right) \). The support of the price distribution is between \( \left[ \frac{5}{32}, 1 \right] \) and \( \left[ \frac{5}{24}, 1 \right] \) in the sense of weak set order.

**Part 2: Prices and Second-Time Searchers**

As the slope of \( h \) is \( -\frac{3}{8} \) for prices \( p \in [p_2, \bar{p}] \) and as \( h \) declines from zero to \( -\frac{5}{32} \) between \( p_2 \) and \( \bar{p} \), the price \( p_2 \) at which the firms start obfuscating again is given by

\[
-\frac{5}{32} = -\frac{3}{8},
\]

and, hence,

\[
p_2 = \frac{7}{12} \approx 0.583 \text{ with } F(p_2) = \frac{3}{2} - 2\frac{\pi + \frac{3}{32}}{p_2} = \frac{9}{14} \approx 0.643.
\]

The price \( p_1 \) at which the firms stop obfuscating is a root of \( h \)

\[
\left( \frac{19}{40} - \frac{1}{16}(1 + F(p^2)) \right) p_1 + \frac{1}{4} \pi - \frac{3}{24} = 0,
\]

and, thus,

\[
p_1 = \frac{385}{1668} \approx 0.231 \text{ with } F(p_1) = \frac{1}{2} + (1 + F(p_2)) \frac{1}{4} - \frac{\pi}{p_1} = \frac{18}{77} \approx 0.234.
\]

This pins down the probability of a new search \( F(p_2) - F(p_1) = \frac{9}{72} \) and the lowest price

\[
\bar{p} = \frac{4\pi}{(3 + F(p^2) - F(p^1))} = \frac{11}{60} \approx 0.183.
\]

It also confirms that the the maximizer of the opportunity cost of obfuscation \( p_0 + \frac{5}{32} = \frac{35}{96} \approx 0.365 \) lies indeed within \((p_1, p_2)\).

The resulting price distribution and the opportunity cost of obfuscation can now be expressed explicitly as

\[
F(p) = \begin{cases} 
\frac{75}{44} - \frac{5}{16} \bar{p}, & \text{for } p \in \left[ \frac{11}{60}, \frac{5}{24} \right), \\
\frac{111}{220} - \frac{1}{16} \bar{p}, & \text{for } p \in \left[ \frac{5}{24}, \frac{385}{1668} \right), \\
\frac{51}{56} - \frac{5}{32} \bar{p}, & \text{for } p \in \left[ \frac{385}{1668}, \frac{7}{12} \right), \\
\frac{3}{2} - \frac{1}{2} \bar{p}, & \text{for } p \in \left[ \frac{7}{12}, 1 \right],
\end{cases}
\]

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and

\[
\begin{align*}
    h(p) &= \begin{cases} 
        \frac{417}{880} p - \frac{7}{16}, & \text{for } p \in (\frac{5}{24}, \frac{385}{1668}), \\
        \frac{417}{128} p - \frac{11}{25}, & \text{for } p \in [\frac{385}{1668}, \frac{35}{96}), \\
        -\frac{51}{224} p + \frac{17}{128}, & \text{for } p \in [\frac{35}{96}, \frac{7}{12}), \\
        \frac{3}{8} p + \frac{7}{32}, & \text{for } p \in [\frac{7}{12}, 1].
    \end{cases}
\end{align*}
\]

Part 3: Value of Search and Expected Level of Obfuscation

Now that seller behavior is clear, it remains to check that it is in accordance with buyer behavior. The values of continuing the search are based on the integration of type

\[
V(p) = \int_{p_d}^{p_u} F(p) dp = \int_{p_d}^{p_u} (a + b \frac{1}{p}) dp = a(p_u - p_d) + b \ln \left( \frac{p_u}{p_d} \right).
\]

Therefore, the function \( V \) is given by

\[
V(p) = \begin{cases} 
    \frac{75}{12} (p - \frac{11}{60}) - \frac{5}{16} \ln \left( \frac{p}{60} \right), & \text{for } p \in [\frac{11}{60}, \frac{5}{24}), \\
    V(\frac{5}{24}) + \frac{111}{220} (p - \frac{5}{24}) - \frac{1}{16} \ln \left( \frac{p}{24} \right), & \text{for } p \in [\frac{5}{24}, \frac{385}{1668}), \\
    V(\frac{385}{1668}) + V(\frac{5}{24}) + \frac{51}{224} (p - \frac{385}{1668}) - \frac{5}{32} \ln \left( \frac{p}{1668} \right), & \text{for } p \in [\frac{385}{1668}, \frac{7}{12}), \\
    V(\frac{7}{12}) + V(\frac{385}{1668}) + V(\frac{5}{24}) + \frac{3}{8} (p - \frac{7}{12}) - \frac{1}{2} \ln \left( \frac{p}{12} \right), & \text{for } p \in [\frac{7}{12}, 1].
\end{cases}
\]

where

\[
V(\frac{5}{24}) \approx 0.003, \quad V(\frac{385}{1668}) \approx 0.008, \quad V(\frac{7}{12}) \approx 0.184, \quad \text{and } V(1) \approx 0.539.
\]

The expected level of obfuscation could now, if needed, be obtained based on the following integration

\[
E_t(t) = \int_{\frac{385}{1668}}^{\frac{7}{12}} t(p) f(p) dp = \int_{\frac{385}{1668}}^{\frac{7}{12}} \ln \left( \frac{V(p)}{\sigma e^{\frac{1}{2} \left( e^{\frac{1}{2} t} - 1 \right)} \left( V\left( \frac{5}{24} \right) \right)} \right) \frac{5}{32} \frac{1}{p^2} dp = \int_{\frac{385}{1668}}^{\frac{7}{12}} \ln \left( \frac{V(p)}{V\left( \frac{5}{24} \right)} \right) \frac{5}{32} \frac{1}{p^2} dp,
\]

where the scale parameter \( \sigma \) is such that the consumers search anew unless the firms obfuscate for prices above \( \frac{5}{24} \).

\[
\sigma = \frac{V\left( \frac{5}{24} \right)}{e^{\frac{1}{2} \left( e^{\frac{1}{2} t} - 1 \right)}} \in (0.0000, 0.0025) \quad \text{for all } E_t(t).
\]
The upper bound of $\sigma$ is given by $E_t(t) = 0$.

For example, if it took about an hour to identify and locate the cheapest futon in an average furniture retailer, the scale parameter would be $\sigma \approx 0.0005$. The first hour of search would cost $\sigma e^1 \approx 0.017$, and a work day of search would consume more than the full value of the futon $\sigma e^8 \approx 1.384$.

Observe that the searchers always find it profitable to enter the market. The expected level of obfuscation $E_t(t)$ can be approximated from above by the probability of obfuscation $\left( F\left( \frac{385}{1068} \right) - F\left( \frac{5}{21} \right) \right) + \left( 1 - F\left( \frac{7}{12} \right) \right)$ multiplied by the maximum obfuscation $t(1) = \ln \left( \frac{V(1)}{V(\frac{5}{21})} \right)$. It is clear that $V(1) > \sigma \left( e^{\frac{1}{m}} + E_t(t) \right)$, which is enough for entry.
Appendix B: Figures

Fig. 1. Florid (above) and simple (below) patterns of the opportunity cost of obfuscation in transparency-obfuscation equilibria. The price intervals with continued search are highlighted in gray.
Fig 2. The example of a transparency-obfuscation equilibrium. The price intervals with continued search are highlighted in gray.