Residential solar power profitability with thermal energy storage and carbon-corrected electricity prices

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A R T I C L E I N F O

Keywords:
- Residential solar PV
- Load control
- Social cost of carbon

A B S T R A C T

We study the economic profitability of residential solar photovoltaic (PV) systems in Finland. We show a moderate rate of returns (1.0% in Northern and 1.4% in Southern Finland) for the PV system investments with time-of-use hot water heating. Optimized hot water heating increases the rate of return by 0.6 percentage points. We internalize the negative externalities of greenhouse gas emissions from electricity generation by presenting the hourly electricity prices as a function of emission permit costs. A 10 €/tCO₂ increase in carbon price improves the PV investment rate of return by 0.3 percentage points.

1. Introduction

The EU aims to place consumers at the heart of the modern energy markets with distributed electricity generation and demand response. The residential solar photovoltaic (PV) system is one of the key technologies to empower consumers and make them more active market participants. Countries with poor solar radiation conditions have adopted various support mechanisms in order to increase the adoption rate of residential PV (La Monaca and Ryan, 2017). However, the subsidy expenditure of an additional installed capacity may become more expensive when the PV technology becomes less expensive (Williams et al., 2020). A higher solar value implies that the subsidy is allocated increasingly to customers who would have invested in solar anyway, without a support policy in place.

Finland has set an ambitious target of reaching carbon neutrality by 2035 (Finnish Government, 2019). Transition to carbon neutral electricity generation and heating of buildings play a key role in achieving the target, despite the already high amount of renewable electricity generation. The importance of nuclear, wind and solar power is emphasized in this transition, in which all technologies have differing purposes. For instance, it is forecasted that solar power output is approximately 2 700 GWh by 2030 and over 13 000 GWh by 2050 in Finland. This would represent a huge growth, as solar power output is approximately 2700 GWh by 2030 and over 13 000 GWh by 2050 in Finland. This would represent a huge growth, as solar power output is approximately 2700 GWh by 2030 and over 13 000 GWh by 2050 in Finland. According to the Finnish long-term renovation strategy (Finnish Government, 2020; Kangas et al., 2020), which is a part of the Energy Performance of Buildings Directive, a large share (29%) of this future solar power capacity is expected to be installed in residential detached houses.

Fortunately, solar power is becoming an economically viable option without subsidies also in Northern Europe due to three trends: decreasing installation costs of PV technology (IRENA, 2019), increasing price of emission allowances (World Bank Group, 2019) and the adoption of load control devices in residential buildings (O’Shaughnessy et al., 2018). To provide further insights on the abovementioned matters, this study addresses the following research questions using Finland as a case study of a country with poor solar irradiance conditions:

- What is the economic profitability of residential PV in Finland?
- How much does the profitability improve by utilizing the demand response potential of a residential hot water heater?
- How much does the profitability increase if the social costs of carbon (SCC) are internalized in the electricity prices?

The profitability of PV investments is calculated for the Northern, Central and Southern locations in Finland to account for differences in the solar irradiance conditions. Net present value (NPV) and internal rate of return (IRR) are used to assess the economic viability of the small-scale PV investment with time-of-use (ToU) and optimized hot water heating strategies. In addition, the differences between levelized
cost of electricity (LCOE) and NPV metrics in the investment profit-
ability estimation are quantified and discussed. Finally, the effect of
emission allowance price on PV investment profitability is quantified
in this article.

This study contributes to the existing literature by adding knowl-
edge on the profitability of residential PV systems in a country with
low level of solar irradiance, no direct subsidy mechanisms, and low
correlation between PV output and own electricity consumption. Our
study adds understanding on how thermal storage and carbon pricing
affect the profitability of PV investment. In contrast to many previous
studies of this topic, our focus is not on the evaluation of solar power
support mechanisms. As solar power business can support achieving
the renewable energy and carbon neutrality targets, and diversify the
renewable energy portfolio also in northern locations, there is a clear
need at household level for an objective evaluation on the profitability
of PV investments.

The remaining sections of this paper are organized as follows.
Section 2 presents relevant literature. Section 3 provides the data and
market descriptions. In Section 4, the estimation of carbon-corrected
electricity prices is explained. Section 5 presents the optimization
model. The results are presented and discussed in Section 6. Section 7
concludes the paper with policy implications.

2. Literature

A large body of literature has investigated the profitability of PV
investments as well as the suitability of policy instruments to boost the
adoption (see, e.g., La Monaca and Ryan (2017), Bertsch et al.
(2017), Simola et al. (2018), Hirvonen et al. (2015), Mondol et al.
(2009) and Koskela et al. (2019).

Studies suggest that the upfront investment costs, the unfavourable
insolation conditions, the low conversion efficiency of solar cells and the
mismatches between electricity production and consumption may prevent
the investment decisions (La Monaca and Ryan, 2017; Bertsch et al.,
2017; Simola et al., 2018; Mondol et al., 2009; Koskela et al.,
2019). Equally important are the availability of policy support mech-
nisms, such as feed-in tariffs, and electricity retail prices in deciding
whether or not to invest in solar power (Bertsch et al., 2017; Hirvonen
et al., 2015; Crago and Chernyakhovskiy, 2017).

In recent years, PV investments have also received more attention in
countries with lower levels of solar irradiance because of decreasing
investment costs (La Monaca and Ryan, 2017). The lowest solar
irradiance conditions are found in Northern European countries, such as
Denmark, Latvia, Ireland, Estonia, Sweden and Finland (Onradscek
et al., 2015). Even within the Northern European countries, the solar
irradiance conditions differ quite significantly. The differences arise
mainly from geographical locations, as the amount of solar irradiance is
lower in the most northern locations. Other reasons for locally different
irradiance conditions are related to the cloudiness and other shadowing
obstacles, such as buildings or trees. Investments become increasingly
less profitable as we go north (Simola et al., 2018). In particular, the
mismatch between solar power generation and residential electricity
consumption affects the viability of PV investment (Simola et al., 2018;
Hirvonen et al., 2015; Koskela et al., 2019).²

To address the prior profitability challenges, the flexibility of the
electricity consumption can be increased with energy storage. En-
ergy storage enables storing the surplus solar power generation for
times of low generation, which improves the value of distributed
PV (O’Shaughnessy et al., 2018). While the price of the traditional
battery storage remains high, utilizing an electric hot water heater
(EHWH) as a thermal energy storage does not require major invest-
ments. Self-consumption can be increased with an EHWH-assisted PV
system (Salpakari and Lund, 2016) and the load-use of EHWH can eas-
ily be changed without significant loss of comfort (Vanhournout et al.,
2012). Most of the existing PV-storage research focuses on the electrical
battery-assisted solar systems (see, e.g., Parra and Patel (2016), Vieira
et al. (2017) and Schopfer et al. (2018), whereas the EHWH-assisted
systems have gained less attention (O’Shaughnessy et al., 2018).

Different kinds of subsidy mechanisms have been implemented and
studied to improve the relative competitiveness of PV systems
(for reviews of these, see, e.g., Bertsch et al. (2017), Hirvonen et al.
(2015) and Polzin et al. (2019). Few studies exist where the relative
competitiveness of renewable energy is improved by internalizing the
damages caused by the greenhouse gas emissions in electricity prices
(Gavard, 2016; Best and Burke, 2018). Generally, carbon pricing has
been shown to increase investments in solar and wind (Abolhosseini
and Heshmati, 2014; Best and Burke, 2018). However, the level of the
carbon price should be high enough to provide proper incentives to
make the investment (Gavard, 2016).

3. Data and market description

Solar power penetration is fairly low in Finland. The total installed
PV capacity was 133.5 MW by the end of 2018 (Ahola, 2018). However,
the solar PV capacity has been increasing rapidly,³ as the installed
PV capacity by the end of 2016 was only 27 MW (Ahola, 2016).
On average, the solar irradiance on an optimally inclined plane in
conditions in Finland is one of the lowest in Europe (Martins, 2017).
There are no direct subsidies, such as feed-in tariffs, paid for solar
power in Finland. The generation fed back into the power grid is
compensated by the hourly day-ahead spot price less a margin collected
by the customer’s electricity provider.

The framework for solar PV profitability modelling is shown in
Fig. 1. In Sections 3 and 4, we describe the data that is input to the
simulation model presented in Section 5.

3.1. Electricity pricing

The electricity bill of a Finnish electricity consumer consists of
three components: energy fee in the day-ahead market (DAM) \( p_D^DAM \),
transmission and distribution (T&D) fee \( p_T&D \) and electricity tax \( t_e \).
All the components are subject to a value added tax (VAT). To simplify,
the energy fee \( p_D^DAM \) is determined for each hour of year \( t \) in the forward
auction market Elspot in the Nordic power market. The total cost of
electricity for the end-user is:

\[
p_t = (p_D^DAM + t_e + p_T&D)(1 + t_{VAT}).
\]

In year 2016, the value added tax was 24% \( (t_{VAT} = 0.24) \), the
electricity tax \( t_e \) (incl. VAT) was 27.94 €/MWh and the transmission
and distribution fee \( p_T&D \) (incl. VAT) was 36.41 €/MWh (Finnish
Energy Authority, 2017). The mean hourly electricity price (incl. VAT)
was 40.24 €/MWh, with a standard deviation of 16.31 €/MWh (Nord
Pool Spot, 2020).

² Mismatch arises from the high need for space heating in winter, whereas the cooling demand is lower than in more southern locations. For example, total hourly electricity consumption in Finland may exceed 15 000 MWh in coldest winter hours, while the lowest consumption of just over 6000 MWh are achieved in the summertime (Finnish Energy, 2020). On the other hand, the period of high solar power output in the summer is much shorter than in the southern locations (for examples one can refer to the open database in Photovoltaic Geographical Information System (2020).
Table 1
Heating degree days of the estimated space heating in Southern (Helsinki), Central (Jyväskylä) and Northern (Sodankylä) Finland.

<table>
<thead>
<tr>
<th>Location</th>
<th>Finland average</th>
<th>Southern</th>
<th>Central</th>
<th>Northern</th>
</tr>
</thead>
<tbody>
<tr>
<td>HDDs</td>
<td>4 698</td>
<td>3 878</td>
<td>4 832</td>
<td>6 180</td>
</tr>
<tr>
<td>Heating coefficient</td>
<td>1.000</td>
<td>0.825</td>
<td>1.029</td>
<td>1.315</td>
</tr>
<tr>
<td>Space heating (kWh)</td>
<td>15 950</td>
<td>13 159</td>
<td>16 412</td>
<td>20 975</td>
</tr>
</tbody>
</table>

3.2. Space heating and other electricity consumption

The investment profitability analysis in this study is conducted in three regions representing the meteorological heterogeneity in Finland. Solar power profitability is quantified in Southern (Helsinki), Central (Jyväskylä) and Northern (Sodankylä) Finland (see Fig. 2). Helsinki is the capital city of Finland, with over 1.55 million people living in the metropolitan area of greater Helsinki. Jyväskylä and Sodankylä are more sparsely populated municipalities with 141,000 and 8,400 residents (Statistics Finland, 2019b).

In this study, the electricity consumption of a representative household is modelled. An electric heated detached house with a floor area of 145 m² is considered, which reflects the average size of new detached house in Finland in 2010 (Statistics Finland, 2019a). It is assumed that two adults and two children live in the house. Total electricity consumption of the household is divided into three different components: space heating, household water heating and other electricity consumption such as use of home appliances. The electricity consumption profile is based on a type load consumption profile that shows the load for each hour of the day each month, distinguishing between weekdays and weekends (Finlex Data Bank, 2009).

The representative household’s space heating consumption profile is scaled according to the heating degree days (HDD) in Southern, Central and Northern Finland (Finnish Meteorological Institute, 2010). Table 1 shows that the selected locations depict well the meteorological heterogeneity in Finland. Jyväskylä (Central) represents the average number of HDDs in Finland. The number of HDDs in Sodankylä (Northern) is 32% higher than the Finnish average, while in Helsinki (Southern) it is 17% lower. Based on Motiva (2011), we use 110 kWh/m² as a representative level of space heating consumption in Finland, so the estimated average annual space heating consumption is 15 950 kWh. Based on the HDD heating coefficients, the space heating consumption is scaled for the locations in Northern, Central and Southern Finland.
We assume that a representative household has a 3 kW electric hot water heater with a volume of 290 l.\(^4\) Thus, the maximum hourly heating power is \(E = x \times 3\) kWh, and the maximum energy storage capacity is given by
\[
E = (c_p \times m \times dT) \times (1/3600) = 21.15\text{ kWh},
\]
where \(c_p = 4.2\) kJ/(kg°C) is the specific heat of water, \(m = 290\) kg and (1/3600) is the conversion rate from kJ to kWh. The input water temperature is assumed to be 5 °C, and it is heated to 67.5 °C, so that \(dT = 62.5^\circ\) C is the required water temperature increase inside the EWH.

Each individual is assumed to consume 50 l of hot water per day.\(^5\) As a result, the sum of daily energy needed to heat 200 l of water is 11.67 kWh. Given the fixed daily consumption, the representative household uses 4 260 kWh of electricity annually for water heating. Electricity consumption related to household water heating is the same in all regions as we assume that the inlet water temperature is constant at 5 °C. The hourly hot water consumption profile is drawn from the hot water profile generator DHWscale (Jordan and Vajen, 2011).

Other electricity consumption is calculated as a residual after space and household water heating are deducted from the estimated total consumption. According to the information in Statistics Finland (2017), on average, 68% of total electricity consumption in Finnish electric-powered detached houses is related to space heating. Thus, hot water heating represents 18.2% of the total average consumption. The residual consumption is 13.8% of the total electricity consumption of a household, which translates to an annual consumption of 3246 kWh.

The estimated total consumption (space heating, water heating, other consumption) for the representative household is thus 20 665 kWh in Southern Finland, 23 936 kWh in Central Finland and 28 481 kWh in Northern Finland. In these benchmark cases, the need for space heating is modelled for a conventional Finnish house, built under normal building guidelines.

### 3.3. Solar power

A parametric solar power output model, described in detail by Böök et al. (2020), is utilized for estimating the solar power output at each specific location. The input datasets consisted of five years (2013–2017) of hourly meteorological observations of local 2-metre (above ground level; AGL) air temperature, 10-metre (AGL) wind speed, global horizontal irradiance (GHI), and diffuse horizontal irradiance (DHI). In addition, direct normal irradiance (DNI), calculated from GHI and DHI, is utilized. Due to the volatile nature of calculated DNI, a quality control (QC) method, extensively documented by Böök et al. (2020), based on several years of observed DNI values from three separate locations in Finland, is implemented for the calculated DNI values:
\[
DN_{IQC} = -851e^{-0.109n_a} + 949, \text{[W/m}^2]\)
\[
DN_{LCIQC} = \min(DN_{IQC} \text{calculated}, DN_{IQClimt}).
\]

where \(DN_{IQClimt}\) is the maximum allowed limit for DNI, \(DN_{IQC}\) is the value chosen by the QC and \(n_a\) is the solar elevation angle in degrees. It was also assumed that no direct solar irradiance is present with solar elevation angles equal or below 0.5 degrees. This elevation angle can, however, be considerably larger depending on the horizon of each specific PV site.

The modelled PV systems were southward-oriented C-Si panels. Optimal slope angles, defined or extrapolated from Photovoltaic Geographical Information System (2020) data, were used for each location.

<table>
<thead>
<tr>
<th>Location</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Slope angle</th>
<th>Missing hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Southern</td>
<td>67.3666</td>
<td>26.6290</td>
<td>49</td>
<td>78</td>
</tr>
<tr>
<td>Central</td>
<td>62.3976</td>
<td>25.6709</td>
<td>42</td>
<td>200</td>
</tr>
<tr>
<td>Northern</td>
<td>60.3267</td>
<td>24.9568</td>
<td>39</td>
<td>30</td>
</tr>
</tbody>
</table>

This information, together with missing modelled solar power output hours, caused by gaps in input data, are listed in Table 2. The missing solar power output values account for less than 1% of all daytime (\(a > 0\)) data.\(^6\)

We consider two different sizes of PV installations: a 10-panel system with a 2.70 kW peak power capacity and a 18-panel system with a 4.68 kW peak power capacity. Table 3 shows the annual solar power output in Southern, Central and Northern Finland (see the map in Fig. 2). The capacity factor of PV in Southern Finland is 12.5%, whereas in the northern locations, the capacity factor decreases to 11.1% in Central Finland and to 10.2% in Northern Finland. When all of the solar power output can be used for its own consumption, the 2.70 (4.86) kWp system meets 22.2% (40.0%) of the household’s electricity consumption in Southern Finland and 13.3% (24.0%) of that in Northern Finland.

The unit investment cost is set to be 1 925 €/kWp for the smaller system and 1 568 €/kWp for the larger system. The costs are based on a winning bid value of a public tender in Finland. These cost estimates are in line with the investment cost estimates of 1 300–2 000 €/kWp in 2016 for installed system sizes below 10 kWp (Finsolar, 2016). They are also in the higher end of the cost estimates of 1 050–1 610 €/kWp for 5–10 kWp systems in 2018 (Ahola, 2018).

The expected lifetime levelized costs of PV electricity (LCOEs) in three locations in Finland for two system sizes are shown in Fig. 3. LCOE is calculated as:
\[
LCOE = \sum_{i=1}^{A} \frac{I_i + IC_s}{1 + r^i} \sum_{a=1}^{1} \frac{1}{(1+r)^a}
\]
where \(I_i\) is the investment cost, \(IC_s\) is the cost of the inverter change, \(AE_s\) is the annual electricity generation, \(r\) is the discount rate and \(A\) is the panel lifetime. We set the inverter change cost \(IC_s\) as 10% of the system investment cost in year \(a = 12\), the discount rate \(r = 3\%\) and the panel lifetime \(A = 25\) years.

To assess the economic rationality of the PV investment, LCOE estimates can be compared to the average cost of electricity from the grid (10.45 cent/kWh in year 2016). Fig. 3 shows that the 2.70 kWp system in Southern Finland is in parity with the electricity from the grid, but the investments in PV in Central and Northern Finland cannot be justified with the 3% discount rate. Given the lower unit investment cost of the larger 4.86 kWp system, the LCOE values are lower than those with 2.70 kWp system. Investment in Southern Finland seems profitable, but LCOEs in Central and Northern Finland are still above the average cost of electricity from the grid.

The profitability analysis based on comparing the LCOE values to the average electricity cost bought from the grid relies on two

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\(^4\) For example, Finnish hot water heater manufacturer Jäipi recommends a 220–300-litre water heater for a household with four residents.

\(^5\) Average water consumption is 125 l per day per person, and approximately 40% of this is hot water (Keskisuomen Energiajohtoinisto, 2015).

\(^6\) Based on the solar power output data, we fit PV output probability distributions for each combination of hour-of-day index (1, . . . , 24) and month index (1, . . . , 12) (see Section 5). Thus, despite the missing data points, we are able to draw representative solar power output probability distributions for each location.
important assumptions. First, the household pays a fixed price for the electricity that it buys from the grid. Second, the household is able to use all of the solar power output for its own consumption; i.e., electricity is never sold back to the grid. This approach does not, however, account for the hourly changing electricity prices and the possibility to have excess solar power output. The first assumption is valid, because the majority of households have a fixed rate electricity contract. However, to assess the market based potential of PV investment, the timing of solar power output matters as the value of electricity varies over the year. The second assumption applies when the size of the PV system is small. In principle, small sizing leads to high unit investment costs and thus the optimally sized PV system may generate excess electricity especially during the summer time in Finland.

We focus on the market based profitability of the PV investment (see Section 6) in this study. We take into account the timing of solar power output and the possibility of selling the excess output to the grid. The profitability of PV investment is calculated under passive ToU hot water heating and under optimized electric hot water heating.

4. Greenhouse gas emissions in electricity generation

Different kinds of subsidy mechanisms have been implemented to improve the relative competitiveness of solar power systems compared to generation technologies using fossil fuels. However, supporting policies can be costly to governments (La Monaca and Ryan, 2017), and subsidy payments can reward customers who would have purchased the technology without the subsidy (Williams et al., 2020).

Alternatively, the relative competitiveness of solar power could be improved by internalizing the pollution damages of CO$_2$ emissions in the power system. The emission price corrects the negative externality of CO$_2$ emissions by directly increasing the price of electricity that is generated by CO$_2$-intensive power plants. The value of solar power increases when it replaces more expensive electricity from the grid. Additionally, the emission pricing mechanism does not require direct subsidies from the government. Given these favourable features, we focus on the emission price perspective in this article. We estimate carbon-corrected electricity prices using historical data and run simulations over different emission permit price scenarios.

The CO$_2$ emissions can be priced directly by a carbon-tax or indirectly by a cap-and-trade system (Weitzman, 1974). Climate policy in the European Union is based on the cap-and-trade mechanism, and the power generation sector operates under the European Union Emissions Trading System (EU ETS) (Salant, 2016). Under the EU ETS, the regulated companies have to surrender EUAs per ton of CO$_2$ emitted. These companies receive or buy EUAs, and the allowances can be traded.

The EUA price remained below 10 €/tCO$_2$ during the beginning of the third trading period (2013–2018) but has increased$^7$ to 20–25 €/tCO$_2$ in 2018 (World Bank Group, 2019).

One way to assess the carbon price is the social cost of carbon (SCC), which describes the monetized damage caused by one additional unit of CO$_2$ to the atmosphere (van den Bijgaart et al., 2016). Essentially, defining the carbon price according to the SCC provides the correct economic incentive for reducing current CO$_2$ emissions. The SCCs are conventionally obtained with integrated assessment models$^8$ (IAMs) (Nordhaus (2014, 2017). A meta-analysis of SCC studies shows a mean value of 54.70 €/tCO$_2$ for the SCC (Wang et al., 2019). In this article, we assess emission price scenarios of 25 €/tCO$_2$ and 50 €/tCO$_2$, which are in line with the EUA price dynamics and the estimated SCC values.

4.1. Estimation of marginal emission factors

Consider that the bid prices by electricity producers are equal to their short-run marginal cost of production (MC), which is determined by the efficiency of the power plant ($\eta$), price of fuel ($p_f$), emission permit price ($\sigma$), the fuel emission factor ($e_f$) and the operation and maintenance costs ($oc$). The marginal production cost function can be written as:

$$MC = \frac{p_f + \sigma e_f}{\eta} + oc.$$  (6)

In other words, the emission costs of a power plant depend on the conversion efficiency of primary energy into electricity, the fuel emission factor and the price of the emissions. Hypothetically, if the carbon price $\sigma$ was increased, marginal production costs would rise, leading to higher bid prices and electricity prices.

CO$_2$ emissions are generated in the process of burning fossil fuels for electricity production. The average emission factors (AEFs) of electricity generation represent the average amount of CO$_2$ emissions$^9$ per produced amount of electricity. The AEFs imply that all produced (and consumed) units of electricity contribute similarly to the amount of total emissions. Utilization of the AEFs in describing the average contribution of consumption on CO$_2$ emissions is well justified because

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$^7$ A market stability reserve began operating in 2019, and a surplus of emission allowances is transferred to the reserve.

$^8$ IAMs are often criticized for their imperfections of damage functions, the handling of catastrophic events and the results’ sensitivity to the choices of the key parameter values (see, e.g., van den Bijgaart et al. (2016), Pindyck (2019) and Weitzman (2011).

$^9$ The emissions include the part of fuel usage that can be allocated to electricity generation in combined heat and power plants.
it may be difficult to allocate certain marginal emissions to the different consumption units. However, applying the AEF to calculate the change in total emissions when deviating from an existing equilibrium may underestimate the effect of marginal consumption on total emissions. Therefore, one could alternatively consider using marginal emission factors (MEFs) in estimating the change in total emissions arising from a unit change in consumption. To examine the changes in emissions from distributed solar power generation and shifting electricity consumption (load shifting), we estimate the MEFs of the Finnish electricity production in 2016.

The following approach for estimating the MEFs has been proposed earlier, for instance, in Graff Zivin et al. (2014) and Holladay and LaRiviere (2017). It is assumed that the load shifting affects the electricity generation in Finland. In other words, we assume that the imported generation is not at the margin and is still consumed as usual. Based on Fig. 4, it is expected that the marginal emissions per produced MWh of electricity are increasing with the production level.

We aim to capture this nonlinearity by including the second-degree polynomial of the production level. Week indicators (Week indicator = 1 for w = 1, …, 51, and 0 otherwise for all t) are included to capture the seasonal time-dependence of production. The model is written as:

\[ E_m = \beta_0 + \beta_1 \text{Production}_t + \beta_2 \text{Production}_t^2 + \sum_{w=1}^{51} \theta_w \text{Week}_w + \epsilon_t, \]  

where \( t = (1, \ldots, T) \) are the hours in the sample year, \( E_m \) is the amount of \( CO_2 \) emissions from electricity generation in Finland in hour \( t \), \( \text{Production}_t \) is the amount of generated electricity in Finland at hour \( t \) and \( \epsilon_t \) is a normally distributed error term.

Based on the Augmented Dickey–Fuller test we reject the null hypothesis of unit roots in the emissions and production series at 1% significance level. The adjusted \( R^2 \) of Model 2 is 0.928. Newey–West type of heteroskedasticity and autocorrelation consistent standard errors (Newey and West, 1987) with 24-hour lag \( 1 \) are shown in the parentheses (Table 4). Marginal emission factors are calculated from the estimated equation as:

\[ \frac{\partial E_m}{\partial \text{Production}_t} = MEF = \beta_1 + 2\beta_2 \text{Production}_t. \]  

The average of hourly AEFs is 0.106 t\( CO_2 \)/MWh with a standard deviation of 0.046 t\( CO_2 \)/MWh. The minimum and maximum are 0.023 and 0.219 t\( CO_2 \)/MWh, respectively. The average, standard deviation, minimum and maximum of the estimated MEFs are 0.238, 0.027, 0.175 and 0.219 t\( CO_2 \)/MWh, respectively. The estimated MEFs are utilized in determining the cost of emissions at the cross-section of domestic production and consumption in Section 4.2.

4.2. The share of carbon prices in the pricing of electricity

The share of costs related to emission permits are calculated from the Finnish area prices in 2016. The estimated MEFs are used in estimating the amount of emissions per unit of electricity around the realized equilibrium. The emissions were converted into emissions permit costs \( EC_j (€/MWh) \) by multiplying them with the average emissions permit price \( e \) of 5.14 €/t\( CO_2 \) in year 2016 and then dividing with the amount of shifted load (MWh). The historical hourly day-ahead market (DAM) price \( p_{DAM} \) is then separated into non-carbon price-related parts and carbon price-related parts as:

\[ p_{DAM} = p_{non-carbon} + EC_j = p_{non-carbon} + \sigma \cdot MEF_j. \]  

We model the emission price scenarios \( \sigma = (25.0, 50.0) €/tCO_2 \) by calculating the new day-ahead market prices according to Eq. (9). These emission price levels are chosen such that they follow the recent EUA price development and the mean value of SCC estimates.

5. Optimization model

The aim of the model is to minimize a household’s total annual net electricity costs. Net cost consists of two components. First, a household pays price \( p \) for each energy unit consumed. As is shown in Eq. (1), the energy price is the sum of the hourly day-ahead market price, the transmission and distribution fee and the energy tax, all subject to VAT. Second, a household receives revenue from each unit of excess solar power output sold to the grid. The household receives the hourly day-ahead market price \( p'_{DAM} \) less the electricity retailer’s margin \( \mu \) of its excess output sold to the grid.

The optimal net cost minimization strategy thus aims to maximize the on-site PV use since own solar power output replaces more expensive energy from the grid: \( p'_{DAM} < p \). Additionally, the energy use from the grid should be scheduled such that the low-price hours are utilized as much as possible. The energy content of the hot water heater allows the use of the heater as a thermal buffer.

The optimization problem is formulated as a discrete-time model with time steps of one hour, \( t \in \{1, \ldots, T\} \), and a time frame of one year, \( T = 8760 \). Hourly hot water heating energy from the electricity grid \( x_t \) (kWh) and from own solar power output \( s_{DWH} \) (kWh) are chosen such that the sum of the hourly net electricity cost \( f_j \) is minimized over the annual period:

\[ \min_{x_t^{DWH}, s_{DWH}} \sum_{t=1}^{T} f(x_t, s_{DWH}), \quad \forall t = 1, \ldots, T. \]  

The model takes into account the hourly total cost of electricity \( p_1 (€/MWh) \), day-ahead market prices \( p'_{DAM} \), exogenously determined other electricity consumption \( C_j \) (kWh), hot water consumption \( h_j \)

---

10 Finland is a net importer from Sweden and Russia and net exporter to Estonia in the electricity exchange. In the Nordic power market, the electricity flows from a higher-priced to a lower-priced area until there is no price difference or the transmission lines become fully congested. As prices in Finland tend to be higher than in Sweden, even if the transmission lines are fully congested, our assumption seems justified. The same applies for Russia. However, as electricity is exported from Finland to Estonia, part of emissions from the Finnish production should not be allocated to the consumption in Finland since the final consumption occurs in Estonia. However, the possible bias is small, as the exports to Estonia represented only 4.6% of total generation in Finland in 2016.

11 This because the day-ahead market demand and supply bids are made for all 24 h in the next day. Similar choice is done, for instance, in Graff Zivin et al. (2014). The partial autocorrelation function of the error terms in Model 2 is presented in Appendix A.

---

Table 4

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CO(_2) emissions (tons)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(1)</td>
</tr>
<tr>
<td>Finnish production</td>
<td>0.202**</td>
<td>0.103*</td>
</tr>
<tr>
<td>(Finnish production)(^2)</td>
<td>0.005</td>
<td>0.009**</td>
</tr>
<tr>
<td>Constant</td>
<td>−1.003.654***</td>
<td>−588.321*</td>
</tr>
<tr>
<td>Week indicators</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>8,784</td>
<td>8,784</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.746</td>
<td>0.928</td>
</tr>
</tbody>
</table>

Note: Regressors in MWh. Newey–West standard errors with a 24-hour lag are reported in the parentheses.

* \( p < 0.1 \)  
** \( p < 0.05 \)  
*** \( p < 0.01 \)
The maximum hot water heater power \( x \) allocated to hot water heating is the second state variable where the EHWH heat energy content is the first state variable 
\[
\dot{x}_t \leq S_t, \quad \dot{S}_t = E_t - x_t - (\alpha + \beta E_t), \quad \forall t = 1, \ldots, T.
\]
Fig. 4. The relationship between consumption of domestic production and CO\(_2\) emissions.

Consequently, we draw \( 24 \times 12 = 288 \) combination of hour-of-day index \( (1, \ldots, 24) \) and month index \( (1, \ldots, 12) \). Probability distributions illustrated in Fig. 5 show that the uncertainty related to solar power distributions are discretized into \( N = 6 \) points. Probability distributions for the solar power output realization is low during the winter (January as an example) and high during the summer (June as an example). The solar power output data, uncertainty with respect to the hourly solar power output realizations are introduced in the model by fitting a solar power output distribution for each combination\(^{12}\) of hour-of-day index \( (1, \ldots, 24) \) and month index \( (1, \ldots, 12) \). Consequently, we draw \( 24 \times 12 = 288 \) probability distributions \( \phi_i \). The distributions are discretized into \( N = 6 \) points.

\[
\Phi = \{\phi_1, \phi_2, \ldots, \phi_N\}, \quad \phi_i \geq 0, \quad \sum_i \phi_i = 1.
\]

where thermal conductance is set to \( UA = 1.05 \) (W/K) and the temperature difference is \( \Delta T_{\text{env}} = 47.5 \) K.

The transition functions are the following:
\[
E_{t+1} = E_t - h_t - L(E_t) + x_t + \epsilon_t^\text{DWH},
\]
for the hot water heater energy content and
\[
P(S_{t+1} = S_t = \phi(\text{month}_{t+1}, \text{hour} - o f \text{day}_{t+1}, S_t), \quad i = 1, 2, \ldots, N.
\]
for the probability of the solar power output realization \( S_t \) in the next hour.

The amount of solar energy used for the hot water heating \( \epsilon_t^\text{DWH} \) defines the solar energy used for other consumption \( s_{t}^\text{own} \) and solar energy sold to grid \( s_{t}^\text{grid} \) as follows:

1. Solar power output used for other household’s own consumption is \( s_{t}^\text{own} = S_t - \epsilon_t^\text{DWH} \), when solar power can be used for the household’s own consumption other than for hot water heating \( C_t > s_{t}^\text{own} \).
2. Solar power output sold to the grid is \( s_{t}^\text{grid} = S_t - \epsilon_t^\text{DWH} - s_{t}^\text{own} \), when solar power output exceeds a household’s own consumption.

6. Results and discussion
In this section, the profitability of PV investment is quantified in Southern, Central and Northern Finland. Two PV size options (2.70 kWp and 4.86 kWp) and two water heating cases (Time-of-Use and optimization based on hourly day-ahead market prices) are considered. In Section 6.1 we show that the optimized water heating allocation differs from the benchmark nighttime heating allocation. The PV investment
6.1. ToU and spot price optimizing hot water heating profiles

The average daily water consumption, solar power output and water heating profiles are presented in Fig. 6. A low-irradiance month (January) is presented in the graphs on the left, and a high-irradiance month (June) is presented in the graphs on the right. The optimized water heating profile (solid, blue) corresponds to the ToU heating profile (dashed, black) to some extent, as less expensive nighttime hours are mainly used also in the water heating optimization. However, instead of a constant profile used in the ToU heating, optimized heating profile utilizes certain nighttime hours more intensively. Importantly, a fraction of the daily solar power output is used for water heating (dotted, red) during the summer months that reduces the amount of electricity bought from the grid.

The heating profiles in Fig. 6 imply that the ToU heating is not necessarily the cost minimizing strategy. Table 5 shows that the correlation between the two heating profiles decreases from Northern to Southern Finland and with larger PV system sizing. The reason is that the optimal strategy is to maximize the on-site solar power use (see Section 5). Consequently, as the solar power output potential increases (larger size and/or southern irradiance conditions), part of the solar power output is allocated to hot water heater during the daytime, although it uses some of the energy storage potential available during the nighttime hours when water heating is less expensive. These results indicate that the passive night-heating strategy diverges more from the cost-minimizing solution the larger the output potential of the household PV investment becomes.

6.2. The PV investment profitability under ToU and optimized hot water heating strategies

Solar power output varies among the three latitudes in Finland (see Fig. 2 and Table 3). The expected annual solar power output potential decreases from Southern (1093 kWh/kWp) to Northern (893 kWh/kWp) Finland. Simultaneously, heating demand increases at higher latitudes, meaning that the total electricity consumption in electric heated buildings is highest in Northern Finland and lowest in Southern Finland (see Table 1). As a result, it is expected that the monetary gains from hot water heating optimization vary between the locations, as its main benefit is to reduce the amount of solar power output sold to the power grid.

Hot water heating optimization generates certain benefits when compared to the ToU heating (see Table 6). Without optimization, the share of solar power output sold to the grid varies from 11.8–26.8% (30.8–47.5%) with the smaller (larger) size option. First, optimization lowers the amount of solar power output sold to the power grid. The decrease is 9 (10) percentage points in Southern Finland with a smaller (larger) size option, while the corresponding values are 4.5 and 7.3 percentage points in Northern Finland. Second, the revenue received from the solar power output sold to the grid (€/MWh) is higher when water heating is optimized. That is, optimization enables the households to sell excess output to the grid in higher-priced hours.

### Table 5
Correlation between ToU and optimized hot water heating profiles.

<table>
<thead>
<tr>
<th>Size (kWp)</th>
<th>Southern</th>
<th>Central</th>
<th>Northern</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.70</td>
<td>0.504</td>
<td>0.508</td>
<td>0.513</td>
</tr>
<tr>
<td>4.86</td>
<td>0.492</td>
<td>0.498</td>
<td>0.510</td>
</tr>
</tbody>
</table>

### Table 6
Share of solar power output sold to grid and its average revenue under ToU and optimized hot water heating in Southern, Central and Northern Finland.

<table>
<thead>
<tr>
<th>Size (kWp)</th>
<th>Southern</th>
<th>Central</th>
<th>Northern</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar to grid (%)</td>
<td>26.8</td>
<td>17.8</td>
<td>18.9</td>
</tr>
<tr>
<td>Solar revenue (€/MWh)</td>
<td>3.42</td>
<td>3.48</td>
<td>3.50</td>
</tr>
<tr>
<td>4.86</td>
<td>Southern</td>
<td>Central</td>
<td>Northern</td>
</tr>
<tr>
<td>Solar to grid (%)</td>
<td>47.5</td>
<td>37.5</td>
<td>39.4</td>
</tr>
<tr>
<td>Solar revenue (€/MWh)</td>
<td>3.33</td>
<td>3.35</td>
<td>3.37</td>
</tr>
</tbody>
</table>
Table 7
Annual electricity cost savings (€/a) of the PV investment under ToU and optimized hot water heating in Southern, Central and Northern Finland.

<table>
<thead>
<tr>
<th></th>
<th>Southern</th>
<th>Central</th>
<th>Northern</th>
</tr>
</thead>
<tbody>
<tr>
<td>ToU</td>
<td>Optim</td>
<td>ToU</td>
<td>Optim</td>
</tr>
<tr>
<td>2.70 kWp</td>
<td>261.7</td>
<td>278.1</td>
<td>246.8</td>
</tr>
<tr>
<td>4.86 kWp</td>
<td>387.3</td>
<td>420.1</td>
<td>370.6</td>
</tr>
</tbody>
</table>

Fig. 6. Average daily hot water heating under ToU and optimized heating strategies.

The results presented in Table 6 imply that hot water heating optimization may improve the profitability of PV investments relative to ToU heating. The first evidence of this implication is given in Table 7, where the annual savings with PV investment are shown. Generally, savings are higher with a larger size option in the southern locations. On average, the annual savings are 4.3% and 6.8% lower in Central and Northern Finland than in Southern Finland, respectively. Heating optimization increases the annual savings by 16.4–32.8 € in the southern, 10.7–25.6 € in the central and 6.5–19.5 € in the northern locations (see Fig. 7). The larger PV capacity, the location in the south with better solar irradiance conditions and the lower electricity consumption highlight the benefits of water heating optimization. Consequently, the cost savings potential related to the optimal use of a hot water heater increases with the combination of larger annual solar power output and smaller electricity consumption.

The annual savings achieved with PV investment are not an adequate measure to assess the investment’s lifetime profitability. Therefore, we calculate net present values (NPVs) of the lifetime savings. Comparing the NPV of savings with the investment cost reveals whether or not the investment reaches the rate of return required by the investor. The lifetime $A$ of the system is assumed to be 25 years, and the discount rate $r$ is 3%, which is the required rate of return. The net present value is the sum of the discounted stream of yearly savings:

$$NPV = \sum_{a=1}^{A} \frac{R_a}{(1+r)^a}. \quad (17)$$

Fig. 8 shows that the investment is not profitable when $r = 3\%$. In other words, the NPV of savings is always lower than the investment cost even when hot water heating is optimized (the bars do not reach the investment cost). Gaps between the NPV of savings and investment cost can be inferred as required cost reduction in PV systems. For instance, the investment cost of a 2.70 kWp (4.86 kWp) system should be 814 € (1129 €) lower for the investment to break even in the southern location; on the other hand, heating optimization reduces these values to 516 € and 531 €, respectively. Cost reduction requirements are higher in the northern locations.

The NPVs calculated with an LCOE principle in Fig. 8 show that the 2.70 kWp system is profitable in Southern Finland and that the 4.86 kWp system is profitable in all locations (see the bars that reach the investment cost). The LCOE values therefore indicate a better PV investment profitability than the more detailed analysis based on hourly market conditions (see Section 3). Two key factors explain the difference. First, the LCOE calculation ignores the possibility of excess solar power output by assuming that each solar energy unit replaces an energy unit bought from the grid. At this point, it must be noted that the household receives the hourly spot price less the retailer margin from excess solar power output sold to the grid, whereas the household saves the energy price, transmission costs and taxes when solar power output can be utilized by itself. Second, the LCOE calculation does not take into account the hourly varying value of electricity replaced by the household’s own solar power output. Correlation between the solar power output and hourly prices is 0.13 in Southern, 0.12 in Central and 0.11 in Northern Finland. The low correlation indicates that the

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13 The inverter change cost is 10% of the investment cost. The inverter is changed in year $a = 12$, and the cost is included in the annual savings.

14 Annual savings are calculated assuming that no solar power output is sold to the grid and that the household pays a fixed electricity price.
actual value of the solar power output for the household is lower when measured with hourly prices than with an average price over the hours.

As pointed out, the required reduction in investment cost to reach a certain rate of return can be quantified with the NPV analysis. Alternatively, the profitability of an investment can be assessed based on the internal rate of return (IRR). The IRR measures the rate of return \( r \), under which the discounted sum of yearly savings \( R_i \) and investment cost \( I_0 \) are equal:

\[
\sum_{i=1}^{n} \frac{R_i}{(1 + r)^i} - I_0 = 0.
\]  

(18)

IRR provides a simple metric quantifying an investment’s profitability, as it is comparable with rates of returns of other investment possibilities available for households. The IRRs calculated with the NPVs of savings and investment costs are shown in Fig. 9. Generally, the IRRs are higher for the 4.86 kWp systems and when the households are located in Southern Finland. Moreover, water heating optimization improves the IRRs in each location. The highest IRR (2.1%) is achieved with the larger PV system in the southern location. Compared with the ToU heating (1.4%), the IRR is increased in this case by 0.7 percentage points. In Northern Finland, the rate of return of the 4.86 kWp system is 1.5% (1.0% with ToU heating).
The rate of returns in Fig. 9 indicate that the small-scale PV investments may not yet be economically profitable enough to reach rapid capacity growth in locations with low solar irradiance, such as Finland. Similar findings have been presented in the previous literature (see, e.g., Hirvonen et al. (2015); Koskela et al. (2019); La Monaca and Ryan (2017); Simola et al. (2018). For example, a study on residential prosumers in the European Energy Union (European Commission, 2017) assumes a required rate of return of 6.2% on investments. Referring to these traditional metrics, our estimated IRRs are far from being sufficient.

However, three recent trends may change this initial conclusion. First, a PV system is a relatively riskless investment, which could well be considered as a substitute for government bonds. For example, a Finnish 30-year bond yield has been below 0.5% since the summer of 2019. Second, households may put weight on other factors related to PV systems, such as emissions reductions (Crago and Chernyakhovskiy, 2017; Ruokamo et al., 2019). To assess this point, we analysed how solar power output reduces households’ CO₂ emissions of electricity consumption (see Fig. 10). The emissions reduction varies from 7.0% (north) to 12.0% (south) with the smaller size options, while the reductions are between 12.5% (north) and 21.5% (south) with the larger option.

Thirdly, the future electricity price level is difficult to forecast and creates uncertainty in estimating the investments’ profitability. For instance, the average emission allowance price σ in year 2016 was only 5.1 €/tCO₂, while recently, the price in the EU ETS has increased up to 20.0–25.0 €/tCO₂ (World Bank Group, 2019). The economic analysis of PV investment based on 2016 prices may thus undermine the profitability of PV investment because the higher emission allowance price increases the cost of grid electricity replaced by the household’s own solar power generation.

### 6.3. Carbon-corrected electricity prices and PV investment profitability

In this section, we study the profitability of PV investment in scenarios with higher emissions price levels of 25.0 €/tCO₂ and 50.0 €/tCO₂. A higher emission price increases electricity prices, ceteris paribus, and the correlation between the marginal emission factors and the hourly spot prices (Table 8). This second feature implies that the profitability of PV investment does not change one-to-one with the average electricity price level.

As discussed earlier, the timing of solar power output matters when conducting an analysis on markets with hourly resolution. The average correlation between the solar power output profiles and the hourly marginal emission factors is −0.14, meaning that solar power generation does not take place during hours with high emissions intensity (tCO₂/MWh) in the power system in Finland. This result is illustrated in Fig. 11, where the middle graph shows the electricity price difference arising from an increase in the emission allowance price from 5.1 €/tCO₂ to 50.0 €/tCO₂. Because the emissions increase with higher demand (see Fig. 4), electricity prices increase clearly during the winter months when the cold outdoor temperature drives up the electricity demand in Finland. Solar power output (North: top graph, South: bottom graph), on the other hand, takes place mostly during the spring, summer and autumn months.

Internal rates of return with a 50.0 €/tCO₂ carbon price are presented in Fig. 12. A higher carbon price improves the profitability in all locations, as expected. With ToU heating, the highest IRR (2.8%) is achieved with the larger PV system in Southern Finland, which corresponds to a 50% improvement in the profitability compared with the IRR with the original carbon prices. Moreover, water heating optimization further improves the IRR to 3.5%. Additionally, the 2.70 kWp system reaches the 3% rate of return with water heating optimization in Southern Finland. In Central and Northern Finland, the rates of return remain always below 3%.

Last, the sensitivity of IRR to emission allowance price level is illustrated with the 4.86 kWp system in Fig. 13. The results show a fairly linear relationship between the IRR and carbon price, where a 10 €/tCO₂ increase in carbon price improves the rate of return by 0.3% points with both water heating profiles.

![Fig. 9. Internal rate of return of savings with PV under ToU and optimized hot water heating.](image-url)

### Table 8

<table>
<thead>
<tr>
<th>CO₂ price (€/tCO₂)</th>
<th>5.1</th>
<th>25.0</th>
<th>50.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean(𝑝𝑡) (€/MWh)</td>
<td>32.45</td>
<td>37.18</td>
<td>43.14</td>
</tr>
<tr>
<td>Corr(𝑚𝑒𝑓, 𝑝𝑡)</td>
<td>0.37</td>
<td>0.41</td>
<td>0.45</td>
</tr>
</tbody>
</table>
7. Conclusions and policy implications

This article provides several interesting results on residential PV profitability in Finland in terms of the solar irradiance conditions close to the Arctic Circle. Using simulation, we obtain NPV, IRR and LCOE metrics for PV investments. In addition, emissions reductions are quantified, and the profitability is evaluated under carbon-corrected electricity prices.

The findings of this study imply that LCOE-based analysis can provide misleadingly high profitability for residential PV investments because the approach ignores the excess solar power output and the price variation of electricity. It is important to provide objective analysis on the economic potential of solar power in Finland. In order to better assess the market-based value of PV investment, households should have access to a PV investment profitability calculator, which would take into account the household’s own hourly electricity consumption profile, expected solar power output profile and the electricity price profile. The calculator should be maintained by a neutral party, so that the incentive for providing correct PV investment valuation is not distorted.

The mismatch between a household’s own electricity consumption and solar power output is one of the key barriers for profitability
in the Northern European countries. To mitigate the mismatch, we show that the optimization of hot water heater electricity consumption in response to a household’s own solar power generation moderately increases the profitability of PV investment. To enable the results of this study, updates in the Energy Performance of Buildings Directive by the European Union could be implemented so that a certain level of automation is required in residential buildings allowing for the solar power usage optimization. A new generation of smart metres should be designed so that the connection of devices (in this case, the EHWHs) to the internet would be possible with low costs. In other words, the characteristics of new building automation technologies could also enhance the profitability of PV investments.

Increase in variable renewable energy generation requires flexibility in other parts of power systems when maintaining power balance. Flexibility has typically been provided on the supply-side, but recently more attention has been paid also on the demand-side. As a result, residential consumers can be considered as a valuable source of flexibility providers in the future power systems. Residential solar power
generation, combined with automated consumption optimization, adds another layer to this discussion. Besides improving the economic profitability of solar PV investment, consumption optimization may have system-wide impacts on maintaining the power balance. From a policy perspective, it should be taken care that the private incentives are aligned with power system benefits with respect to consumption optimization. This issue is left for future research.

This study also demonstrates that there exist alternative ways to increase the profitability of residential PV investments on top of commonly used feed-in tariffs and investment subsidies. Internalizing the social cost of carbon into electricity prices would incentivize PV investments. We show that the profitability of PV investment in Northern Europe can rise by close to 3% if the emission allowance prices increase to 50 €/tCO₂. The pace of reduction of the emission allowances and the role of the Market Stability Reserve in the EU Emission Trading System are critical in this respect. A strengthened EU ETS policy serves as a clear driver for renewable energy investments.

Acknowledgements

Funding from the Academy of Finland Strategic Research Council, Finland project BCDC Energy (AKA292854), Academy of Finland project EcoRiver (323810), Academy of Finland project Regulation and dynamic pricing for energy systems (288957), Yrjö Jahnsson Foundation, Finland, Tauno Tönning Foundation, Finland and Kerttu Saalasti Foundation, Finland is gratefully acknowledged.

Appendix. Partial autocorrelation function

See Fig. A1.

Fig. A1. Partial autocorrelation function of the Model 2 error terms.

References


