PHOTOMETRIC ANALYSIS FOR THE SLOW ROTATING ASTEROID (103) HERA USING CONVEX INVERSION AND LOMMEL-SEELIGER ELLIPSOID METHODS

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(Received; Revised; Accepted)
Submitted to PSS

ABSTRACT

The shapes and rotational states of main-belt asteroids are important for understanding their formation and evolution. Available photometric data of asteroids are biased due to selection effects, including the relative paucity of analyses of slowly rotating objects. In order to get photometric data of slowly rotating asteroids, an international joint observation project has been carried out since 2015 using Chinese and SARA (Southeastern Association for Research in Astronomy) telescopes. In this paper, the photometric data of one of this project targets – (103) Hera were analyzed using the convex inversion method and Lommel-Seeliger ellipsoid model. Combining existing and new photometric data, we re-calculated the shape and spin parameters for (103) Hera. Using a convex shape method, a pair of poles are derived for (103) Hera – (83.0°, 39.0°) and (269.7°, 56.8°) in ecliptic frame. The spin periods corresponding to these poles are very close – 23.74264 h and 23.74267 h respectively. Meanwhile, the same data were analyzed using the Lommel-Seeliger ellipsoid inversion method and a pair of pole solutions – (74.1°, 39.0°) and (263.1°, 51.0°) with a spin period of 23.74262 h and 23.74263 h respectively are derive. Based on the derived shape of (103) Hera, we have fitted the $H, G_1, G_2$ phase function using the calibrated data after removing effects of aspheric shape. As a result, we estimated its absolute magnitude $H = 8.92$ mag with two phase function parameters $G_1 = 0.13$ and $G_2 = 0.45$.

Keywords: convex, ellipsoid — shape — asteroid

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* Supported by the National Natural Science Foundation of China.
1. INTRODUCTION

Asteroids are thought to be remnants of planetesimals in the solar system. Their basic physical properties, such as spin status and shape, can provide important constraints on the formation and evolution of these small bodies themselves as well as the entire solar system (Wang et al. 2015). Apart from a few asteroids that were directly imaged by spacecraft, most of them are observed by ground-based techniques. Especially, the photometric observations of asteroids, which can be traced back to one hundred years ago, became the main source to extract asteroids’ basic physical properties such as shapes and rotation parameters. At present, time-resolved photometry is still widely used to study asteroid physical properties, in particular to determine shapes and rotation parameters. From the statistical analysis for the asteroid’s spin rate, the mean spin rate for the largest asteroids (mean $D \sim 200$ km) locates around $3.0 \text{ d}^{-1}$ and decreases to $1.8 \text{ d}^{-1}$ as $D$ decreases to $\sim 100$ km (Pravec et al. 2002). While it increases to about $4.0 \text{ d}^{-1}$ when $D$ decreases down to $\sim 10$ km. For asteroids with diameter less than 10 km, the spin rate is increasing with decreasing diameter (Pravec et al. 2002). So, for these investigated asteroids, most part or even the whole of their phased lightcurves can be obtained in one night’s observation.

By using a variety of lightcurve inversion methods developed in recent years (see below) astronomers can derive asteroids’ shapes and spin parameters from photometric data. As a result, more and more asteroids’ shapes and spin parameters have been determined this way. A larger number of samples allow us to study the physical properties of asteroid populations, such as main-belt asteroids and asteroid families (Hamáček et al. 2013). However, the available sample of well studied asteroids is burdened with substantial selection effects (Marciniak et al. 2015). Besides well known strong selection effects caused by small, low-albedo and distant asteroids, there is also another selection effect caused by slowly rotating asteroids according to Marciniak et al. (2015).

For slowly rotating asteroids, it is difficult to obtain a complete phased lightcurve in one night’s observation with only one telescope. Especially for asteroids with spin periods close to a half day, one day or two days, the same part of phased lightcurve is given during the observation with only one telescope. That’s why we lack the slowly rotating samples in existing photometric database. In order to expand the sample of long-period asteroids, we carried out international coordinated observations for some long-period asteroids several years ago using telescopes at Yunnan Observatories of China and telescopes of SARA (Southeastern Association for Research in Astronomy – a consortium of colleges and universities in the United States partnered with Lowell Observatory, the Chilean National Telescope Allocation Committee, and the Instituto de Astrofísica de Canarias) (Keel et al. 2017).

(103) Hera as one target of this long-period asteroids’ observation project, were observed in 2015 and 2017 using telescopes at Yunnan Observatory in China, Kitt Peak National Observatory, USA, and Holcomb Observatory on Butler University Campus, USA. Asteroid (103) Hera is an S-type (Tholen & Barucci 1989) main-belt asteroid with a rotation period around 23.74 hours (Harris & Young 1983; Pilcher & Higgins 2011). With the help of newly obtained and published lightcurves, we will analyze the spin parameters and shape for asteroid (103) Hera using two lightcurve inversion methods: the Lommel-Seeliger ellipsoid inversion method (Muinonen et al. 2015; Muinonen & Lumme 2015) and the convex inversion method (Kaasalainen & Torppa 2001; Kaasalainen et al. 2001). By comparing the solutions of spin parameters derived from two different shape models, we try to understand the difference between these two sets of spin parameters and whether a simple shape model (like Lommel-Seeliger ellipsoid) can give a reliable pole solution in the case of asteroids that are difficult to obtain sufficient observational data such as long-period or faint asteroids.

Based on the newly obtained shapes and spin parameters, the phase function (Muinonen et al. 2010; Penttilä et al. 2016) of (103) Hera is fitted using calibrated data (Ďurech et al. 2016) after removing the effect of aspheric shape of Hera.

Therefore, the contents of this paper include the information on observations and data reduction for (103) Hera in Section 2, the solutions of spin parameters and shapes using the convex inversion and Lommel-Seeliger ellipsoid inversion methods in Section 3 and the analysis for (103) Hera’s phase function in Section 4. Finally, there is a summary in Section 5

2. OBSERVATION AND DATA REDUCTION

To obtain sufficient photometric data for (103) Hera, we carried out a joint photometric observation campaign in 2015 and 2017 using three telescopes – a 1.0 m telescope at Yunnan observatories in China, a 0.91 m telescope at
SARA-KP observatories (Keel et al. 2017) and a 0.94 m telescope at Holcomb Observatory 1 on Butler University campus in the United States.

The photometric data were obtained through clear filter or Johnson-Cousins $R$ filter depending on the weather conditions. During the observations on 16 March 2017 using SARA-KP 0.91 m telescope, we changed its FOV once in order to follow the asteroid. As a result, we reduced the two segments of photometric data separately. We analyzed the photometric images obtained at Holcomb 0.94 m telescope on 21 March 2017 in the same manner.

We reduced all newly obtained images using the Image Reduction and Analysis Facility 2 (IRAF) software according to the standard procedure with bias subtraction and flat correction. We identified the occasional cosmic rays on the images by a criterion of five times the standard deviation of sky background and then removed them. We measured the instrumental magnitudes of reference stars and asteroid using aperture photometry with the help of APPHOT package in IRAF. Based on the Full width at half maximum (FWHM) of stars’ point spread function (PSF), we usually tried 3 to 5 apertures which ranging 2 to 2.3 times FWHM to find the optimal aperture. Then, some systematic effects in the photometric data were simulated with reference stars in the field of asteroid and removed from the target’s photometric data (Wang et al. 2013; Tamuz et al. 2005; Collier Cameron et al. 2006). To make the photometric data available for model inversion, we needed to calculate the light-time corrected JD epoch and the ecliptic asteroid-centric cartesian coordinates of Sun and observers in AU (Ďurech et al. 2009, 2010). With the help of Ephemeris Service provided by the Minor Planet Center 3, we can conveniently get the distance from observers to asteroid and the equatorial asteroid-centric cartesian coordinates of Sun and observers in AU. We also downloaded the published data of (103) Hera from Database of Asteroid Models from Inversion Techniques 4 (DAMIT) website. Table 1 shows information on the photometric observations of asteroid (103) Hera.

**Table 1. Information on the Photometric Observations of Asteroid (103) Hera**

<table>
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<th>$\Delta$ r (AU)</th>
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1 [https://www.butler.edu/holcomb-observatory](https://www.butler.edu/holcomb-observatory)
2 IRAF is distributed by the National Optical Astronomy Observatories, which are operated by the Association of Universities for Research in Astronomy, Inc., under cooperative agreement with the National Science Foundation.
3 [https://minorplanetcenter.net/iau/MPEph/MPEph.html](https://minorplanetcenter.net/iau/MPEph/MPEph.html)
### Table 1 (continued)

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Note—$\Delta$ and $r$ are the geocentric and heliocentric distances of asteroid. $\alpha$ means the phase angles of the object and V-mag is the predicted visual magnitude.

### 3. DETERMINATION OF SPIN PARAMETERS AND SHAPES

The lightcurve inversion approach provides us opportunities to understand physical properties (shapes, spin states and scattering parameters of surface) of asteroids. To derive the shapes and spin parameters of (103) Hera from the photometric data, we used the Lommel-Seeliger ellipsoid inversion method (Muinonen et al. 2015; Muinonen & Lumme 2015) and the convex inversion method (Kaasalainen & Torppa 2001; Kaasalainen et al. 2001, 2002). By comparing the inversion results (spin parameters and shapes) of these two methods, we can then investigate the pole solution difference between the simple ellipsoid model and the more complex convex shape model.

Hanuš et al. (2016) recently made an analysis for (103) Hera with the convex inversion method based on dense photometric data at two apparitions and sparse data (Ďurech et al. 2016). A pair of poles ((85°, 24°) and (270°, 40°)) and corresponding convex shapes were given in their analysis. To refine the shape and spin parameters of Hera, we carried out photometric observations in 2015 and 2017. In the lightcurve inversion of Hera, we combined all existed data (the dense photometric data of Harris & Young (1983); Pilcher & Higgins (2011) and sparse data (Ďurech et al. 2016)) and our new data. In all, 40 dense lightcurves in four apparitions and sparse data are involved in our analysis. During the analysis, we found the large RMS of fitting (around 0.03, brought mainly by the sparse data) prevents us...
In recent years, we have applied Lommel-Seeliger ellipsoid model on a C-type main-belt asteroid (585) Bilkis – an asteroid with relatively regular shape (light curve near sinusoidal) and obtained a set of model parameters that produces good fit to the photometric data (Wang et al. 2016). Compared with (585) Bilkis, (103) Hera’s lightcurves show a more irregular shape. Nevertheless, we did the Lommel-Seeliger ellipsoid inversion based on 40 dense lightcurves of Hera with a similar procedure of Wang et al. (2016); Muinonen et al. (2015).

Figure 1. Lightcurves of (103) Hera. Symbol dots are observed data, solid lines are modeled ones using convex model and dash lines are modeled ones using Lommel-Seeliger ellipsoid model. α is the phase angle, θ_1 is the angle between the pole orientation and the direction of the Sun in the ecliptic asteroid-centric cartesian coordinates and θ_2 is the angle between the pole orientation and the direction of the observer in the ecliptic asteroid-centric cartesian coordinates. We plot parts of the published lightcurves (Top) and all of the newly obtained lightcurves (Bottom) considering rotational phase coverage.

to find the significant best solution of lightcurve inversion. So, during the procedure of shape inversion of Hera, these sparse data are excluded. Later, these calibrated sparse data are used to fit the phase function of Hera when we have its refined shape and spin parameters.

3.1. Lommel-Seeliger ellipsoid inversion

For an asteroid with a long spin period or with a spin period close to one or two days, only part of rotational phase lightcurve can be obtained with one telescope at one apparition. In this case (e.g., (103) Hera – an asteroid with spin period of 23.74 h) the lightcurve inversion of asteroid becomes ambiguous. To overcome this difficulty, we carried out a joint observation campaign for (103) Hera using telescopes at Yunnan Observatory, Kitt Peak National Observatory and Holcomb Observatory in 2015 and 2017. As a result, we obtained new lightcurves that cover most part of its rotational phase.

The Lommel-Seeliger ellipsoid inversion method (Muinonen et al. 2015) can be used to investigate the pole and coarse shape of asteroid when photometric data are sparse, e.g. gaia observation, or dense data at one or two apparitions are available because much less free parameters are involved.

In detail, the unknown vector (represented by $\mathbf{P} = (\text{per}_p, \lambda_p, \beta_p, \phi_0, a, b, c, p, G_1, G_2, D)^T$) in the Lommel-Seeliger ellipsoid model comprises: the rotation period $\text{per}_p$, ecliptic longitude of pole $\lambda_p$, ecliptic latitude of pole $\beta_p$, rotational phase $\phi_0$ at $t_0$, size of three semi major axis of ellipsoid $a, b, c (a \geq b \geq c)$, geometric albedo $p$, parameters of phase function $G_1, G_2$, and equivalent diameter of asteroid $D$. In principle, we can infer these unknown parameters by comparing the observed intensities with the modeled ones. However, in the case of relative intensities, only spin parameters (rotation period and orientation of pole) and ellipsoid shape (taken $a = 1$) are analyzed (geometric albedo, parameters of phase function and equivalent diameter of asteroid are fixed).

In recent years, we have applied Lommel-Seeliger ellipsoid model on a C-type main-belt asteroid (585) Bilkis – an asteroid with relatively regular shape (light curve near sinusoidal) and obtained a set of model parameters that produces good fit to the photometric data (Wang et al. 2016). Compared with (585) Bilkis, (103) Hera’s lightcurves show a more irregular shape. Nevertheless, we did the Lommel-Seeliger ellipsoid inversion based on 40 dense lightcurves of Hera with a similar procedure of Wang et al. (2016); Muinonen et al. (2015).
Figure 2. Period distributions of (103) Hera vs. RMS by the Lommel-Seeliger ellipsoid model(left) and by the convex model (right).

We scanned the period interval from 5.0 hour to 50 hour with a step of $\frac{\text{per}}{2\Delta T}$ (see Figure 2). The time interval $\Delta T$ is the time span of the involved photometric data. For each of scanned period value, hundreds of different pole orientations distributed as uniformly as possible on the unit sphere are tested. The local optimised pole orientation, rotational phase and axial ratios are derived with the Nelder-Mead downhill simplex method at each scanning step. Optionally, the scattering parameters can also be estimated if having the calibrated photometric data. Finally, the most probable period obtained by us was around 23.7426 hours. This value is very close to the results of previous period determinations (Harris & Young 1983; Pilcher & Higgins 2011; Hanuš et al. 2016). Setting this period as initial, the most possible pole ($\lambda_p, \beta_p$) is searched by scanning over the whole spherical surface with a high resolution of several degrees step in longitude and latitude directions. The detail information on the brightness model of the Lommel-Seeliger ellipsoid and the procedure of shape inversion can refer the papers (Muinonen et al. 2015; Muinonen & Lumme 2015).

It is worth noting that there are usually two pole solutions for one asteroid during the lightcurve inversion due to a limited geometry (an asteroid orbiting too close to the ecliptic, see Kaasalainen & Lamberg (2006) for details). Here, we got two sets of model parameters – (period = 23.74262 h, ecliptic longitude = 74.1°, ecliptic latitude = 39.0°, axial ratio b/a = 0.79, c/a = 0.40) and (period = 23.74263 h, ecliptic longitude = 263.1°, ecliptic latitude = 51.0°, axial ratio b/a = 0.79, c/a = 0.40) with comparable RMS. In order to understand the uncertainties of the estimated parameters, an MCMC simulation (Muinonen et al. 2012, 2015) is carried out for photometric data of (103) Hera. In detail, proposal distributions of the parameters are derived by a random-walk Metropolis-Hastings algorithm based on the important samples of parameters. For these two sets of model parameters, the 1-σ uncertainties for $\text{per, } \lambda_p, \beta_p, b/a, c/a$ are estimated as $4.5 \times 10^{-6}$ h, 0.7°, 1.2°, $1.6 \times 10^{-3}$ and $5.5 \times 10^{-3}$ respectively. The modeled lightcurves are plotted in Figure 1 as dash lines.

3.2. Convex inversion

The convex inversion method developed by Kaasalainen & Torppa (2001); Kaasalainen et al. (2001) is a powerful tool to extract information on spin parameters, shape and scattering law of asteroids. It is based on the optimization of unknown parameters of the shape (modeled as a convex hull), the rotational state, and the scattering law. The code we used is downloaded from DAMIT (Durech et al. 2009, 2010) web page.

Since we used only relative lightcurves, the parameters of scattering law were fixed as $a = 0.5, d = 0.1$ and $k = -0.5$ (where $a$ and $d$ are the amplitude and scale length of the opposition effect, and $k$ is the overall slope of the phase curve (Kaasalainen et al. 2001, 2003)). For asteroids, Lambert’s coefficient $c$ has usually only minor effect on the solution. So we fixed it as $c = 0.1$. Therefore, the coefficients of truncated spherical harmonic series (degree $l$ and order $m$ of Laplace series expansion, usually set $l = m = 6$) and spin parameters were actually estimated by the Levenberg-Marquardt algorithm. Similarly, the period scan procedure (see Figure 2) based on the convex shape was done with the same range and sampling step as it has been done during the ellipsoid inversion procedure. In the procedure of each period scanning steps, six initial poles are tested. Then the most significant minimum of RMS was

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5 This is derived from the fact that the maxima and minima of a double-sinusoidal lightcurve for periods $P$ and $P \pm \Delta P$ are at the same epochs after $\Delta T$ time.

6 $\lambda_p$ means ecliptic longitude of pole orientation while $\beta_p$ means ecliptic latitude.

found around 23.7426 h. Using this best period as the input value for the convexinv procedure, we then ran this procedure with different initial poles (usually grid points of several tens degree over unit spherical surface) to search for the most probable pole orientation (Durech et al. 2010). During this procedure, the fine shape represented with spherical-harmonic series up to order 8 was inferred. After that, the 3D convex shape of (103) Hera is constructed using the minkowski procedure (based on the spherical-harmonic series obtained) (Durech et al. 2010). We finally obtained a pair of poles (83.0°, 39.0°) and (269.7°, 56.8°) with periods of 23.74264 h and 23.74267 h respectively. Here, we utilize an approximate version of the MCMC-V method developed for orbital inversion by Muinonen et al. (2012) to estimate the uncertainties of spin parameters (Wang et al. 2015). In detail, numbers of virtual photometric data sets are generated by adding Gaussian noise to the original data. The respective virtual least-squares solutions of convex inversion constitute a certain distribution of the unknown parameters. Convolution of this distribution by itself then provides a symmetric proposal distribution for a random-walk Metropolis-Hastings algorithm (Wang et al. 2015). For these two sets of spin parameters, the derived uncertainty of period, pole longitude and latitude is about $3.6 \times 10^{-6}$ h, $0.5^\circ$ and $2.3^\circ$ respectively. Table 2 lists the new derived value of spin parameters for (103) Hera, and values derived by Hanuš et al. (2016). It is easy to see that the spin periods from our analysis and Hanuš et al. (2016) are very close, ours the pole orientation is slightly different in latitude direction from that of Hanuš et al. (2016).

The modeled lightcurves of the convex shape were also shown in Figure 1 as solid lines. After adding 11 new lightcurves obtained by us on 9 nights in 2015 and 2017, the new derived convex shape of (103) Hera (shown in Figure 3) looks more oblate than that given by Hanuš et al. (2016). Using three-axis ellipsoid (semi-axis $a > b > c$) to fit new derived convex shapes, we get $a/b = 1.12$, $b/c = 1.6$ for shape 1 (top in Figure 3) and $a/b = 1.2$, $b/c = 1.7$ for shape 2 (bottom in Figure 3).

<table>
<thead>
<tr>
<th></th>
<th>$\frac{p}{r_1}$</th>
<th>$\lambda_{p_1}$</th>
<th>$\beta_{p_1}$</th>
<th>$\frac{p}{r_2}$</th>
<th>$\lambda_{p_2}$</th>
<th>$\beta_{p_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lommel-Seeliger</td>
<td>23.74262</td>
<td>74.1</td>
<td>39.0</td>
<td>23.74263</td>
<td>263.1</td>
<td>51.0</td>
</tr>
<tr>
<td>Convex</td>
<td>23.74264</td>
<td>83.0</td>
<td>39.0</td>
<td>23.74267</td>
<td>269.7</td>
<td>56.8</td>
</tr>
<tr>
<td>Hanuš et al. (2016)</td>
<td>23.74265</td>
<td>85</td>
<td>24</td>
<td>23.74266</td>
<td>270</td>
<td>40</td>
</tr>
</tbody>
</table>

Note—The subscripts 1 and 2 represent the pair of pole solutions.

4. PHASE FUNCTION ANALYSIS

The shape of the photometric phase curve (magnitude as a function of the phase angle) can serve as a proxy for the taxonomic type of the asteroid in cases when spectral information is not available (Penttilä et al. 2016; Oszkiewicz 2016).
et al. 2012; Shevchenko et al. 2016). The International Astronomical Union (IAU) adopted the $H$-$G$ in 1989 as the standard photometric system of asteroid (Bowell et al. 1989; Penttilä et al. 2016). Muinonen et al. (2010) proposed a modified $H$-$G_1$-$G_2$ system which was adopted as a new photometric system by IAU in 2012. This new function improved especially the backscattering behavior of phase curve for high- and low-albedo asteroids. Here, the calibrated Lowell sparse photometric data of (103) Hera (Ďurech et al. 2016; Hanuš et al. 2016) were re-analyzed after removing the effect of aspheric shape of asteroid (different size of visual area).

We downloaded all the available calibrated photometric data of (103) Hera from DAMIT web page \(^8\) which contains 516 data points observed from 1998 to 2011. For determination of the $H$, $G_1$, $G_2$ phase function, we used the equivalent sphere magnitudes (Wang et al. 2017) corresponding to each sparse data point. To do this, we computed modeled intensity using the derived shape and spin parameters. We calculated the non-spherical correction in magnitude by the quantity

$$a_i = I_i/I_e$$

(1)

where $I_i$ is the $i$-th modeled intensity assuming the new derived convex shape, pole of (103) Hera and the Lommel-Seeliger scattering law, $I_e$ is the corresponding modeled intensity of a equivalent sphere. For each calibrated photometric data points, we calculated the individual magnitude deviation $\Delta M_i$ of a convex shape from its equivalent sphere by the following equation. Then, they were subtracted from original photometric data.

$$\Delta M_i = -2.5 \log(a_i)$$

(2)

In the next step, we fitted the $H$, $G_1$, $G_2$ phase function of (103) Hera using the sparse data assuming the average error of 0.03 mag. The $H - G_1 - G_2$ phase function in magnitude form is following:

$$V(\alpha) = H - 2.5 \log_{10}[G_1 \Phi_1(\alpha) + G_2 \Phi_2(\alpha) + (1 - G_1 - G_2) \Phi_3(\alpha)]$$

(3)

where $\Phi_1(\alpha)$, $\Phi_2(\alpha)$ and $\Phi_3(\alpha)$ are the basis functions. The basis functions consist of linear parts, constant part and parts defined by cubic splines (see Muinonen et al. (2010); Penttilä et al. (2016) for more detail on the basis functions). With the help of the HG1G2tools \(^9\), the $H$, $G_1$, $G_2$ phase function is fitted using the linear least-squares method. As for the errors of derived parameters, they are estimated using Monte Carlo simulation by computing standard deviation from the simulated samples (see Penttilä et al. (2016) for more detail). We estimated its absolute magnitude $H = 8.92 \pm 0.02$ mag with two phase function parameters $G_1 = 0.13 \pm 0.01$ and $G_2 = 0.45 \pm 0.01$ (one-sigma errors given by the HG1G2tools \(^10\)). The corresponding photometric phase curve is presented as line in Figure 4.

We note that although we had done the non-spherical correction for the sparse data, the improvement was not as significant as expected (RMS of phase function fitting reduced from 0.14 mag to 0.13 mag). This was mainly caused by the large dispersion of the Lowell sparse data and the non-convexity of asteroid (103) Hera. The value of $H$ changes at different apparitions due to the fact that at different apparitions the aspect angle is different (as a consequence the visible, illuminated area of this object changes). In the case of (103) Hera, we used the non-spherical correction to deal with the variations of value of $H$. However, the convex shape we used is an approximation to the real situation of asteroid. For example, it cannot deal with the shadow effect of non-convex shape. Besides our non-spherical correction, there can be other different approaches in the determination of $H$. For example, one can subdivide the data into different groups based on the aspect angle and determine the value of $H$ separately. That means one can also derive a value of $H$ corresponding to some well-defined viewing geometry such as when aspect angle = 90°.

5. SUMMARY

From the spin parameters of (103) Hera derived assuming ellipsoid and convex shape, we found the solutions from the two lightcurve inversion methods were consistent with each other in the case of (103) Hera. Both ellipsoid and convex modeling led to a rotation period about 23.7426 h. As for the pole orientation, the ellipsoid solution (74.1°, 39.0°) with a mirror pole (263.1°, 51.0°) was close to the convex solution (83.0°, 39.0°) with a mirror pole (269.7°, 56.8°) considering the typical uncertainty of 10° for the pole estimated by Hanuš et al. (2011). From the axial ratio (b/a = 0.8, c/a = 0.4) derived using ellipsoid lightcurve inversion, the shape of (103) Hera is an oblate ellipsoid. The convex shape (see Figure 3) derived from convex lightcurve inversion supported this.


\(^9\) http://www.helsinki.fi/project/psr/HG1G2/

\(^10\) https://wiki.helsinki.fi/display/PSR/HG1G2+tools
Based on the convex shapes and corresponding spin parameters, we have fitted the $H, G_1, G_2$ phase function of (103) Hera using the Lowell sparse photometric data. Although our efforts to remove the rotational effects due to non-spherical shape for the Lowell data did not yield the desired results because of the large data dispersion, it provides a scheme for making use of calibrated sparse data together with dense photometric data. Dense photometric data obtained by ground-based telescopes is suitable for determining shapes and spin parameters of asteroids while calibrated sparse data, e.g. GAIA data (Mignard et al. 2007) is very useful for fitting the phase function of asteroids.

In the future, more efforts are needed to counteract the selection effects mentioned previously and international collaboration is an essential part of these efforts.

ACKNOWLEDGEMENTS

This work was funded by the National Natural Science Foundation of China (Grant Nos. 11073051 and 11673063). We would like to thank Dr. Frank H. Levinson for his generous financial support that enabled Butler University joining the SARA telescope consortium and upgrading the Holcomb Observatory. We would like to thank Prof. Alberto Cellino and another anonymous reviewer for their constructive suggestions.
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