There Are No Mathematical Explanations

Kuorikoski, Jaakko

2021-04-01

Kuorikoski , J 2021 , ' There Are No Mathematical Explanations ' , Philosophy of Science , vol. 88 , no. 2 , pp. 189-212 . https://doi.org/10.1086/711479

http://hdl.handle.net/10138/334303
https://doi.org/10.1086/711479

acceptedVersion

There Are No Mathematical Explanations

(Forthcoming in Philosophy of Science)
Penultimate draft, please do not cite this version

Jaakko Kuorikoski

University of Helsinki

Abstract:

If ontic dependence is the basis of explanation, there cannot be mathematical explanations. Accounting for the explanatory dependency between mathematical properties and empirical phenomena poses insurmountable metaphysical and epistemic difficulties, and the proposed amendments to the counterfactual theory of explanation invariably violate core commitments of the theory. Instead, mathematical explanations are either abstract mechanistic constitutive explanations or reconceptualizations of the *explanandum* phenomenon in which mathematics as such does not have an explanatory role. Explanation-like reasoning within mathematics, distinction between explanatory and non-explanatory proofs, and comparative judgements of mathematical depth can be fully accounted for by a concept of *formal understanding*.

Acknowledgements:

Early versions of this paper have been presented at EPSA 2015 in Düsseldorf, Stockholm Philosophy Colloquium (2017), Philosophy Research Seminar in Tampere (2017), and POS seminar in Helsinki (2018). I thank the audiences in these meetings for their comments. I would especially like to thank Juha Saatsi, Markus Pantsar, and two anonymous reviewers for their invaluable and constructive criticism and helpful suggestions.

1. Introduction

Are there distinctively mathematical explanations of empirical phenomena? No one denies that mathematics is explanatory and useful in most, perhaps all, fields of science, and the current philosophical literature on scientific explanations is replete with examples of apparently non-causal explanations referring to mathematical facts. But are there really cases in which it is the mathematical fact that explains something in the empirical world – *distinctly* mathematical explanations? How can such otherworldly things as mathematical objects, properties or facts explain something in our contingent, space-timey world?

In the following, I argue that they cannot – at least not *if* one accepts a broadly ontic counterfactual view of explanation. I argue that, despite recent heroic attempts to the contrary, such views cannot accommodate distinctively mathematical explanations. The main reason is that accounting for the explanatory dependency between mathematical properties and empirical phenomena supporting the explanatory counterfactuals poses insurmountable metaphysical and epistemic difficulties. I argue that the proposed amendments to the counterfactual theory, aimed at alleviating these difficulties, invariably end up violating core commitments of the ontic counterfactual account. My argument is therefore strictly conditional (please excuse the somewhat overreaching title), and the main conclusion is that the 'liberal consensus' admitting various non-causal explanations and a monist view of explanation are not as easily reconciled as many authors have taken them to be (cf. Reutlinger 2016; Povich 2018). Instead of explaining empirical phenomena with mathematical facts, I argue that the central examples discussed in the literature are either highly abstract mechanistic explanations or cases of pseudo-explanation in which the apparent increase in understanding provided by the putative mathematical explanation is either a matter of learning more about our representational tools or of correcting false explanatory presuppositions.

Although my primary focus is on putative distinctly mathematical explanations of empirical phenomena, skepticism towards explanatory abstract dependencies bleeds necessarily also into matters of explanation within mathematics. In order to alleviate worries that skepticism concerning mathematical dependencies would lead to an implausible picture of mathematical practice, I will concede that many applications of mathematics do involve explanation-like reasoning and develop a corresponding notion of *formal understanding.* The concept of formal understanding can account for the (at least practically) indispensable role for mathematics in explanation, differences in depth of mathematical understanding, the distinction between explanatory and non-explanatory proofs within mathematics, and comparative judgements of mathematical depth. In the latter part of this essay, I provide reasons for why we should nevertheless draw a sharp distinction between this formal understanding and explanation proper.

2. Mathematical explanation and abstract dependencies

According to a widely accepted view, explanation is about tracking systematic patterns of objective dependency relations (Pincock 20150; Povich 2018; Reutlinger 2016; Woodward 2003; Ylikoski and Kuorikoski 2010): explanations show what the thing to be explained objectively depends on. This dependency is most often understood in terms of counterfactuals: *explanandum* depends on the *explanans* iff, had the *explanans* been different, the *explanandum* would have been different as well. Explanatory information therefore has an essential modal quality: what sets explanatory information apart from purely ‘inert’ descriptive information is that explanations provide answers to what-if-things-had-been-different questions concerning counterfactual states of affairs.

An important aspect of this account of explanation is that the explanatory relationships are objective and ‘ontic’: explanatory relationships do not hold between representations, concepts or descriptions, but between the things in the world being represented, conceptualized and described. The counterfactuals supported by explanations are not about what should be *inferred* if we were to find out that such and such is the case, but about what the world itself would be like if such and such were the case. Holding an ontic dependency view does not mean holding an ontic view of explanation in the sense that explanations are to be directly *identified* with facts about dependencies (cf. Pincock 2018, 50). Instead, and I think more in line with Salmon’s characterization (Salmon 1998, 320), the ontic dependency view acknowledges that explanation as an activity is an epistemic practice we engage in, but insists that the explanatory information itself is *about* features of the world existing independently of these practices.

Although this ontic dependency view has been developed furthest in the case of causal explanation, many have recently proposed that the general idea is also applicable to various non-causal forms of explanation (e.g., Huneman 2010; Jansson and Saatsi 2019; Kuorikoski 2012; Rice 2015; Saatsi and Pexton 2013), including explanations that many take to be distinctively mathematical (Baron et al. 2017; Pincock 2015; Reutlinger 2016;

2018; Povich 2018). The philosophical literature includes many examples of putative distinctively mathematical explanations of empirical phenomena, and some of these will be analyzed in detail below, but let us start with the simplest one offered by Marc Lange. Why is it that a mother cannot divide 23 strawberries equally between her three children? The explanation for this puzzling failure is, of course, that 23 is not divisible by three. This seems like a perfectly good answer to an explanation-seeking why question. The answer also does not make references to any causes of the mother's failure, or any other empirical fact for that matter. There is nothing in the world preventing the division: it is simply something that cannot be done. Accordingly, Lange (2013, 2017) takes the distinguishing feature of mathematical explanations to be their modal strength: truly distinctively mathematical explanations show how the *explanandum* could *not* have been otherwise due to mathematical necessity (which is stronger than causal or nomological necessity). Not surprisingly, Lange originally (2013) opted for a *modal* conception of explanation (later expanded into an independent account of explanation by constraint in his [2017]), according to which explanations show how the *explanandum* was necessary, instead of an ontic dependence conception, which he explicitly denies as being applicable to such distinctively mathematical explanations. Such a modal conception of explanation has a respectable pedigree in its own right, having been proposed before, e.g., by G.H von Wright. Sam Baron (forthcoming) has put forward a theory of distinctively (or extra-mathematical) mathematical explanations based on the distinctively *epistemic* DN-model. In his *Deductive Mathematical* model, the central idea of subsumption by a deductive argument is retained, but the explanatory natural law is replaced with a mathematical fact.

Many philosophers of mathematics have suggested another epistemic view, *unification*, as the view best suited to analyze mathematical explanations, both within as well as outside of mathematics (e.g., Colyvan and Lyon 2008; Bangu 2013).

But suppose one would not want to be a (strong) explanatory pluralist and were convinced of the validity of the ontic dependency conception outside the realm of distinctively mathematical explanations. It is easy to see how mathematical necessity can provide modal information (and even answers to certain forms of what-if-things-had-been-different questions), and how mathematical structures can unify reasoning about otherwise disjoint phenomena, but it is harder to square with the idea that explanations show what the *explanandum depends* upon. How can the *explanandum* depend on anything, if it could not have been otherwise by mathematical necessity? Despite these daunting challenges, many philosophers have recently attempted to apply the ontic dependency conception to mathematical explanations. These accounts are the primary target of this paper.[1]

[1] For example, I do not address the idea that mathematical explanations are program explanations (Lyon 2012, for criticism of this proposal, see Saatsi 2012), various structural mapping accounts, aimed mainly at exploring the indispensability argument, nor Batterman's argument for the importance of limits and singularities *in* explanation (2009). I also do not address such putative explanatory ontological dependencies, which are analyzed in terms of essences rather counterfactuals (such as Fine 1994), as this literature is rather disconnected from the literature on *scientific* explanation. Also, Jansson and Saatsi (2019) argue that insofar as highly abstract explanations explain, they do so by providing

One of the first detailed proposals for such an account is by Christopher Pincock (2015), who argued that there is a class of explanation which explains empirical phenomena with the properties of abstract objects and that such explanations work by tracing *abstract* dependencies. His example is the explanation of the three laws describing the formation of soap-film surfaces among soap bubbles, previously established through empirical experimentation.[2] The bare essentials of the complicated example are the following. The *explanans* for the laws can be roughly divided into three steps: i) a connected network of surfaces is 'soap-film-like' iff it cannot be made to have a smaller area by any small deformation that leaves the frame in which the surfaces are held fixed (this area minimization clause in the definition is crucial for the explanation), ii) such surfaces can be treated as 'solution measures' and iii) such surfaces fit together in configurations known as 'almost minimal sets' and all such configurations must, by mathematical necessity, satisfy

information about dependence. They try to reconstruct standard examples of mathematical explanations in ways that do *not* invoke problematic dependencies between empirical and mathematical facts and remain agnostic about the metaphysics of such dependencies.

[2] 1) Compound bubbles or films suspended on a wire frame are flat or smoothly curved surfaces smoothly joined together. 2) Surfaces can meet in only two ways: three surfaces meet along a smooth curve or six surfaces meet at a vertex. 3) When surfaces meet along curves or when curves and surfaces meet at points, they do so at equal angles (three surfaces meeting along a curve meet at angles of 120 degrees and four curves meeting at a point meet at angles close to 109 degrees).

the three geometric principles constituting the *explanandum*. In fact, being an instance of an almost minimal set is necessary and sufficient for satisfying Plateau's laws.

Now, the way in which soap bubbles attach to each other is clearly an empirical phenomenon, and almost minimal sets are clearly abstract objects. Pincock argues that the properties of these almost minimal sets are explanatorily indispensable, and a crucial step in the explanation works by tracing an objective dependency between an abstract feature of the empirical phenomenon and the more abstract mathematical structure, of which the *explanandum* is an instance. Such relations of abstract dependence are arguably objective matters of fact that science (or at least mathematics) investigates, itself independent of our inferential and representational activities. It must be noted that Pincock does not explicitly endorse a *counterfactual* analysis of dependence and in fact refers approvingly to Kathrin Koslicki's (2012) non-counterfactual analysis of ontological dependence. We will return to this issue in the next section.

In a similar fashion, Saatsi and Pexton (2013) analyzed what they call *distinctively geometric explanations* of regularities tracing objective relations of dependence (understood in counterfactual terms) between empirical regularities and highly abstract and general properties. Although Saatsi and Pexton are careful not to claim that their proposed explanatory relationship would hold between an empirical fact and a platonically understood abstract object, I present their case here for reasons I will return to in the final section below. Their example is an explanation of Kleiber's law, an observed regularity linking the mass of an organism to its metabolic rate. Kleiber's law states that the metabolic rate of an organism (kcal/h on a logarithmic scale) is proportional to its mass to the power of ¾. What makes Kleiber's law striking is that the ¾ exponent is remarkably accurate all the way from single cell organisms to the biggest mammals. Surely there has to be an explanation for why the exponent is so stable, and why it is ¾ rather than something else?

The geometric explanation of the exponent refers to general properties of networks. The distribution of resources (nutrients, oxygen, etc.) within the organism is carried out by a network, and the metabolic rate is related to the fluid flow through this network. If the supply of resources is assumed to be 'democratic' (i.e., it serves all parts of the organism), the final branches of the network are size-invariant, and the energy required to distribute resources is minimized, then the geometry of the resource distribution network is fractal-like and, most importantly, the scaling exponent turns out to be the puzzling ¾. Saatsi and Pexton argue that the central explanatory dependency in this network explanation is between the dimensionality of the organism(s), the fractality of the resource network, and the exponent. Had the dimensionality been different, the exponent would had been different as well – although the dimensionality cannot be manipulated and the explanation is therefore clearly not causal.

Alex Reutlinger generalizes the counterfactual dependence view to cover distinctly mathematical explanations. Reutlinger (2016; 2018) simply demands that the explanatory generalization supports at least one counterfactual of the following form: had the initial or boundary conditions been different than they actually are (in at least one way deemed possible in the light of the generalizations), then the *explanandum* (or its conditional probability) would have been different as well. Reutlinger applies the generalized counterfactual theory to Euler's famous explanation of why no one has succeeded in crossing all the bridges of Königsberg exactly once. Here the explanatory generalization is Euler's theorem and the supported counterfactuals are of the form "if all parts of Königsberg were connected to an even number of bridges, or if exactly two parts of town were connected to an odd number of bridges, then people would not have failed to cross all of the bridges exactly once."

The most developed account of counterfactual ontic dependence view is by Baron, Colyvan and Ripley, who have recently proposed (2017) a specifically interventionist dependency analysis of one of the most-discussed examples of mathematical explanation - the North American periodical cicadas - originally introduced into the discussion by Alan Baker (2005). The *explanandum* is the curious fact that there are a number of cicada species with a particularly long larval stage, and all these species have a periodicity of either 13 or 17 years – both prime numbers. In a nutshell, the putative mathematical explanation of the curious primeness of the long cicada periods is the following. Long life-cycle periods help cicadas avoid predation and minimize the intersection of the period with that of other shorter periods (of possible periodical predators), and will therefore be evolutionarily advantageous. Prime periods minimize intersection with other periods (a number theoretic fact). Hence cicadas have evolved to have these prime periods (given other biological constraints). Baron et al. claim that there is an explanatory dependency between the number theoretic fact and the empirical periodicity, and that this dependency can be analyzed in a similar interventionist manner as causal explanations.

And coming back to the humble explanation of the indivisible strawberries, the corresponding ontic counterfactual analysis of the explanation would be that there is an objective abstract dependency between the indivisibility of 23 by 3 and the empirical failure of the mother to divide her strawberries. Such a view has indeed been recently proposed by Mark Povich (forthcoming), who, following Lange, wants to reserve the term 'distinctly mathematical explanation' for cases in which the *explanandum* can be construed so as to be dependent *only* on a mathematical fact. If such narrowly construed distinctively mathematical explanations exist, we need an account of this ontic dependency relation between abstract entities and worldly phenomena.

3. There are no abstract dependencies

I will next argue that the metaphysical and epistemological difficulties related to the concept of ontic mathematical or abstract dependence are simply insurmountable, and that the proposals appealing to such entities have underestimated these difficulties. How could a necessary mathematical fact have been different? This really is my argument in all its simplicity. If one accepts that all genuine explanations trace objective ontic dependencies, then there cannot be mathematical explanations because there are no suitable dependencies to trace. Note that this does not commit one to defending a purely causal account of explanation: there might still be perfectly objective ontic explanatory relations of dependence other than causation, such as constitution and perhaps grounding.

The first problem with abstract dependencies lies in providing truth conditions for the relevant counterfactuals. As Lange noted, relations between mathematical entities are modally stronger than relations of dependence between contingent entities in space-time. The counterfactual case in which the *explanans* is different (the abstract object in question has different properties that it in fact has) is either inconsistent or definitional of some other abstract object. This is deeply problematic regardless of whether or not one entertains a realist metaphysics of abstract or mathematical entities. On the one hand, realist metaphysics is more congenial to the idea of true ontic dependence between abstract entities, but the epistemology and metaphysics of such dependencies are murkier than the corresponding problems related to the mere existence of abstract objects and knowledge thereof.

On the other hand, if one is a non-realist with respect to abstract objects, then one faces the challenge of explaining how abstract dependencies meet the requirement of specifically *ontic* dependence, i.e., the requirement that explanatory relationships relate things in the world in contrast to merely epistemic relations of dependence between our representations of things in the world. This contrast is essential for the ontic view and motivates many of the features of Woodward's theory that are often wrongly attributed as being definitional of specifically causal explanation. Baron et al., Pincock and Reutlinger all adapt the counterfactual theory to mathematical explanation by simply dropping some of the interventionist requirements from Woodward's theory. I will now focus on this line of argument and argue that all such proposed generalizations end up losing the explanatory baby with the interventionist bathwater.

Pincock readily admits that abstract dependencies are metaphysically curious beasts but nevertheless insists that scientific practice forces us to accept the existence of such things, and that the philosophical task at hand is to develop a viable theory about them. In order to circumvent the taunting problems in figuring out the metaphysics and semantics of the impossible antecedent in the relevant counterfactual defining the explanatory dependency, Pincock's solution is to discard Woodward's requirement that these counterfactuals have to be 'same-object counterfactuals'. It is, of course, logically impossible that a mathematical object (such as an almost minimal set) could really have different mathematical properties than it actually has, but if the antecedent of the counterfactual characterizing the putative explanatory dependency is allowed to 'be about' some *other* mathematical object equipped with the appropriate contrasting mathematical property, this problem could be avoided altogether. Pincock refers approvingly to Koslicki's analysis of *ontological* dependence (2012), which defines dependence in terms of essences, rather than counterfactuals.

Pincock argues that the 'same-object counterfactual' requirement can be dropped for abstract explanations because the function of the requirement in Woodward's theory is to distinguish between explanations of token events and explanatory generalizations, and this distinction is not really applicable to abstract explanations to begin with. The problem with this suggestion is that the role of the 'same-object counterfactual' requirement has not so much to do with the type-token distinction, but with ensuring that the explanatory dependency and the supported counterfactuals are appropriately ontic, not epistemic. The requirement states that the intervention in the antecedent of the counterfactual refers to the very object, the properties of which we are explaining. This guarantees that the counterfactual is appropriately *change-relating*, that it tracks a real dependency which is itself independent of our ways of representing and classifying things. Without the requirement, counterfactuals (and the corresponding what-if inferences) can easily track the *inferential* consequences of our ways of *classifying,* or what would be reasonable *to believe*, given counterfactual evidence which we do not in fact have. Take the following, slightly modified[3] example from Woodward (2003, 279-280; originally by Ardon Lyon (1977)): suppose that a museum has a policy that all and only the Sisleys are hung in room 18. Ignorant of this rule, you ask whether an appropriately Sisley-looking painting in room 17 is in fact a Sisley and are told in response, 'If this painting were in room 18, then it would be a Sisley.' There is certainly an intuitive epistemic sense in which the counterfactual is true in that, given the museum's policy, being hung in room 18 would be good reason *to believe* that any given painting were a Sisley. But this counterfactual does not track an ontic dependency as one cannot turn a particular painting into a Sisley by placing it into room 18. The property of being a Sisley does not depend on the place in which it is hung, but on who painted it. The plausible counterfactual tracks a perfectly

[3] In the original example, the supported counterfactual is the converse: 'If this painting were a Sisley, it would be in room 18.' Note that there is an interpretation of *this* counterfactual that tracks a perfectly good ontic dependency in that if a Sisley were introduced into the museum, it would eventually be hung in room 18. This counterfactual is true due to the (causal) classificatory practices of the museum, not due to the inferential connections inherent in the classificatory practices of the evaluator of the counterfactual.

good reason for *believing* that a given painting is or is not a Sisley, not what *makes* – and therefore explains – a painting to be a Sisley.

The same point applies to counterfactual changes in mathematical objects: if there is no difference between *changing* a specific property of a mathematical object into something and else and simply contemplating the properties of a *different* mathematical object, we lose the very distinction between explanatory and classificatory information. Hence the counterfactual theory cannot simply be weakened by dropping the same-object requirement without at the same time losing the core ontic idea of what it is that makes information explanatory. Pincock could retort to this by stipulating that the classificatory other-object dependency is still explanatory and that an analysis in terms of essence-based ontological dependence guarantees that the dependence is still appropriately 'ontic'. This, however, would mean that Pincock owed us an alternative account of what distinguishes explanatory from non-explanatory information (or risk turning the question of the possibility of explanatory monism purely terminological).[4] An alternative account of the intuitive explanatoriness of such classificatory feats more in line with a strict ontic explanatory monism is outlined in section 5.

---

[4] In later work (2018), Pincock also adopts contrastive difference-making ideas in order to make room for explanatory pluralism (different kinds of explanations can legitimately explain different contrastive facts). Such ideas follow immediately from a counterfactual analysis of dependence, but not, at least as directly, from an essentialist understanding of dependence.

The most detailed account of invariant explanatory dependencies between mathematical facts and empirical phenomena is by Baron et al. (2017), which is also adopted by Povich (forthcoming). In contrast to Pincock, Baron et al. do not shy away from the change-relating nature of explanatory dependencies and claim that sense can be made of the idea that the properties of a mathematical object could have been different. Their central idea is that the appropriate explanatory dependencies between mathematical facts and empirical phenomena are structure-preserving mappings (like iso- or homomorphism). Let us, for the sake of the argument, put aside Goodmanian concerns about the objectivity of such mappings and accept that there indeed are objective, mind-independent structure-preserving mappings between mathematical structures and empirical phenomena. Can these morphisms act as explanatory dependencies by supporting counterfactuals with the right properties?

Baron et al. boil down the evaluation of specifically explanatory counterfactuals into three steps: holding appropriate things fixed, fiddling with the explanatory factor, and tracing the ramifications of the fiddling, given that some of the other factors are held fixed. The role of the explanatory morphism is that when it is held fixed, "local" counterpossible fiddles in mathematical *explanantia* ramify into counterfactual physical changes in the worldly *explananda*. The key question now is whether the operations of holding fixed, fiddling, and tracing ramification can be made sense of in a way that preserves the original rationale of the ontic counterfactual theory.

The bold claim by Baron et al. is that, although counterfactual reasoning in mathematics may not be as familiar as that in causal reasoning, nothing essentially different is involved and similar rules of reasoning apply. The obvious problem is that any local counterfactual mathematical 'change' necessarily leads to a contradiction, and hence it is difficult to see how the tracing of ramifications is supposed to work. Baron et al. rebut this worry by pointing out that *all* counterfactual reasoning is potentially contradictory (with history, laws of nature etc.) - unless the *right* things are held fixed in the evaluation. But what about the necessity of mathematical truths? The answer given is that many – perhaps most – interventions characteristic of ordinary causal counterfactuals are also impossible to carry out in practice. We cannot intervene on the past, for example. Again, there is no in principle difference w.r.t. ordinary causal counterfactuals.

The proposed procedure for locally fiddling with mathematical facts goes as follows. Twiddle on the mathematical fact/property of interest and hold some portion of the embedding mathematical structure fixed. If a contradiction arises, 'release' some more 'nearby' structure so that it is allowed to vary with the twiddle, and then re-check whether new contradictions arise further afield. Reiterate until one gets to the maximum extent to which the structure can be kept fixed without resulting in a contradiction with the introduced 'fiddle' (i.e., determine the most conservative alteration to the mathematical structure consistent with the contemplated change). Now keep the structure-preserving mapping between the mathematical structure and the physical *explanandum* phenomenon fixed (invariant), and check what the physical phenomenon would be like according to the changed mathematical structure. Thus we end up with a change-relating counterfactual dependency relation between a physical and a mathematical fact. If one is worried about the semantics and inferential properties of such counterpossible conditionals, one can take a pick from the various proposed semantics for counterpossibilities (e.g., Brogaard and Salerno 2013) and variants of paraconsistent logics, respectively.

This is how the procedure works in the case of the periodical cicadas. First, twiddle with the explanatory mathematical fact: the primeness of, say, 13. This should be done in the most conservative way. It turns out that one can hold all of number theory fixed except for the twiddle to 13, if one is prepared to change the way multiplication works; 13 now has factors two and six. Of course, one would expect that changing multiplication ends up creating inconsistencies all over the place, but Baron et al. shrug off these looming complications by pointing out that we only need to worry about those contradictions that are relevant to the evaluation of the specific counterfactual (as all counterfactuals imply contradictions somewhere). Next, we hold fixed the morphism between number theory and cicadas. This means that 13-year cicadas would coincide with periodical predators with cycles of two and six. Hence, a periodicity of 13 would be selected against, and there would not be 13-year cicadas.

This rather cavalier introduction of 'local' changes to mathematical structure is justified by Baron et al. on the basis that such selective 'fixing' of background variables is everyday practice in the analysis of causal claims and in causal reasoning more generally. What Baron et al. fail to appreciate, however, is that the principles for what should be held fixed and what allowed to vary in the analysis of causal counterfactuals are derived from general properties of causal structures, especially from the *directionality* of causal influence and the generally assumed *modularity* of causal structures. Directionality of causation dictates that the evaluation of causal counterfactuals should exclude backtracking inferences to counterfactual values of causally upstream variables and prohibits the fixing of causally downstream variables. Independent manipulability of distinct parts of a causal structure is widely (though not universally) taken to be a necessary condition for the applicability of causal reasoning. Furthermore, in the analysis of token ('actual') causation, selective fixing of off-path variables is motivated by the assumption of *spatiotemporal contiguity* of causal processes. Mathematical structures do not, as far as I know, possess analogous properties (directionality, modularity, let alone spatiotemporal contiguity) that would rationalize the practice of locally fixing and varying mathematical facts. Also counterfactually intervened causal structures remain *causally possible*, even if they do create local inconsistencies with the actual causal history, whereas mathematical interventions lead to *mathematically impossible* structures. The introduction of outright inconsistencies in causal reasoning is definitely not as easily digestible as Baron et al. claim it is.

But the fundamental problem in the generalization of interventionist reasoning proposed by Baron et al. is the same one that plagued Pincock's original suggestion for generalizing the ontic dependency account: without an ontological rationale based on the properties of (ontic) mathematical dependencies, the kinds of counterpossibilities entertained by Baron et al. fail to distinguish ontic explanatory inferences from epistemic classificatory inferences. The requirement that the antecedent of the explanatory causal counterfactual is *changed* by a causal intervention guarantees that the counterfactual tracks an objective dependency in the world, not an epistemic inference about what it would have been rational to *believe* if the antecedent had been such and such. In contrast, the local mathematical changes proposed by Baron et al. cannot distinguish whether we are contemplating an alteration on a mathematical structure or simply another mathematical structure altogether (modulo the added difficulty of this alternative structure being inconsistent). Baron et al. foresee this objection and claim that the worry is unfounded as we can keep the reference fixed simply as a matter of stipulation – factorizable 13 would still be the same 13, because this is simply *a part* of the considered counterfactual scenario - and that such stipulation is common to all forms of counterfactual reasoning (Baron et al. 2017, 4-6). But, again, the latter claim is much too quick. *Causal* counterfactuals are meaningful only under a restricted set of contemplated counterfactual changes and, although the exact nature of these restrictions is still debated (see, e.g., Woodward 2016), they rule out, e.g., simple wholesale changes in the identity of the "bearer" of the causal variable. These restrictions are in place to ensure that the explanatory counterfactual tracks the right things. Although Kripke claimed that the stipulation of the stability of reference is a necessary and non-problematic part of the evaluation of counterfactuals, *explanatory* counterfactuals nevertheless have to distinguish between ontic and epistemic grounds for such evaluation – at least according to the ontic dependency view. This is why causal and constitutive explanatory counterfactuals are about counterfactual *physical changes* in the very objects in question, and the meaning of the corresponding explanations becomes more precise, the better these counterfactual changes are defined in terms of detailed hypothetical experiments (Woodward 2003, 114-117). In contrast, in the mathematical case in which the identity of the bearer of the property is merely stipulated, there seem to be no *stipulation-independent* grounds for distinguishing between changing the property itself and changing which property (and object) is under analysis, and no corresponding possibility of refining the meaning of explanatory claims.[5] Without such a difference, there is no difference between explanatory and descriptive/classificatory information. And without this difference, it does not really make sense to talk about explanation to begin with.

4. Dividing strawberries and other best loved fables

[5]Furthermore, the proposal by Baron et al. is hard to reconcile with structuralist views about the nature of numbers and other mathematical objects. More generally, my complaint against Baron et al. and Povich could have been put in terms of not providing identity conditions for mathematical objects. However, such identity conditions necessarily imply substantial commitments within philosophy of mathematics, whereas the discussion on mathematical explanation has mostly steered clear of such specific commitments. Note also that a closer analogy to a causal intervention in the mathematical case would actually be an application of an ordinary mathematical operation (such as addition) to the mathematical object in question, not an "extra-mathematical" structural change proposed by Baron et al. (interventions are ordinary causal processes). The fact that such an ordinary mathematical "change" necessarily changes the identity of the object to which it is applied goes further to show that it is nonsensical to distinguish between "ontic" and epistemic/inferential changes in mathematics. We return to this theme in section 5.

If there really are no mathematical explanations, what about the central examples used in the literature? I now argue that most examples of non-causal mathematical explanations presented in the literature (Baker's periodical cicadas, Saatsi and Pexton's geometrical explanation, Pincock's example of the Plateau's laws for soap films, etc.) are best interpreted as mechanistic constitutive explanations, albeit very abstract ones. The relevant explanatory dependency is that of constitution, a suitably ontic and earthly relation between the whole and its parts, analyzable with only minimal alterations to Woodward's theory.

Let us start with the periodical cicadas, although we will return to the critters yet again in the last section. If we take the general gist of the primeness explanation to be plausible, but the primeness of 13 and 17 to be simply unchangeable, then the specific periodicities of 13 and 17 have to depend on something else. In fact, suitable candidates are not hard to find. According to the explanation, periods of 13 and 17 minimize the chance that the emergence of the brood coincides with that of a periodical predator. If the period of a specific cicada species had been different, then that species would have been subjected to stronger predation and hence more likely to be driven to extinction. What could have been different are the periods and the features of the selection environment. As Saatsi has pointed out (2011), the number theoretic fact *justifies* the explanatory assumption that periods of 13 and 17 minimize the coincidence with other random periods, but no reason has been given why it would not be the facts about intersecting periods and selection that do the explaining. Although highly abstract, contrastive claims about intersecting *periods of organisms* imply contrastive claims about possible evolutionary trajectories (given the assumption of selection environment including predators with randomly distributed periodicities). As Lange admits (2017, 24-25) *this* mathematical explanation therefore looks very much like an ordinary causal-mechanistic explanation – albeit an extremely abstract and sketchy one.

Next we will look at the geometric explanation of Kleiber's law. According to Saatsi and Pexton, the explanation tracks an abstract dependency between the dimensionality of the organisms and the puzzlingly stable scaling exponent. What kind of counterfactual inferences does this dependency license? If the dimensionality of the organisms had been different, the scaling exponent would have been different as well. The antecedent of the counterfactual is certainly not logically impossible and might, under some interpretations, be even physically contingent. This explanatory dependency holds because of the way in which resources are most efficiently distributed in a fractal-like network. The explanation thus works by appealing to the functions of the parts (the vessels transmitting resources), their mutual organization, and an optimality assumption. The explanation can therefore be understood as a mechanistic explanation, albeit an extremely general and abstract one. The explanation applies to all efficient fractal-like resource distribution networks, but as such is quite comparable in its level of abstraction to mechanistic network explanations in other fields (see, e.g., Kuorikoski and Marchionni 2014; Kuorikoski and Ylikoski 2013). This analysis also, arguably, applies to Hüneman's topological explanations (although space here does not permit a full argument). The corresponding explanatory dependency is that of constitution relating worldly things, properties of the parts and their organization, to another worldly thing, the property of the whole.[6] There is no need to consider worlds beyond the physically possible ones, nor dependencies between objects not located in space-time.

What about the explanation of Plateau's laws? Pincock posits that the explanation refers to an abstract dependency between the properties of almost minimal sets and surfaces of soap-like films. What could have been otherwise? If we do not accept that the properties of almost minimal sets really could have been something else other than they in fact are, the explanatory difference-makers have to be found somewhere else. As Pincock emphasizes, an essential element in the explanation is that the connecting surfaces minimize their area, and that therefore not all geometrically equivalent structures fall under the explanation. What therefore makes the difference as to whether the laws apply or not is the physics of the soap films: if the films did not have the physical property of minimizing their surface area, the configuration would not (necessarily) be as described by Plateau's laws. This is a constitutive explanation of the property of the whole by the properties of the parts and their mutual organization. Granted, this quick summarization of the complex abstract

[6] In his later work, Pincock (2018) uses the term constitutive explanations in discussing purely aggregative explanations of properties of wholes, in which the organization does not make a difference to the explandum and claims that the proper relation of dependence underwriting abstract relations is that of *instantiation*. Although space does not allow for a full analysis here, his examples of such *structural* explanations also fall, arguably, neatly under constitutive *mechanistic* explanations.

explanation seems to leave out much of the intuitive ‘explanatoriness’ of the explanation, but we will return to this missing residual in the next section.

It is worth noting that the proposals above render the explanations *not* distinctly mathematical in the sense required by Lange and Povich, as both require that all of the empirical facts appealed to in the explanation should be “built into” the *explanans*, and would therefore have to stay fixed in the assessment of the explanatory counterfactuals. For Lange and Povich, the target of the distinctly mathematical explanation of the periodical cicadas is “in connection with predators having periodic life- cycles, cicadas with prime periods tend to suffer less from predation than cicadas with composite periods do” (Lange 2017, 25). The causal-mechanistic interpretations therefore shift the explananda from the original “narrowly” construed phenomena to something else and that an analysis of the original distinctly mathematical explanations is still wanting. I will next argue that the “narrow” *explananda* are not proper targets of explanation to begin with.

Let us return to Lange’s mathematical explanation demonstrating the mathematical necessity of the *explanandum*, the impossibility of dividing 23 strawberries by three. Although Lange’s modal (constraint) account of explanation is not a target of this paper, an ontic dependence account should provide some resources for analyzing also this case and Povich claims that a straightforward ontic analysis can be given. There is an obvious way in which the why question can be contrastively specified so as to allow for a non-problematic answer in terms of a mundane ontic dependency: if the number of strawberries had been 21 rather than 23, the mother could have divided them by three (Jansson and Saatsi 2019, Skow 2016, 112-113). This turns the explanation into a causal explanation, because the number of strawberries is clearly a variable that can be physically intervened upon.[7] I believe this trivializes Lange's puzzlement, however. As Lange himself states (2013, 498-7), even if the number of strawberries were considered a cause, what distinguishes such examples is that the dependency itself is mathematically necessary. But why is it the case that 21 can be divided by three whereas 23 cannot? This time one has to admit that it is hard to locate a highly abstract causal explanatory mechanism in the scenario, something that would link the persistent failure in an even division to some other this-worldly fact in the right sort of way. Povich insists that there is a suitable dependency between the empirical failure and an other-worldly mathematical fact, and suggests grounding and instantiation as plausible candidates for fulfilling this role. Nevertheless, he does not offer full account of either and, as we saw above, at least the analysis of the relevant counterpossibles by Baron et al. adopted by Povich does not really work.

My suggestion is that we can get by without extravagant metaphysics of dependencies between abstract and concrete objects and problematic counterpossibles by admitting that there is no true dependence to be found. The fact that the mother cannot divide her strawberries does not really depend on anything. There quite literally is nothing keeping

[7] The bridges of Königsberg can also be transformed (trivialized) into a causal explanation by considering possible interventions on the number of bridges (cf. Jansson and Saatsi 2019). Also Skow proposes to defuse the strawberry example in a similar fashion by claiming that the number of strawberries can be a cause, even though the reason why this is so is not itself a cause, but a mathematical fact (Skow 2016, 115-116).

her from carrying out the task. Demonstrations of mathematical impossibility simply cannot be reconciled with the requirement of ontic dependency and, according to a monist view, they are therefore not genuine explanations. Even though a demonstration of a mathematical impossibility is not an explanation as such, it is certainly not an explanatorily inert achievement. I hold that in such cases, the *explanandum* is reconceptualized in such a way as to make it explicit that there was, in fact, nothing to be explained to begin with. Demonstration of an impossibility is therefore not an explanation, but *a correction of a mistaken explanatory presupposition*.[8]

Consider the classic example of the regression fallacy provided by Daniel Kahneman (2002). Military flight instructors in Israel noticed that praise given for an exceptionally smooth landing was usually followed by a comparatively rougher landing (and criticism for an exceptionally rough landing likewise followed by an improved performance). This naturally led to the idea that positive praise was counterproductive and harsh criticism effective in improving landings. In reality, neither is (at least necessarily) the case, since a better-than-average performance is likely to be followed by a worse performance purely due to 'regression to the mean'. The idea that positive and negative feedback causes, and hence explains, the differences in consecutive performances presupposes that there is something in the world that causes these differences – that the differences between

[8] In contrast, Strevens (2018) holds that such pure mathematical impossibility is still on the continuum of abstraction and, consequently, of modal reach common to all scientific explanations.

subsequent landings systematically *depend on* something. In reality the differences are solely due to the stochastic nature of the process (due to noise from countless and unsystematic disturbing factors). The differences do not need explaining, and reconceptualizing the *explanandum* phenomenon as a stochastic process implies that there is, in fact, nothing to be explained.[9] The explication of the stochastic nature of the process therefore increases our understanding of the phenomenon by correcting an explanatory presupposition that an explanation is needed.

In the same way, pointing out that 23 is not divisible by three corrects an explanatory presupposition that the mother's persistent failure in dividing her strawberries requires an explanation to begin with, that there is something from which this failure depends. Once the simple mathematical fact is grasped and the *explanandum* phenomenon reconceptualised accordingly, asking for an explanation for the empirical failure *does not make sense* to begin with. The mathematical fact is therefore not explanatorily inert, it just is not an explanation.

[9] Reutlinger (2018) presents this case as an example of a non-causal statistical explanation and that the explanandum is (the probability of) relative improvement or decline in individual performance. I claim (though cannot here extensively argue) that my analysis better captures the moral of Kahneman's anecdote. In contrast, the difference between my account and Lange's analysis of really statistical explanations as showing the *explanandum* to be "just a statistical fact of life" borders on terminological (cf. Lange 2017, 190-196).

5. Formal understanding

If there are no mathematical explanations, is there no such thing as specifically mathematical understanding? The denial of the existence of mathematical dependencies seems to rule out the possibility of explanation also within mathematics. Yet, as has been repeatedly pointed out in the philosophy of mathematics, mathematicians make comparative judgements about the relative depth or explanatory power of proofs and theorems. Often the mere demonstration that a mathematical fact is the case is deemed unsatisfactory if a further argument demonstrating why it holds – in some intuitive sense of 'why'– is not given. Broader sets of mathematical 'phenomena' are sometimes said to be explained by 'deeper' mathematical facts or by a novel reconceptualization of the field in question. The deduction of Plateau's laws from the properties of minimal sets seems to track *something* and increase our overall understanding of *why* the world is the way it is. Simply denying that such expert intuitions made any sense would amount to fairly outrageous philosophical hubris. Although explanation *within* mathematics is not the main topic of this paper, I will next sketch an account of mathematical understanding that saves these intuitions in the face of the non-existence of mathematical dependencies. I thus aim to show that the skeptical stance towards distinctly mathematical explanations is, on balance, more palatable than the metaphysical and epistemological problems implied by their acknowledgement.

The classic account of explanatory reasoning within mathematics is Steiner's 1978 article 'Mathematical Explanation', in which Steiner analyses the difference between explanatory and unexplanatory proofs. I will focus on Steiner's account not because it is universally accepted (it certainly is not), but because it explicitly appeals to the problematic notion of mathematical dependence. According to Steiner, what sets explanatory proofs apart from non-explanatory ones is that they appeal to the 'characteristic property' of the entity or the structure bearing the property being demonstrated, and then show how the property being proved is dependent on this characteristic property. The idea is therefore analogous to Aristotelian explanation by essence. Counterfactual information is also powerfully invoked in a further requirement that an explanatory proof is to be generalizable in that varying ('deforming') the characteristic property in the proof gives rise to an array of similar theorems of different mathematical properties.

Steiner's original account has recently been further developed by Frans and Weber (2014), who explicitly link the idea of dependence on characteristic property to a Woodwardian account of explanation. They analyze proofs of a geometric 'butterfly theorem' and argue that proofs can be explanatory by showing what properties of mathematical 'entities' are difference-makers to the *explanans* in the theorem. At least some explanations within mathematics therefore provide similar answers to what-if-things-had-been-different questions concerning the effect of counterfactual interventions as causal-mechanistic explanations. Their proposal is therefore very similar to the account given by Baron et al., although Frans and Weber do not provide details about how the local mathematical interventions are supposed to work.

Let us grant that Steiner, Frans and Weber capture legitimate intuitions concerning differences in understanding provided by mathematical proofs. Yet the very existence of mathematical dependencies supposedly grounding such understanding was summarily dismissed above. Fortunately there is no need to deny the existence of such explanation-like reasoning within mathematics, as analyzed by Steiner, Frans and Weber, even though, strictly speaking, there are no explanatory mathematical dependencies. As the analysis of Frans and Weber shows, some proofs provide more grounds to answer what-if questions than others. Proofs based on 'characteristic properties' that can be 'locally varied' are generalizable to closely related mathematical structures and therefore increase the inferential capabilities of the mathematician more than 'isolated' proofs without such generalizable implications. 'Brute force' proofs are a limiting case in that they, as such, do not provide any resources for making further inferences beyond the specific set of mathematical facts demonstrated, and as such provide no understanding *why* these facts hold. In the same way as the ability to answer what-if-things-had-been-different questions can be seen as constituting the understanding provided by a causal mechanistic explanation, the increased inferential ability to answer what-if questions concerning the properties of related mathematical structures are constitutive of the mathematical understanding provided by a proof – and consequently a reasonable measure of its 'depth' (cf. Ylikoski and Kuorikoski 2010). It is just that the what-if answers do not concern *counterpossible* mathematical structures under interventions, but *analogical* inferential connections between slightly different but related systems of mathematical inference. Additional inferences made by possible by the use of analogical structures arguably captures the intuitive explanatoriness of the examples of viewing-as-explanations offered by d'Alessandro (2018). Furthermore, and in line with Lange's analysis of mathematical depth (2015), mathematical facts with many connections to diverse areas of mathematics offer more inferential implications than more 'isolated' mathematical properties, and are therefore, justifiably, judged as having more depth and providing greater understanding.[10] Similar analysis of the explanatory quality of proofs appealing to symmetries (Lange 2014) immediately suggests itself, but is not pursued further here.

Mathematical understanding as the ability to make inferential connections beyond the immediate property, structure or theorem, is a species of what can be called *formal understanding*, i.e., an understanding of our systems of reasoning and representation. Such understanding is constituted by the abilities in making correct what-if inferences *about* systems of reasoning.[11] It is formal in that the relevant abilities concern the form of the

---

[10] Although sometimes "pure" proofs, which do not invoke "alien" mathematical resources from neighboring fields, are judged to be more explanatory than impure ones (Detlefsen and Arana 2011). The validity of these intuitions is an interesting question, but not pursued further here.

[11] If there are explanation-independent reasons for holding a realist stance towards mathematical entities, one can readily admit that these inferences also refer to abstract objects. As before, the account of mathematical understanding presented here aims to be metaphysically non-committal and should definitely not be taken as an endorsement of a formalist philosophy of mathematics in general.

inferences. What would follow from special relativity if the symmetry group of space-time was de Sitter rather than Poincaré? What would the properties of a competitive equilibrium be if preferences were non-homothetic? These kinds of questions concern primarily our systems of reasoning and representation, not directly what we are representing and reasoning about (i.e., space-time or specific markets). The answers are nevertheless fully objective and ontic in that they are grounded on the objective features of our representational systems – what could justifiably be inferred from given assumptions if the properties of the system of reasoning were different. This also provides a natural way of interpreting the apparently counterfactual reasoning involved in the tracing of the inferential consequences of the truth or falsity of the Riemann hypothesis, used as an example of counterfactual reasoning in mathematics by Baron et al. (2017), without having to refer to alternative counterpossible mathematical structures. In the context of model-based explanation, Kuorikoski and Ylikoski (2015) make a similar distinction between understanding a model and understanding with a model. The former is evaluated in terms of the ability to answer what-if questions concerning the consequences of (possibly counterfactual) changes in the model, whereas the latter, in terms of broadened abilities in answering counterfactual what-if questions concerning the modelled system made possible by the model.

The residual explanatory contribution made by the link between almost minimal sets and Plateau's laws is, I argue, captured by the increase in formal understanding. The intimate connection of being an instance of an almost minimal set and Plateau's laws is a genuine discovery about our representational tools. If an object can be characterized in terms of almost minimal sets, then it follows that the object can also be characterized by Plateau's laws. Correcting for the explanatory presupposition in the case of the strawberries was epistemically trivial in that the impossibility of division was transparent, but not all such demonstrations of impossibility in terms of non-applicability of concepts are as transparent. But why should a difference in the cognitive effort required for *grasping* an explanation be indicative of a fundamental difference in what *grounds* the explanation? Think of the impossibility results in theoretical economics, such as Arrow's theorem. The theorem states that it does not make sense to apply a formal concept, an aggregate social welfare function, to systems with particular properties. The theorem is clearly non-trivial, and its discovery taught us important lessons about the limits of our economic tools of reasoning. The theorem increased our formal understanding and this increase in understanding is, I suggest, easily mistaken for a special kind of explanatoriness.

Formal understanding and the associated explanatory-like reasoning within mathematics also nicely capture, I believe, what Saatsi (2011; 2016) describes as the thin explanatory role of mathematics. According to Saatsi, mathematics can play an indispensable role in representation and reasoning without this entailing any necessary ontological commitments to mathematical entities. Mathematics is explanatorily indispensable in the articulation of explanatory dependencies, but, according to the line or argument developed here, does not ontologically enter into these dependencies itself. As was the case with the cicadas and the intersecting periods, Saatsi also points out that mathematics can have a role in *justifying* explanatory assumptions without *being* explanatory. This is also very much in the same spirit as Strevens' 'representational' view of mathematical explanations (Strevens 2018), although based on a completely different understanding of causality and explanation.

6. Reflections or does any of this matter?

The argument against mathematical explanation is based on a specific philosophical theory of explanation – the ontic counterfactual dependence view. D'Alessandro, Lange and Pincock all demand, quite reasonably, that mathematicians' own expert intuitions about what is and what is not explanatory should not be summarily dismissed without additional arguments. Moreover, these arguments should ideally be independent of one's favourite philosophical account of explanation. As the concept of formal understanding leaves plenty of room for mathematical reasoning being rather similar to explanatory reasoning, is the argument presented here merely a pointless terminological crusade? What is gained by insisting that, strictly speaking, there really are no mathematical explanations, despite contrary expert testimony?

First, there are the purely philosophical benefits. *If* one wishes to remain committed to explanatory monism in general and to the ontic dependence view in particular, then doing away with abstract and mathematical explanations saves one from answering a whole host of uncomfortable epistemological and metaphysical questions concerning mathematical or abstract dependencies. As I have argued, weakening the requirements for the counterfactuals characterizing the corresponding explanatory dependencies is not as easy as Pincock, Povich, Reutlinger, and Baron et al. have thought it to be. But, of course, this by itself will not satisfy the requirement of theory-independent grounds for dismissing the experts' talk of mathematical explanations.

Second, and very much relatedly, the denial of mathematical explanations is in line with a broadly empiricist tradition of understanding science and knowledge. By insisting that mathematical facts as such can only have a thin representational/inferential role, all explanatory progress proper remains a staunchly empirical achievement. If the arguments above against the weakening of the interventionist conditions for explanatory dependence are valid, then explanation in fact turns out to be even more intimately connected to contingent empirical matters than perhaps previously thought. This, of course, will not satisfy anyone who is not committed to such a view to begin with, and many proponents of mathematical explanations are favorably inclined towards rationalist and Platonist views concerning the place of mathematics in science and the world more generally. Luckily, there are also more philosophy-independent reasons for favoring the account given here over the admittance of mathematical explanations into our cognitive arsenal.

Third, denying the existence of mathematical explanations arguably leads to a more analytical, fine-grained picture of mathematical reasoning – in science as well as mathematics. The account given here distinguishes between empirical (constitutive) explanations, inferentially fertile analogical reasoning within mathematics, and corrections of explanatory presuppositions, and distributes the epistemic achievements of the canonical examples into empirical explanatory progress and to an increased understanding of our systems of representation and reasoning. I believe such analytical clarity to be of value in itself.

Fourth, and perhaps most importantly, denying that there are mathematical explanations functions as a counterbalance to the seductive nature of elegant mathematical reasoning. Let us return one more time to the mysterious periodical cicadas. The mathematical explanation was that a prime periodicity minimizes the chance that the life cycle of the cicadas will coincide with that of a possible periodical predator. This explanation is simple and elegant, and evokes a strong sense of understanding. It is also controversial and lacking in empirical evidence in its favor. Other live explanation candidates for the periodicity in general are that it evolved as a response to parasites or Pleistocene climatic change, or as a way to avoid the adverse effects of low population density in mating. The primeness could simply be a coincidence. The periods of 13 and 17 years seem to have evolved separately in the different periodical cicada species, but this could well have resulted from a common inherited regulatory system. Recent phylogenetic analyses suggest that the divergence into the present (multiple) 13- and 17-year-period cicada species is associated with climatic fluctuations, thus lending some support to the climate hypothesis (Sota et al. 2013; see also Saatsi 2011 for further criticism). The point here is not to debate the empirics of this particular case, but to note that the elegance and the intellectual satisfaction provided by the mathematical explanation are not, by themselves, evidence for its truth.

Evocative mathematical explanations may also provide unjustified warrant to the very existence of the *explanandum*. This may be the case with Kleiber's law, as many statistical studies have come up with an exponent of 2/3 rather than the ¾ putatively explained by the network explanation. An authoritative review suggests strongly that the exponent seems to vary substantially between 2/3 and 1 across taxons (Glazier 2010, see also Raerinne 2013). So, not only are there alternative explanations for the 'law' available, there is considerable disagreement whether there is a stable generalization to be explained by the network explanation to begin with. Again, the point here is not to enter the empirical debate concerning the stability of said allometries, but to note the danger of mistaking an elegant mathematical explanation for a demonstration of the reality of the *explanandum*.

Even when the mathematical explanation is true, the intellectual satisfaction of the mathematical explanation may lead us to think that we have explained more than we in fact have. The powerful sense of understanding accompanying the grasping of a mathematical explanation may lead to thinking that the particular abstract *explanandum* is the most important aspect of the phenomenon in question. The explanation of the hexagonal shape of honeycombs is arguably a case in point (cf. Lyon and Colyvan 2008). Even though the 'mathematical' optimization explanation for the hexagonal shape is probably on the right track, the intellectual satisfaction derived from this easily blinds us to a quite obvious complication: what really ought to be optimized by the bees is the *volume*, not the surface area, of the honeycombs. And the honeybees actually fail in this optimization task. There are therefore important aspects of the geometry of the honeycomb that are explained by other constraints. Distinctly mathematical explanations are therefore not only epistemologically and metaphysically puzzling - they also present extra challenges to the evaluation of their explanatory merits. We may be better off without them.

References

Baker, Alan. 2005. “Are There Genuine Mathematical Explanations of Physical Phenomena?” *Mind* 114 (454):223-238.

Bangu, Sorin. 2013. “Indispensability and Explanation.” *British Journal for the Philosophy of Science* 64:255-277.

Baron, Sam. 2019. “Mathematical Explanation by Law.” *British Journal for the Philosophy of Science* 70 (3): 683–717.

Baron, Sam, Mark Colyvan, and David Ripley. 2017. “How Mathematics Can Make a Difference.” *Philosophers' Imprint* 17 (3): 1–29.

Batterman, Robert. 2010. “On the Explanatory Role of Mathematics in Empirical Science.” British Journal for the Philosophy of Science 61 (1): 1–25.

Brogaard, Berit, and Joe Salerno. 2013. “Remarks on counterpossibles.” *Synthese* 190: 639-660.

D'Alessandro, William. 2020. “Viewing-As Explanations and Ontic Dependence.” Philosophical Studies 177 (3): 769–92.

Detlefsen, Michael, and Andrew Arana. 2011. “Purity of methods.” *Philosophers' Imprint* 11 (2): 1–20.

Fine, Kit. 1994. “Ontological Dependence.” *Proceedings of the Aristotelian Society*, 95: 269–290.

Frans, Joachim, and Erik Weber. 2014. “Mechanistic Explanation and Explanatory Proofs in Mathematics.” *Philosophia Mathematica* 22:231-248.

Glazier, Douglas S. 2010. “A unifying explanation for diverse metabolic scaling in animals and plants.” *Biological Reviews* 85:111–138.

Huneman, Philippe. 2010. “Topological Explanations and Robustness in Biological Sciences.” *Synthese* 177 (2):213-245.

Jansson, Lena, and Juha Saatsi. 2019. ”Explanatory Abstractions.” *British Journal for the Philosophy of Science* 70:817-844.

Kahneman, Daniel. 2002. “Autobiography (written for the Riksbank Memorial Prize for Alfred Nobel).” https://www.nobelprize.org/prizes/economic-sciences/2002/kahneman/biographical/ , accessed 30.8.2019.

Koslicki, Kathrin. 2012. “Varieties of Ontological Dependence.” In *Metaphysical Grounding: Understanding the Structure of Reality*, ed. Fabrice Correira and Benjamin Schnieder, 186-213. Cambridge and New York: Cambridge University Press.

Kuorikoski, Jaakko. 2012. “Mechanisms, Modularity and Constitutive Explanation.” *Erkenntnis* 77 (3):361-380.

Kuorikoski, Jaakko, and Caterina Marchionni. 2014. “Unification and Mechanistic Detail as Drivers of Model Construction: Models of Networks in Economics and Sociology.” Studies in History and Philosophy of Science A 48:97–104.

Kuorikoski, Jaakko, and Petri Ylikoski. 2013. "How Organization Explains." In EPSA11 Perspectives and Foundational Problems in Philosophy of Science, ed. Vasillios Karakostas and Dennis Dieks, 69–80. Cham: Springer.

Kuorikoski, Jaakko and Petri Ylikoski. 2015. "External Representations and Scientific Understanding." *Synthese* 192: 3817-3837.

Lange, Marc. 2013. "What Makes a Scientific Explanation Distinctively Mathematical?" *British Journal for the Philosophy of Science* 64 (3):485-511.

——— 2014. "Aspects of Mathematical Explanation: Symmetry, Unity, and Salience." *The Philosophical Review* 123: 485-531.

——— 2015. "Depth and Explanation in Mathematics." *Philosophia Mathematica* 23: 196–214.

——— 2017. *Because Without Cause*. Oxford and New York: Oxford University Press.

Lyon, Aidan. 2012. "Mathematical Explanations of Empirical Facts, and Mathematical Realism." Australasian Journal of Philosophy 90 (3): 559–78.

Lyon, Aidan and Mark Colyvan. 2008. "The Explanatory Power of Phase Spaces." *Philosophia Mathematica* 16:227-43.

Lyon, Ardon. 1977. "The Immutable Laws of Nature." *Proceedings of the Aristotelian Society* 77 (1):107–126.

Pincock, Christopher. 2015. "Abstract Explanations in Science." *British Journal for the Philosophy of Science* 66:857-882.

——— 2018. “Accommodating Explanatory Pluralism.” in Reutlinger and Saatsi 2018, 39-56.

Povich, Mark. (forthcoming). “The Narrow Ontic Counterfactual Account of Distinctively Mathematical Explanation.” *The British Journal for the Philosophy of Science*.

Raerinne, Jani. 2013. “Explanatory, Predictive, and Heuristic Roles of Allometries and Scaling Relationships.” *BioScience* 63 (3):191–198.

Reutlinger, Alexander. 2016. “Is There A Monist Theory of Causal and Noncausal Explanations? The Counterfactual Theory of Scientific Explanation.” *Philosophy of Science* 83 (5):733-745.

——— 2018. Extending the Counterfactual Theory of Explanation. In Reutlinger and Saatsi 2018, 74-95.

Reutlinger, Alexander and Juha Saatsi, eds. 2018. *Explanation Beyond Causation: Philosophical Perspectives on Non-Causal Explanations*. Oxford and New York: Oxford University Press.

Rice, Collin. 2015. “Moving beyond Causes: Optimality Models and Scientific explanation.” Noûs 49 (3): 589–615.

Saatsi, Juha. 2011. “The Enhanced Indispensability Argument: Representational Versus Explanatory Role of Mathematics in Science.” *British Journal for the Philosophy of Science* 62: 143-154.

——— 2016. “On the ‘Indispensable Explanatory role’ of Mathematics.” *Mind* 125: 1045-1070.

Saatsi, Juha and Mark Pexton. 2013. “Reassessing Woodward's Account of Explanation: Regularities, Counterfactuals, and Noncausal Explanations.” *Philosophy of Science* 80 (5):613-624.

Sota, Teiji, Satoshi Yamamoto, John Cooley, Kathy Hill, Chris Simon and Jin Yoshimura. 2013. “Independent divergence of 13- and 17-y life cycles among three periodical cicada lineages.” *PNAS* 110:6919-6924.

Steiner, Mark. 1978. “Mathematical Explanation.” *Philosophical Studies* 34:135–151.

Strevens, Michael. 2018. “The Mathematical Route to Causal Understanding.” In Reutlinger and Saatsi 2018, 96-116.

Woodward, James. 2003. *Making Things Happen*. Oxfrod and New York: Oxford University Press.

Ylikoski, Petri and Jaakko Kuorikoski. 2010. “Dissecting Explanatory Power.” *Philosophical Studies* 148 (2):201–219.