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Kudinov, A. A.

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Interpretive Summary

Metafounder approach for single-step genomic evaluations of Red Dairy cattle. By Kudinov et al. Change from the multi-step to the single-step genomic prediction approach in routine evaluations is complicated. In this study, we show the advantage of the metafounders approach in the single-step prediction of milk performance in dairy cattle. In addition, we also test the effect of markers selection on creating a metafounders relationship matrix.

METAFOUNDERS IN RED DAIRY CATTLE EVALUATIONS

Metafounder approach for single-step genomic evaluations of Red Dairy cattle

A.A. Kudinov*,†, E.A. Mäntysaari*, G.P. Aamand‡, P. Uimari†, and I. Strandén*

* Natural Resources Institute Finland (Luke), Jokioinen, Finland, FI-31600
† Department of Agricultural Science, University of Helsinki, Helsinki, Finland, FI-00014
‡ Nordic Cattle Genetic Evaluation, Aarhus, Denmark, DK-8200

Corresponding author: Andrei Kudinov

e-mail: andrei.kudinov@luke.fi
Single-step genomic BLUP (ssGBLUP) is a powerful approach for breeding value prediction in populations with a limited number of genotyped animals. However, conflicting genomic (G) and pedigree (A_{22}) relationship matrices complicate the implementation of ssGBLUP into practice. The metafounder (MF) approach is a recently proposed solution for this problem and has been successfully used on simulated and real multi-breed pig data. Advantages of the method are easily seen across breed evaluations, where pedigrees are traced to several pure breeds, which are thereafter used as MF. Application of the MF method to ruminants is complicated due to multi-breed pedigree structures and the inability to transmit existing unknown parent groups (UPG) to MF. In this study, we apply the MF approach for ssGBLUP evaluation of Finnish Red Dairy cattle treated as a single breed. Relationships among MF were accounted for by a (co)variance matrix (\Gamma) computed using estimated base population allele frequencies. The attained \Gamma was used to calculate a relationship matrix A_{22}^\Gamma for the genotyped animals. We tested the influence of SNP selection on the \Gamma matrix by applying a minor allele frequency (MAF) threshold (T_{MAF}) where accepted markers had an MAF \geq 0.05. Elements in the T_{MAF} matrix were slightly lower than in the \Gamma matrix. Correlation between diagonal elements of the genomic and pedigree relationship matrices increased from 0.53 (A_{22}) to 0.76 (A_{22}^\Gamma and A_{22}^{\Gamma_{MAF}}). Average diagonal elements of A_{22}^\Gamma and A_{22}^{\Gamma_{MAF}} matrices increased to the same level as in the G matrix. ssGBLUP breeding values (GEBV) were solved using either the original 236 or redefined 8 UPG, or 8 MF computed with or without the MAF threshold. For bulls, the GEBV validation test results for the 8 UPG and 8 MF gave the same adjusted $R^2$ (0.31) and over-dispersion (0.73, measured by regression coefficient $b_1$). No significant $R^2$ increase was observed in cows. Thus, the MF greatly influenced the pedigree relationship matrices but not the GEBV.
Selection of SNPs according to MAF had a notable effect on the $\Gamma$ matrix and made the $A_{22}$ and $G$ matrices more similar.

**Key Words**

Genetic groups, single-step genomic BLUP, metafounders, base population.

**INTRODUCTION**

Single-step genomic BLUP (ssGBLUP) is an elegant approach for estimating genomic breeding values (GEBV) that uses pedigree ($A$) and genomic ($G$) relationship matrices (Aguilar et al., 2010; Christensen and Lund, 2010). The approach has two important theoretical assumptions concerning the $A$ and $G$ matrices: the same scale and equal base population (Christensen, 2012). These assumptions complicate the application of ssGBLUP in dairy cattle breeding. In order to meet the assumptions, several methods have been proposed that make $G$ to be like $A$. For example, base population allele frequencies ($AF$) are used (VanRaden, 2008), and elements of $G$ are scaled and centered to have on average the same diagonal and off-diagonal elements as in $A$ (Vitezica et al., 2011; Christensen et al., 2012). In practice, base population AF are unknown and the $G$ matrix is often constructed using AF observed in the genotyped population.

Commercial dairy cattle pedigree can seldom be traced to a genetically homogeneous base population because the pedigree often has a complicated breed structure with unknown parent information (VanRaden, 1992; Sponenberg and Bixby, 2007). To solve the problem of incomplete pedigree, Thompson (1979) and Quaas (1988) developed the concept of phantom parents or unknown parent groups (UPG), for animals with unknown parent(s). UPG are typically assigned according to selection pathways and share the same genetics allowing
more accurate estimation of genetic trend in traditional genetic evaluation (Theron et al., 2002). In ssGBLUP, Misztal et al. (2013) observed bias in UPG solutions. The bias increased with an increase in the number of genotyped animals.

The metafounder (MF) approach was proposed by Legarra et al. (2015) to achieve compatibility in the pedigree and genomic relationship matrices. The MF approach combines the idea of using AF equal to 0.5 for all markers when calculating the $G$ matrix (Christensen, 2012) and assigning unknown parents to MF or pseudo-individuals with self-relationships in the $A$ matrix. MF are similar to UPG, but allow a related base population with non-zero inbreeding coefficients. The relationships within and between the MF are modeled by a gamma matrix ($\Gamma$), which is used in forming the relationship matrix ($A^\Gamma$). The $\Gamma$ matrix may be constructed using an estimated base or observed genotyped population AF (e.g. Legarra et al., 2015; Garcia-Bacciano et al., 2017). However, the $\Gamma$ matrix may be poorly estimated when certain AF are estimated inaccurately due to the low number of rare alleles. The large number of UPG increases chances that an UPG is associated with a low number of rare allele genotypes.

Legarra et al. (2015) and Garcia-Bacciano et al. (2017) showed the advantage of the MF approach in GEBV estimation using simulated data. Xiang et al. (2017) used the MF method for ssGBLUP evaluation in the crossbreed performance in pigs. According to their results, the MF approach successfully combined two breeds in a GEBV evaluation. Pig evaluations clearly focus on the youngest generation and, thus, fewer UPG are needed than in dairy cattle (Arnold et al., 1992). MF approach studies have mostly focused on crossbred and admixture populations (Bradford et al., 2019; van Grevenhof et al., 2019) because the approach may help with implementing ssGBLUP for complicated pedigree populations such as in pigs and poultry. However, implementing the MF approach for dairy cattle may be challenging.
because of the frequently large number of UPG. The few published studies have used simulated dairy cattle data to estimate the $\Gamma$ matrix and its influence on ssGBLUP (Garcia-Bacciano et al., 2017; Bradford et al., 2019), but had only a few MF.

We used the MF approach in the ssGBLUP evaluation of 305-d milk production in Finnish Red dairy cattle. We present two approaches to estimate the $\Gamma$ matrix, using different numbers of markers. We compared values in the two $\Gamma$ matrices. The effect of various $\Gamma$ matrices is shown using model validation statistics from ssGBLUP evaluations having either UPG or MF.

**MATERIALS AND METHODS**

**ssGBLUP models**

The joint relationship matrix of genotyped and non-genotyped animals in ssGBLUP is commonly denoted as $H$ (Aguilar et al., 2010; Christensen and Lund, 2010). The $H^{-1}$ matrix needed in the mixed model equations of ssGBLUP is

$$H^{-1} = A^{-1} + \begin{pmatrix} 0 & 0 \\ 0 & G^{-1} - A_{22}^{-1} \end{pmatrix},$$

where $A$ is the full pedigree relationship matrix, $G$ is the genomic relationship matrix, and $A_{22}$ is a pedigree relationship matrix of the genotyped animals.

**Single step with UPG in A.** Mean genetic levels of animals with missing parental information were modeled using pedigree-based UPG proposed by Quaas and Pollack (1981). In the UPG model, unknown parents are assumed to be unrelated and completely outbred. UPG effects in the model only account for possible non-zero expectations in the breeding values of parent groups. There are alternative ways to account for UPG in forming $H^{-1}$. The standard way is to replace the original $H^{-1}$ matrix with an augmented one, where the UPG are included as
“phantom parents” (Westell et al., 1988). Matilainen et al. (2018), following Misztal et al. (2013), formed the $H^{-1}$ matrix without groups, and, thereafter, included the UPG via so-called QP transformation (Quaas and Pollack, 1981) into the final augmented $H^{-1}$. However, Masuda et al. (2019) recommended omitting the terms involving $G^{-1}$ in the UPG coefficient part of the augmented $H^{-1}$ matrix. In our UPG models, the genomic relationship matrix was constructed using VanRaden (2008) method 1 ($G_{PvR1}$), where base population AF were used to center and scale the marker data. Base population AF were estimated with the GLS model (McPeek et al., 2004) using the Bpop v. 0.30 program (Strandén and Mäntysaari, 2019), which is based on the computational approach described in Strandén et al. (2017). The genomic information was assumed to account for 90% of the variation in breeding values, i.e. the polygenic proportion was 10%. This was attained using a modified $G$ matrix obtained by averaging original $G$ and $A_{22}$ matrices with weights of 0.9 and 0.1, respectively.

**Single step with metafounders.** In the MF approach, the $H^{-1}$ matrix is replaced by a modified $(H^\Gamma)^{-1}$ matrix described by Legarra et al. (2015) and Christensen et al. (2015) as

$$(H^\Gamma)^{-1} = (A^\Gamma)^{-1} + \begin{pmatrix} 0 & 0 \\ 0 & G_w^{-1} - (A_{22}^\Gamma)^{-1} \end{pmatrix},$$

where $G_w = (1 - w)G_{05} + wA_{22}^\Gamma$, $w$ is the proportion of genetic variance not explained by the markers, $G_{05} = (Z_{101}Z_{101}^\prime)_m^2$, $Z_{101}$ is an $n \times m$ marker matrix with genotypes coded by $\{-1,0,1\}$, $m$ is the number of SNP markers, $n$ is the number of genotyped animals, $A^\Gamma$ is pedigree relationship matrix formed with a $\Gamma$ matrix, and $A_{22}^\Gamma$ is a submatrix of $A^\Gamma$ for the genotyped animals. We used a 10% polygenic proportion, i.e. $w = 0.1$, as in Garcia-Baccino et al. (2017). The variance covariance structure of the MF can be estimated by $\Gamma = 8 \text{Cov}(P)$, as presented in the Appendix of Christensen et al. (2015), where $P$ is an $m \times r$ matrix of AF and $r$ is the number of MF.
Test data and model validation

We used Red Dairy Cattle (RDC) milk production data provided by Nordic Cattle Genetic Evaluations (NAV). The data sample was extracted from the NAV production evaluation database by including all cows from 426 Finnish herds with at least 10 genotyped cows. This gave 112,479 cows with first-lactation 305-d milk production records produced during 1988–2018. The pedigree included 226,012 animals born in 1960–2016 consisting of 86% RDC, 12% Holstein (HOL), 2% Finn cattle (FIN, an indigenous Finnish cattle population), and a total of 1% of other breeds (Red Holstein, Jersey, Brown Swiss etc.). There were 236 UPG definitions were the same as those used in the Nordic TD evaluations in November 2018 (Lidauer et al., 2015) and were provided by NAV.

Genotypes were available for 19,757 animals (3,571 bulls and 16,186 cows), which either had observations or were in the pedigree of the animals with observations. Bulls were genotyped using Illumina Bovine SNP50 Bead Chip (Illumina, San Diego, USA) and the cows using a lower-density EuroG 10k chip (http://www.eurogenomics.com/) that had been imputed to the 50K density by NAV. There were 46,914 markers from the 29 bovine autosomes available for the analysis.

Cow and bull validation data sets were created by removing milk production records for either the last year or for four of the previous production years, respectively, as in Gao et al. (2018) and Mäntysaari et al. (2010). We included 101 and 3,551 genotyped test bulls and cows, respectively. Daughter yield deviations (DYD) and yield deviations (YD) were attained using the full data and an animal model by the MiX99 software (Strandén and Lidauer, 1999), as in Gao et al. (2018). The calculated DYD and YD were used for bulls and cows, respectively, in validation regression models \((D)YD = b_0 + b_1 \times GEBV\), with weights for
the DYD observations. The weight for DYD was \( EDC / (EDC + \lambda) \), where \( \lambda = (4 - h^2) / h^2 \), 

\( h^2 \) is heritability, and EDC is the bull’s effective daughter contributions (https://interbull.org/ib/cop_appendix4) in evaluation with the full data set. To attain adjusted validation reliability, we divided the model coefficient of determination (\( R^2 \)) by the average weight. The regression coefficient \( b_1 \) for the bulls was multiplied by two because DYD only represents the sire effect. All the analyses used \( h^2 \) of 0.44, which is a parameter derived from the NAV milk production test day model for 305-d milk yield.

**Unknown parents and metafounders**

Eight groups were defined according to the full pedigree structure and replaced the original 236 UPG. We included six groups for RDC (birth years <1971, 1971–1980, 1981–1990, 1991–2000, 2001–2010, 2011–2016), a HOL group, and a group for the other breeds. These eight groups were treated as UPG or MF. In the MF approach, the base population AF, used to calculate the \( \Gamma \) matrix, were estimated using a GLS approach. The GLS model was \( \mathbf{m}_i = \mathbf{Q}\mu_i + \mathbf{e}_i \), where \( \mathbf{m}_i \) is an \( n \) by 1 vector of marker \( i \) genotypes, \( \mathbf{Q} \) is an \( n \) by 8 matrix, the rows of which sum up to 1, and that assigns individuals to fractions of MF, \( \mu_i \) is an 8 by 1 vector of group means, and \( \mathbf{e}_i \sim (\mathbf{0}, \mathbf{A}_{22}\sigma^2) \) where \( \mathbf{A}_{22} \) was the pedigree relationship matrix for the genotyped animals and \( \sigma^2 \) is the common variance. In allele frequency estimation, the common variance need not be known (e.g. Garcia-Baccino et al., 2017). Estimated base population AF for the MF are \( \mathbf{p}_i = \frac{1}{2} \bar{\mu}_i \) for each marker \( i = 1, ..., m \).

To estimate AF for the MF in the GLS model, the \( \mathbf{A}_{22}^t \) matrix was based on a truncated pedigree, where one parent generation at most was accepted to the genotyped animals. The pedigree truncation guaranteed that the young genotyped animals would contribute to the recent birth year MF and not to the old birth year MF. In addition, the truncation used more genomic information than the full pedigree because genotyped animals had less genotyped
ancestors but instead a young birth year MF. It can be proven that the GLS method will ignore genotype of an animal whose both parents are genotyped and the animal is not an ancestor to a genotyped animal.

The eight columns of base population AF in the $\mathbf{P}$ matrix were used to estimate the variance covariance structure of the eight MF or the $\mathbf{\Gamma}$ matrix, $\mathbf{\Gamma} = 8 \text{Cov}(\mathbf{P})$. The effect of minor allele frequencies (MAF) on the MF covariances were tested by creating two alternative $\mathbf{\Gamma}$ matrices. In the first scenario, the full $\mathbf{P}$ matrix was used to calculate the $\mathbf{\Gamma}$ matrix, denoted $\mathbf{\Gamma}_8$. In the other scenario, denoted $\mathbf{\Gamma}_8\text{MAF}$, only those markers with MAF greater or equal to 0.05 in all RDC cattle MF were included in the $\mathbf{P}$ matrix. The MAF requirement eliminated 3,783 markers and left 43,131 markers that were used to calculate the $\mathbf{\Gamma}_8\text{MAF}$ matrix.

**ssGBLUP computation**

All ssGBLUP calculations used the full pedigree with 226,012 animals and genomic relationship matrices ($\mathbf{G}_{\text{PrR1}}$ or $\mathbf{G}_{05}$) for the 19,757 animals. For the ssGBLUP with MF, the augmented additive relationship matrix of genotyped animals ($\mathbf{A}_{22}$) was calculated using the modified RelaX2 v. 1.83 program (Strandén and Vuori, 2006). The ($\mathbf{G}_{\text{PrR1}}^{-1} - \mathbf{A}_{22}^{-1}$) and ($\mathbf{G}_{w}^{-1} - (\mathbf{A}_{22}^{-1})^{-1}$) matrices were calculated using the HGinv v. 0.87 program (Strandén and Mäntysaari, 2018). The latest MiX99 v. 17.1107 (Strandén and Lidauer, 1999) was used to solve the GEBV using the four ssGBLUP models. Two of the evaluations were UPG models with either 236 UPG (ssGBLUP$_{236\text{UPG}}$) or 8 UPG (ssGBLUP$_{8\text{UPG}}$) in $\mathbf{A}$. UPG were treated as random by adding the inverse of genetic variance to the diagonal of group equations in the mixed model equations. The other two ssGBLUP evaluations were MF models that had eight MF, and the pedigree relationship matrices were based on $\mathbf{\Gamma}_8$ (ssGBLUP$_{\text{F}_8}$) or $\mathbf{\Gamma}_8\text{MAF}$ (ssGBLUP$_{\text{F}_8\text{MAF}}$). Genetic variance parameters from the model with unrelated founders were used to estimate corresponding parameters for the model.
with MF. The variance of breeding values in base population descending from MF \((\sigma^2_{a,k})\) in ssGBLUP\(_{G8}\) and ssGBLUP\(_{G8МАF}\) models were calculated using the scaling parameter \(k\), i.e.,
\[
\sigma^2_{a,k} = \sigma^2_\delta / k,
\]
where \(k = (1 + \text{tr}(\Gamma)/(2n) - 1\text{tr}(\text{1}1^T/n^2))\) and \(\text{tr}(\Gamma)\) is the sum of diagonal elements of the \(\Gamma\) matrix (Legarra et al. 2015).

**Comparisons**

Two traditional ssGBLUP evaluations were computed using different numbers of UPG, and two MF-based ssGBLUP evaluations were computed using different \(\Gamma\) matrices and inbreeding coefficients. We present the two \(\Gamma\) matrices such that the direct effect of the MAF threshold marker selection is seen in elements of the \(\Gamma\) matrices. The MF approach is expected to give more similar pedigree and genomic relationship matrices than the traditional pedigree and genomic relationship matrices. In addition, the off-diagonal elements in the pedigree relationship matrix by the MF approach are expected to be higher than in the traditional pedigree relationship matrix. We assessed differences in the diagonal elements (related to the definition of inbreeding) and off-diagonals (related to relatedness) of \(A_{22}^\Gamma\), \(A_{22}\), \(G_{05}\), and \(G_{PvR1}\) by correlations and mean differences between these matrices. To identify differences in trends of diagonals to the pedigree and genomic matrices (that are related to breeding selection and changes in inbreeding), average diagonal elements of \(A_{22}^\Gamma\), \(G_{05}\), \(G_{PvR1}\), and \(A_{22}\) were plotted by birth year.

The two UPG definitions and two MF \(\Gamma\) matrices gave four sets of ssGBLUP predictions. Validation tests used GEBV from the ssGBLUP evaluations separately from the groups of genotyped bulls and cows. Approximately 80% of bulls born in 1990 to 2014 were genotyped. Thus, differences between the ssGBLUP models may be largest in the genetic trends of the bulls. Averages and standard deviation of selected bull GEBVs by birth year were plotted for comparison purposes. The bulls selected for plotting had at least 10
daughters each. Average cow GEBVs by birth year were plotted using GEBVs from all cows
to illustrate the genetic trend in the general population.

RESULTS AND DISCUSSION

Elements of $\Gamma$, $A_{22}$, $G_{0.5}$, $G_{PvR1}$, and $A_{22}$

Table 1 has elements of the $\Gamma_8$ and $\Gamma_{8MAF}$ matrices. Elements of the $\Gamma_{8MAF}$ matrix were
slightly lower than corresponding elements in the $\Gamma_8$ matrix. All diagonal elements in the $\Gamma$
matrices were less than one, which corresponds to negative inbreeding of MF (Table 2)
calculated as $F = \gamma - 1$, where $\gamma$ is the relationship across gametes (diagonal element of $\Gamma$).
All elements in the calculated $\Gamma_8$ and $\Gamma_{8MAF}$ matrices were from 0.452 to 0.797.

Because the MF were partially formed by breed, the greater than zero off-diagonal elements
suggest shared genetics between breeds. Average mean relationship between the RDC and
HOL metafounders was 0.564 and 0.473 in $\Gamma_8$ and $\Gamma_{8MAF}$, respectively. Off-diagonal
elements of the $\Gamma$ matrix between Holstein and Jersey cattle in Legarra et al. (2015) was 0.48,
which is close to the value we obtained in $\Gamma_{8MAF}$. They calculated the $\Gamma$ matrix using
published statistics in VanRaden et al. (2011), which included only SNP markers with MAF
$\geq 0.05$ (Wiggans et al., 2009). The self-relationships in the HOL and RDC metafounders in
our study were also comparable to 0.55 presented for the HOL and Jersey breeds in Legarra
et al. (2015). In our study, an exception to this was the RDC $< 1970$ group, which had a
diagonal value of 0.618 and 0.719 in $\Gamma_{8MAF}$ and $\Gamma_8$, respectively. The larger diagonal value in
the oldest RDC group may be due to changes in the Finnish RDC breeding program. Before
1970, breeding in the RDC group was mostly limited to Ayrshire cattle with only a low
number of imported animals. After 1970, importation began changing the population to more
resemble a mixed Nordic RDC breed. Diagonal elements in the group of other breeds were
high in both of the $\mathbf{\Gamma}$ matrices (0.740 and 0.797). This may be due to the influence of Finn Cattle having only a small number of animals, which may produce unreliable AF estimates.

Table 3 shows correlations between (off-)diagonal elements of $\mathbf{A}_{22}, \mathbf{G}_{05}$, $\mathbf{G}_{PvR1}$, $\mathbf{A}_{22}^{\Gamma_{8}}$, and $\mathbf{A}_{22}^{\Gamma_{8MAF}}$ matrices. Constructing $\mathbf{A}_{22}$ using $\Gamma_{8}$ and $\Gamma_{8MAF}$ increased the correlation between the diagonal elements of $\mathbf{G}_{05}$ and $\mathbf{A}_{22}$ from 0.66 to 0.76. The diagonal element correlation between elements of $\mathbf{A}_{22}^{\Gamma_{8MAF}}$ and $\mathbf{A}_{22}$ was higher (0.84) than between $\mathbf{A}_{22}^{\Gamma_{8}}$ and $\mathbf{A}_{22}$ (0.81). The correlation between diagonal elements of $\mathbf{G}_{PvR1}$ and $\mathbf{A}_{22}$ decreased from 0.53 to 0.33 and 0.37 for $\mathbf{A}_{22}^{\Gamma_{8MAF}}$ and $\mathbf{A}_{22}$, respectively. Despite the high correlation of 0.99 between the diagonal elements of $\mathbf{A}_{22}^{\Gamma_{8}}$ and $\mathbf{A}_{22}^{\Gamma_{8MAF}}$, average diagonal elements by the birth year of an animal (Figure 1) were at a higher level for $\mathbf{A}_{22}^{\Gamma_{8}}$ than for $\mathbf{A}_{22}^{\Gamma_{8MAF}}$ or $\mathbf{G}_{05}$. Average diagonal elements for both augmented matrices ($\mathbf{A}_{22}^{\Gamma_{8}}$ and $\mathbf{A}_{22}^{\Gamma_{8MAF}}$) were at the same level as $\mathbf{G}_{05}$, i.e., from 1.30 to 1.38, while the average diagonals of $\mathbf{A}_{22}$ and $\mathbf{G}_{PvR1}$ were in range from 0.98 to 1.08. According to the summary statistics in Table 4, values for the off-diagonal elements of the pedigree relationship matrix $\mathbf{A}_{22}$ increased when using $\mathbf{\Gamma}$ to make $\mathbf{A}_{22}^{\Gamma_{8}}$. Hence, all elements in the $\mathbf{G}_{05}$, $\mathbf{A}_{22}^{\Gamma_{8}}$, and $\mathbf{A}_{22}^{\Gamma_{8MAF}}$ matrices were higher on average than those in the $\mathbf{A}_{22}$ and $\mathbf{G}_{PvR1}$ matrices. Interestingly, both the diagonal and off-diagonal element mean, minimum, and maximum values of $\mathbf{G}_{05}$ and $\mathbf{A}_{22}^{\Gamma_{8MAF}}$ agreed very well.

Average inbreeding coefficients in the $\mathbf{A}_{22}$ and $\mathbf{G}_{05}$ matrices were 0.02 and 0.31, respectively. This difference of 0.29 was close to the 0.272 reported in VanRaden et al. (2011) for HOL cattle (0.056 for $\mathbf{A}_{22}$ and 0.328 for $\mathbf{G}_{05}$). The average inbreeding coefficient increased from 0.02 in $\mathbf{A}_{22}$ to 0.34 and 0.29 in $\mathbf{A}_{22}^{\Gamma_{8}}$ and $\mathbf{A}_{22}^{\Gamma_{8MAF}}$, respectively. Following Legarra et al. (2015), a diagonal element value less than one in the $\mathbf{\Gamma}$ matrix means a negative individual inbreeding coefficient for MF. In all RDC MF, all elements of $\text{diag}(\mathbf{\Gamma})^{-1}$ ranged
from -0.38 to -0.43. We observed the highest self-relationships and corresponding MF inbreeding coefficients in the other breed group, which could be explained by the relatively closed small-scale selection program for FinnCattle.

Use of the $\Sigma$ matrix to make the pedigree-based relationship matrix $A^\Sigma_{22}$ or $A^\Sigma_{22}$ increased the correlation between elements of the pedigree and genomic relationship matrices when compared to the correlation between traditionally formed matrices ($G_{PrR1}$ and $A_{22}$). Correlation between diagonal elements of $A^\Sigma_{22}$ and $G_{05}$, as well as between $A^\Sigma_{22}$ and $G_{05}$, was 0.76, which is higher than the correlation of 0.53 between the diagonal elements of $G_{PrR1}$ and $A_{22}$. Correlation between the off-diagonal elements of $A^\Sigma_{22}$ ($A^\Sigma_{22}$) and $G_{05}$ was 0.91, which is a bit higher than the same correlation (0.89) between $G_{PrR1}$ and $A_{22}$. Thus, using the $\Sigma$ matrix to form the relationship matrix lifted the diagonal elements of $A^\Sigma_{22}$ matrix to the same level as in the $G_{05}$ matrix (Figure 1).

The average diagonal of the $A^\Sigma_{22}$ matrix was at a higher level than the average diagonal of the $G_{05}$ matrix (Figure 1). Use of the MAF threshold to make $\Sigma_{MAF}$ for $A^\Sigma_{22}$ gave lower average diagonal values than those in $G_{05}$. In constructing the $\Sigma_{MAF}$ matrix, we deleted the low MAF markers to omit markers with highly uncertain or erroneous AF estimates. This, however, may lead to deleting nearby markers and accepting more markers from certain regions of the genome, particularly if a MAF threshold value higher than 5% is used. Consequently, AF from various MF may become more similar. For example, two breeds may differ due to more intense selection in one of the breeds, leading to the MAF criterion favoring unselected or highly polymorphic markers clustered in certain regions of the genome. Consequently, the $\Sigma$ matrix may show inflated covariances between the MF of these breeds. Linkage Disequilibrium (LD) criteria, in which markers are chosen to minimize LD, is an alternative approach to SNP pruning (Hill and Robertson, 1968). Patterns of LD are
widely used in marker data quality control and in the analysis of population history for various species (Porto-Neto et al., 2014; Makina et al., 2015; Cañas-Álvarez et al., 2016). Multiple studies have shown persistence in LD levels of various breeds and populations (de Roos, 2008; Xu et al., 2019), making LD a potential tool for marker selection.

**ssGBLUP estimation & validation results**

The correction factor \( k \) used to calculate the variance of breeding values in base population descending from metafounders \( (\sigma^2_{a,k}) \) in the GEBV calculations for ssGBLUP\(_{P_B}\) and ssGBLUP\(_{PM_AF}\) was 0.72 and 0.77, respectively. Averages and standard deviations of bull GEBV by birth year are shown in Figures 2 and 3 and the average cow GEBV are shown in Figure 4. We centered the average GEBV trends of cows and bulls, so that the mean GEBV of animals born in 2009 equaled zero. Average bull GEBV in Figure 2 had a similar shape in all the models. The SD level in Figure 3 for bulls born in 2012–2014 was 20 kg (3%) higher in the MF models than in the UPG models. Average cow GEBV by birth year had a similar shape in all models (Figure 4).

Validation test statistics for the approaches are shown in Table 5. Regression coefficients \( (b_1) \) were generally slightly higher using MF than UPG. In the bull validation set, we obtained similar adjusted model reliability by ssGBLUP\(_{UPG}\), ssGBLUP\(_{P_B}\), and ssGBLUP\(_{PM_AF}\), and the gain was 0.04 in comparison to ssGBLUP\(_{236UPG}\). In the cow validation set, the validation reliabilities using MF were 0.01 higher than achieved by the UPG models. To exclude pre-selection bias, we conducted the validation tests for bulls also using DYD computed from ssGBLUP\(_{236UPG}\). The adjusted model reliabilities did not change from those in Table 5.

Genetic trends in GEBV from the UPG and MF models had a similar shape, showing no effect of the alternative group or founder definitions. We assumed that the inadequate
definition of groups would reduce the genetic trend estimate (Tsuruta et al., 2014) but this
was not observed. Each of the bulls included in the yearly means in Figures 2 and 3 had at
least 10 daughters and, therefore, may be less affected by MF. Perhaps ssGBLUP predictions
where most of the sires are genotyped are robust against the definition of UPG or MF. Meyer
and Tier (2018) reported a slightly higher estimated genetic trend with the MF approach
compared to ssGBLUP without groups. However, females were the most often genotyped
group in their data. Also, the SDs of the GEBV were fairly similar between all evaluations
(Figure 3). The unstandardized genetic levels in the MF models were at a higher level
compared to the UPG models. This difference did not affect the animal rankings by GEBV
but indicate that the models defined base populations differently. We observed a high
correlation of bull GEBVs between the MF model and the original 236 UPG model (0.972),
while correlation of GEBVs between the MF model and the 8 UPG model was much lower
(0.931; correlations not given in Tables).

We used pedigree-based UPG in the ssGBLUP model via incomplete QP transformation
(Quaas and Pollak, 1981), i.e. QP transformation for $A^{-1}$ instead of $H^{-1}$. In case of a multi-
breed structure, i.e. for the joint Nordic (Denmark, Finland, Sweden) RDC genetic
evaluation, Matilainen et al. (2018) proposed to use QP transformation in $H^{-1}$ (Misztal et al.,
2013). Bradford et al. (2019) observed that the incomplete QP transformation in ssGBLUP
may be applied successfully by accounting for $A^{-1}$ only, when a purebred population is
analyzed. The MF approach used in this study could be a smooth way to implement the
ssGBLUP model for the joint Nordic evaluation.

Estimation of allele frequencies

Defining the base population is the greatest challenge in the MF approach. We focused on
two issues: the number of MF and the genetic change in time. Simply replacing current UPG
by MF is often impossible in genetic evaluations of large commercial populations, which
have many UPG and animals with missing parents. We combined all UPG by breed and split
the RDC-based UPG by decade to form eight MF. For the HOL and OTHER breeds, the
limited number of animals and absence of phenotypic data were the key reasons for using
only one MF per breed. By using multiple MF in RDC, we could account for a possible
change in AF with time.

Base population AF for the MF are needed to calculate the $\Gamma$ matrix. Garcia-Baccino et al.
(2017) presented three approaches for estimating base population AF to be used for
populations with crossbreed animals. All of these methods use genotypes and a pedigree
relationship matrix or matrices. We used the genetics group model utilizing GLS. An
alternative GLS approach allows differences between gene content variances across breeds
and relies on a multi-breed model presented in Garcia-Cortes et al. (2006). All the pedigree-
based approaches only need the pedigree of ancestors to the genotyped animals, and the base
population groups are defined by MF through pedigree information. However, the
unbalanced distribution of genotyped animals to UPG or MF in the full pedigree affects all
base population AF estimation methods that rely on the pedigree relationship matrix.

In our study, a major part of the genotyped animals (75%) contributed to the oldest RDC
group (RDC < 1971) when the full pedigree was used, although most of the genotyped
animals (90.6%) were born after 2000. Thus, the contribution gained from genotypes of
animals born in 2000–2016 to the recent year groups would be small and would depend on
pedigree incompleteness. Consequently, the base population AF of the oldest RDC groups
would be well estimated with, possibly, a small influence from young animal genotypes. To
solve these issues in the base population AF estimation for the MF, we limited the length of
the pedigree of genotyped animals by only accepting ungenotyped animals with genotyped offspring.

In our study, we calculated the base population AF of HOL and the other breeds group using the ancestor structure of genotyped RDC animals only. We tested the applicability of the chosen GLS approach by estimating an additional $\Gamma$ matrix ($\Gamma_{RDC\&HOL}$, Table 6). The matrix was calculated using HOL AF (Koivula 2019, personal communication). We estimated these AF with HOL breed genotypes and the pedigree used in Koivula et al. (2018). The estimated $\Gamma_{RDC\&HOL}$ was compared with the presented $\Gamma_8$ and $\Gamma_{8MAF}$ matrices (Table 1), which were only based on genotyped RDC animals. The closeness of the average diagonal values in the HOL MF of $\Gamma_{RDC\&HOL}$ (0.615), $\Gamma_8$ (0.661), and $\Gamma_{8MAF}$ (0.593) suggest that we were able to estimate the $\Gamma$ matrices fairly well without including the pure HOL population genotypes. In addition, the MAF-based marker selection gave the closest value to the HOL genotypes-derived value. Using the truncated pedigree is one possible reason for the good estimation of HOL AF using RDC data. The aim of the pedigree truncation was to distribute available genotypes evenly across MF. Pruning the pedigree appeared to solve two important problems: unequal distribution of genotyped animals across MF and the mixture of AF breed groups.

Off-diagonal elements of the $\Gamma$ matrix suggested fairly high similarity between all founder groups. We tested a $\Gamma$ matrix where the off-diagonal elements were half of those in the estimated $\Gamma$ matrix (results not shown). This half-reduced off-diagonal element $\Gamma$ matrix nearly gave the same GEBVs solutions, with a correlation of 0.998. Thus, for this data set, the MF-based ssGBLUP evaluation does not seem to be very sensitive to the off-diagonal element values in the $\Gamma$ matrix. Further work is needed to ascertain that this can be generalized to data sets with more genotyped animals and different population structure.
We observed differences in the $\mathbf{G}$ matrix depending on the set of markers used to estimate the $\mathbf{G}$ matrix. When markers were required to have an MAF above a certain limit, values in the $\mathbf{G}$ matrix were lower than when all the markers were used. This is to be expected because the $\mathbf{G}$ matrix is estimated by the variance of AF and the MAF threshold reduced range of marker AF is used to calculate the variance. The case is similar to that in Chen et al. (2011) where increasing the MAF threshold in the marker selection decreased the values of (off-)diagonal elements in the genomic relationship matrix. The $\mathbf{G}$ matrix is a function of the chosen MAF threshold as a consequence of the marker selection. We must therefore be careful when making interpretations of values in the estimated $\mathbf{G}$ matrix. For example, the MAF threshold was applied to all of the RDC-based MF, but the set of selected markers will change if the HOL animals have genotypes.

The pedigree pruning approach allowed estimation of base population AF for the breed groups despite all the genotyped animals being from the RDC breeding program. Still, it is impossible to model AF changes in base populations and MF before the first genotyped parent generations. One possibility is to assume that the AF changes have continuity and that the changes can also be extrapolated to early years before the genotyping began. Then the variance structures of $\mathbf{G}$ in the observed base populations, i.e. parents of genotyped animals, could be extended to describe variances of unobservable MF using covariance functions (Kirkpatrick et al. 1994) with appropriate breeds and birth years.

**CONCLUSIONS**

We tested the metafounder approach on RDC data with a complicated multi-breed structure. The original 236 UPG were replaced by eight MF and tested in ssGBLUP evaluation. Use of MF increased correlation between elements of the pedigree and genomic relationship
matrices. Introduction of MAF-based marker selection before computing the \( \Gamma \) matrix for the MF gave \( A_{22}^{\text{MAF}} \) an advantage over the original \( A_{22}^{\text{g}} \) in correlations with elements of the genomic relationship matrix. The reduction of UPG groups from 236 to eight reduced the inflation in the predictions and increased validation accuracy. The GEBVs from models with eight MF gave almost the same validation results and genetic trends as the eight UPG. Future development should focus on ways to increase the number of MF closer to the number of UPG.

**ACKNOWLEDGEMENTS**

We acknowledge Viking Genetics (Randers, Denmark) and Nordic Cattle Genetic Evaluation (Aarhus, Denmark) for providing the genotype data and participating in project financing.

**REFERENCES**


Strandén, I., E.A. Mäntysaari. 2018. HGinv Program. Natural Resources Institute Finland (LUKE).


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Table 1. Estimated $\Gamma_8$ (lower) and $\Gamma_{8\text{MAF}}$ (upper) triangle for the metafounders. The diagonal includes diagonals (i.e. self-relationships of metafounders) of $\Gamma_8$ (in brackets) and $\Gamma_{8\text{MAF}}$.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>RDC$^i$ &lt;1970</td>
<td>0.618 (0.719)</td>
<td>0.555</td>
<td>0.563</td>
<td>0.566</td>
<td>0.566</td>
<td>0.471</td>
<td>0.453</td>
<td></td>
</tr>
<tr>
<td>RDC$^i$ 1971–1980</td>
<td>0.659 (0.670)</td>
<td>0.569</td>
<td>0.566</td>
<td>0.564</td>
<td>0.562</td>
<td>0.473</td>
<td>0.454</td>
<td></td>
</tr>
<tr>
<td>RDC$^i$ 1981–1990</td>
<td>0.668 (0.710)</td>
<td>0.609</td>
<td>0.588</td>
<td>0.589</td>
<td>0.585</td>
<td>0.473</td>
<td>0.452</td>
<td></td>
</tr>
<tr>
<td>RDC$^i$ 1991–2000</td>
<td>0.667 (0.689)</td>
<td>0.690</td>
<td>0.587</td>
<td>0.585</td>
<td>0.583</td>
<td>0.473</td>
<td>0.455</td>
<td></td>
</tr>
<tr>
<td>RDC$^i$ 2001–2010</td>
<td>0.671 (0.701)</td>
<td>0.692</td>
<td>0.688</td>
<td>0.598</td>
<td>0.597</td>
<td>0.474</td>
<td>0.452</td>
<td></td>
</tr>
<tr>
<td>RDC$^i$ 2011–2016</td>
<td>0.671 (0.705)</td>
<td>0.688</td>
<td>0.686</td>
<td>0.699</td>
<td>0.603</td>
<td>0.474</td>
<td>0.453</td>
<td></td>
</tr>
<tr>
<td>HOL$^i$</td>
<td>0.563</td>
<td>0.564</td>
<td>0.564</td>
<td>0.566</td>
<td>0.566</td>
<td>0.593 (0.661)</td>
<td>0.479</td>
<td></td>
</tr>
<tr>
<td>OTHER$^i$</td>
<td>0.544</td>
<td>0.544</td>
<td>0.545</td>
<td>0.544</td>
<td>0.545</td>
<td>0.552</td>
<td>0.740 (0.797)</td>
<td></td>
</tr>
</tbody>
</table>

$^i$Red dairy cattle (RDC) has been divided into metafounders by birth year, Holstein (HOL) cattle has one metafounder, and the other breeds (OTHER) have been combined into one metafounder.
Table 2. Inbreeding coefficients of metafounders calculated using $\Gamma_8$ and $\Gamma_{8MAF}$.

<table>
<thead>
<tr>
<th>Groups</th>
<th>$\Gamma_8$</th>
<th>$\Gamma_{8MAF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RDC &lt;1970</td>
<td>-0.28</td>
<td>-0.38</td>
</tr>
<tr>
<td>RDC 1971–1980</td>
<td>-0.33</td>
<td>-0.43</td>
</tr>
<tr>
<td>RDC 1981–1990</td>
<td>-0.29</td>
<td>-0.39</td>
</tr>
<tr>
<td>RDC 1991–2000</td>
<td>-0.31</td>
<td>-0.41</td>
</tr>
<tr>
<td>RDC 2001–2010</td>
<td>-0.29</td>
<td>-0.40</td>
</tr>
<tr>
<td>RDC 2011–2016</td>
<td>-0.29</td>
<td>-0.39</td>
</tr>
<tr>
<td>HOL</td>
<td>-0.34</td>
<td>-0.40</td>
</tr>
<tr>
<td>OTHER</td>
<td>-0.34</td>
<td>-0.26</td>
</tr>
</tbody>
</table>

1Red dairy cattle (RDC) has been divided into metafounders by birth year, Holstein (HOL) cattle has one metafounder, and the other breeds (OTHER) have been combined into one metafounder.
Table 3. Correlation of diagonal (upper triangle) and off-diagonal (lower triangle) elements of $A_{22}$, $G_{05}$, $G_{PvR1}$, $A_{22}^{\Gamma_0}$, and $A_{22}^{\Gamma_{MAF}}$.

<table>
<thead>
<tr>
<th></th>
<th>$A_{22}$</th>
<th>$A_{22}^{\Gamma_2}$</th>
<th>$A_{22}^{\Gamma_{MAF}}$</th>
<th>$G_{05}$</th>
<th>$G_{PvR1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{22}$</td>
<td>1</td>
<td>0.81</td>
<td>0.84</td>
<td>0.66</td>
<td>0.53</td>
</tr>
<tr>
<td>$A_{22}^{\Gamma_2}$</td>
<td>0.89</td>
<td>1</td>
<td>0.99</td>
<td>0.76</td>
<td>0.33</td>
</tr>
<tr>
<td>$A_{22}^{\Gamma_{MAF}}$</td>
<td>0.92</td>
<td>0.99</td>
<td>1</td>
<td>0.76</td>
<td>0.37</td>
</tr>
<tr>
<td>$G_{05}$</td>
<td>0.83</td>
<td>0.91</td>
<td>0.91</td>
<td>1</td>
<td>0.70</td>
</tr>
<tr>
<td>$G_{PvR1}$</td>
<td>0.89</td>
<td>0.86</td>
<td>0.88</td>
<td>0.88</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 4. Mean, minimum (Min), and maximum (Max) element values of $A_{22}$, $G_{05}$, $G_{PVR1}$, $A_{22}^R$, and $A_{22}^{R_{MAF}}$ from diagonal and off-diagonal.

<table>
<thead>
<tr>
<th>Elements</th>
<th>Matrix</th>
<th>Mean</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Diagonal</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_{22}$</td>
<td></td>
<td>1.02</td>
<td>1.00</td>
<td>1.29</td>
</tr>
<tr>
<td>$G_{05}$</td>
<td></td>
<td>1.31</td>
<td>1.24</td>
<td>1.48</td>
</tr>
<tr>
<td>$G_{PVR1}$</td>
<td></td>
<td>1.01</td>
<td>0.91</td>
<td>1.30</td>
</tr>
<tr>
<td>$A_{22}^R$</td>
<td></td>
<td>1.35</td>
<td>1.27</td>
<td>1.51</td>
</tr>
<tr>
<td>$A_{22}^{R_{MAF}}$</td>
<td></td>
<td>1.31</td>
<td>1.23</td>
<td>1.50</td>
</tr>
<tr>
<td><strong>Off-diagonal</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_{22}$</td>
<td></td>
<td>0.07</td>
<td>0.06</td>
<td>0.81</td>
</tr>
<tr>
<td>$G_{05}$</td>
<td></td>
<td>0.63</td>
<td>0.47</td>
<td>1.29</td>
</tr>
<tr>
<td>$G_{PVR1}$</td>
<td></td>
<td>0.05</td>
<td>-0.11</td>
<td>0.99</td>
</tr>
<tr>
<td>$A_{22}^R$</td>
<td></td>
<td>0.72</td>
<td>0.54</td>
<td>1.22</td>
</tr>
<tr>
<td>$A_{22}^{R_{MAF}}$</td>
<td></td>
<td>0.62</td>
<td>0.45</td>
<td>1.16</td>
</tr>
</tbody>
</table>
Table 5. GEBV validation test regression coefficients and validation reliabilities of single-step GBLUP GEBVs for genotyped bulls and cows.

<table>
<thead>
<tr>
<th>Validation set</th>
<th>Model $^1$</th>
<th>$b_0$</th>
<th>SE</th>
<th>$b_1^2$</th>
<th>SE</th>
<th>$R^2$ $^3$</th>
<th>$R_{EDC}^2$ $^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulls</td>
<td>ssGBLUP$_{236UPG}$</td>
<td>70</td>
<td>16</td>
<td>0.61</td>
<td>0.06</td>
<td>0.23</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>ssGBLUP$_{8UPG}$</td>
<td>18</td>
<td>16</td>
<td>0.73</td>
<td>0.06</td>
<td>0.26</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>ssGBLUP$_{8b}$</td>
<td>-22</td>
<td>22</td>
<td>0.72</td>
<td>0.06</td>
<td>0.26</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>ssGBLUP$_{8MAF}$</td>
<td>-27</td>
<td>23</td>
<td>0.73</td>
<td>0.06</td>
<td>0.26</td>
<td>0.31</td>
</tr>
<tr>
<td>Cows</td>
<td>ssGBLUP$_{236UPG}$</td>
<td>118</td>
<td>9</td>
<td>0.89</td>
<td>0.03</td>
<td>0.16</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>ssGBLUP$_{8UPG}$</td>
<td>150</td>
<td>8</td>
<td>0.89</td>
<td>0.03</td>
<td>0.16</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>ssGBLUP$_{8b}$</td>
<td>12</td>
<td>13</td>
<td>0.90</td>
<td>0.03</td>
<td>0.16</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>ssGBLUP$_{8MAF}$</td>
<td>-0.2</td>
<td>13</td>
<td>0.93</td>
<td>0.04</td>
<td>0.16</td>
<td>0.37</td>
</tr>
</tbody>
</table>

$^1$Model ssGBLUP$_{236UPG}$ (ssGBLUP$_{8UPG}$) had 236 (8) unknown parent groups; ssGBLUP$_{8b}$ had 8 metafounders with the metafounder $\Gamma$ matrix calculated using all markers; ssGBLUP$_{8MAF}$ used markers with a minor allele frequency $\geq 0.05$ in the metafounder $\Gamma$ matrix calculation.

$^2$Regression coefficient $b_1$ in equation $DYD = b_0 + b_1 \times GEBV$ for the bulls has been multiplied by 2.

$^3$ $R^2$ is the coefficient of determination from the validation regression, $R_{EDC}^2$ is adjusted by the average reliability of phenotypes in the validation group.
Figure 1. Average diagonal elements of $A_{22}$ (black cross), $G_{Pvr1}$ (red cross), $G_{05}$ (green circles), $A_{22}^{p}$ (orange circles), and $A_{22}^{eMAF}$ (blue circles) by the birth year of an animal. The left side of the y-axis has a scale for $G_{05}$, $A_{22}^{p}$ and $A_{22}^{eMAF}$ and the right side has a scale for $A_{22}$ and $G_{Pvr1}$. 
Figure 2. Average genomic breeding value of bulls by birth year in 305-d milk yield (kg).

Each bulls had at least 10 daughters. The lines above each other are from the unknown parent group models ssGBLUP\textsubscript{236UPG} (black square) and ssGBLUP\textsubscript{8UPG}, (green triangle) and from the metafounders models ssGBLUP\textsubscript{8} (blue diamond) and ssGBLUP\textsubscript{I\textsubscript{0}MAF} (red cross).
Figure 3. Standard deviation of bull genomic breeding values by birth year in 305-d milk yield, kg. Each bull had at least 10 daughters. Trends are from the unknown parent group models ssGBLUP$_{236UPG}$ (black square) and ssGBLUP$_{8UPG}$ (green triangle) and from the metafounders models ssGBLUP$_{8}$ (blue diamond) and ssGBLUP$_{8MAF}$ (red cross).
**Figure 4.** Average genomic breeding value of cows by birth year in 305-d milk yield (kg).

The lines above each other are from the unknown parent group models ssGBLUP$_{236UPG}$ (black square) and ssGBLUP$_{8UPG}$ (green triangle) and from the metafounders models ssGBLUP$_{\Phi}$ (blue diamond) and ssGBLUP$_{\PhiMAF}$ (red cross).
Table 6. Gamma matrix created using base population allele frequencies calculated from Red Dairy Cattle (RDC) and Holstein (HOL) cattle genotypes.

<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>RDC&lt;1970</td>
<td>0.825</td>
<td>0.613</td>
<td>0.602</td>
<td>0.604</td>
<td>0.604</td>
<td>0.603</td>
<td>0.536</td>
<td>0.521</td>
<td>0.533</td>
<td>0.524</td>
<td>0.516</td>
<td>0.515</td>
</tr>
<tr>
<td>RDC 1971–1980</td>
<td>0.638</td>
<td>0.629</td>
<td>0.629</td>
<td>0.627</td>
<td>0.622</td>
<td>0.539</td>
<td>0.521</td>
<td>0.539</td>
<td>0.526</td>
<td>0.516</td>
<td>0.515</td>
<td>0.512</td>
</tr>
<tr>
<td>RDC 1981–1990</td>
<td>0.665</td>
<td>0.665</td>
<td>0.657</td>
<td>0.648</td>
<td>0.543</td>
<td>0.520</td>
<td>0.538</td>
<td>0.525</td>
<td>0.515</td>
<td>0.514</td>
<td>0.512</td>
<td></td>
</tr>
<tr>
<td>RDC 1991–2000</td>
<td>0.670</td>
<td>0.664</td>
<td>0.654</td>
<td>0.543</td>
<td>0.520</td>
<td>0.538</td>
<td>0.525</td>
<td>0.516</td>
<td>0.515</td>
<td>0.512</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RDC 2001–2010</td>
<td>0.676</td>
<td>0.668</td>
<td>0.542</td>
<td>0.520</td>
<td>0.538</td>
<td>0.525</td>
<td>0.515</td>
<td>0.515</td>
<td>0.512</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RDC 2011–2016</td>
<td>0.666</td>
<td>0.547</td>
<td>0.521</td>
<td>0.539</td>
<td>0.526</td>
<td>0.517</td>
<td>0.516</td>
<td>0.514</td>
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<td></td>
</tr>
<tr>
<td>OTHER&lt;1970</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.813</td>
<td>0.511</td>
<td>0.525</td>
<td>0.518</td>
<td>0.509</td>
<td>0.507</td>
</tr>
<tr>
<td>HOL 1970–1980</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.581</td>
<td>0.559</td>
<td>0.579</td>
<td>0.586</td>
<td>0.587</td>
</tr>
<tr>
<td>HOL 1981–1990</td>
<td></td>
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<td></td>
<td>0.574</td>
<td>0.567</td>
<td>0.562</td>
<td>0.561</td>
<td>0.560</td>
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<tr>
<td>HOL 1991–2000</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>0.595</td>
<td>0.594</td>
<td>0.595</td>
<td>0.598</td>
<td></td>
</tr>
<tr>
<td>HOL 2001–2010</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.613</td>
<td>0.615</td>
<td>0.621</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HOL 2011–2016</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.628</td>
<td>0.638</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*RDC and HOL cattle have been divided into metafounders by birth year, while the other breeds (OTHER) have been combined into one metafounder.*